Revisiting collisional Penrose processes in terms of escape probabilities for spinning particles

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We first study the escape probability of a spinning particle emitted from a Kerr black hole and find that the escape probability increases with the spin of the particle around the extreme Kerr black hole; in contrast, the escape probability decreases at the position near the horizon but increases at the position far from the horizon with the increasing spin of the particle. We then probe the relation between the escape probabilities and the energy extraction efficiencies of collisional Penrose processes for particles with varying spin. For an extreme Kerr black hole, the efficiency increases with the escape probability; for a nonextreme Kerr black hole, the near-horizon efficiency decreases with the escape probability, while the efficiency may increase with the escape probability in the ergosphere. In the event horizon limit, we also find that the average escape probability of the spinning particle produced in the collisional Penrose process decreases with the rotation parameter of the Kerr black hole.

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I. INTRODUCTION

The well-known Penrose process (PP) [1] states that an escaped particle can carry more energy than the one it is disintegrated from in a background of Kerr spacetime. A more realistic scenario for overcoming the seeming implausibility of disintegration in PP is believed to be the collisional Penrose process [2], where two particles plunge into the ergoregion and collide. The energy extraction efficiency in the collisional Penrose process, however, was verified to be qualitatively similar to the PP [3]. There is another process, where one of the infalling particles with sufficient angular momentum turns past the rotating black hole and collides on its outgoing orbit with the other infalling particle [4], that can work with relatively higher efficiency. Furthermore, the super Penrose process [5–7], where a head-on collision takes place between one outgoing particle and one ingoing particle, can reach an infinite efficiency at the horizon limit.

The Bañados-Silk-West mechanism, which states that the center-of-mass energy for two spinning particles (one with a critical angular momentum) can be arbitrarily high after a collision near the horizon of an extreme rotating black hole [8–10], has reinvigorated the investigation of energy extraction from the black hole, as the effect of spin carried by the collisional particles has been discussed qualitatively [11,12] and quantitatively [13]. In addition, the effect of the charge carried by the collisional particles has also been examined [14].

The observability of the collisional events around the black hole depends on how often a particle can escape from the black hole to spatial infinity [15]. The astrophysical process in the strong gravity field of the black hole can be further understood by using the notion of the escape probability for the particle, by which we can know which portion of the radiation emitted from the particle source is trapped while the complementary portion escapes to spatial infinity [16]. In this paper, we will first briefly review the equations of motion for the spinning particle in the Kerr spacetime in Sec. II for later use. Then we will calculate the escape probability for the spinning particle which is supposed to emit isotropically from a particle source in Sec. III. In Sec. IV, we will investigate the relation between the energy extraction efficiency of the collisional Penrose process and the escape probability of the produced particle. Section V will be devoted to our conclusions.

II. THE EQUATIONS OF MOTION FOR A SPINNING PARTICLE IN THE KERR SPACETIME

The motion of an astronomical test particle whose pole/depole moment is considered in curved spacetime can be described by the well-known Mathisson-Papapetrou-Dixon equations [17]

$$\frac{DP^a}{D\tau} = -\frac{1}{2} R^a{}_{bcd} v^b S^{cd}, \tag{1}$$

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$$\frac{DS^{ab}}{D\tau} = 2P^{[a}v^{b]}, \tag{2}$$

where τ is the parameter along the world line of the particle. R_{abcd} is the Riemannian curvature tensor of the spacetime geometry. The four-momentum P^a is related to the particle's mass $\mathcal{M}$ by [18,19]

$$P^a P_a = -\mathcal{M}^2 \tag{3}$$

in the zero three-momentum frame, and it together with the particle's four-velocity v^a also defines the other mass m in the zero three-velocity frame by [18,19]

$$P_a v^a = -m. \tag{4}$$

The normalized four-momentum of the particle is

$$u^a \equiv \frac{P^a}{\mathcal{M}}. \tag{5}$$

The dynamical mass of the particle can be ensured to be conserved by using the well-known Tulczyjew condition [20–22]

$$S^{ab}P_b = 0. \tag{6}$$

Also, the magnitude of the spin S can be invariable in the condition [23]

$$S^{ab}S_{ab} = 2S^2. \tag{7}$$

As $\mathcal{M} = m + \mathcal{O}(S^2)$ [24], we can have $v^a u_a = -1$ by reparametrizing τ [25–27]. Accordingly, the four-momentum of the particle can be obtained as [17–19,28]

$$v^a = u^a + \frac{2S^{ab}u^c R_{bcde}S^{de}}{S^{bc}R_{bcde}S^{de} + 4\mathcal{M}^2}. \tag{8}$$

We consider the Kerr spacetime in this paper. After choosing the unit $c = G = 1$, the Kerr line element can be written in Boyer-Lindquist coordinates as

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 - \frac{4Mar}{\Sigma}\sin^2\theta dt d\phi + \frac{\Xi}{\Sigma}\sin^2\theta d\phi^2, \tag{9}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta,$$
$$\Delta = r^2 - 2Mr + a^2,$$
$$\Xi = (a^2 + r^2)^2 - a^2\Delta\sin^2\theta.$$

M is the mass of the black hole and a is the rotation parameter defined by J/M, with J the angular momentum of the black hole. The event horizon and the stationary limit are located at

$$r_+ = M + \sqrt{M^2 - a^2}, \tag{10}$$

$$r_e = M + \sqrt{M^2 - a^2\cos\theta^2}. \tag{11}$$

The collisional Penrose processes that will be discussed in Sec. IV take place in the ergosphere $r_+ < r_* < r_e$. The Kerr spacetime admits conserved energy and angular momentum for the particle as

$$e = \frac{1}{2\mathcal{M}}S^{tb}\nabla_b\xi_t - \xi_t u^t, \tag{12}$$

$$j = -\frac{1}{2\mathcal{M}}S^{\phi b}\nabla_b\xi_\phi + u^\phi\xi_\phi, \tag{13}$$

where

$$\xi_t \equiv \left(\frac{\partial}{\partial t}\right)^a, \qquad \xi_\phi \equiv \left(\frac{\partial}{\partial\phi}\right)^a.$$

In an orthogonal normalized tetrad $\{e_a^{(\mu)}\}$, which reexpresses the metric (9) as

$$ds^2 = \eta_{(i)(j)}e_a^{(i)}e_b^{(j)}, \tag{14}$$

with $\eta_{(i)(j)} = \mathrm{diag}(-1,1,1,1)$, we can introduce the spin vector $s^{(a)}$ of the particle as

$$S^{(c)(d)} = \mathcal{M}\epsilon^{(c)(d)}{}_{(a)(b)}u^{(a)}s^{(b)}, \tag{15}$$

where we have the completely antisymmetric tensor $\boldsymbol{\epsilon}$ as $\epsilon_{(0)(1)(2)(3)} = 1$. Considering that the motion of the spinning particle is confined to the equatorial plane, we then only have a nonvanishing component of the spin vector $s^{(2)} = -s$, with s the magnitude of the spin and $s > 0$ corresponding to a spin direction parallel to that of the Kerr black hole. As a result, we get nonvanishing components of the spin tensor $S^{(a)(b)}$ as

$$S^{(0)(1)} = -\mathcal{M}su^{(3)}, \tag{16a}$$

$$S^{(0)(3)} = \mathcal{M}su^{(1)}, \tag{16b}$$

$$S^{(1)(3)} = \mathcal{M}su^{(0)}. \tag{16c}$$

We now choose the Carter frame to calculate the equations of motion for the spinning particle in the Kerr spacetime. In the Carter frame

$$e_a^{(0)} = \sqrt{\frac{\Delta}{\Sigma}}(dt - a\sin^2\theta d\phi), \tag{17a}$$

$$e_a^{(1)} = \sqrt{\frac{\Sigma}{\Delta}}dr, \tag{17b}$$

$$e_a^{(2)} = \sqrt{\Sigma}d\theta, \tag{17c}$$

$$e_a^{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}[-adt + (a^2 + r^2)d\phi], \tag{17d}$$

the normalized four-momentum of the spinning particle is

$$u^{(0)} = \frac{[er^5 + (ea + es - j)ar^3 + (aeM - jM)sr^2]}{\sqrt{\Delta}\mathcal{X}}, \tag{18}$$

$$u^{(3)} = \frac{r^3(j - ea - es)}{\mathcal{X}}, \tag{19}$$

$$u^{(1)} = \sigma\sqrt{-1 + (u^{(0)})^2 - (u^{(3)})^2} = \sigma\sqrt{O}, \tag{20}$$

where $\mathcal{X} = r^4 - Mrs^2$, $\sigma = 1$ corresponds to a radially outgoing particle and $\sigma = -1$ for a radially ingoing one, and where O is the radial effective potential of the particle. By using Eq. (8), we can obtain the four-velocity of the spinning particle as

$$v^{(0)} = \frac{r^4 - Ms^2r}{-3Mr(u^{(3)})^2 - s^2Mr + r^4}u^{(0)}, \tag{21}$$

$$v^{(1)} = \frac{r^4 - Ms^2r}{-3Mr(u^{(3)})^2 - s^2Mr + r^4}u^{(1)}, \tag{22}$$

$$v^{(3)} = \frac{r^4 + 2Mrs^2}{-3Mr(u^{(3)})^2 - s^2Mr + r^4}u^{(3)}. \tag{23}$$

The equations of motion for the spinning particle are [26]

$$\frac{dt}{d\tau} = \frac{\mathcal{X}(a^2\mathcal{P}_2\mathcal{X} + a\Delta r\mathcal{P}_3 + \mathcal{P}_2r^2\mathcal{X})}{\sqrt{\Delta}[-3M{\mathcal{P}_1}^2s^2r^5 + \mathcal{X}^2r^4 - Mr\mathcal{X}^2s^2]}, \tag{24}$$

$$\frac{dr}{d\tau} = \sqrt{\frac{\Delta}{\Sigma}}v^{(1)}, \tag{25}$$

$$\frac{d\phi}{d\tau} = \frac{1}{a\sin\theta^2}\left(\frac{dt}{d\tau} - \sqrt{\frac{\Sigma}{\Delta}}v^{(0)}\right), \tag{26}$$

where

$$\mathcal{P}_1 = r[j - e(a + s)],$$
$$\mathcal{P}_2 = a^2er^2 - a[es(-Mr + r^2) + jr^2] + r^4e - jsMr,$$
$$\mathcal{P}_3 = 2Mrs^2 + r^4.$$

When $s = 0$, the equations reduce to the equations of motion for a spinless massive particle.

To make the motion of the spinning particle physical, we should constrain the ranges of the parameters. The particle's motion should comply with the timelike condition $v_{(a)}v^{(a)} < 0$ and the forward-in-time condition $dt/d\tau > 0$ [29]. Besides, we should keep $s \lesssim r_0 \ll r_+ \leq 2M$ [23], where r_0 denotes the size of the particle. Based on the necessity of the physical reasonability, we also restrict the radial effective potential $O \geq 0$. Starting from this condition, it can be proved that only the particle with a conserved angular momentum $j \leq 2e \equiv j_c$ can reach the horizon in the extreme Kerr geometry. A particle is critical if it holds an angular momentum $j = j_c$. Otherwise, the cases $j < j_c$ and $j > j_c$ correspond to a subcritical particle and a supercritical particle, respectively. We will exhaustively discuss the nonextreme case in the following.

III. THE ESCAPE PROBABILITY OF THE SPINNING PARTICLE

Without loss of generality, we now set the Kerr black hole mass as $M = 1$. As the radial and angular equations of motion for the spinning particle cannot be separated, we consider here the escape probability of the spinning particle in the equatorial plane [30,31]. By solving $O = 0$, we can obtain the critical conserved angular momenta of the particles as

$$j_+ = \frac{a^2(\mathcal{Y}_1 - 2er^2s) + e(r-3)r^4s + (r-2)r\mathcal{Y}_1 - \mathcal{Y}_2}{r(-2ars + (r-2)r^3 - s^2)}, \tag{27}$$

$$j_- = \frac{-a^2(2er^2s + \mathcal{Y}_1) + e(r-3)r^4s - (r-2)r\mathcal{Y}_1 - \mathcal{Y}_2}{r(-2ars + (r-2)r^3 - s^2)}, \tag{28}$$

where

$$\mathcal{Y}_1 = (r^3 - s^2)\sqrt{\frac{2ars + 2r^3 + s^2}{a^2 + (r-2)r}}, \tag{29}$$

$$\mathcal{Y}_2 = aer(2r^3 + (r+1)s^2). \tag{30}$$

At the event horizon limit, the two branches join to one point

$$j_+(r = r_+) = j_-(r = r_+) = \frac{e(a^4s + 2a^3r_+ - a^2r_+s + ar_+s^2 + 4r_+s)}{a^4 + 2as + s^2}. \tag{31}$$

As shown in Fig. 1, we denote the minimal value of the critical angular momentum j_+ for the spinning particle as

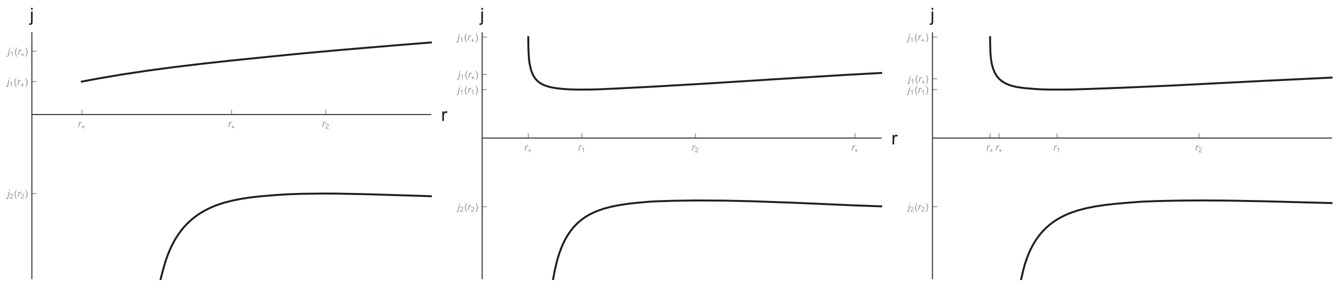

FIG. 1. The critical angular momentum which makes $O = 0$. Left panel: the extreme Kerr black hole case. Middle and right panels: the nonextreme Kerr black hole cases where $r_* > r_1$ and $r_* < r_1$, respectively.

$j_+(r_1)$, with r_1 the corresponding radial position, we also denote the maximal value of the critical angular momentum j_- as $j_-(r_2)$, with r_2 the corresponding radial position. We set the particle source at the position r_*. If the Kerr black hole is extreme, we can have $r_+ < r_*$; if the Kerr black hole is nonextreme, we have $r_+ < r_1 < r_*$ or $r_+ < r_* \leq r_1$. In the background of the extreme Kerr black hole, the spinning particle at the position r_* can escape to spatial infinity irrespective of its initial velocity if $j_+(r_+) < j < j_+(r_*)$, and it can escape to spatial infinity only with initially outgoing velocity in condition of $j_-(r_2) < j < j_+(r_+)$. In the background of the nonextreme Kerr black hole, if the spinning particle with $j_+(r_1) < j < j_+(r_*)$ is located at $r_* > r_1$, it can escape to spatial infinity irrespective of the sign of its initial velocity, and the outgoing particle with $j_-(r_2) < j < j_+(r_1)$ can emit to spatial infinity. With the condition $r_* \leq r_1$, no particle with initial ingoing velocity can go to spatial infinity and only an outgoing particle with $j_-(r_2) < j < j_+(r_1)$ can escape to spatial infinity.

The emission angle α can be introduced for the particle produced by a source at the Carter's frame, which can be defined by the spinning particle's four-momentum as [16,32]

$$\sin\alpha = \frac{p^{(\phi)}}{\sqrt{(p^{(r)})^2 + (p^{(\phi)})^2}}, \tag{32}$$

$$\cos\alpha = \frac{p^{(r)}}{\sqrt{(p^{(r)})^2 + (p^{(\phi)})^2}}. \tag{33}$$

The critical angles at which the spinning particle can escape to spatial infinity are

$$\alpha_{\mathrm{I}} \equiv \alpha[\sigma = -1, j = j_+(r_+)], \text{ extreme case}, \tag{34}$$

$$\alpha_{\mathrm{I}} \equiv \alpha[\sigma = -1, j = j_+(r_1)], \text{ nonextreme case}, \tag{35}$$

$$\alpha_{\mathrm{II}} \equiv \alpha[\sigma = -1, j = j_+(r_*)], \tag{36}$$

$$\alpha_{\mathrm{III}} \equiv \alpha[\sigma = 1, j = j_+(r_*)], \tag{37}$$

$$\alpha_{\mathrm{IV}} \equiv \alpha[\sigma = 1, j = j_-(r_2)]. \tag{38}$$

Note that in the case of $r_* < r_1$, we do not have α_{II} and α_{III}. It is obvious that [33]

$$\sin\alpha_{\mathrm{II}} = \sin\alpha_{\mathrm{III}} = 1, \tag{39}$$

$$\cos\alpha_{\mathrm{II}} = \cos\alpha_{\mathrm{III}} = 0. \tag{40}$$

So

$$\alpha_{\mathrm{II}} = \alpha_{\mathrm{III}} = \frac{\pi}{2}. \tag{41}$$

By specific calculation, we know that $\alpha_{\mathrm{IV}} < \alpha_{\mathrm{III}} = \alpha_{\mathrm{II}} < \alpha_{\mathrm{I}}$. The escape probability of the spinning particle can thus be defined by

$$\rho \equiv \frac{\alpha_{\mathrm{I}} - \alpha_{\mathrm{IV}}}{2\pi}. \tag{42}$$

For the extreme Kerr black hole, $j_+(r_+) = 2e$, we can analytically calculate the escape probability in the linear order of the particle's spin as

$$\rho = \mathcal{Z}_1 + \mathcal{Z}_2 s + \mathcal{O}(s^2), \tag{43}$$

where

$$\mathcal{Z}_1 = \frac{1}{2} - \frac{1}{2\pi}\arcsin\left(\frac{e}{\sqrt{e^2(r+1)^2 - r^2}}\right),$$

$$\mathcal{Z}_2 = \frac{e^3(r^3 + 2r^2 + 2r + 1) - er^3}{r[e^2(r+1)^2 - r^2]\sqrt{r[e^2(r+2) - r]}} > 0.$$

So we can know that the escape probability of the particle increases with the spin of the particle. This can be further confirmed by the left panel of Fig. 2, where we have taken the timelike condition and the forward-in-time condition into consideration. If $e = 1$, we have

$$\lim_{r_* \to 1} \rho \to \frac{1}{2} - \frac{\arcsin(\frac{1}{\sqrt{3}})}{2\pi} \sim 0.402$$

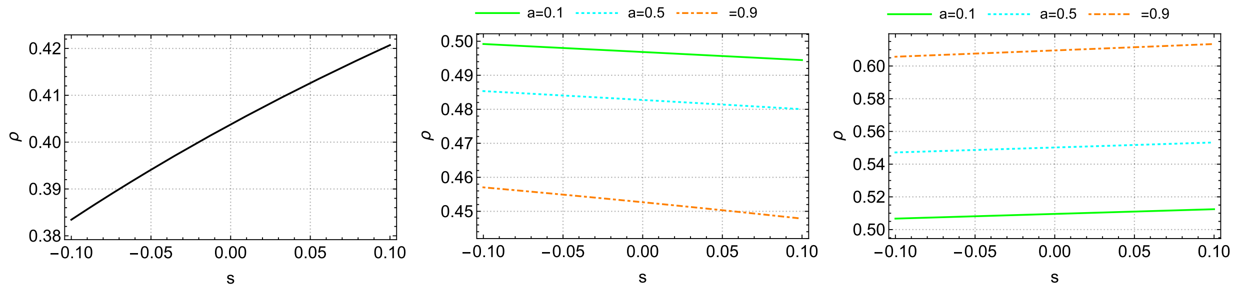

FIG. 2. Escape probabilities of spinning particles from the Kerr black hole for (left and middle panels) $M = 1$, $e = 1$, $r_* = 1.01r_+ < r_1$, (right panel) $r_* = 10r_+ > r_1$. The left panel applies to the extreme Kerr black hole case and the others apply to the nonextreme Kerr black hole case.

for the spinless particle, which is close to but not equal to the one ($\rho \sim 0.412$) obtained in Ref. [33]. For this reason, we have chosen the Carter frame for the particle here even though the locally nonrotating frame was used there. This tells us that the escape probabilities of the particles vary with the reference frame we choose. However, we reckon that different selections of observer's frames will not qualitatively change the results we report on in this paper. Detailed investigations on this are presented in [34]. At the same time, we can see that the escape probability of the particle is dependent upon its energy.

For the nonextreme Kerr black hole, we can numerically calculate the escape probabilities of the spinning particles. We individually show the variations of the escape probabilities with respect to the spin of the particles for cases $r_* < r_1$ and $r_* > r_1$ in the middle and right panels of Fig. 2. We can see that the escape probability increases with the spin for $r_* > r_1$ but decreases with the spin for $r_* < r_1$.

IV. THE ESCAPE PROBABILITY AND THE COLLISIONAL PENROSE PROCESS

In this section, we will study the relation between the escape probability and the maximum energy extraction efficiency in the collisional Penrose process for the spinning particles. We consider that an outgoing particle 1 collides with an ingoing particle 2 in the ergosphere of the Kerr black hole and suppose that the mass, the angular momentum j_k, and the energy e_k of the two particles are equal when they collide.

The maximum energy extraction efficiency of this kind of process around the extreme Kerr black hole has been explored in Ref. [11] in the case in which the two produced particles are both massive. We will generally calculate the maximum energy extraction efficiency in both the nonextreme and extreme Kerr black hole backgrounds.

Without loss of generality, we denote the spins of both particle 1 and particle 2 as s_0, and we suppose that both the produced outgoing massive particle 3 and the ingoing massive particle 4 are endowed with the same spin s_0 based on the conservation of the spin. The total radial momentum is conserved, which gives

$$u_1^{(1)} - u_2^{(1)} = u_3^{(1)} - u_4^{(1)} = \epsilon, \tag{44}$$

where ϵ denotes the total radial momentum of the particles. Owing to the conservation of energy and the angular momentum, we have

$$e_1 + e_2 = e_3 + e_4 = 2e_1, \tag{45}$$

$$j_1 + j_2 = j_3 + j_4 = 2j_1. \tag{46}$$

Substituting $e_4 = 2e_1 - e_3$ and $j_4 = 2j_1 - j_3$ into Eq. (44), we can obtain

$$u_3^{(1)}(a, e_3, j_3, s = s_0, r) - u_4^{(1)}(a, e_1, j_1, e_3, j_3, s = s_0, r) = \epsilon. \tag{47}$$

Then we obtain

$$j_3 = j_3(a, e_1, j_1, e_3, s_0, r, \epsilon). \tag{48}$$

Substituting it into the effective potential O for particle 3 and using the conditions

$$O_3(a, e_1, j_1, e_3, s_0, r, \epsilon) \geq 0, \tag{49}$$

$$v_{(a)} v^{(a)}\big|_{a,s_0,r,\epsilon,e=e_3,j=j_3} < 0, \tag{50}$$

$$\frac{dt}{d\tau}\bigg|_{a,s_0,r,\epsilon,e=e_3,j=j_3} > 0, \tag{51}$$

we have

$$O_3 = O_3(a, e_1, j_1, e_3, s_0, r, \epsilon) = \mathcal{A}e_3^2 + \mathcal{B}e_3 + \mathcal{C} \geq 0, \tag{52}$$

where $\mathcal{A} < 0$. The physically reasonable maximum value of the escaping massive particle 3 is

$$e_3 = e_3^{\max}(a, e_1, j_1, s_0, r, \epsilon) = \frac{-\mathcal{B} + \sqrt{\mathcal{B}^2 - 4\mathcal{A}\mathcal{C}}}{2\mathcal{A}}, \tag{53}$$

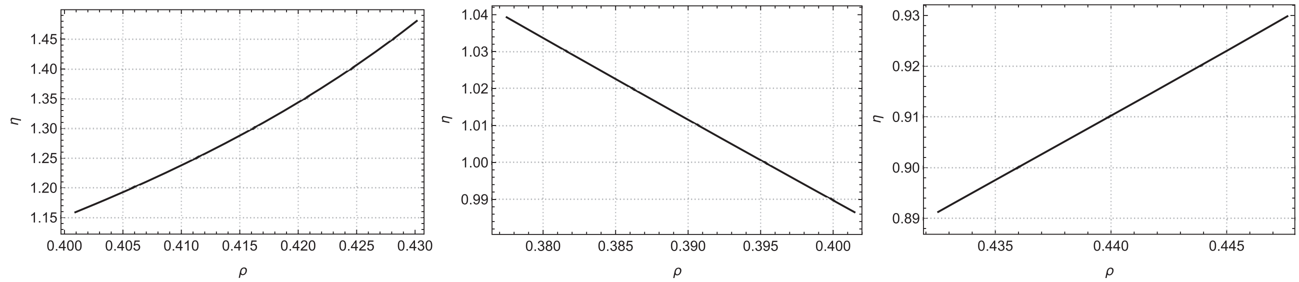

FIG. 3. Variations of the energy extraction efficiencies in the collisional Penrose process with respect to the escaping probability of the produced particle for (left panel) $M = 1$, $\epsilon = 0$, $a = 1$, (middle panel) 0.65, (right panel) 0.9, (left panel) $r_* = 1.01r_+$, (middle panel) $1.1r_+$, (right panel) $1.35r_+$, $j_1 = j_2 = 2$, $e_1 = e_2 = 1$, $-0.1 < s_0 < 0.1$. Different from the cases in Fig. 2, the energy of the escaping particle here cannot be set to be 1 and it depends on the collisional process.

and the maximum energy extraction efficiency η of the collisional Penrose process is

$$\eta = \frac{e_3}{e_1 + e_2}. \tag{54}$$

For the produced escaping particle 3, we can also calculate its escape probability, which is related to its energy e_3. Then it is intriguing to analyze the relation between the escape probability of the particle and the efficiency of the energy extraction process.

We show the relation between the escape probability of the produced emitted particle and the maximum energy extraction efficiency of the collisional Penrose process in Fig. 3. From the diagrams, we can know that, for the extreme Kerr black hole, the energy extraction efficiency increases with an increasing escape probability. For the nonextreme Kerr black hole, there are two different variation trends. If the radius of the collision point is less than the one which makes j_+ minimal, the energy extraction efficiency decreases with the increasing escape probability; if the radius of the collision point is greater than the one which makes j_+ minimal but less than r_e, the energy extraction efficiency increases with the increasing escape probability.

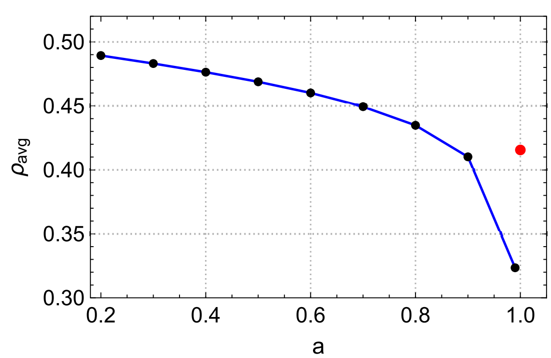

FIG. 4. Variation of the average escape probability $\rho_{\rm avg}$ of the particle produced in the collisional Penrose process with respect to the rotation parameter a of the black hole for $M = 1$, $\epsilon = 0$, $j_1 = j_2 = 2$, $e_1 = e_2 = 1$, $-0.1 < s_0 < 0.1$. The escape probability is roughly calculated by $[\eta(s_0 = -0.1) + \eta(s_0 = 0.1)]/2$, as we have chosen $-0.1 < s_0 < 0.1$ and η almost linearly changes with ρ and s. As we have chosen $r_* = 1.01r_+$, the rotation parameter of the black hole should be $a > 0.198$ so that the collisional point is inside the ergosphere. Note that the red point corresponds to the extreme black hole case.

However, we can see that the particle's escape probability in the collisional Penrose process does not change significantly with the particle's spin. In this regard, different energy extraction efficiencies of the collisional Penrose process can almost correspond to an average escape probability of the spinning particle. Following this, we show the average escape probability $\rho_{\rm avg}$ of the particle produced in the collisional events which take place near the event horizon of the black hole in terms of the black hole rotation parameter in Fig. 4. We see that the average escape probability of the spinning particle produced in the collisional Penrose process near the event horizon of the black hole decreases with the rotation parameter of the Kerr black hole, except for the extreme case (corresponding to the red point). There are subtle properties one should notice here. Because $r_* = 1.01r_+$, we can see a jump of $\rho_{\rm avg}$ from $\rho_{\rm avg}(a = 0.99)$ to $\rho_{\rm avg}(a = 1)$. In fact, there is a turning point for the curve between $a = 0.99$ and $a = 1$, after which $\rho_{\rm avg}$ increases with a, as we will have $r_* > r_1$ if a increases to a certain value very close to 1. For instance, we have $\rho_{\rm avg}(a = 0.9999) = 0.344$. In any case, if we choose $r_* \to r_+$, we can obtain a monotonically decreasing curve from $a \to 0^+$ to $a = 1$, as we always have $r_* \leq r_1$ and the equal sign is for $a = 1$.

V. CONCLUSIONS

In this paper, we revisited the collisional Penrose process in term of the escape probability for the spinning particle. To that end, we first studied the law of the escape probability for the spinning particle around the Kerr black hole. We found that the escape probability ρ of the spinning particle increases with the particle's spin s around the extreme Kerr black hole. In the nonextreme Kerr black hole background, ρ decreases with s if the particle source is located at $r_* < r_1$ and ρ increases with s if the particle source is located at $r_* > r_1$, where r_1 is the position which makes the impact parameter of the particle minimal.

We then investigated the relation between the energy extraction efficiency η of the collisional Penrose process and the escape probability ρ of the produced particle with varying spin. Note that the escape probability of the particle is affected by the particle's energy, so we cannot obtain the law directly. By calculation, we discovered that η increases with ρ for the extreme Kerr black hole. However, for the nonextreme Kerr black hole, η decreases with ρ if the collisional point is located at $r_* < r_1$, and η increases with ρ if the collisional point is located at $r_* > r_1$.

Noticing that the change of the particle's escape probability is relatively minuscule, we further studied the average escape probability for the spinning particle produced in the collisional Penrose process. As a result, we found that the particle's escape probability decreases with the rotation parameter of the Kerr black hole in the horizon limit.

Our discussion is based on viewing the particle as an extended object which has small varying spin. We can know that r_1 is a critical position where properties of the escape probability and the energy extraction efficiency change qualitatively for the nonextreme Kerr black hole. In the extreme Kerr black hole case, r_1 coincides with the event horizon. Our results will be beneficial to astrophysical observation investigations. For astrophysically relevant black holes, $a \lesssim 0.998$. Our results predict a near-horizon physical scenario around the astrophysical rotating black hole: (1) the escape probability of the spinning particle decreases with the pole/depole spin angular momentum of the particle, and (2) the energy extraction efficiency decreases with the minusculely increasing escape probability of the spinning particle.

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