

# Thermal quantum gravity condensates in group field theory cosmology

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The condensate cosmology program of group field theory has produced several interesting results. The key idea is in the suggestion that a macroscopic homogeneous spacetime can be approximated by a dynamical condensate phase of the underlying microscopic system of an arbitrarily large number of candidate quanta of geometry. In this work, we extend the standard treatments in two ways: by using a class of thermal condensates, the coherent thermal states, which encode statistical fluctuations in quantum geometry; and by introducing a suitable class of smearing functions as nonsingular, well-behaved generalizations for relational clock frames in group field theory. In particular, we investigate an effective relational cosmological dynamics for homogeneous and isotropic spacetimes, extracted from a class of free group field theory models, and subsequently investigate aspects of its late and early times evolution. We find the correct classical limit of Friedmann equations at late times, with a bounce and an accelerated expansion at early times. Specifically, we find additional correction terms in the evolution equations corresponding to the statistical contribution of the new thermal condensates in general, and a higher upper bound on the number of  $e$ -folds, even without including any interactions.

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## I. INTRODUCTION

The ultimate goal of any theory of quantum gravity is to describe the known physics, while also providing novel falsifiable physical predictions on a measurable scale. One of the most important arenas in this respect is cosmology, with features such as singularity resolution and inflation representing crucial checkpoints for any viable model based on an underlying theory of quantum gravity. It is thus important for any candidate theory to find a suitable continuum and semiclassical regime within the full theory, in which standard cosmology can be approximated, up to effective corrections of quantum gravitational origin. In fact, in the group field theory (GFT) approach, such a regime has been suggested via a class of condensate phases of the system [1–3].

Group field theory is a statistical field theory defined formally by a partition function,

$$Z_{\text{GFT}} = \int [D\varphi D\bar{\varphi}] e^{-S(\varphi, \bar{\varphi})}, \quad (1.1)$$

where the fields  $\varphi$  and  $\bar{\varphi}$  are defined over a Lie group base manifold  $G$ ,

$$\varphi: G \rightarrow \mathbb{C}: g \mapsto \varphi(g), \quad (1.2)$$

and  $S$  is a generically nonlocal function of the fields dictating the system's dynamics. The choice of the base manifold we are interested in is  $SU(2)^d \times \mathbb{R}$ , with  $d \geq 3$ . This choice is understood as considering a model of discrete gravity associated with  $SU(2)^d$  [4–9], coupled to a scalar matter field taking values in  $\mathbb{R}$  providing, for instance, the possibility of using the matter field  $\phi$  to define a relational frame of reference [1–3, 10, 11]. The group field is invariant under a diagonal right action of  $SU(2)$  on  $SU(2)^d$ ,

$$\varphi(\vec{g}, \phi) = \varphi(g_i h, \phi), \quad \forall h \in SU(2), \quad (1.3)$$

where  $\vec{g} = (g_i) \in SU(2)^d$  with  $i = 1, \dots, d$ , and  $\phi \in \mathbb{R}$ . This symmetry encodes the geometric condition of closure of polyhedra with  $d$  faces labeled by  $SU(2)$  data, associated with the group fields. These polyhedra are the discrete fundamental quanta of geometry building up spacetime, which can then be

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modeled as a peculiar, background independent<sup>1</sup> quantum many-body system [12–17]. In fact, the GFT partition function (1.1) dynamically generates discrete quantum spacetimes that are labeled 2-complexes (spin foams), with boundary states given by labeled graphs (spin networks) [4–9,18]. Bulk processes and boundary states of group field theories are thus dual to polyhedral complexes, in most commonly studied models based on loopless combinatorics [15].

Such a many-body perspective, with the polyhedral quanta of geometry being excitations of the field  $\varphi$ , has been very useful. It has allowed for tangible explorations of connections of group field theory (which in turn is strictly related to several other approaches, including loop quantum gravity [19–22], spin foams [23,24], causal dynamical triangulations [25], tensor models [26], and lattice quantum gravity [27]) with quantum information theory [28–32], and also with quantum statistical mechanics and thermal physics [14,33–37]. It has further allowed for importing ideas and tools from condensed matter theory, which has been crucial, for instance, in the development of quantum condensate cosmology [1–3].

The present work concerns the incorporation of statistical fluctuations in GFT condensate cosmology, directly building on works in [11,33,38], and evaluating the consequences of the presence of such fluctuations in the effective cosmological evolution equations. To this end, we use the framework of thermal representations for GFT and thermal condensates, i.e., coherent thermal states, introduced in [33]. In general, the notion of thermal states in a background independent system is a particularly subtle issue, mainly due to the absence of an absolute notion of time (see, for instance, Ref. [14], and references therein). Nevertheless, thermal condensates [33] can be defined in the present quantum gravitational system using a generalized notion of Gibbs statistical equilibrium and certain quantum many-body techniques [14,34,36]. These condensates are constructed to mathematically include statistical fluctuations in a given (set of) observable(s), not necessarily energy, and without relying on specific geometric interpretations of the associated quantities, e.g., generalized temperature, free energy, entropy [14,33,34,36]. As will be discussed below, the observable of interest here is spatial volume. The resulting state encodes fluctuations in the underlying quantum geometry, with its corresponding generalized temperature being (by construction) a statistical parameter controlling the strength of these fluctuations [33,36]. We understand the ensuing system as describing a phase of the

universe in which not all quanta of geometry have condensed.

In the context of cosmology, a thermal phase seems likely in any reasonable geometrogenesis scenario, in which the universe transitions from a primordial pregeometric hot thermal phase, to a phase with an approximate notion of continuum and macroscopic geometry (here, encoded in the notion of a condensate [2]), and in general with a leftover thermal cloud of quanta that have not condensed. In other words, here, we understand a pure, zero temperature GFT condensate that has been used extensively in previous works [1,3,11,38], as describing a suitable macroscopic phase only at very late times of the system’s evolution and not throughout. What we are working with instead is an intermediary phase that would be expected to arise in a transition between a hot pregeometric phase and a pure condensate. Thus, in this work we present a tentative picture of a universe being modeled as a quantum gravitational condensate of elementary quanta of geometry along with a thermal cloud of the same quanta over it, and in which an early time phase dominated by the thermal cloud and a late time phase dominated by the condensate are generated dynamically.

We work with a free GFT model and thermal condensates with a static (nondynamical) thermal cloud, to derive effective generalized evolution equations for homogeneous and isotropic cosmology, which include correction terms originating in the underlying quantum gravitational and statistical properties of the system. At late times we recover the correct general relativistic limit, while at early times we get a bounce between contracting and expanding phases, along with a phase of accelerated expansion characterized by an increased number of  $e$ -folds compared with previously reported numbers for the same class of free models.

The article is organized as follows. In Sec. II we present a summary of the construction of thermal representations in GFT associated with generalized Gibbs states, and define coherent thermal states as candidates for thermal quantum gravitational condensates for applying in condensate cosmology. In Sec. III we analyze a free GFT model for effective cosmology in the presence of thermal fluctuations introduced via coherent thermal states. We start with explicating the choice of the state in Sec. III A, based on which we derive the GFT effective equations of motion in Sec. III B. In Secs. III C and III D, we reformulate the effective dynamics in terms of relational “clock” functions, implemented as smearing functions along the  $\phi$  direction of the base manifold  $G$ . As we will discuss, this provides a suitable nonsingular generalization of a relational frame used in previous works in terms of the coordinate  $\phi$ . We further derive the effective generalized Friedmann equations for flat homogeneous and isotropic cosmology in Sec. III E, recover the correct classical general relativistic limit in a late time regime in Sec. III E 1, and characterize the early time evolution through an assessment of

<sup>1</sup>Background independence in nonperturbative discrete quantum gravity approaches, such as group field theory, is in the radical sense of having no spacetime (and related continuum geometric) structures *a priori*. General relativistic spacetime (modulo effective quantum gravitational corrections) must then emerge from an underlying nonspatiotemporal, thus manifestly background independent, theory of quantum gravity.

singularity resolution and inflation in Sec. III E 2. Finally, we close with a discussion of various aspects surrounding the inclusion of statistical fluctuations, interactions, and its implications at the level of the effective GFT models, and suggest further extensions of this work in Sec. IV.

## II. THERMAL QUANTUM GRAVITY CONDENSATES

### A. Bosonic group field theory

The quantum operator group field theory for bosonic<sup>2</sup> quanta is based on the commutation algebra,

$$[\varphi(\vec{g}, \phi), \varphi^\dagger(\vec{g}', \phi')] = \mathbb{I}(\vec{g}, \vec{g}') \delta(\phi - \phi') \quad (2.1)$$

with  $[\varphi(\vec{g}, \phi), \varphi(\vec{g}', \phi')] = [\varphi^\dagger(\vec{g}, \phi), \varphi^\dagger(\vec{g}', \phi')] = 0$ . Here,  $\delta$  is a delta distribution for functions on  $\mathbb{R}$ ,  $\mathbb{I}$  is a delta distribution for gauge-invariant functions on  $SU(2)^d$ , and we have dropped the hats over operators to simplify our notation. The Hilbert space for a single gauge-invariant quantum is

$$\mathcal{H} = L^2(SU(2)^d/SU(2)) \otimes L^2(\mathbb{R}), \quad (2.2)$$

where the quotient by  $SU(2)$  ensures the gauge invariance of Eq. (1.3). This is the state space of a single quantum  $d$ -polyhedron labeled with a real number  $\phi$ .

In order to work with formally well-defined quantities, we smear the operator-valued distributions  $\varphi(\vec{g}, \phi)$  with a suitable basis of functions in  $\mathcal{H}$ ,

$$f_{J\alpha}(\vec{g}, \phi) = D_J(\vec{g}) \otimes T_\alpha(\phi), \quad (2.3)$$

where  $\{T_\alpha(\phi)\}_\alpha$  is any orthonormal basis of complex-valued smooth functions in  $L^2(\mathbb{R})$  for the scalar field, labeled by a discrete index  $\alpha$ , and  $\{D_J(\vec{g})\}_J$  is the complete set of orthonormal Wigner functions for gauge-invariant functions on  $SU(2)^d$ , given by  $D_J(\vec{g}) = \sum_{\vec{n}} C_{i,\vec{n}}^{\vec{j}} \prod_{i=1}^d D_{m_i, n_i}^{j_i}(g_i)$ . The basis elements are labeled by  $J \equiv (\vec{j}, \vec{m}, \iota)$ , with irreducible representations of  $SU(2)$  indexed by  $j_i \in \mathbb{N}/2$ , internal representation index  $m_i \in (-j_i, \dots, j_i)$  for each  $j_i$ , and intertwiners  $C_{i,\vec{n}}^{\vec{j}}$  [i.e.,  $SU(2)$  gauge-invariant tensors] indexed by  $\iota$ . Then the ladder operators in this new basis (labeled by  $J, \alpha$ ) are given by smearing the operators in the original basis (labeled by  $\vec{g}, \phi$ ), that is,

<sup>2</sup>Bosonic statistics corresponds to an invariance under particle exchanges in a generic many-body wave function. In the present context, this corresponds to a graph automorphism under node exchanges, since a generic many-body wave function here describes the state of a labeled graph.

$$a_{J\alpha} := \varphi(f_{J\alpha}) = \int_{SU(2)^d \times \mathbb{R}} d\vec{g} d\phi \bar{D}_J(\vec{g}) \bar{T}_\alpha(\phi) \varphi(\vec{g}, \phi), \quad (2.4)$$

$$a_{J\alpha}^\dagger := \varphi^\dagger(f_{J\alpha}) = \int_{SU(2)^d \times \mathbb{R}} d\vec{g} d\phi D_J(\vec{g}) T_\alpha(\phi) \varphi^\dagger(\vec{g}, \phi), \quad (2.5)$$

where  $\bar{D}_J(\vec{g})$  denotes the adjoint matrix to  $D_J(\vec{g})$ . Being simply a change of basis, the algebra is unchanged, now taking the form

$$[a_{J\alpha}, a_{J'\alpha'}^\dagger] = \delta_{JJ'} \delta_{\alpha\alpha'} \quad (2.6)$$

and  $[a_{J\alpha}, a_{J'\alpha'}] = [a_{J\alpha}^\dagger, a_{J'\alpha'}^\dagger] = 0$ . The vacuum state  $|0\rangle$ , which is specified by the annihilation operators,

$$a_{J\alpha}|0\rangle = 0 \quad \forall J, \alpha \quad (2.7)$$

generates a Fock space  $\mathcal{H}_F$  (for symmetric, bosonic states), via cyclic actions of the algebra generators  $\{a_{J\alpha}, a_{J\alpha}^\dagger, 1\}$ , given by

$$\mathcal{H}_F = \bigoplus_{N \geq 0} \text{sym}(\mathcal{H}^{\otimes N}). \quad (2.8)$$

Last, we recall that a useful class of states in  $\mathcal{H}_F$  are the coherent states, defined by

$$|\sigma\rangle := D_a(\sigma)|0\rangle, \quad (2.9)$$

where  $D_a$  is the displacement operator,

$$D_a(\sigma) = e^{a^\dagger(\sigma) - a(\sigma)}, \quad (2.10)$$

and

$$a(\sigma) = \sum_{J,\alpha} \bar{\sigma}_{J\alpha} a_{J\alpha} = \int d\vec{g} d\phi \bar{\sigma}(\vec{g}, \phi) \varphi(\vec{g}, \phi), \quad (2.11)$$

$$a^\dagger(\sigma) = \sum_{J,\alpha} \sigma_{J\alpha} a_{J\alpha}^\dagger = \int d\vec{g} d\phi \sigma(\vec{g}, \phi) \varphi^\dagger(\vec{g}, \phi) \quad (2.12)$$

are the smeared operators, for any single-particle wave function  $\sigma \in \mathcal{H}$ .

### B. Thermal representations

The operator setup summarized above has been used to show that effective cosmological dynamics, featuring various aspects such as early time bounces, inflation, cyclic phases etc. [1–3], can be supported by GFT coherent condensates of the form (2.9) above (with varying details depending on the dynamical model, of course). Particularly in the context of this article, it has been shown that an effective Friedmann dynamics [11,38–41] can be extracted

from an underlying GFT dynamics for coherent states (2.9), thus exhibiting the viability of such condensates as suitable quantum gravitational phases for the cosmological sector [2].

However, as is evident from Eq. (2.9), this phase is built over a degenerate and unentangled vacuum (2.7), characterized by uncorrelated quanta. Furthermore, it does not incorporate statistical fluctuations in the underlying degrees of freedom. Now from a physical point of view, one would expect that thermal fluctuations play a role in the dynamical evolution of physical observables describing any macroscopic system, including a spacetime built from a large number of elementary quanta of geometry. In other words, studying statistical, thermal properties of quantum spacetimes would be valuable [14,33–36], especially in strong gravity regimes such as for the early-time Planck-scale era in the context of quantum cosmology models. Along these lines, thermal condensates of the form (2.25) have recently been suggested to be a suitable class of states to consider [33].

In general, thermal effects are encoded in statistical mixed states, represented by density operators defined on a representation Hilbert space (here,  $\mathcal{H}_F$ ), and of particular interest are statistical equilibrium states. In the present background independent context of GFT, these are given by generalized Gibbs states of the form

$$\rho_{\{\beta_\ell\}} = \frac{1}{Z_{\{\beta_\ell\}}} e^{-\sum_\ell \beta_\ell \mathcal{O}_\ell}, \quad (2.13)$$

where  $\{\mathcal{O}_\ell\}$  is a finite set of observables of interest and  $\{\beta_\ell\}$  is a multivariable generalized inverse temperature [14,34,36]. One can extend this formalism for generalized Gibbs states even further in order to address the question of including thermal fluctuations in condensates, by constructing thermal representations as is done for standard finite-temperature quantum field theories. Such an extension of the GFT formalism to include thermal representations was introduced in [33], by using tools from thermofield dynamics [42–46]. Below we review the main details of this construction.

A simple class of generalized Gibbs states that are properly normalizable,<sup>3</sup> and are of interest to us particularly in the present context of cosmology, is

$$\rho_\beta = \frac{1}{Z} e^{-\beta \mathcal{P}}, \quad (2.14)$$

where  $\mathcal{P}$  is a positive, extensive operator on  $\mathcal{H}_F$ , and  $\beta \in \mathbb{R}_+$ . In the context of gravity, for instance,  $\mathcal{P}$  could be chosen to be a spatial volume operator [14,36], as will be

<sup>3</sup>We refer to [33,36] for the details of proof of normalization for this class of states.

done later in Sec. III. For now, we leave it as this more general class of states, along the lines in [33].

We first define the zero temperature phase ( $\beta = \infty$ ), which is given in terms of the Hilbert space,

$$\mathcal{H}_\infty = \mathcal{H}_F \otimes \tilde{\mathcal{H}}_F. \quad (2.15)$$

Here, the Hilbert space  $\tilde{\mathcal{H}}_F$  is conjugate to  $\mathcal{H}_F$  under the action of an antiunitary (modular conjugation or tilde conjugation) operator [43,46–49] and is the Hilbert space of the tilde degrees of freedom. For a detailed discussion on the tilde and nontilde degrees of freedom, their relations, and possible meanings, we refer to [33] and references therein. For now, we note that the zero temperature Hilbert space  $\mathcal{H}_\infty$  can be understood as describing a bipartite system, with the nontilde degrees of freedom residing in  $\mathcal{H}_F$ , and its associated algebra generated by  $\{a, a^\dagger, 1\}$ , being our original system of interest. Thus also, the observables of interest here are those that belong to the algebra of nontilde  $a$  (or equivalently,  $\varphi$ ) operators.

The space  $\mathcal{H}_\infty$  is a Fock space on the cyclic vacuum

$$|0_\infty\rangle = |0\rangle \otimes |\tilde{0}\rangle \quad (2.16)$$

with ladder operators  $\{a_{J,\alpha}, a_{J,\alpha}^\dagger, \tilde{a}_{J,\alpha}, \tilde{a}_{J,\alpha}^\dagger\}_{\beta=\infty}$  that satisfy

$$[a_{J,\alpha}, a_{J',\alpha'}^\dagger] = [\tilde{a}_{J,\alpha}, \tilde{a}_{J',\alpha'}^\dagger] = \delta_{JJ'} \delta_{\alpha\alpha'} \quad (2.17)$$

and all other commutators, including those between tilde and nontilde operators, vanish. The vacuum  $|0_\infty\rangle$  satisfies

$$a_{J,\alpha}|0_\infty\rangle = \tilde{a}_{J,\alpha}|0_\infty\rangle = 0 \quad \forall J, \alpha. \quad (2.18)$$

The thermal algebra  $\{b_{J,\alpha}, b_{J,\alpha}^\dagger, \tilde{b}_{J,\alpha}, \tilde{b}_{J,\alpha}^\dagger\}_{0 < \beta < \infty}$  is then introduced via thermal Bogoliubov transformations of the generators (2.17), given by

$$b_{J,\alpha} := \cosh[\theta_{J,\alpha}(\beta)] a_{J,\alpha} - \sinh[\theta_{J,\alpha}(\beta)] \tilde{a}_{J,\alpha}^\dagger, \quad (2.19)$$

$$\tilde{b}_{J,\alpha} := \cosh[\theta_{J,\alpha}(\beta)] \tilde{a}_{J,\alpha} - \sinh[\theta_{J,\alpha}(\beta)] a_{J,\alpha}^\dagger, \quad (2.20)$$

and analogous expressions for their adjoints  $b_{J,\alpha}^\dagger$  and  $\tilde{b}_{J,\alpha}^\dagger$ . The parameters  $\theta_{J,\alpha}(\beta)$  encode complete information about the statistical state, being functions of  $\beta$  and eigenvalues of the observables characterizing the state. The temperature-dependent annihilation operators define a new vacuum,

$$b_{J,\alpha}|0_\beta\rangle = \tilde{b}_{J,\alpha}|0_\beta\rangle = 0 \quad \forall J, \alpha, \quad (2.21)$$

called a thermal vacuum, at inverse temperature  $\beta$ . It is cyclic, in turn generating a thermal Hilbert space  $\mathcal{H}_\beta$  via the action of the  $\beta$ -ladder operators, which create and annihilate  $b$  quanta over  $|0_\beta\rangle$ . It is, in fact, inequivalent to all the different vacua at different temperatures, including the zero

temperature  $|0_\infty\rangle$  vacuum [33]. Thus also their corresponding thermal representations labeled by the parameter  $\beta$  are all inequivalent, each representing a distinct statistical phase of the system. Notice that  $|0_\beta\rangle$  is an entangled state with quantum correlations between pairs of  $a$  and  $\tilde{a}$  quanta.

In the present case, the functions  $\theta_{J\alpha}(\beta)$  are uniquely associated with the Gibbs states  $\rho_\beta$  of Eq. (2.14). In practice, they are usually determined by the following condition for the number operator<sup>4</sup>:

$$\text{Tr}_{\mathcal{H}_F}(\rho_\beta a_{J\alpha}^\dagger a_{J\alpha}) = \langle 0_\beta | a_{J\alpha}^\dagger a_{J\alpha} | 0_\beta \rangle_{\mathcal{H}_\beta}, \quad (2.22)$$

where evaluating the right-hand side (using inverse Bogoliubov transformations) gives

$$\text{Tr}_{\mathcal{H}_F}(\rho_\beta a_{J\alpha}^\dagger a_{J\alpha}) = \sinh^2[\theta_{J\alpha}(\beta)] \quad (2.23)$$

for all  $J, \alpha$ . Last, Bogoliubov transformations are canonical. Thus, the  $\beta$ -ladder operators satisfy the same bosonic commutation algebra as before, i.e.,

$$[b_{J\alpha}, b_{J'\alpha'}^\dagger] = [\tilde{b}_{J\alpha}, \tilde{b}_{J'\alpha'}^\dagger] = \delta_{JJ'} \delta_{\alpha\alpha'}, \quad (2.24)$$

and all other commutators vanish.

### C. Coherent thermal states

Coherent thermal states are the canonical coherent states over a thermal vacuum  $|0_\beta\rangle$  which are invariant under the tilde conjugation [33,46,50–52]. They are elements of the thermal Hilbert space  $\mathcal{H}_\beta$ , obtained through the action of displacement operators of the form (2.10), and are defined as

$$|\sigma, \bar{\sigma}; \beta\rangle := D_a(\sigma) D_{\tilde{a}}(\bar{\sigma}) |0_\beta\rangle, \quad (2.25)$$

where  $D_{\tilde{a}}$  is a displacement operator of the same form as (2.10) but for the  $\tilde{a}$  ladder operators. Coherent thermal states are eigenstates of  $\beta$ -annihilation operators with temperature-dependent eigenfunctions,

$$b_{J\alpha} |\sigma, \bar{\sigma}; \beta\rangle = (\cosh[\theta_{J\alpha}] - \sinh[\theta_{J\alpha}]) \sigma_{J\alpha} |\sigma, \bar{\sigma}; \beta\rangle, \quad (2.26)$$

$$\tilde{b}_{J\alpha} |\sigma, \bar{\sigma}; \beta\rangle = (\cosh[\theta_{J\alpha}] - \sinh[\theta_{J\alpha}]) \bar{\sigma}_{J\alpha} |\sigma, \bar{\sigma}; \beta\rangle, \quad (2.27)$$

where  $\sigma_{J\alpha} = (f_{J\alpha}, \sigma)_{\mathcal{H}}$ . They are not, however, the eigenstates of the original annihilation operators  $a_{J\alpha}$ , belonging to our system of interest. This fact induces the emergence of nontrivial thermal contributions, along with coherence properties, in the expectation values of operators originally defined on  $\mathcal{H}_\infty$ , or its relevant subspace  $\mathcal{H}_F$ . For example,

the average number density of the mode  $J\alpha$  in a coherent thermal state is

$$\langle \sigma, \bar{\sigma}; \beta | a_{J\alpha}^\dagger a_{J\alpha} | \sigma, \bar{\sigma}; \beta \rangle = |\sigma_{J\alpha}|^2 + \sinh^2[\theta_{J\alpha}], \quad (2.28)$$

which contains both the usual coherent number density and an additional thermal contribution.

It is important to remark here that our use of the basis  $f_{J\alpha}$ , in particular of the countable basis  $\{T_\alpha\}_{\alpha \in \mathbb{N}}$  for  $L^2(\mathbb{R})$ , in order to develop the finite-temperature GFT formalism in terms of the ladder operators [Eqs. (2.4), (2.5), (2.19), and (2.20)], was crucial. The observables that one might consider in a chosen model must admit domains of definition which contain the sector of Hilbert space that one is interested in, here coherent thermal states. Otherwise, no calculation could be carried out without running into divergences and ill-defined expressions. This, in particular, applies to  $\phi$ -dependent operators, not smeared with  $T_\alpha$ . For instance, if one considers the number density operator as a function of  $\phi$ ,

$$\begin{aligned} N_J(\phi) &= \int d\vec{g} d\vec{g}' \bar{D}_J(\vec{g}') D_J(\vec{g}) \phi^\dagger(\vec{g}, \phi) \phi(\vec{g}', \phi) \\ &= a_{J\alpha}^\dagger(\phi) a_{J\alpha}(\phi), \end{aligned} \quad (2.29)$$

then the calculation of the expectation value in a coherent thermal state would give

$$\langle N_J(\phi) \rangle_{\sigma, \bar{\sigma}; \beta} = |\sigma_J(\phi)|^2 + \sinh^2[\theta_J(\phi)] \delta(\phi - \phi), \quad (2.30)$$

which is clearly ill-defined due to the presence of the Dirac delta distribution  $\delta(0)$  in the thermal part evaluated at the singular point. We thus need to consider smeared observables such as the operator  $a_{J\alpha}^\dagger a_{J\alpha}$  in (2.28), where now the thermal contribution contains a well-defined Kronecker delta  $\delta_{\alpha\alpha}$  coefficient instead.

At this point, we have introduced the necessary kinematical aspects of the theory in order to discuss an effective cosmological model incorporating statistical fluctuations of quantum geometry, which is the subject of the rest of the paper.

## III. THERMAL CONDENSATE COSMOLOGY

We start by presenting the effective free GFT dynamics in a condensate phase with fluctuating geometric volume. We then introduce the notion of a reference clock function and reformulate the setup, including the effective dynamics in terms of functional quantities and equations of motion, with respect to a generic class of these clock functions. Based on this, we present an effective, relational, homogeneous and isotropic cosmological model, and discuss the late and early times evolution it describes.

<sup>4</sup>In principle, the condition (2.22) of identifying observable averages in the two representations, must be satisfied by the full algebra [33,42–46].

### A. Condensates with volume fluctuations

Since we are interested in the homogeneous and isotropic cosmological sector, the main observable of interest is the volume operator, in particular, the volume associated with a (spatial) submanifold given by a foliation parametrized by a clock function (see Sec. III C). Recall that the GFT volume operator associated with a generic many-body state in  $\mathcal{H}_F$  is

$$V := \sum_{J,\alpha} v_J a_{J\alpha}^\dagger a_{J\alpha}, \quad (3.1)$$

where  $v_J \in \mathbb{R}_{\geq 0}$  is the volume assigned to a single quantum with a configuration  $J = (\vec{j}, \vec{m}, \iota)$ . This is an extensive positive operator on  $\mathcal{H}_F$ , and its action on any multiparticle state gives the total volume by summing up the volume contribution  $v_J$  from each quantum.

Further, we select a statistical state of the form (2.14) such that the generator  $\mathcal{P}$  is the above volume operator, that is, a volume Gibbs state [14,36] of the form

$$\rho_\beta = \frac{1}{Z_\beta} e^{-\beta V}, \quad (3.2)$$

which encodes a statistically fluctuating volume of quantum spacetime.

The quantum gravity condensate that we are interested in is thus a coherent thermal state of the form (2.25), but associated specifically with the volume Gibbs state in Eq. (3.2). In other words, we are interested in using a state  $|\sigma, \bar{\sigma}; \beta\rangle$ , specified by two functions, the condensate wave function  $\sigma \in \mathcal{H}$ , and the Bogoliubov parameter  $\theta_{J\alpha}(\beta)$  that is identified by the Bose number distribution of the state (3.2),

$$\sinh^2 [\theta_{J\alpha}(\beta)] = \frac{1}{e^{\beta v_J} - 1}. \quad (3.3)$$

Notice that since the spectrum of  $V$  is independent of the modes  $T_\alpha$ , the functions  $\theta_{J\alpha}$  are also independent of them. Thus  $\theta_{J\alpha} = \theta_J$ , and we will drop the labels  $\alpha$  in quantities associated with  $\theta$  from here on. Also, notice the following important property of our chosen state,

$$\lim_{\beta \rightarrow \infty} |\sigma, \bar{\sigma}; \beta\rangle = |\sigma, \bar{\sigma}\rangle = D_a(\sigma) D_{\bar{a}}(\bar{\sigma}) |0, \tilde{0}\rangle, \quad (3.4)$$

thanks to which all results of the previous works in GFT cosmology are reproduced when the fluctuations are turned off completely.

In the present context of extracting effective cosmological models from a candidate background independent theory of quantum gravity, the use of relational observables is of utmost importance. As mentioned previously, past works in GFT cosmology have interpreted and used the base manifold coordinate  $\phi$  as a relational matter clock, and considered quantities such as  $N(\phi)$  as relational

observables. However, we have also noticed above that in the present setting such quantities (see equation (2.30)) contain UV divergences related to occurrences of the ill-defined  $\delta(\phi = 0)$  distributions, which had in turn prompted us to change the basis<sup>5</sup> to  $T_\alpha$ , as a first step in the inclusion of thermal fluctuations in the context of GFT condensates. It follows that in this basis, we are interested in  $\alpha$ -dependent quantities defined by a partial sum over  $J$ , such as

$$V_\alpha = \sum_J v_J a_{J\alpha}^\dagger a_{J\alpha}, \quad (3.5)$$

and its statistical average in the thermal condensate (which itself now includes statistical fluctuations in volume),

$$\langle V_\alpha \rangle_{\sigma, \bar{\sigma}; \beta} = \sum_J v_J (|\sigma_{J\alpha}|^2 + \sinh^2 [\theta_J(\beta)]). \quad (3.6)$$

Finally, in the context of the relational dynamics discussed later in Sec. III C, we are essentially considering the case of a nondynamical thermal cloud, with only the condensate part of the full system being dynamical. This is understood as a first approximation of the more general case with dynamical thermal fluctuations. We will return to this point later in Secs. III E 2 and IV.

### B. Effective group field theory dynamics

A generic GFT action with a local kinetic term and a nonlocal interaction term (higher than quadratic order in the fields) takes the form

$$S = \int d\vec{g} d\phi \bar{\varphi}(\vec{g}, \phi) \mathcal{K}(\vec{g}, \phi) \varphi(\vec{g}, \phi) + S_{\text{int}}[\varphi, \bar{\varphi}] \quad (3.7)$$

and gives the following classical equation of motion:

$$\mathcal{K}(\vec{g}, \phi) \varphi(\vec{g}, \phi) + \frac{\delta S_{\text{int}}[\varphi, \bar{\varphi}]}{\delta \varphi(\vec{g}, \phi)} = 0. \quad (3.8)$$

In the corresponding quantum theory on  $\mathcal{H}_F$ , the equation of motion is

$$\mathcal{K}(\vec{g}, \phi) \hat{\varphi}(\vec{g}, \phi) + \frac{\delta \widehat{S}_{\text{int}}[\varphi, \bar{\varphi}]}{\delta \varphi(\vec{g}, \phi)} = 0 \quad (3.9)$$

with some choice of operator ordering (and the hat notation reinstated temporarily). An effective equation of motion can then be derived from the above operator equation by taking its expectation value in a special class of quantum states implementing a notion of semiclassical and

<sup>5</sup>Further details about this aspect and the definition of a basis-independent clock are discussed in Sec. III C.

continuum approximations.<sup>6</sup> Here, we take the coherent thermal states introduced above as this class of states. We thus consider

$$\langle \sigma, \bar{\sigma}; \beta | \mathcal{K}(\vec{g}, \phi) \hat{\varphi}(\vec{g}, \phi) + \frac{\delta S_{\text{int}}[\varphi, \bar{\varphi}]}{\delta \bar{\varphi}(\vec{g}, \phi)} | \sigma, \bar{\sigma}; \beta \rangle = 0. \quad (3.10)$$

As a first step to investigate the role of statistical fluctuations of quantum geometry in condensate cosmology, we focus here only on the free part. This would allow us to display clearly the impact of nonzero thermal fluctuations. In other words, any difference in results that we find, as compared to previous zero temperature free theory studies, could then be attributed directly to the presence of these statistical fluctuations. Therefore, restricting to the free kinetic term, we obtain

$$\langle \sigma, \bar{\sigma}; \beta | \mathcal{K}(\vec{g}, \phi) \varphi(\vec{g}, \phi) | \sigma, \bar{\sigma}; \beta \rangle = \mathcal{K}(\vec{g}, \phi) \sigma(\vec{g}, \phi). \quad (3.11)$$

Further using the Peter-Weyl decomposition for  $\sigma$ ,

$$\sigma(\vec{g}, \phi) = \sum_J D_J(\vec{g}) \sigma_J(\phi), \quad (3.12)$$

and considering the following kinetic term (which is a standard choice; see, for instance, Ref. [53] and references therein):

$$\mathcal{K} = \mathcal{K}_0(\vec{g}) + \mathcal{K}_1(\vec{g}) \partial_\phi^2, \quad (3.13)$$

such that

$$\mathcal{K}_0(\vec{g}) (D_J(\vec{g}) \sigma_J(\phi)) = B_J D_J(\vec{g}) \sigma_J(\phi), \quad (3.14)$$

$$\mathcal{K}_1(\vec{g}) \partial_\phi^2 (D_J(\vec{g}) \sigma_J(\phi)) = A_J D_J(\vec{g}) \partial_\phi^2 \sigma_J(\phi), \quad (3.15)$$

we obtain the following equations of motion:

$$\partial_\phi^2 \sigma_J(\phi) - M_J \sigma_J(\phi) = 0, \quad \forall J, \quad (3.16)$$

where  $M_J := -\frac{B_J}{A_J}$ .

Thus, we see that the free GFT dynamical equation of motion in a coherent thermal state is identical to the case where one considers a simple coherent state (2.9) in  $\mathcal{H}_F$ , with no thermal cloud. But as we can already anticipate, observable averages (such as volume) will have thermal contributions in general, consequently modifying their evolution equations.

<sup>6</sup>In line with previous works, we understand the implementation of semiclassical and continuum approximations in the specific sense of using the class of coherent states (which are well-known to be the most classical quantum states, with a peak on a pair of classical conjugate variables), and a condensate phase (described by a collective condensate variable, and with a nonzero order parameter), respectively.

This concludes the derivation of the effective GFT equation of motion using a thermal coherent state. However, as we have mentioned earlier, calculations with observables in a  $(\vec{g}, \phi)$  basis leads to singularities in the  $\phi$ -dependent quantities. This brings us to the following section where we address the question of defining and applying a suitable time reference frame (a clock), and offer a preliminary interpretation of the resultant quantities.

### C. Smearing functions and reference clocks

As we have emphasized before, the use of  $\phi$  as a reference clock is not possible here since the quantities of interest, such as  $\langle V(\phi) \rangle$ , are mathematically ill-defined, which prompted us to define quantities such as  $\langle V_\alpha \rangle$  instead. Below, we generalize this even further and introduce generic<sup>7</sup> square-integrable, complex-valued smooth functions,

$$t(\phi) = \sum_\alpha t_\alpha T_\alpha(\phi), \quad (3.17)$$

in order to define observables and their dynamics as functionals of  $t(\phi)$  (which will later be interpreted as relational). This brings us to the aspect of smearing.

In the quantum operator setup summarized in Secs. II A and II B, instead of smearing the operators with a set of basis functions  $f_{J\alpha}(\vec{g}, \phi)$ , we could instead smear the algebra generators with a complete set of more general smearing functions  $F(\vec{g}, \phi)$  (usually also satisfying additional analyticity and sufficient decay properties). In particular, for the  $\phi$  variable, this would amount to smearing with smooth functions, say  $t(\phi)$ . This would result in an equivalent, but basis-independent algebraic setup, as commonly encountered in Weyl C\*-algebraic theory associated with bosonic quanta.<sup>8</sup> For our actual purposes, we retain the use of the Wigner basis  $D_J(\vec{g})$ , in order to retain also the associated geometric interpretation of (functions of) the spin labels  $J$  as it is standard in both GFT and loop quantum gravity, while in the  $\phi$  direction we smear with a function  $t(\phi)$ . In other words, we are interested in smeared operators of the form

$$a_J(t) := \int d\vec{g} d\phi \bar{D}_J(\vec{g}) \bar{t}(\phi) \varphi(\vec{g}, \phi), \quad (3.18)$$

$$a_J^\dagger(t) := \int d\vec{g} d\phi D_J(\vec{g}) t(\phi) \varphi^\dagger(\vec{g}, \phi), \quad (3.19)$$

which are now understood as *functional* (relational) ladder operators, with respect to the *function*  $t(\phi)$ . By extension, the observable of interest is the volume operator, which now takes the form

<sup>7</sup>While satisfying certain boundary conditions, see Eqs. (3.29).

<sup>8</sup>See [36,37] for details of a Weyl algebraic formulation in group field theory.

$$V_t := \sum_J v_J a_J^\dagger(t) a_J(t), \quad (3.20)$$

which is interpreted as the operator associated with a spatial slice labeled by the function  $t$ .

Notice that in general,  $t$ -relational operators, and their expectation values, are nonlocal functions of their  $\phi$ -relational counterparts. For instance, the average volume in a thermal condensate state is

$$\langle V_t \rangle_{\sigma, \bar{\sigma}; \beta} = \sum_J v_J (|\sigma_J(t)|^2 + \sinh^2[\theta_J] \|t\|^2), \quad (3.21)$$

where  $\|t\|^2 = (t, t)_{L^2(\mathbb{R})}$  and

$$\sigma_J(t) := \int_{\mathbb{R}} d\phi \bar{t}(\phi) \sigma_J(\phi) = (t, \sigma_J)_{L^2(\mathbb{R})}. \quad (3.22)$$

One can show that the quantity  $\langle V_t \rangle_{\sigma, \bar{\sigma}; \beta}$  can be expressed in terms of a nonlocal function of  $\phi$ , namely,

$$\langle V_t \rangle_{\sigma, \bar{\sigma}; \beta} = \sum_J v_J \int_{\mathbb{R}^2} d\phi d\phi' t(\phi) \bar{t}(\phi') \langle N_J(\phi, \phi') \rangle_{\sigma, \bar{\sigma}; \beta}, \quad (3.23)$$

where

$$N_J(\phi, \phi') := a_J^\dagger(\phi) a_J(\phi') \quad (3.24)$$

is the off-diagonal number density (2-point) operator with expectation value

$$\langle N_J(\phi, \phi') \rangle_{\sigma, \bar{\sigma}; \beta} = \bar{\sigma}_J(\phi) \sigma_J(\phi') + \sinh^2[\theta_J] \delta(\phi - \phi'). \quad (3.25)$$

This generic nonlocality of  $t$ -relational quantities with respect to  $\phi$ , for instance in (3.23), is reasonable to expect simply as a technical feature that is characteristic of changing reference frames in general.

Lastly, the smearing functions  $t(\phi)$  are understood as defining reference clock frames, the reasons for which will be made more clear in the next section. For now, we note that such a treatment is compatible with GFTs technically being background independent *field* theories. Unlike in standard quantum field theory and particle physics, in any background independent field theory, a relational clock variable is expected to be defined as a genuine function, such as  $t(\phi)$ , rather than a coordinate on the base manifold, such as  $\phi$ . One common way to tackle this is to introduce an additional dynamical field into the system, deparametrize the full dynamics with respect to it, and use it as a clock, e.g., the Brown-Kuchar dust model in general relativity. In this case then, we notice that relational clocks are *fields* over the spacetime base manifold. Hence, in GFTs one expects a relational clock to also be defined as a field over

the base (Lie group) manifold. This is what we partially achieve in this work, by the use of functions  $t(\phi)$ . Having said that, we strictly refrain from assigning any further *physical* interpretation to the function  $t$ , especially from the spacetime point of view, unlike the coordinate  $\phi$  which has been motivated as a minimally coupled scalar matter field in previous works (see, for instance, Ref. [10]).

## D. Relational functional dynamics

At this point, we come to the important task of expressing the effective GFT equations of motion in terms of the smearing functions. The goal is to arrive at a consistent dynamical description of the present system in a  $t$ -relational reference frame. Let us first reiterate our main line of reasoning. Smearing functions  $t(\phi)$  are used in order to avoid divergences in the  $\phi$ -frame, e.g., in relational quantities such as  $\langle V(\phi) \rangle_{\sigma, \bar{\sigma}; \beta}$ . This leads to observables such as  $\langle V_t \rangle_{\sigma, \bar{\sigma}; \beta}$ . The condensate functional  $\sigma_J(t)$  defined in (3.22) then naturally takes on the role of the dynamical collective variable, instead of  $\sigma_J(\phi)$ . Therefore, the equations of motion (3.16) in terms of the variable  $\phi$  must be rewritten suitably in terms of functions  $t$ , as follows.

We are seeking a differential equation of motion for  $\sigma_J(t)$ , encoding the same dynamics as (3.16). We begin by noticing that the mass term in (3.16) can be written in terms of  $\sigma_J(t)$  simply as

$$M_J \sigma_J(t) = \int_{\mathbb{R}} d\phi M_J \bar{t}(\phi) \sigma_J(\phi) \quad (3.26)$$

using the smearing. Therefore, as before, we see that smearing might offer us a way forward. We then smear Eqs. (3.16) with an arbitrary square-integrable complex-valued smooth function  $t(\phi)$ , obtaining

$$\int_{\mathbb{R}} d\phi \bar{t}(\phi) \partial_\phi^2 \sigma_J(\phi) - M_J \sigma_J(t) = 0, \quad \forall J. \quad (3.27)$$

Now, in order to get a description completely in the  $t$ -frame, we require a suitable derivative operator with a well-defined action on functionals of  $t$ . For this, notice that

$$\begin{aligned} \int_{\mathbb{R}} d\phi \bar{t}(\phi) \partial_\phi^2 \sigma_J(\phi) &= \left( - \int_{\mathbb{R}} d\phi \partial_\phi \bar{t} \frac{\delta}{\delta \bar{t}(\phi)} \right)^2 \sigma_J(t) \\ &=: \mathbf{d}_t^2 \sigma_J(t), \end{aligned} \quad (3.28)$$

where we have used integration by parts, and the following boundary conditions for the smearing functions:

$$\lim_{\phi \rightarrow \pm\infty} t(\phi) = 0, \quad \lim_{\phi \rightarrow \pm\infty} \partial_\phi t(\phi) = 0. \quad (3.29)$$

The operator  $\mathbf{d}_t$  might seem to be a good choice for the functional derivative [54,55] that we are looking for. However, recall that we are working with complex-valued



smearing functions. Thus, in our context, generic functionals of them depend on both  $t$  and  $\bar{t}$ , which are considered to be independent variables; e.g., the norm  $|\sigma_J(t)|^2 = \overline{\sigma_J(t)}\sigma_J(t)$  depends on two variables,  $t$  and  $\bar{t}$ . The operator  $\mathbf{d}_t$  must thus be extended by the conjugate term to obtain the Hermitian differential operator

$$\nabla_t := - \int_{\mathbb{R}} d\phi \left( \partial_{\phi} \bar{t} \frac{\delta}{\delta \bar{t}(\phi)} + \partial_{\phi} t \frac{\delta}{\delta t(\phi)} \right). \quad (3.30)$$

Notice that, as required, we get an equation in terms of  $\nabla_t$  that is analogous to (3.28) above, i.e.,

$$\nabla_t^2 \sigma_J(t) = \int_{\mathbb{R}} d\phi \bar{t}(\phi) \partial_{\phi}^2 \sigma_J(\phi). \quad (3.31)$$

Therefore, the equations of motion (3.16) can be equivalently expressed as

$$\nabla_t^2 \sigma_J(t) - M_J \sigma_J(t) = 0, \quad \forall J \quad (3.32)$$

for all square-integrable smooth functions  $t(\phi)$  satisfying the boundary conditions (3.29).

Note that if one is working with a dynamical model based on higher (than 2) order derivatives in  $\phi$  or in general is interested in extending this setup to include arbitrary higher order generalizations of Eq. (3.31) above, then the boundary conditions (3.29) must be supplemented by vanishing of all higher order derivatives of  $t$  in the limit  $\phi \rightarrow \pm\infty$ . In such a case then one could work with the space of Schwartz functions, for instance, as the relevant set of smearing functions. However, in the present analysis, we do not need to restrict to this special subspace of smooth functions, and the conditions (3.29) are sufficient.

Few remarks are in order concerning the operator  $\nabla_t$  and the associated  $t$ -relational setup. The operator  $\nabla_t$  is a functional differential operator, consisting of functional derivatives with respect to  $\bar{t}$  and  $t$  [54,55]. The flow induced by it is not on the GFT base manifold (in contrast with  $\phi$ ), nor on a given spacetime, but rather on the space of smearing functions. Recalling that functional derivatives can be understood as generalizations of directional derivatives, then  $\nabla_t$  essentially defines a flow with components along the directions of  $(-\partial_{\phi} \bar{t})$  and  $(-\partial_{\phi} t)$ . Further, by construction this operator satisfies

$$\nabla_t \ell(t) = \int_{\mathbb{R}} d\phi \bar{t}(\phi) \partial_{\phi} \ell(\phi) \quad (3.33)$$

for any function  $\ell(\phi) \in L^2(\mathbb{R})$ , where  $\ell(\phi)$  satisfies (3.29), and  $\ell(t) := (t, \ell)_{L^2(\mathbb{R})}$ . Notice that Eq. (3.33) straightforwardly gives Eq. (3.31) used above. The property in (3.33) is important because it motivates the use of smearing functions as relational clock fields. As we have shown above, the  $t$ -relational dynamical quantities and equations are derived

from an appropriate smearing of their (possibly nonlocal)  $\phi$ -relational counterparts. In particular, the  $t$ -functional equations of motion (3.32) are simply the smearing of the  $\phi$ -dependent equations (3.16). The interpretation of the smearing can then be clarified, as a first step, by considering a limiting case where the  $t$ -relational setup reduces to the usual  $\phi$ -relational one. Namely, if one takes a delta distribution<sup>9</sup> peaked on  $\phi$ , that is,  $t(\phi') = \delta(\phi' - \phi)$ , then the full  $t$ -relational setup introduced above naturally reduces to the  $\phi$ -relational one that is used in all previous works in GFT cosmology. For instance, all the smeared quantities take their usual forms as functions of  $\phi$ , e.g.,  $\sigma_J(t) = \sigma(\phi)$ ,  $a_J(t) = a_J(\phi)$ ,  $\langle N_J(t) \rangle_{\sigma, \bar{\sigma}; \beta} = \langle N_J(\phi) \rangle_{\sigma, \bar{\sigma}; \beta}$ .

Along these lines, one can motivate specific choices of smooth clock functions peaked around points of the base manifold, namely values of  $\phi$ , for instance, Gaussian functions. Such choices could then be interpreted as the implementation of a *deparametrization* procedure at the level of the background independent quantum theory. One could further understand the selection of a relational clock as a restriction to a special sector of physical states in the full (nondeparametrized) quantum theory, as was suggested in [36]. However, in general, one would expect to be able to realize such mechanisms in possibly different ways. For instance, in the present setting, this would correspond to a special choice of smearing functions  $t$ ; while a different possibility is explored in [56], in the context of zero temperature ( $\beta = \infty$ ) GFT condensate cosmology. The complete details of mechanisms for deparametrization, how they relate to each other, and if there could be preferred choices, are interesting queries that are left for future investigations. In this article, however, we proceed without any further restriction to a specific class of  $t$  functions and work with the general case. We note that the added generality may also allow for potential switching between relational reference frames in GFT, which is an expected feature of any background independent system devoid of an absolute notion of time or space (see, for instance, Ref. [57], and references therein).

Furthermore, we notice that the  $t$ -relational setup presented here is constructed from the full nondeparametrized operator formulation of GFT, with the algebra satisfying (2.1), and the deparametrization with respect to a relational clock field is implemented via introduction of smearing functions  $t(\phi)$ , as discussed above. Specifically, the kinematic description of the system is fully covariant, i.e., no preferred clock parameter  $\phi$  (from possibly several ones [36]) or function  $t(\phi)$  is chosen as *the* clock. The dynamical description [Eqs. (3.8)–(3.10)] is derived using

<sup>9</sup>Note that a distribution would not satisfy the boundary conditions (3.29), and also the operator  $\nabla_t$  would not be well-defined. However, this peculiar case is to be understood only as a limit, for instance, by considering the limit of vanishing width for a family of Gaussian functions.

the principle of least action, without the use of any relational Hamiltonian. This setup is then technically different from the one used in some recent works such as [58,59]. The relational frame used in these other studies, as in all previous works in GFT cosmology [1–3], is defined with respect to the parameter  $\phi$ , which as discussed above (see also [33]) may lead to divergences. Also, the studies in [58,59] are based on a canonical quantization of already deparametrized classical GFT models. Specifically, the kinematic description is canonical with respect to a chosen clock variable  $\phi$ , with the algebra based on equal  $\phi$ -time commutation relations. Subsequently, the dynamical description is derived from a clock Hamiltonian. Having said that, the descriptions based on these two, *a priori* technically different, setups could eventually be related, since they encode the physics of a given system before and after deparametrization. This question is, however, tightly connected to the open issue of time in quantum gravity. The investigation of this possible relation may help in addressing the question of how physical time emerges in the present background independent theory of quantum gravity.

Returning to the equations of motion (3.32), let us use the standard polar decomposition

$$\sigma_J(t) = \zeta_J^t e^{i\eta_J^t}, \quad (3.34)$$

where  $\zeta_J^t = \sqrt{\sigma_J(t)\overline{\sigma_J(t)}}$  is the modulus and  $\eta_J^t = \tan^{-1}(\frac{\text{Im}\sigma_J(t)}{\text{Re}\sigma_J(t)})$  is the phase of the condensate functional  $\sigma_J(t) \in \mathbb{C}$ . Note that the quantities  $\zeta_J^t$  and  $\eta_J^t$  do not correspond to the smearing of the modulus  $\zeta_J(\phi)$  and the phase  $\eta_J(\phi)$  of the condensate function  $\sigma_J(\phi)$ , which were used in the context of GFT cosmology in previous works [11]. Separating the real and imaginary parts of Eqs. (3.32), we obtain

$$\nabla_t^2 \zeta_J^t - \zeta_J^t (\nabla_t \eta_J^t)^2 - M_J \zeta_J^t = 0, \quad (3.35)$$

$$2\nabla_t \zeta_J^t \nabla_t \eta_J^t + \zeta_J^t \nabla_t^2 \eta_J^t = 0 \quad (3.36)$$

for all  $J$ . These two equations imply the existence of two constants of motion, as in the case of  $\beta = \infty$  free theory [11], given by

$$E_J = (\nabla_t \zeta_J^t)^2 + (\zeta_J^t)^2 (\nabla_t \eta_J^t)^2 - M_J (\zeta_J^t)^2, \quad (3.37)$$

$$Q_J = (\zeta_J^t)^2 \nabla_t \eta_J^t, \quad (3.38)$$

satisfying  $\nabla_t E_J = 0$  and  $\nabla_t Q_J = 0$ .

### E. Effective dynamics for homogeneous and isotropic cosmology

Now, our investigation is based on four ingredients: the choice of quantum states in the full theory (here, the class of

coherent thermal states based on the chosen Gibbs state), the choice of dynamics, the choice of relational observables, and finally the choice of a subclass of condensate wave functions. We have addressed the first three points in Secs. III A–III D, which brings us to the last one, which we address as follows. A notion of homogeneity in the present nonspatiotemporal background independent setting resides in the following: (i) the use of a coherent condensate as the relevant phase for studying the effective cosmology extracted from a GFT model, and (ii) having an additional left diagonal symmetry on the condensate wave function  $\sigma(hg_i, \phi) = \sigma(g_i, \phi)$ ,  $\forall h \in SU(2)$ . In other words, it resides in the facts that the collective dynamics is encoded in a left- and right-invariant single-particle wave function  $\sigma$ , which is also the order parameter of the condensate  $\langle a_J(t) \rangle_{\sigma, \bar{\sigma}, \beta} = \sigma_J(t)$ , where now  $J \equiv (\vec{j}, l_L, l_R)$ , and that each  $a$ -quantum in the condensate is being described by the same wave function  $\sigma$ . Further, a notion of isotropy is implemented by fixing the spins at each vertex to be equal, fixing the two intertwiners to be equal (the geometric interpretation of which remains to be understood), and choosing a special class of intertwiners, namely the eigenvectors of the volume operator with the highest eigenvalue. We refer to past works for detailed discussions on these aspects, for instance, [1,2,11,60–62].

These restrictions imply that the condensate function is entirely determined by the value of a single spin  $j$ . It follows that the equations of motion (3.32) reduce to one equation for each value of the  $SU(2)$  spin label  $j$ ,

$$\nabla_t^2 \sigma_j(t) - M_j \sigma_j(t) = 0, \quad \forall j \in \mathbb{N}/2. \quad (3.39)$$

Consequently we have,  $\forall j \in \mathbb{N}/2$

$$\nabla_t^2 \zeta_j^t - \zeta_j^t (\nabla_t \eta_j^t)^2 - M_j \zeta_j^t = 0, \quad (3.40)$$

$$2\nabla_t \zeta_j^t \nabla_t \eta_j^t + \zeta_j^t \nabla_t^2 \eta_j^t = 0 \quad (3.41)$$

with the same conserved charges (3.37) and (3.38), now labeled by the spin  $j$ .

Having set all the ingredients for a dynamical analysis, we can now proceed with the derivation of the effective dynamical equations for the average volume  $\langle V_t \rangle$  in a coherent thermal state of the form (2.25), which include geometric volume fluctuations as discussed in Sec. III A. For simplicity of notation, we will drop the label  $t$  on relational quantities (such as  $\zeta$ ,  $\eta$ , and volume averages) in the following. Relational volume average is given by

$$\mathbf{V} := \langle V_t \rangle = \sum_j v_j (\zeta_j^2 + s_j^2 \|t\|^2), \quad (3.42)$$

where  $s_j := \sinh[\theta_j(\beta)]$ . Using the effective equations of motion (3.40) and (3.41), and the expressions for the constants of motion (3.37) and (3.38), we obtain

$$\begin{aligned}
 \mathbf{V}' &:= \nabla_t \mathbf{V} \\
 &= 2 \sum_j v_j \zeta_j \nabla_t \zeta_j \\
 &= 2 \sum_j v_j \zeta_j \operatorname{sgn}(\zeta'_j) \sqrt{E_j - \frac{Q_j^2}{\zeta_j^2} + M_j \zeta_j^2}, \quad (3.43)
 \end{aligned}$$

$$\mathbf{V}'' := \nabla_t^2 \mathbf{V} = 2 \sum_j v_j (E_j + 2M_j \zeta_j^2), \quad (3.44)$$

where we have used  $\nabla_t \|t\|^2 = 0$ . From here on we shall assume  $\|t\|^2 = 1$  for convenience.

Then, the effective generalized Friedmann equations, including both quantum gravitational and statistical volume corrections are

$$\left(\frac{\mathbf{V}'}{3\mathbf{V}}\right)^2 = \frac{4}{9} \left( \frac{\sum_j v_j \zeta_j \operatorname{sgn}(\zeta'_j) \sqrt{E_j - \frac{Q_j^2}{\zeta_j^2} + M_j \zeta_j^2}}{\sum_j v_j \zeta_j^2 + \sum_j v_j s_j^2} \right)^2, \quad (3.45)$$

$$\frac{\mathbf{V}''}{\mathbf{V}} = \frac{2 \sum_j v_j (E_j + 2M_j \zeta_j^2)}{\sum_j v_j \zeta_j^2 + \sum_j v_j s_j^2}. \quad (3.46)$$

These equations represent the relational evolution for the volume associated with a foliation labeled by the function  $t$ . Compared to the analogous equations obtained in [11], the main difference arises due to the expression (3.42) for the average volume where there appears an additional statistical contribution  $s_j^2$ , which as we have described above originates directly from the quantum statistical mechanics of the underlying quantum gravity theory.

### 1. Late times evolution

In the following, we will make use of the quantities below that are formally defined as number densities corresponding to the different parameters characterizing the different phases of the system:

$$n_{\text{co}}(j) = \zeta_j^2, \quad n_E(j) = \frac{E_j}{M_j}, \quad (3.47)$$

$$n_{\text{th}}(j) = s_j^2, \quad n_Q(j) = \frac{Q_j}{\sqrt{M_j}}, \quad (3.48)$$

where  $n_{\text{co}}$  and  $n_{\text{th}}$  [equal to (3.3)] are the actual number densities (thus are non-negative) of the condensate and thermal parts of the full system. Different physical regimes can then be described in terms of relative strengths of these parameters.

The domain  $n_E(j), n_Q(j) \ll n_{\text{co}}(j)$  is understood as a classical limit where the volume is large but curvature is small [11]. In this regime, we have

$$\mathbf{V}' \approx 2 \sum_j \operatorname{sgn}(\zeta'_j) v_j \sqrt{M_j \zeta_j^2}, \quad (3.49)$$

$$\mathbf{V}'' \approx 4 \sum_j v_j M_j \zeta_j^2, \quad (3.50)$$

giving the corresponding generalized evolution equations,

$$\left(\frac{\mathbf{V}'}{3\mathbf{V}}\right)^2 = \frac{4}{9} \left( \frac{\sum_j \operatorname{sgn}(\zeta'_j) v_j \sqrt{M_j \zeta_j^2}}{\sum_j v_j \zeta_j^2 + \sum_j v_j s_j^2} \right)^2, \quad (3.51)$$

$$\frac{\mathbf{V}''}{\mathbf{V}} = \frac{4 \sum_j v_j M_j \zeta_j^2}{\sum_j v_j \zeta_j^2 + \sum_j v_j s_j^2}. \quad (3.52)$$

Note that the thermal contribution  $\mathbf{V}_{\text{th}} := \sum_j v_j n_{\text{th}}(j)$  is invariant under variations in the time function  $t$ , that is,  $\nabla_t \mathbf{V}_{\text{th}} = 0$ . Hence, if the full system evolves such that the condensate number density  $n_{\text{co}}(j)$  increases monotonously in time, then eventually we will reach the domain where the condensate part,  $\mathbf{V}_{\text{co}} := \sum_j v_j n_{\text{co}}(j)$ , dominates the thermal cloud, that is,  $\mathbf{V}_{\text{co}} \gg \mathbf{V}_{\text{th}}$ . Thus considering

$$n_{\text{co}} \gg n_Q, n_E, n_{\text{th}} \quad (3.53)$$

and

$$\forall j, \operatorname{sgn}(\zeta'_j) = \pm 1, \quad M_j \equiv M = 3\pi G, \quad (3.54)$$

where  $G$  is Newton's gravitational constant, we obtain

$$\left(\frac{\mathbf{V}'}{3\mathbf{V}}\right)^2 = \frac{4\pi G}{3}, \quad (3.55)$$

$$\frac{\mathbf{V}''}{\mathbf{V}} = 12\pi G, \quad (3.56)$$

which are known to be the relational Friedmann equations in general relativity, for spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime with a minimally coupled massless scalar field [11]. These are the equations of motion of the quantum gravity system in a thermal condensate phase where both (3.53) and (3.54) are valid. Physically, this regime where the condition (3.53) is satisfied, i.e., the contribution coming from the condensate is dominant while the statistical fluctuations are subdominant, corresponds to a phase that effectively mimics a system in a zero temperature condensate.<sup>10</sup> Consequently, as shown above, in this regime we simply get the zero

<sup>10</sup>Note that there could also be a classical regime where the statistical fluctuations are not subdominant. In other words, statistical fluctuations may be important even in regimes where quantum fluctuations are negligible. In the present setting, this could correspond to the case when  $n_{\text{co}} \gg n_E, n_Q$  holds true, while the interplay between  $n_{\text{co}}$  and  $n_{\text{th}}$  is still relevant.

temperature condensate cosmology obtained in previous works [11,39]. In this sense, the use of zero temperature condensates such as  $|\sigma\rangle$  can be understood more generally as a thermal quantum gravity condensate being in a dynamical regime where the condensate dominates,  $n_{\text{co}} \gg n_{\text{th}}$ .

## 2. Early times evolution

We can also look at the evolution equations (3.45) and (3.46) in a different phase, in particular where the thermal contributions and the quantum corrections become relevant. For consistency, the choice (3.54), which recovers the classical limit giving the correct late time behavior, is assumed.

Observe that the expression (3.43) for  $\mathbf{V}'$  admits roots. Namely, there exist solutions  $\{\zeta_j^o\}_j$  such that

$$\mathbf{V}' = 0. \quad (3.57)$$

The solution is given explicitly in terms of  $M$  and the constants of motion  $E_j, Q_j$  as

$$n_{\text{co}}^o(j) = -\frac{1}{2}n_E(j) + \sqrt{\frac{1}{4}n_E(j)^2 + n_Q(j)^2}, \quad \forall j, \quad (3.58)$$

where we have ignored the negative solutions, since  $n_{\text{co}}$  is the number density of the condensate and must be non-negative in general. At this stationary point, the total volume is

$$\mathbf{V}^o = \sum_j v_j (n_{\text{co}}^o(j) + n_{\text{th}}(j)), \quad (3.59)$$

which is clearly nonzero due to the nonvanishing thermal contribution in the present finite  $\beta$  case, even if the condensate part were to vanish. However, as it happens, even  $n_{\text{co}}^o$  does not vanish as long as  $E \neq 0$ . In particular,  $n_{\text{co}}^o \neq 0$  even if  $Q = 0$ . To see this, notice that if  $Q = 0$ , then  $E < 0$ , which is evident from their expressions in Eqs. (3.37) and (3.38), assuming a positive  $M$  as required by the correct classical limit (3.54). In this case then,  $n_{\text{co}}^o = |n_E|$ . In the more general case with  $Q \neq 0$ , both positive and negative  $E$  are allowed in principle, but in each case we again have  $n_{\text{co}}^o > 0$ . Thus, the expectation value  $\mathbf{V}$  does not vanish when  $\mathbf{V}' = 0$ , implying the existence of a nonvanishing minimum<sup>11</sup>  $\mathbf{V}^o$  of  $\mathbf{V}$  throughout the evolution. Physically, in the context of homogeneous and isotropic cosmology, this means that the singularity has been resolved, and that the effective evolution displays a bounce, with a nonzero minimum of the spatial volume.

Further, this ensures a transition between two phases of the universe characterized by the sign  $\text{sgn}(\zeta_j')$ , describing a

contracting universe [ $\text{sgn}(\zeta_j') < 0$ ] and an expanding one [ $\text{sgn}(\zeta_j') > 0$ ]. Each of these phases behaves according to the general relativistic FLRW evolution in the classical and nonthermal limits (3.53), that is, when  $\zeta_j$  (equivalently the condensate contribution to the volume) becomes very large with respect to all the constants of motion and the thermal contributions. However, as expected, these two phases display a nonstandard evolution in general, especially when close to the bounce. This is the regime where  $\zeta_j$  is comparable in magnitude to the other quantities present in the model. This leads to the particularly important question about the presence of a phase of accelerated expansion and its magnitude.

To address this question, we proceed with a simplified analysis, where we make the approximation of selecting one spin mode [11,38], thus dropping the sum over all spins in the various expressions. In this case, the generalized equations of motion (3.45) and (3.46) reduce to

$$\left(\frac{\mathbf{V}'}{\mathbf{V}}\right)^2 = 4M + 4 \left( \frac{E_j \zeta_j^2 - Q_j^2 - 2M \zeta_j^2 s_j^2 - M s_j^4}{(\zeta_j^2 + s_j^2)^2} \right), \quad (3.60)$$

$$\frac{\mathbf{V}''}{\mathbf{V}} = 4M + 2 \left( \frac{E_j - 2M s_j^2}{\zeta_j^2 + s_j^2} \right). \quad (3.61)$$

Now, the magnitude of a phase of accelerated expansion can be estimated in terms of the number of  $e$ -folds [38] given by

$$\mathcal{N} := \frac{1}{3} \ln \left( \frac{\mathbf{V}_{\text{end}}}{\mathbf{V}_{\text{beg}}} \right) = \frac{1}{3} \ln \left( \frac{n_{\text{co}}^{\text{end}} + n_{\text{th}}}{n_{\text{co}}^{\text{beg}} + n_{\text{th}}} \right), \quad (3.62)$$

where  $\mathbf{V}_{\text{beg}}$  and  $\mathbf{V}_{\text{end}}$  are the average total volumes at the beginning and end of the phase of accelerated expansion, respectively, and in terms of an acceleration [38] given by

$$\alpha := \frac{\mathbf{V}''}{\mathbf{V}} - \frac{5}{3} \left( \frac{\mathbf{V}'}{\mathbf{V}} \right)^2. \quad (3.63)$$

Using Eqs. (3.60) and (3.61) above, we get

$$\begin{aligned} \alpha = & -\frac{8}{3}M + 2M \left( \frac{n_E - 2n_{\text{th}}}{n_{\text{co}} + n_{\text{th}}} \right) \left( 1 - \frac{10}{3} \frac{n_{\text{co}}}{n_{\text{co}} + n_{\text{th}}} \right) \\ & + \frac{20}{3}M \left( \frac{n_Q^2 + n_{\text{th}}^2}{(n_{\text{co}} + n_{\text{th}})^2} \right). \end{aligned} \quad (3.64)$$

Assuming that the bounce is the starting point of inflation, we have

$$n_{\text{co}}^{\text{beg}} = n_{\text{co}}^o \quad (3.65)$$

for any  $j$ . It can be checked straightforwardly that acceleration is positive at the beginning, i.e.,  $\alpha|_{\text{beg}} > 0$  as

<sup>11</sup>It can be verified that this is indeed a minimum since  $\mathbf{V}''|_{n_{\text{co}}^o} > 0$ .

required. Now, the end of accelerated expansion is characterized by  $\alpha|_{\text{end}} = 0$ , which gives

$$n_{\text{co}}^{\text{end}} = \frac{3}{4}n_{\text{th}} - \frac{7}{8}n_E + \sqrt{\frac{49}{64}n_E^2 + \frac{5}{2}n_Q^2 + \frac{9}{16}(n_{\text{th}}^2 - n_E n_{\text{th}})} \quad (3.66)$$

assuming an expanding phase of the universe, i.e.,  $V_{\text{end}} > V_{\text{beg}}$ , and non-negativity of  $n_{\text{co}}^{\text{end}}$  even when  $n_{\text{th}}$  is negligible. The number of  $e$ -folds can thus be estimated by

$$e^{3\mathcal{N}} = \frac{\frac{7}{4}(n_{\text{th}} - \frac{1}{2}n_E) + \sqrt{\frac{49}{64}n_E^2 + \frac{5}{2}n_Q^2 + \frac{9}{16}(n_{\text{th}}^2 - n_E n_{\text{th}})}}{(n_{\text{th}} - \frac{1}{2}n_E) + \sqrt{\frac{1}{4}n_E^2 + n_Q^2}}. \quad (3.67)$$

Since the quantities  $n_E$ ,  $n_Q$ , and  $n_{\text{th}}$  are all independent, we can observe three interesting regimes, giving approximate numerical values for  $\mathcal{N}$ :

$$n_{\text{th}}, n_Q \ll n_E: \quad \mathcal{N} \approx \frac{1}{3} \ln\left(\frac{7}{4}\right) \approx 0.186, \quad (3.68)$$

$$n_{\text{th}}, n_E \ll n_Q: \quad \mathcal{N} \approx \frac{1}{6} \ln\left(\frac{5}{2}\right) \approx 0.152, \quad (3.69)$$

$$n_E, n_Q \ll n_{\text{th}}: \quad \mathcal{N} \approx \frac{1}{3} \ln\left(\frac{5}{2}\right) \approx 0.305. \quad (3.70)$$

The upper bound on  $\mathcal{N}$  in the previous zero temperature free theory analysis [38] is 0.186, while here for finite  $\beta$  free theory it is 0.305, achieved in an early time limit. This difference is attributed to the only new aspect that we have introduced in the model, the thermal cloud of quanta of geometry. This shows that the number of  $e$ -folds can be increased, even without a nonlinear dynamics. This fact is in contrast with the previous conclusions [38] that nonlinear interaction terms in the GFT action are necessary to increase  $\mathcal{N}$  (in which case, the interaction terms are naturally accompanied by their corresponding coupling constants, which are free parameters that can be fine-tuned to essentially give the desired value for  $\mathcal{N}$ ). However, it remains that the increase in  $\mathcal{N}$  in our case is very minimal and still not sufficient to match the physical observations, estimated at  $\mathcal{N} \sim 60$ . Nevertheless, this may be overcome if one goes beyond a static thermal cloud. In other words, a dynamical thermal cloud, which would be expected to be left over from a geometrogenesis phase transition (of an originally unbroken, pregeometric phase), could have the potential to provide a viable mechanism for an extended phase of geometric inflation. The implementation of dynamical statistical fluctuations would require special care in order to avoid pathological behaviors with regards

to the use of relational clock functions  $t(\phi)$ , in addition to the standard requirement of having stability via sufficiently subdominant or even decaying fluctuations in the relevant observables of the system, throughout the effective evolution of the universe. We do not treat this case in the present article and leave it for future works.

Finally, we note that the approximation of restricting to a single spin mode in the calculation above, does not alter the qualitative conclusion that a static thermal cloud is not sufficient to generate a satisfactory number of  $e$ -folds to match the observational estimate. However, a relaxation of the condition (3.54) for  $\text{sgn}(\zeta'_j)$ , by considering a nonhomogeneous distribution of the sign with respect to the modes  $j$  while preserving the classical limit manifest in the emergence of Friedmann equations at large volumes, might give rise to a larger ratio between the volumes at the end and at the beginning of inflation and consequently a larger number of  $e$ -folds.

#### IV. DISCUSSION AND OUTLOOK

In this work we have studied some implications of the presence of statistical fluctuations in the context of group field theory by using coherent thermal states for condensate cosmology. We have modeled the quantum gravitational phase of the universe as a thermal condensate consisting of a condensate part representing an effective macroscopic homogeneous spacetime, and a static thermal cloud representing quantum geometric statistical fluctuations over it. This work provides the first steps toward building a GFT thermal condensate cosmology.

The model that we have presented recovers the expected cosmological dynamics at late times (when the thermal part is dominated by the condensate), but displays differences, with respect to earlier works with pure nonthermal condensates, at early times, when the thermal cloud dominates the condensate, and in the presence of quantum corrections. In particular, we have shown that the singularity is generically resolved with a bounce between a contracting and an expanding phase of the universe, and that there exists an early phase of accelerated expansion with an increased number of  $e$ -folds compared to those achieved in previous zero temperature analysis of free GFTs. This increase in the number of  $e$ -folds, obtained in the absence of interactions, is attributed to the presence of the thermal cloud.

For our analysis, we have introduced an appropriate generalization of the relational clock frame in GFT, by considering clock functions  $t(\phi)$ , implemented as smearing functions. Consequently, we have formulated the effective equations of motion and dynamical quantities as functionals of  $t$ . A more complete understanding of such relational frames in GFT and their precise relation to the concept of deparametrization is left to future work.

Since the thermal cloud that we have considered in our study is static, considering a dynamical thermal cloud

would be an important extension of this work, and would allow one to investigate further the dynamical implications of the presence of a thermal cloud. Another valuable extension would be to consider an interacting GFT model in the presence of thermal fluctuations, even with a static cloud. In fact, these aspects of having a dynamical thermal cloud and an overall interacting theory are intimately related, as would be expected. We discuss some of these issues below, and suggest further lines of investigations.

There are three main features motivating our specific choice of state, namely a coherent thermal state of the form (2.25) at inverse temperature  $\beta$ . First,  $\beta$  is assumed to be constant. Second, the state is such that the average number of quanta split neatly into a condensate and a noncondensate part, such that the zero temperature limit gives a pure  $\beta$ -independent condensate [see also Eq. (3.4)]. That is,

$$\langle a^\dagger a \rangle_\beta = n_{\text{co}} + n_{\text{non-co}}, \quad (4.1)$$

$$\lim_{\beta \rightarrow \infty} \langle a^\dagger a \rangle_\beta = n_{\text{co}}, \quad (4.2)$$

where in the present work the noncondensate part is taken to be thermal and at  $\beta$ -equilibrium. Having such a split is not only convenient in doing computations but also adds clarity to expressions in the subsequent analyses when considering the interplays between the two. Third, the expectation value of the field operator in a coherent thermal state is temperature independent, i.e.,

$$\langle a_{J\alpha} \rangle_\sigma = \sigma_{J\alpha} = \langle a_{J\alpha} \rangle_{\sigma, \bar{\sigma}, \beta}, \quad (4.3)$$

thus being identical to the zero temperature case. Now at first sight this may seem contrary to our expectation that the condensate would be affected by the presence of a thermal cloud in general. But, in fact, this choice of state, satisfying the three properties noticed above, is entirely compatible with our approximation of neglecting interactions and using free dynamics. When temperature is switched on, quanta from the condensate are depleted into the thermal cloud. In a generally interacting case, both the thermal cloud and the condensate are interacting and dynamical by themselves, while also interacting with each other. However, in the present case of free dynamics, the thermal cloud will not interact with the condensate part, in addition to the quanta being free also within each part separately. Thus even though our state includes a thermal cloud, the coherent condensate part (described fully by its order parameter  $\sigma$ ) will be unaffected by it, as depicted in Eq. (4.3) above. This is further reasonable in light of constant  $\beta$ . If temperature were to change, say to increase, then we would expect more quanta to be depleted into the thermal cloud, and thus expect the state of the condensate to change. Since the condensate is characterized completely by the order parameter, it would also need to change.

So overall, considering the class of states in which the order parameter is temperature independent is a reasonable approximation when the temperature is constant and interactions are neglected. Excluding interactions ensures that the thermal cloud does not affect the condensate, while constant  $\beta$  ensures that the amount of depletion is also constant, so together the condensate can indeed be approximated by a  $\beta$ -independent order parameter. In such a case we may be missing out on some interesting physics; however, we take our case as a first step toward more advanced investigations in the future.

From our discussion above, a physically important and interesting extension concerns the case of a dynamically changing  $\beta$ . In this case, the expected dominance of the condensate part over the thermal cloud at late times would not only be determined by a dynamically increasing condensate (as is the case in the present work), but also by what would be a dynamically decreasing temperature as the universe expands.

Furthermore, one could extend the previous studies in GFT condensate cosmology [1–3] to the present setting with thermal fluctuations, and investigate various aspects including the dynamical analysis of quantum fluctuations, perturbations, anisotropies, and inhomogeneities.

Finally, from a physical point of view, we presently understand  $\beta$  as a statistical parameter that controls the extent of depletion of the condensate into the thermal cloud, and overall the strength of statistical fluctuations of observables in the system. The question remains whether it also admits a geometrical interpretation. Taking guidance from classical general relativity, we know that spatial volume generates a dynamical evolution in constant mean curvature foliations, wherein the temporal evolution is given by the so-called York time parameter. Constant York time slices are thus constant extrinsic curvature scalar (mean curvature) slices, and the two quantities are proportional to each other. In this case, one could attempt to understand  $\beta$  as the periodicity in York time, equivalently in scalar extrinsic curvature (both of which are conjugates to the spatial volume). In particular for homogeneous and isotropic spacetimes, York time is further proportional to the Hubble parameter. A detailed investigation of such aspects and their implications would be interesting and is left for future work.

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- [1] A. G. A. Pithis and M. Sakellariadou, Group field theory condensate cosmology: An appetizer, *Universe* **5**, 5 (2019).
- [2] D. Oriti, The universe as a quantum gravity condensate, *C. R. Phys.* **18**, 235 (2017).
- [3] S. Gielen and L. Sindoni, Quantum cosmology from group field theory condensates: A review, *SIGMA* **12**, 082 (2016).
- [4] D. Oriti, Quantum gravity as a quantum field theory of simplicial geometry, in *Quantum Gravity*, edited by B. Fauser, J. Tolksdorf, and E. Zeidler (Birkhäuser Verlag, Basel, 2006), [https://doi.org/10.1007/978-3-7643-7978-0\\_6](https://doi.org/10.1007/978-3-7643-7978-0_6).
- [5] L. Freidel, Group field theory: An overview, *Int. J. Theor. Phys.* **44**, 1769 (2005).
- [6] D. Oriti, The group field theory approach to quantum gravity, in *Approaches to Quantum Gravity: Toward a New Understanding of Space, Time and Matter*, edited by D. Oriti (Cambridge University Press, Cambridge, England, 2009).
- [7] D. Oriti, The microscopic dynamics of quantum space as a group field theory, in *Proceedings, Foundations of Space and Time: Reflections on Quantum Gravity: Cape Town, South Africa* (Cambridge University Press, 2011), pp. 257–320, <https://doi.org/10.1017/CBO9780511920998.012>.
- [8] D. Oriti, Disappearance and emergence of space and time in quantum gravity, *Stud. Hist. Philos. Sci. B* **46**, 186 (2014).
- [9] D. Oriti, Group field theory as the 2nd quantization of loop quantum gravity, *Classical Quantum Gravity* **33**, 085005 (2016).
- [10] Y. Li, D. Oriti, and M. Zhang, Group field theory for quantum gravity minimally coupled to a scalar field, *Classical Quantum Gravity* **34**, 195001 (2017).
- [11] D. Oriti, L. Sindoni, and E. Wilson-Ewing, Emergent Friedmann dynamics with a quantum bounce from quantum gravity condensates, *Classical Quantum Gravity* **33**, 224001 (2016).
- [12] D. Oriti, Spacetime as a quantum many-body system, in *Many-body Approaches at Different Scales: A Tribute to Norman H. March on the Occasion of his 90th Birthday*, edited by G. G. N. Angilella and C. Amovilli (Springer International Publishing, Cham, 2018), pp. 365–379.
- [13] E. Bianchi, P. Dona, and S. Speziale, Polyhedra in loop quantum gravity, *Phys. Rev. D* **83**, 044035 (2011).
- [14] I. Kotecha, Thermal quantum spacetime, *Universe* **5**, 187 (2019).
- [15] D. Oriti, J. P. Ryan, and J. Thuerigen, Group field theories for all loop quantum gravity, *New J. Phys.* **17**, 023042 (2015).
- [16] A. Barbieri, Quantum tetrahedra and simplicial spin networks, *Nucl. Phys.* **B518**, 714 (1998).
- [17] J. C. Baez and J. W. Barrett, The quantum tetrahedron in three-dimensions and four-dimensions, *Adv. Theor. Math. Phys.* **3**, 815 (1999).
- [18] M. P. Reisenberger and C. Rovelli, Space-time as a Feynman diagram: The connection formulation, *Classical Quantum Gravity* **18**, 121 (2001).
- [19] A. Ashtekar and J. Lewandowski, Background independent quantum gravity: A status report, *Classical Quantum Gravity* **21**, R53 (2004).
- [20] T. Thiemann, *Modern Canonical Quantum General Relativity*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 2007).
- [21] M. Han, W. Huang, and Y. Ma, Fundamental structure of loop quantum gravity, *Int. J. Mod. Phys. D* **16**, 1397 (2007).
- [22] N. Bodendorfer, An elementary introduction to loop quantum gravity, [arXiv:1607.05129](https://arxiv.org/abs/1607.05129).
- [23] A. Perez, The spin foam approach to quantum gravity, *Living Rev. Relativity* **16**, 3 (2013).
- [24] C. Rovelli and F. Vidotto, *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory* (Cambridge University Press, Cambridge, England, 2014).
- [25] R. Loll, Quantum gravity from causal dynamical triangulations: A review, *Classical Quantum Gravity* **37**, 013002 (2020).
- [26] R. Gurau and J. P. Ryan, Colored tensor models—A review, *SIGMA* **8**, 020 (2012).
- [27] H. W. Hamber, Quantum gravity on the lattice, *Gen. Relativ. Gravit.* **41**, 817 (2009).
- [28] G. Chirco, F. M. Mele, D. Oriti, and P. Vitale, Fisher metric, geometric entanglement and spin networks, *Phys. Rev. D* **97**, 046015 (2018).
- [29] G. Chirco, A. Goeßmann, D. Oriti, and M. Zhang, Group field theory and holographic tensor networks: Dynamical corrections to the Ryu-Takayanagi formula, *Classical Quantum Gravity* **37**, 095011 (2020).
- [30] G. Chirco, D. Oriti, and M. Zhang, Ryu-Takayanagi Formula for symmetric random tensor networks, *Phys. Rev. D* **97**, 126002 (2018).
- [31] G. Chirco, D. Oriti, and M. Zhang, Group field theory and tensor networks: towards a Ryu-Takayanagi formula in full quantum gravity, *Classical Quantum Gravity* **35**, 115011 (2018).
- [32] F. Anza and G. Chirco, Typicality in spin-network states of quantum geometry, *Phys. Rev. D* **94**, 084047 (2016).
- [33] M. Assanioussi and I. Kotecha, Thermal representations in group field theory: Squeezed vacua and quantum gravity condensates, *J. High Energy Phys.* **02** (2020) 173.

- [34] G. Chirco, I. Kotecha, and D. Oriti, Statistical equilibrium of tetrahedra from maximum entropy principle, *Phys. Rev. D* **99**, 086011 (2019).
- [35] G. Chirco and I. Kotecha, Generalized Gibbs ensembles in discrete quantum gravity, in *Geometric Science of Information 2019*, edited by F. Nielsen and F. Barbaresco, Lecture Notes in Computer Science (Springer, Cham, 2019).
- [36] I. Kotecha and D. Oriti, Statistical equilibrium in quantum gravity: Gibbs states in group field theory, *New J. Phys.* **20**, 073009 (2018).
- [37] A. Kegeles, D. Oriti, and C. Tomlin, Inequivalent coherent state representations in group field theory, *Classical Quantum Gravity* **35**, 125011 (2018).
- [38] M. de Cesare, A. G. A. Pithis, and M. Sakellariadou, Cosmological implications of interacting group field theory models: Cyclic universe and accelerated expansion, *Phys. Rev. D* **94**, 064051 (2016).
- [39] D. Oriti, L. Sindoni, and E. Wilson-Ewing, Bouncing cosmologies from quantum gravity condensates, *Classical Quantum Gravity* **34**, 04LT01 (2017).
- [40] M. de Cesare and M. Sakellariadou, Accelerated expansion of the Universe without an inflaton and resolution of the initial singularity from group field theory condensates, *Phys. Lett. B* **764**, 49 (2017).
- [41] A. G. A. Pithis and M. Sakellariadou, Relational evolution of effectively interacting group field theory quantum gravity condensates, *Phys. Rev. D* **95**, 064004 (2017).
- [42] Y. Takahashi and H. Umezawa, Thermo field dynamics, *Collective Phenomena* **2**, 55 (1975).
- [43] H. Matsumoto, Thermo field dynamics and its development, in *Progress in Quantum Field Theory*, edited by H. Ezawa and S. Kamefuchi (Elsevier Science Publishers B. V., 1985), ISBN-10: 0444869905, ISBN-13: 978-0444869906.
- [44] H. Umezawa, H. Matsumoto, and M. Tachiki, *Thermo Field Dynamics and Condensed States* (North-holland, Amsterdam, Netherlands, 1982).
- [45] H. Umezawa, *Advanced Field Theory: Micro, Macro, and Thermal Physics* (AIP, New York, 1993).
- [46] F. C. Khanna, A. P. C. Malbouisson, J. M. C. Malbouisson, and A. R. Santana, *Thermal Quantum Field Theory—Algebraic Aspects and Applications* (World Scientific, New Jersey, 2009).
- [47] I. Ojima, Gauge fields at finite temperatures: Thermo field dynamics, KMS condition and their extension to gauge theories, *Ann. Phys. (N.Y.)* **137**, 1 (1981).
- [48] N. P. Landsman and C. G. van Weert, Real and imaginary time field theory at finite temperature and density, *Phys. Rep.* **145**, 141 (1987).
- [49] E. Celeghini, S. De Martino, S. De Siena, A. Iorio, M. Rasetti, and G. Vitiello, Thermo field dynamics and quantum algebras, *Phys. Lett. A* **244**, 455 (1998).
- [50] S. M. Barnett and P. L. Knight, Thermofield analysis of squeezing and statistical mixtures in quantum optics, *J. Opt. Soc. Am. B* **2**, 467 (1985).
- [51] A. Mann and M. Revzen, Thermal coherent states, *Phys. Lett. A* **134**, 273 (1989).
- [52] J. Oz-Vogt, A. Mann, and M. Revzen, Thermal coherent states and thermal squeezed states, *J. Mod. Opt.* **38**, 2339 (1991).
- [53] S. Gielen, Inhomogeneous universe from group field theory condensate, *J. Cosmol. Astropart. Phys.* **02** (2019) 013.
- [54] E. Engel and R. M. Dreizler, *Density Functional Theory: An Advanced Course*, Theoretical and Mathematical Physics (Springer, Berlin, Heidelberg, 2013).
- [55] R. G. Parr and W. Yang, *Density-functional Theory of Atoms and Molecules* (Oxford University Press, Oxford, 1994).
- [56] L. Marchetti and D. Oriti, Relational observables and their fluctuations in group field theory condensate cosmology, to appear.
- [57] P. A. Höhn, Switching internal times and a new perspective on the ‘wave function of the Universe’, *Universe* **5**, 116 (2019).
- [58] E. Adjei, S. Gielen, and W. Wieland, Cosmological evolution as squeezing: A toy model for group field cosmology, *Classical Quantum Gravity* **35**, 105016 (2018).
- [59] E. Wilson-Ewing, A relational Hamiltonian for group field theory, *Phys. Rev. D* **99**, 086017 (2019).
- [60] S. Gielen, Quantum cosmology of (loop) quantum gravity condensates: An example, *Classical Quantum Gravity* **31**, 155009 (2014).
- [61] S. Gielen, D. Oriti, and L. Sindoni, Cosmology from Group Field Theory Formalism for Quantum Gravity, *Phys. Rev. Lett.* **111**, 031301 (2013).
- [62] S. Gielen, D. Oriti, and L. Sindoni, Homogeneous cosmologies as group field theory condensates, *J. High Energy Phys.* **06** (2014) 013.