Spin networks and the big bang singularity avoidance in the AdS/CFT correspondence

Carlos Silva[*]

Instituto Federal de Educação Ciência e Tecnologia do Ceará (IFCE),
Campus Tianguá—Av. Tabelião Luiz Nogueira de Lima, s/n—Santo Antônio,
Tianguá, Ceará CEP 62324-075, Brazil.

We propose a new version of the AdS/CFT correspondence, where polymer structures similar to loop quantum cosmology spin networks can be induced on an isotropic and homogeneous flat Randall-Sundrum II brane, corresponding to the holographic duals of closed string states living in the bulk. Such polymer structures drive a discrete evolution of the braneworld spacetime that leads to the regularization of the big bang singularity. The present results show that the AdS/CFT conjecture can be a possible bridge between string theory and loop quantum gravity.

Amidst the difficulties to obtain a quantum description of gravity, the holographic principle emerges as a possible guide [1,2]. Such principle, which has been supported by observational evidence [3], appears in its most successful form, as the so-called AdS/CFT conjecture, which has established that a gravitational theory in the AdS_5 bulk is dual to a CFT with a UV cut-off on the bulk boundary [4].

As a definition of nonperturbative string theory, the AdS/CFT correspondence has offered the interesting possibility to solve gravitational singularities, by transfer the discussion from the bulk to the dual field theory on its boundary. However, in the important case of the big bang singularity resolution, no clear picture has emerged by now in this context, despite much effort has gone. See, e.g., [5–8] for more recent discussions on this issue. Particularly, in [7,8] loop quantum gravity (LQG) ideas have been used to this purpose.

The situation becomes even more cloudy if we consider that the bulk boundary consists of a Randall-Sundrum II brane, where the *CFT* is coupled to gravity [9,10]. In fact, in such a scenario, we have no idea what the boundary theory would be. Notwithstanding, an important hint has been pointed out that, whatever such theory is, it should be hot and have a nonzero energy density and pressure [11]. It is because bulk black holes could emit Hawking radiation that would heat the brane, which by its turn, would be forced by the Israel equations to turn into an Friedmann-Robertson-Lemaître-Walker (FRLW) universe driven by an energy density that dilutes as a finite temperature CFT.

More recently, such a thermodynamical aspect of the boundary theory has been reinterpreted by an interesting perspective that arises when we locate the black holes on the brane itself. In this context, the thermal radiation emitted by such holographic black holes provides the thermal nature of the boundary theory, assuming an interesting place in the AdS/CFT dictionary. In fact, this radiation corresponds to the holographic dual of gravitational bremsstrahlung in the AdS_5 bulk [12–14]. Such results bring us a reformulation of the AdS/CFT correspondence in the form of the so-called black hole holographic conjecture (BHHC).

In the present work, we shall demonstrate that the BHHC can be used to introduce a regularization scheme whereby the big bang singularity can be solved, in an AdS/CFT scenario where the boundary theory lives on an isotropic and homogeneous flat brane universe. In such a regularization scheme, the gravitational degrees of freedom belonging to the theory on the boundary can be associated with polymer structures similar to loop quantum cosmology (LQC) spin networks. Such structures will appear as the holographic duals of string states in the bulk and turn the big bang singularity into a bounce. Such results show that the AdS/CFT conjecture can be a possible bridge between string theory and LQG, the two main approaches to quantum gravity up to now.

I. SPACETIME DEGREES OF FREEDOM ON A BRANE: A BHHC PERSPECTIVE

In field theory, a regularization scheme consists of the determination of degrees of freedom, in such a way, those that lead to divergences are cut out from the theory [15]. In this sense, by considering the Bekenstein bound [16], the discussion about the degrees of freedom present in the theory induced on a brane must be connected with what $4D$

[*]carlosalex.phys@gmail.com

solution, describing a black hole, we have localized on it. Such a solution must be given by a suitable slicing of a $5D$ accelerating black hole metric, known as the C-metric in $4D$ [17].

Several approaches have been proposed in the literature, where induced $4D$ black hole metrics of the form

$$ds^2 = -F(r)^2 dt^2 + G(r)^2 dr^2 + H(r)^2 d\Omega^2 \qquad (1)$$

has been used [18–22]. Unfortunately, none of these proposals has been satisfactory so far.

Among the points to be clarified, we note that all the approaches to construct black hole solutions on a brane have the common assumption that the radial coordinate must be fixed by the "area gauge" $H(r) = r$. Consequently, the area of the 2-spheres surrounding the black hole behaves as $A(r) = 4\pi r^2$, increasing monotonically between the horizon and the spacelike infinity.

On the other hand, a different perspective has been driven by a more recent version of the AdS/CFT conjecture, proposed by Tanaka [12], and independently by Emparan, Fabbri and Kaloper [13], which states that for regimes above the AdS length scale: A classical 5-dimensional gravitational theory in the bulk corresponds to a quantum corrected (semiclassical) 4D braneworld spacetime.

This new perspective has been named by Gregory et al. as the "black hole holographic conjecture" (BHHC) [23]. In the context of such idea, black hole solutions localized on the brane, given as solutions of the classical bulk equations in AdS_{D+1} with the brane boundary conditions, cannot be static but must consist in evaporating solutions, corresponding to quantum-corrected black holes in D dimensions, rather than classical ones. In this way, in the AdS/CFT dictionary, extradimensional effects in the bulk will be translated into semiclassical corrections to the boundary theory. Some possibilities for observational confirmation of the BHHC have been pointed in [24,25].

The use of the BHHC has led to the idea that the area function $A(r)$ of the 2-spheres, in the geometry induced on the brane, must be considered to be not monotonic [23]. The main reason for such an assumption comes from the presumed higher dimensional C-metric. Such a metric would consist of an accelerating black hole being "pulled" by a cosmic string. Since the appropriate higher-dimensional metric for a Poincaré invariant string has a turning point in the area function, and the "horizon" is singular, it is expected that a braneworld black hole solution must share such features. A turning point in the area function of a holographic black hole also appears in [26], where it has been demonstrated that, due to backreaction effects, an antievaporating phase must take place at the end of the black hole evaporation process.

It is possible to introduce a scenario, as prescribed by the BHHC, by considering a fluctuation of a black hole metric written in the area gauge ds_{ag}^2 in the following way

$$ds^2 = \frac{H^2}{r^2} ds_{ag}^2, \qquad (2)$$

where we must consider an appropriate choice of the function $H(r)$. In this case, if H/r is a twice differentiable function of the spacetime coordinates, and $0 < H/r < \infty$, we can interpret the equation above as a conformal transformation.

To obtain the desired scenario, a possible choice is

$$H^2(r) = \left(r^2 + \frac{l^4}{r^2} \right), \qquad (3)$$

in a way that the metric ds^2 will possess a not monotonic area function, with a turning point at $r = l$, and singular at $r = 0$. In this scenario, the length l will correspond to a high energy length scale and will enclose the information about higher-dimensional effects, translated as semiclassical corrections on the brane. A similar conformal factor has been used in [27,28] to solve the Schwarzschild black hole singularity.

Interestingly, due to the conformal relationship between the metrics ds^2 and ds_{ag}^2, the way how the black hole temperature and entropy are related with the black hole mass is the same in both spacetimes [27]. However, since the event horizon surface area A changes by conformal transformations, the black hole entropy-area relation must be modified. In fact, by considering the Eq. (3), a new radial coordinate in the ds^2 spacetime will be defined as

$$R = \left(r^2 + \frac{l^4}{r^2} \right)^{1/2}. \qquad (4)$$

Consequently, the rescaled event horizon area will be given by

$$A = 4\pi \left(r_+^2 + \frac{l^4}{r_+^2} \right), \qquad (5)$$

where r_+ is the black hole event horizon radius in the ds_{ag}^2 spacetime.

We can fix the bare metric ds_{ag}^2 in a way that the spacetime described by it obeys the usual Bekenstein-Hawking entropy-area relation, in four dimensions [29] (see, for example, the general class of braneworld black holes introduced in [30]). In this way, we obtain for the ds^2 spacetime:

$$S = \frac{A + \sqrt{A^2 - 2A_l^2}}{8}$$

$$= \frac{\sqrt{A^2 - A_l^2}}{4} + \mathcal{O}(\geq A_l^6), \qquad (6)$$

where $A_l = 4\sqrt{2\pi}l^2$.

In the present work, we shall consider that the presence of a turn-point in the area function of a black hole, in the context of the BHHC, allows us to introduce a cutoff on the degrees of freedom that can be encoded on the bulk boundary, in a way that their counting will be given by the expression (6). Such a cutoff will be implemented through the parameter A_l.

We must cite that the expression (6) has also appeared in the context of loop quantum black holes [31], and has been used to derive the LQC dynamical equations from thermodynamical arguments [32]. Moreover, the present considerations must be valid for regimes above the AdS length scale. Below such scale, all dimensions must be seen on the same footing and the full influence of the extra dimensions must be taken into account. Other choices for the function $H(r)$ can be done in order to implement regularization schemes different from that we have adopted here.

II COSMOLOGY AND THE BIG BANG SINGULARITY AVOIDANCE ON A BRANE

To address the cosmological evolution of the brane spacetime in the light of the BHHC, let us consider the flux of Hawking radiation through the universe horizon. For this flux, we can associate the following temperature [33–35]:

$$T = (2\pi \tilde{r}_H)^{-1}. \tag{7}$$

In the expression above, $\tilde{r}_H$ is the physical radius associated with the universe horizon, given for the flat universe case by

$$\tilde{r}_H = \frac{1}{H}, \tag{8}$$

where we have considered an FLRW metric induced on the brane:

$$ds^2_{\text{FRLW}} = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega_2^2, \tag{9}$$

with $h_{ab} = \text{diag}(-1, a^2)$, $\tilde{r} = a(t)r$, and $H = \dot{a}/a$.

As usual, we shall consider that the energy-momentum tensor of the matter-energy contend of the boundary theory can be written as the ones for a perfect fluid, which gives us an amount of energy that goes through the horizon during a time dt as [36]:

$$dQ = A\psi = A(\rho + p)\tilde{r}H\left(1 + \frac{\rho}{\sigma}\right)dt, \tag{10}$$

where $A = 4\pi \tilde{r}_H^2$, and σ is the brane tension.

Now, we observe that from a global point of view, the universe horizon enters in the bulk, in a way that higher-dimensional effects must be taken into account to the calculation of its entropy. However, according to BHHC,

for regimes above the AdS length scale, such effects can be interpreted as semiclassical corrections on the brane. In this case, we shall borrow the expression (6) from the last section, and use it to express the entropy associated with the universe horizon.

In this way, by taking into account the first law of thermodynamics, $dQ = TdS$, and the temperature (7), we obtain, by discarding higher-order corrections in A_l:

$$\dot{H} = 4\pi \frac{\sqrt{A^2 - A_l^2}}{A}(\rho + p)\left(1 + \frac{\rho}{\sigma}\right), \tag{11}$$

which, by the use of the continuity equation, give us:

$$\frac{8\pi}{3}\frac{d\rho}{dt}\left(1 + \frac{\rho}{\sigma}\right) = \frac{A}{\sqrt{A^2 - A_l^2}}\frac{d(H^2)}{dt}. \tag{12}$$

Finally, by integration, we obtain the Friedmann equation:

$$H^2 = \frac{4\pi}{A_l}\cos(\Theta), \tag{13}$$

where $\Theta = \pm[\frac{2A_l}{3}\rho(1 + \frac{1}{2}\frac{\rho}{\sigma}) - \alpha]$, and α is a phase constant.

Note that, in Eq. (13), the effective density term, in the form of a harmonic function of the classical density, bring us a scenario where a bounce takes the place of the big bang initial singularity, when the universe density assumes a critical value, differently from usual braneworld cosmology [37].

To explore the physical significance of the result above, let us expand the right-hand side of the Eq. (13) in a Taylor series. We get, discarding higher-order terms in A_l and $1/\sigma$ (the high energy contributions),

$$H^2 = A(\alpha)\rho^2 + B(\alpha)\rho + C(\alpha), \tag{14}$$

where

$$A(\alpha) = \frac{4\pi}{9}\left(\frac{3\sin(\alpha)}{\sigma} - 2A_l\cos(\alpha)\right),$$

$$B(\alpha) = \frac{8\pi}{3}\sin(\alpha),$$

$$C(\alpha) = \frac{4\pi}{A_l}\cos(\alpha). \tag{15}$$

The Eq. (14) can be written in the form

$$H^2 = \frac{8\pi}{3}\rho_{\text{tot}}\left(1 - \frac{\rho_{\text{tot}}}{\rho_c}\right), \tag{16}$$

where we have taken $\rho_{\text{tot}} = \rho + \Lambda$, with Λ as a cosmological constant, and

$$\rho_c^{-1} = \frac{1}{6}\left(2A_l\cos(\alpha) - \frac{3\sin(\alpha)}{\sigma}\right), \qquad (17)$$

$$1 - \frac{2\Lambda}{\rho_c} = \sin(\alpha), \qquad (18)$$

$$\cos(\alpha) = \frac{2A_l\Lambda}{3}\left(1 - \frac{\Lambda}{\rho_c}\right). \qquad (19)$$

The Raychaudhuri equation can also be obtained for this case

$$\dot{H} = -4\pi(\rho_{\text{tot}} + p_{\text{tot}})\left(1 - \frac{2\rho_{\text{tot}}}{\rho_c}\right), \qquad (20)$$

where $p_{\text{tot}} = p - \Lambda$.

If we take the limit where $A_l \to 0$ in the Eqs. (17), (18), and (19), and substitute the results in the Eqs. (16) and (20), we recover the usual braneworld cosmology equations [37], as expected (preserving the condition $\Lambda < \rho_c$).

On the other hand, if we fix $l = \beta l_{\text{AdS}}$ in A_l, for $\beta \neq 0$, we can obtain

$$\rho_c = 2\sigma\left(1 + \frac{\pi}{2\beta^2}\right)^{1/2}; \qquad (21)$$

and

$$\Lambda = 2\sigma\left[\left(1 + \frac{\pi}{\beta^4}\right)^{1/2} - 1\right]. \qquad (22)$$

In the limit where we have a small cosmological constant ($\beta \gg 1$) [38,39], we also get

$$\rho_c \approx 2\sigma > 0. \qquad (23)$$

From such a result, ρ_c assumes the role of a critical energy density in the Eqs. (16) and (20), which have semiclassical corrections similar to that appears in LQC [40–42].

III. POLYMER STRUCTURES ON A BRANE AND THEIR HOLOGRAPHIC DUALS

In order to investigate the microscopic structure of the bounce, we have that using the techniques introduced by Singh and Soni [43], we can obtain from the Raychaudhuri equation (20), the following Hamiltonian for gravity:

$$\mathcal{H}_{grav} = \frac{-3V}{32\pi\lambda^2}(2 - e^{ip\sqrt{\Delta}} - e^{-ip\sqrt{\Delta}}), \qquad (24)$$

with p as the conjugate momentum to the volume V.

Moreover,

$$\lambda = (3/(32\pi\rho_c))^{1/2}; \quad \Delta = 6\pi/\rho_c, \qquad (25)$$

where λ and Δ possess dimensions of length squared, and ρ_c is a constant-energy density to be provided by the underlying theory [43].

In the present work, ρ_c is equal to twice the brane tension, according to equation (23). Consequently, we obtain

$$\Delta = \frac{3\pi}{\sigma}. \qquad (26)$$

It is important to observe that, the Hamiltonian (24) is not defined in terms of the conjugate momentum to the volume p, but in terms of its complex exponentials. Here, it is a key point since such exponentials consist of holonomies, the building blocks of spin networks in LQG.

As emphasized in [43], the appearance of such holonomies is not possible in the case of the usual braneworld cosmology. But in the present scenario, it makes the microscopic description of the brane spacetime naturally related to the polymer quantization methodology [44]. In fact, simply promoting the exponentials in the Hamiltonian (24) to operators, we obtain:

$$\hat{\mathcal{H}}_{\text{grav}} = \frac{-3V}{32\pi G\lambda^2}[2\mathbb{I} - \widehat{e^{ip\sqrt{\Delta}}} - \widehat{e^{-ip\sqrt{\Delta}}}], \qquad (27)$$

where in the equation above, the shift operators $\widehat{e^{\pm ip\sqrt{\Delta}}}$ are defined by their action on the states $\psi_{x_n} = e^{ipx_n}$, in the momentum representation:

$$\widehat{e^{\pm ip\sqrt{\Delta}}}\psi_x = e^{\pm i\sqrt{\Delta}p}e^{ipx} = e^{i(x\pm\sqrt{\Delta})p} = \psi_{x\pm\sqrt{\Delta}}. \qquad (28)$$

As we can observe, the action of the shift operators will correspond to a finite displacement equals to $\sqrt{\Delta}$. Consequently, the Hamiltonian (27) gives us that the brane spacetime degrees of freedom can be associated with a polymer structure, defined by a graph, in the form of a regular lattice

$$\gamma_{\sqrt{\Delta}} = \{x \in \mathbb{R}|x = n\sqrt{\Delta}, \quad \forall n \in \mathbb{Z}\}. \qquad (29)$$

Such polymer structure, which is similar to spin networks in LQC [45], brings us a discreteness in the position x, with a discreteness parameter $\sqrt{\Delta}$, which from the Eq. (26) is defined by the brane tension. It imposes superselection rules for the gravitational sector on the brane, in a way that the universe will evolve through discrete increments of the scale factor (or some a function of it, such as an area or volume).

Such superselection rules will also affect the string dynamics in the bulk. It is because, we remember that the brane couples gravitationally to the bulk through closed strings, that leaves and cross the brane, and whose couplings, g_s, are related with the brane tension as $g_s \sim 1/\sigma$ [46]. In this case, from the Eq. (26), we obtain

$$g_s \sim \Delta. \tag{30}$$

in a way that, the spectrum of closed string modes in the bulk will be constrained by the discrete spacetime evolution on the brane.

IV. REMARKS AND CONCLUSIONS

In the present work, we have shown that, in the AdS/CFT context, spacetime degrees of freedom of the gravitational theory on the AdS boundary can be described in terms of polymer structures, similar to LQC spin networks, which correspond to the holographic duals of string states living in the bulk. Such a description of the boundary theory provides us with superselection rules to gravity in a way that the cosmological evolution in a AdS/CFT scenario becomes free from the big bang singularity, which is replaced by a bounce.

We note that the quantum gravity effects will make the bounce to occur before the universe gets in a regime below the AdS length scale, avoiding scenarios where the BHHC should not be valid. In fact, from the Eqs. (23) and (26), the discreteness parameter $\sqrt{\Delta}$, that will correspond to the minimal size of the universe in the present scenario, is such that $\sqrt{\Delta} \approx 8,89 l_{\text{AdS}}$.

Some issues related to the results found out in the present letter deserve more discussion in future works. At first, we must remember that, from the bulk point of view, the big bang singularity is produced by the uncontrolled back-reaction of the dilaton field, as it diverges when the closed string coupling vanishes [47–49]. However, Eq. (30) tells us that the discrete spacetime evolution on the brane constraints g_s to assume only finite nonvanishing values. Consequently, the string modes that would lead to the dilaton divergency and, consequently, to the big bang singularity, must be cut out. The future analysis must improve our understanding of how the string dynamics in the bulk is affected by the discrete evolution of the brane spacetime.

Second, it is needed to note that the polymer structures found out here differ from those appear in LQC, only because they are defined in terms of the brane tension, and not in terms of the Barbero-Immirzi parameter. Such a fact may be interesting for future discussions of an ancient problem in LQG, the so-called Immirzi ambiguity [50]. It is because the brane tension can be dynamically determined [46]. The present results can match the concern by several authors that a solution to the Immirzi ambiguity problem could stay in a possible dynamical determination of the Barbero-Immirzi parameter [51–53].

Thirdly, the semiclassical results presented in the Eqs. (13), (16) and (20) can be easily extended to the case of non-flat universes. However, since the LQC methods have been shown robust only for the flat universe case [54], in the present work we can advocate the big bang singularity resolution only in this situation. On the other hand, the present results have opened the doors for future connections between LQC and AdS/CFT, where more general cases can be analyzed.

At last, based on the present results, a reformulation of the AdS/CFT conjecture can be proposed, where the $4D$ gravitational theory on the brane must be promoted from a semiclassical theory, as prescribed by the BHHC, to a quantum gravity theory. In this case, a possible connection between string theory and LQG can be traced through the following holographic correspondence, relating a gravitational theory in the AdS_5 bulk to a CFT coupled to gravity on the brane: A classical 5-dimensional spacetime in the AdS_5 bulk, described by string theory, must correspond to a quantum 4D braneworld spacetime described by spin networks.

Further investigations must deepen such connection, by considering for example the relationship between string theory and more general spin network states, as those appear in full LQG. Moreover, the conjecture above can be used to analyze other situations where quantum gravity must be important, as for the case of black holes.

ACKNOWLEDGMENTS

[1] G. 't Hooft, arXiv:gr-qc/9310026.

[2] L. Susskind, J. Math. Phys. (N.Y.) **36**, 6377 (1995).

[3] N. Afshordi, C. Coriano, L. Delle Rose, E. Gould, and K. Skenderis, Phys. Rev. Lett. **118**, 041301 (2017).

[4] J. M. Maldacena, Int. J. Theor. Phys. **38**, 1113 (1999); Adv. Theor. Math. Phys. **2**, 231 (1998).

[5] N. Engelhardt, T. Hertog, and G. T. Horowitz, Phys. Rev. Lett. **113**, 121602 (2014).

[6] N. Engelhardt, T. Hertog, and G. T. Horowitz, J. High Energy Phys. 07 (2015) 044.

[7] N. Bodendorfer, A. Schfer, and J. Schliemann, J. High Energy Phys. 06 (2019) 043.

[8] N. Bodendorfer, F. M. Mele, and J. Mnch, Classical Quantum Gravity **36**, 245013 (2019).

[9] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).

[10] M. J. Duff and J. T. Liu, Phys. Rev. Lett. **85,** 2052 (2000).

[11] I. Savonije and E. P. Verlinde, Phys. Lett. B **507,** 305 (2001).

[12] T. Tanaka, Prog. Theor. Phys. Suppl. **148,** 307 (2002).

[13] R. Emparan, A. Fabbri, and N. Kaloper, J. High Energy Phys. 08 (2002) 043.

[14] P. R. Anderson, R. Balbinot, and A. Fabbri, Phys. Rev. Lett. **94,** 061301 (2005).

[15] S. Weinberg, *The Quantum Theory of Fields. Vol. 1: Foundations* (Cambridge University Press, New York, 1995).

[16] J. D. Bekenstein, Phys. Rev. D **23,** 287 (1981).

[17] W. Kinnersley and M. Walker, Phys. Rev. D **2,** 1359 (1970).

[18] N. Dadhich, R. Maartens, P. Papadopoulos, and V. Rezania, Phys. Lett. B **487,** 1 (2000).

[19] A. Chamblin, S. W. Hawking, and H. S. Reall, Phys. Rev. D **61,** 065007 (2000).

[20] R. Casadio, A. Fabbri, and L. Mazzacurati, Phys. Rev. D **65,** 084040 (2002).

[21] M. Visser and D. L. Wiltshire, Phys. Rev. D **67,** 104004 (2003).

[22] K. A. Bronnikov, V. N. Melnikov, and H. Dehnen, Phys. Rev. D **68,** 024025 (2003).

[23] R. Gregory, R. Whisker, K. Beckwith, and C. Done, J. Cosmol. Astropart. Phys. 10 (2004) 013.

[24] R. Emparan, J. Garcia-Bellido, and N. Kaloper, J. High Energy Phys. 01 (2003) 079.

[25] R. Dey, S. Chakraborty, and N. Afshordi, Phys. Rev. D **101,** 104014 (2020).

[26] R. Casadio and C. Germani, Prog. Theor. Phys. **114,** 23 (2005).

[27] C. Bambi, L. Modesto, S. Porey, and L. Rachwal, J. Cosmol. Astropart. Phys. 09 (2017) 033.

[28] C. Bambi, L. Modesto, and L. Rachwal, J. Cosmol. Astropart. Phys. 05 (2017) 003.

[29] J. D. Bekenstein, Phys. Rev. D **7,** 2333 (1973).

[30] E. Abdalla, B. Cuadros-Melgar, A. B. Pavan, and C. Molina, Nucl. Phys. **B752,** 40 (2006).

[31] L. Modesto and I. Premont-Schwarz, Phys. Rev. D **80,** 064041 (2009).

[32] C. A. S. Silva, Eur. Phys. J. C **78,** 409 (2018).

[33] X. H. Ge, Phys. Lett. B **651,** 49 (2007).

[34] Y. Gong and A. Wang, Phys. Rev. Lett. **99,** 211301 (2007).

[35] R. G. Cai, L. M. Cao, and Y. P. Hu, Classical Quantum Gravity **26,** 155018 (2009).

[36] R. G. Cai and S. P. Kim, J. High Energy Phys. 02 (2005) 050.

[37] R. Maartens, Living Rev. Relativity **7,** 7 (2004).

[38] S. Weinberg, Phys. Rev. Lett. **59,** 2607 (1987).

[39] N. Aghanim *et al.* (Planck Collaboration), arXiv:1807 .06209.

[40] V. Taveras, Phys. Rev. D **78,** 064072 (2008).

[41] M. Bojowald, Phys. Rev. Lett. **86,** 5227 (2001).

[42] A. Ashtekar, T. Pawlowski, and P. Singh, Phys. Rev. Lett. **96,** 141301 (2006).

[43] P. Singh and S. K. Soni, Classical Quantum Gravity **33,** 125001 (2016).

[44] A. Corichi, T. Vukasinac, and J. A. Zapata, Phys. Rev. D **76,** 044016 (2007).

[45] J. Mielczarek, Springer Proc. Phys. **157,** 555 (2014).

[46] K. Becker, M. Becker, and J. H. Schwarz, *String Theory and M-Theory: A Modern Introduction* (Cambridge University Press, New York, 2007).

[47] Due the *S*-duality, it is equivalent to the situation where the string coupling goes to infinity.

[48] N. Engelhardt and G. T. Horowitz, Phys. Rev. D **93,** 026005 (2016).

[49] D. Bak, Phys. Rev. D **75,** 026003 (2007).

[50] C. Rovelli and T. Thiemann, Phys. Rev. D **57,** 1009 (1998).

[51] T. Jacobson, Classical Quantum Gravity **24,** 4875 (2007).

[52] V. Taveras and N. Yunes, Phys. Rev. D **78,** 064070 (2008).

[53] S. Mercuri, Phys. Rev. Lett. **103,** 081302 (2009).

[54] A. Ashtekar, A. Corichi, and P. Singh, Phys. Rev. D **77,** 024046 (2008).