

Nonleptonic two-body weak decays of Λ_b in a modified MIT bag model

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We study the nonleptonic two-body weak decays of Λ_b by modifying the MIT bag model without introducing new parameters to construct the momentum eigenstates of the baryons. We find that the branching ratios of $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-$, $\Lambda_c^+ K^-$, $p\pi^-$, and pK^- are $(4.5 \pm 0.2) \times 10^{-3}$, $(3.4 \pm 0.1) \times 10^{-4}$, $(5.0 \pm 0.5) \times 10^{-6}$, and $(6.0 \pm 0.7) \times 10^{-6}$, which are all consistent with the current experimental data, respectively. We also explore P and CP asymmetries for the decays of $\Lambda_b^0 \rightarrow p(\pi^-, K^-)$. In particular, we obtain that the direct CP -violating rate asymmetries in $\Lambda_b^0 \rightarrow p\pi^-$ and $\Lambda_b^0 \rightarrow pK^-$ are around -4.4% and 6.7% , in comparison with $(-2.5 \pm 2.9)\%$ and $(-2.5 \pm 2.2)\%$ from the Particle Data Group in 2020, respectively.

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I. INTRODUCTION

There have been many interesting measurements in the decays of b baryons by the LHCb Collaboration, including those for the charmful modes of $\Lambda_b^0 \rightarrow \Lambda_c^+ M$ [1] and charmless ones of $\Lambda_b^0 \rightarrow pM$ [2] ($M = \pi^-, K^-$) as well as the discoveries of the hidden-charm pentaquarks in $\Lambda_b^0 \rightarrow J/\psi pM$ [3,4] and double-charm baryon state of Ξ_{cc}^{++} via $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ [5]. In particular, based on the new experimental data from LHCb [6], the Particle Data Group (PDG) of 2020 [7] has updated the average values for the direct CP -violating rate asymmetries (\mathcal{A}_{CPs}) in $\Lambda_b^0 \rightarrow p\pi^-$ and $\Lambda_b^0 \rightarrow pK^-$ to be

$$\begin{aligned} \mathcal{A}_{CP}(\Lambda_b^0 \rightarrow p\pi^-)_{\text{PDG}} &= (-2.5 \pm 2.9)\%, \\ \mathcal{A}_{CP}(\Lambda_b^0 \rightarrow pK^-)_{\text{PDG}} &= (-2.5 \pm 2.2)\%, \end{aligned} \quad (1)$$

respectively. In the standard model, these two decay amplitudes contain V_{ub} and loop-induced penguin operators [8], which could provide weak and strong phases, respectively, resulting in nonvanishing direct CP -violating rate asymmetries (\mathcal{A}_{CPs}). Previous theoretical studies of \mathcal{A}_{CPs} in Eq. (1) have been performed in various QCD models, such as the perturbative QCD method [9],

generalized factorization approach [10], and light-front quark model (LFQM) [11]. These calculations in the literature are important to check if the results in the standard model are consistent with the experimental measurements. In this work, we would like also to explore the CP -violating asymmetries with the MIT bag model.

It is known that to calculate the decay processes of the baryons, we need to know the details of the baryon wave functions. In the MIT bag model, the quarks in the baryon are confined in a static bag. The model enjoys various successes in its simple structure to explain the mass spectra and magnetic dipole moments of the baryons [12–15]. However, the construction of the baryon wave functions in the MIT bag model is localized in a fixed space. As a result, the center motions of the baryons are not moved. In other words, the baryon wave functions are not in the momentum eigenstates. Such a defect makes the model questionable when one discusses the decay processes involving essentially the momentum eigenstates. Nonetheless, many calculations in the bag model have been done by ignoring this problem [16–22]. In this work, we will modify the MIT bag model to construct the momentum eigenstates for the baryons to study the decays without introducing extra parameters.

On the other hand, it is known that one can examine the heavy quark symmetry in the charmful decays of $\Lambda_b^0 \rightarrow \Lambda_c^+ M$. Particularly, the baryonic matrix element is related to the Isgur Wise function of $\xi(\omega)$ [23], given by

$$\langle \Lambda_c^+ | \bar{c} \gamma^\mu (1 - \gamma_5) b | \Lambda_b^0 \rangle = \bar{u}_{\Lambda_c^+} \xi(1 - \gamma_5) u_{\Lambda_b^0}, \quad (2)$$

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where $\omega = v_1 \cdot v_2$ with $v_{1,2}$, the velocities of the two baryons, and $u_{\mathbf{B}}$ is the four-component Dirac spinor for the corresponding baryon of \mathbf{B} . In the heavy quark limit, in which the relative velocity between the two baryons vanishes, we have that $\xi(\omega) = 1$ with $\omega = v_1 \cdot v_2$. We will check if the heavy quark symmetry is valid in our modified MIT bag model.

This paper is organized as follows. In Sec. II, we introduce the decay amplitudes and parameters. In Sec. III, we constitute the baryon wave functions in the modified MIT bag model, where we sum over the localized baryon wave functions with different centers. In Sec. IV, we compute the form factors accordingly. In Sec. V, we present our numerical results and compare them with the experimental data as well as the theoretical evaluations in the literature. We conclude our study in Sec. VI.

II. DECAY AMPLITUDES AND PARAMETERS

We start with the two-body decays of $\mathbf{B}_i \rightarrow \mathbf{B}_q M$, where $\mathbf{B}_{i(q)}$ is the initial (final) baryon with spin 1/2 and M represents the pseudoscalar meson. In this study, we concentrate on the corresponding decays with $\mathbf{B}_i = \Lambda_b^0$, $\mathbf{B}_q = (\Lambda_c^+, p)$, and $M = (\pi^-, K^-)$. The spin-dependent amplitude for $\mathbf{B}_i \rightarrow \mathbf{B}_q M$ can be written as

$$\mathcal{A}(\mathbf{B}_i \rightarrow \mathbf{B}_q M) = \bar{u}_{\mathbf{B}_q} (A - B\gamma_5) u_{\mathbf{B}_i}, \quad (3)$$

where A and B are the s -wave and p -wave amplitudes, corresponding to the parity violating and conserving ones, and $u_{\mathbf{B}_{i,q}}$ are the baryon Dirac spinors, respectively. In general, A and B are not relatively real, resulting in CP -violating effects. The decay branching ratio and forward-backward asymmetry for the initial baryon in the rest frame are given as [24]

$$\begin{aligned} \Gamma(\mathbf{B}_i \rightarrow \mathbf{B}_q M) &= \frac{|\vec{p}_q|}{8\pi} \left(\frac{(M_{B_i} + M_{B_q})^2 - m_M^2}{M_{B_i}^2} |A|^2 \right. \\ &\quad \left. + \frac{(M_{B_i} - M_{B_q})^2 - m_M^2}{M_{B_i}^2} |B|^2 \right), \\ \alpha_P(\mathbf{B}_i \rightarrow \mathbf{B}_q M) &= \frac{2\kappa \text{Re}(A^* B)}{|A|^2 + \kappa^2 |B|^2}, \quad \kappa = \frac{|\vec{p}_q|}{E_{B_q} + m_{B_q}}, \end{aligned} \quad (4)$$

where m_M is the meson mass and \vec{p}_q represents the 3-momentum of the daughter baryon \mathbf{B}_q , while α_P describes the decay asymmetry between the helicity states of \mathbf{B}_q , defined by

$$\alpha_P(\mathbf{B}_i \rightarrow \mathbf{B}_q M) = \frac{\Gamma(\hat{p}_q \cdot \hat{s}_q = 1) - \Gamma(\hat{p}_q \cdot \hat{s}_q = -1)}{\Gamma(\hat{p}_q \cdot \hat{s}_q = 1) + \Gamma(\hat{p}_q \cdot \hat{s}_q = -1)} \quad (5)$$

with $\hat{p}_q(\hat{s}_q)$ the unit vector in the $\vec{p}_q(\vec{s}_q)$ direction of \mathbf{B}_q , provided with the initial baryon \mathbf{B}_i unpolarized. We can

also define the decay asymmetries for the antiparticles, given by

$$\bar{\alpha}_P(\mathbf{B}_i \rightarrow \mathbf{B}_q M) = \alpha_P(\bar{\mathbf{B}}_i \rightarrow \bar{\mathbf{B}}_q \bar{M}). \quad (6)$$

The nonzero mean value between α_P and $\bar{\alpha}_P$ is a consequence of CP violation, which is defined by [25,26]

$$\bar{\mathcal{A}}_{CP}(\mathbf{B}_i \rightarrow \mathbf{B}_q M) = \frac{\alpha_P(\mathbf{B}_i \rightarrow \mathbf{B}_q M) + \bar{\alpha}_P(\mathbf{B}_i \rightarrow \mathbf{B}_q M)}{\alpha_P(\mathbf{B}_i \rightarrow \mathbf{B}_q M) - \bar{\alpha}_P(\mathbf{B}_i \rightarrow \mathbf{B}_q M)}. \quad (7)$$

On the other hand, the direct CP -violating rate asymmetries of the decays are given by

$$\mathcal{A}_{CP}(\mathbf{B}_i \rightarrow \mathbf{B}_q M) = \frac{\Gamma(\mathbf{B}_i \rightarrow \mathbf{B}_q M) - \Gamma(\bar{\mathbf{B}}_i \rightarrow \bar{\mathbf{B}}_q \bar{M})}{\Gamma(\mathbf{B}_i \rightarrow \mathbf{B}_q M) + \Gamma(\bar{\mathbf{B}}_i \rightarrow \bar{\mathbf{B}}_q \bar{M})}. \quad (8)$$

In the present work, to relate α_P , $\bar{\mathcal{A}}_{CP}$, and \mathcal{A}_{CP} directly to the weak interactions, we ignored the final state interactions in our calculations. As a result, the complex phases of A and B solely come from the Wilson coefficients and the Cabibbo-Kobayashi-Maskawa (CKM) elements. We note that possible sizable CP -violating effects in $\Lambda_b \rightarrow p(\pi^-, K^-)$ could be induced due to the weak phase from the CKM matrix element of V_{ub} .

III. BARYON WAVE FUNCTIONS

In the MIT bag model, the quarks in the baryon are constrained in a certain bag with radius R . Inside the bag, each quark obeys the free Dirac equation, given by

$$i\partial/\psi - m_q\psi = 0, \quad (9)$$

due to the mean field approximation of the gluon field, where m_q is the current quark mass. The boundary condition on the surface of the bag is given as

$$in_i\partial_i\psi = \psi, \quad (10)$$

where n_i is the unit vector toward the surface.

In this work, we only consider the ground states of the baryons with their angular momenta to be $J = 1/2$, where the spatial parts of the quark wave functions satisfy the spherical symmetry. Consequently, by inserting the boundary condition of Eq. (10) into Eq. (9), we get

$$\tan(p_q R) = \frac{p_q R}{1 - m_q R - E_q R}, \quad (11)$$

where p_q is the magnitude of the 3-momentum for the quark and $E_q = \sqrt{p_q^2 + m_q^2}$. The lowest momentum given in Eq. (11) corresponds to the ground state. The quark wave function centered in $\vec{x} = 0$ is written as

$$\psi(x) = \phi(\vec{x}) e^{-iE_q t} = \frac{N}{\sqrt{4\pi}} \left(\begin{array}{c} \omega_+ j_0(p_q r) \chi \\ i\omega_- j_1(p_q r) \hat{r} \cdot \vec{\sigma} \chi \end{array} \right) e^{-iE_q t}, \quad (12)$$

where $\omega_{\pm} \equiv \sqrt{1 \pm m_q/E_q}$, χ is the two-component spinor of the quark, describing the orientation of the angular momentum, j_i represents the i th spherical Bessel function of the first kind, and N stands for the normalized factor, given by

$$N = \sqrt{\frac{E_q(E_q - m_q)}{R^3 j_0^2(p_q R) [2E_q(E_q - \frac{1}{R}) + \frac{m_q}{R}]}}. \quad (13)$$

Since the baryon is made of the product of the three quarks, we write its wave function in term of the quark wave functions ψ_{q_i} , given by

$$\begin{aligned} \Psi(\vec{x}_{q_1}, \vec{x}_{q_2}, \vec{x}_{q_3}, t) \\ = \phi_{q_1}(\vec{x}_{q_1}) \phi_{q_2}(\vec{x}_{q_2}) \phi_{q_3}(\vec{x}_{q_3}) e^{-i(E_{q_1} + E_{q_2} + E_{q_3})t}, \end{aligned} \quad (14)$$

where q_i ($i = 1, 2, 3$) denote the i th quarks and E_{q_i} are the corresponding energies. Here, the spin, flavor, and color configurations have not been written down explicitly.

However, the above construction of the baryon wave function is valid only around $\vec{x} = 0$, which is mainly used to discuss the mass spectrum and magnetic dipole moment of the baryon. To calculate the dynamical processes, we need the baryon state to be in the particular 4-momentum eigenstate of the spacetime translation.

On the other hand, in Eq. (14) one assumes that the baryons are at rest, and the centers of the initial and final baryons locate in the same spot at a particular time, $t = t'$. However, if the baryons are not at relatively rest, the centers will be not coincident at $t \neq t'$ and the results of the dynamical factors will no longer be the same. Clearly, the baryon wave function in Eq. (14) is not the eigenstate of the spacetime translation.

To construct the baryon wave function to be invariant under the space translation, we have to modify the MIT bag model. To do that, we first integrate Eq. (14) with different center locations, leading to

$$\begin{aligned} \Psi(x_{q_1}, x_{q_2}, x_{q_3}) \\ = \mathcal{N} \int d^3\vec{x} \prod_{i=1,2,3} \phi_{q_i}(\vec{x}_{q_i} - \vec{x}) e^{-iE_{q_i} t_{q_i}}, \end{aligned} \quad (15)$$

where \mathcal{N} is the normalization factor. We then consider the space translation

$$\begin{aligned} \Psi(x_{q_1} + d, x_{q_2} + d, x_{q_3} + d) \\ = \mathcal{N} \int d^3\vec{x} \prod_{i=1,2,3} \phi_{q_i}(\vec{x}_{q_i} + \vec{d} - \vec{x}) e^{-iE_{q_i} t_{q_i}}, \end{aligned} \quad (16)$$

where d has only the spatial component. By changing the integration variable \vec{x} to $\vec{x} - \vec{d}$, we see that the wave function is indeed invariant under the space translation.

The energy of the baryon at rest essentially corresponds to the mass of the baryon, which can be read off from Eq. (15) to be $M = E_{q_1} + E_{q_2} + E_{q_3}$. We note that the bag energy has not been included in our study. In principle, one can calculate the energy contribution from the complicated gluon and gluon-quark interactions. However, we will not include such effects. Instead, we will simply use $M = E_{q_1} + E_{q_2} + E_{q_3}$ in the integral associated with the energy-momentum conservation as an approximation.

The wave function for the baryon at rest is given in Eq. (15). To obtain the function in a certain 4-momentum, we boost it in the z direction, given by

$$\begin{aligned} \Psi_v(x_{q_1}, x_{q_2}, x_{q_3}) \\ = S_v^{\mathbf{B}} \Psi(x_{q_1}^v, x_{q_2}^v, x_{q_3}^v) \\ = \mathcal{N} \int d^3\vec{x} \prod_{i=1,2,3} S_v^q \phi_{q_i}(\vec{x}_{q_i}^v - \vec{x}) e^{-i\gamma E_{q_i} (t_{q_i} - v z_{q_i})}. \end{aligned} \quad (17)$$

Here, $S_v^{\mathbf{B}(q)}$ is the pure Lorentz boost matrix for the baryon (quark) 4 spinor in the z direction with S_v^q given by

$$S_v^q = \begin{pmatrix} a_+ I & a_- \sigma_z \\ a_- \sigma_z & a_+ I \end{pmatrix}, \quad (18)$$

where I is the 2×2 unity matrix, $a_{\pm} = \sqrt{\frac{1}{2}(\gamma \pm 1)}$ and $\gamma = \sqrt{1/(1-v^2)}$. In addition, v in the superscript of S_v^q indicates the Lorentz transformation of the coordinate, given explicitly as

$$\begin{aligned} (x_{q_i}^v)_0 &= \gamma(x_{q_i})_0 - \gamma v (x_{q_i})_3, \\ (x_{q_i}^v)_1 &= (x_{q_i})_1, \quad (x_{q_i}^v)_2 = (x_{q_i})_2, \\ (x_{q_i}^v)_3 &= \gamma(x_{q_i})_3 - \gamma v (x_{q_i})_0, \end{aligned} \quad (19)$$

where $(x_{q_i})_0$ is the time component of the coordinate.

To obtain the proper normalization factor, we calculate the overlap between the two baryon wave functions with different speeds at time t , given by

$$\begin{aligned}
& \int \Psi_v^\dagger(\vec{x}_{q_1}, \vec{x}_{q_2}, \vec{x}_{q_3}, t) \Psi_v(\vec{x}_{q_1}, \vec{x}_{q_2}, \vec{x}_{q_3}, t) d^3\vec{x}_{q_1} d^3\vec{x}_{q_2} d^3\vec{x}_{q_3} \\
&= e^{-iM(\gamma-\gamma')t} \mathcal{N}^2 \int d^3\vec{x} d^3\vec{x}' \prod_{i=1,2,3} d^3\vec{x}_{q_i} \phi^\dagger(\vec{x}_{q_i}^{v'} - \vec{x}') \\
&\quad \times S_v^q S_v^q \phi(\vec{x}_{q_i}^v - \vec{x}) e^{iE_{q_i}(\gamma v - \gamma' v') z_{q_i}}, \quad (20)
\end{aligned}$$

where $S_v^{q\dagger} = S_v^q$ for the pure Lorentz boost and $M = E_{q_1} + E_{q_2} + E_{q_3}$ for the mass of the baryon. To simplify the integral, we adopt the following variables:

$$\begin{aligned}
\vec{x}_{q_i}^r &= \vec{x}_{q_i}^v - \frac{1}{2}(\vec{x} + \vec{x}'), \\
\vec{x}_\Delta &= \vec{x} - \vec{x}', \\
\vec{x}_A &= \frac{1}{2}(\vec{x} + \vec{x}'). \quad (21)
\end{aligned}$$

Now, the overlap integral is read as

$$\begin{aligned}
& e^{-iM(\gamma-\gamma')t} \frac{\mathcal{N}^2}{\gamma^3} \int d^3\vec{x}_\Delta d^3\vec{x}_A \prod_{i=1,2,3} d^3\vec{x}_{q_i}^r \phi^\dagger\left(\vec{x}_{q_i}^r + \frac{1}{2}\vec{x}_\Delta\right) \\
&\quad \times S_v^{q2} \psi\left(\vec{x}_{q_i}^r - \frac{1}{2}\vec{x}_\Delta\right) e^{iE_{q_i}^B(v-v')z_{q_i}^r} e^{iE_{q_i}(v-v')z_A} \\
&= \mathcal{N}^2 \gamma (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \int d^3\vec{x}_\Delta \prod_{i=1,2,3} d^3\vec{x}_{q_i}^r \\
&\quad \times \phi^\dagger\left(\vec{x}_{q_i}^r + \frac{1}{2}\vec{x}_\Delta\right) \phi\left(\vec{x}_{q_i}^r - \frac{1}{2}\vec{x}_\Delta\right), \quad (22)
\end{aligned}$$

where $1/\gamma^3$ comes from the Jacobian in Eq. (21), and \vec{p} and \vec{p}' are the 3-momenta of the baryons. Here, we have used the fact that the integral does not vanish if and only if $v = v'$ to reduce the complexity in ϕ .

By normalizing the baryon wave function as $\langle p' | p \rangle = \gamma (2\pi)^3 \delta^3(\vec{p} - \vec{p}')$, we find that

$$\frac{1}{\mathcal{N}^2} = \int d^3\vec{x}_\Delta \prod_{i=1,2,3} d^3\vec{x}_{q_i}^r \phi^\dagger\left(\vec{x}_{q_i}^r + \frac{1}{2}\vec{x}_\Delta\right) \phi\left(\vec{x}_{q_i}^r - \frac{1}{2}\vec{x}_\Delta\right), \quad (23)$$

which is clearly independent of the velocity by the construction of the baryon wave functions.

It is worthwhile to take a look at the physical interpretation of the baryon wave function in Eq. (15). In contrast to the original wave function in Eq. (14) with the quarks confined in a static bag located at $\vec{x} = 0$, those quarks for Eq. (15) distribute all over the space to fulfill the translation-invariant requirement. Note that the distance between two arbitrary quarks is limited within $2R$ in the bag model. For the case in Eq. (15), it is due to the fact that if $|\vec{x}_{q_1} - \vec{x}_{q_2}| > 2R$, then either $|\vec{x}_{q_1} - \vec{x}|$ or $|\vec{x}_{q_2} - \vec{x}|$ will exceed R , resulting in the vanishing integral. So, the quarks

are entangling to each other in the spatial part of the wave function, which is not the case in Eq. (14).

The average distance between the quarks is given by

$$\sqrt{\langle (\vec{x}_{q_1} - \vec{x}_{q_2})^2 \rangle} = \sqrt{\langle \vec{x}_{q_1}^2 \rangle + \langle \vec{x}_{q_2}^2 \rangle - 2\langle \vec{x}_{q_1} \cdot \vec{x}_{q_2} \rangle}. \quad (24)$$

In the original MIT bag model, as the quark positions are independent to each other, we have that $\langle \vec{x}_{q_1} \cdot \vec{x}_{q_2} \rangle = \langle \vec{x}_{q_1} \rangle \cdot \langle \vec{x}_{q_2} \rangle = 0$. However, they do not vanish in our modified MIT bag model. For the proton with three massless light quarks, the average distance in our modified model is roughly 20% shorter than the original one with the same bag radius.

IV. BARYON FORM FACTORS

To calculate the Λ_b^0 decays, we use the factorization approach. In this approach, the amplitudes for $\Lambda_b^0 \rightarrow \Lambda_c^+ M$ can be written as

$$\begin{aligned}
\mathcal{A}_{(\Lambda_b^0 \rightarrow \Lambda_c^+ M)} &= \frac{G_F}{\sqrt{2}} a_1 V_{cb}^* V_{uq} \langle M | \bar{u} \gamma^\mu (1 - \gamma_5) q | 0 \rangle \\
&\quad \times \langle \Lambda_c^+ | \bar{c} \gamma_\mu (1 - \gamma_5) b | \Lambda_b^0 \rangle \\
&= i \frac{G_F}{\sqrt{2}} a_1 V_{cb}^* V_{uq} f_M q^\mu \langle \Lambda_c^+ | \bar{c} \gamma_\mu (1 - \gamma_5) b | \Lambda_b^0 \rangle \\
&= i \frac{G_F}{\sqrt{2}} a_1 V_{cb}^* V_{uq} f_M [(m_b - m_c) \langle \Lambda_c^+ | \bar{c} b | \Lambda_b^0 \rangle \\
&\quad + (m_b + m_c) \langle \Lambda_c^+ | \bar{c} \gamma_5 b | \Lambda_b^0 \rangle], \quad (25)
\end{aligned}$$

where G_F is the Fermi constant, $a_1 = c_1 + c_2/3 = 1.02$ [8] with $c_{1,2}$ the Wilson coefficients, V_{ij} represent the CKM elements with $q = d(s)$ corresponding to $M = \pi^-(K^-)$, f_M is the meson decay constant, and the quarks operators are evaluated at $x = 0$. For the decay of $\Lambda_b^0 \rightarrow pM$, the amplitude is given by [10]

$$\begin{aligned}
\mathcal{A}(\Lambda_b^0 \rightarrow pM) &= i \frac{G_F}{\sqrt{2}} m_b f_M [\alpha_M \langle p | \bar{u} b | \Lambda_b^0 \rangle + \beta_M \langle p | \bar{u} \gamma_5 b | \Lambda_b^0 \rangle], \quad (26)
\end{aligned}$$

where α_M (β_M) and α_V in Eq. (29) are defined by

$$\begin{aligned}
\alpha_M &= V_{ub} V_{uq}^* a_1 - V_{ib} V_{iq}^* (a_4 + r_M a_6), \\
\beta_M &= V_{ub} V_{uq}^* a_1 - V_{ib} V_{iq}^* (a_4 - r_M a_6), \quad (27)
\end{aligned}$$

with $r_M \equiv 2m_M^2/[m_b(m_q + m_u)]$ and $a_i \equiv c_i^{\text{eff}} + c_{i\pm 1}^{\text{eff}}/N_c^{\text{(eff)}}$ for $i = \text{odd}$ (even), composed of the effective Wilson coefficients c_i^{eff} defined in Ref. [27].

Now, we are left with the matrix elements of the scalar and pseudoscalar operators in Eqs. (25) and (26), which can be parametrized as

$$\begin{aligned} \langle \mathbf{B}_q | \bar{q} b(0) | \Lambda_b^0 \rangle &= f_s^{\mathbf{B}_q} \bar{u}_{\mathbf{B}_q} u_{\Lambda_b^0} \\ \langle \mathbf{B}_q | \bar{q} \gamma_5 b(0) | \Lambda_b^0 \rangle &= f_p^{\mathbf{B}_q} \bar{u}_{\mathbf{B}_q} \gamma_5 u_{\Lambda_b^0}, \end{aligned} \quad (28)$$

where \mathbf{B}_q represents $\Lambda_c^+(p)$ with \bar{q} being $\bar{c}(\bar{u})$.

We evaluate the form factors of $f_s^{\mathbf{B}_q}$ and $f_p^{\mathbf{B}_q}$ in the Briet frame, in which the initial and final baryons have opposite velocities, i.e., $\vec{v}_{1,2} = -\vec{v}$. In the derivations of the matrix elements, one actually deals with the quark operators in the x dependence, given by

$$\begin{aligned} &\int \langle \mathbf{B}_q | \bar{q}(\gamma_5) b(x) e^{i p_M x} | \Lambda_b^0 \rangle d^4 x \\ &= \langle \mathbf{B}_q | \bar{q}(\gamma_5) b(0) | \Lambda_b^0 \rangle (2\pi)^4 \delta(p_i - p_q - p_M), \end{aligned} \quad (29)$$

for the scalar (pseudoscalar) matrix element with p_M being the 4-momentum of the M meson. Here, we have used that the initial and final baryons are in the momentum eigenstates to reduce the integral with the Dirac δ function of $(2\pi)^4 \delta(p_i - p_q - p_M)$. Clearly, one can evaluate the form factors with either the quark operators located at $x = 0$ in Eq. (28) or the x -dependent ones in the left-hand side of Eq. (29). We start with the baryon wave functions in Eq. (17), given by

$$\begin{aligned} &\int \langle \mathbf{B}_q | \bar{q} b(x_{q_3}) e^{i p_M x_{q_3}} | \Lambda_b \rangle d^4 x_{q_3} \\ &= \mathcal{N}_{\Lambda_b} \mathcal{N}_{B_q} \int d^3 \vec{x} d^3 \vec{x}' d^4 x_{q_3} \bar{\phi}_q(\vec{x}_{q_3}^v - \vec{x}') \\ &\quad \times S_{-v}^{q_2} \phi_b(\vec{x}_{q_3}^{-v} - \vec{x}) e^{i[\gamma(E_q - E_b) + p_0]t} e^{-i[\gamma v(E_q + E_b) + p_3]z_3} \\ &\quad \times \prod_{j=1,2} \phi_{q_j}^\dagger(\vec{x}_{q_j}^v - \vec{x}') \phi_{q_j}(\vec{x}_{q_j}^{-v} - \vec{x}) e^{-2i\gamma v E_{q_j} v z_{q_j}}, \end{aligned} \quad (30)$$

with $S_v^{q_1} \gamma_0 = \gamma_0 S_{-v}^{q_2}$ for the Lorentz boost and $(q_1, q_2) = (u, d)$. Similar to the case in Eq. (20), we adopt the variable transformations with some modifications, given by

$$\begin{aligned} (\vec{x}_{q_i}^r)_j &= (\vec{x}_{q_i})_j - \frac{1}{2}(\vec{x}' + \vec{x})_j, \\ (\vec{x}_{q_i}^r)_3 &= \gamma(\vec{x}_{q_i})_3 - \frac{1}{2}(\vec{x}' + \vec{x})_3, \\ \vec{x}_A &= \frac{1}{2}(\vec{x} + \vec{x}'), \\ (\vec{x}_\Delta)_j &= (\vec{x} - \vec{x}')_j, \\ (\vec{x}_\Delta)_3 &= (\vec{x} - \vec{x}')_3 - 2\gamma v t, \end{aligned} \quad (31)$$

where $i = 1, 2, 3$ and $j = 1, 2$. Subsequently, we have that

$$\begin{aligned} &\mathcal{Z} \int d^3 \vec{x}_\Delta d^3 \vec{x}_{q_3}^r \bar{\phi}_q\left(\vec{x}_{q_3}^r + \frac{1}{2}\vec{x}_\Delta\right) S_{-v}^{q_2} \phi_b\left(\vec{x}_{q_3}^r - \frac{1}{2}\vec{x}_\Delta\right) \\ &\quad \times e^{i v(M_{B_f} + M_{\Lambda_b} - E_q - E_b) z_{q_3}} \prod_{j=1,2} D_{q_j}(\vec{x}_\Delta), \\ &\mathcal{Z} \equiv (2\pi)^4 \delta^4(p_i - p_f - q_M) \frac{\mathcal{N}_{\Lambda_b} \mathcal{N}_{B_f}}{\gamma^2}, \\ &D_{q_j}(\vec{x}_\Delta) \equiv \int d^3 \vec{x} \phi_{q_j}^\dagger\left(\vec{x} + \frac{1}{2}\vec{x}_\Delta\right) \phi_{q_j}\left(\vec{x} - \frac{1}{2}\vec{x}_\Delta\right) e^{-2i E_{q_j} v z_{q_j}}. \end{aligned} \quad (32)$$

Alternatively, one can evaluate the integral with b and q quarks located at $x = 0$ in Eq. (28), given as

$$\begin{aligned} &\mathcal{Z} \gamma^2 \int d^3 \vec{x} d^3 \vec{x}' \bar{\phi}_q(-\vec{x}') S_{-v}^{q_2} \phi_b(-\vec{x}) \\ &\quad \times \prod_{j=1,2} \phi_{q_i}^\dagger(\vec{x}_{q_i}^v - \vec{x}') \phi_{q_i}(\vec{x}_{q_i}^{-v} - \vec{x}) e^{-2i\gamma E_{q_j} v z_{q_j}}. \end{aligned} \quad (33)$$

After changing the integral variables by

$$\begin{aligned} (\vec{x}_{q_i}^r)_k &= (\vec{x}_{q_i})_k - \frac{1}{2}(\vec{x} + \vec{x}')_k, \\ (\vec{x}_{q_i}^r)_3 &= \gamma(\vec{x}_{q_i})_3 - \frac{1}{2}(\vec{x}' - \vec{x})_3, \\ \vec{x}_B &= -\frac{1}{2}(\vec{x} + \vec{x}'), \\ \vec{x}_\Delta &= \vec{x} - \vec{x}', \end{aligned} \quad (34)$$

where $k, j = 1, 2$, one obtains the identical equation as the one in Eq. (32) with $M = E_{q_1} + E_{q_2} + E_{q_3}$.

Similarly, the pseudoscalar part can be given as

$$\begin{aligned} &\int \langle \mathbf{B}_q | (\bar{q} \gamma_5 b)(x) e^{i p_M x} | \Lambda_b^0 \rangle d^4 x \\ &= \mathcal{Z} \int d^3 \vec{x}_\Delta d^3 \vec{x}_{q_3}^r \bar{\phi}_q\left(\vec{x}_{q_3}^r + \frac{1}{2}\vec{x}_\Delta\right) \gamma_5 S_{-v}^{q_2} \phi_b\left(\vec{x}_{q_3}^r - \frac{1}{2}\vec{x}_\Delta\right) \\ &\quad \times e^{i v(M_{B_q} + M_{\Lambda_b} - E_q - E_b) z_{q_3}} \prod_{j=1,2} D_{q_j}(\vec{x}_\Delta). \end{aligned} \quad (35)$$

With the normalization in Eq. (23), we derive that

$$\begin{aligned} f_s &= \frac{\eta}{\gamma(2\pi)^4 \delta^4(p_i - p_f - p)} \int \langle \mathbf{B}_q | (\bar{q} b)(x) e^{i p_M x} | \Lambda_b^0 \rangle d^4 x, \\ f_p &= \frac{\eta}{\gamma(2\pi)^4 \delta^4(p_i - p_f - p)} \int \langle \mathbf{B}_q | (\bar{q} \gamma_5 b)(x) e^{i p_M x} | \Lambda_b^0 \rangle d^4 x, \end{aligned} \quad (36)$$

where η is the overlap factor of the spin-flavor configuration. For $\Lambda_b \rightarrow \Lambda_c^+(p)$, we have $\eta = 1(\sqrt{3}/2)$ [17].

V. NUMERICAL RESULTS AND DISCUSSIONS

We use the bag radius of the Λ_c^+ baryon as $R(\Lambda_c^+) = 4.8 \text{ GeV}^{-1}$ from Ref. [14]. In the limit of the heavy quark symmetry, the baryon wave functions for Λ_b^0 and Λ_c^+ can be taken to be the same. As a result, the bag radius for the Λ_b^0 should be also around 4.8 GeV. For the proton, it is usually chosen to be 5 GeV^{-1} . However, to simplify our numerical calculations, we choose the same bag radiuses for Λ_b^0 , Λ_c^+ and p , i.e., $R = R(\Lambda_b^0) = R(\Lambda_c^+) = R(p) = 4.8 \pm 0.2 \text{ GeV}^{-1}$, where the uncertainty is to account for the dependence of the bag radius. In general, an increase of the bag radius would reduce the form factors. In addition, we take that $m_u = m_d = 5 \text{ MeV}$. Note that the variation of the light quark masses from 0 to 10 MeV does not make much of a difference for the numerical values of the form factors. To determine the heavy quark masses, we assume that the baryon mass differences are related to the corresponding quark energies, i.e.,

$$\begin{aligned} E_b &= M_{\Lambda_b^0} - M_p + E_u, \\ E_c &= M_{\Lambda_c^+} - M_p + E_u, \end{aligned} \quad (37)$$

where E_b , E_c , and E_u are the energies of b , c , and u quarks in the bag, and $M_{\Lambda_b^0}$, $M_{\Lambda_c^+}$, and M_p are the corresponding baryons masses, respectively. With $R = 4.8 \text{ GeV}^{-1}$, we find $(m_b, m_c) = (5.1, 1.8) \text{ GeV}$, which satisfy the relation of the heavy quark symmetry, given by $m_b - m_c = M_{\Lambda_b^0} - M_{\Lambda_c^+}$.

The form factors associated with the vector and axial-vector currents are defined by

$$\begin{aligned} \langle \mathbf{B}_q | \bar{q} \gamma_\mu b | \Lambda_b^0 \rangle &= \bar{u}_f (f_1 \gamma_\mu - f_2 i \sigma_{\mu\nu} (P_M)^\nu + f_3 (P_M)_\mu) u_{\Lambda_b^0}, \\ \langle \mathbf{B}_q | \bar{q} \gamma_\mu \gamma_5 b | \Lambda_b^0 \rangle &= \bar{u}_f (g_1 \gamma_\mu - g_2 i \sigma_{\mu\nu} (P_M)^\nu + g_3 (P_M)_\mu) \gamma_5 u_{\Lambda_b^0}. \end{aligned} \quad (38)$$

By using the equations of motion and comparing Eqs. (28) and (38), we obtain the form factors associated with the vector and axial currents from the scalar and pseudoscalar ones, given by

$$\begin{aligned} f_1 &= (m_b - m_q) / (M_{\Lambda_b^0} - M_{\mathbf{B}_q}) f_s, \\ g_1 &= (m_b + m_q) / (M_{\Lambda_b^0} + M_{\mathbf{B}_q}) f_p, \end{aligned} \quad (39)$$

TABLE I. Form factors for $\Lambda_b^0 \rightarrow \Lambda_c^+ / p$.

p_M^2	$f_s^{\Lambda_c^+}$	$f_p^{\Lambda_c^+}$	$f_1^{\Lambda_c^+}$	$g_1^{\Lambda_c^+}$	f_s^p	f_p^p	f_1^p	g_1^p
$(M_{\Lambda_b^0} - M_{\Lambda_c^+})^2$	1.02 ± 0.00	1.20 ± 0.00	1.02 ± 0.00	1.04 ± 0.01	1.32 ± 0.00	2.10 ± 0.04	1.44 ± 0.00	1.64 ± 0.03
M_π^2	0.50 ± 0.00	0.58 ± 0.01	0.50 ± 0.00	0.50 ± 0.00	0.14 ± 0.01	0.18 ± 0.01	0.16 ± 0.01	0.14 ± 0.01
M_K^2	0.51 ± 0.01	0.59 ± 0.01	0.51 ± 0.01	0.51 ± 0.00	0.14 ± 0.01	0.18 ± 0.01	0.16 ± 0.01	0.14 ± 0.00

respectively. Here, we have neglected the contributions from f_3 and g_3 , which are suppressed based on the heavy baryon mass of Λ_b . Because of the relation of $m_b - m_c = M_{\Lambda_b^0} - M_{\Lambda_c^+}$, we find that $f_s^{\Lambda_c^+} = f_1^{\Lambda_c^+}$, whereas f_p is larger than g_1 in all cases.

Our numerical results of the form factors with different values of p_M^2 are listed in Table I, where we only consider the uncertainties from the bag radius. For $\Lambda_b^0 \rightarrow \Lambda_c^+$, with the heavy quark symmetry limit we have that

$$\begin{aligned} f_1^{\Lambda_c^+}(p_M^2) &= g_1^{\Lambda_c^+}(p_M^2), \\ f_1^{\Lambda_c^+}(p_M^2 = (M_{\Lambda_b^0} - M_{\Lambda_c^+})^2) &= g_1^{\Lambda_c^+}(p_M^2 = (M_{\Lambda_b^0} - M_{\Lambda_c^+})^2) = 1, \end{aligned} \quad (40)$$

which are well consistent with our numerical values. The results with the heavy quark symmetry indicate that our constructions for baryon wave functions are reasonable, while the form factors of $f_3^{\Lambda_c^+}$ and $g_3^{\Lambda_c^+}$ can be safely neglected. It is interesting to note that $f_1^{\Lambda_c^+}$ and $g_1^{\Lambda_c^+}$ correspond to the Isgur Wise function in the context of the heavy quark symmetry. The first derivatives of $f_1^{\Lambda_c^+}$ and $g_1^{\Lambda_c^+}$ are found to be

$$\begin{aligned} \rho_V^2 &= -\frac{d}{d\omega} f_1(\omega) \Big|_{\omega=1} = 1.96 \pm 0.04, \\ \rho_A^2 &= -\frac{d}{d\omega} g_1(\omega) \Big|_{\omega=1} = 2.07 \pm 0.04 \end{aligned} \quad (41)$$

where the slight difference between ρ_V^2 and ρ_A^2 can be viewed as the $(1/m_b)$ correction. Our results in Eq. (41) are consistent with $\rho^2 = \rho_V^2 = \rho_A^2 = 1.3\text{--}3.7$ in the literature based on the heavy quark symmetry [18,28–30]. On the other hand, the experimental measurement on $\Lambda_b^0 \rightarrow \Lambda_c^+ \ell^- \bar{\nu}_\ell$ gives [31]

$$\rho^2 = 2.04 \pm 0.46(\text{stat})_{-1.00}^{+0.72}(\text{syst}), \quad (42)$$

in which the central value is very close to our values in Eq. (41).

The decay widths and asymmetries are shown in Table II. As $f_1 = g_1$ from the heavy quark symmetry and $m_M^2 \approx 0$ due to the soft meson limit, we expect that $A = \kappa B$, leading to $\alpha_P(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-, \Lambda_c^+ K^-) = 1$, as given in the table.

TABLE II. Decay widths and symmetries.

Channel	$\Gamma(\text{s}^{-1})$	α_P (%)	$\bar{\alpha}_P$ (%)	$\bar{\mathcal{A}}_{CP}$ (%)	\mathcal{A}_{CP} (%)
$\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-$	$(3.03 \pm 0.07) \times 10^9$	100.0 ± 0.0	-100.0 ± 0.0	0	0
$\Lambda_b^0 \rightarrow \Lambda_c^+ K^-$	$(2.33 \pm 0.06) \times 10^8$	100.0 ± 0.0	-100.0 ± 0.0	0	0
$\Lambda_b^0 \rightarrow p \pi^-$	$(3.41 \pm 0.38) \times 10^6$	85.6 ± 0.0	-83.2 ± 0.1	1.4 ± 0.0	-4.4 ± 0.1
$\Lambda_b^0 \rightarrow p K^-$	$(4.11 \pm 0.49) \times 10^6$	-29.7 ± 0.3	44.4 ± 0.4	-19.6 ± 0.2	6.7 ± 0.0

TABLE III. Decay branching ratios and direct CP asymmetries.

	Our results	[10]	[11]	[9]	[32]	PDG [7]
$10^3 \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)$	4.5 ± 0.2	$4.16^{+2.43}_{-1.73}$	4.9 ± 0.4
$10^4 \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ K^-)$	3.4 ± 0.1	$3.1^{+1.8}_{-1.3}$	3.6 ± 0.3
$10^6 \mathcal{B}(\Lambda_b^0 \rightarrow p \pi^-)$	5.0 ± 0.5	4.2 ± 0.7	4.30	$5.2^{+2.5}_{-1.9}$...	4.5 ± 0.8
$10^6 \mathcal{B}(\Lambda_b^0 \rightarrow p K^-)$	6.0 ± 0.7	4.8 ± 0.7	2.17	$2.0^{+1.0}_{-1.3}$...	5.4 ± 1.0
$10^2 \mathcal{A}_{CP}(\Lambda_b^0 \rightarrow p \pi^-)$	-4.4 ± 0.1	-3.9 ± 0.2	$-3.37^{+0.29}_{-0.37}$	-31^{+43}_{-1}	...	-2.5 ± 2.9
$10^2 \mathcal{A}_{CP}(\Lambda_b^0 \rightarrow p K^-)$	6.7 ± 0.0	5.8 ± 0.2	$10.1^{+1.3}_{-2.0}$	-5^{+26}_{-5}	...	-2.5 ± 2.2

TABLE IV. Values (10^{-2}) of $R = \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ K^-)/\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)$ in various approaches.

Our result	U-spin	Factorization	LFQM [32]	LHCb [33]	PDG [7]
7.6 ± 0.1	5.3	7.7	7.5	7.21 ± 0.22	7.35 ± 0.86

In Table III, we compared our results with those of Refs. [9–11,32] in the literature as well as the experimental data [7]. In the literature, the form factors are evaluated by fitting the experimental data in the generalized factorization approach [10], considering the LFQM for the baryon wave functions [11,32], and using the perturbative QCD method with the hybrid scheme [9]. As shown in Table III, the decay branching ratios for $\Lambda_b^0 \rightarrow \Lambda_c^+(\pi^-, K^-)$ from the modified bag model are close to those in LFQM [32] as well as the experimental data. We also find that our predicted branching ratio for $\Lambda_b^0 \rightarrow p K^-$ is about 1.2 times larger than that for $\Lambda_b^0 \rightarrow p \pi^-$, which agrees with the data and that in the generalized factorization approach [10], but is different from the results of perturbative QCD [9] and LFQM [11]. On the other hand, our results for the direct CP -violating rate asymmetries of $\Lambda_b^0 \rightarrow p(\pi^-, K^-)$ are sizable, which are consistent with all other theoretical predictions, and the experimental data except $\mathcal{A}_{CP}(\Lambda_b^0 \rightarrow p K^-)_{\text{PDG}}$. As the experimental value of $\mathcal{A}_{CP}(\Lambda_b^0 \rightarrow p K^-)_{\text{PDG}}$ in Eq. (1) is consistent with zero with a negative central value, whereas our prediction of $+6.7\%$ along with the others in Refs. [10,11] is positive, it is very interesting to see if such CP asymmetry can be measured precisely by the ongoing experiment at LHCb. In addition, we see that $\bar{\mathcal{A}}_{CP}(\Lambda_b^0 \rightarrow p K^-)$ is predicted to be $(-19.6 \pm 0.2)\%$, which is very large.

In Table IV, we illustrate the ratio of $R = \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ K^-)/\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)$ in various approaches. In the table, the result of the U-spin symmetry is based on the $SU(2)$ symmetry between d and s quarks, which leads to the naive relation for R , given by

$$R_{\text{U-spin}} = \left| \frac{V_{us}}{V_{ud}} \right|^2 \approx 5.3\%. \quad (43)$$

In the factorization approach, R receives an extra factor due to the meson decay constants, read as

$$R_{\text{Factorization}} = \left| \frac{V_{us} f_K}{V_{ud} f_\pi} \right|^2 \approx 7.7\%, \quad (44)$$

which is consistent with our result and that in LFQM [32] as well as the data [7,33]. Clearly, it shows the evidence that the decays of $\Lambda_b^0 \rightarrow \Lambda_c^+(\pi^-, K^-)$ are factorizable.

VI. CONCLUSIONS

We have studied the decays of $\Lambda_b^0 \rightarrow \Lambda_c^+(\pi^-, K^-)$ and $\Lambda_b^0 \rightarrow p(\pi^-, K^-)$ in the modified MIT bag model. We have provided a new way to construct the baryon momentum eigenstates in the bag model without introducing new parameters. In particular, we have summed over the localized baryon wave function in Eq. (14) with different

centers to fulfill the requirement of the invariant for the space translation.

For $\Lambda_b^0 \rightarrow \Lambda_c^+(\pi^-, K^-)$, we have found that the decay branching ratios are $(4.5 \pm 0.2) \times 10^{-3}$ and $(3.4 \pm 0.1) \times 10^{-4}$ with the uncertainties only from the bag radius, which agree well with the experimental data of $(4.9 \pm 0.4) \times 10^{-3}$ and $(3.6 \pm 0.3) \times 10^{-4}$, respectively. We have also shown that our results of the first derivatives for the form factors $f_1(\omega)$ and $g_1(\omega)$ in Eq. (41) match with the data as well as those in the literature, indicating the validation of the heavy quark symmetry in the decay processes.

For $\Lambda_b^0 \rightarrow p(\pi^-, K^-)$, our predicted decay branching ratios of $(5.0 \pm 0.5) \times 10^{-6}$ and $(6.0 \pm 0.7) \times 10^{-6}$ are consistent with the current data of $(4.5 \pm 0.8) \times 10^{-6}$ and $(5.4 \pm 1.0) \times 10^{-6}$ [7], respectively. In addition, we have

explored the CP -violating asymmetries for the decays. Particularly, we have obtained that $\mathcal{A}_{CP}(\Lambda_b^0 \rightarrow p\pi^-)$ and $\mathcal{A}_{CP}(\Lambda_b^0 \rightarrow pK^-)$ are $(-4.4 \pm 0.1)\%$ and $(6.7 \pm 0.0)\%$, in comparison with $(-2.5 \pm 2.9)\%$ and $(-2.5 \pm 2.2)\%$ from the Particle Data Group in 2020, respectively. It is also interesting to note that $\bar{\mathcal{A}}_{CP}(\Lambda_b^0 \rightarrow pK^-)$ is predicted to be $(-19.6 \pm 0.2)\%$, which is very large. It is clear that more precise future experimental measurements on these CP -violating asymmetries are needed.

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