Structural identities in the first-order formulation of quantum gravity

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We study the self-consistency of the first-order formulation of quantum gravity, which may be attained by introducing, apart from the graviton field, another auxiliary quantum field. By comparing the forms of the generating functional Z before and after integrating out the additional field, we derive a set of structural identities, which must be satisfied by the Green's functions at all orders. These are distinct from the usual Ward identities, being necessary for the self-consistency of the first-order formalism. They relate the Green's functions involving the additional quantum field to those containing a certain composite graviton field, which corresponds to its classical value. Thereby, the structural identities lead to a simple interpretation of the auxiliary field.

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I. INTRODUCTION

The first-order formulation of gauge theories has a simple form involving only cubic interactions, which are momentum independent. This simplifies the computations of the quantum corrections in the usual second-order gauge theories that involve momentum dependent three-point as well as higher-point vertices [1–12]. In quantum gravity, for example, the first-order formulation allows to replace an infinite number of complicated multiple graviton couplings present in the second-order Einstein–Hilbert (EH) action with a small number of simple cubic vertices [7,8]. The EH action has the form

$$S = -\frac{1}{16\pi G_N}\int d^d x\sqrt{-g}g^{\mu\nu}R_{\mu\nu}(\Gamma), \tag{1.1}$$

where G_N is Newton's constant, and the affine connection $\Gamma^\lambda_{\mu\nu}$ may be written in terms of the metric $g_{\mu\nu}$ as

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}(g_{\mu\sigma,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma}). \tag{1.2}$$

The Ricci tensor $R_{\mu\nu}(\Gamma)$ is given by

$$R_{\mu\nu}(\Gamma) = \Gamma^\rho_{\mu\rho,\nu} - \Gamma^\rho_{\mu\nu,\rho} - \Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\sigma\rho} + \Gamma^\rho_{\mu\sigma}\Gamma^\sigma_{\nu\rho}. \tag{1.3}$$

As noted by Einstein and Palatini [13] at the classical level, it is possible to treat both $g_{\mu\nu}$ and $\Gamma^\lambda_{\mu\nu}$ as being independent quantities. In this first-order action, the equation of motion for $\Gamma^\lambda_{\mu\nu}$ yields Eq. (1.2). At the quantum level, it has been shown [7,8] that the radiative corrections computed using the first-order and second-order EH actions are the same.

In a previous paper [14], we examined a set of structural identities, which are necessary for the consistency of the first-order formulation of the Yang–Mills theory. The purpose of the present work is to extend this analysis to quantum gravity, where the corresponding structural identities ensure the self-consistency of the first-order formulation.

To this end, we introduce a source $j_{\mu\nu}$ for the graviton field and also a source $J^\lambda_{\mu\nu}$ for the other auxiliary field, which is treated as an independent field, and consider the generating functional $Z[J,j]$ of Green's functions. We then compare the functional dependence of $Z[J,j]$ on the sources in the original first-order formalism with that obtained after making a suitable shift, which enables integrating out the auxiliary field. The equality of these functional forms leads to a set of structural identities among the Green's functions, which must be satisfied to all orders. Such identities are complementary but distinct from the usual Ward identities, being necessary for the internal

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consistency of the first-order formulation of quantum gravity.

These identities show that in the first-order formalism the Green's functions containing only external graviton fields are the same as the corresponding ones, which occur in the second-order formulation. Furthermore, these identities relate the Green's functions involving external auxiliary fields to those involving a certain composite graviton field. This combination, which corresponds to the classical value of the auxiliary field, contains graviton fields which are pinched at the same spacetime point. It is well known [15–17] that composite fields can lead to short-distance singularities. In the present case, such singularities are important for the cancellations of ultraviolet (UV) divergences arising from loop diagrams, which are necessary for the implementation of the structural identities.

Since calculations in quantum gravity have a great algebraic complexity, in Sec. II we recast the analysis done in [14] into an alternative form, which is based on a simpler diagonal representation of the first-order formulation of the Yang–Mills theory [18]. Such a representation exhibits similar features to those in quantum gravity, yet it is easier to handle algebraically. With this insight, we consider in Sec. III the Lagrangian and the generating functional of Green's functions in a corresponding diagonal representation of the first-order formulation of quantum gravity [8]. In Sec. IV, we derive a transparent structural identity, which has been explicitly verified to one-loop order that clarifies the meaning of the auxiliary field in this formulation. In Sec. V, we study another structural identity satisfied by the Green's functions and examine the cancellations between the loop UV divergences and the short-distance singularities arising from the tree diagrams involving composite fields. A brief discussion of the results is given in Sec. VI. Several details of one-loop calculations are outlined in the Appendix.

II. STRUCTURAL IDENTITIES IN YANG–MILLS THEORY

The first-order formulation of the Yang–Mills theory involves the gluon A_μ^a and the auxiliary fields $F_{\mu\nu}^a$ whose dynamics are described by the Lagrangian:

$$\tilde{\mathcal{L}}_{\rm YM}^{(1)} = \frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu} - \frac{1}{2} F^{a\,\mu\nu}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c). \tag{2.1}$$

This form has a single vertex $\langle FAA \rangle$ but leads to a rather involved nondiagonal matrix propagator containing the (AA), (FF) and the mixed (FA), (AF) propagators. On the other hand, if we make in Eq. (2.1) the change of variable

$$F_{\mu\nu}^a = \tilde{H}_{\mu\nu}^a + \partial_\mu A_\nu^a - \partial_\nu A_\mu^a, \tag{2.2}$$

one obtains the Lagrangian

$$\tilde{\mathcal{L}}_{\rm YM}^{\rm I} = \frac{1}{4} \tilde{H}_{\mu\nu}^a \tilde{H}^{a\mu\nu} - \frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 - \frac{g}{2} f^{abc}(\tilde{H}_{\mu\nu}^a + \partial_\mu A_\nu^a - \partial_\nu A_\mu^a) A^{b\,\mu} A^{c\,\nu}, \tag{2.3}$$

which involves two cubic vertices $\langle AAA \rangle$, $\langle \tilde{H}AA \rangle$ as well as two simple propagators (AA), $(\tilde{H}\tilde{H})$. The Becchi-Rouet-Stora-Tyutin renormalization of this diagonal formulation of the Yang–Mills theory has been implemented to all orders in Ref. [18] (see also [19–21]). The complete Lagrangian density for this formulation in covariant gauges is

$$\mathcal{L}_{\rm YM}^{\rm I} = \tilde{\mathcal{L}}_{\rm YM}^{\rm I} - \frac{1}{2\xi}(\partial_\mu A^{\mu a})^2 + \partial^\mu \bar{\eta}^a(\delta^{ab}\partial_\mu - g f^{abc} A_\mu^c)\eta^b, \tag{2.4}$$

where ξ is a gauge-fixing parameter and $\bar{\eta}^a$, η^b are ghost fields. In addition, we will also introduce the external sources $\tilde{J}_{\mu\nu}^a$ and $\tilde{j}_\mu^a$ as follows:

$$\mathcal{L}_{\rm source} = \tilde{J}_{\mu\nu}^a \tilde{H}^{a\,\mu\nu} + \tilde{j}_\mu^a A^{a\mu}. \tag{2.5}$$

The generating functional for Green's functions is given by the path integral

$$Z[J,j] = N \int \mathcal{D}\eta \mathcal{D}\bar{\eta} \mathcal{D}\tilde{H} \mathcal{D}A \times \exp i\left[S + \int d^d x \tilde{J}_{\mu\nu}^a \tilde{H}^{a\,\mu\nu} + \tilde{j}_\mu^a A^{a\mu}\right], \tag{2.6}$$

where N is a normalization factor and $S = \int d^d x \mathcal{L}_{\rm YM}^{\rm I}$. This equation has a form which is suitable for functional differentiation with respect to $\tilde{J}$ and $\tilde{j}$ and, therefore, for obtaining the Green's functions.

If we were to set $\tilde{J}_{\mu\nu}^a = 0$ at the outset (so that we would consider Green's functions with only external fields A_μ^a) and make the change of variable in the functional integral

$$\tilde{H}_{\mu\nu}^a \to \tilde{H}_{\mu\nu}^a + g f^{abc} A_\mu^b A_\nu^c, \tag{2.7}$$

then one can integrate out the $\tilde{H}_{\mu\nu}^a$ field and find that

$$Z[\tilde{J} = 0, \tilde{j}] = Z_2[\tilde{j}], \tag{2.8}$$

where $Z_2[\tilde{j}]$ is the generating functional for the second-order theory, characterized by the Lagrangian density

$$\mathcal{L}_{\rm YM}^{\rm II} = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c)^2 - \frac{1}{2\xi}(\partial_\mu A^{\mu a})^2 + \partial^\mu \bar{\eta}^a(\delta^{ab}\partial_\mu - g f^{abc} A_\mu^c)\eta^b, \tag{2.9}$$

together with the source term $\tilde{j}^a_\mu A^{a\mu}$. This establishes the important property that the Green's functions with only external gluon fields are the same in both approaches.

We now consider using $Z[\tilde{J},\tilde{j}]$ with $\tilde{J}\neq 0$ and examine what changes occur in the first-order formalism when there are external fields $\tilde{H}^a_{\mu\nu}$. To this end, we consider in place of Eq. (2.7), the shift

$$\tilde{H}^a_{\mu\nu} \to \tilde{H}^a_{\mu\nu} + gf^{abc}A^b_\mu A^c_\nu - 2\tilde{J}^a_{\mu\nu}. \tag{2.10}$$

This leads, after integrating out the $\tilde{H}^a_{\mu\nu}$ field, to the alternative form of the generating functional

$$Z'[J,j] = N\int \mathcal{D}\eta\mathcal{D}\bar{\eta}\mathcal{D}A \exp i\left[\int d^dx(\mathcal{L}^{\mathrm{II}}_{\mathrm{YM}} + gf^{abc}\tilde{J}^a_{\mu\nu}A^{b\mu}A^{c\nu} - \tilde{J}^a_{\mu\nu}\tilde{J}^{a\mu\nu} + \tilde{j}^a_\mu A^{a\mu})\right]. \tag{2.11}$$

This equals to $Z_2[\tilde{j}]$ in Eq. (2.8) if we set $\tilde{J}^a_{\mu\nu}=0$. It is interesting to note the unusual dependence of $Z'[\tilde{J},\tilde{j}]$ on $\tilde{J}$ in Eq. (2.11).

Comparing the forms of Eqs. (2.6) and (2.11) of the generating functionals and differentiating these with respect to $\tilde{J}$ and $\tilde{j}$, leads to a set of structural identities among the Green's functions, which must be satisfied to all orders. Such structural identities lead to relations between the Green's functions involving $\tilde{H}$-fields and the Green's functions that contain the composite fields $gf^{abc}A^b_\mu(x)A^c_\nu(x)$. These identities hold both for the finite as well as for the UV divergent parts of the Green's functions. We have verified them explicitly for the divergent contributions to one-loop order using dimensional regularization in $4-2\epsilon$ dimensions.

Taking the functional derivatives of Eqs. (2.6) and (2.11) with respect to $\tilde{J}^{a\mu\nu}(x)$ and $\tilde{j}^{b\alpha}(y)$ at $\tilde{J}=\tilde{j}=0$ and equating the results, we obtain the relation

$$\langle 0|T\tilde{H}^a_{\mu\nu}(x)A^b_\alpha(y)|0\rangle = gf^{ade}\langle 0|TA^d_\mu(x)A^e_\nu(x)A^b_\alpha(y)|0\rangle. \tag{2.12}$$

Equation (2.12) represents a quantum mechanical extension of the relation $\tilde{H}^a_{\mu\nu}=gf^{abc}A^b_\mu A^c_\nu$, which holds at the classical level. This structural identity is clearly satisfied in the tree approximation since the mixed $(\tilde{H}A)$ propagator vanishes in our theory. The right-hand side of Eq. (2.12) also vanishes at the tree level. To order g^2, we find that the divergent parts on both sides of Eq. (2.12) are, in momentum space, equal to

$$-\frac{g^2C_{\mathrm{YM}}}{16\pi^2\epsilon}\frac{5+\xi}{4}\frac{\delta^{ab}}{k^2}(k_\mu\eta_{\nu\alpha}-k_\nu\eta_{\mu\alpha}). \tag{2.13}$$

Applying $\delta^2/\delta\tilde{J}^{a\mu\nu}(x)\delta\tilde{J}^{b\alpha\beta}(y)$ to Eqs. (2.6) and (2.11) and equating the results, leads to

$$\langle 0|T\tilde{H}^a_{\mu\nu}(x)\tilde{H}^b_{\alpha\beta}(y)|0\rangle = 2iI_{\mu\nu,\alpha\beta}\delta^d(x-y) + g^2f^{ab'c'}f^{bd'e'}\langle 0|TA^{b'}_\mu(x)A^{c'}_\nu(x)A^{d'}_\alpha(y)A^{e'}_\beta(y)|0\rangle, \tag{2.14}$$

where

$$I_{\mu\nu,\alpha\beta} = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta}-\eta_{\nu\alpha}\eta_{\mu\beta}). \tag{2.15}$$

This identity is also manifestly satisfied at tree level, where the first term on the right-hand side of Eq. (2.14) is just equal to the tree propagator $(\tilde{H}\tilde{H})$. To order g^2, one can verify, in momentum space, that the divergent part on both sides of Eq. (2.14) are equal to

$$-i\frac{g^2C_{\mathrm{YM}}}{16\pi^2\epsilon}(1+\xi)\delta^{ab}I_{\mu\nu,\alpha\beta}. \tag{2.16}$$

It is worth pointing out that in the identities Eqs. (2.12) and (2.14), the origin of the divergent contributions is different. On the left-hand side of these equations, UV divergences come from one-loop graphs, whereas on their right-hand side short-distance singularities arise from the pinched tree graphs.

Further differentiations of Eqs. (2.6) and (2.11) with respect to $\tilde{J}$ and $\tilde{j}$ yield a set of structural identities, which are complementary to the usual Ward identities. One can compare the above identities with the ones found in Ref. [14] in the usual first-order formulation of the Yang–Mills theory (see, for example, Eqs. (3.1)–(3.2) and (4.1)–(4.2) in [14]). One can see that the structural identities obtained in the diagonal representation have a much simpler form. This feature will be especially useful for the derivation of the corresponding identities in quantum gravity.

III. DIAGONAL FORMULATION OF FIRST-ORDER PALATINI ACTION

Instead of using $g_{\mu\nu}$ and $\Gamma^\lambda_{\mu\nu}$ as independent fields in the action of Eq. (1.1), it turns out to be more useful to employ the independent combinations [7]

$$h^{\mu\nu} = \sqrt{-g}g^{\mu\nu} \tag{3.1}$$

and

$$G^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \frac{1}{2}(\delta^{\lambda}_{\mu}\Gamma^{\sigma}_{\nu\sigma} + \delta^{\lambda}_{\nu}\Gamma^{\sigma}_{\mu\sigma}). \tag{3.2}$$

Thus, we arrive at the following Lagrangian density in d spacetime dimensions

$$\tilde{\mathcal{L}}^{(1)}_{\rm EH} = \frac{h^{\mu\nu}}{\kappa^2}\left(G^{\lambda}_{\mu\nu,\lambda} + \frac{1}{d-1}G^{\lambda}_{\mu\lambda}G^{\sigma}_{\nu\sigma} - G^{\lambda}_{\mu\sigma}G^{\sigma}_{\nu\lambda}\right). \tag{3.3}$$

In order to proceed, $h^{\mu\nu}$ is expanded about a flat metric $\eta^{\mu\nu}$ ($\kappa = \sqrt{16\pi G_N}$),

$$h^{\mu\nu}(x) = \eta^{\mu\nu} + \kappa\phi^{\mu\nu}(x). \tag{3.4}$$

Equation (3.3) yields a basic vertex $\langle\phi GG\rangle$ (see Eqs. (3.4) and (3.12) below). However, it leads to an involved non-diagonal matrix propagator containing $(\phi\phi)$, (GG) and the mixed propagator (ϕG). As in the Yang–Mills theory, it proves convenient to use a diagonal formulation of the first-order EH action [8]. This may be achieved by making the change of variable [compare with Eq. (2.2)]

$$G^{\lambda}_{\mu\nu} = H^{\lambda}_{\mu\nu} + (M^{-1})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}(h=\eta)h^{\pi\tau}_{,\rho}, \tag{3.5}$$

where

$$(M^{-1})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}(h) = -\frac{1}{2(d-2)}h^{\lambda\rho}h_{\mu\nu}h_{\pi\tau} + \frac{1}{4}h^{\lambda\rho}(h_{\pi\mu}h_{\tau\nu} + h_{\pi\nu}h_{\tau\mu}) - \frac{1}{4}(h_{\tau\mu}\delta^{\rho}_{\nu}\delta^{\lambda}_{\pi} + h_{\pi\mu}\delta^{\rho}_{\nu}\delta^{\lambda}_{\tau} + h_{\tau\nu}\delta^{\rho}_{\mu}\delta^{\lambda}_{\pi} + h_{\pi\nu}\delta^{\rho}_{\mu}\delta^{\lambda}_{\tau}). \tag{3.6}$$

In this way, the Lagrangian density Eq. (3.3) may be written in the form

$$\begin{aligned}\tilde{\mathcal{L}}^{\rm I}_{\rm EH} &= \frac{1}{2}H^{\lambda}_{\mu\nu}M^{\mu\nu}_{\lambda}{}^{\pi\tau}_{\rho}(\eta)H^{\rho}_{\pi\tau} - \frac{1}{2}\phi^{\mu\nu}_{,\lambda}(M^{-1})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}(\eta)\phi^{\pi\tau}_{,\rho} \\ &\quad + \frac{\kappa}{2}[H^{\lambda}_{\mu\nu} + \phi^{\alpha\beta}_{,\rho}(M^{-1})^{\rho}_{\alpha\beta}{}^{\lambda}_{\mu\nu}(\eta)]M^{\mu\nu}_{\lambda}{}^{\pi\tau}_{\sigma}(\phi)[H^{\sigma}_{\pi\tau} + (M^{-1})^{\sigma}_{\pi\tau}{}^{\upsilon}_{\gamma\delta}(\eta)\phi^{\gamma\delta}_{,\upsilon}],\end{aligned} \tag{3.7}$$

where $M^{\mu\nu}_{\lambda}{}^{\pi\tau}_{\sigma}(\phi)$ is given by

$$\begin{aligned}M^{\mu\nu}_{\lambda}{}^{\pi\tau}_{\sigma}(\phi) = \frac{1}{2}\bigg[&\frac{1}{d-1}(\delta^{\nu}_{\lambda}\delta^{\tau}_{\sigma}\phi^{\mu\pi} + \delta^{\mu}_{\lambda}\delta^{\tau}_{\sigma}\phi^{\nu\pi} + \delta^{\nu}_{\lambda}\delta^{\pi}_{\sigma}\phi^{\mu\tau} + \delta^{\mu}_{\lambda}\delta^{\pi}_{\sigma}\phi^{\nu\tau}) \\ &- (\delta^{\tau}_{\lambda}\delta^{\nu}_{\sigma}\phi^{\mu\pi} + \delta^{\tau}_{\lambda}\delta^{\mu}_{\sigma}\phi^{\nu\pi} + \delta^{\pi}_{\lambda}\delta^{\nu}_{\sigma}\phi^{\mu\tau} + \delta^{\pi}_{\lambda}\delta^{\mu}_{\sigma}\phi^{\nu\tau})\bigg].\end{aligned} \tag{3.8}$$

Thus, we see that the Lagrangian Eq. (3.7) involves three cubic vertices $\langle H\phi H\rangle$, $\langle\phi H\phi\rangle$, and $\langle\phi\phi\phi\rangle$. On the other hand, it leads only to two uncoupled propagators $(\phi\phi)$ and (HH).

Using the Lagrangian in Eq. (3.7) in the Euler–Lagrange equation for the field $H^{\lambda}_{\mu\nu}$, we obtain the classical solution

$$H^{\lambda}_{\mu\nu} = -[(M(\eta) + \kappa M(\phi))^{-1}\kappa M(\phi)M^{-1}(\eta)]^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}\phi^{\pi\tau}_{,\rho}. \tag{3.9}$$

Since $M(\eta) + \kappa M(\phi) = M(\eta + \kappa\phi)$, this can be written as

$$H^{\lambda}_{\mu\nu} = [M^{-1}(\eta + \kappa\phi) - M^{-1}(\eta)]^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}\phi^{\pi\tau}_{,\rho}. \tag{3.10}$$

Substituting (3.10) back into (3.7), we obtain (using $\phi^{\mu\nu}_{,\lambda} = h^{\mu\nu}_{,\lambda}$)

$$-\frac{1}{2}h^{\mu\nu}_{,\lambda}(M^{-1})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}(h)h^{\pi\tau}_{,\rho}, \tag{3.11}$$

which is just the classical second-order Einstein–Hilbert Lagrangian. This demonstrates the classical equivalence of the two formalisms [8].

In order to obtain the propagator of the $\phi^{\mu\nu}$ field, we use the gauge fixing Lagrangian

$$\mathcal{L}_{\rm gf} = -\frac{1}{2\xi}(\phi^{\mu\nu}_{,\nu})^2. \tag{3.12}$$

With this gauge fixing, the contributions coming from the vector ghost fields d_ν, $\bar{d}_\mu$ are [22]

$$\begin{aligned}\mathcal{L}_{\rm ghost} = \bar{d}_{\mu}[&\partial^2\eta^{\mu\nu} + (\phi^{\rho\sigma}_{,\rho})\partial_{\sigma}\eta^{\mu\nu} - (\phi^{\rho\mu}_{,\rho})\partial^{\nu} + \phi^{\rho\sigma}\partial_{\rho}\partial_{\sigma}\eta^{\mu\nu} \\ &- (\partial_{\rho}\partial^{\nu}\phi^{\rho\mu})]d_{\nu}.\end{aligned} \tag{3.13}$$

Thus, the complete diagonal first-order Lagrangian density becomes

$$\mathcal{L}^{\rm I}_{\rm EH} = \tilde{\mathcal{L}}^{\rm I}_{\rm EH} + \mathcal{L}_{\rm gf} + \mathcal{L}_{\rm ghost}. \tag{3.14}$$

Next, we will also introduce the external sources $J^{\mu\nu}_{\lambda}$ and $j_{\mu\nu}$ as follows:

$$\mathcal{L}_{\rm source} = J^{\mu\nu}_{\lambda}H^{\lambda}_{\mu\nu} + j_{\mu\nu}\phi^{\mu\nu}. \tag{3.15}$$

Using the above results, the generating functional for Green's functions will be given by the Feynman path integral

$$Z[J,j] = N\int \mathcal{D}d\mathcal{D}\bar{d}\mathcal{D}H\mathcal{D}\phi \exp i\left[S + \int d^dx(J_\lambda^{\mu\nu}H_{\mu\nu}^\lambda + j_{\mu\nu}\phi^{\mu\nu})\right], \tag{3.16}$$

where N is a normalization factor and $S = \int d^dx\mathcal{L}_{\mathrm{EH}}^{\mathrm{I}}$. This equation has a form which is appropriate for generating the Green's functions through the application of functional differentiations with respect to $J_\lambda^{\mu\nu}$ and $j_{\mu\nu}$.

Performing the following shift in the functional integral (3.16)

$$H_{\mu\nu}^\lambda \to H_{\mu\nu}^\lambda + [M^{-1}(\eta+\kappa\phi) - M^{-1}(\eta)]_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho}\phi_{,\rho}^{\pi\tau} - (M^{-1})_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho}(\eta+\kappa\phi)J_\rho^{\pi\tau}, \tag{3.17}$$

we obtain

$$Z'[J,j] = N\int \mathcal{D}d\mathcal{D}\bar{d}\mathcal{D}\phi\mathcal{D}H \exp i\int d^dx\left\{\frac{1}{2}H_{\mu\nu}^\lambda[M(\eta)+\kappa M(\phi)]_\lambda^{\mu\nu}{}_\sigma^{\pi\tau}H_{\pi\tau}^\sigma + \mathcal{L}_{\mathrm{EH}}^{\mathrm{II}}\right.$$
$$\left. + J_\lambda^{\mu\nu}[M^{-1}(\eta+\kappa\phi) - M^{-1}(\eta)]_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho}\phi_{,\rho}^{\pi\tau} - \frac{1}{2}J_\lambda^{\mu\nu}(M^{-1})_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho}(\eta+\kappa\phi)J_\rho^{\pi\tau} + j_{\mu\nu}\phi^{\mu\nu}\right\}. \tag{3.18}$$

This enables one to integrate out the auxiliary field $H_{\mu\nu}^\lambda$ and leads to the alternative form of the generating functional[1]

$$Z'[J,j] = N\int \mathcal{D}d\mathcal{D}\bar{d}\mathcal{D}\phi \exp i\int d^dx\left\{\mathcal{L}_{\mathrm{EH}}^{\mathrm{II}} + J_\lambda^{\mu\nu}[M^{-1}(\eta+\kappa\phi) - M^{-1}(\eta)]_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho}\phi_{,\rho}^{\pi\tau}\right.$$
$$\left. - \frac{1}{2}J_\lambda^{\mu\nu}(M^{-1})_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho}(\eta+\kappa\phi)J_\rho^{\pi\tau} + j_{\mu\nu}\phi^{\mu\nu}\right\}, \tag{3.19}$$

where $\mathcal{L}_{\mathrm{EH}}^{\mathrm{II}}$ is the second-order EH Lagrangian with ghosts and gauge fixing, which may be written as

$$\mathcal{L}_{\mathrm{EH}}^{\mathrm{II}} = -\frac{1}{2}\phi_{,\lambda}^{\mu\nu}(M^{-1})_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho}(\eta+\kappa\phi)\phi_{,\rho}^{\pi\tau} - \frac{1}{2\xi}(\phi_{,\nu}^{\mu\nu})^2 + \mathcal{L}_{\mathrm{ghost}}. \tag{3.20}$$

We remark that the alternative generating functional (3.19) has a certain similarity to the corresponding functional in the Yang–Mills theory given by (2.11). The analogy is even more pronounced if we note that the coefficient of the source J is just the result found at the classical level given in Eq. (3.10) for the auxiliary field. We also note though that unlike Eq. (2.11), the term quadratic in the source J for the auxiliary field contains field dependency. Using a similar procedure to that employed in the Yang–Mills theory [see Eqs. (2.7)–(2.9)], one can show that the Green's function with only external gravitons are the same in the first and second-order formulations.

IV. CONSISTENCY CONDITION FOR THE AUXILIARY FIELD

Taking the functional derivatives of Eqs. (3.16) and (3.19) with respect to $J_\lambda^{\mu\nu}$ and $j_{\pi\tau}$ at $J_\lambda^{\mu\nu} = j_{\pi\tau} = 0$ and equating the results, we obtain the structural identity

$$\langle 0|TH_{\mu\nu}^\lambda(x)\phi^{\pi\tau}(y)|0\rangle$$
$$= \langle 0|T[M^{-1}(\eta+\kappa\phi) - M^{-1}(\eta)]_{\mu\nu}^{\lambda}{}_{\alpha\beta}^{\rho}\phi_{,\rho}^{\alpha\beta}(x)\phi^{\pi\tau}(y)|0\rangle, \tag{4.1}$$

where $(M^{-1})_{\mu\nu}^{\lambda}{}_{\alpha\beta}^{\rho}$ is defined in Eq. (3.6). Equation (4.1) is manifestly satisfied at tree level because its left-hand side vanishes since there is no mixed $H\phi$ propagator in the theory. Similarly, the right-hand side of (4.1) vanishes in the tree approximation (order zero in κ).

To one-loop order, the contribution to the left-hand side of Eq. (4.1) arises from the Feynman diagrams shown in Fig. 1. Using dimensional regularization, the contribution from the graph in Fig. 1(a) actually vanishes while the divergent contribution coming from the graph in Fig. 1(b) is given in momentum space in the gauge $\xi = 1$ by

$$\frac{-\kappa^2}{16\pi^2\epsilon}\left[\frac{31}{96}k^\lambda(\delta_\mu^\pi\delta_\nu^\tau + \delta_\nu^\pi\delta_\mu^\tau) + \dots\right], \tag{4.2}$$

where … stands for terms with other tensor structures, which are given in a general gauge in the Appendix.

[1]The Green's functions which involves the field $H_{\mu\nu}^\lambda$ necessarily will have $H_{\mu\nu}^\lambda$ appearing in a closed loop. But the propagator for the field $H_{\mu\nu}^\lambda$ is momentum independent and hence the associated loop momentum integrals vanish if we use dimensional regularization [8].

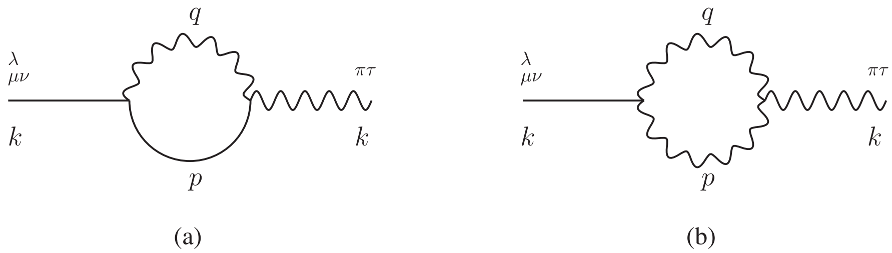

FIG. 1. One-loop contributions to the propagator $\langle H^{\lambda}_{\mu\nu}\phi^{\pi\tau}\rangle$. We are free to choose either $p = q - k$ or q as the loop integration momentum.

In order to obtain the contribution coming from the right-hand side of Eq. (4.1), one must expand the expression in the square bracket in a power series of $\kappa\phi$. Using for simplicity a schematic notation, we obtain

$$\begin{aligned} &M^{-1}(\eta+\kappa\phi) - M^{-1}(\eta) \\ &= -\kappa M^{-1}(\eta)M(\phi)M^{-1}(\eta) \\ &\quad + \kappa^2 M^{-1}(\eta)M(\phi)M^{-1}(\eta)M(\phi)M^{-1}(\eta) + \cdots, \end{aligned} \tag{4.3}$$

where $M(\phi)$ is a linear function of ϕ, which is given by Eq. (3.8). Substituting this result in the right-hand side of Eq. (4.1), one gets up to order κ^2 two terms that involve, respectively, a product of three and four ϕ fields. Using Wick's theorem, we can verify that the contribution from the cubic term comes from the Feynman graph shown in Fig. 2(a). This diagram corresponds to a three-point tree Green's function, which has, however, two coordinates pinched at the same spacetime point x.

As we have mentioned earlier, such a composite field leads to an ultra-violet (short distance) contribution. Using the appropriate expression for the three-point graviton vertex [8], one can evaluate in momentum space the contribution from Fig. 2(a). The result turns out to be in agreement with the one given in Eq. (4.2). One must also consider the contribution involving four ϕ fields in (4.1), which arises due to the last term in Eq. (4.3). This is represented by the Feynman diagram shown in Fig. 2(b). However, such a pinched contribution vanishes upon using dimensional regularization.

Thus, we see that the features, which appear in the structural identity (4.1), are similar to those which occur in the Yang–Mills theory via the identity (2.12). It is straightforward to generalize Eq. (4.1) to an arbitrary number of graviton fields, namely

$$\begin{aligned} &\langle 0|TH^{\lambda}_{\mu\nu}(x)\phi^{\pi_1\tau_1}(y_1)\cdots\phi^{\pi_n\tau_n}(y_n)|0\rangle \\ &= \langle 0|T[M^{-1}(\eta+\kappa\phi) - M^{-1}(\eta)]^{\lambda}_{\mu\nu}{}^{\rho}_{\alpha\beta}\phi^{\alpha\beta}_{,\rho}(x) \\ &\quad \times \phi^{\pi_1\tau_1}(y_1)\cdots\phi^{\pi_n\tau_n}(y_n)|0\rangle. \end{aligned} \tag{4.4}$$

This relation may be interpreted as being, in quantum gravity, a quantum-mechanical extension of the relation (3.10), which holds at the classical level.

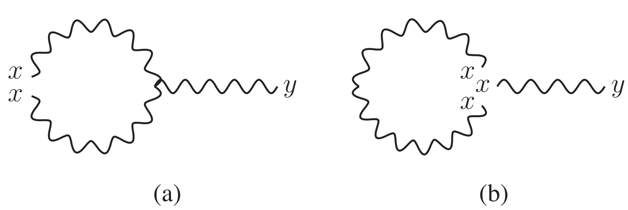

FIG. 2. Pinched contributions to the right-hand side of Eq. (4.1).

V. A SECOND STRUCTURAL IDENTITY

Applying $\delta^2/\delta J^{\mu\nu}_{\lambda}(x)\delta J^{\pi\tau}_{\rho}(y)$ to Eqs. (3.16) and (3.19) and equating the results, yields

$$\begin{aligned} &\langle 0|TH^{\lambda}_{\mu\nu}(x)H^{\rho}_{\pi\tau}(y)|0\rangle \\ &= i\langle 0|T(M^{-1})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}(\eta+\kappa\phi)(x)|0\rangle\delta^d(x-y) \\ &\quad + \langle 0|T\Delta^{\lambda}_{\mu\nu}[\phi(x)]\Delta^{\rho}_{\pi\tau}[\phi(y)]|0\rangle, \end{aligned} \tag{5.1}$$

where we have introduced the shorthand notation

$$\Delta^{\lambda}_{\mu\nu}[\phi(x)] = [M^{-1}(\eta+\kappa\phi) - M^{-1}(\eta)]^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}\phi^{\pi\tau}_{,\rho}(x). \tag{5.2}$$

The structural identity (5.1) is clearly satisfied at tree level, where the (HH) propagator is precisely equal to $(M^{-1})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}(\eta)\delta^d(x-y)$. We will now examine the perturbative expansion of each side of Eq. (5.1). To one-loop order, the contributions to the left-hand side of this equation arise from the Feynman diagrams shown in Fig. 3.

Using dimensional regularization, the contribution from the graph in Fig. 3(a) vanishes while the divergent part of the contribution from the graph in Fig. 3(b) is given in momentum space in the gauge $\xi = 1$ by

$$\frac{i\kappa^2k^2}{16\pi^2\epsilon}\left[\frac{1}{24}\delta^{\lambda}_{\nu}\delta^{\rho}_{\tau}\left(\eta_{\pi\mu} + 6\frac{k_\pi k_\mu}{k^2}\right) + \ldots\right], \tag{5.3}$$

where … denotes terms with other tensorial structures, which are explicitly given in the Appendix.

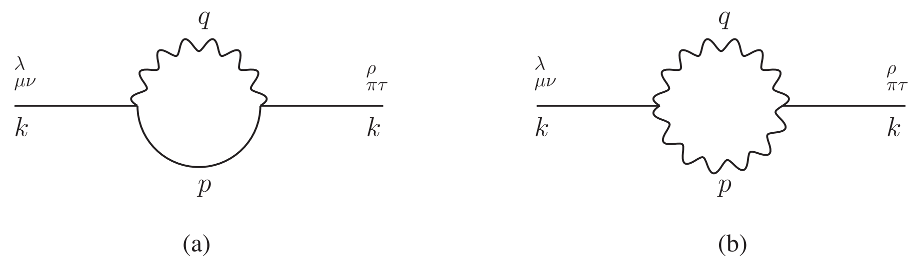

FIG. 3. One-loop contributions to the propagator $(H^{\lambda}_{\mu\nu} H^{\rho}_{\pi\tau})$.

Next, let us examine the contributions of order κ^2, which come from the terms on the right-hand side of Eq. (5.1). Such a contribution could arise from the first term, but this vanishes upon using dimensional regularization. Thus, we must evaluate only the κ^2 contribution coming from the last term. This part arises by considering the terms of order κ, which occur in each of the factors appearing in the last expression on the right-hand side of Eq. (5.1). Using the expansion indicated in Eq. (4.3), one gets from the last term in Eq. (5.1) the Eq. (A16) in the Appendix.

We note here that these composite field contributions are pinched at the spacetime points x and y. The Feynman diagrams associated with such Green's functions are shown in Fig. 4. The divergent contributions coming from Fig. 4(a) [there is an additional graph with $x \leftrightarrow y$ on the left side] turn out to add up to a result which agrees with that given in Fig. 3(b). We have also verified this identity at one-loop order for any dimension d in a general gauge (see the Appendix). On the other hand, the contributions coming from Fig. 4(b) vanish upon using dimensional regularization in momentum space. Verifying this result beyond order κ^2 becomes exceedingly difficult as it would involve going beyond one-loop order.

We remark that the structural identity (5.1) resembles the identity (2.14), which holds in the Yang–Mills theory. Therefore, as we have seen in the previous examples, the structural identities in the diagonal representation of the first-order Yang–Mills and gravity theories exhibit many similar features, though they are not identical.

VI. DISCUSSION

We have examined the structural identities which ensure the self-consistency of the first-order formulation of quantum gravity. Since calculations in this theory are quite involved even at one-loop order, we have studied first the structural identities in the diagonal representation of Yang–Mills theory, which are simpler. It turns out that these identities in the Yang–Mills theory have many features similar to the ones which occur in the diagonal representation of the first-order quantum gravity. With this insight, we have compared the forms of the generating functionals $Z[J, j]$ of Green's functions in quantum gravity before and after integrating out the auxiliary field $H^{\lambda}_{\mu\nu}$. Differentiations of these two forms with respect to $J^{\lambda}_{\mu\nu}$ and $j_{\mu\nu}$ yield a set structural identities given in Eqs. (4.1) and (5.1), which are complementary but distinct from the usual Ward identities. These identities show that the Green's functions containing only external graviton (gluon) fields are the same in the first and second-order formulations.

These identities also lead to connections between the Green's functions involving the field $H^{\lambda}_{\mu\nu}$ and the Green's functions in second-order formulation containing a composite graviton field that corresponds to the classical value of the auxiliary field. Equation (4.4) provides a simple interpretation of the auxiliary field $H^{\lambda}_{\mu\nu}$. An interesting feature is that the implementation of the structural identities requires cancellations between UV divergences, which appear in one-loop diagrams and the short-distance singularities that occur in the tree graphs, which are pinched at the same spacetime points. This shows that the singularities arising at the tree level from the composite graviton field are necessary for the first-order formulation of quantum gravity to be consistent. These identities have also a practical utility as they allow us to compute more efficiently, in the second-order formulation, some involved composite field expectation values in terms of those containing the local auxiliary field.

Recently [23,24], we have introduced a Lagrange multiplier field, which restricts the path integral in quantum gravity to the field configurations that satisfy the classical equations of motion. It was shown that such a method has the effect of eliminating all multiloop corrections beyond one-loop order and doubling of the usual one-loop contributions. This makes it possible to renormalize the EH action while retaining unitarity. Such a treatment was employed both in the second-order as well as in the first-order formulations of quantum gravity. In the later

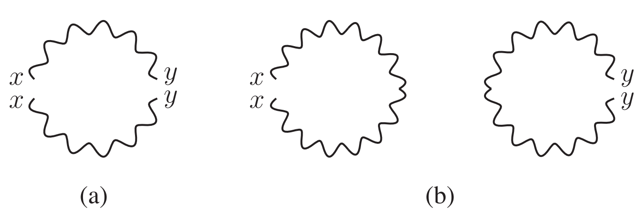

FIG. 4. Pinched contributions associated with the last term in Eq. (5.1).

case, one may also expect to have a corresponding set of structural identities, which are necessary for the consistency of the theory. This is an interesting issue, which requires further study.

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APPENDIX: ONE-LOOP RESULTS

We employ the same Feynman rules, procedures, and conventions as in Sec. 3 of [8] with the replacements $\phi^{\mu\nu} \to \kappa\phi^{\mu\nu}$, $G^{\lambda}_{\mu\nu} \to \kappa H^{\lambda}_{\mu\nu}$ and $S \to S/\kappa^2$ (S is the action) so that the coupling constant κ is shown explicitly in the vertices and in the resulting Green's functions.

1. The general approach for the calculation of massless one-loop self-energies

Let us consider some generic field theory for fields ϕ_a, where a represents a collection of Lorentz indices, or indices for internal degrees of freedom such as in the case of Yang–Mills theories. The most general form of the momentum space massless self-energy is

$$\Pi_{ab}(k) = \int \frac{d^d p}{(2\pi)^d} I_{ab}(p,q) = \sum_{i=1}^{n} C^i T^i_{ab}(k); \quad (q \equiv k + p), \tag{A1}$$

where n is the number of independent tensors, which can be obtained from the general symmetry properties of $\Pi_{ab}(k)$ (for instance, in the case of the photon self-energy there are the two independent tensors $\eta_{\mu\nu}$ and $k_\mu k_\nu$). Upon contracting Eq. (A1) with each of the n tensors, we obtain n linear equations for the coefficients C^i containing several scalar integrals of the following type (using Einstein summation convention for the labels a and b)

$$\int \frac{d^d p}{(2\pi)^d} I_{ab}(p,q) T^i_{ab}(k). \tag{A2}$$

Next, we simplify the n scalars $I_{ab}(p,q)T^i_{ab}(k)$ using the relations

$$p \cdot k = \frac{1}{2}(q^2 - p^2 - k^2),$$
$$q \cdot k = \frac{1}{2}(q^2 - p^2 + k^2) \quad \text{and} \quad p \cdot q = \frac{1}{2}(p^2 + q^2 - k^2) \tag{A3}$$

so that all the scalar integrals acquire the form

$$I^{rs} = \mu^{4-d} \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2)^r (q^2)^s}, \tag{A4}$$

where μ is an arbitrary mass parameter. In the simplest cases $r = s = 1$. When considering gauge theories with a general gauge fixing parameter, we can have $r = 1$, 2 and $s = 1$, 2. Since we are using a dimensional regularization procedure, the only nonvanishing integrals are the following:

$$I^{11} = i \frac{(k^2/\mu^2)^{d/2-2}}{2^d \pi^{d/2}} \frac{\Gamma(2-\frac{d}{2})\Gamma(\frac{d}{2}-1)^2}{\Gamma(d-2)} \equiv I, \tag{A5a}$$

$$I^{12} = I^{21} = \frac{3-d}{k^2} I, \tag{A5b}$$

$$I^{22} = \frac{(3-d)(6-d)}{k^4} I \tag{A5c}$$

(we have a factor of i relative to Eq. (3.31a) of [8], which takes into account that we are Wick rotating back to Minkowski space).

Once we have all the relevant scalar integrals in Eq. (A5), we may solve the linear system of algebraic equations for constants C^i in (A1). In general, this procedure would be of no practical use unless we make use of computer algebra algorithms, as we have done in the present work (for example, the tensor basis for the self-energy of the H-field has 22 rank 6 tensors). Using this procedure, we have previously obtained the expression for the graviton self-energy in the diagonalized first-order formalism [8].

For $d = 4 - 2\epsilon$, Eq. (A5) yields the following UV pole part

$$I^{\mathrm{UV}} \equiv \frac{i}{16\pi^2 \epsilon}. \tag{A6}$$

It is worth mentioning that the present approach is as an example of the Passarino–Veltman reduction method [25].

2. The $H\phi$ self-energy

Figures 1(a) and 1(b), without the external free propagators, are the two contributions for the mixed $H\phi$-fields self-energy. Since the internal H-field propagator in Fig. 1(a) has no momentum dependence, the loop momentum integration vanishes when using dimensional regularization. The self-energy contribution from Fig. 1(b) can be expressed as

TABLE I. Coefficients for the mixed $H\phi$ self-energy [see Eq. (A7)] in units of $i\kappa^2 k^2 I$, where I is given by Eq. (A5a).

	1	$(\xi-1)$	$(\xi-1)^2$
$C^{H\phi}_{(1)}$	$-\frac{1}{16(d-1)}$	$-\frac{1}{16(d-1)}$	0
$C^{H\phi}_{(2)}$	$-\frac{(d-2)(d+2)}{16(d-1)}$	$\frac{3d^2-18d+16}{16(d-1)}$	$-\frac{d^3-8d^2+22d-14}{16(d-1)}$
$C^{H\phi}_{(3)}$	$\frac{d^2+2d+2}{32(d-2)(d-1)}$	$-\frac{7d^2-41d+32}{32(d-2)(d-1)}$	$\frac{5d^3-37d^2+96d-60}{64(d-2)(d-1)}$
$C^{H\phi}_{(4)}$	0	$-\frac{1}{32(d-1)}$	0
$C^{H\phi}_{(5)}$	$\frac{d^2+d-1}{16(d-1)}$	$\frac{5-d}{8}$	$\frac{d-2}{8(d-1)}$
$C^{H\phi}_{(6)}$	$-\frac{d^2+6d-4}{32(d-2)(d-1)}$	$\frac{d-5}{8(d-2)}$	$\frac{d-6}{32(d-1)}$
$C^{H\phi}_{(7)}$	$\frac{(d-2)^2}{32(d-1)}$	$-\frac{d-2}{16(d-1)}$	$\frac{(d-2)^2}{32(d-1)}$
$C^{H\phi}_{(8)}$	$\frac{(d-2)(d+2)}{16(d-1)}$	$-\frac{4d^2-23d+18}{16(d-1)}$	$\frac{d^3-8d^2+21d-12}{8(d-1)}$
$C^{H\phi}_{(9)}$	$-\frac{1}{16}(d+2)$	$\frac{4d^2-23d+22}{32(d-1)}$	$-\frac{d-2}{8(d-1)}$
$C^{H\phi}_{(10)}$	0	$\frac{(d-4)d}{16(d-1)}$	$-\frac{(d-4)^2 d}{32(d-1)}$
$C^{H\phi}_{(11)}$	$-\frac{d^2-10d+4}{32(d-1)}$	$-\frac{3(d^2-6d+4)}{32(d-1)}$	$\frac{1}{64}(d-2)^2$
$C^{H\phi}_{(12)}$	$\frac{d}{8(d-2)(d-1)}$	$\frac{d-2}{32(d-1)}$	$-\frac{(d-4)^2 d}{32(d-2)(d-1)}$

$$(\Pi^{H\phi})^{\lambda}_{\mu\nu}{}^{\pi\tau} = \sum_{i=1}^{12} C^{H\phi}_{(i)} (T^{H\phi}_i)^{\lambda}_{\mu\nu}{}^{\pi\tau}, \tag{A7}$$

where the tensors $(T^{H\phi}_i)^{\lambda}_{\mu\nu}{}^{\pi\tau}$, $i = 1\ldots 12$ are given by

$$(T^{H\phi}_1)^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{4}(k^\pi \delta^\lambda_\nu \delta^\tau_\mu + k^\pi \delta^\lambda_\mu \delta^\tau_\nu + \delta^\pi_\nu k^\tau \delta^\lambda_\mu + \delta^\pi_\mu k^\tau \delta^\lambda_\nu), \tag{A8a}$$

$$(T^{H\phi}_2)^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{2} k^\lambda (\delta^\pi_\nu \delta^\tau_\mu + \delta^\pi_\mu \delta^\tau_\nu), \tag{A8b}$$

$$(T^{H\phi}_3)^{\lambda}_{\mu\nu}{}^{\pi\tau} = k^\lambda \eta^{\pi\tau} \eta_{\mu\nu}, \tag{A8c}$$

$$(T^{H\phi}_4)^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{2} \eta_{\mu\nu} (k^\pi \eta^{\lambda\tau} + \eta^{\lambda\pi} k^\tau), \tag{A8d}$$

$$(T^{H\phi}_5)^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{4}(\delta^\pi_\nu k_\mu \eta^{\lambda\tau} + \eta^{\lambda\pi} k_\mu \delta^\tau_\nu + \delta^\pi_\mu k_\nu \eta^{\lambda\tau} + \eta^{\lambda\pi} k_\nu \delta^\tau_\mu), \tag{A8e}$$

$$(T^{H\phi}_6)^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{2} \eta^{\pi\tau} (k_\mu \delta^\lambda_\nu + k_\nu \delta^\lambda_\mu), \tag{A8f}$$

$$(T^{H\phi}_7)^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{2k^2} k^\pi k^\tau (k_\mu \delta^\lambda_\nu + k_\nu \delta^\lambda_\mu), \tag{A8g}$$

$$(T^{H\phi}_8)^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{4k^2} k^\lambda (k^\pi k_\mu \delta^\tau_\nu + k^\pi k_\nu \delta^\tau_\mu + \delta^\pi_\nu k_\mu k^\tau + \delta^\pi_\mu k_\nu k^\tau), \tag{A8h}$$

$$(T^{H\phi}_9)^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{2k^2} k_\mu k_\nu (k^\pi \eta^{\lambda\tau} + \eta^{\lambda\pi} k^\tau), \tag{A8i}$$

TABLE II. The UV parts of the coefficients for the mixed $H\phi$ self-energy [see Eq. (A7)] in units of $i\kappa^2 k^2 I^{\mathrm{UV}}$, where I^{UV} is given by Eq. (A5a).

	1	$(\xi-1)$	$(\xi-1)^2$
$C^{H\phi}_{(1)}$	$-\frac{1}{48}$	$-\frac{1}{48}$	0
$C^{H\phi}_{(2)}$	$-\frac{1}{4}$	$-\frac{1}{6}$	$-\frac{5}{24}$
$C^{H\phi}_{(3)}$	$\frac{13}{96}$	$\frac{5}{48}$	$\frac{13}{96}$
$C^{H\phi}_{(4)}$	0	$-\frac{1}{96}$	0
$C^{H\phi}_{(5)}$	$\frac{19}{48}$	$\frac{1}{8}$	$\frac{1}{12}$
$C^{H\phi}_{(6)}$	$-\frac{3}{16}$	$-\frac{1}{16}$	$-\frac{1}{48}$
$C^{H\phi}_{(7)}$	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$
$C^{H\phi}_{(8)}$	$\frac{1}{4}$	$\frac{5}{24}$	$\frac{1}{3}$
$C^{H\phi}_{(9)}$	$-\frac{3}{8}$	$-\frac{1}{16}$	$-\frac{1}{12}$
$C^{H\phi}_{(10)}$	0	0	0
$C^{H\phi}_{(11)}$	$\frac{5}{24}$	$\frac{1}{8}$	$\frac{1}{16}$
$C^{H\phi}_{(12)}$	$\frac{1}{12}$	$\frac{1}{48}$	0

$$(T^{H\phi}_{10})^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{k^4} k^\lambda k_\mu k_\nu k^\pi k^\tau, \tag{A8j}$$

$$(T^{H\phi}_{11})^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{k^2} k^\lambda \eta_{\mu\nu} k^\pi k^\tau, \tag{A8k}$$

$$(T^{H\phi}_{12})^{\lambda}_{\mu\nu}{}^{\pi\tau} = \frac{1}{k^2} k^\lambda k_\mu k_\nu \eta^{\pi\tau}. \tag{A8l}$$

Using the Feynman rules given in Ref. [8], we obtain the equivalent of $I_{ab}(p,q)$ in Eq. (A1). Next, using the general approach described in Appendix A 1, we obtain the coefficients for the $H\phi$ self-energy shown in Table I. These expressions have an UV part, which arises when $d = 4 - 2\epsilon$ and $\epsilon \to 0$, given by the numbers in Table II.

3. The *H*-field self-energy

Figures 3(a) and 3(b), without the external free propagators, are the two contributions for the H-field self-energy. Since the internal H-field propagator in Fig. 3(a) has no momentum dependence, the loop momentum integration vanishes when using dimensional regularization. The self-energy contribution from Fig. 3(b) can be expressed as

$$(\Pi^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \sum_{i=1}^{22} C^{HH}_{(i)} (T^{HH}_i)^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}, \tag{A9}$$

where the tensors $(T^{HH}_i)^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau}$, $i = 1\ldots 22$ are given by

$$(T^{HH}_1)^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4}(\delta^\rho_\pi \delta^\lambda_\nu \eta_{\mu\tau} + \delta^\rho_\pi \delta^\lambda_\mu \eta_{\nu\tau} + \eta_{\pi\nu} \delta^\lambda_\mu \delta^\rho_\tau + \eta_{\pi\mu} \delta^\lambda_\nu \delta^\rho_\tau), \tag{A10a}$$

$$(T_{2}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{2}\eta^{\lambda\rho}(\eta_{\pi\nu}\eta_{\mu\tau} + \eta_{\pi\mu}\eta_{\nu\tau}), \tag{A10b}$$

$$(T_{3}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \eta_{\pi\tau}\eta^{\lambda\rho}\eta_{\mu\nu}, \tag{A10c}$$

$$(T_{4}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4}(\eta_{\pi\nu}\delta^{\lambda}_{\tau}\delta^{\rho}_{\mu} + \delta^{\lambda}_{\pi}\delta^{\rho}_{\mu}\eta_{\nu\tau} + \eta_{\pi\mu}\delta^{\lambda}_{\tau}\delta^{\rho}_{\nu} + \delta^{\lambda}_{\pi}\eta_{\mu\tau}\delta^{\rho}_{\nu}), \tag{A10d}$$

$$(T_{5}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4}(\delta^{\rho}_{\pi}\delta^{\lambda}_{\tau}\eta_{\mu\nu} + \delta^{\lambda}_{\pi}\eta_{\mu\nu}\delta^{\rho}_{\tau} + \eta_{\pi\tau}\delta^{\lambda}_{\nu}\delta^{\rho}_{\mu} + \eta_{\pi\tau}\delta^{\lambda}_{\mu}\delta^{\rho}_{\nu}), \tag{A10e}$$

$$(T_{6}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4k^4}(k_{\pi}k^{\lambda}k_{\mu}k_{\nu}\delta^{\rho}_{\tau} + \delta^{\rho}_{\pi}k^{\lambda}k_{\mu}k_{\nu}k_{\tau} + k_{\pi}k_{\mu}k^{\rho}k_{\tau}\delta^{\lambda}_{\nu} + k_{\pi}k_{\nu}k^{\rho}k_{\tau}\delta^{\lambda}_{\mu}), \tag{A10f}$$

$$(T_{7}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4k^4}k^{\lambda}k^{\rho}(k_{\pi}k_{\mu}\eta_{\nu\tau} + k_{\pi}k_{\nu}\eta_{\mu\tau} + \eta_{\pi\nu}k_{\mu}k_{\tau} + \eta_{\pi\mu}k_{\nu}k_{\tau}), \tag{A10g}$$

$$(T_{8}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4k^4}(k_{\pi}k_{\mu}k_{\nu}k^{\rho}\delta^{\lambda}_{\tau} + \delta^{\lambda}_{\pi}k_{\mu}k_{\nu}k^{\rho}k_{\tau} + k_{\pi}k^{\lambda}k_{\mu}k_{\tau}\delta^{\rho}_{\nu} + k_{\pi}k^{\lambda}k_{\nu}k_{\tau}\delta^{\rho}_{\mu}), \tag{A10h}$$

$$(T_{9}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{2k^4}k^{\lambda}k^{\rho}(\eta_{\pi\tau}k_{\mu}k_{\nu} + k_{\pi}k_{\tau}\eta_{\mu\nu}), \tag{A10i}$$

$$(T_{10}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4k^2}(k_{\pi}\delta^{\rho}_{\tau} + \delta^{\rho}_{\pi}k_{\tau})(k_{\mu}\delta^{\lambda}_{\nu} + k_{\nu}\delta^{\lambda}_{\mu}), \tag{A10j}$$

$$(T_{11}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4k^2}\eta^{\lambda\rho}(k_{\pi}k_{\mu}\eta_{\nu\tau} + k_{\pi}k_{\nu}\eta_{\mu\tau} + \eta_{\pi\nu}k_{\mu}k_{\tau} + \eta_{\pi\mu}k_{\nu}k_{\tau}), \tag{A10k}$$

$$(T_{12}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4k^2}(k_{\pi}\delta^{\lambda}_{\tau} + \delta^{\lambda}_{\pi}k_{\tau})(k_{\mu}\delta^{\rho}_{\nu} + k_{\nu}\delta^{\rho}_{\mu}), \tag{A10l}$$

$$(T_{13}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4k^2}(\delta^{\rho}_{\pi}k_{\mu}k_{\nu}\delta^{\lambda}_{\tau} + \delta^{\lambda}_{\pi}k_{\mu}k_{\nu}\delta^{\rho}_{\tau} + k_{\pi}k_{\tau}\delta^{\lambda}_{\nu}\delta^{\rho}_{\mu} + k_{\pi}k_{\tau}\delta^{\lambda}_{\mu}\delta^{\rho}_{\nu}), \tag{A10m}$$

$$(T_{14}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{2k^2}\eta^{\lambda\rho}(\eta_{\pi\tau}k_{\mu}k_{\nu} + k_{\pi}k_{\tau}\eta_{\mu\nu}), \tag{A10n}$$

$$(T_{15}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{8k^2}(\delta^{\rho}_{\pi}k^{\lambda}k_{\mu}\eta_{\nu\tau} + k_{\pi}k^{\rho}\delta^{\lambda}_{\mu}\eta_{\nu\tau} + \delta^{\rho}_{\pi}k^{\lambda}k_{\nu}\eta_{\mu\tau} + \eta_{\pi\nu}k^{\lambda}k_{\mu}\delta^{\rho}_{\tau} \tag{A10o}$$

$$+\eta_{\pi\mu}k^{\lambda}k_{\nu}\delta^{\rho}_{\tau} + k_{\pi}k^{\rho}\delta^{\lambda}_{\nu}\eta_{\mu\tau} + \eta_{\pi\nu}k^{\rho}k_{\tau}\delta^{\lambda}_{\mu} + \eta_{\pi\mu}k^{\rho}k_{\tau}\delta^{\lambda}_{\nu}), \tag{A10p}$$

$$(T_{16}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{2k^2}k^{\lambda}k^{\rho}(\eta_{\pi\nu}\eta_{\mu\tau} + \eta_{\pi\mu}\eta_{\nu\tau}), \tag{A10q}$$

$$(T_{17}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4k^2}(\eta_{\pi\tau}k^{\lambda}k_{\mu}\delta^{\rho}_{\nu} + \eta_{\pi\tau}k^{\lambda}k_{\nu}\delta^{\rho}_{\mu} + k_{\pi}k^{\rho}\delta^{\lambda}_{\tau}\eta_{\mu\nu} + \delta^{\lambda}_{\pi}k^{\rho}k_{\tau}\eta_{\mu\nu}), \tag{A10r}$$

$$(T_{18}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{8k^2}(k_{\pi}k^{\lambda}\eta_{\mu\tau}\delta^{\rho}_{\nu} + \eta_{\pi\mu}k^{\lambda}k_{\tau}\delta^{\rho}_{\nu} + k_{\pi}k^{\lambda}\delta^{\rho}_{\mu}\eta_{\nu\tau} + \eta_{\pi\nu}k_{\mu}k^{\rho}\delta^{\lambda}_{\tau} \tag{A10s}$$

$$+\delta^{\lambda}_{\pi}k_{\mu}k^{\rho}\eta_{\nu\tau} + \eta_{\pi\mu}k_{\nu}k^{\rho}\delta^{\lambda}_{\tau} + \delta^{\lambda}_{\pi}k_{\nu}k^{\rho}\eta_{\mu\tau} + \eta_{\pi\nu}k^{\lambda}k_{\tau}\delta^{\rho}_{\mu}), \tag{A10t}$$

$$(T_{19}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{4k^2}(k_{\pi}k^{\lambda}\eta_{\mu\nu}\delta^{\rho}_{\tau} + \eta_{\pi\tau}k_{\mu}k^{\rho}\delta^{\lambda}_{\nu} + \eta_{\pi\tau}k_{\nu}k^{\rho}\delta^{\lambda}_{\mu} + \delta^{\rho}_{\pi}k^{\lambda}k_{\tau}\eta_{\mu\nu}), \tag{A10u}$$

$$(T_{20}^{HH})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{k^6}k_{\pi}k^{\lambda}k_{\mu}k_{\nu}k^{\rho}k_{\tau}, \tag{A10v}$$

TABLE III. Coefficients for the H-field self-energy [see Eq. (A9)] in units of $\kappa^2 k^2 I$, where I is given by Eq. (A5a).

	1	$(\xi - 1)$	$(\xi - 1)^2$
$C^{HH}_{(1)}$	$-\frac{1}{2(d-2)}$	$\frac{3(d-3)d}{16(d-2)(d-1)}$	$-\frac{1}{4(d-1)(d+1)}$
$C^{HH}_{(2)}$	$-\frac{d^2-2d-2}{4(d-2)(d-1)}$	$-\frac{4d^3-17d^2+31d-32}{16(d-2)(d-1)}$	$-\frac{d+2}{8(d-1)(d+1)}$
$C^{HH}_{(3)}$	$\frac{1}{2(d-1)}$	$\frac{d(3d-5)}{16(d-2)(d-1)}$	$\frac{d}{16(d-1)(d+1)}$
$C^{HH}_{(4)}$	$\frac{d^2-2}{4(d-2)(d-1)}$	$-\frac{4d^3-23d^2+33d-16}{16(d-2)(d-1)}$	$-\frac{1}{4(d-1)(d+1)}$
$C^{HH}_{(5)}$	$-\frac{1}{(d-2)(d-1)}$	$\frac{d^2-3d-8}{8(d-2)(d-1)}$	$\frac{d}{4(d-1)(d+1)}$
$C^{HH}_{(6)}$	0	$-\frac{d-4}{2(d-1)}$	$\frac{(d-4)(d-2)d}{4(d-1)(d+1)}$
$C^{HH}_{(7)}$	0	$-\frac{(d-4)(d^2-31d+24)}{16(d-1)}$	$-\frac{(d-4)(4d^3-16d^2-3d+14)}{4(d-1)(d+1)}$
$C^{HH}_{(8)}$	0	$\frac{(d-4)^2(d-3)}{8(d-1)}$	$-\frac{(d-4)(d-2)(d+2)}{4(d-1)(d+1)}$
$C^{HH}_{(9)}$	0	$4-d$	$\frac{(d-4)(4d^3-17d^2-4d+20)}{8(d-1)(d+1)}$
$C^{HH}_{(10)}$	$\frac{d^2-4d+2}{2(d-2)(d-1)}$	$\frac{3d^2-12d+8}{4(d-2)(d-1)}$	$\frac{(d-2)d}{4(d-1)(d+1)}$
$C^{HH}_{(11)}$	$\frac{d(d^2-d-4)}{4(d-2)(d-1)}$	$\frac{d^3+15d^2-50d+16}{16(d-2)(d-1)}$	$\frac{(d-2)(2d+1)}{4(d-1)(d+1)}$
$C^{HH}_{(12)}$	$\frac{d^3-2d^2-4d+4}{4(d-2)(d-1)}$	$-\frac{(d-4)(d^2-4d+2)}{4(d-2)(d-1)}$	$\frac{(d-2)d}{4(d-1)(d+1)}$
$C^{HH}_{(13)}$	$-\frac{(d-4)d}{2(d-2)(d-1)}$	$\frac{3d^3-23d^2+50d-16}{8(d-2)(d-1)}$	$-\frac{(d-2)(d+2)}{4(d-1)(d+1)}$
$C^{HH}_{(14)}$	$-\frac{d}{2(d-1)}$	$\frac{d^2-11d+12}{8(d-1)}$	$-\frac{(d-2)(d+2)}{8(d-1)(d+1)}$
$C^{HH}_{(15)}$	$\frac{d^2}{2(d-2)(d-1)}$	$-\frac{3d^3-15d^2+18d-16}{8(d-2)(d-1)}$	$-\frac{d-2}{2(d-1)(d+1)}$
$C^{HH}_{(16)}$	$\frac{2d^2-d-2}{4(d-1)}$	$-\frac{11d^2-89d+72}{16(d-1)}$	$\frac{4d^4-28d^3+55d^2+32d-52}{8(d-1)(d+1)}$
$C^{HH}_{(17)}$	$\frac{d}{2(d-1)}$	$-\frac{(d-7)d}{8(d-1)}$	$\frac{(d-2)d}{4(d-1)(d+1)}$
$C^{HH}_{(18)}$	$-\frac{2d^2+d-2}{2(d-1)}$	$\frac{11d^2-73d+72}{8(d-1)}$	$-\frac{(d-2)(2d+3)}{2(d-1)(d+1)}$
$C^{HH}_{(19)}$	-1	-1	$-\frac{(d-2)(d+2)}{4(d-1)(d+1)}$
$C^{HH}_{(20)}$	0	0	$\frac{(d-6)(d-4)(d-2)d}{16(d-1)(d+1)}$
$C^{HH}_{(21)}$	0	$-\frac{(d-4)(d-3)d}{16(d-1)}$	$\frac{(d-4)(d-2)d}{16(d-1)(d+1)}$
$C^{HH}_{(22)}$	1	$\frac{3d-4}{4(d-1)}$	$\frac{8d^3-23d^2-10d+24}{16(d-1)(d+1)}$

TABLE IV. Coefficients for the UV part of the H-field self-energy [see Eq. (A9)] in units of $\kappa^2 k^2 I^{\mathrm{UV}}$, where I^{UV} is given by Eq. (A6).

	1	$(\xi-1)$	$(\xi-1)^2$
$C^{HH}_{(1)}$	$-\frac{1}{4}$	$\frac{1}{8}$	$-\frac{1}{60}$
$C^{HH}_{(2)}$	$-\frac{1}{4}$	$-\frac{19}{24}$	$-\frac{1}{20}$
$C^{HH}_{(3)}$	$\frac{1}{6}$	$\frac{7}{24}$	$\frac{1}{60}$
$C^{HH}_{(4)}$	$\frac{7}{12}$	$-\frac{1}{24}$	$-\frac{1}{60}$
$C^{HH}_{(5)}$	$-\frac{1}{6}$	$-\frac{1}{12}$	$\frac{1}{15}$
$C^{HH}_{(6)}$	0	0	0
$C^{HH}_{(7)}$	0	0	0
$C^{HH}_{(8)}$	0	0	0
$C^{HH}_{(9)}$	0	0	0
$C^{HH}_{(10)}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{15}$
$C^{HH}_{(11)}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{3}{10}$
$C^{HH}_{(12)}$	$\frac{5}{6}$	0	$\frac{2}{15}$
$C^{HH}_{(13)}$	0	$\frac{1}{6}$	$-\frac{1}{5}$
$C^{HH}_{(14)}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{10}$
$C^{HH}_{(15)}$	$\frac{4}{3}$	$-\frac{1}{6}$	$-\frac{1}{15}$
$C^{HH}_{(16)}$	$\frac{13}{6}$	$\frac{9}{4}$	$\frac{47}{30}$
$C^{HH}_{(17)}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{15}$
$C^{HH}_{(18)}$	$-\frac{17}{3}$	$-\frac{11}{6}$	$-\frac{11}{15}$
$C^{HH}_{(19)}$	-1	-1	$-\frac{1}{5}$
$C^{HH}_{(20)}$	0	0	0
$C^{HH}_{(21)}$	0	0	0
$C^{HH}_{(22)}$	1	$\frac{2}{3}$	$\frac{8}{15}$

$$(T^{HH}_{21})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{k^4} k_\pi k_\mu k_\nu k_\tau \eta^{\lambda\rho}, \tag{A10w}$$

$$(T^{HH}_{22})^{\lambda}_{\mu\nu}{}^{\rho}_{\pi\tau} = \frac{1}{k^2} \eta_{\pi\tau} k^\lambda k^\rho \eta_{\mu\nu}. \tag{A10x}$$

Using the Feynman rules given in Ref. [8], we obtain the equivalent of $I_{ab}(p,q)$ in Eq. (A1). Next, using the general approach described in subsection A 1, we obtain the coefficients for the HH self-energy shown in Table III. These expressions have an UV part, which arises when $d = 4 - 2\epsilon$ and $\epsilon \to 0$, given by the numbers in Table IV.

4. Propagators

a. Mixed $H\phi$ propagator

The mixed $H\phi$ propagator $(M^{-1}\Pi^{H\phi}\mathcal{D})^{\lambda}_{\mu\nu}{}^{\pi\tau}$ can also be expressed in terms of the tensor basis in Eq. (A8) as

$$(M^{-1}\Pi^{H\phi}\mathcal{D})^{\lambda}_{\mu\nu}{}^{\pi\tau} = \sum_{i=1}^{12} P^{H\phi}_{(i)} (T^{H\phi}_i)^{\lambda}_{\mu\nu}{}^{\pi\tau}. \tag{A11}$$

The coefficients $P^{H\phi}_{(i)}$ are obtained by solving the system of 12 algebraic equations, which results from the contractions of Eq. (A11) with $(T^{H\phi}_j)^{\lambda}_{\mu\nu}{}^{\pi\tau}$, $j = 1\ldots12$. A straightforward computer algebra calculation generates relations between $P^{H\phi}_{(i)}$ and $C^{H\phi}_{(i)}$. Then, using the results for $C^{H\phi}_{(i)}$ given in Table I, we obtain the entries of Table V for the mixed $H\phi$ propagator. Table VI shows the UV part of the mixed $H\phi$ propagator, which arises when $d = 4 - 2\epsilon$ and $\epsilon \to 0$, obtained from Table V making $d = 4$.

TABLE V. Coefficients for the mixed $H\phi$ propagator [see Eq. (A11)] in units of $\kappa^2 I$, where I is given by Eq. (A5a).

	1	$(\xi-1)$	$(\xi-1)^2$
$P^{H\phi}_{(1)}$	0	$-\frac{i}{16(d-1)}$	$-\frac{i}{16(d-1)}$
$P^{H\phi}_{(2)}$	$\frac{i(2d^2+d-5)}{16(d-1)}$	$-\frac{i(5(d-6)d+26)}{16(d-1)}$	$\frac{i(d((d-8)d+24)-18)}{16(d-1)}$
$P^{H\phi}_{(3)}$	$-\frac{i(d((d-5)d+19)-6)}{32(d-2)(d-1)}$	$\frac{i(23d^2-147d+122)}{64(d^2-3d+2)}$	$-\frac{i(d(d(10d-73)+193)-134)}{64(d-2)(d-1)}$
$P^{H\phi}_{(4)}$	$-\frac{i}{16(d-1)}$	$-\frac{3i}{32(d-1)}$	$-\frac{i}{32(d-1)}$
$P^{H\phi}_{(5)}$	$-\frac{i(d^2-4)}{8(d-1)}$	$\frac{i(3(d-6)d+16)}{8(d-1)}$	$-\frac{i(d((d-8)d+22)-14)}{8(d-1)}$
$P^{H\phi}_{(6)}$	$\frac{i((d-6)d-3)}{16(d-1)}$	$\frac{1}{32}i(4d-27)$	$-\frac{i(d((d-7)d+21)-17)}{32(d-1)}$
$P^{H\phi}_{(7)}$	$-\frac{i((d-10)d+4)}{16(d-1)}$	$-\frac{i(d(3d-26)+26)}{16(d-1)}$	$\frac{i(d((d-9)d+34)-28)}{32(d-1)}$
$P^{H\phi}_{(8)}$	$-\frac{1}{8}i(d+2)$	$\frac{i(d(4d-23)+24)}{16(d-1)}$	$-\frac{i(3d-10)}{16(d-1)}$
$P^{H\phi}_{(9)}$	$\frac{i(d+2)(2d-3)}{16(d-1)}$	$-\frac{i(d(12d-71)+54)}{32(d-1)}$	$\frac{i(d(4(d-8)d+85)-50)}{32(d-1)}$
$P^{H\phi}_{(10)}$	0	$\frac{i(d-6)d}{16(d-1)}$	$-\frac{i((d-5)(d-4)d+4)}{32(d-1)}$
$P^{H\phi}_{(11)}$	$\frac{i(d((d-5)d+28)-20)}{32(d-2)(d-1)}$	$-\frac{i(7d^2-48d+44)}{16(d^2-3d+2)}$	$\frac{i(d(d(5d-39)+110)-80)}{32(d-2)(d-1)}$
$P^{H\phi}_{(12)}$	$-\frac{i(d-2)}{32(d-1)}$	$\frac{i((d-9)d+2)}{64(d-1)}$	$-\frac{i(d(d(2d-15)+43)-34)}{64(d-1)}$

TABLE VI. UV part of the coefficients for the mixed $H\phi$ propagator [see Eq. (A11)] in units of $\kappa^2 I^{\mathrm{UV}}$, where I^{UV} is given by Eq. (A6).

	1	$(\xi-1)$	$(\xi-1)^2$
$P^{H\phi}_{(1)}$	0	$-\frac{i}{48}$	$-\frac{i}{48}$
$P^{H\phi}_{(2)}$	$\frac{31i}{48}$	$\frac{7i}{24}$	$\frac{7i}{24}$
$P^{H\phi}_{(3)}$	$-\frac{9i}{32}$	$-\frac{49i}{192}$	$-\frac{55i}{192}$
$P^{H\phi}_{(4)}$	$-\frac{i}{48}$	$-\frac{i}{32}$	$-\frac{i}{96}$
$P^{H\phi}_{(5)}$	$-\frac{i}{2}$	$-\frac{i}{3}$	$-\frac{5i}{12}$
$P^{H\phi}_{(6)}$	$-\frac{11i}{48}$	$-\frac{11i}{32}$	$-\frac{19i}{96}$
$P^{H\phi}_{(7)}$	$\frac{5i}{12}$	$\frac{5i}{8}$	$\frac{7i}{24}$
$P^{H\phi}_{(8)}$	$-\frac{3i}{4}$	$-\frac{i}{12}$	$-\frac{i}{24}$
$P^{H\phi}_{(9)}$	$\frac{5i}{8}$	$\frac{19i}{48}$	$\frac{17i}{48}$
$P^{H\phi}_{(10)}$	0	$-\frac{i}{6}$	$-\frac{i}{24}$
$P^{H\phi}_{(11)}$	$\frac{19i}{48}$	$\frac{3i}{8}$	$\frac{7i}{24}$
$P^{H\phi}_{(12)}$	$-\frac{i}{48}$	$-\frac{3i}{32}$	$-\frac{13i}{96}$

b. *H*-field propagator

The HH propagator $(M^{-1}\Pi^{HH}M^{-1})_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho}$ can also be expressed in terms of the tensor basis in Eq. (A10) as

$$(M^{-1}\Pi^{HH}M^{-1})_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho} = \sum_{i=1}^{22} P^{HH}_{(i)} (T^{HH}_i)_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho}. \qquad \text{(A12)}$$

The coefficients $P^{HH}_{(i)}$ are obtained by solving the system of 22 algebraic equations, which results from the contractions of Eq. (A12) with $(T^{HH}_j)_{\mu\nu}^{\lambda}{}_{\pi\tau}^{\rho}$, $j = 1\ldots22$. A straightforward computer algebra calculation generates the relations between $P^{HH}_{(i)}$ and $C^{HH}_{(i)}$. Then, using the results for $C^{HH}_{(i)}$ given in Table III, we obtain the entries of Table VII for the H-field propagator. Table VIII shows the UV part of the H-field propagator, which arises when $d = 4 - 2\epsilon$ and $\epsilon \to 0$, obtained from Table VII making $d = 4$.

TABLE VII. Coefficients for the H-field propagator [see Eq. (A12)] in units of $\kappa^2 k^2 I$, where I is given by Eq. (A5a).

	1	$(\xi-1)$	$(\xi-1)^2$
$P^{HH}_{(1)}$	$\frac{1}{2(d-1)}$	$\frac{d(3d-5)}{16(d-2)(d-1)}$	$\frac{d}{16(d-1)(d+1)}$
$P^{HH}_{(2)}$	$-\frac{2d^2-3d-4}{8(d-2)(d-1)}$	$-\frac{2d^3-7d^2+15d-20}{16(d-2)(d-1)}$	$-\frac{3d+4}{32(d-1)(d+1)}$
$P^{HH}_{(3)}$	$\frac{1}{8(d-2)(d-1)}$	$\frac{d^2-2d+4}{16(d-2)(d-1)}$	$-\frac{3d+4}{64(d-1)(d+1)}$
$P^{HH}_{(4)}$	$\frac{d+1}{4(d-1)}$	$\frac{3d^2-d-8}{16(d-2)(d-1)}$	$\frac{d}{16(d-1)(d+1)}$
$P^{HH}_{(5)}$	$-\frac{1}{2(d-2)}$	$-\frac{d^2-d+4}{8(d-2)(d-1)}$	$\frac{d}{16(d-1)(d+1)}$
$P^{HH}_{(6)}$	0	$\frac{4-d}{2}$	$\frac{(d-4)(4d^3-17d^2-4d+20)}{16(d-1)(d+1)}$
$P^{HH}_{(7)}$	0	$-\frac{(d-4)(d-3)d}{16(d-1)}$	$\frac{(d-4)(d-2)d}{16(d-1)(d+1)}$
$P^{HH}_{(8)}$	0	$\frac{(d-4)(d-3)(d-2)}{8(d-1)}$	$-\frac{(d-4)(d-2)(3d+4)}{16(d-1)(d+1)}$
$P^{HH}_{(9)}$	0	$\frac{(d-4)(d+2)}{8(d-2)(d-1)}$	$-\frac{(d-4)(5d^3-14d^2-20d+8)}{32(d-2)(d-1)(d+1)}$
$P^{HH}_{(10)}$	1	$\frac{3d-4}{4(d-1)}$	$\frac{8d^3-23d^2-10d+24}{16(d-1)(d+1)}$
$P^{HH}_{(11)}$	$\frac{d^2}{4(d-1)}$	$-\frac{3d^2-41d+32}{16(d-1)}$	$\frac{4d^4-28d^3+57d^2+30d-56}{16(d-1)(d+1)}$
$P^{HH}_{(12)}$	$\frac{(d-2)(d+1)}{4(d-1)}$	$\frac{5-d}{2}$	$\frac{4d^4-28d^3+53d^2+34d-48}{16(d-1)(d+1)}$
$P^{HH}_{(13)}$	$\frac{d}{2(d-1)}$	$-\frac{d^2-9d+6}{8(d-1)}$	$\frac{(d-2)(3d+2)}{16(d-1)(d+1)}$
$P^{HH}_{(14)}$	0	$\frac{1-d}{8}$	$-\frac{(d-2)(d+2)}{32(d-1)(d+1)}$
$P^{HH}_{(15)}$	$-\frac{d}{2(d-1)}$	$\frac{d^2-11d+12}{8(d-1)}$	$-\frac{(d-2)(d+2)}{8(d-1)(d+1)}$
$P^{HH}_{(16)}$	$\frac{4d^3-7d^2-8d+8}{8(d-2)(d-1)}$	$-\frac{9d^3-87d^2+202d-120}{16(d-2)(d-1)}$	$\frac{4d^4-28d^3+69d^2+18d-80}{32(d-1)(d+1)}$
$P^{HH}_{(17)}$	$\frac{d}{2(d-2)(d-1)}$	$\frac{d^3-5d^2+8d+4}{8(d-2)(d-1)}$	$-\frac{(d-2)(d+2)}{16(d-1)(d+1)}$
$P^{HH}_{(18)}$	$-d-1$	$\frac{11d^2-81d+72}{8(d-1)}$	$-\frac{4d^4-28d^3+59d^2+30d-64}{8(d-1)(d+1)}$
$P^{HH}_{(19)}$	$\frac{3}{d-2}$	$-\frac{7d^2-44d+36}{4(d-2)(d-1)}$	$\frac{3(4d^4-25d^3+44d^2+28d-48)}{16(d-2)(d-1)(d+1)}$
$P^{HH}_{(20)}$	0	0	$\frac{(d-6)(d-4)(d-2)d}{64(d-1)(d+1)}$
$P^{HH}_{(21)}$	0	$-\frac{(d-4)(d^2-12d+12)}{16(d-1)}$	$-\frac{(d-4)(16d^3-69d^2-10d+72)}{64(d-1)(d+1)}$
$P^{HH}_{(22)}$	$-\frac{d(2d^2+d-24)+20}{8(d-2)^2(d-1)}$	$\frac{d(d(5d-49)+130)-88}{8(d-2)^2(d-1)}$	$\frac{d(d(d((97-8d)d-390)+468)+360)-576}{64(d-2)^2(d^2-1)}$

TABLE VIII. The UV pole part of the coefficients for the H-field propagator (see Eq. (A12) in units of $\kappa^2 k^2 I^{\rm UV}$, where $I^{\rm UV}$ is given by Eq. (A6).

	1	$(\xi-1)$	$(\xi-1)^2$
$P^{HH}_{(1)}$	$\frac{1}{6}$	$\frac{7}{24}$	$\frac{1}{60}$
$P^{HH}_{(2)}$	$-\frac{1}{3}$	$-\frac{7}{12}$	$-\frac{1}{30}$
$P^{HH}_{(3)}$	$\frac{1}{48}$	$\frac{1}{8}$	$-\frac{1}{60}$
$P^{HH}_{(4)}$	$\frac{5}{12}$	$\frac{3}{8}$	$\frac{1}{60}$
$P^{HH}_{(5)}$	$-\frac{1}{4}$	$-\frac{1}{3}$	$\frac{1}{60}$
$P^{HH}_{(6)}$	0	0	0
$P^{HH}_{(7)}$	0	0	0
$P^{HH}_{(8)}$	0	0	0
$P^{HH}_{(9)}$	0	0	0
$P^{HH}_{(10)}$	1	$\frac{2}{3}$	$\frac{8}{15}$
$P^{HH}_{(11)}$	$\frac{4}{3}$	$\frac{7}{4}$	$\frac{13}{15}$
$P^{HH}_{(12)}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{7}{10}$
$P^{HH}_{(13)}$	$\frac{2}{3}$	$\frac{7}{12}$	$\frac{7}{60}$
$P^{HH}_{(14)}$	0	$-\frac{3}{8}$	$-\frac{1}{40}$
$P^{HH}_{(15)}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{10}$
$P^{HH}_{(16)}$	$\frac{5}{2}$	$\frac{4}{3}$	$\frac{41}{60}$
$P^{HH}_{(17)}$	$\frac{1}{3}$	$\frac{5}{12}$	$-\frac{1}{20}$
$P^{HH}_{(18)}$	-5	$-\frac{19}{6}$	$-\frac{29}{15}$
$P^{HH}_{(19)}$	$\frac{3}{2}$	$\frac{7}{6}$	$\frac{6}{5}$
$P^{HH}_{(20)}$	0	0	0
$P^{HH}_{(21)}$	0	0	0
$P^{HH}_{(22)}$	$-\frac{17}{24}$	$-\frac{1}{3}$	$\frac{1}{120}$

5. Explicit verification of the structural identities

The right side of Eq. (4.1), at order κ^2, can be written as

$$\kappa \mathcal{M}^{\lambda\ \ \rho}_{\mu\nu\ \alpha\beta\ \gamma\delta}\langle 0|T\phi^{\gamma\delta}(x)\phi^{\alpha\beta}_{,\rho}(x)\phi^{\pi\tau}(y)|0\rangle, \qquad (A13)$$

where $\mathcal{M}$ is defined in such a way that

$$-\kappa(M^{-1}(\eta)M(\phi)M^{-1}(\eta))^{\lambda\ \ \rho}_{\mu\nu\ \alpha\beta} \equiv \kappa\phi^{\gamma\delta}\mathcal{M}^{\lambda\ \ \rho}_{\mu\nu\ \alpha\beta\gamma\delta} \qquad (A14)$$

with $M(\phi)$ given by (3.8).

In momentum space, Eq. (A13) can be written as

$$-i\kappa\mathcal{M}^{\lambda\ \ \rho}_{\mu\nu\ \alpha\beta\ \gamma\delta}\left[\int\frac{d^dp}{(2\pi)^d}p_\rho\mathcal{D}^{\alpha\beta\sigma_1\theta_1}(p)\mathcal{D}^{\gamma\delta\sigma_2\theta_2}(q)\right.$$
$$\left.\times\mathcal{V}_{\sigma_1\theta_1\sigma_2\theta_2\sigma_3\theta_3}(-p,q,-k)\right]\mathcal{D}^{\sigma_3\theta_3\pi\tau}(k). \qquad (A15)$$

We are using the same notation employed for the self-energies (p is the integration momentum, k is an external momentum, and $q=p+k$); note that $\mathcal{D}^{\mu\nu\rho\sigma}(p)$ is the graviton propagator, and $\mathcal{V}_{\mu\nu\alpha\beta\gamma\delta}(p,q,r)$ is the cubic graviton vertex given, respectively, by Eqs. (3.25a) and (3.25e) of [8].[2] Since $\mathcal{M}$ is just a combination of products of ηs and δs, each of the several terms in Eq. (A15) can be cast in the same form as (A11) in terms of the tensor basis given by Eqs. (A8). After a straightforward calculation, we have obtained a result which coincides with one-loop contribution to the mixed $H\phi$ propagator (the same structure constants shown in Table V), which confirms the identity (4.1) for any dimension and gauge parameter.

Similarly, the second term on the right side of Eq. (5.1), at order κ^2, can be written as

$$\kappa^2\mathcal{M}^{\lambda\ \ \rho_1}_{\mu\nu\ \pi_1\tau_1\ \gamma_1\delta_1}\langle 0|T\phi^{\gamma_1\delta_1}(x)\phi^{\pi_1\tau_1}_{,\rho_1}(x)\phi^{\gamma_2\delta_2}(y)\phi^{\pi_2\tau_2}_{,\rho_2}(y)|0\rangle$$
$$\times\mathcal{M}^{\rho\ \ \rho_2}_{\pi\tau\ \pi_2\tau_2\ \gamma_2\delta_2}. \qquad (A16)$$

In momentum space, Eq. (A16), can be written as

$$-\kappa^2\mathcal{M}^{\lambda\ \ \rho_1}_{\mu\nu\ \pi_1\tau_1\ \gamma_1\delta_1}\left\{\int\frac{d^dp}{(2\pi)^d}[\mathcal{D}^{\pi_1\tau_1\gamma_2\delta_2}(p)\mathcal{D}^{\gamma_1\delta_1\pi_2\tau_2}(q)p_{\rho_1}\right.$$
$$\left.-q_{\rho_1}\mathcal{D}^{\gamma_1\delta_1\gamma_2\delta_2}(p)\mathcal{D}^{\pi_1\tau_1\pi_2\tau_2}(q)]q_{\rho_2}\right\}\mathcal{M}^{\rho\ \ \rho_2}_{\pi\tau\ \pi_2\tau_2\ \gamma_2\delta_2}. \qquad (A17)$$

Equation (A17) can also be cast in the same form as (A12) in terms of the tensor basis given by Eqs. (A10). After a straightforward calculation, we have obtained a result which coincides with one-loop contribution to the H-field propagator (the same structure constants shown in Table VII), which confirms the identity (5.1) for any dimension and gauge parameter.

We point out that these structural identities relate elements of the basic Feynman rules, in each formalism, in a nontrivial way. There is also a practical implication since these identities allow one to compute some rather involved composite field expectation values in a much more efficient way by using the auxiliary field instead.

[2]Both the three graviton interaction vertex and the propagator are the same as in the second-order formalism from the expansion of Eq. (4.3).

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