Froissart bound, diffraction scattering of hadrons, and scaling at asymptotic energies

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The Froissart bound on the total cross section, σ_t, is subjected to test against very high energy data. We have found no clear evidence for its violation. The scaling property of the differential cross section in the diffraction region is investigated. It exhibits scaling in the ISR, SPS, Tevatron, and LHC energy domain which had hitherto remained unexplored. The slope of the diffraction peak is fitted and the data are tested against the rigorous bounds.

I. INTRODUCTION

The high energy scattering of hadrons is of great interest. The total cross sections σ_t^{pp} and $\sigma_t^{p\bar{p}}$, from ISR to LHC energies, are rising and the cosmic ray data continues to follow the trend. The elastic cross sections, σ_{el}, grow with energy as well. The high energy scattering data are endowed with several fascinating features [1]. The elastic differential cross sections exhibit forward peak whose width shrinks as energy grows. The slope of the diffraction peak increases with energy. There are phenomenological models to study high energy hadronic processes. There is another approach to study strong interactions without appealing to any specific model. It is the S-matrix formulation pioneered by Heisenberg [2]. This philosophy, over the years, has evolved into axiomatic field theory approach as known at present. The rigorous results derived in this framework are not based on any model. Indeed, the structure rests on certain fundamental axioms to be respected by relativistic quantum field theories (QFT). The analyticity properties and other attributes of scattering amplitudes are proved starting from the axioms. The hallmark of the axiomatic formulation is that the principal results are presented as upper and lower bounds on experimentally measured quantities. So far there are no experimental evidences for the violation of any of these bounds. One of the most celebrated results is the Froissart-Martin bound on the total hadronic cross sections, σ_t. Throughout this paper the high energy data means the data

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accumulated in the energy range covered by accelerators such as ISR, SPS, Tevatron, and LHC. Whenever the data of cosmic rays are incorporated they will be referred to explicitly. The high energy accelerators have been accumulating data over the years. It is worthwhile to focus attention on such data and study afresh how they put the bounds to tests.

The purpose of this article is to investigate certain areas of strong interaction processes which have remained unexplored in the high energy regime so far. We shall consider three topics as mentioned below. (1) Our intent is to test the validity of the Froissart-Martin bound [3,4]

$$\sigma_t(s) \leq \frac{4\pi}{t_0} \log^2\left(\frac{s}{s_0}\right) \tag{1}$$

in the asymptotic energy regime. Here s is the c.m. energy squared, t_0 is a constant less than or equal to the t-channel threshold. The prefactor in (1) was fixed by Martin [4] from the first principles. s_0 is introduced to make argument of the $\log^2$ dimensionless and it cannot be determined *ab initio*. However, recently, there has been attempts to determine s_0 with additional inputs [5]. The first important point to note is that the total cross section cannot have arbitrary energy dependence. Moreover, this bound is not obtained from any model in a perturbative framework, rather, it is a consequence of general axioms. Therefore, any experimental evidence for violation of the bound (1) would be a matter of concern. We have adopted a strategy to test the Froissart-Martin bound in the very high energy regimes alluded to earlier. We shall outline our prescription in the next section. (2) The scaling of the differential cross section, $\frac{d\sigma}{dt}(s,t)$, in the diffraction region, i.e., in the near forward direction, is an interesting and intriguing feature in high energy reactions. This attribute has been observed in the pre-ISR energy domain in the past. The scaling, in the present

context, is interpreted as follows. The scattering amplitude, $F(s, t)$, is a function of two Lorentz invariant variables s and the momentum transferred squared, t. In the limit, $s \to \infty$, for fixed physical t, $F(s, t) \to f(\tau)$; $f(\tau)$ being a function of a single variable. The mystery about such a scaling lies in the fact that perturbative QCD techniques cannot be applied to explain such an observed scaling. The problem is that in the kinematical region in question, i.e., large s and small $|t|$, the reaction is soft; therefore, the power of the asymptotic freedom is inapplicable. We shall elaborate more on this type of scaling in sequel. In the past, several models [6–9] were proposed which described high energy scattering and investigated scaling. A good review of scaling phenomena with extensive references is presented in [9]. We examine scaling in the aforementioned energy domain. (3) The slope of the diffraction peak, $b(s)$, is a very important parameter in high energy hadronic collisions. It is not only useful to describe diffraction scattering but also it is utilized by the experimentalists to extract σ_t. The interests in energy dependence of $b(s)$ is to test the shrinking of the diffraction peak with energy; furthermore, axiomatic bounds would also be tested. Moreover, in the past, the measured diffraction peak data almost saturated the lower bound for the absorptive part of the differential cross section; leading to the conjecture that the amplitude is dominated by the imaginary part.

Remarks: (i) The analyticity of the 4-point scattering amplitudes is the cornerstone of the bound (1). The rigorous results such as analyticity and crossing symmetry are proved from axiomatic field theories [10–13]; either in the framework of Lehmann-Symanzik-Zimmermann (LSZ) formulation [14] or from Wightman's axioms [15]. We recall the axioms of LSZ for sake of completeness. (a1) The states of the system are represented in a Hilbert space, $\mathcal{H}$. All the physical observables are self-adjoint operators in the Hilbert space, $\mathcal{H}$. (a2) The theory is invariant under inhomogeneous Lorentz transformations. (a3) The energy-momentum of the states are defined. It follows from the requirements of Lorentz and translation invariance that we can construct a representation of the orthochronous Lorentz group. The representation corresponds to unitary operators, $U(a, \Lambda)$, and the theory is invariant under these transformations. Thus there are Hermitian operators corresponding to spacetime translations, denoted as P_μ, with $\mu = 0, 1, 2, 3$ which have following properties:

$$[P_\mu, P_\nu] = 0. \tag{2}$$

If $\mathcal{F}(x)$ is any Heisenberg operator then its commutator with P_μ is

$$[P_\mu, \mathcal{F}(x)] = i\partial_\mu \mathcal{F}(x). \tag{3}$$

It is assumed that the operator does not explicitly depend on spacetime coordinates. If we choose a representation where the translation operators, P_μ, are diagonal and the basis vectors $|p, \alpha>$ span the Hilbert space, $\mathcal{H}$,

$$P_\mu|p, \alpha\rangle = p_\mu|p, \alpha\rangle \tag{4}$$

then we are in a position to make more precise statements:

(a) Existence of the vacuum: there is a unique invariant vacuum state $|0\rangle$ which has the property

$$U(a, \Lambda)|0\rangle = |0\rangle \tag{5}$$

The vacuum is unique and is Poincaré invariant.

(b) The eigenvalue of P_μ, p_μ, is lightlike, with $p_0 > 0$. We are discussing only the case of massive states. If we implement infinitesimal Poincaré transformation on the vacuum state then

$$P_\mu|0\rangle = 0, \quad \text{and} \quad M_{\mu\nu}|0\rangle = 0 \tag{6}$$

from above postulates; note that $M_{\mu\nu}$ are the generators of Lorentz transformations. (a4) The locality of theory implies that a (bosonic) local operator at spacetime point x^μ commutes with another (bosonic) local operator at x'^μ when their separation is spacelike i.e., if $(x - x')^2 < 0$. Our Minkowski metric convention is $(1, -1, -1, -1)$. The fixed-t dispersion relations for, $F(s, t)$, is proved from the above axiomatic theories and t must lie within the Lehmann-Martin ellipse [16,17]. Moreover, there are several theorems, presented as upper and lower bounds which have stood the experimental tests so far [18–20]. (ii) The aforementioned scaling lies in a kinematical domain (large s and small $|t|$) such that the perturbative QCD techniques cannot be applied to explain it. In the context of this scaling, the axiomatic field theory provides a precise and rigorous definition and proves the *raison d'etre* of the scaling [21]. We have chosen two different types of scaling variables to investigate how the data respond to our propositions. The choice is motivated under certain mathematical grounds and it is not of purely phenomenological as we shall explain later. (iii) The combined data of σ_t^{pp} and $\sigma_t^{p\bar{p}}$ are used to test the validity of Froissart bound. The point to note is that the rising of σ_t^{pp} and $\sigma_t^{p\bar{p}}$ was first observed at ISR. It was noted that $\Delta\sigma = \sigma_t^{pp} - \sigma_t^{p\bar{p}}$ is small and it decreases with growing ISR energy. The Pomeranchuk's theorem [22–24] provides a clue for us from the above observed feature. The original theorem stated that the particle-particle and the particle-antiparticle total cross sections would tend to equal values asymptotically; however, an important assumption was that [22] total cross sections attain constant values at asymptotic energies. We recall that σ_t^{pp} and $\sigma_t^{p\bar{p}}$ continue to rise at ISR, SPS, Tevatron and LHC energies; cosmic ray confirms the same trend. Note that the Pomeranchuk theorem is not a consequence of axioms of the QFT. The generalized theorem for the case of rising cross sections has been proved [25]. We recall that only ISR has measured the two cross sections, σ_t^{pp}

and $\sigma_t^{p\bar{p}}$, at the same energy and therefore, the theorem can be tested there. When fits to pp and $p\bar{p}$ cross section data are extrapolated, the two curves show a tendency to converge asymptotically. We justify, therefore, to combine the two sets of data and fit the cross sections.

The article is organized as follows. The next section, Sec. II, is devoted to study of phenomena mentioned earlier. In Sec. II A, we propose a formula to fit the combined total cross section data, σ^{pp} and $\sigma_t^{p\bar{p}}$, from ISR energies all the way up to the cosmic rays. We present a justification for the choice of the fitting formula. The study of the scaling of the elastic differential cross section is carried out in Sec. II B. We briefly recall the essential results on this type of scaling envisaged from rigorous mathematical structure. The results from the axiomatic approach are utilized for our purpose. We examine the scaling of diffractive elastic scattering. We choose two different scaling variables to investigate this phenomenon. Section II C is devoted to the study of the slope of the diffraction peak, $b(s)$. As we have noted earlier, it is an important experimentally measured parameter. We mention in passing that, to our surprise, there has not been much activity to explore the energy dependence of $b(s)$. We fit $b(s)$ from ISR energies to the LHC energy. Moreover, there are rigorous bounds on the slope of the absorptive diffraction differential cross sections, $b^A(s)$. There is no strong reason that why the data should respect these bounds. The motivation lies in the fact that, in the past, the data seemed to saturate the lower bound which led to an interesting conjecture. We test the bounds against the data. The third section contains the summary of our results and conclusions.

II. ANALYSIS OF EXPERIMENTAL DATA

We proceed to fit σ_t, investigate the scaling of differential cross sections, and the energy dependence of the slope parameter.

A. The fit to total cross sections

The purpose is to fit the total cross section data in order to test the Froissart-Martin bound. We would like to draw attention to the following facts. The PDG [1] has presented a very good fit to total cross sections of a large number of hadronic reactions over a wide range of energies. Let us discuss their fit to σ_t^{pp}. They considered data from 5 GeV all the way up to LHC energies; moreover, the cosmic ray data are also incorporated in the fit. We recall that the Froissart-bound saturating energy dependence is incorporated in their fit. In other words, the term is $\log^2 \frac{s}{s_0}$. The coefficient of this term, the Heisenberg constant, H, and s_0 are treated as floating parameters. In addition there is a constant term, P, associated with Pomeranchuk trajectory and several other terms corresponding to the contributions of Regge trajectories were added. It is important to note that the total cross sections are flat from 5 GeV till the pre-ISR energy domain

and there are vast data points measured with very good precision. Moreover, in the energy interval noted above, the Regge trajectory, such as ρ, A_2.. contributions are important although they fall off as power of s compared to the Pomeranchuk term and the Heisenberg term. In the fit to σ_t, the vast set of data points where σ_t is constant, terms subdominant compared to $\log^2 s$ term, play a crucial role. Moreover, the fit does not set out to test the Froissart-Martin bound since its exponent is fixed as 2. Our objective is to test the bound; as a consequence, we take the power of the exponent to be a floating parameter. A point is in order. It was claimed, in the past, in a fit to σ_t^{pp}, that there is evidence for violation of the Froissart bound [26]. Subsequently this claim was refuted by Block and Halzen [27]. Note that those two papers covered the same energy range as was covered by the PDG [1], i.e., from 5 GeV to cosmic ray energies. There is a recent comprehensive review [28] on high energy collisions which covers diverse aspects of hadronic processes and we refer it to the interested reader.

Let us discuss our proposal to fit the total cross section data. We choose the following parametrization to fit the combined data.

$$\sigma_t = H \log^\alpha \left(\frac{s}{s_o} \right) + P. \tag{7}$$

H and P are the Heisenberg and Pomeranchuk constants, respectively. P is the contribution of the Pomeranchuk trajectory in the Regge pole parlance. The constants H, P and α, are free parameters and are determined from the fits. We fix $s_0 = 16.00 \text{ GeV}^2$, taking a hint from the PDG fit. PDG adopted the following strategy to fit σ_t data. For the fit to σ_t^{pp} the chosen energy range was from 5 GeV to cosmic ray regime. The Froissart-bound-saturating energy dependence is incorporated in their fitting procedure. Note that in the pre-ISR energy regime the measured cross sections are flat and measured with very good precision. Moreover, Regge pole contributions, with subleading power behaviors, should be included in the pre-ISR energy domain. However, in the energy range starting from ISR, the Regge contributions are negligible. It is worth while to discuss and justify the reasons for not including the contributions of subleading Regge poles to σ_t in the energy range starting from ISR point and beyond (where our interests lie). Moreover, the subleading Regge poles have an important roles to play in fitting σ_t where the data are collected from the accelerators of those period. We refer to [18] and to the review article of Leader [29] for detailed discussions. We recall that σ_t is almost constant in the "low energy" (pre-ISR) region, however, contribution of the Regge tail is also necessary. Let us consider the case of pp scattering to get a concrete idea. The Pomeranchuk trajectory contributes a constant term to σ_t and its intercept is $\alpha_P(0) = 1$. Then there are subleading trajectories corresponding to ω, ρ, A_2,

ϕ, etc. When a fit to σ_t^{pp} was considered by Rarita *et al.* [30], they concluded, from numerical fits, that the ω trajectory dominates [29,30] and the contributions of other Regge trajectories is quite small [31]. They found that the Regge residue (interpreted as the Regge trajectory coupling) is $R_{pp\omega} \approx 15.5$ mb and $\alpha_\omega(0) \approx 0.45$ and the Regge scale, to define a dimensionless ratio (say $\frac{s}{s_*}$) is $s_* = 1$ GeV^2. Let us estimate what is the contribution of the ω-trajectory to σ_t^{pp} at the ISR energy. The contribution of the ω-trajectory to σ_t is quite small in the energy range from ISR to LHC. For example, at ISR energy of $\sqrt{s} = 23.5$ GeV, the ω-Regge pole contribution to σ_t is approximately 0.5 mb whereas at LHC, for $\sqrt{s} = 8$ TeV, it is ≈ 0.001 mb; the corresponding σ_t are ≈ 39 mb and ≈ 103 mb at 23.5 GeV and 8 TeV respectively. The parametrizations of [30] is used for the above estimates. Consequently, for our purpose, the parametrization (7) is well justified.

We considered the combined data of σ_t^{pp} and $\sigma_t^{p\bar{p}}$ for the energy range as mentioned earlier. ISR data are from $\sqrt{s} = 23.5$ to 63 GeV [32–41]. SPS data are at 540, 541, 546, and 900 GeV [42–48], whereas Tevatron data points are at 546 GeV, 1.02 TeV, and 1.8 TeV [49–56]. The LHC data points are at 7, 8 and 13 TeV [57–64]. Cosmic ray points are at 8, 14, 24, 30, 57, and 95 TeV [65–68]. The measured values of cross sections against $\sqrt{s}$, along with the fitted curve, are shown in Fig. 1. The fitted values for the parameters are $P = 36.4 \pm 0.3$ mb, $H = 0.22 \pm 0.02$ mb, and $\alpha = 2.07 \pm 0.04$. The quality of the fit, as reflected by the χ^2/n.d.f. is found to be moderate due to inclusion of both σ_t^{pp} and $\sigma_t^{p\bar{p}}$ measurements from ISR. A fit excluding $\sigma_t^{p\bar{p}}$ from ISR, as shown in Fig. 1 (lower), improves the fit quality without significantly changing the value of the fit parameters. We find no conclusive evidence for the violation of the Froissart bound.

B. Scaling of the diffraction cross section

The scaling observed in the diffraction region is an unexpected feature. This scaling phenomena is mysterious in the sense that we are unable to explain it from perturbative QCD perspectives. The kinematical region of interests (i.e., large s and small $|t|$) does not fall into the purview of the perturbative QCD terrain. Moreover, we draw attention to the fact that so far there is no careful analysis of the scaling in the diffraction region of very high energy regime which we intend to explore. Therefore, it deserves a thorough investigation. A precise definition of scaling and its derivation was provided from axiomatic field theory approach. The necessary ingredients stated and proved from the axiomatic field theory are [21]: (i) The scattering amplitude, $F(s,t)$, is holomorphic in the disk $|t| < t_0$ for any s in cut s plane, where t_0 is a constant less than or equal to the t-channel threshold. (ii) The polynomial boundedness: $|F(s,t)|$ is bounded by s^N, $N \in Z$ and N is
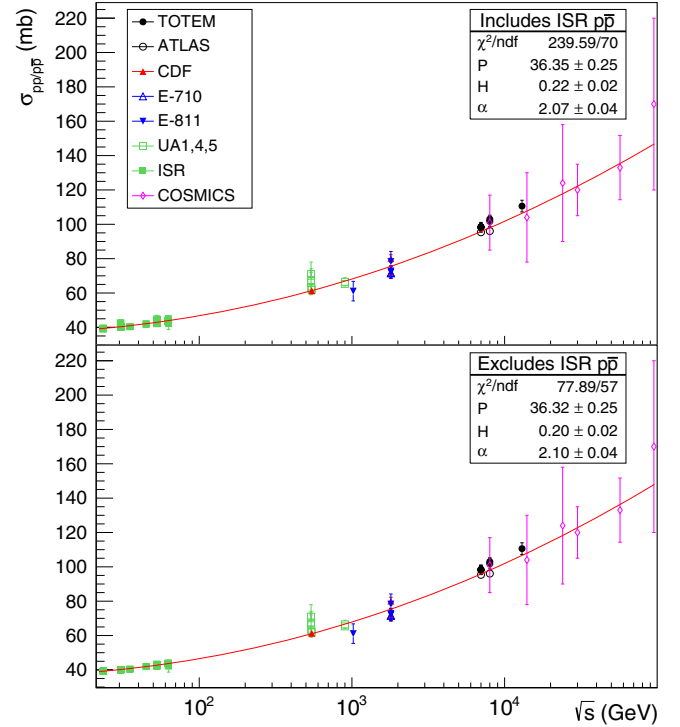

FIG. 1. σ_t^{pp} and $\sigma_t^{p\bar{p}}$ against $\sqrt{s}$, measured by various experiments. The data points are fitted to a function defined in (7). The upper plot includes both pp and $p\bar{p}$ data points, while the lower one excludes $p\bar{p}$ data points from ISR experiments.

finite, for $|t| < t_0$ and $s \rightarrow +\infty$. (iii) $F(s,t)$ satisfies unitarity in the s-channel. It follows from the results (i)-(iii) that $F(s,t)$ satisfies the bound

$$|F(s,t)| \leq \left[\frac{4\pi\sigma_{el}(s)}{t_0}\right]^{1/2} s\log(s) \exp\left[\left(\frac{|t|}{t_0}\right)^{1/2} \log(s)\right].$$

(8)

The Pomeranchuk-theorem-violating amplitude (see [21]) asymptotically behaves as

$$F(s,0) \sim i\sigma s + Cs\log(s), \quad 0 < |C| < (4\pi\sigma/t_0)^{1/2} \quad (9)$$

where σ is the total cross section. There is a more refined mathematical statement on the behavior of $F(s,0)$ in [21]. A general class of amplitudes of the form

$$F(s,0) \sim (i\alpha + \beta)s(\log(s))^{-2} \quad (10)$$

was considered by [21]; where $0 < \alpha \leq (\frac{4\pi}{t_0})$, $|\beta| \leq (\frac{4\pi\alpha}{t_0})^{1/2}$ and $F(s,0)$ respects the Froissart bound. Our sole interest is to recapitulate the scaling property succinctly derived and proved in [21]. A function

$$f(s,\tau) = \frac{F(s,-t_0\tau(\log(s))^{-2})}{F(s,0)} \quad (11)$$

was introduced in [21]. The scaling is interpreted in the following sense: define $f(\tau) = \lim_{s \to \infty} f(s, \tau)$. It was proved that $f(\tau)$ is an entire function with following properties

$$f(0) = 1, \quad |f(\tau)| < \left(\frac{4\pi C_0}{t_0}\right)^{(1/2)} e^{\sqrt{\tau}}, \quad \text{for all } \tau \qquad (12)$$

where $C_0 = \frac{\alpha}{\alpha^2 + \beta^2}$; α and β are as in (10). In a nutshell, it summarizes the scaling phenomena as derived from the axiomatic perspectives.

We have chosen two different types of scaling variables based on certain theoretical consideration. (a) The first choice is $\tau_1 = \frac{1}{16\pi} \frac{\sigma_t^2}{\sigma_{el}} t$. The technique of group contraction was adopted to describe high energy diffraction scattering [69]. It was demonstrated that the scaling occurs in the small $|t|$ domain at high energies. The high energy collision of spinless particles, in small angles, was envisaged. Let us consider a sphere of radius γ with polar angles θ and ϕ. The amplitude, for scattering of scalars, depends only on θ. Let us focus on scattering in the forward regions such that $\gamma\theta \to$ finite as $\theta \to 0$ and $\gamma \to \infty$. The group contraction technique [70] was invoked as follows. $O(3)$ is the little group of the Lorentz group $O(3, 1)$. In the limit, $\gamma \to \infty$, $O(3)$ contracts to E_2, the group of translations. Let $L_i, i = 1, 2, 3$ be generators of $O(3)$ satisfying $[L_i, L_j] = i\epsilon_{ijk}L_k$. Define $L_2 = \gamma Q_2$ and $L_1 = -\gamma Q_1$ with L_3 unaltered. The angular momentum algebra assumes the form $[L_3, Q_2] = iQ_1, [L_3, Q_1] = -iQ_2$ and $[Q_1, Q_2] = 0$ in the limit $\gamma \to \infty$. Notice that Q_1, Q_2 are translation operators of E_2. L_3 generates rotation around the 3rd axis. If q_a and q_b are the momenta defining a representation of E_2, then the eigenvalue of Casimir of $O(3)$, $l(l+1) \to (l + 1/2)^2$ in the limit $\gamma \to \infty$. The partial wave amplitude, $f_l(s) \to f(q, s)$ where $q = \sqrt{(q_1^2 + q_2^2)}$. The expression for the scattering amplitude [69] is derived from the following ingredients: (i) Partial wave unitarity. (ii) Optical theorem to relate the absorptive forward scattering amplitude to σ_t. And (iii) utilize the standard relation $\sigma_{el} = \frac{4\pi}{p^2} \sum_0^\infty (2l + 1)|f_l(s)|^2$, p being the c.m. momentum. Next step is to replace $f_l(s)$ by $f(q, s)$ and the sum over l goes over to an integral in q. Moreover, $f(q, s)$ is assumed to have a Gaussian distribution in q. Finally, the normalized differential cross section, after a few algebraic steps, was derived to be

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt}\bigg|_{t=0} e^{t\frac{\sigma_t^2}{16\pi\sigma_{el}}}. \qquad (13)$$

The identification of $\tau_1 = |t|\frac{\sigma_t^2}{16\pi\sigma_{el}}$ as a scaling variable was noted in [69]. Indeed the diffraction scattering data, available at that juncture, for $\pi^\pm p$, $k^\pm p$, pp and $p\bar{p}$ over a wide energy range exhibited the scaling behavior. The objective is to examine whether the present data exhibits the

scaling. (b) The other choice is $\tau_2 = |t|b(s)$, where $b(s)$ is the experimentally measured slope parameter. Cornille and Martin [71] have demonstrated rigorously the existence of this scaling variable. The data prior to SPS, Tevatron, LHC era exhibited the scaling [71]. Subsequent works [72–75] investigated and tested the scaling with the vast data of that era. Notice that the data in the forward direction is conveniently parametrized as

$$\frac{d\sigma}{dt}(s, t) = \frac{d\sigma}{dt}\bigg|_{t=0} e^{B(s,t)} \qquad (14)$$

where $B(s, t)$ is usually expanded in a power series in t, around $t = 0$, as $B(s, t) = b(s)t + c(s)t^2 + \dots$; where $b(s)$ is the slope of the diffraction peak, $c(s)$ is the curvature and so on. Thus the $b(s)t$ term dominates in the small t, near forward region, while fitting $\frac{d\sigma}{dt}$. We restrict the values of $|t|$ to the range that is used in the corresponding measurements to extract the values of $b(s)$ assuming a purely exponential dependence of $\frac{d\sigma}{dt}$ on t. If we include larger values of $|t|$ then the data favor the retention of the higher curvature terms in the expansion of $B(s, t)$ in (14).

We have displayed scaling in the diffraction region for the choice $\tau_1 = \frac{|t|\sigma_t^2}{16\pi\sigma_{el}}$ and $\tau_2 = |t|b(s)$, in Fig. 2. We use $|t|$ in our definition of scaling variables for convenience (t is always negative in the physical region for s-channel reactions). We have not included a set of ISR data points in a very small $|t|$ region; this point is discussed in detail in the context of extraction of $b(s)$ in sequel. If those points are included in the scaling plot they deviate from the scaling displayed by rest of the data. It is important to note that the rest of the data exhibits scaling so nicely from ISR to LHC energy range. Now let us compare and contrast the scaling phenomena with the two scaling variables. For the purpose of examining the scaling for the measured values of $d\sigma/dt$ as function of $|t|$ data are taken from [42,57,59,76–83]. We analyze the scaling property with $\tau_1 = \frac{1}{16\pi} \frac{\sigma_t^2}{\sigma_{el}} |t|$ as the scaling variable and the covered range of τ_1 is: $0.02 \leq \tau_1 \leq 3.6$. The scaling is very good for the case of τ_1 for small values of $|t|$ as depicted in Fig. 2(a). Note that the scaling is excellent for the τ_2 variable in the small $|t|$ region and $0.05 \leq \tau_2 \leq 9.0$, which is displayed in Fig. 2(b). There is a study of scaling in hadronic scattering [84]; however, their approach is different and their main focus is not in the kinematical domain of our interests.

C. The slope of the diffraction peak

The slope of the diffraction peak is defined to be

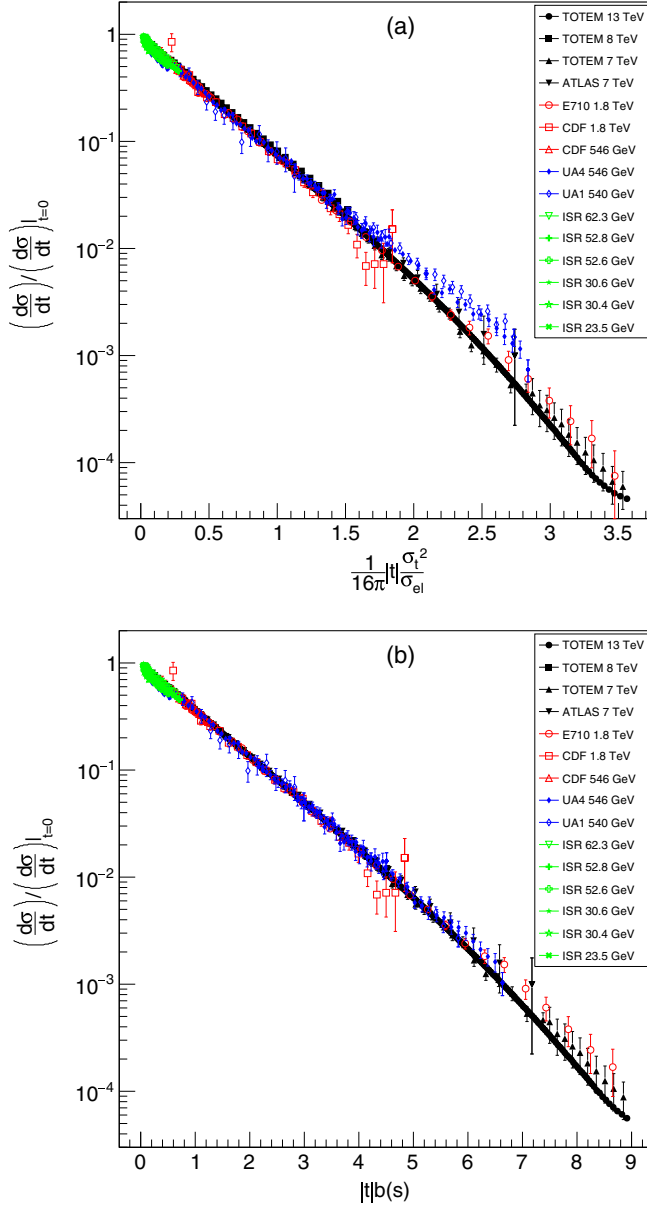$$b(s) = \frac{d}{dt} \log\left(\frac{d\sigma}{dt}\right)\bigg|_{t=0} \qquad (15)$$
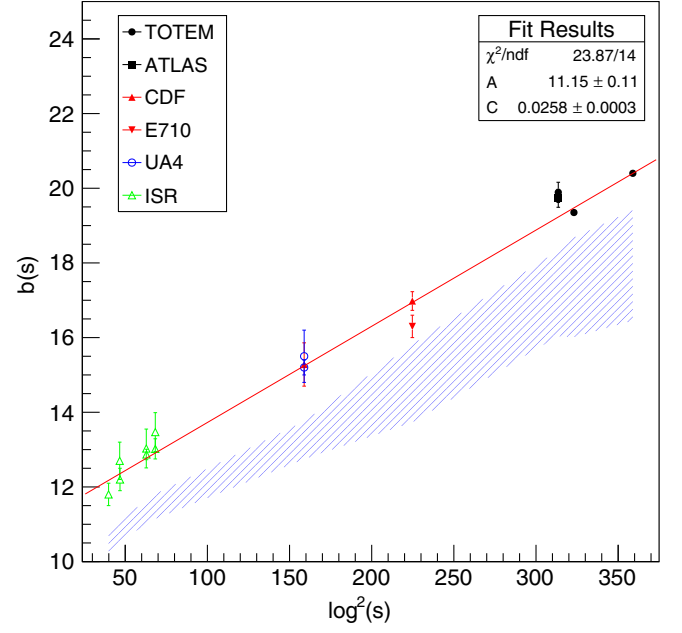
FIG. 3. Plot of measured value of $b(s)$ as function of $\log^2(s)$ as obtained from various experiments. The error band is the lower limit as defined in (17).

FIG. 2. Distributions of the ratio $\frac{d\sigma}{dt}(s,t)/\frac{d\sigma}{dt}\big|_{t=0}$ as function of (a) $\frac{1}{16\pi}|t|\frac{\sigma_t^2}{\sigma_{el}}$ and (b) $|t|b(s)$, measured by various experiments for pp and $p\bar{p}$ collisions.

and it is experimentally extracted from diffraction scattering data. Another object of interests is the slope of absorptive differential cross section

$$b^A(s) = \frac{d}{dt}\log\left(\frac{d\sigma^A}{dt}\right)\bigg|_{t=0}. \quad (16)$$

There are rigorous lower and upper bounds on $b^A(s)$.

$$\frac{2}{9}\left[\frac{\sigma_t^2}{4\pi\sigma_{el}} - \frac{1}{k^2}\right] \leq b^A(s) \leq \frac{1}{2(t_0 - \epsilon)}\left[\log\left(\frac{s}{\sigma_t}\right)\right]^2. \quad (17)$$

The lower bound is due to Martin and MacDowell [85] whereas the upper bound was proved by Singh [86]. We have fitted the slope with the parameterization $b(s) = A + C\log^2\left(\frac{s}{s'}\right)$ and chose $s' = 1$ GeV^2 so that the argument of the log is dimensionless. The fit is depicted in Fig. 3. Note the following: the measurement of $\frac{d\sigma}{dt}(s,t)$ exhibits a sharp rise in the range $0.0004 \leq |t| \leq 0.004$. In this $|t|$ interval, $\frac{d\sigma}{dt}$ cannot be fitted by parametrizing it with only a $b(s)$. The experiment [76] fits the data by adding the proton form factor term to account for the Coulomb effect. Block and collaborators [87] consider an eikonal model to study pp scattering. They argue that in order to explain behavior of $\frac{d\sigma}{dt}$ in the said $|t|$ region, the effects of higher curvature terms be considered. We leave aside this set of data points of the particular ISR experiment to fit $b(s)$ taking into account the values of $b(s)$ as presented in various experiments. These experiments have extracted $b(s)$ from the fits to $\frac{d\sigma}{dt}$ in near forward direction. Let us examine how the measured $b(s)$ stands up against the bounds (17). We remind that the bounds (17) are proved for $b^A(s)$. Therefore, the data need not respect them. When the lower bound [85] was proved and was tested against experiments, the data nearly saturated the bound. Consequently, it was conjectured that the amplitude is dominantly imaginary [85]. In order to compare (experimentally) measured $b(s)$ against MacDowell-Martin bound we used measured values of σ_t and σ_{el}. Thus the bound depicted in Fig. 3 is a band due to the uncertainties of σ_t and σ_{el} in measurements. The data satisfies the Martin-MacDowell bound, however. We remark that the upper bound (17) lies way above the

data due to the presence of the factor $2t_0$, $t_0 = 4m_\pi^2$, in the denominator. The measured values of $b(s)$ are taken from [42,57,59,76,79–83], which were used for the fit.

III. SUMMARY AND CONCLUSIONS

One of our objectives was to test the Froissart bound against experiments. We selected the set of data points where the total cross sections start rising. Since $\Delta\sigma$ is quite small at ISR energy and keeps diminishing with growing s we justified the inclusion of both pp and $p\bar{p}$ data in the fitting of σ_t. We set out to verify scaling of the diffraction data at asymptotic energies with small $|t|$. We note that this attribute of elastic scattering at asymptotic energy has hitherto escaped attentions and scrutiny. It is quite satisfying that the data exhibit very good scaling for both the choices: τ_1 and τ_2. The slope $b(s)$ is a very important parameter in the study of hadronic collisions and is used to extract σ_t since the point $t = 0$ is not accessed in experiments. We have presented a fit to $b(s)$ data. The axiomatic bounds of $b^A(s)$ was compared with the data, although the data need not necessarily respect them; however, they are consistent with the bounds.

There is an interesting paper on the ultra high energy neutrino-nucleon deep inelastic scattering [88]. In this work the authors study the behavior of the structure function. They show that the νN scattering cross section exhibits $(\log(s))^2$ behavior.

The following remarks are in order. It is an important issue to ponder over the violation of the Froissart-Martin bound. So far the experimental data are consistent with the bound. If, in future, the experiments unambiguously confirm violation of the bound then it will be a matter of concern. One possible explanation would be to question the axioms of local field theories. However, these field theories do not lead only to the bound on total cross sections. There are other important inferences drawn from the axiomatic field theory. For example, the CPT theorem [15] is another important outcome which has been put to tests. Therefore, it will not be so easy to abandon any of the axioms for local field theories. On the other hand, if we accept the axioms of local field theory, then new physics paradigms might have to be introduced. Thus observation of the violation of the bound on σ_t would lead to very important consequences. In conclusion, we have examined several interesting and important aspects of very high energy scatterings and we have utilized the data to explore some of the phenomena which had not been explored in the recent years. We tested the validity of the sacred Froissart bound and it still stands.

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