Restricted Weyl symmetry

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We discuss the physics of a restricted Weyl symmetry in a curved space-time where a gauge parameter $\Omega(x)$ of Weyl transformation satisfies a constraint $\Box\Omega = 0$. First, we present a model of QED where we have a restricted gauge symmetry in the sense that a $U(1)$ gauge parameter $\theta(x)$ obeys a similar constraint $\Box\theta = 0$ in a flat Minkowski space-time. Next, it is precisely shown that a global scale symmetry must be spontaneously broken at the quantum level. Finally, we discuss the origin of the restricted Weyl symmetry and show that its symmetry can be derived from a full Weyl symmetry by taking a gauge condition $R = 0$ in the BRST formalism.

I. INTRODUCTION

Both local and global scale symmetries are very mysterious symmetries in that they are ubiquitous in nature from particle physics to cosmology, but in the real world they usually emerge as approximate symmetries which are broken either explicitly by anomalies or badly by the presence of a built-in scale in theories.[1] If the scale symmetries are exact ones which are only spontaneously broken as in the gauge symmetries in the standard model (SM) of particle physics, they might shed some light on various important unsolved problems such as cosmological constant problem and the gauge hierarchy problem etc.

In this short article, we explore an idea that there might be an intermediate scale symmetry between local and global scale symmetries, which is dubbed a restricted Weyl symmetry in a curved space-time [4–6]. In the restricted Weyl symmetry, a gauge parameter, which is nothing but a conformal factor $\Omega(x)$, is constrained by a condition $\Box\Omega = 0$ whereas in a conventional or full Weyl transformation the conformal factor is an unconstrained and free parameter. In the restricted Weyl symmetry, we are allowed to work with a generic dimensionless action as in a scale symmetry, which should be contrasted to the situation in the full Weyl symmetry where only the conformal tensor

squared is an invariant action if we neglect the other fields except for the metric $g_{\mu\nu}$.

The structure of this article is the following: In Sec. II, we review a restricted gauge symmetry in QED which appears after we fix the gauge invariance by the Lorenz condition, i.e., the Lorenz gauge. The restricted gauge symmetry resembles the restricted Weyl symmetry in the sense that both the symmetries are constrained by an equation $\Box\Theta = 0$ where $\Box$ denotes the d'Alembertian operator and Θ stands for a generic gauge parameter. Of course, an obvious difference between the restricted gauge symmetry and the restricted Weyl one lies in the fact that the former is defined in a flat Minkowski space-time while the latter is so in a curved Riemannian space-time. Their similarity, however, gives us some hints about the spontaneous symmetry breakdown and the origin of the restricted Weyl symmetry, which are dealt with in Secs. III and IV, respectively. The final section is devoted to conclusion.

II. RESTRICTED GAUGE SYMMETRY

Let us start with QED which is gauge-fixed by the Lorenz gauge:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + eA_\mu\bar{\psi}\gamma^\mu\psi$$
$$+ B\partial_\mu A^\mu + \frac{\alpha}{2}B^2, \tag{1}$$

where A_μ, ψ and B are respectively the electromagnetic field, spinor field, and Nakanishi-Lautrup field, the field strength is defined as $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, and α is a real number which can be chosen for our convenience. This Lagrangian density has a restricted $U(1)$ gauge invariance given by

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[1]We sometimes call a local and global scale symmetry a Weyl symmetry and scale symmetry, respectively [1–3].

$$\delta A_\mu = \partial_\mu \theta, \qquad \delta \psi = ie\theta \psi,$$
$$\delta \bar{\psi} = -ie\theta \bar{\psi}, \qquad \delta B = 0, \qquad (2)$$

where $\Box \theta = 0$ where $\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu$ is the d'Alembertian operator in a flat Minkowski space-time.

Then, let us ask ourselves what solution to the constraint $\Box \theta(x) = 0$ is. Since $\Box \theta = 0$ is nothing but the Klein-Gordon equation for a massless real scalar field, a general solution is given by

$$\theta(x) = \int \frac{d^3 k}{\sqrt{(2\pi)^3 2k_0}} [a(k)e^{-ikx} + a^\dagger(k)e^{ikx}], \quad (3)$$

where $k_0 = |\vec{k}|$. In the absence of the spinor field $\psi = 0$ and with the $\alpha = 1$ gauge, the gauge field also obeys the same field equation $\Box A_\mu = 0$ as in $\Box \theta = 0$, so we can then gauge away one component in A_μ by means of the residual symmetry (2). Incidentally, if we are interested in only respecting the restricted gauge invariance, and we do not inquire its origin and ignore an issue of nonrenormalizability, we could add any invariant terms such as $(\partial_\mu A^\mu)^n$ with integers $n \geq 2$ to the Lagrangian density (1).

Now we are interested in only zero-mode solutions. The constraint $\Box \theta = 0$ is then easily solved to be [7]

$$\theta = a_\mu x^\mu + b, \qquad (4)$$

where a_μ, b are infinitesimal constants. With this solution, the restricted gauge transformation can be rewritten as

$$\delta A_\mu = a_\mu, \qquad \delta \psi = ie(a_\mu x^\mu + b)\psi,$$
$$\delta \bar{\psi} = -ie(a_\mu x^\mu + b)\bar{\psi}, \qquad \delta B = 0. \qquad (5)$$

Since the transformation (5) is a global symmetry, following the Noether theorem [8] we can derive the Noether currents $j_\mu{}^\rho, j^\rho$ corresponding to the parameters a_μ, b, respectively:

$$j_\mu{}^\rho = F_\mu{}^\rho + \delta_\mu^\rho B - x_\mu e\bar{\psi}\gamma^\rho \psi, \qquad j^\rho = -e\bar{\psi}\gamma^\rho \psi. \quad (6)$$

The Noether charges Q_μ, Q are respectively defined as

$$Q_\mu = \int d^3 x \, j_\mu{}^0, \qquad Q = \int d^3 x \, j^0. \qquad (7)$$

Next, let us show that a vectorial charge Q_μ is necessarily broken spontaneously [7]. Actually, we find that

$$\delta A_\nu = [i(a^\mu Q_\mu + bQ), A_\nu] = \partial_\nu \theta = a_\nu, \qquad (8)$$

which implies that

$$[iQ_\mu, A_\nu] = \eta_{\mu\nu}, \qquad [iQ, A_\nu] = 0. \qquad (9)$$

Taking the vacuum expectation value of the former relation leads to

$$\langle 0|[iQ_\mu, A_\nu]|0\rangle = \eta_{\mu\nu}. \qquad (10)$$

Equation (10) means that a global symmetry generated by Q_μ is spontaneously broken. As a result, A_μ includes a massless Nambu-Goldstone (NG) boson, but it may be either a scalar or a vector particle. It has been already proved that each possibility exactly corresponds to either a symmetry generated by Q is spontaneously broken or the symmetry remains unbroken [7]. In particular, in the latter case we can regard the photon as a NG boson coming from the SSB of Q_μ [9].

III. RESTRICTED WEYL SYMMETRY

In this section, we consider a gravitational theory coupled to a $U(1)$ gauge theory with a complex scalar field where there is a restricted Weyl invariance in the sense that a gauge parameter of Weyl transformation, $\Omega(x)$, satisfies a constraint, $\Box \Omega(x) = 0$[2]:

$$\mathcal{L} = \sqrt{-g}\left(\alpha R^2 - \xi R|\Phi|^2 - |D_\mu \Phi|^2 - \lambda |\Phi|^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right), \qquad (11)$$

where the covariant derivative is defined as $D_\mu \Phi \equiv (\partial_\mu - ieA_\mu)\Phi$. Without loss of generality, in this article we drop the gauge field A_μ and we work with the following Lagrangian density [5]:

$$\mathcal{L} = \sqrt{-g}(\alpha R^2 - \xi R|\Phi|^2 - |\partial_\mu \Phi|^2 - \lambda |\Phi|^4). \quad (12)$$

Indeed, this Lagrangian density is invariant under the restricted Weyl transformation

$$g_{\mu\nu} \to g'_{\mu\nu} = \Omega^2(x)g_{\mu\nu}, \qquad \Phi \to \Phi' = \Omega^{-1}(x)\Phi, \qquad (13)$$

where the gauge parameter obeys a constraint $\Box \Omega = 0$. In order to prove the invariance, we need to use the following transformation of the scalar curvature under (13):

$$R \to R' = \Omega^{-2}(R - 6\Omega^{-1}\Box\Omega). \qquad (14)$$

We are at present interested in zero-mode solution to the equation $\Box \Omega = 0$. It is obvious that $\Omega(x) = $ const. is a zero mode solution corresponding to a global scale invariance. Then, let us pay our attention to the scale invariance. In an infinitesimal form $\Omega = e^\Lambda$ with $|\Lambda| \ll 1$, the infinitesimal gauge parameter Λ must obey a constraint $\Box \Lambda = 0$ as well, so the zero-mode solution is given by

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[2]We follow the conventions and notation of the MTW textbook [10].

$$\Lambda(x) = b, \qquad (15)$$

where b is a constant.

Since there is a global invariance associated with the parameter b, we can construct a conserved Noether charge Q. To derive a conserved current according to the Noether theorem [8], let us first recall that the Lagrangian density is assumed to contain only up to the first derivatives of fields. For this aim, we will first rewrite a αR^2 term into the form $\varphi R - \frac{1}{4\alpha}\varphi^2$ where φ is a scalar field with the dimension of mass squared:

$$\mathcal{L} = \sqrt{-g}\left(\varphi R - \frac{1}{4\alpha}\varphi^2 - \xi R|\Phi|^2 - |\partial_\mu\Phi|^2 - \lambda|\Phi|^4\right). \qquad (16)$$

Then, we will perform the integration by parts to make the second derivative in R change the first derivative. Following the calculation [3,11], it turns out that the conserved current for dilatation reads

$$J^\mu = \sqrt{-g}g^{\mu\nu}\partial_\nu[6\varphi + (1-6\xi)|\Phi|^2]. \qquad (17)$$

We can also verify that this current is conserved, $\partial_\mu J^\mu = 0$, by using the field equations as desired. One might wonder why no derivatives of the metric appear in the expression of J^μ. This is because the derivatives of φ and Φ are mixed with the metric, thus making J_0 serve as a generator of the metric transformation. Moreover, in case of a conformal coupling $\xi = \frac{1}{6}$ and the absence of a R^2 term, the conserved current is identically vanishing [12,13]. In other words, the nonvanishing current requires us to treat with a R^2 term and/or a scalar matter field Φ with a nonconformal coupling $\xi \neq \frac{1}{6}$.

Using the corresponding Noether charge defined as $Q = \int d^3x J^0$, we find that

$$\delta g_{\mu\nu} = [ibQ, g_{\mu\nu}] = 2bg_{\mu\nu}, \qquad (18)$$

from which we have

$$[iQ, g_{\mu\nu}] = 2g_{\mu\nu}, \qquad (19)$$

Assuming $\langle 0|g_{\mu\nu}|0\rangle = \eta_{\mu\nu}$ [3] and taking the vacuum expectation value of Eq. (19) leads to

$$\langle 0|[iQ, g_{\mu\nu}]|0\rangle = 2\eta_{\mu\nu}. \qquad (20)$$

Equation (20) clearly implies that the global scale invariance must be broken spontaneously at the quantum

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[3] Of course, we can consider a more general fixed background $\bar{g}_{\mu\nu}$ which satisfies $\langle 0|g_{\mu\nu}|0\rangle = \bar{g}_{\mu\nu}$, but a flat Minkowski background assures that a $GL(4)$ symmetry is spontaneously broken to an $SO(1,3)$ and consequently the graviton is a Nambu-Goldstone tensor boson in quantum gravity [14].

level [15]. Note that this finding is obtained by using the zero-mode solution to the constraint $\Box\Omega = 0$.

Next, let us verify explicitly that this is the case by moving from the Jordan frame to the Einstein frame [16]. To do so, we will move to the Einstein frame by implementing a local conformal transformation

$$g_{\mu\nu} \to g_{*\mu\nu} = \Omega^2(x)g_{\mu\nu}, \qquad \Phi \to \Phi_* = \Omega^{-1}(x)\Phi. \qquad (21)$$

Under this conformal transformation we have [3]

$$\sqrt{-g} = \Omega^{-4}\sqrt{-g_*},$$
$$R = \Omega^2(R_* + 6\Box_*f - 6g_*^{\mu\nu}f_\mu f_\nu), \qquad (22)$$

where we have defined

$$f \equiv \log\Omega, \qquad \Box_*f \equiv \frac{1}{\sqrt{-g_*}}\partial_\mu(\sqrt{-g_*}g_*^{\mu\nu}\partial_\nu f),$$
$$f_\mu \equiv \partial_\mu f = \frac{\partial_\mu\Omega}{\Omega}. \qquad (23)$$

Then, the Lagrangian density (16) is cast to the form

$$\mathcal{L} = \sqrt{-g_*}\left[(\varphi\Omega^{-2} - \xi|\Phi_*|^2)(R_* + 6\Box_*f - 6g_*^{\mu\nu}f_\mu f_\nu)\right.$$
$$\left. - \frac{1}{4\alpha}\varphi^2\Omega^{-4} - \Omega^{-2}g_*^{\mu\nu}\partial_\mu(\Omega\Phi_*^\dagger)\partial_\nu(\Omega\Phi_*) - \lambda|\Phi_*|^4\right]. \qquad (24)$$

To reach the Einstein frame, we have to choose a conformal factor $\Omega(x)$ to satisfy a relation

$$\varphi\Omega^{-2} = \xi|\Phi_*|^2 + \frac{M_{\text{Pl}}^2}{2}, \qquad (25)$$

where M_{Pl} is the reduced Planck mass. As a result, with a redefinition $\omega(x) \equiv \sqrt{6}M_{\text{Pl}}f(x)$, we obtain a Lagrangian density in the Einstein frame:

$$\mathcal{L} = \sqrt{-g_*}\left[\frac{M_{\text{Pl}}^2}{2}R_* - \frac{1}{2}g_*^{\mu\nu}\partial_\mu\omega\partial_\nu\omega - \frac{1}{16\alpha}M_{\text{Pl}}^4 - |\partial_\mu\Phi_*|^2\right.$$
$$- \frac{\xi}{4\alpha}M_{\text{Pl}}^2|\Phi_*|^2 - \left(\lambda + \frac{\xi^2}{4\alpha}\right)|\Phi_*|^4$$
$$\left. + \left(\frac{1}{\sqrt{6}M_{\text{Pl}}}\Box_*\omega - \frac{1}{6M_{\text{Pl}}^2}g_*^{\mu\nu}\partial_\mu\omega\partial_\nu\omega\right)|\Phi_*|^2\right]. \qquad (26)$$

It is worthwhile to notice that spontaneous symmetry breakdown for a scale invariance has occurred and consequently we have a massless Nambu-Goldstone boson $\omega(x)$, which is often called "dilaton." As a bonus, a gauge symmetry is also spontaneously broken if we choose the parameters to be

$$\frac{\xi}{4\alpha} < 0, \qquad \lambda + \frac{\xi^2}{4\alpha} > 0. \tag{27}$$

Also note that the last two nonrenormalizable terms in Eq. (26) are suppressed by the Planck mass so that they would make only a small contribution at low energies $E \ll M_{\mathrm{Pl}}$.

IV. ORIGIN OF RESTRICTED WEYL SYMMETRY

We wish to understand the origin of a restricted Weyl symmetry. We will see that the restricted Weyl symmetry emerges as a residual symmetry of Weyl symmetry in a similar way that a restricted gauge symmetry appears as a residual symmetry of the conventional gauge symmetry after we take a Lorenz gauge in QED.

For generality, let us work with a general theory which is invariant under a full Weyl transformation:

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{12}R\phi^2 + c_1 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{\lambda}{4!}\phi^4\right), \tag{28}$$

where ϕ denotes a (ghostlike) scalar field, $C_{\mu\nu\rho\sigma}$ is a conformal tensor and c_1 is a constant. It is well known that this Lagrangian density is invariant under a Weyl transformation without an additional constraint on $\Omega(x)$:

$$g_{\mu\nu} \to g'_{\mu\nu} = \Omega^2(x)g_{\mu\nu}, \qquad \phi \to \phi' = \Omega^{-1}(x)\phi. \tag{29}$$

In order to derive the restricted Weyl invariance from the full Weyl invariance, one has to fix the Weyl symmetry in such a way that the gauge fixing condition breaks the full Weyl invariance but leaves the restricted Weyl invariance unbroken. A suitable gauge condition is $R = 0$.[4] This gauge choice can be achieved as follows: Let us expand $g_{\mu\nu}$ around a flat metric $\eta_{\mu\nu}$ as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $h_{\mu\nu}$ being a small fluctuation ($|h_{\mu\nu}| \ll 0$). In order to show that we can take a gauge condition $R = 0$ for the full Weyl symmetry, we start with $R \neq 0$, and then show that we can arrive at $R' = 0$ by means of a Weyl transformation as seen in the relation (14). With the expansion $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, the scalar curvature reads $R = -\Box h$ ($h \equiv \eta^{\mu\nu}h_{\mu\nu}$) to the linear order in $h_{\mu\nu}$. Using an infinitesimal Weyl transformation $\Omega = e^\Lambda$, the right-hand side (rhs) of Eq. (14) can be rewritten to the linear order in $h_{\mu\nu}$ and Λ as

$$\Omega^{-2}(R - 6\Omega^{-1}\Box\Omega) = -\Box(h + 6\Lambda). \tag{30}$$

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[4]We could take a more general gauge condition $R + k\phi^2 = 0$ (k is a constant) if necessary [15,17]. In this case, the only change of the final result is to replace $\frac{1}{2\alpha}R^2$ with $\frac{1}{2\alpha}(R + k\phi^2)^2$ in (33) and consequently the coefficients in front of $R\phi^2$ and ϕ^4 are modified.

Thus, if we choose the infinitesimal gauge parameter as $\Lambda = -\frac{1}{6}h$, we can certainly achieve $R' = 0$.

We therefore attempt to fix the Weyl invariance in terms of a gauge condition $R = 0$ in the BRST formalism. First of all, the BRST transformation for the Weyl symmetry reads

$$\delta_B g_{\mu\nu} = 2cg_{\mu\nu}, \quad \delta_B\sqrt{-g} = 4c\sqrt{-g}, \quad \delta_B R = -2cR - 6\Box c,$$
$$\delta_B\phi = -c\phi, \quad \delta_B\bar{c} = iB, \quad \delta_B c = \delta_B B = 0. \tag{31}$$

Note that this BRST transformation comes from the local scale symmetry (29) in four dimensions, so it is never broken spontaneously or explicitly.

Next, a Lagrangian density for the gauge condition and the Faddeev-Popov (FP) ghost is of form

$$\mathcal{L}_{\mathrm{GF+FP}} = -i\delta_B\left[\sqrt{-g}\bar{c}\left(R + \frac{\alpha}{2}B\right)\right]$$
$$= \sqrt{-g}\left(\hat{B}R + \frac{\alpha}{2}\hat{B}^2 - 6i\bar{c}\Box c\right)$$
$$= \sqrt{-g}\left(-\frac{1}{2\alpha}R^2 + 6ig^{\mu\nu}\partial_\mu\bar{c}\partial_\nu c\right), \tag{32}$$

where we have defined as $\hat{B} \equiv B + 2i\bar{c}c$ and in the last step we performed the path integral over the auxiliary field $\hat{B}$ and integration by parts [17].

Thus, we arrive at a gauge-fixed and BRST-invariant Lagrangian density given by[5]

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{12}R\phi^2 + c_1 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{\lambda}{4!}\phi^4\right.$$
$$\left. - \frac{1}{2\alpha}R^2 + 6ig^{\mu\nu}\partial_\mu\bar{c}\partial_\nu c\right). \tag{33}$$

It is worthwhile to note that this BRST-invariant Lagrangian density is also invariant under the restricted Weyl transformation. Actually, the first three terms are manifestly invariant under the restricted Weyl transformation since they are so under the full Weyl transformation. The last two terms turn out to be invariant under not the full Weyl transformation but the restricted Weyl transformation. For instance, the invariance of the ghost term can be shown as follows: First, let us assume that both FP-ghost and FP-antighost have the Weyl weight -1, that is, under the Weyl transformation they transform as

$$c \to c' = \Omega^{-1}(x)c, \qquad \bar{c} \to \bar{c}' = \Omega^{-1}(x)\bar{c}. \tag{34}$$

Then, we find that under the Weyl transformation the ghost term transforms as

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[5]Note that the physical content between (28) and (33) is exactly the same since Eq. (32) is BRST-exact.

$$\sqrt{-g}g^{\mu\nu}\partial_\mu\bar{c}\partial_\nu c$$
$$\rightarrow \sqrt{-g}g^{\mu\nu}[\partial_\mu\bar{c}\partial_\nu c + \Omega^{-1}\nabla_\mu\nabla_\nu\Omega\cdot\bar{c}c - \nabla_\mu(\Omega^{-1}\nabla_\nu\Omega\cdot\bar{c}c)].$$

$$(35)$$

Hence, the ghost kinetic term is invariant under the restricted Weyl transformation up to a surface term. In this way, we have succeeded in deriving the restricted Weyl invariance by beginning with a full Weyl invariance by taking a gauge condition $R = 0$. Of course, if we do not pay an attention to the origin of the restricted Weyl invariance, we can add any terms which are invariant under the restricted Weyl symmetry to (33), and remove the ghost term from (33). However, the fact that the restricted Weyl symmetry can be derived from the full Weyl symmetry not only sheds some light on its geometrical structure but also clarifies that the restricted Weyl symmetry is not an *ad hoc* but natural symmetry.

V. CONCLUSION

In this article, we have investigated a restricted Weyl symmetry. In particular, we have clarified two points: First, on the basis of the BRST formalism we have shown that a global scale invariance, which is included in the restricted Weyl invariance, must be broken spontaneously at the quantum level. Second, we have derived a gauge-fixed and BRST-invariant action, which is also invariant under the restricted Weyl symmetry as well as diffeomorphisms, by starting with a Weyl-invariant gravitational theory by fixing the Weyl symmetry by a gauge condition $R = 0$. Our derivation clarifies the origin of the restricted Weyl symmetry.

In addition to them, our article presents new interesting phenomena. First, there is a mismatch of the magnitude of the Higgs mass between (26) and Eq. (6) in [5]. This is because we cannot read off the correct value of the Higgs mass if we move to the exact Einstein frame. In fact, Eq. (6) in [5] is not completely in the Einstein frame owing to the existence of an $R|\Phi|^2$ term.[6] Second, to our knowledge, the gauge condition $R = 0$ for a local scale invariance has been adopted for the first time in this article. This gauge condition, together with the Einstein equation, implies

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[6]This fact was informed to us by T. Kugo, for which we are grateful.

that the cosmological constant is vanishing so this gauge choice might play an important role in the resolution of the cosmological constant problem in future. Finally, as can be seen in (26), if $\lambda|\Phi_*|^4$ were zero, the Higgs potential would have an expression of a perfect square. This fact means that the cosmological constant is vanishing at the minimum of the Higgs potential, which has been recently analyzed in detail in Ref. [18].

Nevertheless, there seem to remain many unsolved problems relevant to the restricted Weyl symmetry. In what follows, we will comment on only two important problems to be understood in future. One of them is related to trace anomaly. It has been recently established that a scale symmetry is a quantum symmetry which is broken only spontaneously and is free from trace anomaly [19–21]. This is done by using a subtraction function $\mu(\omega)$ (ω is a dilaton in Sec. III) instead of a dimensionful subtraction scale μ in the dimensional regularization method.

However, as a price we have to pay, an infinite number of counterterms like $(\frac{|\Phi|}{\omega})^n$ with n being integers would be needed, thereby breaking the property of renormalizability. This issue of nonrenormalizability is not so serious in the context at hand since the Einstein-Hilbert term in (26) is a nonrenormalizable term as well. The model made in Sec. III could be a candidate of physics beyond the standard model (BSM) so one should construct such a BSM in an explicit manner in terms of the manifestly scale invariant regularization scheme to attack the gauge hierarchy problem and the cosmological constant problem etc.

The other problem is related to a constraint $\Box\Omega = 0$ in a restricted Weyl symmetry. As reviewed in Sec. I, a constraint $\Box\theta = 0$ in QED removes one dynamical degree of freedom in A_μ. It is unclear what dynamical degree of freedom can be removed by the constraint $\Box\Omega = 0$ in the case of the restricted Weyl symmetry. It is known that the constraint $\Box\Omega = 0$ has an infinite number of classical solutions, so it should remove one dynamical degree of freedom. To put differently, we do not understand the physical meaning of the constraint $\Box\Omega = 0$ yet. We wish to return these problems in future.

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[1] H. Weyl, in *Gravitation und Elekrizität, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* (1918), pp. 465–480; English translation, in *Gravitation and Electricity* (1918), pp. 24–37 in O'Raifeartaigh's Book.

[2] L. O'Raifeartaigh, *The Dawning of Gauge Theory* (Princeton University Press, Princeton, NJ, 1997).

[3] Y. Fujii and K. Maeda, *The Scalar-Tensor Theory of Gravitation* (Cambridge University Press, Cambridge, England, 2003).

[4] A. Edery and Y. Nakayama, Restricted Weyl invariance in four-dimensional curved spacetime, Phys. Rev. D **90,** 043007 (2014).

[5] A. Edery and Y. Nakayama, Generating Einstein gravity, cosmological constant and Higgs mass from restricted Weyl invariance, Mod. Phys. Lett. A **30,** 1550152 (2015).

[6] A. Edery and Y. Nakayama, Critical gravity from four dimensional scale invariant gravity, J. High Energy Phys. 11 (2019) 169.

[7] T. Kugo, H. Terao, and S. Uehara, Dynamical gauge bosons and hidden local symmetries, Prog. Theor. Phys. Suppl. **85,** 122 (1985).

[8] E. Noether, Invariant variation problems, Gott. Nachr. **1918,** 235 (1918) [Transp. Theory Stat. Phys. **1,** 186 (1971)].

[9] R. Ferrari and L. E. Picasso, Spontaneous breakdown in quantum electrodynamics, Nucl. Phys. **B31,** 316 (1971).

[10] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman and Co (Sd), San Francisco, 1973).

[11] I. Oda, Higgs mechanism in scale-invariant gravity, Adv. Stud. Theor. Phys. **8,** 215 (2014).

[12] R. Jackiw and S.-Y. Pi, Fake conformal symmetry in conformal cosmological models, Phys. Rev. D **91,** 067501 (2015).

[13] I. Oda, Fake conformal symmetry in unimodular gravity, Phys. Rev. D **94,** 044032 (2016).

[14] N. Nakanishi and I. Ojima, Proof of the Exact Masslessness of Gravitons, Phys. Rev. Lett. **43,** 91 (1979).

[15] I. Oda, Spontaneous symmetry breakdown of scale symmetries, arXiv:2003.07947.

[16] I. Oda, Planck scale from broken local conformal invariance in Weyl geometry, Contribution to CORFU2019, arXiv:2003.12256.

[17] I. Oda, Emergence of Einstein gravity from Weyl gravity, arXiv:2003.01437.

[18] I. Oda, Higgs Potential from Weyl conformal gravity, arXiv:2006.10867.

[19] F. Englert, C. Truffin, and R. Gastmans, Conformal invariance in quantum gravity, Nucl. Phys. **B117,** 407 (1976).

[20] M. Shaposhnikov and D. Zenhausern, Quantum scale invariance, cosmological constant and hierarchy problem, Phys. Lett. B **671,** 162 (2009).

[21] D. M. Ghilencea, Manifestly scale-invariant regularization and quantum effective operators, Phys. Rev. D **92,** 045036 (2015).