Charging the O(N) model

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We determine, for the first time, the scaling dimensions of a family of fixed-charge operators stemming from the critical $O(N)$ model in $4-\epsilon$ dimensions to the leading and next to leading order terms in the charge expansion but to all orders in the coupling. We test our results to the maximum known order in perturbation theory while determining higher order terms.

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I. INTRODUCTION

The discovery of the Higgs heralds a new era in our understanding of fundamental interactions. It crowns the Standard Model of particle physics as one of the most successful theories of nature while simultaneously opening new avenues tailored at gaining a deeper understanding of the ultimate laws of nature.

One of the most striking features of the Standard Model is its near scale invariant nature. In fact, at the classical level, the only two operators that explicitly violate scale invariance are the Higgs mass and the cosmological constant. It is therefore natural to start investigating the dynamics of theories of fundamental interactions around their scale invariant limit. Scale invariance is highly intertwined with conformal symmetry which leads to powerful constraints on the dynamics of the theory at hand. It is therefore useful to organize quantum field theories around their conformal limit. Therefore, in our analysis we shall use it as a tool to access important information about the theory.

In Ref. [1] the authors employed a semiclassical approach to determine the scaling dimensions of the fixed-charge ϕ^n operator, Δ_{ϕ^n}, in the $U(1)$ scalar ϕ^4 model at the Wilson-Fisher (WF) fixed point. The Standard Model Higgs is, however, described by a non-Abelian $O(4)$ model, up to gauge and Yukawa interactions. This calls for generalizing the approach to non-Abelian theories which, as we shall see, is quite involved.

We will, therefore, consider $O(N)$ theories and determine the scaling dimensions of a family of fixed-charge operators to the leading and next to leading order terms in the charge expansion but to all orders in the coupling. Our work builds upon the pioneering idea of using the large-charge limit [2,3] to gain relevant information about conformal dynamics [4]. We test our results to the maximum known order in perturbation theory while determining higher order terms. We plan to generalize our results to generic gauge-Yukawa theories that are the backbones of any known theory of fundamental interactions.

II. $O(N)$ AT FIXED CHARGE

As mentioned in the Introduction, in Ref. [1] (see also [5]) the $U(1)$ model was investigated using a semiclassical method in order to compute the scaling dimensions of the fixed-charge ϕ^n composite operator, Δ_{ϕ^n}, in the $\frac{\lambda}{4}\phi^4$ model, with λ the self-coupling. This was performed by analyzing the conformal theory at the WF fixed point obtained by going away from four dimensions via $d=4-\epsilon$ with ϵ positive and tiny. Using the operator-state correspondence [6,7] one can use the conformal map of the theory at the WF fixed point on a cylindrical gravitational background to determine Δ_{ϕ^n} via the expectation value of the evolution operator e^{-HT} on an arbitrary state $|\psi_n\rangle$ carrying charge n with H the Hamiltonian of the system and T the time coordinate.

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To make this explicit, consider the path integral formula for $\langle\psi_n|e^{-HT}|\psi_n\rangle$,

$$\langle\psi_n|e^{-HT}|\psi_n\rangle = \frac{1}{\mathcal{Z}}\int D\chi_i D\chi_f \psi_n(\chi_i)\psi_n^*(\chi_f) \times \int_{\rho=f,\chi=\chi_i}^{\rho=f,\chi=\chi_f} D\rho D\chi e^{-S} \tag{1}$$

where $\mathcal{Z}$ normalizes the vacuum-to-vacuum transition amplitude while the wave functional

$$\psi_n(\chi) = \exp\left(\frac{in}{R^{d-1}\Omega_{d-1}}\int d\Omega_{d-1}\chi\right) \tag{2}$$

fixes the charge of the initial and final states to n. Here ρ and χ are the modulus and phase of the complex scalar field, while f is a constant value. For small values of the quartic coupling, λ, this path integral can be computed via the saddle-point method. The remarkable upshot of [1] is that, similarly to the large-N 't Hooft expansion in gauge theories, the result can be organized as a double expansion in λ and λn with λn fixed. In other words, the anomalous dimension of ϕ^n can be written as

$$\Delta_{\phi^n} = \sum_{\kappa=-1}^{\infty}\lambda^\kappa\Delta_\kappa(\lambda n), \tag{3}$$

where each of the functions $\Delta_\kappa(\lambda n)$ is computed semiclassically to all orders in the fixed 't Hooft λn coupling. A similar analysis, entirely in four dimensions because of the existence of an ultraviolet interacting perturbative fixed point, appeared earlier in [8] for non-Abelian gauge-Yukawa theories but concentrating on the large-charge limit.

A. $O(N)$ model setup and ground state

Here, inspired by the fact that the Standard Model Higgs is described by an $O(4)$ non-Abelian model, we move to analyze the non-Abelian massless $O(N)$ vector model described below:

$$\mathcal{S} = \int d^dx\left(\frac{(\partial\phi_i)^2}{2} + \frac{(4\pi)^2 g_0}{4!}(\phi_i\phi_i)^2\right). \tag{4}$$

In $d = 4-\epsilon$ this theory features an infrared WF fixed point for the renormalized coupling g, and its value at the 3-loop level in the MS scheme reads [9]

$$g^*(\epsilon) = \frac{3\epsilon}{8+N} + \frac{9(3N+14)\epsilon^2}{(8+N)^3} + \frac{\epsilon^3}{(8+N)^5}\left[\frac{3}{8}(4544+1760N+110N^2-33N^3) - 36\zeta(3)(N+8)(5N+22)\right] + \mathcal{O}(\epsilon^4). \tag{5}$$

In the $O(N)$ vector model with even (odd) N, we can fix up to $n = \frac{N}{2}$ $(\frac{N-1}{2})$ charges, which is the rank of the $O(N)$ group. We fix $k \le n$ of these charges and write the path integral expression to determine the ground state energy of this charge configuration on a cylinder. From now on, we focus on the even-N case, since the odd case is similar, and rewrite the action in terms of n complex field variables

$$\varphi_1 = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) = \frac{1}{\sqrt{2}}\sigma_1 e^{i\chi_1}, \tag{6}$$

$$\varphi_2 = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) = \frac{1}{\sqrt{2}}\sigma_2 e^{i\chi_2}, \tag{7}$$

$$\varphi_3 = \cdots. \tag{8}$$

At the WF fixed point g^* we map the action onto the cylinder, $\mathbb{R}^d \to \mathbb{R}\times S^{d-1}$, which now reads

$$\mathcal{S}_{\rm cyl} = \int d^dx\sqrt{g} \times\left(g_{\mu\nu}\partial^\mu\bar{\varphi}_i\partial^\nu\varphi_i + m^2\bar{\varphi}_i\varphi_i + \frac{(4\pi)^2 g_0}{6}(\bar{\varphi}_i\varphi_i)^2\right). \tag{9}$$

A mass term appears, $m^2 = (\frac{d-2}{2R})^2$, stemming from R, the radius of the sphere. Notice that the procedure above is merely employed to ease the computation.

The charges are fixed using k constraints $Q_i = \bar{Q}_i$, where $\{\bar{Q}_i\}$ is a set of fixed constants. Clearly, φ_i $(\bar{\varphi}_i)$ has charge $\bar{Q}_i = 1(-1)$. The solution of the EOM with minimal energy is spatially homogeneous, and it is given by

$$\begin{cases} \sigma_i = A_i, \chi_i = -i\mu t & i = 1,\ldots,k, \\ \varphi_{k+j} = 0 & j = 1,\ldots,n-k. \end{cases} \tag{10}$$

As pointed out in [10], a striking consequence of such a homogeneous solution is the presence of a single chemical potential μ, even if the charges $\bar{Q}_i$ are all different. The parameters A_i and μ are fixed by the EOM and by the expression for the Noether charge as

$$\mu^2 - m^2 = \frac{(4\pi)^2}{6}g_0 v^2 \qquad \frac{\bar{Q}}{\rm vol.} = \mu v^2 \qquad {\rm vol.} = 2\pi^2 R^3, \tag{11}$$

where we have defined

$$v^2 \equiv \sum_{i=1}^{k} A_i^2 \qquad \bar{Q} \equiv \sum_{i=1}^{k} \bar{Q}_i \tag{12}$$

with $\bar{Q}$ the sum of the charges.

The presence of a single chemical potential leaves the $O(2n-2k)\times U(k)$ symmetry of the original $O(2n)$ symmetry unbroken [10]. Then, the vacuum of the theory spontaneously breaks $U(k)$ to $U(k-1)$. In fact it is possible to rotate the ground state to

$$\frac{1}{\sqrt{2}}(A_1,\ldots,A_k,0,\ldots,0) \to (\underbrace{0,\ldots,0}_{k-1},\frac{v}{\sqrt{2}},\underbrace{0,\ldots,0}_{n-k}). \tag{13}$$

Stemming from the considerations above, the saddle-point computation is organized as single coupling 't Hooft expansion in $g^*\bar{Q}$ similar to the Abelian case. The sum of the charges act as a single charge, which is a welcome simplification leading to

$$\langle\psi_{\bar{Q}}|e^{-HT}|\psi_{\bar{Q}}\rangle = \frac{1}{\mathcal{Z}}\int_{\sigma_{N/2}=v}^{\sigma_{N/2}=v} D^n\sigma D^n\chi e^{-\mathcal{S}_{\rm eff}} \tag{14}$$

where

$$\mathcal{S}_{\rm eff} = \int_{-T/2}^{T/2} dt \int d\Omega_{d-1}\left(\frac{1}{2}\partial\sigma_i\partial\sigma_i + \frac{1}{2}\sigma_i^2(\partial\chi_i\partial\chi_i) + \frac{m^2}{2}\sigma_i^2 + \frac{(4\pi)^2}{24}g_0(\sigma_i\sigma_i)^2 + \frac{i}{\rm vol.}\bar{Q}\dot{\chi}_{N/2}\right). \tag{15}$$

The sums over i go from 1 to $n=N/2$; i.e., we have fixed the maximum number of charges to $k=n$. In conclusion, the scaling dimension at the fixed point of the smallest dimension operator carrying a total charge $\bar{Q}$ assumes the form

$$\Delta_{O_{\bar{Q}}} = E_{O_{\bar{Q}}}R = \sum_{j=-1}^{\infty} g^{*j}\Delta_j(g^*\bar{Q}). \tag{16}$$

Here $E_{O_{\bar{Q}}}$ is the ground state energy and R the radius of the sphere.

B. Fixed-charge operators

In CFT with internal global symmetries, operators organize themselves into multiplets transforming according to irreducible representations of the symmetry group. Within any one such multiplet, component operators are further distinguished by their charge configurations, namely, the value of their charges associated with the Cartan generators. Component operators of different charge configurations do not mix under renormalization. Nevertheless, by virtue of the Wigner-Eckart theorem, they necessarily have the same scaling dimension.

We would like to compute scaling dimensions of the lowest-lying operators corresponding to some fixed-charge configuration via a semiclassical expansion and compare the results to ordinary perturbation theory whenever possible. In a massless theory, operators with different engineering dimensions do not mix. We therefore consider the minimal classical scaling dimension (MCSD) for a given charge configuration. Let us start by assuming the generic charge configuration to be $[m]=(m_1,m_2,\ldots,m_{N/2})$, with the m_i's representing the charge associated with the ith Cartan generator. Without loss of generality, we suppose all m_i's are positive. Then the fixed-charge operator with the MCSD is $\mathcal{O}_{[m]}\equiv\prod_{i=1}^{i=N/2}(\varphi_i)^{m_i}$ with φ complex. If some m_i are negative, they would correspond to replacing φ_i with $\bar{\varphi}_i$. Here, $\mathcal{O}_{[m]}$ must be a tensor operator living in the traceless fully symmetric subspace of $O(N)$ transformations, which corresponds to an irreducible representation and therefore has a definite scaling dimension. Note that $\mathcal{O}_{[m]}$ is fully symmetric simply because it is a product of a commuting scalar field. Furthermore, it is traceless and it turns out to be the MCSD operator with this charge configuration.

We, therefore, arrive at the conclusion that operators that have the same total charge and MCSD all belong to the same irreducible representation of $O(N)$ and thus have the same scaling dimension.

We therefore identify such an operator to be the $\bar{Q}$-index traceless symmetric tensor $T_{\bar{Q}}\equiv T^{(\bar{Q})}_{i_1\ldots i_{\bar{Q}}}$. The latter can be represented as $\bar{Q}$-box Young tableaux with one row.

The scaling dimension of $T_{\bar{Q}}$ at the fixed point has been computed to $\mathcal{O}(\epsilon^3)$ in [11,12],

$$\begin{aligned}\Delta_{T_{\bar{Q}}} = {} & \underbrace{\bar{Q}+\left(-\frac{\bar{Q}}{2}+\frac{\bar{Q}(\bar{Q}-1)}{8+N}\right)}\epsilon - \left[\frac{184+N(14-3N)}{4(8+N)^3}\bar{Q} + \underbrace{\frac{(N-22)(N+6)}{2(8+N)^3}\bar{Q}^2} + \underbrace{\frac{2}{(8+N)^2}\bar{Q}^3}\right]\epsilon^2 + \left[\underbrace{\frac{8}{(8+N)^3}\bar{Q}^4}\right. \\ & + \underbrace{\frac{-456-64N+N^2+2(8+N)(14+N)\zeta(3)}{(8+N)^4}\bar{Q}^3} \\ & - \frac{-31136-8272N-276N^2+56N^3+N^4+24(N+6)(N+8)(N+26)\zeta(3)}{4(N+8)^5}\bar{Q}^2 \\ & \left. + \frac{-65664-8064N+4912N^2+1116N^3+48N^4-N^5+64(N+8)(178+N(37+N))\zeta(3)}{16(N+8)^5}\bar{Q}\right]\epsilon^3 + \mathcal{O}(\epsilon^4).\end{aligned} \tag{17}$$

Terms highlighted with underbrace will be used to test our results stemming from the semiclassical computation below.

III. SEMICLASSICAL APPROACH TO THE $O(N)$ MODEL

We now have all the instruments to proceed with the computation of $\Delta_{T_{\bar{Q}}}$ semiclassically.

A. Classical contribution

Here we focus on the leading term Δ_{-1}, which is given by the effective action (15) evaluated on the classical trajectory (10) at the fixed point

$$\frac{1}{g^*}\frac{\Delta_{-1}(g^*\bar{Q})}{R} = \frac{\bar{Q}}{4}\left(3\mu + \frac{m^2}{\mu}\right). \tag{18}$$

By inserting the second equation in (11) into the first one and setting $d = 4$, we obtain

$$R^3\mu^3 - R\mu = \frac{4}{3}\bar{Q}g^* \tag{19}$$

with the solution

$$R\mu = \frac{3^{\frac{1}{3}} + (6g^*\bar{Q} + \sqrt{-3+36(g^*\bar{Q})^2})^{\frac{2}{3}}}{3^{\frac{2}{3}}(6g^*\bar{Q} + \sqrt{-3+36(g^*\bar{Q})^2})^{\frac{1}{3}}}. \tag{20}$$

Thus, the leading contribution is

$$\frac{4\Delta_{-1}}{g^*\bar{Q}} = \frac{3^{\frac{2}{3}}(x+\sqrt{-3+x^2})^{\frac{1}{3}}}{3^{\frac{1}{3}} + (x+\sqrt{-3+x^2})^{\frac{2}{3}}} + \frac{3^{\frac{1}{3}}(3^{\frac{1}{3}} + (x+\sqrt{-3+x^2})^{\frac{2}{3}})}{(x+\sqrt{-3+x^2})^{\frac{1}{3}}} \tag{21}$$

where $x \equiv 6g^*\bar{Q}$. The expansion for small $g^*\bar{Q}$ reads

$$\frac{\Delta_{-1}}{g^*} = \bar{Q}\left[1 + \frac{1}{3}g^*\bar{Q} - \frac{2}{9}(g^*\bar{Q})^2 + \frac{8}{27}(g^*\bar{Q})^3 + \mathcal{O}((g^*\bar{Q})^4)\right]. \tag{22}$$

The leading term Δ_{-1} matches exactly the $U(1)$ result [1]. This is a direct consequence of having a single chemical potential, and it is consistent with the fact that the leading power of the charge at a given loop order in the perturbative expression for $\Delta_{T_{\bar{Q}}}$ does not depend on N. This can be easily seen by rewriting Eq. (17) as a coupling expansion by means of Eq. (5). Our result suggests that this behavior continues at higher loop orders.

B. Quantum corrections

The time is ripe to determine the leading quantum corrections Δ_0 to be added to the classical result (21). To this end, we expand around the saddle-point configuration (10) considering the ground state in (13). We parametrize the fluctuations as

$$\begin{cases} \chi_i = -i\mu t + \frac{1}{v}p_i(x) & i = 1,\ldots,\frac{N}{2}-1 \\ \chi_{N/2} = -i\mu t + \frac{1}{v}\pi(x) & \\ \sigma_i = s_i(x) & i = 1,\ldots,\frac{N}{2}-1 \\ \sigma_{N/2} = v + r(x). & \end{cases} \tag{23}$$

Expanding the Lagrangian (15) to the quadratic order in the fluctuations, we arrive at

$$\mathcal{L}_2 = \frac{1}{2}(\partial\pi)^2 + \frac{1}{2}(\partial r)^2 + (\mu^2 - m^2)r^2 - 2i\mu r\dot{\pi} + \frac{1}{2}\partial s_i\partial s_i + \frac{1}{2}\partial p_i\partial p_i - 2i\mu s_i\dot{p}_i. \tag{24}$$

The spectrum for the non-Abelian case contains states that are also seen in the Abelian case, corresponding to one relativistic (type I) Goldstone boson (the conformal mode) $\chi_{N/2}$ and one massive state $\sigma_{N/2}$ with mass $\sqrt{6\mu^2 - 2m^2}$. Their dispersion relations read

$$\omega_\pm(l) = \sqrt{J_\ell^2 + 3\mu^2 - m^2 \pm \sqrt{4J_\ell^2\mu^2 + (3\mu^2 - m^2)^2}}, \tag{25}$$

with the negative sign applying to the Goldstone boson. Additionally, the non-Abelian case also features $n-1 = \frac{N}{2}-1$ nonrelativistic (type II) Goldstone bosons χ_i and $n-1$ massive states σ_i with mass 2μ,

$$\omega_{\pm\pm}(l) = \sqrt{J_\ell^2 + \mu^2} \pm \mu \tag{26}$$

with $J_\ell^2 = \ell(\ell + d - 2)/R^2$ the eigenvalues of the Laplacian on the sphere.

The counting of Goldstone modes can be understood by recalling that the symmetry breaking pattern is $U(\frac{N}{2}) \to U(\frac{N}{2}-1)$. Naively, one would have expected $\dim(U(\frac{N}{2})/U(\frac{N}{2}-1)) = N-1$ relativistic Goldstone modes. However, the explicit Lorentz symmetry breaking due to the fixed charge modifies some of the type I Goldstone bosons into fewer type II Goldstones. Each type II counts as two type I Goldstones in the counting of the degrees of freedom with respect to the number of broken generators [13]. Thus we have

$$1 + 2\times\left(\frac{N}{2}-1\right) = N - 1 = \dim\left(U\left(\frac{N}{2}\right)/U\left(\frac{N}{2}-1\right)\right). \tag{27}$$

Here, Δ_0 is determined by the fluctuation functional determinant, and it is given by

$$\Delta_0 = \frac{R}{2}\sum_{\ell=0}^{\infty} n_\ell\left[\omega_+(\ell)+\omega_-(\ell) + \left(\frac{N}{2}-1\right)(\omega_{++}(\ell)+\omega_{--}(\ell))\right] \tag{28}$$

where $n_\ell = (1+\ell)^2$ is the Laplacian multiplicity on the 3-sphere.

The difference with respect to the Abelian case is that now we have to include the contributions of all the $2\times(\frac{N}{2}-1)$ new modes. As a result, the rank of the $O(N)$ group n enters in the computation and leads to a nontrivial dependence of the leading quantum corrections on the number of scalars N.

Following the procedure of [1], we obtain

$$\Delta_0(g^*\bar{Q}) = -\frac{15\mu^4R^4+6\mu^2R^2-5}{16} + \frac{1}{2}\sum_{\ell=1}^{\infty}\sigma(\ell) + \frac{\sqrt{3\mu^2R^2-1}}{\sqrt{2}} - \frac{1}{16}\left(\frac{N}{2}-1\right)[7+R\mu(-16+6R\mu+3R^3\mu^3)]. \tag{29}$$

The last term and sum $\sigma(\ell)$ distinguish the non-Abelian case from the Abelian one with

$$\sigma(\ell) = R(1+\ell)^2\left[\omega_+(\ell)+\omega_-(\ell)+\left(\frac{N}{2}-1\right)(\omega_{++}(\ell)+\omega_{--}(\ell))\right] + \frac{1}{8}(N+8)(R^2\mu^2-1)^2\frac{1}{\ell} + \frac{1}{2}(2-N-(N+2)R^2\mu^2) + \frac{1}{2}(2-5N-(2+N)R^2\mu^2)\ell - 3N\ell^2 - N\ell^3, \tag{30}$$

where all the quantities are evaluated in $d=4$ dimensions. The above constitutes our main achievement.

As a nontrivial test of our result, we now compare it with the perturbative results [the red contributions in (17)] to the maximum known order in perturbation theory. To do this we expand the result for small $g\bar{Q}$, where the sum above can be computed analytically,

$$\Delta_0(g^*\bar{Q}) = -\left(\frac{5}{3}+\frac{N}{6}\right)g^*\bar{Q} + \left(\frac{1}{3}-\frac{N}{18}\right)(g^*\bar{Q})^2 + \frac{1}{27}[N-36+28\zeta(3)+2N\zeta(3)](g^*\bar{Q})^3 + \mathcal{O}((g^*\bar{Q})^4). \tag{31}$$

The sum of the classical contribution (22) and the leading quantum correction (31) is

$$\frac{\Delta_{-1}(g^*\bar{Q})}{g^*} + \Delta_0(g^*\bar{Q}) = \bar{Q} - \frac{g^*\bar{Q}}{6}(10+N-2\bar{Q}) + \frac{g^{*2}\bar{Q}^2}{18}(6-N-4\bar{Q}) + \frac{g^{*3}\bar{Q}^3}{27}[N-36+8\bar{Q}+2(14+N)\zeta(3)] + \mathcal{O}(g^{*4}\bar{Q}^5). \tag{32}$$

To better visualize our results, now rewrite the above at the WF fixed point (5) so that we can directly compare with the leading and next leading order terms in the charge expansion of (17) (underbraced terms) as follows:

$$\Delta_{T_{\bar{Q}}}^{\mathrm{LO+NLO}} = \bar{Q} + \left(-\frac{\bar{Q}}{2}+\frac{\bar{Q}(\bar{Q}-1)}{8+N}\right)\epsilon - \left[\frac{(N-22)(N+6)}{2(8+N)^3}\bar{Q}^2 + \frac{2}{(8+N)^2}\bar{Q}^3\right]\epsilon^2 + \left[\frac{8}{(8+N)^3}\bar{Q}^4 + \frac{-456-64N+N^2+2(8+N)(14+N)\zeta(3)}{(8+N)^4}\bar{Q}^3\right]\epsilon^3. \tag{33}$$

The above remarkably shows the power of the approach given that we can now predict the classical and quantum correction for the higher perturbative loops of the anomalous dimension. To help future checks we give the explicit result up to order g^{*6}.

$$\text{4-loops:}\quad \left(-\frac{14}{27}\bar{Q} + \frac{1}{81}[4(73-N)-2\zeta(3)(65+6N)-5\zeta(5)(30+N)]\right)(g^*\bar{Q})^4, \tag{34}$$

$$\text{5-loops:}\quad \left(\frac{256}{243}\bar{Q} + \frac{1}{243}[3(-800+7N)+28\zeta(3)(28+3N)+40\zeta(5)(22+N)+14\zeta(7)(62+N)]\right)(g^*\bar{Q})^5, \tag{35}$$

$$\text{6-loops}: \left(-\frac{572}{243}\bar{Q}+\frac{2}{279}[10191-64N-2\zeta(3)(1327+160N)-2\zeta(5)(1441+80N) -70\zeta(7)(46+N)-21\zeta(9)(126+N)]\right)(g^*\bar{Q})^6. \tag{36}$$

Since the result is valid for any N, one can now directly apply it to the Higgs sector of the Standard Model for which $N=4$ and up to Yukawa and gauge interactions.

In this paper we focused on the limit $g^*\bar{Q}$ that is fixed and small. This allowed us to determine the all-order perturbative contributions to the quantum scaling dimension related to the fixed charge. Another interesting and complementary limit, already much explored in the literature [10], is the one in which $g^*\bar{Q}$ is large, which can be straightforwardly obtained from (21) and (29).

In conclusion, we used the semiclassical approach to determine, for the first time, the scaling dimensions for the critical $O(N)$ model in $4-\epsilon$ dimensions. We determined the scaling dimension to the leading order and next to leading order terms in the charge expansion but to all orders in the coupling. This work constitutes the stepping stone towards generalizing the approach to gauge-Yukawa theories.

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Note added.—Recently, a related study in $d=6-\epsilon$ was investigated in [14].

[1] G. Badel, G. Cuomo, A. Monin, and R. Rattazzi, J. High Energy Phys. 11 (2019) 110.
[2] S. Hellerman, D. Orlando, S. Reffert, and M. Watanabe, J. High Energy Phys. 12 (2015) 071.
[3] L. Alvarez-Gaume, D. Orlando, and S. Reffert, J. High Energy Phys. 12 (2019) 142.
[4] D. Orlando, S. Reffert, and F. Sannino, Phys. Rev. D **101**, 065018 (2020).
[5] G. Arias-Tamargo, D. Rodriguez-Gomez, and J. Russo, J. High Energy Phys. 10 (2019) 201.
[6] J. L. Cardy, J. Phys. A **17**, L385 (1984).
[7] J. L. Cardy, J. Phys. A **18**, L757 (1985).
[8] D. Orlando, S. Reffert, and F. Sannino, J. High Energy Phys. 08 (2019) 164.
[9] H. Kleinert and V. Schulte-Frohlinde, *Critical Properties of ϕ^4-Theories* (World Scientific, Singapore, 2001).
[10] L. Alvarez-Gaume, O. Loukas, D. Orlando, and S. Reffert, J. High Energy Phys. 04 (2017) 059.
[11] D. J. Wallace and R. K. P. Zia, J. Phys. C **8**, 839 (1975).
[12] S. K. Kehrein, Nucl. Phys. **B453**, 777 (1995).
[13] H. B. Nielsen and S. Chadha, Nucl. Phys. **B105**, 445 (1976).
[14] G. Arias-Tamargo, D. Rodriguez-Gomez, and J. G. Russo, arXiv:2003.13772.