

## Gravitational particle production in loop quantum cosmology

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We investigate the gravitational particle production in the bounce phase of loop quantum cosmology (LQC). We perform both analytical and numerical analysis of the particle production process in a LQC scenario with a Bunch-Davies vacuum initial condition in the contracting phase. We obtain that if we extend the validity of the dressed metric approach beyond the limit of small backreaction in which it is well justified, this process would lead to a radiation dominated phase in the preinflationary phase of LQC. Our results indicate that the test field approximation, which is required in the truncation scheme used in the dressed metric approach, might not be a valid assumption in a LQC scenario with such initial conditions.

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### I. INTRODUCTION

It is well known that in the description of the very early Universe and tracing back the time evolution, we enter in a Planck scale regime where quantum gravity effects can dominate. On such scales, modifications of general relativity (GR) are expected to be important. In the theoretical explorations of the early Universe, one generally uses the Friedmann-Lemaître-Robertson-Walker (FLRW) solutions to the Einstein equations (with appropriate matter sources) as background spacetimes. However, the FLRW spacetimes of interest are incomplete in the past due to the big bang singularity, when matter fields and spacetime curvature diverge. General relativity is not a suitable theory once curvature reaches the Planck scale, where quantum effects become important. Therefore, we cannot expect reliable results for quantum fields evolving in classical backgrounds. The natural way is to look for a quantum theory of gravity, which, e.g., can provide the quantization of spacetime.

There are some viable candidate models of quantum gravity that can be able to describe the physics of the very early Universe. Some of these models share the feature of avoiding the initial singularity, which results in a nonsingular bouncing universe. Although inflation is the dominant paradigm for the early Universe, there are alternative ideas [1–3], like several bouncing models, which can agree with current cosmological observations as well as inflation does. In these models, the quantum effects are important at the Planck scale and are responsible for the bounce, but become less important as the Universe

evolves away from Planck scale, thus eventually recovering GR. Among these quantum gravity candidates, loop quantum gravity (LQG) provides a promising avenue in this direction [4–11]. Loop quantum cosmology (LQC) arises as the result of applying the principles of LQG to cosmological settings [12–14]. In LQC, the quantum geometry creates a repulsive effective force that is totally negligible at low spacetime curvature but grows rapidly in the Planck regime, overwhelming the classical gravitational attraction. In cosmological models, while Einstein's equations hold to an excellent degree of approximation at low curvature, they receive important corrections in the Planck regime. As a consequence of these corrections, any time a curvature invariant grows to the Planck scale, quantum geometry effects dilute it, resolving the problems of singularities of GR.

In addition to the bounce and contraction phases, the inflationary scenario admits an extension for LQC in the expanding phase. Inflation can be important to put several models in agreement with observations in the context of LQC (see, for example, Ref. [15]). Several authors have investigated the naturalness of an inflationary phase after the bounce [16–19]. Despite the different viewpoints, in LQC by starting with a scalar field with a typical potential considered in inflationary models, after the bounce, an inflationary phase will almost inevitably sets in. In this context, it is important to analyze the physical implications of preinflationary dynamics in LQC, i.e., the quantum evolution from the bounce to the onset of the slow roll.

An important aspect to investigate in bouncing cosmologies is the Parker mechanism of gravitational particle production (GPP) [20,21], which has shown to be very efficient in the bounce phase of several models [22–28].

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If we assume that the spacetime is the Minkowski one both at early (prebounce) and at late (postbounce) times, but underwent a period of expansion during an intermediate time interval, then the initial Minkowski vacuum state of a test scalar field  $\chi$  will evolve nontrivially during the intermediate time interval to a final state, which is not equal to the final time Minkowski vacuum. From the point of view of the final Minkowski frame, the final state contains  $\chi$  particles. The same phenomenon applies to linear cosmological perturbations that evolve in a similar way to test scalar fields on the cosmological background. It was shown in Ref. [22] that in the matter bounce scenario, the process of GPP is sufficient to produce a hot early universe. The matter bounce scenario was also investigated in the context of LQC in Ref. [29]. It was also shown in Ref. [28] that GPP, under some conditions, can also be responsible for the emergence of a hot thermal state in the Universe in the new ekpyrotic model, thus avoiding the need to introduce an additional reheating phase. In Refs. [25,26,30,31], the GPP was also analyzed in the context of other bouncing cosmologies.<sup>1</sup>

In this work, we compute the GPP via the Parker mechanism [20,21] in the preinflationary phase of LQC. We compute the energy density stored in the produced particles and compare it with the background energy density, which we consider to be dominated by the kinetic energy of the inflaton field before the slow-roll phase. In our framework, we will consider gravity coupled to scalar fields and study the dynamics of quantum fields on quantum cosmological spacetimes.

Since the interest relies on the dynamics of quantized fields propagating on these quantum cosmological backgrounds, one needs a quantum gravity extension of the standard field equations. The extension we are going to consider lies in the framework of the dressed metric approach, which allows for a description of the field mode equations in a form analogous to the classical one [34]. The dressed metric approach considers a truncation scheme that is assumed to be well justified in the cases in which the backreaction of the produced modes are negligible. By computing the backreaction of the gravitationally produced particles in the preinflationary phase of LQC, we will be able to test the validity of the dressed metric approach in this scenario.

This paper is organized as follows. In Sec. II, we describe the background dynamics of the LQC model considered here. In Sec. III, we analyze the gravitational particle production in LQC. In Sec. IV, we present the analytical and the numerical results for the energy density of the particles produced. Finally, our concluding remarks are presented in Sec. V.

<sup>1</sup>In Refs. [32,33], this production was analyzed specifically in the context of LQC, but in a different framework than the one considered here.

## II. BACKGROUND MODEL

We consider LQC as our cosmological scenario from which the Einstein's equations receive explicit modifications in the Planck regime. The spatial geometry in LQC is encoded in the volume of a fixed fiducial cubic cell, rather than the scale factor  $a$ , and is given by

$$v = \frac{\mathcal{V}_0 a^3 m_{\text{Pl}}^2}{2\pi\gamma}, \quad (2.1)$$

where  $\mathcal{V}_0$  is the comoving volume of the fiducial cell,  $\gamma$  is the Barbero-Immirzi parameter [35] of LQC, whose numerical value is given by  $\gamma \simeq 0.2375$ ,  $m_{\text{Pl}} \equiv 1/\sqrt{G} = 1.22 \times 10^{19}$  GeV is the Planck mass, and  $G$  is the Newton constant of gravitation. The conjugate momentum to  $v$  is denoted by  $b$ , and it is given by  $b = -4\pi\gamma P_{(a)}/(3a^2\mathcal{V}_0 m_{\text{Pl}}^2)$ , where  $P_{(a)}$  is the conjugate momentum to the scale factor.

The solution of the LQC effective equations implies that the Hubble parameter  $H$  can be written as

$$H = \frac{1}{2\gamma\lambda} \sin(2\lambda b), \quad (2.2)$$

where  $\lambda = (48\pi^2\gamma^2/m_{\text{Pl}}^4)^{1/4}$  and  $b$  ranges over  $(0, \pi/\lambda)$ . The energy density,  $\rho$ , relates to the LQC variable  $b$  through  $\rho = 3m_{\text{Pl}}^2 \sin^2(\lambda b)/(8\pi\gamma^2\lambda^2)$ . Then, the Friedman equation in LQC assumes the form [19],

$$\frac{1}{9} \left( \frac{\dot{v}}{v} \right)^2 \equiv H^2 = \frac{8\pi}{3m_{\text{Pl}}^2} \rho \left( 1 - \frac{\rho}{\rho_c} \right), \quad (2.3)$$

where  $\rho$  is the energy density,  $\rho_c = 3m_{\text{Pl}}^2/(8\pi\gamma^2\lambda^2) \approx 0.41m_{\text{Pl}}^4$  is the critical density in LQC, and the dots denote derivatives with respect to the cosmic time. For  $\rho \ll \rho_c$ , we recover GR as expected. The expression (2.3) holds independently of the particular characteristics of the inflationary regime. We can see that the singularity is avoided in this model and, when the energy density approaches the critical density, the Universe undergoes a bounce with  $H = 0$ .

We consider a cosmological scenario in which the energy content of the Universe is dominated by the inflaton field, with a potential  $V(\phi)$ . We also consider in our scenario an extra scalar field  $\chi$  with no interactions to standard model (SM) fields and also no direct coupling to the inflaton. Hence, the field  $\chi$  only couples to  $\phi$  and to the SM particles gravitationally. There are many well motivated candidates for spectator fields in cosmology. For example, dark matter can well be completely decoupled from the visible matter (and also from the inflaton) and interact only gravitationally [36]. The production of spectators can all be gravitational (typically, at the end of inflation in the usual scenarios), or they can also be produced through graviton mediated scatterings (in which case, their interactions with

other fields are naturally suppressed by Planck mass factors). Another example of a spectator field in cosmology is the curvaton (see, e.g., Ref. [37]), which is essentially, in its simplest implementations, decoupled from the inflaton and not contributing to the energy density during inflation, but it can become important later on. We believe that spectator fields are well motivated in cosmology and the GPP of them in the present study is of interest. Here we also extend the hypothesis of  $\chi$  being a spectator field to in the pre-bounce and around the bounce phases. Thus,  $\chi$  has negligible contribution to the background dynamics during these phases. However, due to the marked change in the metric evolution at around the bounce, we expect that  $\chi$  will be produced gravitationally, and it can have important effects in the postbounce subsequent evolution. We want to determine how important will be this contribution of the GPP of  $\chi$  particles to the subsequent postbounce preinflationary dynamics.

Due to the quantum nature of  $H$ , we can explicitly observe the evolution of the scale factor during the cosmological phase close to the bounce, where the Universe is dominated by the inflaton field, which is described by a barotropic fluid with equation of state  $p = \omega\rho$ . This solution reads (see, e.g., Ref. [38])

$$a(t) = a_B \left[ 1 + \frac{6\pi\rho_c}{m_{\text{Pl}}^2} (1 + \omega)^2 t^2 \right]^{\frac{1}{3(1+\omega)}}. \quad (2.4)$$

In this work, we will consider the case of a bounce dominated by the inflaton kinetic energy, which behaves as stiff matter, i.e., like a fluid with equation of state  $\omega \approx 1$ . During this phase, the scale factor is given by

$$a = a_B \left( 1 + \gamma_B \frac{t^2}{t_{\text{Pl}}^2} \right)^{1/6}, \quad (2.5)$$

where  $\gamma_B \equiv 24\pi\rho_c/m_{\text{Pl}}^4 \simeq 30.9$  and  $t_{\text{Pl}} \equiv 1/m_{\text{Pl}}$  is the Planck time. This can be accomplished with the homogeneous scalar field inflaton  $\phi$  subjected to a potential  $V(\phi)$ , whose evolution equation reads

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0. \quad (2.6)$$

In this scenario, with the bounce dominated by the kinetic energy of  $\phi$ , the evolution of the Universe after the contraction, and prior to preheating, can be divided in three different phases according to the behavior of the equation of state of the dominant fluid (see, e.g., Ref. [15]): The bouncing phase, the transition phase, and the slow-roll inflation phase. In the bounce phase, which lasts from the bounce until around  $t \sim 10^4 t_{\text{Pl}}$ , the evolution of  $a(t)$  is independent of the inflationary model, since the inflaton potential energy density is negligible around the bounce. In the transition phase, the kinetic energy of the inflaton decreases very rapidly (about 12 orders from its initial

Planck scale), and the equation of state changes suddenly from  $w(\phi) \sim 1$  to  $w(\phi) \sim -1$ . The transition phase is very short compared to the other phases. It starts at  $t \sim 10^4 t_{\text{Pl}}$  and last until  $t \sim 10^5 t_{\text{Pl}}$ . After that, the slow-roll phase starts with the inflaton potential energy dominating. From the bounce time to the beginning of inflation (i.e., during the preinflationary phase), the Universe expands around 4–5  $e$ -foldings [15,39]. As we will see below, the dynamics of the fields equations is different in each of these three phases.

As already mentioned above, the aim of this work is to see how the GPP of an spectator scalar field  $\chi$  can affect the preinflationary phase of LQC. At the same time, this will also provide us with means to test the validity of the dressed metric approach in this scenario. We begin by describing the mechanism of particle production in the following section.

### III. PARTICLE PRODUCTION IN CURVED SPACE-TIMES

We consider the standard procedure of describing quantum fields on classical, though curved, spacetimes. However, this strategy cannot be directly justified in the quantum gravity era, where curvature and matter densities are of Planck scale. Nevertheless, using techniques from LQG, the standard theory was extended providing us with means to overcome this limitation [34].

#### A. Dressed metric approach

In LQG, we do not have the analog of the full Einstein's equations to perturb. Several strategies have been developed to overcome this issue. Here, we follow the mainstream strategy in LQG, which consists in first truncating the classical theory in a manner appropriate to the physical problem under consideration. Then, the quantization is carried out considering the underlying quantum geometry of LQG. Finally, the consequences of the resulting framework are derived. The full phase space is truncated, keeping only the FLRW background with first-order inhomogeneous perturbations. The truncated phase space is described by  $\Gamma_{\text{Trun}} = \Gamma_0 \times \tilde{\Gamma}_1$ , where  $\Gamma_0$  is the four-dimensional phase space for the FLRW background, and  $\tilde{\Gamma}_1$  is the phase space of gauge invariant perturbations [40–42]. For a universe dominated by the scalar field  $\phi$ , the background phase space  $\Gamma_0$  is coordinatized by  $[\nu, b; \phi, p_{(\phi)}]$  and carries a single Hamiltonian constraint which implies in the equation,

$$S_0[N_{\text{hom}}] = N_{\text{hom}} \left[ -\frac{3b^2\nu}{4\gamma} + \frac{p_{(\phi)}^2 m_{\text{Pl}}^2}{4\pi\gamma\nu} + \frac{2\pi\gamma\nu}{m_{\text{Pl}}^2} V(\phi) \right] = 0, \quad (3.1)$$

where  $N_{\text{hom}}$  represents an homogeneous lapse. By considering in the above equation the choice  $N_{\text{hom}} = 1$ , it gives the

evolution in cosmic time, while  $N_{\text{hom}} = a$  gives it in conformal time,  $N_{\text{hom}} = a^3$  in harmonic time, and  $N_{\text{hom}} = a^3 \mathcal{V}_0 / p_{(\phi)}$  corresponds to using the inflaton field as “time clock” (see, e.g., Ref. [40] for more details).

Since the phase space of the truncated system is a product, the Hilbert space of quantum states has the form  $\mathcal{H} = \mathcal{H}_0 \otimes \mathcal{H}_1$ . The Hilbert space of background fields,  $\mathcal{H}_0$ , consists of wave functions  $\Psi_0(\nu, \phi)$ . The evolution of this quantum state is governed by the LQC quantum Hamiltonian constraint. Thus, the wave function  $\Psi_0(\nu, \phi)$  is subjected to the constraint  $\hat{S}_0 \Psi_0 = 0$  from the Dirac quantization procedure. From this constraint, we are led to the equation  $-i\partial_\phi \Psi_0(\nu, \phi) = \hat{H}_0 \Psi_0(\nu, \phi)$  [40], where  $\hat{H}_0$  is a self-adjoint operator whose explicit form will not be needed here.

Therefore, the evolution of this quantum state with respect to the emergent time variable  $\phi$  is governed by the LQC quantum Hamiltonian constraint. Among several states  $\Psi_0(a, \phi)$  in the LQC Hilbert space, one is interested in a state that is sharply peaked around a classical trajectory at late times. Evolving this state by using the LQC quantum Hamiltonian constraint, it has been shown that it remains sharply peaked during the whole dynamical trajectory, even deep in the Planck era [40]. Consequently, the evolution of the peak of such states can be described by an effective trajectory that is governed by the effective equations. Now, quantum perturbations propagate on quantum geometries, which are regular and free of singularities, being the energy density bounded above by the critical value  $\rho_c \simeq 0.41 m_{\text{pl}}^4$ .

On the other hand, the classical dynamics on the full  $\Gamma_{\text{Trun}}$  is not generated by a constraint; therefore, one cannot recover the quantum dynamics for the total system by imposing a quantum constraint. We do this by first proceeding in the homogeneous sector as mentioned, by reinterpreting the quantum Hamiltonian constraint as an evolution equation in the homogeneous sector, and then, we “lift” the resulting quantum trajectory to the full Hilbert space  $\mathcal{H}$  (see Ref. [40]) considering the truncated space. Nevertheless, in order for the truncation scheme to be valid, we need to ensure that the energy density stored in the perturbations is much smaller than the energy density of the background [41], i.e.,  $\rho_{\text{pert}} / \rho_{\text{bg}} \ll 1$ , all the way back to the bounce. Only then we would be assured of a self-consistent solution, justifying our truncation, which ignores the backreaction. Otherwise, one would have to perform a full quantum gravity theory.

In the scenario considered here, in addition to the dominant inflaton field, we consider a spectator scalar field  $\chi$ , which has absent, or negligible interactions to the other components of the Universe. We follow the analysis of the gravitational production of this scalar field  $\chi$  in the framework of the dressed metric approach [34,40–42], which has the advantage of allowing the description of the main equations in a form analogous to the classical ones.

In the dressed metric approach, the equation of motion of the operators representing scalar perturbations and the scalar field equations are formally the same as the usual equations appearing in a classical spacetime in GR. We analyze the GPP associated with this scalar field  $\chi$ . Considering that  $\chi$  has a sufficiently small or negligible mass (compared, e.g., to  $H$  in the postbounce phase), thus,  $\chi$  behaves essentially as radiation during the whole preinflationary phase.

In the test field approximation, the dynamics of fields propagating on the quantum geometry, which is described by the quantum state  $\Psi_0$ , behave as propagating in a quantum modified effective geometry described by the following dressed metric [40,42]:

$$\tilde{g}_{ab} dx^a dx^b = \tilde{a}^2 (-d\tilde{\eta}^2 + dx_i dx^i), \quad (3.2)$$

where the dressed scale factor  $\tilde{a}$  and the dressed conformal time  $\tilde{\eta}$  are given, respectively, by

$$\tilde{a} = \left( \frac{\langle \hat{H}_0^{-1/2} \hat{a}^4 \hat{H}_0^{-1/2} \rangle}{\langle \hat{H}_0^{-1} \rangle} \right)^{1/4}, \quad (3.3)$$

and

$$d\tilde{\eta} = \langle \hat{H}_0^{-1/2} \rangle \langle \hat{H}_0^{-1/2} \hat{a}^4 \hat{H}_0^{-1/2} \rangle^{1/2} d\phi. \quad (3.4)$$

In the latter equations,  $\hat{H}_0$  is the background Hamiltonian, and the expectation values are taken with respect to the background quantum geometry state, which is given by  $\Psi_0(a, \phi)$ . In this approach, the equations of motion of the operators representing scalar and tensor perturbations are formally the same as the equations appearing in classical spacetimes, which in Fourier space read

$$\mu_k''(\tilde{\eta}) + \left[ k^2 - \frac{\tilde{a}''}{\tilde{a}} + \tilde{U}(\tilde{\eta}) \right] \mu_k(\tilde{\eta}) = 0, \quad (3.5)$$

where primes here denote derivative with respect to the conformal time,  $\mu_k(\tilde{\eta}) = z R_k$ , with  $R_k$  denoting the comoving curvature perturbation,  $z(\tilde{\eta}) = a\dot{\phi}/H$ , and

$$\tilde{U}(\tilde{\eta}) = \frac{\langle \hat{H}_0^{-1/2} \hat{a}^2 \hat{U}(\phi) \hat{a}^2 \hat{H}_0^{-1/2} \rangle}{\langle \hat{H}_0^{-1/2} \hat{a}^4 \hat{H}_0^{-1/2} \rangle}. \quad (3.6)$$

The quantities  $\tilde{a}$ ,  $\tilde{\eta}$  and  $\tilde{U}(\tilde{\eta})$  represent the quantum expectation values in the background state  $\Psi_0(a, \phi)$ . However, for sharply peaked background states, the dressed effective quantities are well approximated by their peaked values  $a$ ,  $\eta$ , and  $U(\eta)$ . Then, the equation of motion for scalar modes, Eq. (3.5), becomes

$$\mu_k''(\eta) + \left[ k^2 - \frac{a''}{a} + U(\eta) \right] \mu_k(\eta) = 0, \quad (3.7)$$

where  $U = a^2(f^2V(\phi) + 2fV_{,\phi}(\phi) + V_{,\phi\phi}(\phi))$  and  $f \equiv \sqrt{2\pi G\dot{\phi}}/\sqrt{\rho}$ . The effective potential  $U$  is negligible in the bounce and transition phases, and it can then be neglected [15]. Thus, the equation of motion in the bounce and transition phases can be simply written as

$$\mu_k''(\eta) + \left[ k^2 - \frac{a''(\eta)}{a(\eta)} \right] \mu_k(\eta) = 0. \quad (3.8)$$

Despite its use in the present study, it is important to mention that there are other alternatives to the dressed metric approach. Other important approaches for treating perturbations in LQC are, e.g., the hybrid quantization approach (see, for example, Refs. [43,44] and references therein), and the deformed algebra approach (see, for example, Ref. [45] and references therein).

### B. Gravitational particle production

By considering the evolution of the Fourier modes  $\mathcal{X}_k$  of the scalar field  $\chi$ , in the dressed metric approach, the form of its equation of motion is quite analogous to the classical spacetime equations. Also, by neglecting the  $\chi$  mass, the evolution equation for the conformal rescaled field mode,  $\chi_k \equiv a\mathcal{X}_k$ , simply takes the same form as that for the perturbation modes Eq. (3.8). Hence, we can simply identify  $\mu_k \rightarrow \chi_k$  in Eq. (3.8), giving

$$\chi_k''(\eta) + \left[ k^2 - \frac{a''(\eta)}{a(\eta)} \right] \chi_k(\eta) = 0. \quad (3.9)$$

Equation (3.9) represents a set of uncoupled oscillators with time variable frequency  $\sqrt{k^2 - a''/a}$ . The time dependence of  $a''/a$  stems from the evolution of the scale factor  $a(\eta)$ , which parametrizes the background evolution. Therefore, for each instant  $\eta$ , we define a different vacuum. Parker [20,21] established conditions for the definition of a time dependent particle number operator  $n(\eta)$ . It can be defined if its vacuum expectation value varies slowly as possible with time as the expansion rate of the Universe is sufficiently slow. Also, this expansion period must occur between two ‘‘Minkowskian’’ vacuum states. One evaluates the effect of GPP occurring between these vacuum states. The mathematical treatment is summarized below.

The Hamiltonian for  $\chi_k(\eta)$  in terms of its Fourier modes reads

$$H(\eta) = \int d^3k (2E_k \hat{a}_k^\dagger \hat{a}_k + F_k \hat{a}_k \hat{a}_{-k} + F_k^* \hat{a}_k^\dagger \hat{a}_{-k}^\dagger), \quad (3.10)$$

where

$$E_k(\eta) = \frac{1}{2} |\chi_k'(\eta)|^2 + \frac{\omega_k^2}{2} |\chi_k(\eta)|^2, \quad (3.11)$$

$$F_k(\eta) = \frac{1}{2} (\chi_k'(\eta))^2 + \frac{\omega_k^2}{2} (\chi_k(\eta))^2, \quad (3.12)$$

where  $\omega_k = k$  in the massless approximation for the  $\chi$  field. We diagonalize the Hamiltonian performing the following Bogoliubov transformation:

$$\hat{b}_{\vec{k}} = \alpha_k(\eta) \hat{a}_{\vec{k}} + \beta_k^*(\eta) \hat{a}_{-\vec{k}}^\dagger, \quad (3.13)$$

where the Bogoliubov coefficients  $\alpha_k(\eta)$  and  $\beta_k(\eta)$  satisfy the constraint  $|\alpha_k(\eta)|^2 - |\beta_k(\eta)|^2 = 1$  due to normalization of the modes. The resulting diagonal Hamiltonian reads

$$H(\eta) = \int d^3k \omega_k b_k^\dagger b_k, \quad (3.14)$$

and Eq. (3.11), for instance, becomes

$$E_k(\eta) = \omega_k \left[ \frac{1}{2} + |\beta_k(\eta)|^2 \right]. \quad (3.15)$$

Defining the vacuum states  $|0_{(a)}\rangle$  and  $|0_{(b)}\rangle$  such that  $a_{\vec{k}}|0_{(a)}\rangle = b_{\vec{k}}|0_{(b)}\rangle = 0$ , we can compute the expectation value of the number operator  $\hat{N}_{\vec{k}}^{(b)} = b_{\vec{k}}^\dagger b_{\vec{k}}$  in the vacuum  $|0_{(a)}\rangle$ ,

$$n_k(\eta) = \langle {}_{(a)}0 | \hat{N}_{\vec{k}}^{(b)} | {}_{(a)}0 \rangle = |\beta_k(\eta)|^2. \quad (3.16)$$

We observe that  $|\beta_k(\eta)|^2$  is exactly the particle number per mode. Therefore, from Eq. (3.15), we obtain that  $E_k(\eta) = \omega_k [1/2 + n_k(\eta)]$  is the energy contribution of GPP. The Bogoliubov coefficients relate the initial Minkowskian vacuum states  $\chi_k^{(i)}$  to the final ones  $\chi_k^{(f)}$  in the following way:

$$\chi_k^{(f)}(\eta) = \alpha_k \chi_k^{(i)}(\eta) + \beta_k \chi_k^{(i)*}(\eta). \quad (3.17)$$

When  $\beta_k = 0$ , there is no particle production and the constraint  $|\alpha_k(\eta)|^2 - |\beta_k(\eta)|^2 = 1$  gives  $\alpha_k = 1$  and, then,  $\chi_k^{(i)} = \chi_k^{(f)}$  for the whole evolution.

From the definition of the particle number per mode, Eq. (3.16), we can obtain the total particle number density  $n_p(\eta)$  and the total energy density  $\rho_p(\eta)$  produced. The total particle number density can be defined as the limit of  $\sum_k |\beta_k(\eta)|^2$  in a box of side  $L \rightarrow \infty$  divided by the volume  $a^3(\eta)L^3$ , which gives

$$\begin{aligned} n_p(\eta) &= \frac{1}{a^3(\eta)L^3} \left( \frac{L}{2\pi} \right)^3 \int_0^\infty d^3k n_k(\eta) \\ &= \frac{1}{2\pi^2 a^3(\eta)} \int_0^\infty dk k^2 |\beta_k(\eta)|^2. \end{aligned} \quad (3.18)$$

Likewise, the total energy density  $\rho_p(\eta)$  can be defined as the momentum sum of  $\omega_k n_k(\eta)$ , which results in

$$\rho_p(\eta) = \frac{1}{2\pi^2 a^4(\eta)} \int_0^\infty dk k^2 \omega_k |\beta_k(\eta)|^2. \quad (3.19)$$

Equation (3.19) allows us to obtain the energy density associated with the particles that are gravitationally produced in the Universe.

It is important to mention that the energy density of produced particles can also be obtained from the expectation value of the energy-momentum tensor of  $\chi$  field at any time  $\eta$ , which reads [46]

$$\rho_p^{\text{EM}}(\eta) = \frac{1}{4\pi^2 a^4(\eta)} \int_0^\infty dk k^2 \left\{ |\chi'_k(\eta)|^2 + \omega_k^2 |\chi_k(\eta)|^2 + \left(\frac{a'}{a}\right)^2 |\chi_k(\eta)|^2 - \frac{a'}{a} [\chi_k^*(\eta) \chi_k(\eta)]' \right\}. \quad (3.20)$$

Far from the bounce, when the expansion rate is negligible, the energy difference due to produced particles with respect to the initial vacuum reduces to Eq. (3.19).

In the following we will directly solve the equations of motion for the fields in each cosmological phase.

### C. The scalar field equation of motion

In order to perform a preliminary analytical analysis, we will neglect the backreaction effect of the produced particles on the cosmological background, and we will solve the equation of motion for the field modes  $\chi_k$ , Eq. (3.9), for each cosmological phase. Backreaction effects will be considered explicitly when we numerically solve for the modes later in Sec. IV B.

To obtain the solution for the modes  $\chi_k$ , we need to solve Eq. (3.9) in each phase (bounce, transition, and slow-roll inflationary phase). We will not compute the particle production in the previous matter contraction phase since, independently of the details of the contracting phase, particles are effectively produced only after they get rid of the influence of the potential  $a''/a$ , which happens soon after the bounce phase in the LQC model here considered. The behavior of the modes in each phase depends on which term,  $k^2$  or  $a''/a$ , dominates the time dependent frequency in Eq. (3.9). To analyze the relevant modes, it is useful to define the characteristic length  $\lambda = \sqrt{a/a''}$ , which plays a role analogous to that of the comoving Hubble radius. We also define the characteristic momentum  $k_B = \sqrt{a''/a}|_{t=t_B}$ , which is the physical energy at the bounce. Note that from Eq. (2.5), we have that

$$k_B \equiv \sqrt{\gamma_B/3} m_{\text{pl}} \simeq 3.21 m_{\text{pl}}, \quad (3.21)$$

where we have set  $a_B = 1$  at the bounce. The relevant field modes  $\chi_k$  are those with  $k \approx k_B$ , as these are the modes

expected to give the main contribution to the GPP. However, the dynamics of these modes have different behavior when they are inside and outside the characteristic length  $\lambda$ . We will focus on the modes that begin well inside  $\lambda$  in the contracting phase, exit  $\lambda$  during the bounce phase, and then enter  $\lambda$  again in the transition phase and then on. We consider the initial Bunch-Davies (BD) vacuum state in the contracting phase. In the following subsections, we describe the solution for  $\chi_k(\eta)$  in each cosmological phase.

#### 1. Bounce phase

Since the equations for the momentum modes for the perturbations, Eq. (3.8), and that for the modes  $\chi_k$ , Eq. (3.9), are essentially identical, we can quite conveniently adapt the results already derived by the authors in Ref. [15] and used in there to solve for Eq. (3.8) to get the bounce contribution to the power spectrum. This starts by realizing that Eq. (3.9) can be seen as analogous to a type of the Schrödinger equation and where the term  $a''(\eta)/a(\eta)$  acts like a potential, which behaves as an effective barrier during the bouncing phase. This potential can be well approximated by a Pöschl-Teller potential at the bounce, i.e.,  $a''(\eta)/a(\eta) \approx k_B^2 \text{sech}^2[\sqrt{6}k_B(\eta - \eta_B)]$ . In this case, the solution of Eq. (3.9) can be put in the form of the solution of a standard hypergeometric equation, and it is given by [15]

$$\begin{aligned} \chi_k(\eta) &= a_k x^{ik/(2\sqrt{6}k_B)} (1-x)^{-ik/(2\sqrt{6}k_B)} \\ &\times {}_2F_1(a_1 - a_3 + 1, a_2 - a_3 + 1, 2 - a_3, x) \\ &+ b_k [x(1-x)]^{-ik/(2\sqrt{6}k_B)} {}_2F_1(a_1, a_2, a_3, x), \end{aligned} \quad (3.22)$$

where  $x \equiv x(\eta) = \{1 + \exp[-2\sqrt{6}k_B(\eta - \eta_B)]\}^{-1}$ ,

$$a_1 \equiv \frac{1}{2} \left( 1 + \frac{1}{\sqrt{3}} \right) - \frac{ik}{\sqrt{6}k_B}, \quad (3.23)$$

$$a_2 \equiv \frac{1}{2} \left( 1 - \frac{1}{\sqrt{3}} \right) - \frac{ik}{\sqrt{6}k_B}, \quad (3.24)$$

$$a_3 \equiv 1 - \frac{ik}{\sqrt{6}k_B}, \quad (3.25)$$

and  $a_k$  and  $b_k$  are integration constants determined by the initial conditions. These will be explicitly determined below.

#### 2. Transition phase

In the transition phase, on the other hand, the term  $k^2$  dominates over  $a''/a$  in Eq. (3.9). The solution of the equation of motion in this phase is given by

$$\chi_k = \frac{1}{\sqrt{2k}} (\tilde{\alpha}_k e^{-ik\eta} + \tilde{\beta}_k e^{ik\eta}), \quad (3.26)$$

where  $\tilde{\alpha}_k$  and  $\tilde{\beta}_k$  are two more integration constants and also determined when matching the different solutions.

### 3. Slow-roll inflationary phase

After the transition phase, in the slow-roll inflationary phase, the Universe is no longer in the quantum regime, and the equation of motion reduces to the usual relativistic one,

$$\chi_k''(\eta) + \left( k^2 - \frac{\nu^2 + 1/4}{\eta^2} \right) \chi_k(\eta) = 0, \quad (3.27)$$

where

$$\nu^2 = \eta^2 \frac{a''(\eta)}{a(\eta)} + \frac{1}{4}. \quad (3.28)$$

In this phase, the approximate analytical solution of Eq. (3.27) is given in terms of the Hankel functions as

$$\chi_k(\eta) \approx \frac{\sqrt{-\pi\eta}}{2} \left[ \alpha_k H_\nu^{(1)}(-k\eta) + \beta_k H_\nu^{(2)}(-k\eta) \right], \quad (3.29)$$

where  $\alpha_k$  and  $\beta_k$  are again two other integration constants.

### 4. Matching phases

To determine the coefficients  $\alpha_k$  and  $\beta_k$  in Eq. (3.29), one needs to match the solutions of each phase in their intermediate regions. During the contracting phase, close to the bounce, all the relevant modes are well inside the characteristic length  $\lambda$ . Then, we can choose the BD vacuum state as the initial conditions of the modes. Therefore,

$$\chi_k^{\text{in}}(\eta) \approx \frac{1}{\sqrt{2k}} e^{-ik\eta}. \quad (3.30)$$

We can consider the solution of the bounce phase, Eq. (3.22), in the limit  $\eta - \eta_B \gg 0$ . By comparing the resulting equation with the above initial condition, we obtain that the coefficients  $a_k$  and  $b_k$  in Eq. (3.22) are given by

$$a_k = 0, \quad b_k = e^{ik\eta_B} / \sqrt{2k}. \quad (3.31)$$

Next, by comparing the solution for  $\chi_k(\eta)$  in the bounce phase, Eq. (3.22), with the solution in the transition phase, Eq. (3.26), we can obtain the expressions for  $\tilde{\alpha}_k$  and  $\tilde{\beta}_k$ . These, compared to the solution in the slow-roll, Eq. (3.29), in the limit  $-k\eta \rightarrow \infty$ , give, after some algebra, that [15]

$$\alpha_k = \tilde{\alpha}_k = \frac{\Gamma(a_3)\Gamma(a_1 + a_2 - a_3)}{\Gamma(a_1)\Gamma(a_2)} e^{2ik\eta_B}, \quad (3.32)$$

$$\beta_k = \tilde{\beta}_k = \frac{\Gamma(a_3)\Gamma(a_3 - a_1 - a_2)}{\Gamma(a_3 - a_1)\Gamma(a_3 - a_2)}. \quad (3.33)$$

From the previous results, it then follows that

$$|\beta_k|^2 = \frac{1}{2} \left[ 1 + \cos\left(\frac{\pi}{\sqrt{3}}\right) \right] \text{csch}^2\left(\frac{\pi k}{\sqrt{6}k_B}\right). \quad (3.34)$$

The coefficients of the slow-roll phase (far future) and the ones of the contracting phase (far past) are related through Eq. (3.17). This relation gives that the coefficient  $\beta_k$  of Eq. (3.34) is exactly the Bogoliubov coefficient. As we have already discussed, the quantity  $|\beta_k|^2$  is the number of particles produced per mode  $k$ ,  $n_k$ . By computing this quantity, we can infer the quantity of particles produced since the initial BD vacuum in the beginning of the contracting phase.<sup>2</sup> This will be computed in the next section both analytically, when neglecting the backreaction effect of the produced particles on the background evolution, and numerically, by fully including its effect.

## IV. RESULTS

We next compute particle production due to the gravitational effects both analytically and numerically. From the analytical point of view, we make use of the results of the previous Sec. III C 4. In this case, the produced particles take no part in the dynamics; i.e., there are no backreaction effects. On the other hand, in the numerical setting, we consistently compute the particle production and include its effects in the Universe dynamics, such that the backreaction effects are self-consistently taken into account.

### A. Analytical results

The energy density of relativistic  $\chi$  particles that are produced due to the bounce is determined by Eq. (3.19) with Eq. (3.34). When performing the momentum integral in this equation, we must establish both ultraviolet (UV) and infrared (IR) cutoffs. This point requires some discussion since there is not a consensus on the choice of these integration limits. For instance, in Ref. [41], the integration limits were chosen such that the range of frequencies

<sup>2</sup>The presence of particles produced before the beginning of inflation have consequences to the power spectrum of the model, which receives a correction factor such that it is written as  $\Delta_{\mathcal{R}}(k) = |\alpha_k + \beta_k|^2 \Delta_{\mathcal{R}}^{GR}(k)$ , where  $\Delta_{\mathcal{R}}^{GR}$  is the GR form of the spectrum [15,41]. A fast oscillatory behavior of the power spectra arises from the final interference term, due to the rapidly changing relative phase. This oscillation is so fast in  $k$  that in any realistic observations they would be averaged out [41]. Consequently, the LQC power spectrum can be simply given by rescaling the standard one from general relativity by a factor of  $(1 + 2|\beta_k|^2)$ . The same equations of motion described in this section were also solved in Refs. [15,41] in the context of the curvature perturbation.

integrated are defined by the window of observable modes in the cosmic microwave background (CMB). This choice implies in a  $k_{\min}$  much higher than  $k_B$ . As discussed in Ref. [41], the dynamics of modes for such high values of  $k$  is largely insensitive to the background geometry, so they essentially evolve as if they were in flat spacetime. In this case, the evolved state is indistinguishable from the BD vacuum at the onset of inflation. In addition, there is nothing, in principle, to prevent particle production already starting at the scale  $\lambda$  of the effective horizon, right after the bounce. The particles that are produced at that moment should contribute to the energy density, specially if they are relativistic (radiationlike). In fact, if a too low IR cutoff  $k_{\min}$  is taken such to be much higher than  $k_B$ , we lose essentially all particle production, since  $k_B$  is exactly the scale around which (and below it) we expected that most of the gravitational particle production, due to the bounce, effectively happens. The associated modes may not be observed today, but they should contribute to the energy density during all the preinflationary era, where their energy density is not redshifted away too fast. For this reason, here we choose a different approach. For our purpose of estimating the energy density stored in the particles produced, we will be interested in the modes such that  $k_{\min}(\eta) \leq k \leq k_B$ , where  $k_{\min}(\eta) = \lambda^{-1} \equiv \sqrt{a''(\eta)/a(\eta)}$  for all  $\eta$ . The produced modes are effectively considered as particles after they reenter the horizon. Those are the modes which exit and reenter the effective horizon  $\lambda$  during the preinflationary phase, and then they will only reexit  $\lambda$  later, in the slow-roll inflationary phase. Hence, those are the modes which are expected to give the largest contribution to the particle production, since those are the ones that exit  $\lambda$  and reenter it before inflation starts. Our UV cutoff will then be given by  $k_B$ , while  $\lambda^{-1}$  yields the natural IR cutoff in the present problem. Therefore, the energy density of the particles created is given by Eq. (3.19), which with the appropriate IR and UV cutoffs, can be written as

$$\rho_p(\eta) = \frac{1}{2\pi^2 a^4(\eta)} \int_{k_{\min}(\eta)}^{k_B} dk k^2 n_k(\eta) \omega_k, \quad (4.1)$$

which upon using  $n_k \equiv |\beta_k|^2$ , with  $|\beta_k|^2$  given by Eq. (3.34) and  $\omega_k \sim k$  for relativistic  $\chi$  particles, we obtain that

$$\rho_p(\eta) = \frac{1 + \cos\left(\frac{\pi}{\sqrt{3}}\right)}{4\pi^2 a^4(\eta)} \int_{k_{\min}(\eta)}^{k_B} dk k^3 \text{csch}^2\left(\frac{\pi k}{\sqrt{6}k_B}\right). \quad (4.2)$$

Since the integral is dominated by the UV limit  $k_B$ , we can simply set the lower limit of integration to zero in the above equation. Thus, Eq. (4.2) gives

$$\rho_p(\eta) \simeq 4.5 \times 10^{-3} \frac{k_B^4}{a^4(\eta)} \simeq 0.012 \frac{m_{\text{pl}}^4}{a^4(\eta)}, \quad (4.3)$$

where we have used the result (3.21) for  $k_B$ . Equation (4.3) gives the energy density of particles produced after the main modes reenter  $\lambda$  after the bounce, i.e., when  $a(\eta) > 1$ . As already previously mentioned, the relevant modes for GPP of relativistic  $\chi$  particles are the ones such that  $k < k_B$ , which are the ones that exit and reenter  $\lambda$  during the bounce phase. These modes will only reexit  $\lambda$  again in the slow-roll inflationary phase. Therefore, we can neglect the particle production after the end of the bounce phase until the beginning of the inflationary phase, since the relevant modes for GPP are all inside  $\lambda$  during this period.

We are considering the cases where the kinetic energy density of the inflaton is the dominant energy component at the bounce. Thus, the background energy evolves like stiff matter,  $\rho_{\text{back}} = \rho_c/a^6$ . On the other hand, the  $\chi$  GPP evolves like relativistic matter  $\sim 1/a^4$ , as seen in Eq. (4.2). Let us first estimate an upper bound on the energy density of GPP by imposing that it remains subdominant up to the beginning of inflation. We write the energy density of particles produced as  $\rho_p = \rho_\chi a^{-4}$ , where  $\rho_\chi$  is the energy density of particles produced soon after the bounce (when the main modes reenter  $\lambda$ ). The elapsed  $e$ -foldings between the bounce and the start of inflation in LQC is around  $N_{\text{pre-infl}} = 4-5$   $e$ -folds (see, e.g., Ref. [15]). Thus, by requiring that at the start of inflation  $\rho_\chi < \rho_{\text{bg}}$ , we readily find the condition  $\rho_\chi \lesssim 2 \times 10^{-5} m_{\text{pl}}^4$  for the energy density of relativistic GPP produced around the bounce *not to dominate* the energy content of the Universe up until before inflation. Let us now estimate  $\rho_\chi$  from the result given by Eq. (4.3). We can take as an estimate the maximum of GPP to happen around the time  $t_s$  where  $a''/a = 0$  in the postbounce phase, which can be estimated from Eq. (2.5) to be  $t_s \simeq 0.3 t_{\text{pl}}$ . Thus, from Eq. (4.3), we can estimate that  $\rho_\chi \equiv \rho_p(t_s) \simeq 5 \times 10^{-3} m_{\text{pl}}^4$ . This result is about 2 orders of magnitude larger than the upper bound estimated above for the relativistic GPP not to dominate until before the beginning of inflation. Given the previous discussion about the validity of the truncation scheme used in the dressed metric approach, we see that what the above result indicates is that this standard procedure cannot be justified in this scenario, being necessary a full quantization approach.

However, the above analytical estimate for the energy density of GPP is expected to overestimate the particle production, since the backreaction of this GPP is fully absent in the derivation of  $\beta_k(\eta)$  that was presented in the previous section and, when included, should affect the evolution dynamics of the background, potentially decreasing the net GPP. In the next section, we compute the particle production by including continuously the backreaction of the GPP on the cosmological background. We then verify whether we can extend the validity of the dressed metric approach beyond the limit which it is strictly justified.



## B. Numerical results

The inclusion of the backreaction effects of the particles that are continuously produced on the cosmological background can only be done numerically. As in any problem of GPP, we have to properly account for how the initial conditions are set, in particular, for the field modes. We can consider two different sets of initial conditions for the modes. One is the BD vacuum, imposed at the contracting phase right before the quantum bounce, and the other is the fourth-order adiabatic vacuum state [46] set at the bounce. In Ref. [15], it was shown that, in the scenarios considered here, they are essentially equivalent and lead to the same results. In the following we take advantage of this finding, and we are going to consider the initial BD vacuum state in our numerical analysis, which makes the numerics much simpler.

As we have mentioned in the latter subsection, we will be interested in the modes such that  $k_{\min}(\eta) \leq k \leq k_B$ , where  $k_{\min}(\eta) \equiv \sqrt{a''(\eta)/a(\eta)}$ . Therefore, using these UV and IR limits, the energy density of produced particles reads like Eq. (4.1) from the previous subsection. The expression for the number density  $n_k$  is given through  $E_k$ , Eq. (3.11), expressed in terms of the field modes  $\chi_k$  and their time derivatives. The field modes  $\chi_k(\eta)$  are explicitly obtained by numerically solving their equation of motion, given by Eq. (3.9). If we fix the scale factor as the one given by Eq. (2.5) and compute the GPP, this procedure must give results similar to Eq. (4.2). This is the case where there is no backreaction in the universe dynamics. However, we are interested in including the effect of particle production on its dynamics, which demands us to add to the numerical system the equation for the stiff matter field evolution, Eq. (2.6), and to include the energy density of produced particles in the Friedmann equation, Eq. (2.3).

It is important to draw attention to some of the aspects of the GPP considered here. We compute particle production soon after the bounce. We consider as particles the modes which get inside  $\lambda$ , after the squeezing during the bounce, i.e., modes that satisfy  $k^2 < a''/a$  during the bounce phase for some time interval. Then, the energy contribution from particle production is computed at the instant these modes reenter  $\lambda$ . For each instant, we consider the modes which have entered  $\lambda$  and integrate on these modes. Near the bounce, modes with  $k \lesssim k_B$  are the relevant ones, whereas those with  $k > k_B$  are neglected as these modes are always *inside*  $\lambda$ ; i.e.,  $k^2 < a''/a$  is never satisfied. As we move away from the bounce, lower energy modes with  $k < k_B$  reenter  $\lambda$  and are also integrated. To compute the energy of produced particles, one can use Eq. (3.19) but keeping in mind that this expression is valid when we compare far past initial and far future final states. Equation (3.19) then gives the net energy production between these states. However, selecting the relevant modes,  $k_{\min}(\eta) \leq k \leq k_B$ , we obtain Eq. (4.1), which accounts only for particle modes. In Sec. IV A, we use Eq. (4.1) in order to obtain an analytical estimate due to its simplicity. However, for more accurate results, we need to observe that the initial

condition is not specified in the far past, but near the bounce. This may result in some spurious energy density contribution in the initial vacuum, which must be subtracted. In this context, we can obtain an exact energy density of produced particles using Eq. (3.20), which is obtained directly from the energy-momentum tensor and is integrated along the same lines of Eq. (4.1). However, this expression does not give the net result but the energy density at each instant of time. To obtain a net result, we must subtract the energy density at the initial vacuum state. Finally, we discuss the aforementioned subtraction and the initial condition. We take our initial condition before the bounce, at the instant  $t_i$  where  $a''/a = 0$ . This is not too far from the bounce to guarantee the precision of Eq. (4.1). Also, the fact that  $a''/a = 0$  at  $t = t_i$  does not guarantee that  $a''/a = 0$  in Eq. (3.20), which may result in non-negligible contributions to the energy density. However, if we consider the subtraction of the vacuum energy for both expressions, the computations are both consistent. Either way, we have explicitly verified that if the above cares are properly taken into account, the results coming from either Eq. (4.1) or Eq. (3.20) are completely equivalent.

Considering the above remarks, the full set of coupled equations to be solved numerically is then

$$\mathcal{H}^2 \equiv \left(\frac{a'}{a}\right)^2 = \frac{8\pi}{3m_{\text{Pl}}^2} a^2 \rho \left(1 - \frac{\rho}{\rho_c}\right), \quad (4.4)$$

$$\rho = \frac{\phi'^2}{2a^2} + V(\phi) + \rho_p, \quad (4.5)$$

$$\phi'' + 2\mathcal{H}\phi' + a^2 V_{,\phi} = 0, \quad (4.6)$$

$$\chi_k'' + \left(k^2 - \frac{a''}{a}\right)\chi_k = 0, \quad (4.7)$$

$$\rho_p = \frac{1}{2\pi^2 a^4} \int_{\sqrt{a''(\eta)/a(\eta)}}^{k_B} dk k^2 \omega_k |\beta_k(\eta)|^2, \quad (4.8)$$

$$|\beta_k(\eta)|^2 = \frac{1}{2\omega_k} [|\chi_k'(\eta)|^2 + \omega_k^2 |\chi_k(\eta)|^2] - \frac{1}{2}. \quad (4.9)$$

With the bounce dominated by the kinetic energy of the inflaton, the potential  $V(\phi)$  can be neglected during the contraction, bounce, and transition phases, and therefore, the explicit form of the potential does not affect the particle production phase. In practice, we consider the chaotic quartic potential for the inflaton,<sup>3</sup>

<sup>3</sup>The use of a quartic inflaton potential here is only meant for illustrative purposes. Any other suitable inflaton potential could also be considered as well. The difference between other choices of inflaton potential is expected only to be mostly important on the slow-roll inflationary dynamics and on the resulting observable quantities. Even though the quartic inflaton potential is ruled out in the simple inflationary scenarios, it can lead to consistent results and be in accordance to the most recent Planck data in the context of the warm inflation scenario (see, e.g., Refs. [39,47] and references therein), where inflation happens in a state characterized by the presence of dissipation and radiation effects.

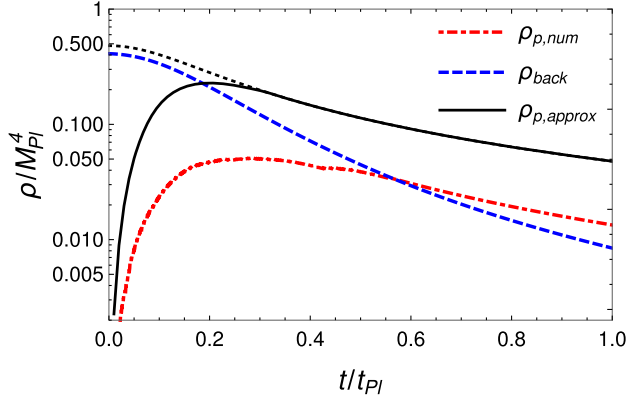


FIG. 1. Evolution of the different energy densities as a function of the cosmic time. The dot-dashed curve is the result obtained by numerically solving the coupled set of Eqs. (4.4)–(4.9). The dashed curve is the background energy density. The solid curve is the result obtained when using the approximation given by Eq. (4.2), while the dotted curve is the result given by Eq. (4.3).

$$V(\phi) = \frac{V_0}{4} \left( \frac{\phi}{m_{\text{pl}}} \right)^4, \quad (4.10)$$

with  $V_0$  fixed by the cosmic microwave background normalization for the amplitude of the power spectrum [48], which gives  $V_0/m_{\text{pl}}^4 \simeq 1.37 \times 10^{-13}$ .

The initial conditions for the  $\chi$  field momentum modes are chosen to be the BD ones,

$$\chi_k(\eta_i) = \frac{1}{\sqrt{2k}} e^{-ik\eta_i}, \quad \chi'_k(\eta_i) = -i\sqrt{\frac{k}{2}} e^{-ik\eta_i}, \quad (4.11)$$

where the initial time  $\eta_i$  is chosen in the prebounce phase, when  $a''/a = 0$ , which, in terms of the cosmic time, is found to be  $t_i \simeq -0.312t_{\text{pl}}$ . The initial conditions for the inflaton field and its derivative are chosen, such that we would have around 60  $e$ -folds of inflation when neglecting the backreaction of the GPP. This gives  $\phi(t_i) \simeq 1.94m_{\text{pl}}$  and  $\dot{\phi}(t_i) \simeq 0.45m_{\text{pl}}^2$  for the case of the potential in Eq. (4.10).

In Fig. 1, we present the behavior of the energy densities relevant for our analysis. The dot-dashed curve is the result for the energy density of produced particles by numerically solving the coupled set of Eqs. (4.4)–(4.9). The dashed curve gives the background energy density. The solid curve is the result obtained when using the approximation given by Eq. (4.2), while the dotted curve is given by Eq. (4.3). We see that the simple result Eq. (4.3) starts to agree with the one from Eq. (4.2) for  $t \gtrsim 0.3t_{\text{pl}}$ . In either case, both results overestimate the one obtained from the complete numerical analysis, which consistently accounts for the backreaction of the GPP. Both results, however, show a peak around  $t \sim 0.2t_{\text{pl}}$ , right after the bounce and the superinflation phase. The consistent inclusion of the backreaction effect smoothly introduces the produced radiation in the background, ceasing the continuous particle

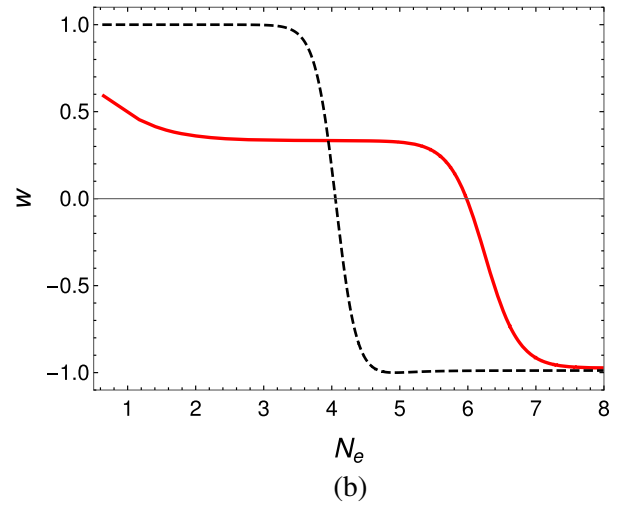
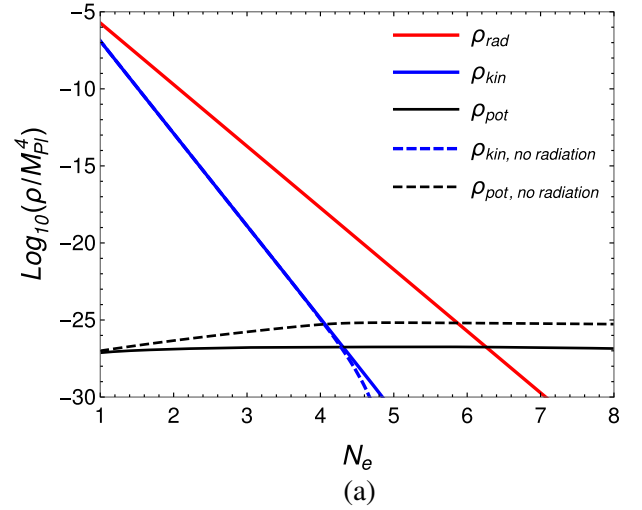


FIG. 2. Panel (a): The radiation energy density due to GPP, the inflaton's kinetic energy  $\dot{\phi}^2/2$  and potential energy  $V(\phi)$  as a function of the number of  $e$ -folds, in the presence of the backreaction effect due to the GPP (solid lines), and in the absence of GPP (dashed lines). Panel (b): The equation of state  $w = p/\rho$ , in the presence of GPP (solid line), and in the absence of it (dashed line).

production. In the numerical calculations,  $\rho_p$  should still grow for a moment after the modes enter  $\lambda$ , due to the time interval for the stabilization of the modes. By the time  $t \gtrsim 0.6t_{\text{pl}}$ , all the modes are already stabilized, the GPP ceases, and  $\rho_p$  just redshifts away as relativistic matter, as expected. At its peak value, we have  $\rho_p \simeq 0.05m_{\text{pl}}^4$ , which is still considerably larger than the upper bound estimated in the previous subsection,  $\rho_\chi \lesssim 2 \times 10^{-5}m_{\text{pl}}^4$ . By extending the validity of the dressed metric approach, the radiation gravitationally produced<sup>4</sup> during the bounce phase in LQC

<sup>4</sup>Here, we denote as radiation all the contributions, which have equation of state  $w = 1/3$ .

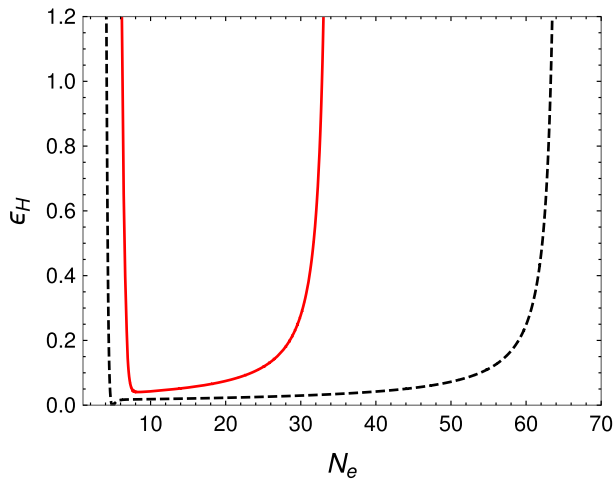


FIG. 3. The Hubble slow-roll parameter  $\epsilon_H = -\dot{H}/H^2$ , from the preinflationary phase up to the end of inflation, as a function of the number of  $e$ -folds, in the presence of the effect due to GPP (solid line) and in the absence of GPP (dashed line).

should not dominate the energy content of the Universe before inflation starts. Nevertheless, the numerical analysis, including the backreaction effects, shows that the radiation domination phase will start already at around  $t \sim 0.5t_{\text{pl}}$ .

By extrapolating the evolution up until the start of the slow-roll phase, in Fig. 2(a), we show the evolution of the kinetic and potential energy densities for the inflaton along with the radiation energy density due to the GPP and that include all the effects of backreaction due to the GPP in their respective evolutions (solid lines). For comparison, we also show the results for the kinetic and potential energies densities in the absence of the effects of the GPP. In Fig. 2(b), we show the equation of state  $w = p/\rho$ , in the presence of GPP (solid line), and in the absence of it (dashed line). The effect of the GPP is clear in the evolution of the quantities shown in Fig. 2. The energy density due to GPP quickly dominates over the kinetic energy soon after the bounce as described above. The kination regime in the preinflationary phase after the bounce is soon replaced by a radiation dominated regime ( $w \simeq 1/3$ ) that lasts until the potential energy dominates, when the slow-roll phase starts. While in the absence of GPP, the preinflationary phase lasts around four  $e$ -folds, the GPP extends it to around six  $e$ -folds, thus delaying the start of the inflationary phase. This also reflects strongly on the duration of the inflationary phase, which can be seen in Fig. 3, where we show the slow-roll parameter  $\epsilon_H = -\dot{H}/H^2$ , already in the preinflationary phase, till the end of inflation. In the absence of GPP and for the choice of initial conditions, we have taken in our numerical example, we have 60

$e$ -folds of inflation. However, the GPP shortens the inflationary phase to around 30  $e$ -folds. This is a common feature expected to happen to other choices of initial conditions for the inflaton: there will be an increase of the duration of the preinflationary phase and a decrease of the inflationary one, whenever radiation dominates the dynamics after the bounce [39].

## V. CONCLUSIONS

We have investigated the process of gravitational particle production in the preinflationary phase of loop quantum cosmology (LQC). We show, for the first time, that if we suppose the validity of the dressed metric approach in a LQC scenario with a BD initial condition in the contracting phase, which is equivalent to the choice of a fourth order adiabatic vacuum at the bounce [15], the backreaction of the produced particles leads to a short radiation dominated phase before the onset of inflation. Both the numerical and analytical analysis agree qualitatively in this conclusion. In a sense, the cosmological scenario we obtain in this case is very similar to the one obtained for LQC in warm inflation, as shown e.g., in Refs. [39,47]. In warm inflation, there is also a preinflationary phase following the bounce that is radiation dominated instead of kination dominated. However, in warm inflation, the radiation is produced by the intrinsic dissipative processes that can drive the decay of the inflaton field in light (relativistic) degrees of freedom. Here, the radiation dominated regime in the preinflationary phase after the bounce is due entirely to gravitational particle production that occurs during the bounce phase.

In fact, our results give an indication that it is not consistent to ignore the backreaction of the gravitationally produced particles in the preinflationary epoch of LQC, and this backreaction leads to a state at the onset of inflation significantly different from the BD vacuum. Since one should not expect that the dressed metric approach should be self-consistent in this regime, our analysis put in check the validity of the dressed metric approach in a LQC model with such initial conditions.

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