New $\Lambda_b(6072)^0$ state as a 2S bottom baryon

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As a result of continuous developments, the recent experimental searches lead to the observations of new particles at different hadronic channels. Among these hadrons are the excited states of the heavy baryons containing single bottom or charmed quark in their valance quark content. The recently observed Λ_b (6072)⁰ state is one of these baryons and possibly 2S radial excitation of the Λ_b state. Considering this information from the experiment, we conduct a QCD sum rule analysis on this state and calculate its mass and current coupling constant considering it as a 2S radially excited Λ_b resonance. For completeness, in the analyses, we also compute the mass and current coupling constant for the ground state Λ_b^0 and its first orbital excitation. We also consider the Λ_c^+ counterpart of each state and attain their mass, as well. The obtained results are consistent with the experimental data as well as existing theoretical predictions.

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I. INTRODUCTION

The progress in experimental facilities and techniques culminated in many exciting observations of the various new particles in recent years. Among these new states, there exist excited states of the heavy baryons at different channels that have been in the focus of much attention. The searches for the properties of these states can play crucial roles in the understanding of the dynamics, nature, and quark-gluon organizations of these states as well as the perturbative and nonperturbative natures of QCD. Investigations of the baryons with single heavy quark and two light quarks contribute to a better understanding of the confinement mechanism and help us test the predictions of not only the quark model and the heavy quark symmetry but also that of other theoretical approaches used to describe these states.

In the last few decades, we witnessed the observations of various excited baryons containing single heavy quark in their quark content. Among these states are the $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, $\Omega_c(3119)^0$ states [\[1\]](#page-6-0)

observed from the investigation of the $\Xi_c^+ K^-$ mass spectrum, $\Xi_b(6227)^-$ [\[2\]](#page-6-1), $\Sigma_b(6097)^{\pm}$ [\[3\]](#page-6-2), $\Xi_b'(5935)^-$,
 $\Xi_b(5955)^-$ [41 A *(5012)⁰ A *(5020)⁰ [51 A (6146)⁰ Ξ_b (5955)[–] [\[4\]](#page-6-3), $\Lambda_b^*(5912)^0$, $\Lambda_b^*(5920)^0$ [\[5\]](#page-6-4), Λ_b (6146)⁰,
 Λ_b (6152)⁰ [6], Ω_b (6316)[–], Ω_b (6330)[–], Ω_b (6340)[–], and $\Lambda_b(6152)^0$ [\[6\]](#page-6-5), $\Omega_b(6316)^-$, $\Omega_b(6330)^-$, $\Omega_b(6340)^-$, and $\Omega_b(6350)$ ⁻ [\[7\]](#page-6-6). A wealth of theoretical investigations accompanied these observations to elucidate their various properties and to enrich our understanding of their structures. Their mass spectrum and decay mechanisms were extensively searched for by quark model [8–[36\],](#page-6-7) heavy hadron chiral perturbation theory [37–[41\],](#page-6-8) relativistic flux tube model [\[42\]](#page-6-9), Bethe-Salpeter formalism [\[43\]](#page-6-10), ${}^{3}P_0$ model $[44–51]$ $[44–51]$, lattice QCD $[52–55]$ $[52–55]$, the bound state picture $[56]$, light cone QCD sum rules [\[57](#page-7-2)–66] and QCD sum rules method [\[67](#page-7-3)–75], etc. For more related discussions about these states, we refer to the Refs. [76–[81\]](#page-7-4) and the references therein.

Nowadays the LHCb Collaboration announced the observation of another new beauty baryon state, which shows consistency with 2S radial excitation of Λ_b^0 baryon, in the $\Lambda_b \pi^+ \pi^-$ invariant mass spectrum with a significance exceeding 14 standard deviations [\[82\]](#page-7-5). Its mass and width were reported as $m_{\Lambda_b^{*0}} = 6072.3 \pm 2.9 \pm 0.6 \pm 0.2$ MeV and $\Gamma = 72 \pm 11 \pm 2$ MeV, respectively, with an interpretation of its being 2S excited state. This observation is tation of its being 2S excited state. This observation is also consistent with the report of CMS collaboration [\[83\]](#page-7-6) indicating a broad excess of events in the region of 6040–6100 MeV. In 2012, the LHCb Collaboration

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announced the observation of two narrow Λ_b states decaying into $\Lambda_b^0 \pi^+ \pi^-$, which are Λ_b $(5912)^0$ and Λ_b (5920)⁰ and these states were interpreted as orbital Λ_b (5920)⁰ and these states were interpreted as orbital excitations of Λ_b^0 baryon [\[5\]](#page-6-4). These baryons were studied using the QCD sum rule approach in the heavy quark effective theory [\[69\].](#page-7-7) Later, in 2019, the LHCb collaboration reported the observation of another Λ_b baryon doublet, namely $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$, with an interpretation of their being 1D-wave state [\[6\]](#page-6-5). The mass predictions in the QCD sum rule method for these states were presented in Refs. [\[70,75\]](#page-7-8). In the present work, we focus our attention on the newly observed state $\Lambda_b(6072)^0$ and perform an analysis on the mass of this particle considering its being first radial excitation, 2S-state, with possible quantum numbers $J^P = \frac{1}{2}^+$, as suggested by the LHCb
Collaboration. To this end, we adopt the OCD sum rule Collaboration. To this end, we adopt the QCD sum rule method [84–[86\]](#page-7-9) with a proper interpolating current that couples the states with considered quantum numbers. This method is a nonperturbative method applied with success to calculate various properties of hadrons, such as their spectroscopic and decay properties, giving consistent results with experimental observations. Thus, the interpolating current used in the calculations not only couples to the considered radially excited state but also to the ground and orbitally excited ones. Therefore in this work, we first calculate the mass and the current coupling constant of the ground state baryon, then we obtain the masses and current

coupling constants of its first orbital and radial excitations. For completeness, we also include in our analyses the charmed counterpart of the considered states. The spectroscopic analyses of the considered states may shed light on the quantum numbers and structure of these states, improve our understanding of the strong interaction and help us test the predictions of the quark model.

The outline of this work is as follows: Sec. [II](#page-1-0) provides the details of the QCD sum rules calculations for the masses and the current coupling constants of the considered states. In Sec. [III](#page-4-0) the numerical analyses and the results are presented. The last section gives a summary of the results and conclusion.

II. QCD SUM RULE CALCULATIONS FOR THE $Λ_b$ AND $Λ_c$ STATES

The states considered in this study are analyzed through the following two-point correlation function:

$$
\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | \mathcal{T} \{ \eta(x) \bar{\eta}(0) \} | 0 \rangle, \tag{1}
$$

where $n(x)$ represents the interpolating current in terms of the related valance quark fields and T is used to represent the time ordering operator. The following interpolating current is used in the calculations:

$$
\eta = \frac{\epsilon_{abc}}{\sqrt{6}} \left[2(u^{aT}Cd^b)\gamma_5 Q^c + 2\beta(u^{aT}C\gamma_5 d^b)Q^c + (u^{aT}CQ^b)\gamma_5 d^c + \beta(u^{aT}C\gamma_5 Q^b)d^c + (Q^{aT}Cd^b)\gamma_5 u^c + \beta(Q^{aT}C\gamma_5 d^b)u^c \right],
$$
\n(2)

where Q represents b (c) quark field for $\Lambda_b(\Lambda_c)$ state; a, b, and c are color indices, C is the charge conjugation operator and the β is an arbitrary parameter to be fixed later. The above interpolating current is written considering all the quantum numbers of the states under study. The three states considered in the present study have all the same quantum numbers and quark contents but different energies, hence, all of these particles couple to the same interpolating field. According to the quark model, Λ_Q belongs to the antitriplet representation of the $SU(3)$ and the current describing it should be antisymmetric with respect to the exchange of the light quark fields. The interpolating field should also be color singlet. Thus, its general form satisfying these conditions can be decomposed as

$$
\eta \sim \epsilon_{abc} \{ (u^{aT} C \Gamma d^b) \tilde{\Gamma} Q^c + (u^{aT} C \Gamma Q^b) \tilde{\Gamma} d^c + (Q^{aT} C \Gamma d^b) \tilde{\Gamma} u^c \},\tag{3}
$$

where Γ and Γ can be any of the matrices 1, γ_5 , γ_μ , $\gamma_5\gamma_\mu$, or $\sigma_{\mu\nu}$. The main task is to determine the Γ and Γ. For this aim let us first consider the transpose of the part $\epsilon_{abc}(u^{a\dot{T}}\nabla\Gamma d^b)$ in the first term:

$$
[\epsilon_{abc}u^{aT}CTd^{b}]^{T} = \epsilon_{abc}d^{bT}C(C\Gamma^{T}C^{-1})u^{a} = -\epsilon_{abc}d^{aT}C(C\Gamma^{T}C^{-1})u^{b}, \qquad (4)
$$

where a simple theorem was used: if $A = BD$, where A, B, and D are matrices, whose elements are Grassmann numbers,
then $A^T = -D^T B^T$. We also used $C^T = C^{-1}$ and $C^2 = -1$. The quantity $C^T C^{-1}$ is Γ for the cases $\Gamma =$ then $A^T = -D^T B^T$. We also used $C^T = C^{-1}$ and $C^2 = -1$. The quantity $C\Gamma^T C^{-1}$ is Γ for the cases $\Gamma = 1$, γ_5 , and $\gamma_5\gamma_\mu$; and it is $-\Gamma$ for the matrices $\Gamma = \gamma$, and σ . The transpose of a one by one ma it is −Γ for the matrices $\Gamma = \gamma_\mu$ and $\sigma_{\mu\nu}$. The transpose of a one by one matrix, i.e., a scalar, must be equal to itself. Thus,

$$
-\epsilon_{abc}d^{aT}C(C\Gamma^{T}C^{-1})u^{b} = -\epsilon_{abc}d^{aT}C\Gamma u^{b} = \epsilon_{abc}u^{aT}C\Gamma d^{b},\tag{5}
$$

which is held for $\Gamma = 1$, γ_5 and $\gamma_5 \gamma_\mu$. Note that, $\epsilon_{abc}u^{aT}C\Gamma d^{b}$ is antisymmetric for the $u \leftrightarrow d$ exchange, which was used in the above relation. The simplest way is to choose the Λ_{Ω} state to have the same total spin/spin projection as the heavy quark Q . Therefore, the spin of the diquark formed by light quarks must be zero. This immediately implies that $\Gamma = 1$ or γ_5 . Thus, the two possible forms of the interpolating field become

$$
\eta_1 = \epsilon_{abc} (u^{aT} C d^b) \tilde{\Gamma}_1 Q^c,
$$

and

$$
\eta_2 = \epsilon_{abc} (u^{aT} C \gamma_5 d^b) \tilde{\Gamma}_2 Q^c.
$$
 (6)

The matrices $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ are determined via the Lorentz and parity considerations. As η_1 and η_2 are Lorentz scalars, one concludes that $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ should be 1 or γ_5 . The parity consideration leads to $\tilde{\Gamma}_1 = \gamma_5$ and $\tilde{\Gamma}_2 = 1$. Therefore, the two possible forms of the interpolating field for the considered term can be written as

$$
\eta_1 = \epsilon_{abc} (u^{aT} C d^b) \gamma_5 Q^c,
$$

and

$$
\eta_2 = \epsilon_{abc} (u^{aT} C \gamma_5 d^b) Q^c.
$$
 (7)

Evidently, the arbitrary linear combination of the above possibilities can better represent the baryon Λ_{Ω} :

$$
\eta \sim \epsilon_{abc} [(u^{aT} C d^b) \gamma_5 Q^c + \beta (u^{aT} C \gamma_5 d^b) Q^c], \qquad (8)
$$

where the general mixing parameter β is introduced to gain the general form of the interpolating field. Repeating similar steps for the second and third terms in Eq. (3) , we finally acquire Eq. [\(2\)](#page-1-2) to interpolate the Λ_{Ω} states. In the present study we make an assumption and take the parameter β the same for all the ground and excited Λ _O resonances.

According to the standard prescriptions of the QCD sum rule method, the correlation function is calculated via two different approaches. First, it is calculated in terms of hadronic degrees of freedom and called the physical or hadronic side of the calculations. The result of this part contains the physical quantities such as mass and current coupling constant of the considered states. The second approach brings out the results in terms of QCD degrees of freedom such as quark-gluon condensates, QCD coupling constant, the masses of the quarks, etc called the QCD side of the calculations. By matching the results of both sides, considering the coefficients of the same Lorentz structures, one gets the QCD sum rules for the physical quantities under question.

For the physical side of the calculations, the correlator, Eq. [\(1\),](#page-1-3) is calculated by inserting complete sets of hadronic states into the appropriate places. This step turns the correlator into the form

$$
\Pi^{\text{Phys}}(q) = \frac{\langle 0|\eta|\Lambda_{Q}(q,s)\rangle\langle\Lambda_{Q}(q,s)|\bar{\eta}|0\rangle}{m^2 - q^2} + \frac{\langle 0|\eta|\tilde{\Lambda}_{Q}(q,s)\rangle\langle\tilde{\Lambda}_{Q}(q,s)|\bar{\eta}|0\rangle}{\tilde{m}^2 - q^2} + \frac{\langle 0|\eta|\Lambda'_{Q}(q,s)\rangle\langle\Lambda'_{Q}(q,s)|\bar{\eta}|0\rangle}{m^2 - q^2} + \cdots
$$
 (9)

The $|\Lambda_{Q}(q, s)\rangle$, $|\tilde{\Lambda}_{Q}(q, s)\rangle$ and $|\Lambda'_{Q}(q, s)\rangle$ are used to represent the one-particle states of the ground and its first represent the one-particle states of the ground, and its first orbital excitation 1P and first radial excitation 2S states, respectively. Here, m , \tilde{m} , and m' are their respective masses and \cdots represents the contributions of the higher states and continuum. The matrix elements in Eq. [\(9\)](#page-2-0) are parametrized as follows:

$$
\langle 0|\eta|\Lambda_{Q}(q,s)\rangle = \lambda u(q,s),
$$

\n
$$
\langle 0|\eta|\tilde{\Lambda}_{Q}(q,s)\rangle = \tilde{\lambda}\gamma_{5}u(q,s),
$$

\n
$$
\langle 0|\eta|\Lambda'_{Q}(q,s)\rangle = \lambda'u(q,s),
$$

\n(10)

where λ , $\tilde{\lambda}$, and λ' are the corresponding current coupling constants and $u(q, s)$ is the Dirac spinor. These matrix elements are used in Eq. [\(9\)](#page-2-0) and summation over spins of Dirac spinors, which is given as

$$
\sum_{s} u(q,s)\bar{u}(q,s) = (q+1), \qquad (11)
$$

is applied. Then, the physical side takes the form:

$$
\Pi^{\text{Phys}}(q) = \frac{\lambda^2(\cancel{q} + m)}{m^2 - q^2} + \frac{\widetilde{\lambda}^2(\cancel{q} - \widetilde{m})}{\widetilde{m}^2 - q^2} + \frac{\lambda'^2(\cancel{q} + m')}{m'^2 - q^2} + \cdots
$$
\n(12)

After the Borel transformation, the final result for the physical side becomes

$$
\tilde{\Pi}^{\text{Phys}}(q) = \lambda^2 (q + m) e^{-\frac{m^2}{M^2}} + \tilde{\lambda}^2 (q - \tilde{m}) e^{-\frac{\tilde{m}^2}{M^2}} + \lambda'^2 (q + m') e^{-\frac{m'^2}{M^2}} + \cdots
$$
 (13)

For the QCD side, one computes the correlation function, Eq. [\(1\)](#page-1-3), using the interpolating current given in Eq. [\(2\)](#page-1-2) explicitly. To perform the calculations, first the possible contractions between the quark fields are carried out via Wick's theorem. For the contracted quark fields the corresponding light and heavy quark propagators presented in coordinate space are used with following explicit forms:

$$
S_{q,ab}(x) = i\delta_{ab}\frac{\not x}{2\pi^2 x^4} - \delta_{ab}\frac{m_q}{4\pi^2 x^2} - \delta_{ab}\frac{\langle\bar{q}q\rangle}{12} + i\delta_{ab}\frac{\not xm_q\langle\bar{q}q\rangle}{48} - \delta_{ab}\frac{x^2}{192}\langle\bar{q}g_s\sigma Gq\rangle + i\delta_{ab}\frac{x^2 \not xm_q}{1152}\langle\bar{q}g_s\sigma Gq\rangle
$$

$$
-i\frac{g_s G_{ab}^{\alpha\beta}}{32\pi^2 x^2}[\not x\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\not x] - i\delta_{ab}\frac{x^2 \not xg_s^2\langle\bar{q}q\rangle^2}{7776},\tag{14}
$$

and

$$
S_Q(x) = \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} - i \frac{m_Q^2 \cancel{x}}{4\pi^2 x^2} K_2(m_Q\sqrt{-x^2}) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 du \left[\frac{\cancel{x} + m_Q}{2(m_Q^2 - k^2)^2} G^{\mu\nu}(ux) \sigma_{\mu\nu} + \frac{u}{m_Q^2 - k^2} x_\mu G^{\mu\nu}(ux) \gamma_\nu \right],
$$
\n(15)

where $G_{\mu\nu}$ is the gluon field strength tensor, K_{ν} is the Bessel function of the second kind and $G_{ab}^{\alpha\beta} = G_A^{\alpha\beta} t_{ab}^A$, with $A = 1, 2, \ldots, 8$, and $t^A = 3^{A/2}$. After the usage of the $A = 1, 2, ..., 8$ and $t^A = \lambda^A/2$. After the usage of the proposators Fourier and Borel transformations are perpropagators, Fourier and Borel transformations are performed. Finally, the continuum subtraction with the help of quark-hadron duality assumption is applied. The result of the QCD side of the sum rule is obtained in the form

$$
\tilde{\Pi}^{\text{QCD}}(s_0, M^2) = \int_{(m_Q + m_u + m_d)^2}^{s_0} ds e^{-\frac{s}{M^2}} \rho(s) + \Gamma, \quad (16)
$$

where, s_0 represents the continuum threshold and $\rho(s)$ is the spectral density that is obtained by taking the imaginary part of the result, $\rho(s) = \frac{1}{\pi} \text{Im}[\Pi^{\text{QCD}}]$. The $\rho(s)$ and Γ are lengthy functions, so we do not present their evolicit forms lengthy functions, so we do not present their explicit forms here.

After the computations of the both sides, the results are matched through the dispersion relations considering the coefficients of the same Lorentz structures, that are \oint and I. The QCD sum rules for the considered quantities are obtained as

$$
\lambda^2 e^{-\frac{m^2}{M^2}} + \tilde{\lambda}^2 e^{-\frac{\tilde{m}^2}{M^2}} + \lambda'^2 e^{-\frac{m'^2}{M^2}} = \tilde{\Pi}^{\text{QCD}}_{\cancel{q}}(s_0, M^2), \quad (17)
$$

and

$$
\lambda^2 m e^{-\frac{m^2}{M^2}} - \tilde{\lambda}^2 \tilde{m} e^{-\frac{\tilde{m}^2}{M^2}} + \lambda'^2 m' e^{-\frac{m'^2}{M^2}} = \tilde{\Pi}_I^{\text{QCD}}(s_0, M^2). \tag{18}
$$

The relation obtained using the ϕ structure is used to derive the QCD sum rules for mass and coupling constant by following the ground state $+$ continuum scheme in which we consider the second and third terms of the left-hand-side of Eq. [\(17\)](#page-3-0) as parts of the continuum. This results in the following equation for the mass of the ground state:

$$
m^{2} = \frac{\frac{d}{d(-\frac{1}{M^{2}})}\tilde{\Pi}_{\cancel{q}}^{\text{QCD}}(s_{0}, M^{2})}{\tilde{\Pi}_{\cancel{q}}^{\text{QCD}}(s_{0}, M^{2})}.
$$
(19)

The current coupling constant is obtained as

$$
\lambda^2 = e^{\frac{m^2}{M^2}} \tilde{\Pi}^{\text{QCD}}_d(s_0, M^2). \tag{20}
$$

Then we consider the first two terms on the left-hand side of Eq. [\(17\)](#page-3-0) by increasing the threshold and the third one is taken in the continuum. By using the results obtained for ground state as inputs, we get the mass and current coupling constant for the first orbitally excited, 1P, state. And finally, the results of the ground and 1P states are used in a similar manner, namely, ground state $+$ first orbitally excited state $\frac{1}{1}$ first radially excited state $+$ continuum approach, to obtain the physical quantities of the radially excited, 2S, state.

III. NUMERICAL ANALYSES

To numerically analyze the results obtained in the previous section we need some input parameters that are presented in Table [I.](#page-4-1) Though our main concern in the present work is the mass of the newly observed 2S $\Lambda_b(6072)^0$ state, we also obtain the masses for the 1S and 1P excited states and the corresponding current couplings for both Λ_b^0 and Λ_c^+ channels. To this end, we also need to fix three auxiliary parameters that are entered the sum rules: β , s_0 , and M^2 . They are fixed based on the standard prescriptions of the method. Thus, we impose the conditions of the mild dependence of the results to the auxiliary parameters, the convergence of the operator product expansion (OPE), and the dominance of the

TABLE I. Some input parameters used in the analyses.

Parameters	Values		
m_c	1.27 ± 0.02 GeV [87]		
m_h	$4.18_{-0.02}^{+0.03}$ GeV [87]		
m_{μ}	2.16 ^{+0.49} MeV [87]		
m_d	$4.67^{+0.48}_{-0.17}$ MeV [87]		
$\langle \bar{q}q \rangle$ (1 GeV)	$(-0.24 \pm 0.01)^3$ GeV ³ [88]		
m_0^2	(0.8 ± 0.1) GeV ² [88]		
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$	(0.012 ± 0.004) GeV ⁴ [89]		

contributions of the states under consideration over the higher states and continuum.

To fix the parameter β , we plot the functions in the QCD side in terms of this parameter and look for the regions that the results have weak dependence on the β . We set $\beta =$ $\tan \theta$ and vary $\cos \theta$ in the interval $[-1, 1]$ to explore the whole region. Figure [1,](#page-4-2) as an example, shows $\tilde{\Pi}^{\text{QCD}}_{\cancel{q}}$ as a function of $cos \theta$. From this figure and our numerical analyses we obtain the following working windows for cos θ, which are valid for all states at Λ_b and Λ_c channels:

$$
-1.0 < \cos \theta < -0.5 \quad \text{and} \quad 0.5 < \cos \theta < 1.0. \tag{21}
$$

From figure [1](#page-4-2), we see that the $\tilde{\Pi}^{\text{QCD}}_{\cancel{q}}$ has a relatively weak dependence on $\cos \theta$ in the above intervals.

The working intervals of Borel parameters, restricted by the convergence of OPE, the pole dominance requirements, and the stability of the results in response to the variation of these parameters, are presented in Table [II.](#page-4-3) For analyses, we take into account the ground-state $\frac{1}{1}$ first orbitally excited state $\frac{1}{1}$ first radially excited state $\frac{1}{1}$ continuum approach and use Eq. [\(17\)](#page-3-0) to move step by step as follows: First, we obtain the mass and current coupling constant for the ground state Λ ^o particles. To achieve these quantities we choose proper threshold parameters considering the ground-state $+$ continuum scheme and the notion that the threshold parameter is related to the energy of the next excited state. Considering that we choose the proper interval for the s_0 as also given in Table [II](#page-4-3). The masses and the current coupling constants obtained in this step are also given in Table [II](#page-4-3) and these are used as inputs in the second step. Second, we consider the ground state \pm first orbitally excited state $+$ continuum scheme, and with the same logic that is used for the determination of s_0 of the previous step, we determine a new s_0 working interval. The results obtained in this step are presented in Table [II,](#page-4-3) as well. And finally, we consider the radially excited 2S state with ground-state $+$ first orbitally excited state $+$ first radially excited state $+$ continuum approach and attain the proper threshold parameter for this approach. The results obtained for 2S states are also depicted in Table [II.](#page-4-3) The errors in the results arise from the errors of the input parameters and the uncertainties coming from the

TABLE II. The auxiliary parameters and the results of masses and current coupling constants.

Particle	State	M^2 (GeV ²)	s_0 (GeV ²)	Mass (MeV)	λ (GeV ³)
Λ_h	$\Lambda_b(\frac{1}{2}^+)(1S)$	$6.0 - 8.0$	$5.86^2 - 5.90^2$	5611.47 ± 27.47	0.042 ± 0.003
	$\Lambda_b(\frac{1}{2}^-)(1P)$	$6.0 - 8.0$	$5.92^2 - 5.96^2$	$5910.56 + 84.54$	0.020 ± 0.008
	$\Lambda_b(\frac{1}{2}^+)(2S)$	$6.0 - 8.0$	$6.18^2 - 6.22^2$	$6073.65 + 93.22$	0.051 ± 0.007
Λ_c	$\Lambda_c(\frac{1}{2}^+)(1S)$	$3.0 - 5.0$	$2.53^2 - 2.57^2$	$2282.42 + 28.38$	0.022 ± 0.001
	$\Lambda_c(\frac{1}{2}^-)(1P)$	$3.0 - 5.0$	$2.63^2 - 2.67^2$	2592.36 ± 53.01	0.014 ± 0.003
	$\Lambda_c(\frac{1}{2}^+)(2S)$	$3.0 - 5.0$	$2.73^2 - 2.77^2$	2765.52 ± 22.29	0.016 ± 0.004

FIG. 2. $m_{\Lambda_b}(2S)$ as a function of M^2 at average values of the $\cos \theta$ and s_0 .

FIG. 3. $m_{\Lambda_b}(2S)$ as a function of s_0 at average values of the $\cos \theta$ and M^2 .

determinations of the working intervals for the auxiliary parameters. We shall remark that the main source of the uncertainties belongs to the variations of the parameters β , s_0 and M^2 M^2 in their working windows. Figures 2 and [3](#page-5-1) show the dependence of, for instance, $m_{\Lambda_b}(2S)$ to M^2 and s_0 at average values of β /cos θ. The weak dependence of the mass on M^2 and s_0 appears as parts of uncertainties presented in Table [II](#page-4-3).

IV. CONCLUSION

Focusing on the recently observed state $\Lambda_b(6072)^0$, we studied the ground states 1S, first orbital 1P and first radial 2S excitations of the spin- $\frac{1}{2} \Lambda_b$ and Λ_c states. The experimentally observed values for the mass of $\Lambda_b(6072)^0$ state is $m_{\Lambda_b^{*0}} = 6072.3 \pm 2.9 \pm 0.6 \pm 0.2$ MeV with a width value $\Gamma = 72 \pm 11 \pm 2$ MeV [\[82\]](#page-7-5). In Ref. [82], it was underlined
that this result is consistent with the predictions of the quark that this result is consistent with the predictions of the quark model for $\Lambda_b(2S)$ state [\[10,13,16\]](#page-6-12). Motivated by this observation, we calculated the masses and current coupling constants for ground 1S, first orbitally excited 1P and first radially excited 2S states of Λ_b and Λ_c particles. For the analyses, we applied a powerful nonperturbative method, QCD sum rule with a suitable interpolating current formed considering the quark content and quantum numbers of the considered states. The results presented in Table [II](#page-4-3) for ground and first orbital excitations of Λ_b and Λ_c baryons are in good agreement with the present experimental findings given as: $m_{\Lambda_b^0}$ =5619.60±0.17MeV [\[87\],](#page-7-10) $m_{\Lambda_b^0}$ (5912)⁰ = $5912.20 \pm 0.13 \pm 0.17 \,\text{MeV}$ [\[87\]](#page-7-10), $m_{\Lambda_c^+}$
0.14 MeV [87] m = - - - 2502.25 + 0.2 $c_c^+ = 2286.46 \pm 28.5$ 0.14 MeV [\[87\]](#page-7-10), $m_{\Lambda_c(2595)^+} = 2592.25 \pm 0.28$ MeV [\[87\].](#page-7-10)

As for the main focus of this work, the mass obtained for $\Lambda_b (6072)^0$ as $m_{\Lambda_b(2S)} = 6073.65 \pm 93.22$ MeV is consistent with the experimental result $m_s = 6073.3 \pm 2.0 \pm 1.0$ tent with the experimental result, $m_{\Lambda_p^0} = 6072.3 \pm 2.9 \pm 0.61162$ M₂V₁921. The result is also applicated with the 0.6 ± 0.2 MeV [\[82\]](#page-7-5). The result is also consistent with the various theoretical predictions given for the radially excited various theoretical predictions given for the radially excited Λ_b state with $J^P = \frac{1}{2}^+$ as $m = 6045$ MeV [\[10\],](#page-6-12) $m = 6106$ MeV [16] $m = 6089$ MeV [13] $m = 6106$ MeV 6.107 GeV [\[16\],](#page-6-13) $m = 6089$ MeV [\[13\],](#page-6-14) $m = 6106$ MeV [\[15\]](#page-6-15), $m = 6153$ MeV [\[17\]](#page-6-16). In Ref. [\[19\]](#page-6-17) the mass for this particle is calculated using the hypercentral quark model with and without first order corrections to the confinement potential as $m = 6.026$ GeV and $m = 6.016$ GeV, respectively. The Ref. [\[90\]](#page-7-13) presented the mass of the particle as $m = 5982-6127$ MeV obtained from the chiral quark model using five different sets of model parameters. As is seen from these results, the mass obtained in this work is in good consistency with the present theoretical predictions within the errors.

The mass for the 2S Λ_c state is also obtained for completeness and its value is attained as $m_{\Lambda_c(2S)} =$ 2765.52 ± 22.29 MeV. This result is also consistent with the mass value for $\Delta (2765)^+$ given as m_{eff} (220). the mass value for $\Lambda_c(2765)^+$ given as $m_{\Lambda_c(2765)^+} =$ 2766.6 \pm 2.4 MeV [\[87\].](#page-7-10) This particle is presented in
PDG as Δ (2765)⁺ or Σ (2765)⁺ with unknown $I(I^P)$ – PDG as $\Lambda_c(2765)^+$ or $\Sigma(2765)^+$ with unknown $I(J^P)$ = $?$ (??) quantum numbers. However in Ref. [\[91\]](#page-7-14) its isospin was determined as zero and name for it was suggested to be $\Lambda_c(2765)^+$. In this work, we obtained the mass for the first radial excitation of the Λ_c state with $J^P = \frac{1}{2}^+$ in consistency with the mass of the $\Lambda_c(2765)^+$ state. Our prediction is also consistent with the theoretical works with the following predictions for 2S wave Λ_c state: $m = 2775$ MeV [\[10\]](#page-6-12), $m = 2772$ MeV [\[12\],](#page-6-18) $m = 2769$ MeV [\[13\],](#page-6-14) $m =$ 2769 MeV [\[26\],](#page-6-19) $m = 2.791$ GeV [\[16\],](#page-6-13) $m = 2772$ MeV [\[30\]](#page-6-20), $m = 2766$ MeV [\[42\],](#page-6-9) $m = 2.758$ GeV [\[20\]](#page-6-21), $m =$ 2857 MeV [\[17\]](#page-6-16), $m = 2785$ MeV [\[15\],](#page-6-15) $m = 2749$ MeV [\[92\]](#page-7-15) and $m = 2654 - 2825$ MeV [\[90\]](#page-7-13) obtained with five different sets of model parameters. These results are in agreement with that of present work within the errors.

A comparison of the result of this work with the present theoretical and experimental findings indicates that the particle $\Lambda_b(6072)^0$ is the first radial excitation of the Λ_b

baryon with the quantum numbers $J^P = \frac{1}{2}^+$. The consistency of the result for the first radial excitation of A, with tency of the result for the first radial excitation of Λ_c with $J^P = \frac{1}{2}^+$ with other theoretical results and the present
experimental value of Λ (2765)⁺ is also considerable experimental value of $\Lambda_c(2765)^+$ is also considerable. Our result indicates that it may be first radial excitation

of Λ_c state with quantum numbers $J^P = \frac{1}{2}^+$. Further studies
on these states, including their masses and decay properties. on these states, including their masses and decay properties, and comparison with the result of the present study, may provide more clarifications on the quantum numbers of these states.

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