T-dualizing de Sitter no-go scenarios

Pramod Shukla*

ICTP, Strada Costiera 11, Trieste 34151, Italy

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In the context of realizing de Sitter vacua and the slow-roll inflation, several no-go conditions have been found in the framework of type IIA (generalized) flux compactifications. In this article, using our recently proposed T-dual dictionary [P. Shukla, A dictionary for the type II non-geometric flux compactifications, arXiv:1909.07391], we translate various such type IIA no-go conditions that subsequently lead to some interesting de Sitter no-go scenarios in the presence of (non)geometric fluxes on the dual type IIB side. We also present the relevance of using $K3/\mathbb{T}^4$-fibered Calabi Yau threefolds in order to facilitate one particular class of the de Sitter no-go conditions. This analysis helps in refining certain corners of the vast nongeometric flux landscape for the hunt of de Sitter vacua.

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I. INTRODUCTION

Recent revival of the swampland conjecture [1,2] has boosted a huge amount of interest toward exploring the (non)existence of de Sitter vacua within a consistent theory of quantum gravity. The original idea of swampland has been proposed to state that de Sitter solutions must be absent in a consistent theory of quantum gravity [3]. This idea has recently been endorsed as a bound involving the scalar potential (V) and its derivatives given in the following manner:

$$\frac{|\nabla V|}{V} \geq \frac{c}{M_p}, \tag{1.1}$$

where the constant c is an order one quantity. This conjecture has been supported by several explicit computations in the context of attempts made for realizing classical de Sitter solutions and inflationary cosmology in the type II superstring flux compactifications [4–23]. Note the de Sitter maxima as well, and several counterexamples were known [7,13,24–26] or have been reported soon after the proposal was made [27–37] reflecting the need of refining the de Sitter swampland conjecture in Eq. (1.1). Subsequently a refined version of the conjecture has been proposed which states that at least one of the following two constraints should always hold [38]:

$$\frac{|\nabla V|}{V} \geq \frac{c}{M_p}, \qquad \min\left[\frac{\nabla_i \nabla_j V}{V}\right] \leq -\frac{c'}{M_p^2}, \tag{1.2}$$

where c and $c' > 0$ are order one constants. Note that these two parameters can be related to the usual inflationary parameters, namely the ϵ_V and η_V parameters, which are needed to be sufficiently small for having the slow-roll inflation (e.g., see [6,39,40]),

$$\epsilon_V \geq \frac{1}{2} c^2, \qquad |\eta_V| \leq c'. \tag{1.3}$$

Therefore, it is rather quite obvious that the conjecture (1.2) poses an obstruction not only to realizing de Sitter vacua but also to realizing slow-roll inflationary scenarios, which demands $\epsilon_V \ll 1$ and $|\eta_V| \ll 1$. However, this definition of the ϵ_V parameter follows from a more general definition given in terms of the Hubble parameter as $\epsilon_H = -\dot{H}/H^2$, which only needs to satisfy $\epsilon_H < 1$ for having an accelerated universe. This leads to a possible window circumventing the conjecture in the multifield inflation with turning trajectories [40,41]. Moreover, given the fact that no universal theoretical quantification of the c and c' parameters being available (though some experimental estimates have been reported in [42]), the order one statement may still keep some window open [43,44].

The question of realizing de Sitters is twofold; the first is about its existence and the second is about the stability, and a plethora of interesting models have been proposed on these lines [4,5,8–10,12,13,15,16,20,26,45]. The swampland conjecture [3] has also been found to be in close connections with the allowed inflaton field range in a trustworthy effective field description as it has been argued that a massive tower of states can get excited after a certain

*pramodmaths@gmail.com

limit to the inflaton excursions [45–59]. The recent surge of developments following the recent swampland proposal can be found in [27–37,41–44,60–69] with an extensive review on the status in [70].

Contrary to the (minimal) de Sitter no-go scenarios, there have been several proposals for realizing stable de Sitter vacua in the context of string model building [71–83]; see [84,85] also for the F-theoretic initiatives taken in this regard. In fact, realizing de Sitter solutions and possible obstructions on the way of doing it have been the center of attraction for decades.[1] Moreover, some interesting models realizing de Sitter vacua in the framework of nongeometric flux compactifications have also been proposed [11,14,87–96]. However, the issues related to fluxes being integral and whether they satisfy all the NS-NS Bianchi identities or not can still be considered to be among some open questions in this regard. In fact, it has been observed that the Bianchi identities are not fully known in the beyond toroidal examples as there have been some inconsistencies observed in two ways of deriving the identities [97–101].

Motivations, goals, and a brief summary of the results

Several de Sitter no-go theorems on the type IIA side have been well known for a decade or so [5,6,8,15], which have also been studied for the type II nongeometric compactifications using a simple isotropic torus case in [11,87]. With a goal of extending the nongeometric flux phenomenology beyond the toroidal cases, the study of generic four-dimensional type II scalar potentials and their ten-dimensional origin have been performed in a series of papers [102–107]. Taking this program one step further, in a companion paper [108], we have presented a one-to-one T-dual mapping of the two type II effective scalar potentials, along with the flux constraints arising from the NS-NS Bianchi identities and the tadpole cancellation conditions, which are also in one-to-one correspondence under T-duality. The main motivations and the goals aimed for in this article can be presented under the following points:

(i) Our so-called “cohomology” or “symplectic” formulation of the scalar potential presented in [108] opens up the window to study the nongeometric models beyond the toroidal constructions, and also enables one to explicitly translate any useful findings of one setup into its T-dual picture. On these lines, we plan to T-dualize the several de Sitter no-go scenarios realized in some purely geometric type IIA frameworks [5,6,8,15]. This helps us in delving into their type IIB counterparts, which turn out to be nongeometric de Sitter no-go frameworks, and those have not been known before. The utility of our approach can be underlined by the fact that although the type IIA no-go scenarios have been known for more than a decade, there have been no de Sitter no-go proposals in the generic nongeometric type IIB framework.

(ii) In our analysis, we show the relevance of considering the complex structure moduli in deriving the T-dual type IIB no-go conditions. Note that all the type IIA no-go results in [5,6,8,15], which we T-dualize, are realized using the extremization conditions only in the “volume/dilaton” plane, and without taking into account the complex structure moduli sector. This illustrates that any claim of evading the no-go originated from the volume/dilaton analysis should be checked by including all the remaining moduli.

(iii) On the lines of classifying type IIA and type IIB models based on their (non)geometric nature via turning on a certain set of fluxes at a time, we present an interesting recipe that corresponds to considering what we call some “special solutions” of the NS-NS Bianchi identities. These solutions are such that they lead to a purely geometric framework as a T-dual of a nongeometric setup on either of the respective IIA or IIB sides. In particular, the type IIA nongeometric model with fluxes allowed as in the special solution of the Bianchi identities is T-dual to a purely geometric type IIB model, which has been known to have de Sitter no-go scenario [16,29], and subsequently our analysis concludes that the corresponding T-dual type IIA model despite having the nongeometric fluxes (still allowed by the special solution) cannot evade the no-go result.

This shows that our approach will be useful for playing with constructing models in search of the de Sitter no-go or against those no-go arguments, given that the most generic nongeometric setup could still be expected to evade the no-go, though there are several specifics to be checked in a given model before arriving at any final conclusion.

(iv) In addition to finding the (non)geometric flux regime or the types of fluxes needed to evade a certain kind of de Sitter no-go result, we also find that if there are some specific geometries involved, such as the $K3/\mathbb{T}^4$-fibered complex threefold, then there can be a restoration of the no-go results despite the inclusion of those fluxes which apparently could be anticipated to evade the respective no-go results. We illustrate this observation for explicit type IIA and IIB toroidal nongeometric setups.

So, our results can be considered as providing some systematics about constructing de Sitter no-go scenarios along with the recipes to find the possibilities of evading them, and at the same time, in looking for some specific

[1]For an updated recent review on realizing de Sitter solutions in string theoretic models along with the status on quintessence, we refer the readers to [86].

geometries of the moduli space which could again restore the de Sitter no-go result, despite the presence of those fluxes that are naively anticipated to evade the no-go. Thus, our analysis presents a playing ground for constructing/evading the de Sitter no-go scenarios.

The article is organized as follows: In Sec. II we present some interesting solutions of the NS-NS Bianchi identities which we further use for deriving the no-go conditions in the upcoming sections. Section III presents a type IIA no-go with standard fluxes and its T-dual type IIB counterpart, which includes nongeometric fluxes as well. In Sec. IV first we rederive the fact that one can evade the type IIA no-go-1 with geometric fluxes and Romans mass, and then we T-dualize it to study the type IIB counterpart. Section V presents the relevance of the $K3/\mathbb{T}^4$-fibered Calabi Yau threefolds, which help in finding a new class of de Sitter no-go scenarios in both the type II theories. Finally we conclude with the results and observations in Sec. VI.

Note: Let us mention at the outset that we will follow the T-dual dictionary from a companion paper [108], which includes the necessary ingredients of the generic formulation of the four-dimensional scalar potentials for the type IIA and the type IIB supergravities with (non)geometric fluxes, and this dictionary is placed in the Appendix. For the current interests in this article, we will directly utilize the scalar potential for the possible applications in the lights of de Sitter and inflationary no-go scenarios. Though we attempt to keep the article self-contained, we encourage the interested readers to follow the other relevant details if necessary, e.g., on the superpotential, D-terms, directly from [108].

II. SOLUTIONS OF BIANCHI IDENTITIES

In this section we aim to present some interesting solutions of the Bianchi identities satisfied by the various fluxes of the type IIA and IIB theories. The full list of allowed NS-NS fluxes, namely $\{\mathrm{H}, w, \mathrm{Q}, \mathrm{R}\}$ in type IIA and $\{H, \omega, Q, R\}$ in type IIB along with the RR fluxes, namely $\{\mathrm{F}_0 \equiv m_0, \mathrm{F}_2 \equiv m^a, \mathrm{F}_4 \equiv e_a, \mathrm{F}_6 \equiv e_0\}$ in type IIA and $\{F_0, F_i, F^i, F^0\}$ in type IIB, and their T-duality relations are collected in Table I, which has been adopted from the Table VIII of the T-dual dictionary given in the Appendix. Here the flux as well as various moduli are counted via the Hodge numbers as $\alpha \in \{1, 2, \ldots, h^{1,1}_+\}$, $a \in \{1, 2, \ldots, h^{1,1}_-\}$ on both sides, while $\Lambda \in \{0, 1, 2, \ldots, h^{2,1}_-\}$ and $J, K \in \{1, 2, \ldots, h^{2,1}_+\}$ on the type IIB side, whereas the splitting of the complex structure indices on the type IIA side is such that the k and λ sum to $h^{2,1}$. The various fluxes appearing in the four-dimensional type IIA supergravity are constrained by the following five classes of NS-NS Bianchi identities [108]:

$$\begin{aligned}
(\mathbf{I}) \quad & \mathrm{H}^\lambda \hat{w}_{\alpha\lambda} = \mathrm{H}_{\hat{k}} \hat{w}_\alpha{}^{\hat{k}}, \\
(\mathbf{II}) \quad & \mathrm{H}^\lambda \hat{\mathrm{Q}}^\alpha{}_\lambda = \mathrm{H}_{\hat{k}} \hat{\mathrm{Q}}^{\alpha\hat{k}}, \qquad w_a{}^\lambda \hat{w}_{\alpha\lambda} = w_{a\hat{k}} \hat{w}_\alpha{}^{\hat{k}}, \\
(\mathbf{III}) \quad & \hat{\mathrm{Q}}^\alpha{}_\lambda w_a{}^\lambda = w_{a\hat{k}} \hat{\mathrm{Q}}^{\alpha\hat{k}}, \qquad \mathrm{Q}^a{}_{\hat{k}} \hat{w}_\alpha{}^{\hat{k}} = \mathrm{Q}^{a\lambda} \hat{w}_{\alpha\lambda}, \\
& \hat{w}_{\alpha\lambda} \hat{\mathrm{Q}}^{\alpha\hat{k}} = \hat{\mathrm{Q}}^\alpha{}_\lambda \hat{w}_\alpha{}^{\hat{k}}, \qquad \hat{w}_{\alpha\lambda} \hat{\mathrm{Q}}^\alpha{}_\rho = \hat{\mathrm{Q}}^\alpha{}_\lambda \hat{w}_{\alpha\rho}, \qquad \hat{w}_\alpha{}^{\hat{k}} \hat{\mathrm{Q}}^{\alpha\hat{k}'} = \hat{\mathrm{Q}}^{\alpha\hat{k}} \hat{w}_\alpha{}^{\hat{k}'}, \\
& \mathrm{R}^\lambda \mathrm{H}_{\hat{k}} - \mathrm{H}^\lambda \mathrm{R}_{\hat{k}} + w_a{}^\lambda \mathrm{Q}^a{}_{\hat{k}} - \mathrm{Q}^{a\lambda} w_{a\hat{k}} = 0, \\
& \mathrm{H}_{[\hat{k}} \mathrm{R}_{\hat{k}']} + \mathrm{Q}^a{}_{[\hat{k}} w_{a\hat{k}']} = 0, \qquad \mathrm{H}^{[\lambda} \mathrm{R}^{\rho]} + \mathrm{Q}^{a[\lambda} w_a{}^{\rho]} = 0, \\
(\mathbf{IV}) \quad & \mathrm{R}^\lambda \hat{w}_{\alpha\lambda} = \mathrm{R}_{\hat{k}} \hat{w}_\alpha{}^{\hat{k}}, \qquad \mathrm{Q}^{a\lambda} \hat{\mathrm{Q}}^\alpha{}_\lambda = \mathrm{Q}^a{}_{\hat{k}} \hat{\mathrm{Q}}^{\alpha\hat{k}}, \\
(\mathbf{V}) \quad & \mathrm{R}^\lambda \hat{\mathrm{Q}}^\alpha{}_\lambda = \mathrm{R}_{\hat{k}} \hat{\mathrm{Q}}^{\alpha\hat{k}}.
\end{aligned} \tag{2.1}$$

Similarly on the type IIB side, we have the following five classes of Bianchi identities [98]:

$$\begin{aligned}
(\mathbf{I}) \quad & H_\Lambda \omega_a{}^\Lambda = H^\Lambda \omega_{\Lambda a}, \\
(\mathbf{II}) \quad & H^\Lambda \hat{Q}_\Lambda{}^\alpha = H_\Lambda \hat{Q}^{\alpha\Lambda}, \qquad \omega_a{}^\Lambda \omega_{b\Lambda} = \omega_b{}^\Lambda \omega_{a\Lambda}, \qquad \hat{\omega}_\alpha{}^K \hat{\omega}_{\beta K} = \hat{\omega}_\beta{}^K \hat{\omega}_{\alpha K}, \\
(\mathbf{III}) \quad & \omega_{a\Lambda} \hat{Q}^{\alpha\Lambda} = \omega_a{}^\Lambda \hat{Q}^\alpha{}_\Lambda, \qquad Q^{aK} \hat{\omega}_{\alpha K} = Q^a{}_K \hat{\omega}^K_\alpha, \\
& H_\Lambda R_K + \omega_{a\Lambda} Q^a{}_K + \hat{Q}^\alpha{}_\Lambda \hat{\omega}_{\alpha K} = 0, \qquad H^\Lambda R_K + \omega_a{}^\Lambda Q^a{}_K + \hat{Q}^{\alpha\Lambda} \hat{\omega}_{\alpha K} = 0, \\
& H_\Lambda R^K + \omega_{a\Lambda} Q^{aK} + \hat{Q}^\alpha{}_\Lambda \hat{\omega}_\alpha{}^K = 0, \qquad H^\Lambda R^K + \omega_a{}^\Lambda Q^{aK} + \hat{Q}^{\alpha\Lambda} \hat{\omega}_\alpha{}^K = 0, \\
(\mathbf{IV}) \quad & R^K \hat{\omega}_{\alpha K} = R_K \hat{\omega}_\alpha{}^K, \qquad \hat{Q}^{\alpha\Lambda} \hat{Q}^\beta{}_\Lambda = \hat{Q}^{\beta\Lambda} \hat{Q}^\alpha{}_\Lambda, \qquad Q^{aK} Q^b{}_K = Q^{bK} Q^a{}_K, \\
(\mathbf{V}) \quad & R_K Q^{aK} = R^K Q^a{}_K.
\end{aligned} \tag{2.2}$$

First we argue how choosing a certain type of involution can project out many flux components and hence can indeed simplify the generic set of identities, for which finding solutions becomes rather easier. Moreover, we present another set of

TABLE I. T-duality transformations among the various fluxes and complex variable.

	Type IIA with $D6/O6$	Type IIB with $D3/O3$ and $D7/O7$
F-term fluxes	H_0, H_k, H^λ, w_{a0}, w_{ak}, $w_a{}^\lambda$, $\mathrm{Q}^a{}_0$, $\mathrm{Q}^a{}_k$, $\mathrm{Q}^{a\lambda}$, R_0, R_k, R^λ, e_0, e_a, m^a, m_0.	H_0, ω_{a0}, $\hat{Q}^\alpha{}_0$, H_i, ω_{ai}, $\hat{Q}^\alpha{}_i$, H^i, $\omega_a{}^i$, $\hat{Q}^{\alpha i}$, $-H^0$, $-\omega_a{}^0$, $-\hat{Q}^{\alpha 0}$, F_0, F_i, F^i, $-F^0$.
D-term fluxes	$\hat{w}_\alpha{}^0$, $\hat{w}_\alpha{}^k$, $\hat{w}_{\alpha\lambda}$, $\hat{\mathrm{Q}}^{\alpha 0}$, $\hat{\mathrm{Q}}^{\alpha k}$, $\hat{\mathrm{Q}}^\alpha{}_\lambda$.	$-R_K$, $-Q^a{}_K$, $\hat{\omega}_{\alpha K}$, $-R^K$, $-Q^{aK}$, $\hat{\omega}_\alpha{}^K$.
Complex moduli	N^0, N^k, U_λ, T^a.	S, G^a, T_α, U^i.

solutions that we call a special solution for both the type IIA and the type IIB theories. They are very peculiar in many aspects as we will elaborate later on.

A. Simple solutions

The set of type IIA Bianchi identities given in Eq. (2.1) suggests that if one chooses the antiholomorphic involution such that the even (1,1)-cohomology sector is trivial, which is very often the case one considers for a simple phenomenological model [99,107,109,110], then only the following Bianchi identities remain nontrivial:

$$\mathrm{R}^\lambda \mathrm{H}_{\hat{k}} - \mathrm{H}^\lambda \mathrm{R}_{\hat{k}} + w_a{}^\lambda \mathrm{Q}^a{}_{\hat{k}} - \mathrm{Q}^{a\lambda} w_{a\hat{k}} = 0,$$
$$\mathrm{H}_{[\hat{k}} \mathrm{R}_{\hat{k}']} + \mathrm{Q}^a{}_{[\hat{k}} w_{a\hat{k}']} = 0, \qquad \mathrm{H}^{[\lambda} \mathrm{R}^{\rho]} + \mathrm{Q}^{a[\lambda} w_a{}^{\rho]} = 0. \tag{2.3}$$

In such a situation, there will be no D-term contributions generated to the scalar potential as all the fluxes relevant for D-terms have $\alpha \in h^{1,1}_+$ indices, and hence are projected out.

For the T-dual of the above type IIA setting, one needs to look at the set of type IIB Bianchi identities given in Eq. (2.2), which suggests that if one chooses the holomorphic involution such that the even (2,1)-cohomology sector is trivial, then only the following Bianchi identities remain nontrivial,

$$H_\Lambda \omega_a{}^\Lambda = H^\Lambda \omega_{\Lambda a}, \qquad H^\Lambda \hat{Q}_\Lambda{}^\alpha = H_\Lambda \hat{Q}^{\alpha\Lambda},$$
$$\omega_a{}^\Lambda \omega_{b\Lambda} = \omega_b{}^\Lambda \omega_{a\Lambda},$$
$$\omega_{a\Lambda} \hat{Q}^{\alpha\Lambda} = \omega_a{}^\Lambda \hat{Q}^\alpha{}_\Lambda, \qquad \hat{Q}^{\alpha\Lambda} \hat{Q}^\beta{}_\Lambda = \hat{Q}^{\beta\Lambda} \hat{Q}^\alpha{}_\Lambda, \tag{2.4}$$

which are in a one-to-one correspondence with those in Eq. (2.3). In such a situation, there will be no D-term generated as all the fluxes with $\{J, K\} \in h^{2,1}_+$ indices are projected out. Moreover, on top of this if the holomorphic involution is chosen to result in a trivial odd (1,1)-cohomology, which corresponds to a situation with the absence of odd moduli G^a on the type IIB side and is also very often studied as a case for being simplistic in nature (e.g., see [100,110–112]), then there are only two Bianchi identities to worry about, and they are given as

$$H^\Lambda \hat{Q}_\Lambda{}^\alpha = H_\Lambda \hat{Q}^{\alpha\Lambda}, \qquad \hat{Q}^{\alpha\Lambda} \hat{Q}^\beta{}_\Lambda = \hat{Q}^{\beta\Lambda} \hat{Q}^\alpha{}_\Lambda. \tag{2.5}$$

This further simplification on type IIB side corresponds to the absence of N^k moduli on the type IIA side, and so is the case for the corresponding fluxes that couple to N^k through the superpotential. This leads to two Bianchi identities on the type IIA side which happen to be T-dual to those presented in Eq. (2.5) and are given as

$$\mathrm{R}^\lambda \mathrm{H}_0 - \mathrm{H}^\lambda \mathrm{R}_0 + w_a{}^\lambda \mathrm{Q}^a{}_0 - \mathrm{Q}^{a\lambda} w_{a0} = 0,$$
$$\mathrm{H}^{[\lambda} \mathrm{R}^{\rho]} + \mathrm{Q}^{a[\lambda} w_a{}^{\rho]} = 0. \tag{2.6}$$

These "simple" solutions of the Bianchi identities based on some specific choice of orientifold involution leads to some interesting scenarios in both type IIA and type IIB theories.

B. IIA with special solution ≡ IIB with geometric-flux ≡ ∃ dS no-go

From the set of type IIA Bianchi identities given in Eq. (2.1), one can observe that several Bianchi identities appear in the form of orthogonal symplectic vectors and therefore half of the flux components can be set to zero by performing appropriate symplectic rotations.[2] The same is equivalent to setting some fluxes, say those with upper $h^{2,1}$ indices, to zero as we present below,

$$\mathrm{H}^\lambda = 0, \qquad \hat{w}_\alpha{}^0 = \hat{w}_\alpha{}^k = w_a{}^\lambda = 0,$$
$$\mathrm{R}^\lambda = 0, \qquad \hat{\mathrm{Q}}^{\alpha 0} = \hat{\mathrm{Q}}^{\alpha k} = \mathrm{Q}^{a\lambda} = 0. \tag{2.7}$$

This is what we call the special solution. Now, using these special flux choices in Eq. (2.7) results in the fact that all the type IIA Bianchi identities except the following three are trivially satisfied:

$$\mathrm{H}_{[0} \mathrm{R}_{k]} + \mathrm{Q}^a{}_{[0} w_{ak]} = 0,$$
$$\mathrm{H}_{[k} \mathrm{R}_{k']} + \mathrm{Q}^a{}_{[k} w_{ak']} = 0,$$
$$\hat{w}_{\alpha\lambda} \hat{\mathrm{Q}}^\alpha{}_\rho = \hat{\mathrm{Q}}^\alpha{}_\lambda \hat{w}_{\alpha\rho}. \tag{2.8}$$

This makes a huge simplification in the generic complicated flux constraints. Now considering the T-dual of the type IIA special flux choice, as given in Eq. (2.7), turns out to be equivalent to switching off the following flux components on the type IIB side:

$$\hat{Q}^\alpha{}_0 = \hat{Q}^\alpha{}_i = Q^a{}_K = 0, \qquad R_K = 0,$$
$$Q^{\alpha i} = \hat{Q}^{\alpha 0} = Q^{aK} = 0, \qquad R^K = 0, \tag{2.9}$$

[2] See [97,98,113] also, for more arguments in this regard relating to dyonic Black hole charges.

which means setting all the nongeometric (Q as well as R) fluxes to zero on the type IIB side. Moreover, using the T-dual flux choice on the type IIB side as given in Eq. (2.9), one finds that the set of Bianchi identities on the type IIB side are reduced into the following three constraints:

$$H_\Lambda \omega_a{}^\Lambda = H^\Lambda \omega_{\Lambda a}, \qquad \omega_a{}^\Lambda \omega_{b\Lambda} = \omega_b{}^\Lambda \omega_{a\Lambda},$$
$$\hat{\omega}_\alpha{}^K \hat{\omega}_{\beta K} = \hat{\omega}_\beta{}^K \hat{\omega}_{\alpha K}, \tag{2.10}$$

which is very much expected as there are no nonzero Q and R flux components present in the current setting. As a side remark, let us point out that if the involutions are considered as per the choices earlier explained as simple solutions, i.e., those without D-terms, then there remain just two identities on the two sides,

$$\textbf{IIA}: \quad \mathrm{H}_{[0}\mathrm{R}_{k]} + \mathrm{Q}^a{}_{[0}w_{ak]} = 0, \qquad \mathrm{H}_{[k}\mathrm{R}_{k']} + \mathrm{Q}^a{}_{[k}w_{ak']} = 0,$$
$$\textbf{IIB}: \quad H_\Lambda \omega_a{}^\Lambda = H^\Lambda \omega_{\Lambda a}, \qquad \omega_a{}^\Lambda \omega_{b\Lambda} = \omega_b{}^\Lambda \omega_{a\Lambda}, \tag{2.11}$$

and even the above ones are absent if one sets $a = 0$, i.e., no G^a moduli in IIB and equivalently no N^k moduli in IIA. Thus with some orientifold setting one can have special solutions in which all the Bianchi identities are trivial. Note that all these identities are well in line with the T-duality transformations inherited from their generic structure before taking any simplification.

A no-go condition for de Sitter and slow-roll inflation

As we have seen, the type IIA nongeometric setup with the special solution leads to a type IIB setup without any nongeometric flux. Now, following from Table XI of the dictionary in Appendix, the type IIB scalar potential can be expressed as a sum of the following pieces:

$$V_{\rm IIB}^{\rm RR} = \frac{e^{4\phi}}{4\mathcal{V}^2\mathcal{U}}[f_0^2 + \mathcal{U} f^i \mathcal{G}_{ij} f^j + \mathcal{U} f_i \mathcal{G}^{ij} f_j + \mathcal{U}^2 (f^0)^2],$$
$$V_{\rm IIB}^{\rm NS1} = \frac{e^{2\phi}}{4\mathcal{V}^2\mathcal{U}}[h_0^2 + \mathcal{U} h^i \mathcal{G}_{ij} h^j + \mathcal{U} h_i \mathcal{G}^{ij} h_j + \mathcal{U}^2 (h^0)^2],$$
$$V_{\rm IIB}^{\rm NS2} = \frac{e^{2\phi}}{4\mathcal{V}^2\mathcal{U}}\Bigg[\mathcal{V}\mathcal{G}^{ab}\Bigg(h_{a0}h_{b0} + \frac{l_i l_j}{4} h_a{}^i h_b{}^j + h_{ai}h_{bj}u^i u^j + \mathcal{U}^2 h_a{}^0 h_b{}^0 - \frac{l_i}{2} h_a{}^i h_{b0} - \frac{l_i}{2} h_{a0} h_b{}^i - \mathcal{U} u^i h_a{}^0 h_{bi} - \mathcal{U} u^i h_b{}^0 h_{ai}\Bigg)\Bigg],$$
$$V_{\rm IIB}^{\rm loc} = \frac{e^{3\phi}}{2\mathcal{V}^2}[f^0 h_0 - f^i h_i + f_i h^i - f_0 h^0],$$
$$V_{\rm IIB}^{D} = \frac{e^{2\phi}}{4\mathcal{V}^2}[t^\alpha t^\beta (\hat{h}_{\alpha J}\mathcal{G}^{JK}\hat{h}_{\beta K} + \hat{h}_\alpha{}^J \mathcal{G}_{JK} \hat{h}_\beta{}^K)], \tag{2.12}$$

where $f_0, f_i, f^i, f^0, h_0, h_i, h^i, h^0, h_{a0}, h_{ai}, h_a{}^0, h_a{}^i, \hat{h}_{\alpha K}$, and $\hat{h}_\alpha{}^K$ are the axionic flux orbits as defined in Table X. However, as they do not depend on any of the saxions, it is not relevant to give their explicit lengthy details. Also note that in this orientifold we have the following axionic flux orbits of Table X being identically zero on the type IIB side:

$$h^\alpha{}_0 = h^\alpha{}_i = h^{\alpha i} = h^{\alpha 0} = 0, \qquad h_K{}^0 = h^{K0} = 0. \tag{2.13}$$

For studying the scalar potential in Eq. (2.12), let us extract the volume factor by introducing a new modulus ρ via defining the two-cycle volume moduli as $t^\alpha = \rho\gamma^\alpha$ where γ^α is the angular Kähler moduli satisfying the constraint $\ell_{\alpha\beta\gamma}\gamma^\alpha\gamma^\beta\gamma^\gamma = 6$. This leads to the overall volume being given as $\mathcal{V} = \rho^3$ and the volume dependent moduli space metric being simplified as

$$\mathcal{G}^{ab} = -\hat{\ell}^{ab} = -\frac{1}{\rho}(\hat{\ell}_{\alpha ab}\gamma^\alpha)^{-1}. \tag{2.14}$$

Also note that the moduli space metric $\mathcal{G}^{JK}$ and its inverse $\mathcal{G}_{JK}$ are independent of any of the volume moduli, and in particular on the ρ modulus as well. Subsequently the scalar potential can be expressed as

$$V = V_1 + V_2 + V_3 + V_4, \tag{2.15}$$

where defining a new variable $\tau = e^{-\phi}\sqrt{\mathcal{V}} = e^{-\phi}\rho^{3/2}$, the above four pieces are given as

$$V_1 = \frac{A_1}{\tau^4}, \qquad V_2 = \frac{A_2}{\tau^2\rho^3}, \qquad V_3 = \frac{A_3}{\tau^2\rho}, \qquad V_4 = \frac{A_4}{\tau^3\rho^{3/2}}. \tag{2.16}$$

Here A_i's depend on the complex structure moduli and the angular Kähler moduli but not on any of the τ and ρ moduli. In addition, one has $A_1 \geq 0$, $A_2 \geq 0$; however, signs of A_3 and A_4 are not fixed. Also note that we have combined the two pieces $V_{\rm IIB}^{\rm NS2}$ and $V_{\rm IIB}^{D}$ as they have the same scaling for the ρ and τ moduli. This leads to the following relation:

$$-3\tau\partial_\tau V - \rho\partial_\rho V = 12V_1 + 9V_2 + 7V_3 + \frac{21}{2}V_4. \tag{2.17}$$

This apparently shows that the necessary condition for the de Sitter no-go scenario, which one usually gets in the (τ, ρ) plane, is evaded. But after checking trace and determinants of the Hessian in the (τ, ρ) plane, one finds that the determinant of the Hessian evaluated at the extremum is never positive, hence confirming a no-go case due to the presence of tachyons. Such a type IIB setup with $D3/D7$ and $O3/O7$ having F_3, H_3 and the geometric flux have also been studied in [16,29], where it was concluded that no stable de Sitter vacua can be realized in this type IIB setting. Thus from our T-duality rules, we conclude the following de Sitter no-go condition on the dual type IIA side:

> ***Type IIA no-go theorem**: In the framework of nongeometric type IIA orientifold compactification with $O6$ planes, one cannot have a de Sitter solution by merely considering the RR flux F_0, F_2, F_4, F_6 along with the special solutions of the NS-NS Bianchi identities.*

Note that given the fact that there are certain nongeometric flux components present in the dual type IIA side despite corresponding to the special solutions of the Bianchi identities, this de Sitter no-go condition would not have been possible to guess *a priori* the explicit computations are done, but from the type IIB side it is not hard to invoke.

C. IIB with special solution $\equiv$ IIA with geometric-flux $\equiv\nexists$ dS no-go

Similar to the type IIA case, one can observe from Eq. (2.2) that many of the type IIB Bianchi identities also appear in the form of orthogonal symplectic vectors, and therefore half of the flux components can be rotated away, as presented below:

$$H^0 = 0 = H^i, \qquad \omega_a{}^0 = 0 = \omega_a{}^i, \qquad \hat{Q}^{\alpha 0} = 0 = \hat{Q}^{\alpha i},$$
$$\hat{\omega}_\alpha{}^K = 0, \qquad Q^{aK} = 0, \qquad R^K = 0. \tag{2.18}$$

Now, one can observe that using the special flux choice in Eq. (2.18) results in the fact that all the type IIB Bianchi identities except the following two are trivially satisfied:

$$H_0 R_K + \omega_{a0} Q^a{}_K + \hat{Q}^\alpha{}_0 \hat{\omega}_{\alpha K} = 0,$$
$$H_i R_K + \omega_{ai} Q^a{}_K + \hat{Q}^\alpha{}_i \hat{\omega}_{\alpha K} = 0. \tag{2.19}$$

Moreover, the type IIB special solution as given in Eq. (2.18) is equivalent to switching-off the following T-dual fluxes on the type IIA side:

$$\mathrm{Q}^a{}_0 = \mathrm{Q}^a{}_k = \mathrm{Q}^{a\lambda} = 0, \qquad \hat{\mathrm{Q}}^{\alpha 0} = \hat{\mathrm{Q}}^{\alpha k} = \hat{\mathrm{Q}}^\alpha{}_\lambda = 0,$$
$$\mathrm{R}_0 = \mathrm{R}_k = \mathrm{R}^\lambda = 0. \tag{2.20}$$

This immediately implies that type IIB special solutions correspond to setting all the nongeometric fluxes to zero on the type IIA side. Further, using the T-duality on the type IIB side, the two constraints given in Eq. (2.19) translates into the following two constraints on the type IIA side:

$$\mathrm{H}^\lambda \hat{w}_{\alpha\lambda} = \mathrm{H}_{\hat{k}} \hat{w}_\alpha{}^{\hat{k}}, \qquad w_a{}^\lambda \hat{w}_{\alpha\lambda} = w_{a\hat{k}} \hat{w}_\alpha{}^{\hat{k}}, \tag{2.21}$$

which is very much expected as there are no nonzero Q- and R-flux components present in this setting. As a side remark, one can observe that for a trivial even (2,1) cohomology on the type IIB side, the special solution is sufficient to satisfy all the flux constraints as the constraints in Eq. (2.19) get trivial. On the T-dual type IIA side, this would mean to have the even-(1,1) cohomology trivial and so trivially satisfying Eq. (2.21). A summary of the results of this section has been presented in Table II.

III. NO-GO-1

In this section we present the de Sitter no-go scenario realized in the context of type IIA flux compactification with the inclusion of the NS-NS H_3 flux, and the standard RR fluxes, namely the F_0, F_2, F_4, F_6 flux [5]. First we revisit the ingredients of the no-go condition and then we will T-dualize the same to investigate the no-go condition in the type IIB theory.

A. Type IIA with RR flux and H_3 flux

In the absence of any geometric and nongeometric fluxes in the type IIA flux compactifications, the generic four-dimensional scalar potential presented in Table XI simplifies to a form given as

TABLE II. Possible nonzero fluxes in the special solutions of Bianchi identities.

Scenario	$\exists$ no-go	Type IIA with $D6/O6$	Type IIB with $D3/O3$ and $D7/O7$
Type IIA with special solutions	Yes	H_0, w_{a0}, $\mathrm{Q}^a{}_0$, R_0, H_k, w_{ak}, $\mathrm{Q}^a{}_k$, R_k, e_0, e_a, m^a, m_0. $\hat{w}_{\alpha\lambda}$, $\hat{\mathrm{Q}}^\alpha{}_\lambda$.	H_0, H_i, H^i, $-H^0$, ω_{a0}, ω_{ai}, $\omega_a{}^i$, $-\omega_a{}^0$, F_0, F_i, F^i, $-F^0$. $\hat{\omega}_{\alpha K}$, $\hat{\omega}_\alpha{}^K$. (Type IIB with geometric flux)
Type IIB with special solution	No	H_0, H_k, H^λ, w_{a0}, w_{ak}, $w_a{}^\lambda$, e_0, e_a, m^a, m_0. $\hat{w}_\alpha{}^0$, $\hat{w}_\alpha{}^k$, $\hat{w}_{\alpha\lambda}$. (Type IIA with geometric flux)	H_0, ω_{a0}, $\hat{Q}^\alpha{}_0$, H_i, ω_{ai}, $\hat{Q}^\alpha{}_i$, F_0, F_i, F^i, $-F^0$. $-R_K$, $-Q^a{}_K$, $\hat{\omega}_{\alpha K}$.

$$V_{\rm IIA} = \frac{e^{4D}}{4\mathcal{V}}\left[{\rm f}_0^2 + \mathcal{V}{\rm f}^a\tilde{\mathcal{G}}_{ab}{\rm f}^b + \mathcal{V}{\rm f}_a\tilde{\mathcal{G}}^{ab}{\rm f}_b + \mathcal{V}^2({\rm f}^0)^2\right] + \frac{e^{2D}}{4\mathcal{V}}\left[\frac{{\rm h}_0^2}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}{\rm h}_{i0}{\rm h}_{j0} + \tilde{\mathcal{G}}_{\lambda\rho}{\rm h}^\lambda{}_0{\rm h}^\rho{}_0\right] + \frac{e^{3D}}{2\sqrt{\mathcal{U}}}\left[{\rm f}^0{\rm h}_0 - \frac{k_\lambda}{2}{\rm f}^0{\rm h}^\lambda{}_0\right], \tag{3.1}$$

where the various "axionic flux orbits" defined in Table IX are simplified to the following form:

$$\begin{aligned}
{\rm f}_0 &= e_0 + {\rm b}^a e_a + \frac{1}{2}\kappa_{abc}{\rm b}^a{\rm b}^b m^c + \frac{1}{6}\kappa_{abc}{\rm b}^a{\rm b}^b{\rm b}^c m_0 \\
&\quad - \xi^0{\rm H}_0 - \xi^k{\rm H}_k - \xi_\lambda{\rm H}^\lambda, \\
{\rm f}_a &= e_a + \kappa_{abc}{\rm b}^b m^c + \frac{1}{2}\kappa_{abc}{\rm b}^b{\rm b}^c m_0, \\
{\rm f}^a &= m^a + m_0{\rm b}^a, \qquad {\rm f}^0 = m_0, \\
{\rm h}_0 &= {\rm H}_0 + {\rm z}^k{\rm H}_k + \frac{1}{2}\hat{k}_{\lambda mn}{\rm z}^m{\rm z}^n{\rm H}^\lambda, \\
{\rm h}_{k0} &= {\rm H}_k + \hat{k}_{\lambda kn}{\rm z}^n{\rm H}^\lambda, \qquad {\rm h}^\lambda{}_0 = {\rm H}^\lambda.
\end{aligned} \tag{3.2}$$

We further introduce a new modulus ρ through a redefinition in the overall volume ($\mathcal{V}$) of the Calabi Yau threefold by considering the two-cycle volume moduli t^a via $t^a = \rho\gamma^a$, where γ^a's denote the angular Kähler moduli satisfying the constraint $\kappa_{abc}\gamma^a\gamma^b\gamma^c = 6$ implying $\mathcal{V} = \rho^3$. Now we can extract the volume factor ρ from the Kähler moduli space metric and its inverse in the following way:

$$\begin{aligned}
\tilde{\mathcal{G}}_{ab} &= \frac{\kappa_a\kappa_b - 4\mathcal{V}\kappa_{ab}}{4\mathcal{V}} = \rho\tilde{g}_{ab}, \\
\tilde{\mathcal{G}}^{ab} &= \frac{2{\rm t}^a{\rm t}^b - 4\mathcal{V}\kappa^{ab}}{4\mathcal{V}} = \frac{1}{\rho}\tilde{g}^{ab},
\end{aligned} \tag{3.3}$$

where $\tilde{g}_{ab}$ and the inverse $\tilde{g}^{ab}$ do not depend on the ρ modulus. Subsequently the scalar potential in Eq. (3.1) can be written as

$$V_{\rm IIA} = \frac{e^{4D}}{4\rho^3}\left[{\rm f}_0^2 + \rho^2{\rm f}_a\tilde{g}^{ab}{\rm f}_b + \rho^4{\rm f}^a\tilde{g}_{ab}{\rm f}^b + \rho^6({\rm f}^0)^2\right] + \frac{e^{2D}}{4\rho^3}\left[\frac{{\rm h}_0^2}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}{\rm h}_{i0}{\rm h}_{j0} + \tilde{\mathcal{G}}_{\lambda\rho}{\rm h}^\lambda{}_0{\rm h}^\rho{}_0\right] + \frac{e^{3D}}{2\sqrt{\mathcal{U}}}\left[{\rm f}^0{\rm h}_0 - \frac{k_\lambda}{2}{\rm f}^0{\rm h}^\lambda{}_0\right]. \tag{3.4}$$

Now for the above potential, one can easily show that the following inequality holds:

$$3\partial_D V_{\rm IIA} - \rho\partial_\rho V_{\rm IIA} = 9V_{\rm IIA} + \frac{e^{4D}}{4\rho^3}\left[6{\rm f}_0^2 + 4\rho^2{\rm f}_a\tilde{g}^{ab}{\rm f}_b + 2\rho^4{\rm f}^a\tilde{g}_{ab}{\rm f}^b\right] \geq 9V_{\rm IIA}, \tag{3.5}$$

where in the last step we have used the fact that all the additional terms in the bracket are guaranteed to be nonnegative. This immediately leads to a de Sitter no-go theorem because at this extremum $\partial_D V_{\rm IIA} = 0 = \partial_\rho V_{\rm IIA}$, the potential is evaluated to take nonpositive values as we see below:

$$V_{\rm IIA}^{\rm ext} = -\frac{1}{9}\times\frac{e^{4D}}{4\rho^3}\left[6{\rm f}_0^2 + 4\rho^2{\rm f}_a\tilde{g}^{ab}{\rm f}_b + 2\rho^4{\rm f}^a\tilde{g}_{ab}{\rm f}^b\right] \leq 0. \tag{3.6}$$

Moreover, one has the following inequality on the inflationary slow-roll ϵ parameter:

$$\begin{aligned}
\epsilon &\geq V_{\rm IIA}^{-2}\left[\frac{\rho^2}{3}(\partial_\rho V_{\rm IIA})^2 + \frac{1}{4}(\partial_D V_{\rm IIA})^2\right] \\
&= V_{\rm IIA}^{-2}\left[\frac{1}{39}(3\partial_D V_{\rm IIA} - \rho\partial_\rho V_{\rm IIA})^2 + \frac{1}{52}(\partial_D V_{\rm IIA} + 4\rho\partial_\rho V_{\rm IIA})^2\right] \geq \frac{27}{13}.
\end{aligned} \tag{3.7}$$

This clearly forbids the slow-roll inflation in this simplistic framework as proposed in [5,8].

B. T-dual de Sitter no-go-1 in type IIB

Now we invoke the T-dual of this type IIA no-go scenario and investigate the type IIB side. The type IIB fluxes that are T-dual to the nonzero type IIA fluxes are given in Table III. This shows that the type IIB side can generically have all the components of the F_3 flux while for the NS-NS sector, there are only the "rigid" fluxes which are allowed, though due to a mixing through the T-duality, there are some (non)geometric flux components present unlike the type IIA case. We call H_0, ω_{a0}, and $\hat{Q}^\alpha{}_0$ "rigid fluxes" because they are the ones that are allowed in a type IIB framework without the complex structure moduli. However, by saying this we do not mean that our T-dual approach is valid for the rigid Calabi Yau compactification as it is well known that the mirror of a rigid Calabi Yau is not a Calabi Yau [114–116]. We have studied the scalar potentials arising in rigid compactifications separately in

TABLE III. Nonzero type IIA fluxes and their respective T-duals for *No-Go-1*.

IIA	e_0	e_a	m^a	m_0	H_0	H_k	H^λ
IIB	F_0	F_i	F^i	$-F^0$	H_0	ω_{a0}	$\hat{Q}^\alpha{}_0$

[101], and throughout this work we assume that the compactifications are on nonrigid threefolds. For the present case, this type IIB scenario only reflects the fact that we have just rigid fluxes turned on while setting others to zero, and for this, a no-go should exist.

Having no (non)geometric fluxes present, there are no Bianchi identities to satisfy in the type IIA side, and the same is true for the type IIB side as well, despite the presence of some rigid (non)geometric fluxes.[3] The dual scalar potential for the type IIB side can be read from Table XI as

$$\begin{aligned} V_{\rm IIB} &= \frac{e^{4\phi}}{4\mathcal{V}^2\mathcal{U}}[f_0^2 + \mathcal{U} f_i \mathcal{G}^{ij} f_j + \mathcal{U} f^i \mathcal{G}_{ij} f^j + \mathcal{U}^2 (f^0)^2] \\ &\quad + \frac{e^{2\phi}}{4\mathcal{V}^2\mathcal{U}}[h_0^2 + \mathcal{V}\mathcal{G}^{ab} h_{a0} h_{b0} + \mathcal{V}\mathcal{G}_{\alpha\beta} h^\alpha{}_0 h^\beta{}_0] \\ &\quad + \frac{e^{3\phi}}{2\mathcal{V}^2}\left[f^0 h_0 - \frac{\ell_\alpha}{2} f^0 h^\alpha{}_0\right], \end{aligned} \tag{3.8}$$

where the simplified axionic flux orbits following from Table X are given as

$$\begin{aligned} f_0 &= F_0 + v^i F_i + \frac{1}{2} l_{ijk} v^j v^k F^i - \frac{1}{6} l_{ijk} v^i v^j v^k F^0 - \omega_{a0} c^a \\ &\quad - \hat{Q}^\alpha{}_0 \hat{c}_\alpha - c_0 \left(H_0 + \omega_{a0} b^a + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b \hat{Q}^\alpha{}_0 \right), \\ f_i &= F_i + l_{ijk} v^j F^k - \frac{1}{2} l_{ijk} v^j v^k F^0, \\ f^i &= F^i - v^i F^0, \qquad f^0 = -F^0, \\ h_0 &= H_0 + \omega_{a0} b^a + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b \hat{Q}^\alpha{}_0, \\ h_{a0} &= \omega_{a0} + \hat{Q}^\alpha{}_0 \hat{\ell}_{\alpha ab} b^b, \qquad h^\alpha{}_0 = \hat{Q}^\alpha{}_0. \end{aligned} \tag{3.9}$$

Although the no-scale structure on type IIB is broken by the presence of the nonzero $\hat{Q}^\alpha{}_0$ flux that couples to T_α moduli in the superpotential and subsequently has been reflected via the appearance of the moduli space metric $\mathcal{G}_{\alpha\beta}$ in the scalar potential (3.8), but that would not lead to a de Sitter solution as suggested by the dual type IIA side. Thus the type IIA no-go condition tells us something interesting and is harder to guess *a priori* on the type IIB side.

To check that this duality-based claim is true, all we need to do is to swap the role of the Kähler moduli with complex structure moduli. On that line, similar to the case of volume modulus $\mathcal{V}$, now we defined a new modulus σ from the saxion of the complex structure moduli such that $u^i = \sigma \lambda^i$, which leads to $\mathcal{U} = \sigma^3$ subject to a condition: $l_{ijk}\lambda^i\lambda^j\lambda^k = 6$ satisfied by the angular complex structure moduli γ^i on the type IIB side. Now we can extract the σ factor from the complex structure moduli space metric and its inverse in the following way:

$$\mathcal{G}_{ij} = \frac{l_i l_j - 4\mathcal{U} l_{ij}}{4\mathcal{U}} = \sigma g_{ij}, \qquad \mathcal{G}^{ij} = \frac{2u^i u^j - 4\mathcal{U} l^{il}}{4\mathcal{U}} = \frac{1}{\sigma} g^{ij}, \tag{3.10}$$

where g_{ij} and g^{ij} depend only on the angular complex structure moduli and not on the σ modulus. Using this information the scalar potential in Eq. (3.8) can be written as

$$\begin{aligned} V_{\rm IIB} &= \frac{e^{4\phi}}{4\mathcal{V}^2\sigma^3}[f_0^2 + \sigma^2 f_i g^{ij} f_j + \sigma^4 f^i g_{ij} f^j + \sigma^6 (f^0)^2] \\ &\quad + \frac{e^{2\phi}}{4\mathcal{V}^2\sigma^3}[h_0^2 + \mathcal{V}\mathcal{G}^{ab} h_{a0} h_{b0} + \mathcal{V}\mathcal{G}_{\alpha\beta} h^\alpha{}_0 h^\beta{}_0] \\ &\quad + \frac{e^{3\phi}}{2\mathcal{V}^2}\left[f^0 h_0 - \frac{\ell_\alpha}{2} f^0 h^\alpha{}_0\right]. \end{aligned} \tag{3.11}$$

Subsequently it is not hard to show that the following inequality holds:

$$\begin{aligned} &3\partial_\phi V_{\rm IIB} - \sigma \partial_\sigma V_{\rm IIB} \\ &\quad = 9 V_{\rm IIB} + \frac{e^{4\phi}}{4\mathcal{V}^2\sigma^3}[f_0^2 + \sigma^2 f_i g^{ij} f_j + \sigma^4 f^i g_{ij} f^j] \geq 9 V_{\rm IIB}, \end{aligned} \tag{3.12}$$

where in the last step we have used the fact that all the additional terms in brackets are guaranteed to be positive semidefinite. This immediately leads to a de Sitter no-go theorem as at this extremum $\partial_\phi V_{\rm IIB} = 0 = \partial_\sigma V_{\rm IIB}$, the potential can only take nonpositive values as we see below:

$$V_{\rm IIB}^{\rm ext} = -\frac{e^{4\phi}}{2\mathcal{V}^2\sigma^3}[6f_0^2 + 4\sigma^2 f_i g^{ij} f_j + 2\sigma^4 f^i g_{ij} f^j] \leq 0. \tag{3.13}$$

Thus we are able to prove an interesting de Sitter no-go theorem on the type IIB side.

> ***Type IIB no-go theorem 1:*** *In the framework of type IIB nongeometric flux compactification with $O3/O7$ orientifold planes, one cannot have a de Sitter solution by considering the RR flux F_3 along with the rigid NS-NS flux components H_0, ω_{a0}, and $\hat{Q}^\alpha{}_0$ only.*

IV. NO-GO-2

In this section we consider another no-go condition found in the type IIA framework, which in addition to the

[3] This is something one would expect from the set of Bianchi identities known to us in the cohomology formulation, though there are several observations based on toroidal examples that there may be a few of the missing identities in this approach [97–101].

ingredients of the no-go-1 scenario, also includes the geometric flux [7–9], and subsequently we will T-dualize the same to invoke its type IIB counterpart.

A. Type IIA with RR flux, H_3 flux, and ω flux

This type IIA de Sitter no-go scenario includes the NS-NS H_3 flux, geometric flux w, and the standard RR fluxes, namely the F_0, F_2, F_4, and F_6 fluxes [7–9]. However, there are no nongeometric fluxes turned on, i.e., $\mathrm{Q}^a{}_{\hat{k}} = \mathrm{Q}^{a\lambda} = \hat{\mathrm{Q}}^{\alpha\hat{k}} = \hat{\mathrm{Q}}^\alpha{}_\lambda = 0$ and $\mathrm{R}_{\hat{k}} = 0 = \mathrm{R}^\lambda$. To get the scalar potential from our generic formula in Table XI one has to simply set the following flux orbits to zero:

$$\mathrm{h}^a = 0 = \mathrm{h}^0, \qquad \mathrm{h}^a{}_k = 0 = \mathrm{h}_k{}^0, \qquad \mathrm{h}^{a\lambda} = 0 = \mathrm{h}^{\lambda 0}, \qquad \hat{\mathrm{h}}^{\alpha 0} = 0 = \hat{\mathrm{h}}^\alpha{}_\lambda, \tag{4.1}$$

where the last two fluxes are parts of the D-term contributions via the Q flux. Setting off these nongeometric fluxes in Eq. (4.1), the generic scalar potential given in Table XI can be simplified to take a form given as

$$\begin{aligned} V_{\mathrm{IIA}} = {} & \frac{e^{4D}}{4\mathcal{V}}[\mathrm{f}_0^2 + \mathcal{V}\mathrm{f}_a\tilde{\mathcal{G}}^{ab}\mathrm{f}_b + \mathcal{V}\mathrm{f}^a\tilde{\mathcal{G}}_{ab}\mathrm{f}^b + \mathcal{V}^2(\mathrm{f}^0)^2] + \frac{e^{2D}}{4\mathcal{V}}\left[\frac{\mathrm{h}_0^2}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}\mathrm{h}_{i0}\mathrm{h}_{j0} + \tilde{\mathcal{G}}_{\lambda\rho}\mathrm{h}^\lambda{}_0\mathrm{h}^\rho{}_0\right] \\ & + \frac{e^{2D}}{4\mathcal{V}}\left[\mathrm{t}^a\mathrm{t}^b\left(\frac{\mathrm{h}_a\mathrm{h}_b}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}\mathrm{h}_{ai}\mathrm{h}_{bj} + \tilde{\mathcal{G}}_{\lambda\rho}\mathrm{h}_a{}^\lambda\mathrm{h}_b{}^\rho\right) + \frac{1}{\mathcal{U}}\left(\mathrm{h}_a - \frac{k_\lambda}{2}\mathrm{h}_a{}^\lambda\right)(\mathcal{V}\tilde{\mathcal{G}}^{ab} - \mathrm{t}^a\mathrm{t}^b)\left(\mathrm{h}_b - \frac{k_\rho}{2}\mathrm{h}_b{}^\rho\right)\right. \\ & \left. + \frac{1}{\mathcal{U}}(\mathcal{U}\hat{\mathrm{h}}_\alpha{}^0 + \mathrm{z}^\lambda\hat{\mathrm{h}}_{\alpha\lambda})\mathcal{V}(\hat{\kappa}_{a\alpha\beta}\mathrm{t}^a)^{-1}(\mathcal{U}\hat{\mathrm{h}}_\beta{}^0 + \mathrm{z}^\rho\hat{\mathrm{h}}_{\beta\rho})\right] + \frac{e^{3D}}{2\sqrt{\mathcal{U}}}\left[(\mathrm{f}^0\mathrm{h}_0 - \mathrm{f}^a\mathrm{h}_a) - (\mathrm{f}^0\mathrm{h}^\lambda{}_0 - \mathrm{f}^a\mathrm{h}^\lambda{}_a)\frac{k_\lambda}{2}\right], \end{aligned} \tag{4.2}$$

where using the simplifications from Eq. (4.1), the various nonzero "axionic flux orbits" can be written out from Table IX and those are simplified as

$$\begin{aligned} \mathrm{f}_0 &= e_0 + \mathrm{b}^a e_a + \frac{1}{2}\kappa_{abc}\mathrm{b}^a\mathrm{b}^b m^c + \frac{1}{6}\kappa_{abc}\mathrm{b}^a\mathrm{b}^b\mathrm{b}^c m_0 - \xi^0(\mathrm{H}_0 + \mathrm{b}^a w_{a0}) - \xi^k(\mathrm{H}_k + \mathrm{b}^a w_{ak}) - \xi_\lambda(\mathrm{H}^\lambda + \mathrm{b}^a w_a{}^\lambda), \\ \mathrm{f}_a &= e_a + \kappa_{abc}\mathrm{b}^b m^c + \frac{1}{2}\kappa_{abc}\mathrm{b}^b\mathrm{b}^c m_0 - \xi^0 w_{a0} - \xi^k w_{ak} - \xi_\lambda w_a{}^\lambda, \\ \mathrm{f}^a &= m^a + m_0\mathrm{b}^a, \qquad \mathrm{f}^0 = m_0, \\ \mathrm{h}_0 &= (\mathrm{H}_0 + \mathrm{b}^a w_{a0}) + \mathrm{z}^k(\mathrm{H}_k + \mathrm{b}^a w_{ak}) + \frac{1}{2}\hat{k}_{\lambda mn}\mathrm{z}^m\mathrm{z}^n(\mathrm{H}^\lambda + \mathrm{b}^a w_a{}^\lambda), \\ \mathrm{h}_{k0} &= (\mathrm{H}_k + \mathrm{b}^a w_{ak}) + \hat{k}_{\lambda kn}\mathrm{z}^n(\mathrm{H}^\lambda + \mathrm{b}^a w_a{}^\lambda), \mathrm{h}^\lambda{}_0 = (\mathrm{H}^\lambda + \mathrm{b}^a w_a{}^\lambda), \\ \mathrm{h}_a &= w_{a0} + \mathrm{z}^k w_{ak} + \frac{1}{2}\hat{k}_{\lambda mn}\mathrm{z}^m\mathrm{z}^n w_a{}^\lambda, \qquad \mathrm{h}_{ak} = w_{ak} + \hat{k}_{\lambda kn}\mathrm{z}^n w_a{}^\lambda, \qquad \mathrm{h}_a{}^\lambda = w_a{}^\lambda, \\ \hat{\mathrm{h}}_{\alpha\lambda} &= \hat{w}_{\alpha\lambda} + \hat{k}_{\lambda km}\mathrm{z}^m\hat{w}_\alpha{}^k - \frac{1}{2}\hat{k}_{\lambda km}\mathrm{z}^k\mathrm{z}^m\hat{w}_\alpha{}^0, \qquad \hat{\mathrm{h}}_\alpha{}^0 = \hat{w}_\alpha{}^0. \end{aligned} \tag{4.3}$$

Note that unlike the previous de Sitter no-go scenario, now there can be nontrivial contributions generated from the D-terms via the geometric fluxes. Similar to the previous case, extracting the factor ρ from the various volume moduli and metrics as in Eq. (3.3), the total scalar potential in Eq. (4.2) simplifies to the following form:

$$\begin{aligned} V_{\mathrm{IIA}} = {} & \frac{e^{4D}}{4\rho^3}[\mathrm{f}_0^2 + \rho^2\mathrm{f}_a\tilde{g}^{ab}\mathrm{f}_b + \rho^4\mathrm{f}^a\tilde{g}_{ab}\mathrm{f}^b + \rho^6(\mathrm{f}^0)^2] + \frac{e^{2D}}{4\rho^3}\left[\frac{\mathrm{h}_0^2}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}\mathrm{h}_{i0}\mathrm{h}_{j0} + \tilde{\mathcal{G}}_{\lambda\rho}\mathrm{h}^\lambda{}_0\mathrm{h}^\rho{}_0\right] \\ & + \frac{e^{2D}}{4\rho}\left[\gamma^a\gamma^b\left(\frac{\mathrm{h}_a\mathrm{h}_b}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}\mathrm{h}_{ai}\mathrm{h}_{bj} + \tilde{\mathcal{G}}_{\lambda\rho}\mathrm{h}_a{}^\lambda\mathrm{h}_b{}^\rho\right) + \frac{1}{\mathcal{U}}\left(\mathrm{h}_a - \frac{k_\lambda}{2}\mathrm{h}_a{}^\lambda\right)(\tilde{g}^{ab} - \gamma^a\gamma^b)\left(\mathrm{h}_b - \frac{k_\rho}{2}\mathrm{h}_b{}^\rho\right)\right. \\ & \left. + \frac{1}{\mathcal{U}}(\mathcal{U}\hat{\mathrm{h}}_\alpha{}^0 + \mathrm{z}^\lambda\hat{\mathrm{h}}_{\alpha\lambda})(\hat{\kappa}_{a\alpha\beta}\gamma^a)^{-1}(\mathcal{U}\hat{\mathrm{h}}_\beta{}^0 + \mathrm{z}^\rho\hat{\mathrm{h}}_{\beta\rho})\right] + \frac{e^{3D}}{2\sqrt{\mathcal{U}}}\left[(\mathrm{f}^0\mathrm{h}_0 - \mathrm{f}^a\mathrm{h}_a) - (\mathrm{f}^0\mathrm{h}^\lambda{}_0 - \mathrm{f}^a\mathrm{h}^\lambda{}_a)\frac{k_\lambda}{2}\right]. \end{aligned} \tag{4.4}$$

TABLE IV. Nonzero type IIA fluxes and their respective T-duals for *No-Go-2*.

IIA	e_0	e_a	m^a	m_0	H_0	H_k	H^λ	w_{a0}	w_{ak}	${w_a}^\lambda$	${\hat{w}_\alpha}^0$	${\hat{w}_\alpha}^k$	$\hat{w}_{\alpha\lambda}$
IIB	F_0	F_i	F^i	$-F^0$	H_0	ω_{a0}	${\hat{Q}^\alpha}_0$	H_i	ω_{ai}	${\hat{Q}^\alpha}_i$	$-R_K$	$-{Q^a}_K$	$\hat{\omega}_{\alpha K}$

Now using the scalar potential in Eq. (4.4) one can show that the following interesting relation holds:

$$\begin{aligned}\partial_D V_{\mathrm{IIA}} - \rho\partial_\rho V_{\mathrm{IIA}} &= 3V_{\mathrm{IIA}} + \frac{e^{2D}}{2\rho^3}\left[\frac{\mathrm{h}_0^2}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}\mathrm{h}_{i0}\mathrm{h}_{j0} + \tilde{\mathcal{G}}_{\lambda\rho}{\mathrm{h}^\lambda}_0{\mathrm{h}^\rho}_0\right]\\ &\quad + \frac{e^{4D}}{4\rho^3}[4\mathrm{f}_0^2 + 2\rho^2\mathrm{f}_a\tilde{g}^{ab}\mathrm{f}_b - 2\rho^6(\mathrm{f}^0)^2]. \qquad (4.5)\end{aligned}$$

One can observe the fact that for $\mathrm{f}^0 = m_0$ being set to zero, all the terms on the right-hand side are non-negative which results in $(\partial_D V_{\mathrm{IIA}} - \rho\partial_\rho V_{\mathrm{IIA}}) \geq 3V_{\mathrm{IIA}}$, and hence in this situation a new no-go condition holds despite the fact that geometric fluxes are included. Moreover, one has the following inequality on the inflationary parameter ϵ:

$$\begin{aligned}\epsilon &\geq V_{\mathrm{IIA}}^{-2}\left[\frac{\rho^2}{3}(\partial_\rho V_{\mathrm{IIA}})^2 + \frac{1}{4}(\partial_D V_{\mathrm{IIA}})^2\right]\\ &= V_{\mathrm{IIA}}^{-2}\left[\frac{1}{7}(3\partial_D V_{\mathrm{IIA}} - \rho\partial_\rho V_{\mathrm{IIA}})^2\right.\\ &\quad \left. + \frac{1}{84}(3\partial_D V_{\mathrm{IIA}} + 4\rho\partial_\rho V_{\mathrm{IIA}})^2\right] \geq \frac{9}{7}. \qquad (4.6)\end{aligned}$$

However, it is also true that the earlier no-go condition is evaded with the simultaneous presence of geometric flux and the Romans mass term. The extremization conditions $\partial_D V_{\mathrm{IIA}} = 0 = \partial_\rho V_{\mathrm{IIA}}$ lead to the following form of the potential:

$$\begin{aligned}V_{\mathrm{IIA}}^{\mathrm{ext}} &= -\frac{e^{2D}}{6\rho^3}\left[\frac{\mathrm{h}_0^2}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}\mathrm{h}_{i0}\mathrm{h}_{j0} + \tilde{\mathcal{G}}_{\lambda\rho}{\mathrm{h}^\lambda}_0{\mathrm{h}^\rho}_0\right]\\ &\quad - \frac{e^{4D}}{12\rho^3}[4\mathrm{f}_0^2 + 2\rho^2\mathrm{f}_a\tilde{g}^{ab}\mathrm{f}_b - 2\rho^6(\mathrm{f}^0)^2],\end{aligned}$$

which clearly opens up the possibility of getting de Sitter via considering a large enough value for the Romans mass parameter $\mathrm{f}^0 = m_0$ [7].

B. T-dual de Sitter no-go-2 in type IIB

Now we want to know the T-dual version of this second type IIA no-go scenario on the type IIB side, and the T-duality from the nonzero fluxes in type IIA gives the flux ingredients of the type IIB setup as given in Table IV. It shows that for this scenario, the dual type IIB side can get fairly complicated with the presence of RR (F_3) flux along with all the (non)geometric NS-NS fluxes unlike the type IIA case. Moreover, given the fact that this scenario corresponds to type IIA without any nongeometric flux, and therefore as we have analyzed in the previous section, this would be dual to type IIB with the special solution of Bianchi identities, in which half of the fluxes can be rotated away by a suitable symplectic transformation. Also, the Bianchi identities to worry about on type IIA and their dual type IIB side are simply the following ones:

$$\begin{aligned}\mathbf{IIA:}\quad & \mathrm{H}^\lambda\hat{w}_{\alpha\lambda} = \mathrm{H}_{\hat{k}}{\hat{w}_\alpha}^{\hat{k}}, \qquad {w_a}^\lambda\hat{w}_{\alpha\lambda} = w_{a\hat{k}}{\hat{w}_\alpha}^{\hat{k}};\\ \mathbf{IIB:}\quad & H_0 R_K + \omega_{a0}{Q^a}_K + {\hat{Q}^\alpha}_0\hat{\omega}_{\alpha K} = 0,\\ & H_i R_K + \omega_{ai}{Q^a}_K + {\hat{Q}^\alpha}_i\hat{\omega}_{\alpha K} = 0. \qquad (4.7)\end{aligned}$$

For implementing the special solution of Bianchi identities in the type IIB scalar potential, we need to switch off the following axionic flux orbits,

$$\begin{aligned}h^0 = 0 = h^i, &\qquad {h_a}^i = 0 = {h_a}^0,\\ h^{\alpha i} = 0 = h^{\alpha 0}, &\qquad {\hat{h}_\alpha}^K = 0 = \hat{h}^K, \qquad (4.8)\end{aligned}$$

where the last two hatted fluxes are parts of the D-term contributions. Using this simplification, and after a bit of reshuffling of terms, the dual scalar potential for the type IIB side can subsequently be read from Table XI and turns out to be given as

$$\begin{aligned}V_{\mathrm{IIB}} &= \frac{e^{4\phi}}{4\mathcal{V}^2\mathcal{U}}[f_0^2 + \mathcal{U}f^i\mathcal{G}_{ij}f^j + \mathcal{U}f_i\mathcal{G}^{ij}f_j + \mathcal{U}^2(f^0)^2] + \frac{e^{2\phi}}{4\mathcal{V}^2\mathcal{U}}\left[h_0^2 + \mathcal{V}\mathcal{G}^{ab}h_{a0}h_{b0} + \mathcal{V}\mathcal{G}_{\alpha\beta}{h^\alpha}_0{h^\beta}_0\right.\\ &\quad + u^i u^j(h_i h_j + \mathcal{V}\mathcal{G}_{\alpha\beta}{h^\alpha}_i{h^\beta}_j + \mathcal{V}\mathcal{G}^{ab}h_{ai}h_{bj}) + (\mathcal{U}\mathcal{G}^{ij} - u^i u^j)\left(h_i - \frac{\ell_\alpha}{2}{h^\alpha}_i\right)\left(h_j - \frac{\ell_\beta}{2}{h^\beta}_j\right)\\ &\quad \left. + \mathcal{U}(\mathcal{V}{\hat{h}_J}^{\,0} - t^\alpha\hat{h}_{\alpha J})(\hat{\ell}_{iJK}u^i)^{-1}(\mathcal{V}{\hat{h}_K}^{\,0} - t^\beta\hat{h}_{\beta K})\right] + \frac{e^{3\phi}}{\mathcal{V}^2}\left[(f^0 h_0 - f^i h_i) - (f^0{h^\alpha}_0 - f^i{h^\alpha}_i)\frac{\ell_\alpha}{2}\right], \qquad (4.9)\end{aligned}$$

where the simplified version of the nontrivial axionic flux orbits are given as below:

$$\begin{aligned}
&f^0 = -F^0, \qquad f^i = F^i - v^i F^0, \\
&f_i = F_i + l_{ijk} v^j F^k - \frac{1}{2} l_{ijk} v^j v^k F^0 - \omega_{ai} c^a - \hat{Q}^\alpha{}_i \hat{c}_\alpha - c_0 h_i, \\
&f_0 = F_0 + v^i F_i + \frac{1}{2} l_{ijk} v^j v^k F^i - \frac{1}{6} l_{ijk} v^i v^j v^k F^0 - \omega_{a0} c^a - \hat{Q}^\alpha{}_0 \hat{c}_\alpha - c_0 h_0, \\
&h_0 = H_0 + \omega_{a0} b^a + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b \hat{Q}^\alpha{}_0 + v^i h_i, \qquad h_i = H_i + \omega_{ai} b^a + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b \hat{Q}^\alpha{}_i, \\
&h_{a0} = \omega_{a0} + \hat{Q}^\alpha{}_0 \hat{\ell}_{\alpha ab} b^b + v^i h_{ai}, \qquad h_{ai} = \omega_{ai} + \hat{Q}^\alpha{}_i \hat{\ell}_{\alpha ab} b^b, \\
&h^\alpha{}_0 = \hat{Q}^\alpha{}_0 + v^i \hat{Q}^\alpha{}_i, \qquad h^\alpha{}_i = \hat{Q}^\alpha{}_i, \\
&\hat{h}_{\alpha K} = \hat{\omega}_{\alpha K} - Q^a{}_K \hat{\ell}_{\alpha ab} b^b + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b R_K, \qquad \hat{h}_K{}^0 = -R_K.
\end{aligned} \tag{4.10}$$

Now similar to the previous no-go-1 case, in order to prove that there is a new de Sitter no-go scenario in the type IIB side with nongeometric flux, all we need to do is to swap the role of the complex-structure and the Kähler moduli. To see it explicitly we extract the σ factor from the complex-structure moduli and the moduli space metrics as given in Eq. (3.10). This leads to the type IIB scalar potential being written as

$$\begin{aligned}
V_{\rm IIB} = &\frac{e^{4\phi}}{4\mathcal{V}^2\sigma^3}[f_0^2 + \sigma^2 f_i g^{ij} f_j + \sigma^4 f^i g_{ij} f^j + \sigma^6 (f^0)^2] + \frac{e^{2\phi}}{4\mathcal{V}^2\sigma^3}[(h_0^2 + \mathcal{V}\mathcal{G}^{ab} h_{a0} h_{b0} + \mathcal{V}\mathcal{G}_{\alpha\beta} h^\alpha{}_0 h^\beta{}_0)] \\
&+ \frac{e^{2\phi}}{4\mathcal{V}^2\sigma}\Bigg[\lambda^i\lambda^j (h_i h_j + \mathcal{V}\mathcal{G}_{\alpha\beta} h^\alpha{}_i h^\beta{}_j + \mathcal{V}\mathcal{G}^{ab} h_{ai} h_{bj}) + (g^{ij} - \lambda^i\lambda^j)\left(h_i - \frac{\ell_\alpha}{2} h^\alpha{}_i\right)\left(h_j - \frac{\ell_\beta}{2} h^\beta{}_j\right) \\
&+ (\mathcal{V}\hat{h}_J{}^0 - t^\alpha \hat{h}_{\alpha J})(\hat{\ell}_{iJK}\lambda^i)^{-1}(\mathcal{V}\hat{h}_K{}^0 - t^\beta \hat{h}_{\beta K})\Bigg] + \frac{e^{3\phi}}{2\mathcal{V}^2}\left[(f^0 h_0 - f^i h_i) - (f^0 h^\alpha{}_0 - f^i h^\alpha{}_i)\frac{\ell_\alpha}{2}\right],
\end{aligned} \tag{4.11}$$

where the angular moduli λ^i's and the metrics g^{ij}, g_{ij} do not have any dependence on the σ modulus. Subsequently it is not hard to show that the following relation holds:

$$\begin{aligned}
&\partial_\phi V_{\rm IIB} - \sigma\partial_\sigma V_{\rm IIB} \\
&\quad = 3V_{\rm IIB} + \frac{e^{2\phi}}{2\mathcal{V}^2\sigma^3}[(h_0^2 + \mathcal{V}\mathcal{G}^{ab} h_{a0} h_{b0} + \mathcal{V}\mathcal{G}_{\alpha\beta} h^\alpha{}_0 h^\beta{}_0)] \\
&\qquad + \frac{e^{4\phi}}{4\mathcal{V}^2\sigma^3}[4f_0^2 + 2\sigma^2 f_i g^{ij} f_j - 2\sigma^6 (f^0)^2].
\end{aligned} \tag{4.12}$$

The last term is the only nonpositive term, and this shows that for $f^0 \equiv -F^0 = 0$ we have the inequality $(\partial_\phi V_{\rm IIB} - \sigma\partial_\sigma V_{\rm IIB}) \geq 3V_{\rm IIB}$. This immediately leads to a de Sitter no-go theorem as at this extremum $\partial_\phi V_{\rm IIB} = 0 = \partial_\sigma V_{\rm IIB}$, the potential is allowed to take only the nonpositive values as long as $f^0 = 0$ as we see below,

$$\begin{aligned}
V_{\rm IIB}^{\rm ext} = &-\frac{e^{2\phi}}{3\mathcal{V}^2\sigma^3}[(h_0^2 + \mathcal{V}\mathcal{G}^{ab} h_{a0} h_{b0} + \mathcal{V}\mathcal{G}_{\alpha\beta} h^\alpha{}_0 h^\beta{}_0)] \\
&-\frac{e^{4\phi}}{2\mathcal{V}^2\sigma^3}[4f_0^2 + 2\sigma^2 f_i g^{ij} f_j - 2\sigma^6 (f^0)^2].
\end{aligned} \tag{4.13}$$

Thus we are able to prove an interesting de Sitter no-go theorem on the type IIB side by T-dualizing the type IIA no-go, and moreover, we have a possible way for finding de Sitter by satisfying the necessary condition $F^0 \neq 0$ for the nongeometric flux with special solutions.

> ***Type IIB no-go theorem 2:*** *In type IIB framework with $O3/O7$ orientifold planes and (non)geometric fluxes along with the standard F_3, H_3 fluxes, one cannot have stable de Sitter minima with special solutions of Bianchi identities, unless the F^0 component of the F_3 flux is nonzero, where $F_3 = F^\Lambda \mathcal{A}_\Lambda - F_\Lambda \mathcal{B}^\Lambda$ and $\Lambda \in \{0, 1, \ldots, h^{2,1}_-\}$.*

V. NO-GO-3

In the previous section, we have seen that after including the Romans mass term in type IIA or equivalently F^0 component of the three-form F_3 flux in type IIB, the necessary condition for getting the de Sitter no-go is violated. This can be taken as a window to hunt for de Sitter solutions. On the other hand, naively speaking, in order to restore the no-go condition or for finding another no-go, one would need to nullify the effects of these respective fluxes in the type IIA and the type IIB scenarios, and therefore one can ask the question if there are certain geometries that could be useful for this purpose. In this section we will show how the $K3$- or $\mathbb{T}^4$-fibered Calabi Yau (CY) threefolds could be useful in this regard as they facilitate a factorization in the moduli space as shown to be needed in [8].

A. Type IIA with $K3$- or $\mathbb{T}^4$-fibered (CY) threefolds

Superstring compactifications using $K3$- or $\mathbb{T}^4$-fibered CY threefolds present an interesting case as there is some kind of factorization guaranteed in the Kähler moduli space. By the theorem of [117,118], such a Calabi Yau threefold will have at least one two-cycle dual to a $K3$ or a $\mathbb{T}^4$ divisor that appears only linearly in the intersection polynomial.[4] In other words, the intersection numbers can be managed to split in the following manner by singling out a component through the splitting of index a as $a = \{1, a'\}$,

$$\kappa_{111} = 0 = \kappa_{11a'}, \qquad \kappa_{1a'b'} \neq 0, \qquad \hat{\kappa}_{1\alpha\beta} \neq 0,$$
$$\hat{\kappa}_{a'\alpha\beta} = 0, \quad \text{where } a' \neq 1 \neq b'. \tag{5.1}$$

On top of that, in particular we also assume that $\kappa_{a'b'c'} = 0$ and note that there is only one nonzero intersection of the type $\hat{\kappa}_{a\alpha\beta}$ with $a = 1$. A concrete example of a $K3$-fibered CY threefold with such even/odd splitting in the intersection numbers (and hence in the corresponding moduli space metrics) can be found in [122]. Recall that a nonzero intersection number of the type $\hat{\kappa}_{a\alpha\beta}$ is also essential for generating the D-terms by coupling through the (non) geometric fluxes.

Let us say that the volume of a two-cycle that is singled out is denoted as $\mathrm{t}^1 = \rho_0$ leaving $\mathrm{t}^{a'}$ the number of volume moduli as the remaining ones, and then the overall volume of the threefold can be written out as

$$\mathcal{V} = \frac{1}{6}\kappa_{abc}\mathrm{t}^a\mathrm{t}^b\mathrm{t}^c = \frac{1}{2}\kappa_{1a'b'}\rho_0\mathrm{t}^{a'}\mathrm{t}^{b'}, \tag{5.2}$$

which leaves the volume form as a homogeneous function of degree 2 in the remaining prime-indexed Kähler moduli. Now we can still assume $\mathrm{t}^{a'} = \rho\gamma^{a'}$ where $\gamma^{a'}$'s are the remaining angular Kähler moduli satisfying $\kappa_{1a'b'}\gamma^{a'}\gamma^{b'} = 2$. This leads to a simple volume form given as

$$\mathcal{V} = \rho_0\rho^2. \tag{5.3}$$

Before we come to the explicit detail on restoring the de Sitter no-go condition by making an appropriate choice of the geometry, let us throw some more light on the motivation of looking at this $K3/\mathbb{T}4$-fibered geometry by considering the following Romans mass term as it appears in the type IIA scalar potential,

$$V_{\mathrm{f}^0} = \frac{e^{4D}}{2}\mathcal{V}(\mathrm{f}^0)^2. \tag{5.4}$$

One can easily be convinced that using Eq. (4.4) in which $\mathcal{V} = \rho^3$ simplification has been made we get the following relations:

$$(\partial_D V_{\mathrm{f}^0} - \rho\partial_\rho V_{\mathrm{f}^0}) = V_{\mathrm{f}^0} \Rightarrow (\partial_D V_{\mathrm{IIA}} - \rho\partial_\rho V_{\mathrm{IIA}})$$
$$= 3V_{\mathrm{IIA}} - 2V_{\mathrm{f}^0} + \cdots, \tag{5.5}$$

where dots have some non-negative pieces as seen while deriving the no-go-2, and this way V_{f^0} appearing with a minus sign on the right-hand side helps in evading the de Sitter no-go condition. Now suppose we have a volume form of the type $\mathcal{V} = \rho_0\rho^2$ instead of $\mathcal{V} = \rho^3$; then the following relations hold:

$$\textbf{(I)} \quad (\partial_D V_{\mathrm{f}^0} - \rho_0\partial_{\rho_0} V_{\mathrm{f}^0}) = 3V_{\mathrm{f}^0} \Rightarrow (\partial_D V_{\mathrm{IIA}} - \rho_0\partial_{\rho_0} V_{\mathrm{IIA}})$$
$$= 3V_{\mathrm{IIA}} + \cdots,$$
$$\textbf{(II)} \quad (2\partial_D V_{\mathrm{f}^0} - \rho\partial_\rho V_{\mathrm{f}^0}) = 6V_{\mathrm{f}^0} \Rightarrow (2\partial_D V_{\mathrm{IIA}} - \rho\partial_\rho V_{\mathrm{IIA}})$$
$$= 6V_{\mathrm{IIA}} + \cdots, \tag{5.6}$$

where we can see that now V_{f^0} can be completely absorbed in V_{IIA} and so a negative piece with V_{f_0} is absent. Here we take an assumption (to be proven in a while) that one can appropriately make the flux choice to be such that all the other pieces inside the dots remain to be non-negative. Thus by considering these simple heuristics, one can anticipate getting another de Sitter no-go with some appropriate choice of fluxes and geometries.

Let us mention that one can also demand the splitting of intersection numbers on the mirror side, i.e., $k_{\lambda\rho\sigma}$ leading to the splitting in the complex structure moduli metric, to balance things from the $(\partial_D V_{\mathrm{f}^0})$ piece [8] rather than considering $(\partial_\rho V_{\mathrm{f}^0})$ via taking a factorizable Kähler moduli space as we are considering. That may result in some new no-go scenarios; however, we will not consider that case in this work.

To explore the details, using the choice for the triple-intersection numbers given in Eq. (5.1) and the definitions of the metric given in Table XI we have the following block-diagonal forms for the (inverse-)moduli space metrics,

$$\mathcal{V}\tilde{\mathcal{G}}^{ab} = \begin{pmatrix} \rho_0^2 & 0 \\ 0 & \rho^2(\gamma^{a'}\gamma^{b'} - \tilde{\kappa}^{a'b'}) \end{pmatrix},$$
$$\mathcal{V}\tilde{\mathcal{G}}_{ab} = \begin{pmatrix} \rho^4 & 0 \\ 0 & \rho_0^2\rho^2(\tilde{\kappa}_{a'}\tilde{\kappa}_{b'} - \tilde{\kappa}_{a'b'}) \end{pmatrix}, \tag{5.7}$$

[4] Such Calabi Yau threefolds with $K3/\mathbb{T}^4$ fibrations have also been useful for realizing fiber inflation models [119–121].

where $a' \in \{2, 3, \ldots, h^{1,1}_-\}$ and the angular quantities with a' indices do not depend on any of the moduli ρ_0 and ρ. From the scalar potential in Eq. (4.2), which is relevant for this type IIA case with geometric flux, we observe that the volume moduli ρ_0 and ρ can appear through factors such as $(\mathcal{V}\tilde{\mathcal{G}}^{ab})$, $(\mathcal{V}\tilde{\mathcal{G}}_{ab})$, $(\mathrm{t}^a\mathrm{t}^b)$, or $(\hat{\kappa}_{a\alpha\beta}\mathrm{t}^a)$. As we have seen from Eq. (5.7), the moduli space metrics are already block diagonal with the splitting of index a as $a = \{1, a'\}$. Also note that the piece with $(\hat{\kappa}_{a\alpha\beta}\mathrm{t}^a)^{-1}$ will only depend on ρ_0 (and not on the ρ) modulus as we have assumed in Eq. (5.1) that $\hat{\kappa}_{1\alpha\beta}$ is the only nonzero intersection with index α, β being in the even (1,1) cohomology. However, scalar potential pieces involving the factor $(\mathrm{t}^a\mathrm{t}^b)$ can generate off-diagonal mixings and so might disturb the balance of pieces in $(\partial_D V_{\mathrm{IIA}} - \rho_0\partial_{\rho_0} V_{\mathrm{IIA}}) = 3V_{\mathrm{IIA}} + \cdots$, so as to keep retaining the pieces hidden in the dots as positive semidefinite, something that was established for the earlier no-go-2. To concretize these arguments, we simplify the geometric type IIA scalar potential given in Eq. (4.2) utilizing the above splitting of the moduli space metrics, and it turns out to be given as

$$\begin{aligned}
V_{\mathrm{IIA}} = {}& \frac{e^{4D}}{4\rho_0\rho^2}[(\mathrm{f}_0)^2 + \rho_0^2(\mathrm{f}_1)^2 + \rho^2\mathrm{f}_{a'}(\gamma^{a'}\gamma^{b'} - \tilde{\kappa}^{a'b'})\mathrm{f}_{b'} + \rho^4(\mathrm{f}^1)^2 + \rho_0^2\rho^2\mathrm{f}^{a'}(\tilde{\kappa}_{a'}\tilde{\kappa}_{b'} - \tilde{\kappa}_{a'b'})\mathrm{f}^{b'} + \rho_0^2\rho^4(\mathrm{f}^0)^2] \\
& + \frac{e^{2D}}{4\rho_0\rho^2}\left[\frac{\mathrm{h}_0^2}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}\mathrm{h}_{i0}\mathrm{h}_{j0} + \tilde{\mathcal{G}}_{\lambda\rho}\mathrm{h}^\lambda{}_0\mathrm{h}^\rho{}_0\right] + \frac{e^{2D}}{4\rho^2}\times\rho_0\left[\left(\frac{\mathrm{h}_1\mathrm{h}_1}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}\mathrm{h}_{1i}\mathrm{h}_{1j} + \tilde{\mathcal{G}}_{\lambda\rho}\mathrm{h}_1{}^\lambda\mathrm{h}_1{}^\rho\right)\right] \\
& + \frac{e^{2D}}{2\rho}\left[\gamma^{a'}\left(\frac{\mathrm{h}_{a'}\mathrm{h}_1}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}\mathrm{h}_{a'i}\mathrm{h}_{1j} + \tilde{\mathcal{G}}_{\lambda\rho}\mathrm{h}_{a'}{}^\lambda\mathrm{h}_1{}^\rho\right)\right] + \frac{e^{2D}}{4\rho_0}\left[\gamma^{a'}\gamma^{b'}\left(\frac{\mathrm{h}_{a'}\mathrm{h}_{b'}}{\mathcal{U}} + \tilde{\mathcal{G}}^{ij}\mathrm{h}_{a'i}\mathrm{h}_{b'j} + \tilde{\mathcal{G}}_{\lambda\rho}\mathrm{h}_{a'}{}^\lambda\mathrm{h}_{b'}{}^\rho\right)\right] \\
& - \frac{e^{2D}}{4\rho_0\mathcal{U}}\left[\left(\mathrm{h}_{a'} - \frac{k_\lambda}{2}\mathrm{h}_{a'}{}^\lambda\right)\tilde{\kappa}^{a'b'}\left(\mathrm{h}_{b'} - \frac{k_\rho}{2}\mathrm{h}_{b'}{}^\rho\right)\right] - \frac{e^{2D}}{2\rho\mathcal{U}}\left[\left(\mathrm{h}_1 - \frac{k_\lambda}{2}\mathrm{h}_1{}^\lambda\right)\gamma^{a'}\left(\mathrm{h}_{a'} - \frac{k_\rho}{2}\mathrm{h}_{a'}{}^\rho\right)\right] \\
& + \frac{e^{2D}}{4\rho_0\mathcal{U}}[(\mathcal{U}\hat{\mathrm{h}}_\alpha{}^0 + \mathrm{z}^\lambda\hat{\mathrm{h}}_{\alpha\lambda})(\hat{\kappa}_{1\alpha\beta}\gamma^1)^{-1}(\mathcal{U}\hat{\mathrm{h}}_\beta{}^0 + \mathrm{z}^\rho\hat{\mathrm{h}}_{\beta\rho})] + \frac{e^{3D}}{2\sqrt{\mathcal{U}}}\left[(\mathrm{f}^0\mathrm{h}_0 - \mathrm{f}^a\mathrm{h}_a) - (\mathrm{f}^0\mathrm{h}^\lambda{}_0 - \mathrm{f}^a\mathrm{h}^\lambda{}_a)\frac{k_\lambda}{2}\right],
\end{aligned} \tag{5.8}$$

where the flux orbits can be read off from Eq. (4.3) after imposing the splitting of indices as $a = \{1, a'\}$ and using the intersection numbers given in Eq. (5.1). Now from this complicated potential we can see the off-diagonal mixing, e.g., arising from the $(\mathrm{t}^a\mathrm{t}^b)$ factor as we discussed before. This issue can be avoided by appropriately setting the respective fluxes coupled in the off-diagonal blocks to zero. That is, by taking either of the following two cases which subsequently leads to the new de Sitter no-go scenarios:

$$\begin{aligned}
&(\mathbf{I}) \quad \mathrm{h}_1 = \mathrm{h}_{1k} = \mathrm{h}_1{}^\lambda = 0 \Leftrightarrow w_{10} = w_{1k} = w_1{}^\lambda = 0, \\
&\qquad \Rightarrow (\partial_D V_{\mathrm{IIA}} - \rho_0\partial_{\rho_0}V_{\mathrm{IIA}}) \geq 3V_{\mathrm{IIA}}; \\
&(\mathbf{II}) \quad \mathrm{h}_{a'0} = \mathrm{h}_{a'k} = \mathrm{h}_{a'}{}^\lambda = \hat{\mathrm{h}}_\alpha{}^0 = \hat{\mathrm{h}}_\alpha{}^k = \hat{\mathrm{h}}_{\alpha\lambda} = 0 \Leftrightarrow \\
&\qquad w_{a'0} = w_{a'k} = w_{a'}{}^\lambda = \hat{w}_\alpha{}^0 = \hat{w}_\alpha{}^k = \hat{w}_{\alpha\lambda} = 0 \Rightarrow (2\partial_D V_{\mathrm{IIA}} - \rho\partial_\rho V_{\mathrm{IIA}}) \geq 6V_{\mathrm{IIA}}.
\end{aligned} \tag{5.9}$$

Also note that in the no-go scenarios corresponding to the above two cases, one has to impose those extra flux conditions about the vanishing of certain fluxes to determine the simplified axionic flux orbits from their generic expressions. However, given their nature of being independent of the saxion, it does not bother us for our purpose as we are only interested in considering the saxionic derivatives of the potential to look for the possible no-go inequalities.

B. *T*-dual de Sitter no-go-3 in type IIB

On the lines of computations done for the explicit *T*-dualization of the two de Sitter no-go scenarios, one can be convinced that the no-go-3 in (5.9) can easily be *T*-dualized to find new no-go scenarios on the type IIB side. For this to happen, the assumption to make is that type IIB compactification should be done on the CY threefolds which have $K3$- or $\mathbb{T}^4$-fibered mirror CYs. So this framework should not be confused with having type IIB compactification on the $K3$- or $\mathbb{T}^4$-fibered CY itself, although there might be a different set of no-go's for that case, but those would not be the ones we are considering as type IIA no-go-3.

Having said the above, now the complex structure side can be studied by the mirror CY, and hence will inherit the splitting of complex-structure moduli space on the type IIB side such that one can single out two complex structure moduli σ_0 and σ such that

$$u^1 = \sigma_0, \qquad u^{i'} = \sigma\lambda^{i'}, \qquad l_{1i'j'}\lambda^{i'}\lambda^{j'} = 2, \qquad \mathcal{U} = \sigma_0\sigma^2,$$
$$\mathcal{U}\mathcal{G}^{ij} = \begin{pmatrix} \sigma_0^2 & 0 \\ 0 & \sigma^2(\lambda^{i'}\lambda^{j'} - \tilde{l}^{i'j'}) \end{pmatrix}, \qquad \mathcal{U}\mathcal{G}_{ij} = \begin{pmatrix} \sigma^4 & 0 \\ 0 & \sigma_0^2\sigma^2(\tilde{l}_{i'}\tilde{l}_{j'} - \tilde{l}_{i'j'}) \end{pmatrix}, \tag{5.10}$$

where the indices i''s denote the remaining complex structure moduli different from u^1 and quantities such as $\tilde{l}_i$ are the ones that only depend on the angular complex structure moduli. Under these circumstances, the type IIB scalar potential can be explicitly given as

$$\begin{aligned} V_{\mathrm{IIB}} = & \frac{e^{4\phi}}{4\mathcal{V}^2\sigma_0\sigma^2}[(f_0)^2 + (\sigma^4(f^1)^2 + \sigma_0^2\sigma^2 f^{i'}(\tilde{l}_{i'}\tilde{l}_{j'} - \tilde{l}_{i'j'})f^{j'}) + (\sigma_0^2(f_1)^2 + \sigma^2 f_{i'}(\lambda^{i'}\lambda^{j'} - \tilde{l}^{i'j'})f_{j'}) + \sigma_0^2\sigma^4(f^0)^2] \\ & + \frac{e^{2\phi}}{4\mathcal{V}^2\sigma_0\sigma^2}\Big[h_0^2 + \mathcal{V}\mathcal{G}^{ab}h_{a0}h_{b0} + \mathcal{V}\mathcal{G}_{\alpha\beta}h^\alpha{}_0 h^\beta{}_0 + \sigma_0^2((h_1)^2 + \mathcal{V}\mathcal{G}_{\alpha\beta}h^\alpha{}_1 h^\beta{}_1 + \mathcal{V}\mathcal{G}^{ab}h_{a1}h_{b1}) \\ & + \sigma^2\lambda^{i'}\lambda^{j'}(h_{i'}h_{j'} + \mathcal{V}\mathcal{G}_{\alpha\beta}h^\alpha{}_{i'}h^\beta{}_{j'} + \mathcal{V}\mathcal{G}^{ab}h_{ai'}h_{bj'}) + \sigma^2(\lambda^{i'}\lambda^{j'} - \tilde{l}^{i'j'})\Big(h_{i'} - \frac{\ell_\alpha}{2}h^\alpha{}_{i'}\Big)\Big(h_{j'} - \frac{\ell_\beta}{2}h^\beta{}_{j'}\Big) \\ & + \sigma^2(\mathcal{V}\hat{h}_J{}^0 - t^\alpha\hat{h}_{\alpha J})(\hat{\ell}_{1JK})^{-1}(\mathcal{V}\hat{h}_K{}^0 - t^\beta\hat{h}_{\beta K})\Big] + \frac{e^{3\phi}}{\mathcal{V}^2}\Big[(f^0 h_0 - f^i h_i) - (f^0 h^\alpha{}_0 - f^i h^\alpha{}_i)\frac{\ell_\alpha}{2}\Big], \end{aligned} \tag{5.11}$$

where the only interest for us at the moment lies in the saxionic moduli σ_0 and σ, though for completion we do provide the explicit expressions for all the axionic flux orbits as

$$\begin{aligned} & f^0 = -F^0, \qquad f^1 = F^1 - v^1 F^0, \qquad f^{i'} = F^{i'} - v^{i'}F^0, \\ & f_1 = F_1 + l_{1i'j'}v^{i'}F^{j'} - \frac{1}{2}l_{1j'k'}v^{j'}v^{k'}F^0 - \omega_{a1}c^a - \hat{Q}^\alpha{}_1\hat{c}_\alpha - c_0 h_1, \\ & f_{i'} = F_{i'} + l_{1i'j'}(v^{j'}F^1 + v^1F^{j'}) - l_{1i'j}v^1v^{j'}F^0 - \omega_{ai'}c^a - \hat{Q}^\alpha{}_{i'}\hat{c}_\alpha - c_0 h_{i'}, \\ & f_0 = F_0 + v^1F_1 + v^{i'}F_{i'} + \frac{1}{2}l_{1i'j'}v^{i'}v^{j'}F^1 + l_{1i'j'}v^1v^{i'}F^{j'} \\ & \qquad - \frac{1}{2}l_{1i'j'}v^{i'}v^{j'}v^1F^0 - \omega_{a0}c^a - \hat{Q}^\alpha{}_0\hat{c}_\alpha - c_0 h_0, \\ & h_0 = H_0 + \omega_{a0}b^a + \frac{1}{2}\hat{\ell}_{\alpha ab}b^ab^b\hat{Q}^\alpha{}_0 + v^1h_1 + v^{i'}h_{i'}, \\ & h_1 = H_1 + \omega_{a1}b^a + \frac{1}{2}\hat{\ell}_{\alpha ab}b^ab^b\hat{Q}^\alpha{}_1, \qquad h_{i'} = H_{i'} + \omega_{ai'}b^a + \frac{1}{2}\hat{\ell}_{\alpha ab}b^ab^b\hat{Q}^\alpha{}_{i'}, \\ & h_{a0} = \omega_{a0} + \hat{Q}^\alpha{}_0\hat{\ell}_{\alpha ab}b^b + v^1h_{a1} + v^{i'}h_{ai'}, \qquad h_{a1} = \omega_{a1} + \hat{Q}^\alpha{}_1\hat{\ell}_{\alpha ab}b^b, \\ & h_{ai'} = \omega_{ai'} + \hat{Q}^\alpha{}_{i'}\hat{\ell}_{\alpha ab}b^b, \qquad h^\alpha{}_0 = \hat{Q}^\alpha{}_0 + v^1\hat{Q}^\alpha{}_1 + v^{i'}\hat{Q}^\alpha{}_{i'}, \qquad h^\alpha{}_1 = \hat{Q}^\alpha{}_1, \qquad h^\alpha{}_{i'} = \hat{Q}^\alpha{}_{i'}, \\ & \hat{h}_{\alpha K} = \hat{\omega}_{\alpha K} - Q^a{}_K\hat{\ell}_{\alpha ab}b^b + \frac{1}{2}\hat{\ell}_{\alpha ab}b^ab^bR_K, \qquad \hat{h}_K{}^0 = -R_K. \end{aligned} \tag{5.12}$$

A close look at the scalar potential in Eq. (5.11) confirms that one can have the following two T-dual cases:

$$\begin{aligned} (\mathbf{I}) \quad & h_1 = h_{a1} = h^\alpha{}_1 = 0 \Leftrightarrow H_1 = \omega_{a1} = \hat{Q}^\alpha{}_1 = 0 \\ & \Rightarrow (\partial_D V_{\mathrm{IIB}} - \sigma_0\partial_{\sigma_0}V_{\mathrm{IIB}}) \geq 3V_{\mathrm{IIA}}, \\ (\mathbf{II}) \quad & h_{i'} = h_{ai'} = h^\alpha{}_{i'} = \hat{h}_{\alpha K} = h^a{}_K = \hat{h}_K{}^0 = 0 \\ & \Leftrightarrow H_{i'} = \omega_{ai'} = \hat{Q}^\alpha{}_{i'} = \hat{\omega}_{\alpha K} = Q^a{}_K = R_K = 0 \\ & \Rightarrow (2\partial_D V_{\mathrm{IIB}} - \sigma\partial_\sigma V_{\mathrm{IIB}}) \geq 6V_{\mathrm{IIB}}. \end{aligned} \tag{5.13}$$

This result can be summarized in the following no-go condition.

TABLE V. Type IIA de Sitter no-go scenarios with $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ having geometric flux.

$\mathrm{h}_1 = \mathrm{h}_1{}^{\lambda} = 0$	$w_{10} = w_1{}^{\lambda} = 0$	$\partial_D V_{\mathrm{IIA}} - \rho_0 \partial_{\rho_0} V_{\mathrm{IIA}} \geq 3 V_{\mathrm{IIA}}$
$\mathrm{h}_2 = \mathrm{h}_2{}^{\lambda} = 0$	$w_{20} = w_2{}^{\lambda} = 0$	$\partial_D V_{\mathrm{IIA}} - \rho_0 \partial_{\rho_0} V_{\mathrm{IIA}} \geq 3 V_{\mathrm{IIA}}$
$\mathrm{h}_3 = \mathrm{h}_3{}^{\lambda} = 0$	$w_{30} = w_3{}^{\lambda} = 0$	$\partial_D V_{\mathrm{IIA}} - \rho_0 \partial_{\rho_0} V_{\mathrm{IIA}} \geq 3 V_{\mathrm{IIA}}$
$\mathrm{h}_{20} = \mathrm{h}_{30} = \mathrm{h}_2{}^{\lambda} = \mathrm{h}_3{}^{\lambda} = 0$	$w_{20} = w_{30} = w_2{}^{\lambda} = w_3{}^{\lambda} = 0$	$2\partial_D V_{\mathrm{IIA}} - \rho \partial_{\rho} V_{\mathrm{IIA}} \geq 6 V_{\mathrm{IIA}}$
$\mathrm{h}_{10} = \mathrm{h}_{30} = \mathrm{h}_1{}^{\lambda} = \mathrm{h}_3{}^{\lambda} = 0$	$w_{10} = w_{30} = w_1{}^{\lambda} = w_3{}^{\lambda} = 0$	$2\partial_D V_{\mathrm{IIA}} - \rho \partial_{\rho} V_{\mathrm{IIA}} \geq 6 V_{\mathrm{IIA}}$
$\mathrm{h}_{10} = \mathrm{h}_{20} = \mathrm{h}_1{}^{\lambda} = \mathrm{h}_2{}^{\lambda} = 0$	$w_{10} = w_{20} = w_1{}^{\lambda} = w_2{}^{\lambda} = 0$	$2\partial_D V_{\mathrm{IIA}} - \rho \partial_{\rho} V_{\mathrm{IIA}} \geq 6 V_{\mathrm{IIA}}$

Type IIB no-go theorem 3: *In type IIB framework with $O3/O7$ orientifold planes and (non)geometric fluxes along with the standard F_3, H_3 fluxes, one cannot have stable de Sitter minima with special solutions of Bianchi identities, if the complex structure moduli spaces exhibit a factorization on top of suitably having some of the flux components set to zero. This can happen when the mirror of the type IIB compactifying CY is a particular type of $K3/\mathbb{T}^4$-fibered CY threefold satisfying Eq. (5.1).*

C. More de Sitter no-go conditions for toroidal examples

This no-go-3 appears to be a rather complicated statement to make; however, it has several interesting implications. To illustrate what it means in a simple way, we consider the toroidal models based on type IIA and type IIB compactifications using orientifold of the $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold. To being with, let us mention that this no-go-3 can be applied directly to these conventional vanilla toroidal orientifold models which have been studied numerous times. This model has the only intersection number nonzero to be

$$\mathrm{IIA}: \quad \kappa_{123} = 1, \qquad k_{123} = 1,$$
$$\mathrm{IIB}: \quad \ell_{123} = 1, \qquad l_{123} = 1, \tag{5.14}$$

while all the other intersection numbers are zero. With the standard orientifold involution there are no D-terms present in type IIA or type IIB settings. So the total scalar potential arises from the F-term contributions itself. In addition, let us note that the even (1,1) cohomology is trivial in type IIA while the odd (1,1) cohomology is trivial in type IIB implying that fluxes/moduli with indices k in type IIA and with index a in type IIB are absent.

1. *Type IIA*

It turns out that 12 axionic flux orbits are identically zero in this construction, which in addition does not include nongeometric Q and R fluxes,

$$\mathrm{h}^a = 0 = \mathrm{h}^0, \quad \mathrm{h}_{k0} = \mathrm{h}_{ak} = \mathrm{h}^a{}_k = \mathrm{h}_k{}^0 = 0, \quad \mathrm{h}^{a\lambda} = 0 = \mathrm{h}^{\lambda 0},$$
$$\hat{\mathrm{h}}_\alpha{}^0 = \hat{\mathrm{h}}_{\alpha\lambda} = \hat{\mathrm{h}}^{\alpha 0} = \hat{\mathrm{h}}^\alpha{}_\lambda = 0. \tag{5.15}$$

As there can be equivalence between the three $\mathbb{T}^2$'s appearing in the six-torus, and therefore one can single out ρ_0 modulus from any of the three t^a's, say we take $t^1 = \rho_0$ and subsequently the remaining 2×2 sector in the Kähler moduli space is block diagonal. In fact, it is completely diagonal in all the three moduli, though we need it only partially. Noting that the only fluxes which can get nonzero values in this model are the following:

$$\mathrm{h}_0, \qquad \mathrm{h}_a, \qquad \mathrm{h}_a{}^{\lambda}, \qquad \mathrm{h}^{\lambda}{}_0, \tag{5.16}$$

our no-go-3 implies that one would end up having de Sitter no-go scenarios if one switches off certain fluxes as mentioned in Table V.

The particular models of Table V present those cases in which one would have de Sitter no-go irrespective of the fact whether the Romans mass term is zero or nonzero. This simply means that these are the examples in which geometric fluxes are not enough to evade the no-go-2 despite having nonzero Romans mass. Moreover, from the observations from Table V it is not hard to guess that if all the geometric fluxes are zero, one gets back to the no-go-1 having an inequality of the type $(3\partial_D V_{\mathrm{IIA}} - \rho \partial_\rho V_{\mathrm{IIA}}) \geq 9 V_{\mathrm{IIA}}$.

2. *Type IIB*

Now an interesting question to ask is what happens to the dual type IIB side that would involve nongeometric fluxes as well, unlike the type IIA case. It turns out that 12 axionic flux orbits are identically zero in this construction, and they are given as

$$h^0 = h^i = 0, \quad h_{a0} = h_{ai} = h_a{}^i = h_a{}^0 = 0, \quad h^{\alpha i} = h^{\alpha 0} = 0,$$
$$\hat{h}_K = \hat{h}_{\alpha K} = \hat{h}_\alpha{}^K = \hat{h}^K = 0. \tag{5.17}$$

Now due to symmetries in the intersection number l_{ijk}, one can single out a σ_0 modulus from any of the three complex structure saxions u^i's, say we take $u^1 = \sigma_0$ and subsequently the remaining 2×2 sector in the complex structure moduli space is block diagonal, and one can write $\mathcal{U} = \sigma_0 \sigma^2$. As before, it is completely diagonal in all three moduli. Noting that the only fluxes which can get nonzero values in this model are the following ones:

$$h_0, \qquad h_i, \qquad h^{\alpha 0}, \qquad h^\alpha{}_i, \tag{5.18}$$

TABLE VI. Type IIB de Sitter no-go scenarios with $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ having (non)geometric fluxes.

$h_1 = h^\alpha{}_1 = 0$	$H_1 = \hat{Q}^\alpha{}_1 = 0$	$(\partial_\phi V_{\rm IIB} - \sigma_0 \partial_{\sigma_0} V_{\rm IIB}) \geq 3V_{\rm IIB}$
$h_2 = h^\alpha{}_2 = 0$	$H_2 = \hat{Q}^\alpha{}_2 = 0$	$(\partial_\phi V_{\rm IIB} - \sigma_0 \partial_{\sigma_0} V_{\rm IIB}) \geq 3V_{\rm IIB}$
$h_3 = h^\alpha{}_3 = 0$	$H_3 = \hat{Q}^\alpha{}_3 = 0$	$(\partial_\phi V_{\rm IIB} - \sigma_0 \partial_{\sigma_0} V_{\rm IIB}) \geq 3V_{\rm IIB}$
$h_2 = h_3 = h^\alpha{}_2 = h^\alpha{}_3 = 0$	$H_2 = H_3 = \hat{Q}^\alpha{}_2 = \hat{Q}^\alpha{}_3 = 0$	$(2\partial_\phi V_{\rm IIB} - \sigma \partial_\sigma V_{\rm IIB}) \geq 6V_{\rm IIB}$
$h_3 = h_1 = h^\alpha{}_3 = h^\alpha{}_1 = 0$	$H_3 = H_1 = \hat{Q}^\alpha{}_3 = \hat{Q}^\alpha{}_1 = 0$	$(2\partial_\phi V_{\rm IIB} - \sigma \partial_\sigma V_{\rm IIB}) \geq 6V_{\rm IIB}$
$h_1 = h_2 = h^\alpha{}_1 = h^\alpha{}_2 = 0$	$H_1 = H_2 = \hat{Q}^\alpha{}_1 = \hat{Q}^\alpha{}_2 = 0$	$(2\partial_\phi V_{\rm IIB} - \sigma \partial_\sigma V_{\rm IIB}) \geq 6V_{\rm IIB}$

our no-go-3 implies that one would end up having de Sitter no-go scenarios if one switches off certain fluxes as mentioned in Table VI. The particular models of Table VI present those cases in which one would have de Sitter no-go irrespective of the fact whether the F^0 components of the RR F_3 flux is zero or nonzero, and moreover despite having some nongeometric fluxes being turned on. This means that these are the examples in which nongeometric fluxes are not enough to evade the no-go-2 due to the presence of some specific geometries inherited from the six-torus.

VI. SUMMARY AND CONCLUSIONS

In this article, we have T-dualized several de Sitter no-go scenarios which have been well known in the type IIA flux compactifications for more than a decade. This subsequently leads to a set of peculiar de Sitter no-go scenarios in the type IIB flux compactifications with (non)geometric fluxes.

Before exploring the de Sitter no-go scenarios, we have studied the solutions of Bianchi identities in the type IIA and type IIB theories as the same is crucial for finding a genuinely effective scalar potential. In this context we present a peculiar class of solutions, what we call the special solutions of Bianchi identities, in each of the two type II theories. The main idea behind the existence of such solutions is the fact that several Bianchi identities can be understood as a set of orthogonal symplectic (flux) vectors, and hence half of the flux components can be rotated away by a symplectic transformation. The possible nonzero fluxes for the special solutions are summarized in Table II. Moreover, after exploring the T-dual versions of these special solutions from type IIA to type IIB and vice versa, we make some very interesting observations as collected in the following points:

(i) The nongeometric type IIA setup with the special solutions of Bianchi identities is equivalent to the type IIB setup without any nongeometric fluxes. Moreover, for such a type IIB geometric setup with $O3/O7$, there is a de Sitter no-go theorem [16,29], which we have also rederived from our approach. This helps us in concluding that the T-dual type IIA setting, which although includes some nongeometric fluxes, cannot result in stable de Sitter vacua, and this is something against the naive expectations.

(ii) The nongeometric type IIB setup with special solutions of Bianchi identities is equivalent to the type IIA setup without any nongeometric fluxes turned on. Such a type IIA setup has been studied in a variety of models in the past, especially regarding the search of de Sitter vacua and their no-go conditions [5,7,8,45].

In this context of type IIA orientifold compactifications with geometric flux, first we have rederived several de Sitter no-go scenarios of [5,8] and have subsequently explored their T-dual counterparts in type IIB theory. In particular, we have T-dualized three classes of type IIA no-go scenarios that are summarized in Table VII. These can be elaborated as follows:

(i) no-go-1: Type IIB nongeometric setup with $O3/O7$ and having RR flux F_3 along with only the rigid

TABLE VII. T-dual fluxes relevant for the three no-go scenarios.

Scenarios		Fluxes in type IIA with $D6/O6$	Fluxes in type IIB with $D3/O3$ and $D7/O7$
No-go-1	F-term fluxes	H_0, H_k, H^λ, e_0, e_a, m^a, m_0.	H_0, ω_{a0}, $\hat{Q}^\alpha{}_0$, F_0, F_i, F^i, $-F^0$.
No-go-2 and no-go-3	F-term fluxes	H_0, H_k, H^λ, w_{a0}, w_{ak}, $w_a{}^\lambda$, e_0, e_a, m^a, m_0.	H_0, ω_{a0}, $\hat{Q}^\alpha{}_0$, H_i, ω_{ai}, $\hat{Q}^\alpha{}_i$, F_0, F_i, F^i, $-F^0$.
	D-term fluxes	$\hat{w}_\alpha{}^0$, $\hat{w}_\alpha{}^k$, $\hat{w}_{\alpha\lambda}$.	$-R_K$, $-Q^a{}_K$, $\hat{\omega}_{\alpha K}$.
No-scale-structure in IIB	F-term fluxes	H_0, w_{a0}, $\mathrm{Q}^a{}_0$, R_0, e_0, e_a, m^a, m_0.	H_0, H_i, H^i, $-H^0$, F_0, F_i, F^i, $-F^0$.

fluxes H_0, ω_{a0}, and $\hat{Q}^\alpha{}_0$ cannot give stable de Sitter vacua.

(ii) no-go-2: Type IIB nongeometric setup with $O3/O7$ and having RR flux F_3 along with only the special solutions of the NS-NS Bianchi identities cannot give stable de Sitter vacua unless the F^0 component of the F_3 flux is nonzero.

(iii) no-go-3: This no-go scenario is rather a restoration of the no-go-2 itself, in the sense of F^0 being zero or nonzero getting irrelevant. This can be done by choosing certain compactification geometries that have factorization in the complex structure moduli space. To be specific, the violation of no-go-2 via including the nonzero F^0 flux (of F_3) can be avoided if the type IIB compactification is made on a CY threefold that admits a $K3/\mathbb{T}^4$-fibered mirror Calabi Yau threefold having some specific triple intersection numbers along with the need of setting a couple of fluxes to zero.

Note that in Table VII we have also collected the T-dual fluxes corresponding to the type IIB no-scale model that has only the F_3 and H_3 fluxes. This subsequently shows that in the dual type IIA side, one has all the RR fluxes and NS-NS fluxes of the rigid type only, for which we have already shown that a de Sitter no-go condition exists.

To conclude, we have shown in this analysis how one can engineer a pair of T-dual setups in type IIA and type IIB theories in which it may be easier to derive some de Sitter no-go conditions that can be translated into the mirror side. By considering multiple examples, we have presented a kind of recipe for evading or further restoring the no-go window depending on the various ingredients, including the compactification geometries, one could use. Thus one of the main advantages of this work can also be taken as where not to look for the de Sitter search, and hence refining the vast nongeometric flux landscape for hunting the de Sitter vacua. Moreover, our analysis can also be extended to utilize/investigate the nongeometric type II models for/against the recently proposed trans-Planckian censorship conjecture [123] and also its possible connection with the swampland distance conjecture. We hope to report on (some of) these issues in the near future [124].

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APPENDIX: A DICTIONARY FOR THE TYPE II NONGEOMETRIC FLUX COMPACTIFICATIONS

TABLE VIII. One-to-one T-duality transformations among the various fluxes, the moduli, and the axions.

	Type IIA with $D6/O6$	Type IIB with $D3/O3$ and $D7/O7$
F-term fluxes	H_0, H_k, H^λ,	H_0, ω_{a0}, $\hat{Q}^\alpha{}_0$,
	w_{a0}, w_{ak}, $w_a{}^\lambda$,	H_i, ω_{ai}, $\hat{Q}^\alpha{}_i$,
	$\mathrm{Q}^a{}_0$, $\mathrm{Q}^a{}_k$, $\mathrm{Q}^{a\lambda}$,	H^i, $\omega_a{}^i$, $\hat{Q}^{\alpha i}$,
	R_0, R_k, R^λ,	$-H^0$, $-\omega_a{}^0$, $-\hat{Q}^{\alpha 0}$,
	e_0, e_a, m^a, m_0.	F_0, F_i, F^i, $-F^0$.
D-term fluxes	$\hat{w}_\alpha{}^0$, $\hat{w}_\alpha{}^k$, $\hat{w}_{\alpha\lambda}$,	$-R_K$, $-Q^a{}_K$, $\hat{\omega}_{\alpha K}$,
	$\hat{\mathrm{Q}}^{\alpha 0}$, $\hat{\mathrm{Q}}^{\alpha k}$, $\hat{\mathrm{Q}}^\alpha{}_\lambda$.	$-R^K$, $-Q^{aK}$, $\hat{\omega}_\alpha{}^K$.
Complex moduli	N^0, N^k, U_λ, T^a.	S, G^a, T_α, U^i.
	$\mathrm{T}^a = \mathrm{b}^a - i\mathrm{t}^\mathrm{a}$,	$U^i = v^i - iu^i$,
	$\mathrm{N}^0 = \xi^0 + i(\mathrm{z}^0)^{-1}$,	$S = c_0 + is$,
	$\mathrm{N}^k = \xi^k + i(\mathrm{z}^0)^{-1}\mathrm{z}^k$,	$G^a = (c^a + c_0 b^a) + isb^a$,
	$\mathrm{U}_\lambda = -\frac{i}{2\mathrm{z}^0}(k_{\lambda\rho\kappa}\mathrm{z}^\rho\mathrm{z}^\kappa - \hat{k}_{\lambda km}\mathrm{z}^k\mathrm{z}^m) + \xi_\lambda$.	$T_\alpha = -\frac{is}{2}(\ell_{\alpha\beta\gamma}t^\beta t^\gamma - \hat{\ell}_{\alpha ab}b^a b^b)$ $+(c_\alpha + \hat{\ell}_{\alpha ab}c^a b^b + \frac{1}{2}c_0\hat{\ell}_{\alpha ab}b^a b^b)$.
Axions	z^k, b^a, ξ^0, ξ^k,	b^a, v^i, c_0, $c^a + c_0 b^a$,
	ξ_λ.	$c_\alpha + \hat{\ell}_{\alpha ab}c^a b^b + \frac{1}{2}c_0\hat{\ell}_{\alpha ab}b^a b^b$.
Saxions	$(\mathrm{z}^0)^{-1}$, z^λ, t^a, $\mathcal{V}$, $\mathcal{U}$.	$s \equiv e^{-\phi}$, t^α, u^i $\mathcal{U}$, $\mathcal{V}$.
Inter-sections	$k_{\lambda\rho\mu}$, $\hat{k}_{\lambda mn}$, κ_{abc}, $\hat{\kappa}_{a\alpha\beta}$.	$\ell_{\alpha\beta\gamma}$, $\hat{\ell}_{\alpha ab}$, l_{ijk}, $\hat{l}_{iJK}$.

TABLE IX. Axionic flux orbits for the type IIA side.

	Type IIA flux orbits
f_0	$\mathbb{G}_0 - \xi^{\hat{k}}\mathcal{H}_{\hat{k}} - \xi_\lambda \mathcal{H}^\lambda$
f_a	$\mathbb{G}_a - \xi^{\hat{k}}\mho_{a\hat{k}} - \xi_\lambda \mho_a{}^\lambda$
f^a	$\mathbb{G}^a - \xi^{\hat{k}}\mathcal{Q}^a{}_{\hat{k}} - \xi_\lambda \mathcal{Q}^{a\lambda}$
f^0	$\mathbb{G}^0 - \xi^{\hat{k}}\mathcal{R}_{\hat{k}} - \xi_\lambda \mathcal{R}^\lambda$
h_0	$\mathcal{H}_0 + \mathcal{H}_k \mathrm{z}^{\mathrm{k}} + \frac{1}{2}\hat{k}_{\lambda mn}\mathrm{z}^{\mathrm{m}}\mathrm{z}^{\mathrm{n}}\mathcal{H}^\lambda$
h_a	$\mho_{a0} + \mho_{ak}\mathrm{z}^{\mathrm{k}} + \frac{1}{2}\hat{k}_{\lambda mn}\mathrm{z}^{\mathrm{m}}\mathrm{z}^{\mathrm{n}}\mho_{\mathrm{a}}{}^\lambda$
h^a	$\mathcal{Q}^a{}_0 + \mathcal{Q}^a{}_k\mathrm{z}^{\mathrm{k}} + \frac{1}{2}\hat{k}_{\lambda mn}\mathrm{z}^{\mathrm{m}}\mathrm{z}^{\mathrm{n}}\mathcal{Q}^{a\lambda}$
h^0	$\mathcal{R}_0 + \mathcal{R}_k\mathrm{z}^{\mathrm{k}} + \frac{1}{2}\hat{k}_{\lambda mn}\mathrm{z}^{\mathrm{m}}\mathrm{z}^{\mathrm{n}}\mathcal{R}^\lambda$
h_{k0}	$\mathcal{H}_k + \hat{k}_{\lambda kn}\mathrm{z}^{\mathrm{n}}\mathcal{H}^\lambda$
h_{ak}	$\mho_{ak} + \hat{k}_{\lambda kn}\mathrm{z}^{\mathrm{n}}\mho_a{}^\lambda$
$\mathrm{h}^a{}_k$	$\mathcal{Q}^a{}_k + \hat{k}_{\lambda kn}\mathrm{z}^{\mathrm{n}}\mathcal{Q}^{a\lambda}$
$\mathrm{h}_k{}^0$	$\mathcal{R}_k + \hat{k}_{\lambda kn}\mathrm{z}^{\mathrm{n}}\mathcal{R}^\lambda$
$\mathrm{h}^\lambda{}_0$	$\mathcal{H}^\lambda$
$\mathrm{h}_a{}^\lambda$	$\mho_a{}^\lambda$
$\mathrm{h}^{a\lambda}$	$\mathcal{Q}^{a\lambda}$
$\mathrm{h}^{\lambda 0}$	$\mathcal{R}^\lambda$
F-term fluxes	$\mathbb{G}_0 = \bar{e}_0 + \mathrm{b}^a\bar{e}_a + \frac{1}{2}\kappa_{abc}\mathrm{b}^a\mathrm{b}^b m^c + \frac{1}{6}\kappa_{abc}\mathrm{b}^a\mathrm{b}^b\mathrm{b}^c m_0,$
	$\mathbb{G}_a = \bar{e}_a + \kappa_{abc}\mathrm{b}^b m^c + \frac{1}{2}\kappa_{abc}\mathrm{b}^b\mathrm{b}^c m_0,$
	$\mathbb{G}^a = m^a + m_0\mathrm{b}^a,$
	$\mathbb{G}^0 = m_0,$
	$\mathcal{H}_{\hat{k}} = \bar{\mathrm{H}}_{\hat{k}} + \bar{w}_{a\hat{k}}\mathrm{b}^a + \frac{1}{2}\kappa_{abc}\mathrm{b}^b\mathrm{b}^c\mathrm{Q}^a{}_{\hat{k}} + \frac{1}{6}\kappa_{abc}\mathrm{b}^a\mathrm{b}^b\mathrm{b}^c\mathrm{R}_{\hat{k}},$
	$\mathcal{H}^\lambda = \bar{\mathrm{H}}^\lambda + \bar{w}_a{}^\lambda\mathrm{b}^a + \frac{1}{2}\kappa_{abc}\mathrm{b}^b b^c\mathrm{Q}^{a\lambda} + \frac{1}{6}\kappa_{abc}\mathrm{b}^a\mathrm{b}^b\mathrm{b}^c\mathrm{R}^\lambda,$
	$\mho_{a\hat{k}} = \bar{w}_{a\hat{k}} + \kappa_{abc}\mathrm{b}^b\mathrm{Q}^c{}_{\hat{k}} + \frac{1}{2}\kappa_{abc}\mathrm{b}^b\mathrm{b}^c\mathrm{R}_{\hat{k}},$
	$\mho_a{}^\lambda = \bar{w}_a{}^\lambda + \kappa_{abc}\mathrm{b}^b\mathrm{Q}^{c\lambda} + \frac{1}{2}\kappa_{abc}\mathrm{b}^b\mathrm{b}^c\mathrm{R}^\lambda,$
	$\mathcal{Q}^a{}_{\hat{k}} = \mathrm{Q}^a{}_{\hat{k}} + \mathrm{b}^a\mathrm{R}_{\hat{k}},\ \mathcal{Q}^{a\lambda} = \mathrm{Q}^{a\lambda} + \mathrm{b}^a\mathrm{R}^\lambda,$
	$\mathcal{R}_{\hat{k}} = \mathrm{R}_{\hat{k}},\ \mathcal{R}^\lambda = \mathrm{R}^\lambda.$
D-term fluxes	$\hat{\mathrm{h}}_{\alpha\lambda} \equiv \hat{\mho}_{\alpha\lambda} = \hat{w}_{\alpha\lambda} + \hat{k}_{\lambda km}\mathrm{z}^m\hat{w}_\alpha{}^k - \frac{1}{2}\hat{k}_{\lambda km}\mathrm{z}^k\mathrm{z}^m\hat{w}_\alpha{}^0$
	$\hat{\mathrm{h}}_\alpha{}^k \equiv \hat{\mho}_\alpha{}^k = \hat{w}_\alpha{}^k - \mathrm{z}^k\hat{w}_\alpha{}^0,\ \hat{\mathrm{h}}_\alpha{}^0 \equiv \hat{\mho}_\alpha{}^0 = \hat{w}_\alpha{}^0,$
	$\hat{\mathrm{h}}^\alpha{}_\lambda \equiv \hat{\mathcal{Q}}^\alpha{}_\lambda = \hat{\mathrm{Q}}^\alpha{}_\lambda + \hat{k}_{\lambda km}\mathrm{z}^m\hat{\mathrm{Q}}^{\alpha k} - \frac{1}{2}\hat{k}_{\lambda km}\mathrm{z}^\lambda\mathrm{z}^k\mathrm{z}^m\hat{\mathcal{Q}}^{\alpha 0},$
	$\hat{\mathrm{h}}^{\alpha k} \equiv \hat{\mathcal{Q}}^{\alpha k} = \hat{\mathrm{Q}}^{\alpha k} - \mathrm{z}^k\hat{\mathrm{Q}}^{\alpha 0},\ \hat{\mathrm{h}}^{\alpha 0} \equiv \hat{\mathcal{Q}}^{\alpha 0} = \hat{\mathrm{Q}}^{\alpha 0}.$

TABLE X. Axionic type IIB flux orbits with their dual type IIA counterpart.

	Type IIB flux orbits	Dual type IIA
f_0	$\mathbb{F}_0 + v^i\mathbb{F}_i + \frac{1}{2}l_{ijk}v^jv^k\mathbb{F}^i - \frac{1}{6}l_{ijk}v^iv^jv^k\mathbb{F}^0$	f_0
f_i	$\mathbb{F}_i + l_{ijk}v^j\mathbb{F}^k - \frac{1}{2}l_{ijk}v^jv^k\mathbb{F}^0$	f_a
f^i	$\mathbb{F}^i - v^i\mathbb{F}^0$	f^a
f^0	$-\mathbb{F}^0$	f^0
h_0	$\mathbb{H}_0 + v^i\mathbb{H}_i + \frac{1}{2}l_{ijk}v^jv^k\mathbb{H}^i - \frac{1}{6}l_{ijk}v^iv^jv^k\mathbb{H}^0$	h_0
h_i	$\mathbb{H}_i + l_{ijk}v^j\mathbb{H}^k - \frac{1}{2}l_{ijk}v^jv^k\mathbb{H}^0$	h_a
h^i	$\mathbb{H}^i - v^i\mathbb{H}^0$	h^a
h^0	$-\mathbb{H}^0$	h^0

(Table continued)

TABLE X. *(Continued)*

	Type IIB flux orbits	Dual type IIA
h_{a0}	$\mho_{a0} + v^i \mho_{ai} + \frac{1}{2} l_{ijk} v^j v^k \mho_a{}^i - \frac{1}{6} l_{ijk} v^i v^j v^k \mho_a{}^0$	h_{k0}
h_{ai}	$\mho_{ai} + l_{ijk} v^j \mho_a{}^k - \frac{1}{2} l_{ijk} v^j v^k \mho_a{}^0$	h_{ak}
$h_a{}^i$	$\mho_a{}^i - v^i \mho_a{}^0$	$\mathrm{h}^a{}_k$
$h_a{}^0$	$-\mho_a{}^0$	$\mathrm{h}_k{}^0$
$h^\alpha{}_0$	$\hat{\mathbb{Q}}_0{}^\alpha + v^i \hat{\mathbb{Q}}_i{}^\alpha + \frac{1}{2} l_{ijk} v^j v^k \hat{\mathbb{Q}}^{\alpha i} - \frac{1}{6} l_{ijk} v^i v^j v^k \hat{\mathbb{Q}}^{\alpha 0}$	$\mathrm{h}^\lambda{}_0$
$h^\alpha{}_i$	$\hat{\mathbb{Q}}_i{}^\alpha + l_{ijk} v^j \hat{\mathbb{Q}}^{\alpha k} - \frac{1}{2} l_{ijk} v^j v^k \hat{\mathbb{Q}}^{\alpha 0}$	$\mathrm{h}_a{}^\lambda$
$h^{\alpha i}$	$\hat{\mathbb{Q}}^{\alpha i} - v^i \hat{\mathbb{Q}}^{\alpha 0}$	$\mathrm{h}^{a\lambda}$
$h^{\alpha 0}$	$-\hat{\mathbb{Q}}^{\alpha 0}$	$\mathrm{h}^{\lambda 0}$
F-term fluxes	$\mathbb{F}_\Lambda = \bar{F}_\Lambda - \bar{\omega}_{a\Lambda} c^a - \overline{\hat{Q}^\alpha}_\Lambda (c_\alpha + \hat{\ell}_{\alpha ab} c^a b^b) - c_0 \mathbb{H}_\Lambda$	
	$\mathbb{F}^\Lambda = F^\Lambda - \omega_a{}^\Lambda c^a - \hat{Q}^{\alpha\Lambda} (c_\alpha + \hat{\ell}_{\alpha ab} c^a b^b) - c_0 \mathbb{H}^\Lambda$	
	$\mathbb{H}_\Lambda = \bar{H}_\Lambda + \bar{\omega}_{a\Lambda} b^a + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b \overline{\hat{Q}^\alpha}_\Lambda$	
	$\mathbb{H}^\Lambda = H^\Lambda + \omega_a{}^\Lambda b^a + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b \hat{Q}^{\alpha\Lambda}$	
	$\mho_{a\Lambda} = \bar{\omega}_{a\Lambda} + \overline{\hat{Q}^\alpha}_\Lambda \hat{\ell}_{\alpha ab} b^b$	
	$\mho_a{}^\Lambda = \omega_a{}^\Lambda + \hat{Q}^{\alpha\Lambda} \hat{\ell}_{\alpha ab} b^b$	
	$\hat{\mathbb{Q}}^\alpha{}_\Lambda = \overline{\hat{Q}^\alpha}_\Lambda,\ \hat{\mathbb{Q}}^{\alpha\Lambda} = \hat{Q}^{\alpha\Lambda}$	
D-term fluxes	$\hat{h}_{\alpha K} \equiv \hat{\mho}_{\alpha K} = \hat{\omega}_{\alpha K} - Q^a{}_K \hat{\ell}_{\alpha ab} b^b + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b R_K$	$\hat{\mathrm{h}}_{\alpha\lambda}$
	$\hat{h}_\alpha{}^K \equiv \hat{\mho}_\alpha{}^K = \hat{\omega}_\alpha{}^K - Q^{aK} \hat{\ell}_{\alpha ab} b^b + \frac{1}{2} \hat{\ell}_{\alpha ab} b^a b^b R^K$	$\hat{\mathrm{h}}^\alpha{}_\lambda$
	$h^a{}_K \equiv \mathbb{Q}^a{}_K = -Q^a{}_K + R_K b^a,\ h^{aK} \equiv \mathbb{Q}^{aK} = -Q^{aK} + R^K b^a$	$\hat{\mathrm{h}}_\alpha{}^k,\ \hat{\mathrm{h}}^{\alpha k}$
	$\hat{h}_K{}^0 \equiv -\mathbb{R}_K = -R_K,\ \hat{h}^{K0} \equiv -\mathbb{R}^K = -R^K$	$\hat{\mathrm{h}}_\alpha{}^0,\ \hat{\mathrm{h}}^{\alpha 0}$

TABLE XI. A one-to-one exchange of the type IIA and type IIB scalar potentials under *T*-duality.

IIA	$V_{\mathrm{IIA}}^{\mathrm{tot}} = \frac{e^{4D}}{4\mathcal{V}} [\mathrm{f}_0^2 + \mathcal{V} \mathrm{f}^a \tilde{\mathcal{G}}_{ab} \mathrm{f}^b + \mathcal{V} \mathrm{f}_a \tilde{\mathcal{G}}^{ab} \mathrm{f}_b + \mathcal{V}^2 (\mathrm{f}^0)^2] + \frac{e^{2D}}{4\mathcal{U}\mathcal{V}} [\mathrm{h}_0^2 + \mathcal{V} \mathrm{h}^a \tilde{\mathcal{G}}_{ab} \mathrm{h}^b + \mathcal{V} \mathrm{h}_a \tilde{\mathcal{G}}^{ab} \mathrm{h}_b + \mathcal{V}^2 (\mathrm{h}^0)^2 + \mathcal{U} \tilde{\mathcal{G}}^{ij} (\mathrm{h}_{i0} \mathrm{h}_{j0} + \frac{\kappa_a \kappa_b}{4} \mathrm{h}_i{}^a \mathrm{h}_j{}^b + \mathrm{h}_{ai} \mathrm{h}_{bj} \mathrm{t}^a \mathrm{t}^b + \mathcal{V}^2 \mathrm{h}_i{}^0 \mathrm{h}_j{}^0 - \frac{\kappa_a}{2} \mathrm{h}^a{}_i \mathrm{h}_{j0} - \frac{\kappa_a}{2} \mathrm{h}_{i0} \mathrm{h}^a{}_j - \mathcal{V} \mathrm{t}^a \mathrm{h}_i{}^0 \mathrm{h}_{aj} - \mathcal{V} \mathrm{t}^a \mathrm{h}_{ai} h_j{}^0) + \mathcal{U} \tilde{\mathcal{G}}_{\lambda\rho} (\mathrm{h}^\lambda{}_0 \mathrm{h}^\rho{}_0 + \frac{\kappa_a \kappa_b}{4} \mathrm{h}^{\lambda a} \mathrm{h}^{\rho b} + \mathrm{t}^a \mathrm{t}^b \mathrm{h}_a{}^\lambda \mathrm{h}_b{}^\rho + \mathcal{V}^2 \mathrm{h}^{\lambda 0} \mathrm{h}^{\rho 0} - \frac{\kappa_a}{2} \mathrm{h}^\lambda{}_0 \mathrm{h}^{\rho a} - \frac{\kappa_a}{2} \mathrm{h}^{\lambda a} \mathrm{h}^\rho{}_0 - \mathcal{V} \mathrm{t}^a \mathrm{h}^{\lambda 0} \mathrm{h}_a{}^\rho - \mathcal{V} \mathrm{t}^a \mathrm{h}_a{}^\lambda \mathrm{h}^{\rho 0}) + \frac{k_\lambda k_\rho}{4} (\mathcal{V} \mathrm{h}^{a\lambda} \tilde{\mathcal{G}}_{ab} \mathrm{h}^{b\rho} + \mathcal{V} \mathrm{h}_a{}^\lambda \tilde{\mathcal{G}}^{ab} \mathrm{h}_b{}^\rho + \mathcal{V} \mathrm{t}^a \mathrm{h}^{\lambda 0} \mathrm{h}_a{}^\rho + \mathcal{V} \mathrm{t}^a \mathrm{h}_a{}^\lambda \mathrm{h}^{\rho 0} - \mathrm{t}^a \mathrm{t}^b \mathrm{h}_a{}^\lambda \mathrm{h}_b{}^\rho + \frac{\kappa_a}{2} \mathrm{h}^\lambda{}_0 \mathrm{h}^{a\rho} + \frac{\kappa_a}{2} \mathrm{h}^{a\lambda} \mathrm{h}^\beta{}_0 - \frac{\kappa_a \kappa_b}{4} \mathrm{h}^{a\lambda} \mathrm{h}^{b\rho}) - 2 \times \frac{k_\lambda}{2} (\mathcal{V} \mathrm{h}^a \tilde{\mathcal{G}}_{ab} \mathrm{h}^{b\lambda} + \mathcal{V} \mathrm{h}_a \tilde{\mathcal{G}}^{ab} \mathrm{h}_b{}^\lambda + \mathcal{V} \mathrm{t}^a \mathrm{h}^0 \mathrm{h}_a{}^\lambda + \mathcal{V} \mathrm{t}^a \mathrm{h}_a \mathrm{h}^{\lambda 0} - \mathrm{t}^a \mathrm{t}^b \mathrm{h}_a \mathrm{h}_b{}^\lambda + \frac{\kappa_a}{2} \mathrm{h}^a \mathrm{h}_0{}^\lambda + \frac{\kappa_a}{2} \mathrm{h}_0 \mathrm{h}^{a\lambda} - \frac{\kappa_a \kappa_b}{4} \mathrm{h}^a \mathrm{h}^{b\lambda}) + [(\mathcal{U} \hat{\mathrm{h}}_\alpha{}^0 + \mathrm{z}^\lambda \hat{\mathrm{h}}_{\alpha\lambda}) \mathcal{V} \tilde{\mathcal{G}}^{\alpha\beta} (\mathcal{U} \hat{\mathrm{h}}_\beta{}^0 + \mathrm{z}^\rho \hat{\mathrm{h}}_{\beta\rho}) + (\mathcal{U} \hat{\mathrm{h}}^{\alpha 0} + \mathrm{z}^\lambda \hat{\mathrm{h}}^\alpha{}_\lambda) \mathcal{V} \tilde{\mathcal{G}}_{\alpha\beta} (\mathcal{U} \mathrm{h}^{\beta 0} + \mathrm{z}^\rho \hat{\mathrm{h}}^\beta{}_\rho)]] + \frac{e^{3D}}{2\sqrt{\mathcal{U}}} [(\mathrm{f}^0 \mathrm{h}_0 - \mathrm{f}^a \mathrm{h}_a + \mathrm{f}_a \mathrm{h}^a - \mathrm{f}_0 \mathrm{h}^0) - (\mathrm{f}^0 \mathrm{h}^\lambda{}_0 - \mathrm{f}^a \mathrm{h}^\lambda{}_a + \mathrm{f}_a \mathrm{h}^{\lambda a} - \mathrm{f}_0 \mathrm{h}^{\lambda 0}) \frac{k_\lambda}{2}].$
	$\tilde{\mathcal{G}}_{ab} = \frac{\kappa_a \kappa_b - 4\mathcal{V}\kappa_{ab}}{4\mathcal{V}},\ \tilde{\mathcal{G}}^{ab} = \frac{2\mathrm{t}^a \mathrm{t}^b - 4\mathcal{V}\kappa^{ab}}{4\mathcal{V}},\ \tilde{\mathcal{G}}_{\alpha\beta} = -\hat{\kappa}_{\alpha\beta},\ \tilde{\mathcal{G}}^{\alpha\beta} = -\hat{\kappa}^{\alpha\beta},$
	$\tilde{\mathcal{G}}_{\lambda\rho} = \frac{k_\lambda k_\rho - 4\mathcal{U} k_{\lambda\rho}}{4\mathcal{U}},\ \tilde{\mathcal{G}}^{\lambda\rho} = \frac{2\mathrm{z}^\lambda \mathrm{z}^\rho - 4\mathcal{U} k^{\lambda\rho}}{4\mathcal{U}},\ \tilde{\mathcal{G}}_{jk} = -\hat{k}_{jk},\ \tilde{\mathcal{G}}^{jk} = -\hat{k}^{jk}.$
IIB	$V_{\mathrm{IIB}}^{\mathrm{tot}} = \frac{e^{4\phi}}{4\mathcal{V}^2\mathcal{U}} [f_0^2 + \mathcal{U} f^i \mathcal{G}_{ij} f^j + \mathcal{U} f_i \mathcal{G}^{ij} f_j + \mathcal{U}^2 (f^0)^2] + \frac{e^{2\phi}}{4\mathcal{V}^2\mathcal{U}} [h_0^2 + \mathcal{U} h^i \mathcal{G}_{ij} h^j + \mathcal{U} h_i \mathcal{G}^{ij} h_j + \mathcal{U}^2 (h^0)^2 + \mathcal{V} \mathcal{G}^{ab} (h_{a0} h_{b0} + \frac{l_i l_j}{4} h_a{}^i h_b{}^j + h_{ai} h_{bj} u^i u^j + \mathcal{U}^2 h_a{}^0 h_b{}^0 - \frac{l_i}{2} h_a{}^i h_{b0} - \frac{l_i}{2} h_{a0} h_b{}^i - \mathcal{U} u^i h_a{}^0 h_{bi} - \mathcal{U} u^i h_b{}^0 h_{ai}) + \mathcal{V} \mathcal{G}_{\alpha\beta} (h^\alpha{}_0 h^\beta{}_0 + \frac{l_i l_j}{4} h^{\alpha i} h^{\beta j} + u^i u^j h^\alpha{}_i h^\beta{}_j + \mathcal{U}^2 h^{\alpha 0} h^{\beta 0} - \frac{l_i}{2} h^\alpha{}_0 h^{\beta i} - \frac{l_i}{2} h^{\alpha i} h^\beta{}_0 - \mathcal{U} u^i h^{\alpha 0} h^\beta{}_i - \mathcal{U} u^i h^\alpha{}_i h^{\beta 0}) + \frac{\ell_\alpha \ell_\beta}{4} (\mathcal{U} h^{\alpha i} \mathcal{G}_{ij} h^{\beta j} + \mathcal{U} h^\alpha{}_i \mathcal{G}^{ij} h^\beta{}_j + \mathcal{U} u^i h^{\alpha 0} h_i{}^\beta + \mathcal{U} u^i h^\alpha{}_i h^{\beta 0} - u^i u^j h^\alpha{}_i h^\beta{}_j + \frac{l_i}{2} h^\alpha{}_0 h^{\beta i} + \frac{l_i}{2} h^{\alpha i} h^\beta{}_0 - \frac{l_i l_j}{4} h^{\alpha i} h^{\beta j}) - 2 \times \frac{\ell_\alpha}{2} (\mathcal{U} h^i \mathcal{G}_{ij} h^{\alpha j} + \mathcal{U} h_i \mathcal{G}^{ij} h^\alpha{}_j + \mathcal{U} u^i h^0 h^\alpha{}_i + \mathcal{U} u^i h_i h^{\alpha 0} - u^i u^j h_i h^\alpha{}_j + \frac{l_i}{2} h^i h^\alpha{}_0 + \frac{l_i}{2} h_0 h^{\alpha i} - \frac{l_i l_j}{4} h^i h^{\alpha j}) + [(\mathcal{V} \hat{h}_J{}^0 - t^\alpha \hat{h}_{\alpha J}) \mathcal{U} \mathcal{G}^{JK} (\mathcal{V} \hat{h}_K{}^0 - t^\beta \hat{h}_{\beta K}) + (\mathcal{V} \hat{h}^{J0} - t^\alpha \hat{h}_\alpha{}^J) \mathcal{U} \mathcal{G}_{JK} (\mathcal{V} \hat{h}^{K0} - t^\beta \hat{h}_\beta{}^K)]] + \frac{e^{3\phi}}{2\mathcal{V}^2} [(f^0 h_0 - f^i h_i + f_i h^i - f_0 h^0) - (f^0 h^\alpha{}_0 - f^i h^\alpha{}_i + f_i h^{\alpha i} - f_0 h^{\alpha 0}) \frac{\ell_\alpha}{2}].$
	$\mathcal{G}_{\alpha\beta} = \frac{\ell_\alpha \ell_\beta - 4\mathcal{V}\ell_{\alpha\beta}}{4\mathcal{V}},\ \mathcal{G}^{\alpha\beta} = \frac{2t^\alpha t^\beta - 4\mathcal{V}\ell^{\alpha\beta}}{4\mathcal{V}},\ \mathcal{G}_{ab} = -\hat{\ell}_{ab},\ \mathcal{G}^{ab} = -\hat{\ell}^{ab},$
	$\mathcal{G}_{ij} = \frac{l_i l_j - 4\mathcal{U} l_{ij}}{4\mathcal{U}},\ \mathcal{G}^{ij} = \frac{2u^i u^j - 4\mathcal{U} l^{ij}}{4\mathcal{U}},\ \mathcal{G}^{JK} = -\hat{l}^{JK},\ \mathcal{G}^{JK} = -\hat{l}^{JK}.$

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