Toward $e^+e^- \to \pi^+\pi^-$ annihilation inspired by higher ρ mesonic states around 2.2 GeV

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Very recently, the *BABAR* Collaboration indicated that there exists an explicit enhancement structure near 2.2 GeV when focusing on the $e^+e^- \to \pi^+\pi^-$ process again, which inspires our interest in studying the production of higher ρ mesonic states. Since the branching ratio of $\pi^+\pi^-$ channel of D–wave ρ states are much smaller than S–wave states, we choose $\rho(1900)$ and $\rho(2150)$ as the intermediate states in $e^+e^- \to \pi^+\pi^-$, where $\rho(1900)$ and $\rho(2150)$ are treated as $\rho(3S)$ and $\rho(4S)$ states, respectively. Our result indicates that the *BABAR*'s data of $e^+e^- \to \pi^+\pi^-$ around 2 GeV can be depicted well, which shows that this enhancement structure near 2.2 GeV existing in $e^+e^- \to \pi^+\pi^-$ can be due to the contribution from two ρ mesons, $\rho(1900)$ and $\rho(2150)$. Additionally, this conclusion can be enforced by the consistence of the extracted values of $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)$ of $\rho(1900)$ and $\rho(2150)$ in the whole fitting processes and the corresponding theoretical calculations. The present study of $e^+e^- \to \pi^+\pi^-$ data may provide valuable information to establish the ρ meson family.

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I. INTRODUCTION

e^+e^- annihilation process can be as an ideal platform to study vector particles. A typical example is the observation of J/ψ charmonium [1]. By adopting initial state radiation method which plays crucial role to the observation of charmoniumlike state $Y(4260)$ from the $e^+e^- \to J/\psi\pi^+\pi^-$ process [2], the *BABAR* Collaboration measured the $e^+e^- \to \pi^+\pi^-$ process [3] by the collected 232 fb^{-1} experimental data, by which the cross section information of $e^+e^- \to \pi^+\pi^-$ from the $\pi^+\pi^-$ threshold to center-of-mass energy of 3 GeV was obtained. Very recently, the *BABAR* Collaboration focused on $e^+e^- \to \pi^+\pi^-$ again, and indicated that there exists an explicit enhancement structure near 2.2 GeV in the $\pi^+\pi^-$ invariant mass spectrum [4]. This phenomenon stimulates our interest in studying $e^+e^- \to \pi^+\pi^-$ since it has a close relation to establish light vector mesons around 2 GeV, which is one part of whole study of light hadron spectroscopy.

In fact, the $\pi^+\pi^-$ final state determines that $e^+e^- \to \pi^+\pi^-$ is a clean process to explore light vector ρ mesons with positive G parity. Generally, light vector mesons can be grouped into isovector ρ meson family, isoscalar ω and ϕ meson families. If checking the mass spectrum of light vector meson [5–7], we may find that some higher ρ, ω, and ϕ states accumulate around 2 GeV mass range, which may result in the difficulty of distinguishing them when analyzing some annihilation processes of e^+e^- into light mesons. It is obvious that the pollution from ω and ϕ mesons can be avoided for $e^+e^- \to \pi^+\pi^-$, which is the reason why we are dedicated to the study of $e^+e^- \to \pi^+\pi^-$ by combing with higher ρ mesons around 2 GeV.

In Ref. [6], Lanzhou group once performed the mass spectrum analysis and calculated these two-body Okuba-Zweig-Iizuka (OZI) allowed decays of ρ meson family. By combining with these reported ρ-like states collected in Particle Data Group (PDG) [8], the possible assignment of these ρ-like states into the ρ meson family was suggested [6], which is crucial step of constructing ρ meson family. However, it is not the end of whole story.

After releasing the detailed data of cross section of $e^+e^- \to \pi^+\pi^-$ [3,4] by *BABAR*, we may continue to carry out the study of the production of higher ρ mesonic states around 2 GeV via $e^+e^- \to \pi^+\pi^-$. Based on the results

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given in Ref. [6], we may select suitable higher ρ mesons around 2 GeV as the intermediate states in $e^+e^- \to \pi^+\pi^-$. And then, by fitting the cross section of $e^+e^- \to \pi^+\pi^-$ around 2.2 GeV under our theoretical approach, we may get the information of the contribution of these ρ mesonic states to the cross section of $e^+e^- \to \pi^+\pi^-$, which is valuable to further test the suggestive assignment of the ρ meson family [6]. We hope that our effort presented in this work can improve our understanding of constructing the ρ meson family, especially for these higher ρ mesonic states around 2 GeV.

This paper is organized as follows. After the Introduction, we will give a concise review of ρ mesons with mass around 2 GeV, and introduce the possible assignment to them (see Sec. II). In Sec. III, we will present our theoretical framework of calculating $e^+e^- \to \pi^+\pi^-$ with these discussed ρ mesons as intermediate states. When fitting the *BABAR* data, we finally extract the magnitude of different ρ meson contributions to the $e^+e^- \to \pi^+\pi^-$ cross section around 2.2 GeV. And then, we give the numerical results in Sec. IV. This paper ends with the summary.

II. THE SITUATION OF ρ MESONS AROUND 2 GeV

There are many ρ-like states reported by experiments. Among these states, $\rho(770)$ is a well established ground state with very broad width. As shown in PDG [8], $\rho(1450)$ can be assigned as the first radial excited state of $\rho(770)$. By the analysis of the mass spectrum [9] and the study of total decay width [6] and the branching ratio of the $\rho(1700) \to 2\pi, 4\pi$ [10] and $e^+e^- \to \omega\pi^0$ process [11], we may find that $\rho(1700)$ as a candidate of the $\rho(1^3D_1)$ meson state is suitable. It is the research status of some low-lying ρ-like states.

$\rho(1900)$ was first observed by the DM2 Collaboration, which corresponds to a dip around 1.9 GeV with analyzing the process $e^+e^- \to 6\pi$ [12]. After that, there were many experiments relevant to $\rho(1900)$ which include the FENICE Collaboration [13], the E687 Collaboration [14,15], the *BABAR* Collaboration [16,17], and the CMD3 Collaboration [18]. In addition, the analysis of experimental data on isovector P–wave of pion-pion scattering also testifies in favor of the existence of $\rho(1900)$ [19]. It is worth nothing that $\rho(1900)$ was identified from 6π peak exactly at the $p\bar{p}$ threshold [18]. Thus, Bugg suggested that this state is likely to be a $\rho(^3S_1)$ captured by the very strong $p\bar{p}$ S–wave but could be a nonresonant cusp effect [20].

In 2013, Lanzhou group [6] indicated that $\rho(1900)$ can be regarded as 3^3S_1 state since the obtained total width overlaps with the *BABAR*'s dada [21]. Here, the main decay channels of $\rho(1900)$ are $\pi\pi$, $\pi a_1(1260)$, $\pi h_1(1170)$, $\pi\pi(1300)$, and $\pi\omega(1420)$ [6]. Therefore, $\rho(1900)$ with a large branching ratio of 4π can be understood.

Clegg and Donnachi jointly analyzed the data on 6π states produced in the e^+e^- annihilation and diffractive photoproduction, and indicated that there exists a resonance with peak near 2.1 GeV [22–24], which corresponds to $\rho(2150)$. Later, $\rho(2150)$ was assigned as the third radial excitation of $\rho(770)$ by fitting the pion form factor [25]. In addition, other experiments like GAMS [26,27], Crystal Barrel [28–31] and *BABAR* [32] confirmed the observation of $\rho(2150)$ in different processes.

TABLE I. The suggested assignment to these observed ρ-like states and the information of their main decay channels from Ref. [6].

State	Assignment	Main decay channels
$\rho(770)$	$\rho(1S)$	$\pi\pi$
$\rho(1450)$	$\rho(2S)$	$\pi\pi$, $\pi a_1(1260)$, $\pi\omega$, $\pi h_1(1170)$
$\rho(1900)$	$\rho(3S)$	$\pi\pi$, $\pi a_1(1260)$, $\pi h_1(1170)$, $\pi\pi(1300)$, $\pi\omega(1420)$
$\rho(2150)$	$\rho(4S)$	$\pi\pi$, $\pi a_1(1260)$, $\pi\omega$, $\pi h_1(1170)$
$\rho(1700)$	$\rho(1D)$	$\pi a_1(1260)$, $\pi h_1(1170)$
$\rho(2000)$	$\rho(2D)$	$\pi\pi(1300)$, $\rho\rho$, $\pi\pi_2(1670)$, $\pi a_1(1260)$
$\rho(2270)$	$\rho(3D)$	$\pi\pi(1300)$, $\pi\pi(1800)$

According to the analysis of mass spectrum [20,33–35], $\rho(2150)$ can be a good candidate of $\rho(4^3S_1)$ meson state. The study of OZI-allowed two-body strong decay behaviors of $\rho(2150)$ supports this assignment [6], since the SPEC's data [31] can be reproduced [6]. Here, the dominant channels of $\rho(2150)$ are $\pi\pi$, $\pi a_1(1260)$, $\pi\omega$ and $\pi h_1(1170)$ [6], which can explain why $\rho(2150)$ was observed in $\pi^+\pi^-$, $\omega\pi^0$, $\eta'\pi\pi$, $f_1(1285)\pi\pi$, and $\omega\pi\eta$ experimentally.

In PDG [8], there are two ρ-like states are listed as further state, which are $\rho(2000)$ and $\rho(2270)$. $\rho(2000)$ was observed in the $p\bar{p} \to \pi\pi$ reaction with the mass around 1988 MeV [36]. Later, a combined fit was presented to the data of $p\bar{p} \to \omega\eta\pi^0$ and $\omega\pi$, by which the existence of $\rho(2000)$ was confirmed [28]. The analysis of the Regge trajectory shows $\rho(2000)$ as the first radial excitation of $\rho(1700)$ [6,31]. The dominant channels of $\rho(2000)$ include $\pi\pi(1300)$, $\rho\rho$, $\pi\pi_2(1670)$ and $\pi a_1(1260)$ indicated in Ref. [6].

In the reaction $\gamma p \to \omega\pi^+\pi^-\pi^0$, a spin-parity analysis shows the existence of a resonance with $J^P = 1^-$ in the $\omega\rho^\pm\pi^\mp$ final state, which has mass around 2.28 ± 0.05 GeV [23]. And then, the Crytal Barrel experiment fitted the $\omega\eta\pi$ data from the $p\bar{p}$ annihilation, where $\rho(2270)$ was confirmed [28]. The Regge trajectory analysis gives that $\rho(2270)$ is a good candidate of the second radial excitation of $\rho(1700)$ [6,31]. The decay information of $\rho(2270)$ was also provided in Ref. [6].

In Table I, we collect the information of these reported ρ-like states, and their assignments and dominant decay channels.

III. DEPICTING THE CROSS SECTION OF $e^+e^- \to \pi^+\pi^-$ AROUND 2 GeV

In this section, we focus on the $e^+e^- \to \pi^+\pi^-$ process at center-of-mass energy around 2 GeV. Due to the constraint of conservation of G parity, $e^+e^- \to \pi^+\pi^-$ can be applied to study ρ-like states. As shown in Fig. 1, there exist two mechanisms working together for $e^+e^- \to \pi^+\pi^-$. The first one is e^+e^- direct annihilation into $\pi^+\pi^-$, where the virtual photon couples with the final state $\pi^+\pi^-$, which provides the background contribution. The second one is that $e^+e^- \to \pi^+\pi^-$ occurs via intermediate ρ states.

When trying to reproduce the line shape of the cross section of $e^+e^- \to \pi^+\pi^-$ process around 2 GeV reported by the *BABAR* Collaboration recently [4], we need to choose suitable intermediate ρ meson states. The collected information in Table I shows that $\rho(1900)$, $\rho(2000)$, $\rho(2150)$, and $\rho(2270)$ should be considered in our calculation.

In this work, we adopt effective Lagrangian approach to calculate these discussed processes shown in Fig. 1. The effective Lagrangian involved in the concrete work include [37]

$$\begin{aligned}\mathcal{L}_{\pi\pi\gamma} &= ieA^\mu(\pi\partial_\mu\pi - \partial_\mu\pi\pi),\\ \mathcal{L}_{\rho\pi\pi} &= ig_{\rho\pi\pi}\rho_i^\mu(\pi\partial_\mu\pi - \partial_\mu\pi\pi),\\ \mathcal{L}_{\gamma\rho} &= -e\frac{m_\rho^2}{f_\rho}\rho^\mu A_\mu.\end{aligned} \tag{1}$$

It is worth noting that the above Lagrangian densities are not unique. For example, for $\mathcal{L}_{\rho\pi\pi}$, the Lorentz structure involving the derivative of the field-strength tensor of the ρ meson field, i.e., $\partial_\mu F_{\mu\nu}(\pi\partial_\nu\pi)$ with $F_{\mu\nu} = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu$, is also allowed. However, the current experimental data from *BABAR* does not support us to consider more Lagrangian densities in a realistic calculation. Of course, we hope that these contributions from other Lagrangian couplings can be included in accurate experimental measurements of $e^+e^- \to \pi^+\pi^-$ in the future.

The amplitudes corresponding to the diagrams in Fig. 1 can be written as

$$\begin{aligned}\mathcal{M}_{\rm Dir} &= [\bar{v}(p_2,m_e)(ie\gamma^\mu)u(p_2,m_e)]\frac{-g_{\mu\nu}}{q^2}[ie(p_4^\nu - p_3^\nu)F_\pi(q^2)],\\ \mathcal{M}_{\rho_i} &= [\bar{v}(p_2,m_e)(ie\gamma_\mu)u(p_2,m_e)]\frac{-g^{\mu\xi}}{q^2}\left(-e\frac{m_{\rho_i}^2}{f_{\rho_i}^2}\right)\\ &\quad\times\frac{-g_{\xi\nu}+q_\xi q_\nu/m_{\rho_i}^2}{q^2-m_{\rho_i}^2+im_{\rho_i}\Gamma_{\rho_i}}[ig_{\rho_i\pi\pi}(p_4^\nu - p_3^\nu)].\end{aligned} \tag{2}$$

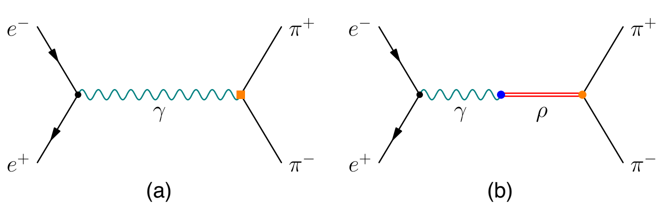

FIG. 1. The diagrams for depicting the $e^+e^- \to \pi^+\pi^-$ process. Here, (a) is direct annihilation process while (b) corresponds to the intermediate ρ state contribution.

Here, F_π is the timelike form factor of charged pion and $q = p_1 + p_2$. ρ_i denote intermediate ρ meson states. m_{ρ_i} and Γ_{ρ_i} are resonance parameter, which can be fixed by the corresponding experimental data [8]. The total amplitude of $e^+e^- \to \pi^+\pi^-$ is superposition of different contribution

$$\mathcal{M}_{\rm Total} = \mathcal{M}_{\rm Dir} + \sum_i e^{i\theta_i}\mathcal{M}_{\rho_i}, \tag{3}$$

where θ_i denotes the phase angle between the amplitudes from direct annihilation and the intermediate ρ state contribution. $g_{\rho_i} = g_{\rho_i\pi\pi}m_{\rho_i}^2/f_{\rho_i}$ and f_{ρ_i} represents decay constant of some ρ meson. With the above amplitude, the differential cross section of $e^+e^- \to \pi^+\pi^-$ can be calculated directly, i.e.,

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s}\frac{1}{|p_{1\ {\rm cm}}|^2}\overline{|\mathcal{M}_{\rm Total}|^2}. \tag{4}$$

When fitting the cross section for $e^+e^- \to \pi^+\pi^-$, we can treat θ_i and g_{ρ_i} as free parameters. Additionally, the form factor of charged pion is not determined since the form factor in the timelike range is a complex function. In general, the form factor changes slowly when q^2 is far away from threshold. For simplicity, we assume the form factor is a constant in the center-of-mass energy considered here. Thus, we argue that the form factor of charged pion can be absorbed into phase angle.[1] In the present work, the form factor is taken as $F_K(s) = ae^{-b\sqrt{s}}$, where a and b are free parameters.

By the effective Lagrangian listed in Eq. (1), the dilepton and $\pi^+\pi^-$ decay widths of these intermediate ρ meson state can be expressed as

[1]Assuming that the pion form factor is the product of a constant term and an exponential term, and complex character of form factor can be reflected in the constant term, i.e.,

$$F_\pi(s) = ae^{-i\phi}e^{-b\sqrt{s}}, \tag{5}$$

where $e^{-i\phi}$ is an any complex number and its modulus can be absorbed into free parameter a. According to the definitions in Eqs. (2) and (3), then the total amplitude can be written as

$$\begin{aligned}\mathcal{M}_{\rm Total} &= e^{-i\phi}\mathcal{M}_{\rm Dir} + \sum_i e^{i\phi_i}\mathcal{M}_{\rho_i} \Rightarrow e^{-i\phi}\mathcal{M}'_{\rm Total}\\ &= e^{-i\phi}\left(\mathcal{M}_{\rm Dir} + \sum_i e^{i(\phi_i+\phi)}\mathcal{M}_{\rho_i}\right).\end{aligned} \tag{6}$$

Because $|e^{-i\phi}\mathcal{M}'_{\rm Total}|^2 = |\mathcal{M}_{\rm Total}|^2 = |\mathcal{M}'_{\rm Total}|^2$, so the above scattering amplitude is equivalent to Eq. (3). That is to say, the complex part of form factor can be absorbed into phase angle between the amplitudes from direct annihilation and the intermediate ρ state contribution. Thus, the form factor can be taken as a real form.

TABLE II. The information of four intermediate ρ state involved in the $e^+e^- \to \pi^+\pi^-$ process around 2.2 GeV. The second and the third columns are resonance parameter. R is the parameter in SHO wave function [see Eq. (11)] which was given in Ref. [6]. $\mathcal{B}(\pi^+\pi^-)$ is branching ratio of $\pi^+\pi^-$ mode calculated via the QPC model [6]. $\Gamma_{e^+e^-}$ is dilepton decay width calculated in this work by Eq. (9). By the numerical results listed in the fifth and the sixth columns, the results of $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)$ can be obtained.

State	$M_{\rm exp}$ (MeV)	$\Gamma_{\rm exp}$ (MeV)	R (GeV^{-1}) [6]	$\mathcal{B}(\pi^+\pi^-)$ [6]	$\Gamma_{e^+e^-}$ (keV)	$\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)$ (keV)
$\rho(1900)$	$1909 \pm 17 \pm 25$ [16]	160 ± 20 [16]	$3.85 \sim 4.28$	$0.1450 \sim 0.3509$	$0.1958 \sim 0.1578$	$0.0284 \sim 0.0554$
$\rho(2150)$	2150 ± 17 [6]	230 ± 50 [31]	$4.74 \sim 4.98$	$0.3889 \sim 0.3396$	$0.0888 \sim 0.0806$	$0.0345 \sim 0.0274$
$\rho(2000)$	2000 ± 30 [38]	260 ± 45 [38]	$4.34 \sim 4.80$	$0.0740 \sim 0.0573$	$0.0204 \sim 0.0160$	$0.0015 \sim 0.0009$
$\rho(2270)$	2265 ± 40 [31]	325 ± 80 [31]	$4.40 \sim 4.80$	$0.0510 \sim 0.0315$	$0.0163 \sim 0.0129$	$0.0008 \sim 0.0004$

$$\Gamma_{e^+e^-} = \frac{e^4 m_{\rho_i}}{12\pi f_{\rho_i}^2},$$
$$\Gamma_{\pi^+\pi^-} = \frac{g_{\rho_i\pi\pi}^2 (m_{\rho_i}^2 - 4m_\pi^2)^{3/2}}{48\pi m_{\rho_i}^2}, \tag{7}$$

respectively. Thus, we may further define the production of dilepton decay width and the branching ratio of $\pi^+\pi^-$ mode of the discussed ρ meson

$$\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-) = \frac{e^4 g_{\rho_i}^2 (m_{\rho_i}^2 - 4m_\pi^2)^{3/2}}{576\pi^2 m_{\rho_i}^5 \Gamma_{\rho_i}}, \tag{8}$$

where m_{ρ_i} and m_π are the masses of intermediate ρ states and final state pion, respectively.

IV. NUMERICAL RESULTS

In the following, we will fit the cross section for $e^+e^- \to \pi^+\pi^-$ measured by the *BABAR* Collaboration [4], where an event accumulation near 2.2 GeV exists in the $\pi^+\pi^-$ invariant mass spectrum. As seen in Table II, $\pi^+\pi^-$ is the most dominant decay channel for the S-wave ρ mesons but not important to the D-wave ρ mesons, which is from the theoretical calculation in Ref. [6]. Especially, we find that the theoretical result of $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)$ for S-wave ρ mesons ($\rho(1900)$ and $\rho(2150)$) is one order of magnitude larger than that for D-wave ρ mesons ($\rho(2000)$ and $\rho(2270)$). Thus, in order to reduce the number of fitting parameters, $\rho(2000)$ and $\rho(2270)$ that are treated as D-wave meson states [6] will not be considered in the following study. In our realistic analysis, we only choose $\rho(1900)$ and $\rho(2150)$ as intermediate resonances, which are assigned as $\rho(3S)$ and $\rho(4S)$ states, respectively. When fitting the experimental data under our theoretical framework, there are six free parameters, a, b, θ_1, θ_2, $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(1900)}$ and $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(2150)}$. Here, the subscripts in $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(1900)}$ and $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(2150)}$ is applied to distinguish the $\rho(1900)$ and $\rho(2150)$ contributions. Additionally, $[m_{\rho(1900)}, \Gamma_{\rho(1900)}]$ and $[m_{\rho(2150)}, \Gamma_{\rho(2150)}]$ as the resonance parameters of $\rho(1900)$ and $\rho(2150)$, respectively, are input parameters which are taken from PDG (see Table II).

In the fitting process, we found that $\rho(2150)$ play dominant role to reproduce the line shape of $e^+e^- \to \pi^+\pi^-$ around 2.2 GeV [4]. We can find two solutions (solution A and solution B), both of which can reproduce the *BABAR*'s data well. In Table III, we list these obtained fitting parameters. And then, in Fig. 2, we further present the fitted results and the comparison with the experimental data. It is worth mentioning that the intermediate resonance $\rho(1900)$ has obvious contribution if describing the line shape corresponding to the center-of-mass energy $\sqrt{s} < 2.2$ GeV.

We also notice obvious difference of the fitted $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(1900)}$ and $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(2150)}$ under two solutions, which makes us to check the reasonability of the obtained $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(1900)}$ and $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(2150)}$ values.

$\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(1900)}$ and $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(2150)}$ can be theoretically calculated. In Ref. [6], Lanzhou group have calculated the widths of these discussed ρ mesons decaying into the $\pi^+\pi^-$ channel by the quark pair creation (QPC) model. Under the framework of the potential model, the general expression of dilepton decay width of the discussed ρ state [9] is

$$\Gamma_{e^+e^-} = \frac{4\pi}{3}\alpha^2 m_{\rho_i} \mathcal{M}_{\rho_i}^2, \tag{9}$$

where $\mathcal{M}_{\rho_i}$ denotes decay amplitude, which is defined as $\mathcal{M}_{\rho_i} = \sqrt{2}V_{\rho_i}$ and $\mathcal{M}_{\rho_i} = (4/3)^{1/2}V'_{\rho_i}$ for S–wave and D–wave ρ mesons, respectively. Here, factor V_{ρ_i} and V'_{ρ_i} read as

TABLE III. The parameters obtained by fitting the cross section of $e^+e^- \to \pi^+\pi^-$ measured by *BABAR* [4].

Parameters	Solution A	Solution B
a	0.22 ± 0.16	0.54 ± 0.02
b (GeV^{-1})	1.06 ± 0.36	1.37 ± 0.01
θ_1 (rad)	1.55 ± 0.49	1.45 ± 0.35
θ_2 (rad)	4.98 ± 0.10	5.17 ± 0.18
$\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(1900)}$ (eV)	15.06 ± 6.10	3.80 ± 0.89
$\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(2150)}$ (eV)	44.53 ± 18.33	2.74 ± 0.79
χ^2/d.o.f	1.10	1.06

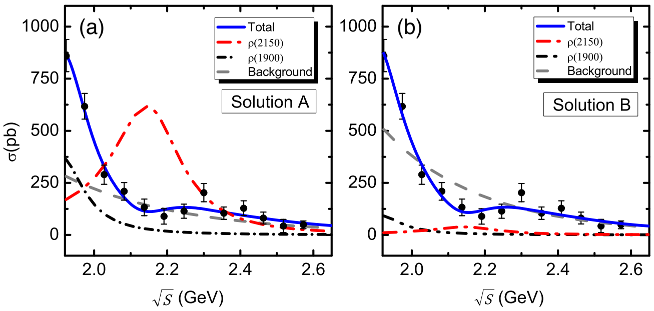

FIG. 2. The fitted result of the cross section of $e^+e^- \to \pi^+\pi^-$. Here, the black dots with error bar is *BABAR* result [4]. We present two solutions [Solution A (left) and Solution B (right)], which can depict the data well.

$$V_{\rho_i} = m_{\rho_i}^{-2}\tilde{m}_{\rho_i}^{1/2}(2\pi)^{3/2}\int d^3p(4\pi)^{-1/2}\phi_{\rho_i}(p)\left(\frac{m_1m_2}{E_1E_2}\right)^{1/2},$$
$$V'_{\rho_i} = m_{\rho_i}^{-2}\tilde{m}_{\rho_i}^{1/2}(2\pi)^{3/2}\int d^3p(4\pi)^{-1/2}\phi_{\rho_i}(p) \times\left(\frac{m_1m_2}{E_1E_2}\right)^{1/2}\left(\frac{p}{E_1}\right)^2. \quad (10)$$

Here, $m_1 = m_2 = 0.22$ GeV [9] is quark mass inside the ρ meson, and E_1 and E_2 are the energy of the corresponding quarks. And then, $\tilde{m}_{\rho_i} = 2\int d^3pE|\phi_{\rho_i}(p)|^2$. The radial part of the spatial wave functions of these involved ρ meson states can be depicted by the radial part of simple harmonic oscillator (SHO) wave function $\phi_{\rho_i}(p)$, i.e.,

$$\phi_{\rho_i}(p) = (-1)^n(-i)^LR^{3/2}e^{-\frac{p^2R^2}{2}}\sqrt{\frac{2n!}{\Gamma(n+L+3/2)}}(pR)^L \times L_n^{L+1/2}(p^2R^2). \quad (11)$$

Here, R that refers to the size of meson state is the parameter of SHO wave function and its possible value has been suggested by Ref. [6], which is summarized in Table II. Combined with the $\Gamma_{\pi^+\pi^-}$ listed in Table II, $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)$ can be directly calculated, which is also dependent on R value (see the seventh column in Table II). Since the fitted results of $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(1900)}$ and $\Gamma_{e^+e^-}\mathcal{B}(\pi^+\pi^-)_{\rho(2150)}$ are comparable with the calculated results from the potential model, the fitted result corresponding to Solution A in Fig. 2 and Table III is more favorable.

V. SUMMARY

The $e^+e^- \to \pi^+\pi^-$ process is a good platform to study ρ-like states due to G-parity conservation. Inspired by the *BABAR* measurement of the cross section of the $e^+e^- \to \pi^+\pi^-$ process around 2 GeV, we study the contribution of higher radial excitations in the ρ meson family to $e^+e^- \to \pi^+\pi^-$. When reproducing the experimental data of $e^+e^- \to \pi^+\pi^-$ around 2.2 GeV, $\rho(2150)$ and $\rho(1900)$ as $\rho(4S)$ and $\rho(3S)$ play important role. Combining with former study of mass spectrum and decay behavior of ρ meson family [6], the present work enforces the assignment of $\rho(2150)$ and $\rho(1900)$ as $\rho(4S)$ and $\rho(3S)$, respectively, which is a crucial step in constructing ρ meson family.

In recent years, BESIII measured some processes of e^+e^- annihilation into light mesons [39–43]. In the following, we may focus on other typical processes of e^+e^- annihilation into light mesons, which have close relation to light vector mesonic states. Since there exist abundant ρ, ω and ϕ higher excitations around 2 GeV, studying these processes of e^+e^- annihilation into light mesons is helpful to better understand the contribution of these light vector meson to these processes. Obviously, the present work is a beneficial attempt on this issue.

We also feel that promoting experimental precision can inspire theoretical progress. The present work is a good example for this point. In the near future, BESIII and Belle II will be main force of the study of light hadron spectroscopy [44,45]. We also expect more precise experimental data of processes of e^+e^- annihilation into light mesons, which is valuable to construct light vector meson family.

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