Special case of the Bañados-Silk-West effect

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If two particles collide near the rotating extremal black hole and one of them is fine-tuned, the energy in the center of mass frame $E_{\mathrm{c.m.}}$ can grow unbounded. This is the so-called Bañados-Silk-West (BSW) effect. Recently, another type of high energy collisions was considered in which all processes happen in the Schwarzschild background with free falling particles. If the Killing energy E of one of particle is sufficiently small, $E_{\mathrm{c.m.}}$ grows unbounded. We show that, however, such a particle cannot be created in any precedent collision with finite energies, angular momenta and masses. Therefore, in contrast to the standard BSW effect, this one cannot be realized if initial particles fall from infinity. If the black hole is electrically charged, such a type of collisions is indeed possible, when a particle with very small E collides with one more particle coming from infinity. Thus the BSW effect is achieved due to collisions of neutral particles in the background of a charged black hole. This requires, however, at least a two-step process.

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I. INTRODUCTION

If two particles collide near a rotating black hole, under certain conditions this leads to the unbounded growth of the energy in the center of mass frame $E_{\mathrm{c.m.}}$ This is the essence of the so-called Bañados-Silk-West (BSW) effect [1] (see also more early works [2–4]). The aforementioned condition requires that one of colliding particles has fine-tuned parameters, so that $X \equiv E - \omega L$ vanishes or is sufficiently small near the horizon. Here, E is the Killing energy, L being the angular momentum, ω a certain metric coefficient responsible for rotation. There is also a static charged counterpart of the BSW effect [5], when the aforementioned condition reads $E - q\varphi \approx 0$ near the horizon. Here, q is the particle's charge, φ being the Coulomb potential of a black hole. When there is neither rotation nor electric charge, the effect, in general, disappears. In particular, if two particles of equal masses with $E_1 = E_2 = m$, collide in the Schwarzschild background, $E_{\mathrm{c.m.}} \leq 2\sqrt{5}m$ [6]. Hereafter, we use subscript "i" to indicate quantities related to particle i.

Meanwhile, there is a rather special case when the criticality condition can be formally satisfied even without rotation or the electric charge. This happens if $E \approx 0$ itself is small that can be satisfied even for the Schwarzschild black hole. This observation was made in [7] (see discussion on p. 3864 before Eq. (35) there). Quite recently, it was rediscovered in [8]. It is worth noting that the criticality condition $E = 0$ was also considered in Sec. II of [9] for $2 + 1$ black holes with the cosmological constant [10]. It appeared as a limit of the angular momentum $L \to 0$ in a more general condition (2.15) there. The corresponding space-time is not asymptotically flat but near the horizon the same features manifest themselves, so $E_{\mathrm{c.m.}}$ grows unbounded when $E \to 0$ for one of two particles.

Meanwhile, there is a problem with physical realization of such a scenario. The corresponding particle cannot come from infinity, where $E \geq m$, m being the particle mass. So small energy can be obtained if a particle is maintained near the horizon. Indeed, the energy of a particle that remains in the rest in the static field $E = m\sqrt{-g_{00}}$, where g_{00} is the corresponding component of the metric tensor. If, in the Schwarzschild metric, $g_{00} \to 0$, the energy $E \to 0$ as well. However, if such a particle is kept fixed, the experienced acceleration $a \sim 1/l$, where l is the proper distance to the horizon. Letting such a particle move freely, one indeed obtains unbounded $E_{\mathrm{c.m.}}$ after its collision with some other particle. But this result is gained by the expense of unbounded forces that were exerted on a particle before it has been released. To a large extent, this deprives the scenario under discussion of physical significance.

If, nonetheless, we want some scenario with small E to be more physical, we must elucidate, whether or not it can be realized by means of particles that move along geodesics or under the action of a finite force. This problem is considered in the present work. It was conjectured in the end of Ref. [8] that particle in question can be obtained in some foregoing collision. We show that for

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the Schwarzschild black hole this is impossible. Instead, this is indeed possible in the background of the extremal Reissner-Nordström (RN) black hole.

The paper is organized as follows. In Sec. II we give the form of the metric and equations of motion in the RN metric. In Sec. III we consider collisions of particles 1 and 2 that turn into particles 3 and 4 and list basic formulas for the energy in the center of mass frame. In Sec. IV we discuss the collision in the special case when particles are neutral. In Sec. V we list basic formulas that enable us to find dynamic characteristics of new particles, given the data of initial ones. These formulas are exact. In Sec. VI we elucidate, whether we can obtain the energy E_3 of particle 3 as small as we like. The answer is negative. Further, we discuss the second collision of neutral particles when particle 5 comes form infinity. In Sec. VII we consider processes in the extremal RN background. The combined fine-tuned (critical) particle 0 (equivalent to $1+2$) decays to particles 3 and 4. We show that we can obtain a neutral particle 3 with very small E_3 that leads to the analogue of the BSW effect in next collision with particle 5. In Sec. VIII we summarize the results.

In what follows, we use the geometric system of units in which fundamental constants $G=c=1$.

II. EQUATIONS OF MOTION

Let us consider the black hole metric

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2, \tag{1}$$

where $d\omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, $f = f(r)$. The largest root $r = r_+$ of equation $f = 0$ corresponds to the event horizon. If a particle moves along the electrogeodesic (i.e., under the action of gravitation and electrostatic force only), for motion within the equatorial plane we have

$$m\dot{t} = \frac{X}{f}, \tag{2}$$

$$m\dot{\phi} = \frac{L}{r^2}, \tag{3}$$

$$X = E - q\varphi, \tag{4}$$

$$m\dot{r} = \sigma P, \qquad P = \sqrt{X^2 - f\tilde{m}^2}, \qquad \tilde{m}^2 = m^2 + \frac{L^2}{r^2}, \tag{5}$$

dot denotes derivative with respect to the proper time, $\sigma = \pm 1$. The forward-in-time condition $\dot{t} > 0$ entails

$$X \geq 0. \tag{6}$$

In the case of the Schwarzschild black hole, $\varphi = 0$, $f = 1 - \frac{r_+}{r}$. For the extremal RN black hole, $\varphi = \frac{r_+}{r}$, $f = (1 - \frac{r_+}{r})^2$, so

$$X = E - q + q\sqrt{f}. \tag{7}$$

In what follows, we use the standard terminology. If $X_H = 0$, a particle is called critical. This is realized if $E = q$. If X_H is separated from zero, it is called usual. If $X_H = O(\sqrt{f_c})$, it is called near-critical. Here, subscript "H" refers to the quantity calculated on the horizon and "c" to that taken in the point of collision.

III. PARTICLE COLLISIONS AND ENERGY IN THE CENTER OF MASS FRAME

If particles 1 and 2 collide, for the energy in the center of mass frame we have

$$\begin{aligned} E^2_{\text{c.m.}} &= -(m_1 u_{1\mu} + m_2 u_{2\mu})(m_1 u^\mu_1 + m_2 u^\mu_2) \\ &= m_1^2 + m_2^2 + 2m_1 m_2 \gamma, \end{aligned} \tag{8}$$

where $\gamma = -u_{1\mu} u^\mu_2$ is the relative Lorentz factor of particle motion, $u^\mu_{1,2}$ are the four-velocities of particles. Then, for motion within the equatorial plane one obtains from (2)–(5) that for motion of both particles in the same direction

$$m_1 m_2 \gamma = \frac{X_1 X_2 - P_1 P_2}{f} - \frac{L_1 L_2}{r^2}. \tag{9}$$

If collision happens near the horizon, particle 1 is critical and particle 2 is usual, we have unbounded growth of γ [5]

$$E^2_{\text{c.m.}} \approx 2\frac{(X_2)_H (E_1 - \sqrt{E_1^2 - \tilde{m}_1^2})}{\sqrt{f}}. \tag{10}$$

IV. SPECIAL SCENARIO

Let us consider collision of neutral particles, so all $q_i = 0$, $X_i = E_i$. Then, in (9), $P_i = \sqrt{E_i^2 - f\tilde{m}_i^2}$. We also assume that E_1 is very small,

$$E_1 = \alpha\sqrt{f_c}, \tag{11}$$

where α is some constant, $f_c \ll 1$ for collision near the horizon. Then,

$$m_1 m_2 \gamma \approx \frac{E_2(\alpha - \sqrt{\alpha^2 - \tilde{m}_1^2})}{\sqrt{f_c}} \tag{12}$$

formally grows unbounded. This just corresponds to the situation described in [7,8,9]. However, we want particle 1 to be created in some precedent collision, preferably due to the process that involves particles coming from infinity. As, in the case under consideration, they both are usual, the first collision must occur with finite $E_{\text{c.m.}}$.

As is already established [11], the BSW effect has a simple explanation. The quantity X (4) obeys the relation

$$X = m\frac{\sqrt{f}}{\sqrt{1-V^2}}, \tag{13}$$

where V is the three-velocity measured by a static observer (see Eq. (29) in the aforementioned work). Then, for a usual particle, the horizon limit $f \to 0$ shows that $V \to 1$. Meanwhile, for the critical or near-critical particle, the left-hand side of (13) has the order $\sqrt{f}$, so this equation is satisfied with $V < 1$. Then, collision of a rapid and slow particles results in the large relative velocity close to the speed of light, and γ grows unbounded.

The case under consideration has its specific feature. Now, $q = 0$, $X = E$. Therefore, the left hand side can be made as small as one likes not due to fine tuning between parameters E and q but due to small value of $E = O(\sqrt{f})$ itself.

V. THE FIRST COLLISION AND THE OVERALL SCHEME

We assume that after collision, new particles 3 and 4 appear. Alternatively, we can consider decay of particle 0 that formally combines particle 1 and 2. The conservation laws in the point of collision tell us

$$E_0 = E_1 + E_2 = E_3 + E_4, \tag{14}$$

$$L_0 = L_1 + L_2, \tag{15}$$

$$q_0 = q_1 + q_2 = q_3 + q_4, \tag{16}$$

$$-P_0 = -P_1 - P_2 = \sigma_3 P_3 + \sigma_4 P_4. \tag{17}$$

It follows from these equations that

$$X_0 = X_1 + X_2 = X_3 + X_4. \tag{18}$$

Now, it is convenient to take advantages of the results already obtained in the previous work [12] and listed there in Eqs. (19)–(25). The only obvious difference is that now instead of ωL, the quantity X contains $q\varphi$ (4). If particle 0 is thought of as a combined one, m_0 coincides with the $E_{\mathrm{c.m.}}$ in the particle collision. Otherwise, m_0 is simply the mass of particle 0. Then, straightforward algebraic manipulation give us

$$(X_3)_c = \frac{1}{2\tilde{m}_0^2}(X_0\Delta_+ + P_0\sqrt{D}\delta)_c, \tag{19}$$

$$(X_4)_c = \frac{1}{2\tilde{m}_0^2}(X_0\Delta_- - P_0\sqrt{D}\delta)_c, \tag{20}$$

where $\delta = 1$ or $\delta = -1$.

$$\Delta_\pm = \tilde{m}_0^2 \pm (\tilde{m}_3^2 - \tilde{m}_4^2). \tag{21}$$

The positivity of $X_{3,4}$ entails

$$\Delta_\pm > 0. \tag{22}$$

$$D = \Delta_+^2 - 4\tilde{m}_0^2\tilde{m}_3^2 = \Delta_-^2 - 4\tilde{m}_0^2\tilde{m}_4^2. \tag{23}$$

It is necessary that

$$D \geq 0, \tag{24}$$

$$\tilde{m}_0 \geq \tilde{m}_3 + \tilde{m}_4. \tag{25}$$

For charged particles, we have 4 conservation laws for 6 unknowns $E_{3,4}$, $L_{3,4}$, $q_{3,4}$. In the above formulas, all quantities related to particles 1 and 2 (hence, those of effective particle 0 as well) are fixed. We also assume that masses $m_{3,4}$ are fixed for any given process. Meanwhile, one of two angular momentum (say, L_3) and one of the charges (say, q_3) remain free parameters.

Below, we are interested in the two-step scenario that, overall, can be described as follows.

(1) The first step. Two particles come from infinity, collide and produce two new ones. The energy $E_{\mathrm{c.m.}}$ is finite in the point of collision.

(2) One of new particles (say, particle 3) has a very small energy E_3.

(3) In point 2 that is more close to the horizon ($f_2 < f_1$) it collides with one more particle 5 coming from infinity. $E_{\mathrm{c.m.}}$ in the second event (collision between particles 3 and 5) is unbounded.

In this scheme, the first step can be replaced with the decay of one particle 0 instead of collisions of two ones.

VI. COLLISIONS OF NEUTRAL PARTICLES

A. Generic subcase

Let all particles be electrically neutral, so all $q_i = 0$, $X_i = E_i$. If particle 0 comes from infinity, it is usual, since $E_0 \geq m_0$. If it is a combined particle, this is true as well, since $E_0 \geq m_1 + m_2$. Now we ask, is it possible to achieve indefinitely small $X_{3,4} = E_{3,4}$, assuming that m_0 is finite and nonzero? If yes, this would mean that decay of particle 0 to 3 and 4 leads to particle (say, 3) with indefinitely small E_3. Then, the second collision between particle 3 and some additional particle 5 coming from infinity would give us indefinitely large $E_{\mathrm{c.m.}}$ as is explained above and was considered in [7,8,9].

For the candidate particle 3, we take $\delta = -1$ in (19), since we want to make E_3 (almost) zero. Using (25), it is convenient to rewrite (19)

$$E_3 = \frac{2\tilde{m}_3^2 P_0^2 + \frac{f}{2}\Delta_+^2}{(E_0\Delta_+ + P_0\sqrt{D})}, \tag{26}$$

where we put $X_0 = E_0$.

As particle 0 is usual, it is seen from (5) that in the horizon limit $f \to 0$, the quantity $P_0 \to X_0$. Thus the numerator tends to $2(\tilde{m}_3^2)_c E_0^2$ and does not vanish for any $\tilde{m}_3 \neq 0$. Then, we cannot achieve indefinitely small E_3, so the scenario under discussion does not work.

B. Special subcases

Is it possible to achieve $E_3 \to 0$ by taking a very small $\tilde{m}_3$? Let, at first, $\tilde{m}_3 = 0$ exactly. From the definition of $\tilde{m}$ (5), it follows that $m_3 = 0$ and $L_3 = 0$. Then, $P_3 = E_3$. If particle 3 collides one more time in point 2 with some usual particle 5 coming from infinity, we have from (8), (9)

$$(E_{\mathrm{c.m.}}^2)_2 = 2E_3 \frac{\left(E_5 - \sqrt{E_5^2 - \tilde{m}_5^2 f_2}\right)}{f_2} + m_5^2, \quad (27)$$

where according to (26) $E_3 \sim f_1$ is small.

In the horizon limit, we obtain that $E_{\mathrm{c.m.}}^2$ is finite,

$$(E_{\mathrm{c.m.}}^2)_2 \approx \frac{E_3 (\tilde{m}_5^2)_H}{E_5} + m_5^2, \quad (28)$$

so again there is no BSW effect.

Instead, we may try to choose $\tilde{m}_3$ to be small but nonzero. In turn, two different subcases should be considered separately. In doing so, $\tilde{m}_3^2 \approx m_3^2 + \frac{L_3}{r_+^2} = \mathrm{const}$.

1. Subcase a

$$\tilde{m}_3^2 f_1 \ll E_3^2 \quad (29)$$

Then, we obtain from the general expressions (8), (9) that

$$(E_{\mathrm{c.m.}}^2)_2 \approx \frac{\tilde{m}_3^2}{E_3} E_5 + m_5^2. \quad (30)$$

Meanwhile, it follows from (26) that

$$\frac{\tilde{m}_3^2}{E_3} \le \frac{2\tilde{m}_3^2 E_0 \Delta_+}{2\tilde{m}_3^2 E_0^2 + \frac{f}{2}\Delta_+^2} \le \frac{\Delta_+}{E_0}, \quad (31)$$

where we put in the horizon limit $P_0 \approx E_0$ and took into account that $D \approx \Delta_+^2$ because of small $\tilde{m}_3^2$. Then, it follows that (30) is finite, there is no BSW effect.

2. Subcase b

$$\tilde{m}_3^2 f_1 = \beta^2 E_3^2. \quad (32a)$$

Here, β is some coefficient $O(1)$. Then, it follows from Eqs. (8) and (9) that,

$$(E_{\mathrm{c.m.}}^2)_2 \approx \frac{2E_5}{f_2}\left(E_3 - \sqrt{E_3^2 - \tilde{m}_3^2 f_2}\right) + m_5^2. \quad (33)$$

This can be rewritten as

$$(E_{\mathrm{c.m.}}^2)_2 \approx \frac{E_5 E_3 F(y, \beta^2)}{f_1} + m_5^2 \quad (34)$$

$$F(y, \beta^2) \equiv \frac{(1 - \sqrt{1 - \beta^2 y})}{y} = \frac{\beta^2}{1 + \sqrt{1 - \beta^2 y}}, \quad (35)$$

where $y = \frac{f_2}{f_1}$, $0 \le y \le 1$.

Obviously, the function F is finite. It is monotonically increasing with y and changes from

$$F(0, \beta^2) = \frac{\beta^2}{2} \quad (36)$$

on the horizon to

$$F(1, \beta^2) = 1 - \sqrt{1 - \beta^2}, \quad (37)$$

if the 2nd collision occurs practically in the same point immediately after the 1st one.

It is seen from (26) that for our limit $f_1 \to 0$, condition (32a) is compatible with (26) for

$$\tilde{m}_3^2 \approx A^2 f_1 \quad (38)$$

only, where A is a constant. Then, $E_3 \approx \frac{A}{\beta} f_1$, where we infer from (26) that

$$A = \beta \frac{2E_0^2 A^2 + \frac{(\Delta_+^2)_H}{2}}{2E_0 (\Delta_+)_H}. \quad (39)$$

This quadratic equation can be solved easily,

$$A = \frac{(\Delta_+)_H}{2E_0 \beta}\left(1 \pm \sqrt{1 - \beta^2}\right). \quad (40)$$

In (34), the numerator and denominator have the same order f_1, so we obtain

$$(E_{\mathrm{c.m.}}^2)_2 \approx \frac{A}{\beta} E_5 F + m_5^2. \quad (41)$$

Again, it is finite, there is no BSW effect.

Thus having enumerated all possible subcases, we come to the conclusion that, starting from particles with finite parameters, one cannot create a suitable particle 3 to arrange the second collision with unbounded $(E_{\mathrm{c.m.}}^2)_2$.

In [8] (see the paragraph before Eq. (10) there), it was assumed by hand that $E_3 \sim \sqrt{f_1}$. However, the requirement of the finiteness of $E_{\mathrm{c.m.}} = m_0$ in the precedent collision in which particle E_3 was created, imposes severe restrictions. As we saw, they give rise to another dependence $E_3 \sim f_1$ that makes $E_{\mathrm{c.m.}}$ finite in the second collision.

VII. PROCESS WITH CHARGED CRITICAL PARTICLES

Now, we consider particle motion in the extremal RN background. Then, for critical particle 0 we have according to (7),

$$X_0 = E_0\sqrt{f}, \qquad P_0 = \sqrt{(E_0^2 - \tilde{m}_0^2)f}. \tag{42}$$

We assume that particle 0 (that effectively can model the combination of particles $1 + 2$) turns into particles 3 and 4. As the initial particle 0 is critical, it follows from (6) that near the horizon X_3 and X_4 are both small. More precisely, if they are created in the collision with finite $E_{\mathrm{c.m.}} = m_0$, they have the same order $\sqrt{f_1}$. We want to elucidate, whether or not the BSW effect is possible in the second collision, if particle 3 is uncharged, $q_3 = 0$.

It follows from (26), that contains now X_0 in the denominator instead of E_0 and (42) that

$$E_3 = C_1\sqrt{f_1}, \tag{43}$$

where $C_1 = C(r_1)$,

$$C = \frac{2\tilde{m}_3^2(E_0^2 - \tilde{m}_0^2) + \frac{\Delta_+^2}{2}}{E_0\Delta_+ + \sqrt{E_0^2 - \tilde{m}_0^2}\sqrt{D}}, \tag{44}$$

$$P_3 = \sqrt{f_1}\sqrt{C^2 - \tilde{m}_3^2\frac{f}{f_1}}. \tag{45}$$

For the process near the horizon, $C_1 \approx C_H$. If particle 3 collides with some usual particle 5,

$$(E_{\mathrm{c.m.}}^2)_2 \approx \frac{2(X_5)_H C_H F(y,\delta)}{\sqrt{f_1}}, \tag{46}$$

where again $y = \frac{f_2}{f_1}$ but now $\delta = \frac{\tilde{m}_3^2}{C_H^2}$. The function F is defined above in (35), so (36) and (37) are still valid, the function F is bounded. However, because of small f_1 in the denominator, $(E_{\mathrm{c.m.}}^2)_2$ can be made unbounded.

The dependence $(E_{\mathrm{c.m.}}^2) \sim f^{-1/2}$ is exactly the same as in the case of the standard BSW effect for charged black holes [5]. However, there is a qualitative difference now. In the standard case, one of two colliding particles should be electrically charged. This is necessary to satisfy the criticality condition $X_H = 0$. Meanwhile, now a particle that plays in the second collision the same role as the (near) critical particle does in the standard BSW effect, is *neutral*. We see that the role of the electric charge in the problem is twofold. On one hand, the charge is required for a black hole. More precisely, the presence of the charge enables one to produce in the first event a particle with $q_3 = 0$ and small E_3, as a result of collision between two critical particles. In accordance with general rules, such a collision leads to finite $E_{\mathrm{c.m.}}$ (see kinematic explanation in Sec. III B of [11]). From the other hand, the charge is irrelevant on the second stage of the process, when a neutral particle collides with a usual one coming from infinity. Then, the BSW effect can be achieved even due to collision of two neutral particles.

VIII. CONCLUSIONS

Thus we considered two complementary cases. (1) Particle motion occurs in the neutral background. In particular, this is valid for the Schwarzschild metric. This holds also for the RN one, provided all particles are electrically neutral. Then, an initial particle 0 (true or effective combined one) is usual. It is shown that, arranging the collision between two initial particles coming from infinity, it is impossible to obtain particle 3 with almost vanishing E_3 to realize the BSW effect in the second collision. This means that the scenario in which particle 3 with $E_3 \approx 0$ is created in the foregoing collision (as outlined in the end of [8]) does not work for the Schwarzschild black hole. (2) Particle 0 is critical. It decays to two fragments, one of which is neutral. Then, the BSW effect is indeed possible in the next collision. This gives a new type of the BSW effect for electrically charged black holes, realized with the help of electrogeodesics and geodesics. The counterpart of the phenomenon discussed in our paper should exist also for rotating neutral black holes, then the role of particle 3 with aforementioned properties will be played by a particle with $L_3 = 0$. For $2 + 1$ dimensional black holes with the cosmological constant it was realized in [9]. Meanwhile, one can expect a similar phenomenon for $3 + 1$ asymptotically flat black holes as well, including the Kerr metric.

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