

Stress-energy of the quantized fields in the spacetime of the Damour-Solodukhin wormhole

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The static traversable wormhole should be made out of some type of exotic matter which satisfies the Morris-Thorne conditions. Although the characteristic size of the region with the exotic matter can be made arbitrary small, the calculations performed so far suggest that the Morris-Thorne conditions are quite restrictive and it is hard to find the matter with the desired properties. Traditionally, the quantized fields are considered as the best candidates because they can violate the weak-energy condition. In this paper we employ the Schwinger-DeWitt expansion to construct and examine the approximate stress-energy tensor of the quantized massive scalar (with an arbitrary curvature coupling), spinor and vector field in the spacetime of the Damour-Solodukhin wormhole. We find that for the scalar field there is a region in a parameter space in which the stress-energy tensor has the desired properties. That means that of the twenty-one cases considered so far (the seven types of the wormhole geometries and the three types of the massive fields) only in the four cases (for certain values of the parameters) the stress-energy tensor does satisfy the Morris-Thorne conditions.

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I. INTRODUCTION

Typically, in order to construct the traversable wormhole some amount of the strange matter is needed. This assertion follows from the analysis of the Einstein field equations for the general static line element describing a spherically symmetric wormhole [1,2] (see also [3,4])

$$ds^2 = -e^{2\Phi(r)} dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (1)$$

where $\Phi(r)$ is a redshift function and $b(r)$ is a shape function. In the static orthonormal frame the Einstein field equations assume simple form

$$\rho = T_{\hat{t}\hat{t}} = \frac{1}{8\pi r^2} \frac{d}{dr} b, \quad (2)$$

$$\tau = -T_{\hat{r}\hat{r}} = \frac{1}{8\pi} \left[\frac{b}{r^3} - \frac{2}{r} \left(1 - \frac{b}{r}\right) \frac{d}{dr} \Phi \right] \quad (3)$$

and

$$p = T_{\hat{\phi}\hat{\phi}} = T_{\hat{\theta}\hat{\theta}} = \frac{r}{2} \left[(\rho - \tau) \frac{d}{dr} \phi - \frac{d}{dr} \tau \right], \quad (4)$$

for the energy density, the tension and the lateral pressure, respectively. Now, denote by ρ_0 and τ_0 the energy density and the tension at the throat. As is well known, the traversability of a wormhole requires that the following minimal set of conditions is satisfied:

$$\frac{\tau_0 - \rho_0}{|\rho_0|} \geq 0, \quad (5)$$

and

$$\tau_0 > 0. \quad (6)$$

The second condition follows from the absence of the event horizon and finiteness of the energy density ρ_0 whereas the first one is essentially the flaring-out condition [1,2]. The problem with the above conditions is that it is not an easy task to find matter fields satisfying them. Because of that the forms of matter satisfying the traversability conditions are called exotic. Since the quantized fields frequently violate the energy conditions they are among the most solid candidates for the exotic matter. In general, to construct such a wormhole one has to solve the semiclassical Einstein field equations with the stress-energy tensor of the quantized field. The first (numerical) calculations of this type were carried out by Hochberg, Popov and Sushkov in Ref. [5]. It is a very important result showing that it is possible to construct the self-consistent solution of the semiclassical Einstein field equations describing a Lorentzian wormhole

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connecting two asymptotically flat regions. Unfortunately, being numerical, this approach does not give much information about the geometry of the thus obtained wormhole. On the other hand however, it seems to be extremely hard, if not impossible, to construct the analytic solution. The main difficulty arises because the right-hand side of the semiclassical Einstein equations should depend functionally on a general metric tensor or, at least, on a wide class of metrics. Moreover, of the all approximations of the stress-energy tensor only the one based on the Schwinger-DeWitt expansion has the desired properties. The second difficulty is related to the fact that the stress-energy tensor is very complicated and involves higher derivatives of the metric, that practically prevents construction of the analytic solutions. However, one still can gain valuable information from the analysis of the stress-energy tensor of the quantized fields. Specifically, one can deduce if the energy density and the tension have the appropriate form support a traversable wormhole spacetime. If the Morris-Thorne conditions are not satisfied the quantum fields tend to destroy a wormhole and make it less operable. Various aspects of the quantum field theory in the wormhole spacetimes have been studied, for example, in Refs. [6–14]. A comprehensive discussion of general properties of wormholes can be found in Refs. [15–18].

It should be emphasized once again that the inequalities (6) and (5) are quite restrictive. Indeed, as have been shown in Refs. [19,20], of the eighteen cases considered there (six types of the wormhole geometries and three types of the massive fields) only in three cases (for certain values of the parameters) the stress-energy tensor does satisfy the Morris-Thorne conditions. In this paper (which can be treated as an extension of Refs. [19,20]) we shall construct the stress-energy tensor of the quantized massive scalar, spinor and vector fields in the spacetime of the Damour-Solodukhin wormhole [21] and check if the matter fields are of sufficient “exoticity.” We shall demonstrate that for the stress-energy tensor of the massive scalar field with the curvature coupling ξ there is a region in the space of parameters in which the conditions (6) and (5) are simultaneously satisfied. Unfortunately, neither the spinor nor the vector fields are of sufficient exoticity.

The line element describing the Damour-Solodukhin wormhole is given by

$$ds^2 = -(f(r) + \omega^2)dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \quad (7)$$

where $f(r) = 1 - \frac{2M}{r}$ and ω is a dimensionless parameter. Although it looks as a small modification of the Schwarzschild line element, it leads to a completely different type of the spacetime. Indeed, when $\omega = 0$ the line element reduces to the Schwarzschild solution, however, for a nonvanishing ω , no matter how small, it describes the Lorentzian wormhole with a throat located

at $r = r_0 = 2M$. The throat joins two asymptotically flat regions $2M \leq r < \infty$. If the parameter ω is small, one expects that the wormhole can mimic some of the observational features of black holes. It has been shown that for exponentially small values of the parameter, say, $\omega \sim e^{-4\pi M^2}$ the solution (7) is able to mimic both classical and quantum properties of the Schwarzschild black holes. Many interesting properties of such configurations are studied in Ref. [22]. However, since in this paper we are less interested in the black hole mimickers we treat ω as a free parameter.

Throughout the paper we use natural units $\hbar = c = G = 1$. The signature of the metric is taken to be “mainly positive”, i.e., +2, and the conventions for the curvature tensor are $\mathcal{R}^a_{bcd} = \partial_c \Gamma^a_{bd} - \dots$, and $\mathcal{R}^a_{bac} = \mathcal{R}_{bd}$.

II. THE STRESS-ENERGY TENSOR OF THE QUANTIZED MASSIVE FIELDS

The mathematical difficulties arising in the attempts to construct the renormalized stress-energy tensor of the quantized fields in a curved background are well known. They are related to the fact that the building blocks of the stress-energy tensor are the operator-valued distributions and the whole procedure is infected with unavoidable divergences. Moreover, the solutions of the equations describing quantum fields in curved spacetime cannot be expressed, except very rare cases, in terms of the known special functions. One is forced, therefore, to refer either to approximate methods or to numerics. The renormalized effective action of the quantized field in a curved background is a nonlocal quantity. However, if the Compton length associated with a quantized massive field is much smaller than a characteristic radius of curvature, then the nonlocal contribution to the total action can be neglected and the remaining (local) part can be expressed as the sum of terms constructed from the integrated Hadamard-Minakshisundaram-DeWitt-Seely coefficients. This is the Schwinger-DeWitt expansion in the powers of m^{-2} , where m is the mass of the field. Once the renormalized effective action is known the stress-energy tensor can be calculated in a standard way [23–26].

In a static and spherically-symmetric background there is a complementary method of constructing the stress-energy tensor of the massive scalar field with an arbitrary curvature coupling devised by Anderson *et al.* [27], in which the approximate WKB solutions of the scalar field equations are summed by means of the Abel-Plana formula. Both methods give precisely the same results as there is a one-to-one correspondence between them at each order of the calculations. Indeed, to obtain the main approximation (i.e., m^{-2} terms) the sixth-order WKB approximation of the mode functions is needed. Similarly, the next-to-leading approximation (i.e., m^{-4} terms) is equivalent to the eighth-order WKB approximation, and so forth.

The Schwinger-DeWitt approach has been employed in various contexts in Refs. [28–37] and the applications span from black hole physics to cosmology and form topological structures to wormholes. Detailed analysis carried out in Refs. [19,27] shows that the Schwinger-DeWitt approximation of the renormalized stress-energy tensor is very good. For example, for the quantized scalar field in the Reissner-Nordström spacetime characterized by the mass M and the charge Q , the deviation of the approximate stress-energy tensor from the exact (numerical) one is always below 1%, provided the condition $mM \geq 2$ is satisfied.

In what follows we shall confine ourselves to the scalar, spinor and vector fields satisfying, respectively

$$(-\square + \xi\mathcal{R} + m^2)\phi^{(0)} = 0, \quad (8)$$

$$(\gamma^a \nabla_a + m)\phi^{(1/2)} = 0 \quad (9)$$

and

$$(\delta_a^b \square - \nabla_a \nabla^b - \mathcal{R}_a^b - \delta_a^b m^2)\phi^{(1)} = 0, \quad (10)$$

where ξ is a curvature coupling constant and γ^a are the Dirac matrices satisfying standard relations $\gamma^a \gamma^b + \gamma^b \gamma^a = 2I g^{ab}$. The approximate one-loop effective action of the quantized massive scalar, spinor and vector fields is given by [38,39]

$$\begin{aligned} W_{(s)}^{(1)} &= \frac{1}{192\pi^2 m^2} \int d^4x g^{1/2} [\alpha_1^{(s)} \mathcal{R} \square \mathcal{R} + \alpha_2^{(s)} \mathcal{R}_{pq} \square \mathcal{R}^{pq} + \alpha_3^{(s)} \mathcal{R}^3 + \alpha_4^{(s)} \mathcal{R} \mathcal{R}_{pq} \mathcal{R}^{pq} + \alpha_5^{(s)} \mathcal{R} \mathcal{R}_{pqab} \mathcal{R}^{pqab} \\ &\quad + \alpha_6^{(s)} \mathcal{R}_q^p \mathcal{R}_a^q \mathcal{R}_p^a + \alpha_7^{(s)} \mathcal{R}^{pq} \mathcal{R}_{ab} \mathcal{R}_p^a \mathcal{R}_q^b + \alpha_8^{(s)} \mathcal{R}_{pq} \mathcal{R}^p \mathcal{R}_{cab} \mathcal{R}^{cab} + \alpha_9^{(s)} \mathcal{R}_{ab} \mathcal{R}^{pq} \mathcal{R}_{pq} \mathcal{R}_{cd} \mathcal{R}^{cd} + \alpha_{10}^{(s)} \mathcal{R}_p^a \mathcal{R}_q^b \mathcal{R}_c^q \mathcal{R}_d^p \mathcal{R}^c \mathcal{R}^d] \\ &= \frac{1}{192\pi^2 m^2} \sum_{i=1}^{10} \alpha_i^{(s)} \mathcal{W}_i, \end{aligned} \quad (11)$$

where \mathcal{W}_i are purely geometric terms, constructed entirely from the Riemann tensor, its contractions and covariant derivatives. The type of the field is encoded in the coefficients $\alpha_i^{(s)}$ tabulated in Table I.

The renormalized stress-energy tensor is given by

$$\langle T^{ab} \rangle = \frac{2}{g^{1/2}} \frac{\delta}{\delta g_{ab}} W_{(s)}^{(1)} = \frac{1}{96\pi^2 m^2 g^{1/2}} \sum_{i=1}^{10} \alpha_i^{(s)} \frac{\delta}{\delta g_{ab}} \mathcal{W}_i \quad (12)$$

and the full form of the functional derivatives of \mathcal{W}_i with respect to the metric tensor was given in Refs. [25,26]. The

TABLE I. The coefficients $\alpha_i^{(s)}$ for the massive scalar, spinor, and vector fields.

	$s = 0$	$s = 1/2$	$s = 1$
$\alpha_1^{(s)}$	$\frac{1}{2}\xi^2 - \frac{1}{5}\xi + \frac{1}{56}$	$-\frac{3}{140}$	$-\frac{27}{280}$
$\alpha_2^{(s)}$	$\frac{1}{140}$	$\frac{1}{14}$	$\frac{9}{28}$
$\alpha_3^{(s)}$	$(\frac{1}{6} - \xi)^3$	$\frac{1}{432}$	$-\frac{5}{72}$
$\alpha_4^{(s)}$	$-\frac{1}{30}(\frac{1}{6} - \xi)$	$-\frac{1}{90}$	$\frac{31}{60}$
$\alpha_5^{(s)}$	$\frac{1}{30}(\frac{1}{6} - \xi)$	$-\frac{7}{720}$	$-\frac{1}{10}$
$\alpha_6^{(s)}$	$-\frac{8}{945}$	$-\frac{25}{378}$	$-\frac{52}{63}$
$\alpha_7^{(s)}$	$\frac{2}{315}$	$\frac{47}{630}$	$-\frac{19}{105}$
$\alpha_8^{(s)}$	$\frac{1}{1260}$	$\frac{19}{630}$	$\frac{61}{140}$
$\alpha_9^{(s)}$	$\frac{17}{7560}$	$\frac{29}{3780}$	$-\frac{67}{2520}$
$\alpha_{10}^{(s)}$	$-\frac{1}{270}$	$-\frac{1}{54}$	$\frac{1}{18}$

final result is quite complicated and will not be presented here.

Because of the simplicity of the line element describing static and spherically symmetric line element one can use the Euler-Lagrange equations to construct the (00) and (11) component of the stress-energy tensor. The angular components can be calculated from the covariant conservation equation $\nabla_a T^{ab} = 0$. Although in this method one avoids calculations of the functional derivatives of the general action with respect to the metric tensor g_{ab} , the result expressed in terms of the two metric potential functions are still too complicated to be presented here.

Now, let us return to the Damour-Solodukhin wormhole. Although the line element describing the Damour-Solodukhin wormhole is a small modification of the Schwarzschild metric tensor, the stress-energy tensor of the quantized massive fields calculated with the aid of Eqs. (11) and (12) is quite complicated and for obvious reasons will not be given here [40]. Instead, we will focus on its behavior at the throat and analyze if the stress-energy tensor can support the existence of the Damour-Solodukhin wormhole.

After some algebra, the stress energy of the massive scalar field with a general curvature coupling ξ can be written in the form

$$T_a^b = \frac{1}{96\pi^2 m^2 M^6} \left(1 - \frac{2}{x} - \omega^2\right)^{-6} \sum_{k=0}^7 \beta_a^{(k)b} \frac{1}{x^{k+8}}, \quad (13)$$

where $x = r/M$ and the numerical coefficients $\beta_a^{(k)b}$ depend parametrically on ω and ξ . By construction it is covariantly

conserved and regular. It can be easily shown that when $\omega = 0$ the tensor reduces to the known result in the Schwarzschild spacetime. On the other hand, at the throat of the Damour-Solodukhin wormhole one has

$$T_a^b = \frac{1}{96\pi^2 m^2 M^6 \omega^6} \mathfrak{T}_a^b, \quad (14)$$

where the tensors \mathfrak{T}_a^b are given by

$$\begin{aligned} \mathfrak{T}_t^t &= \frac{43\omega^6}{35840} + \frac{907\omega^4}{215040} + \frac{23\omega^2}{4480} + \frac{179}{107520} \\ &\quad - \left(\frac{3\omega^2}{64} + \frac{13}{512} \right) \xi^3 + \left(\frac{13\omega^4}{128} + \frac{33\omega^2}{256} + \frac{47}{1024} \right) \xi^2 \\ &\quad - \left(\frac{9\omega^6}{1280} + \frac{59\omega^4}{1536} + \frac{3\omega^2}{64} + \frac{81}{5120} \right) \xi, \end{aligned} \quad (15)$$

$$\begin{aligned} \mathfrak{T}_r^r &= \frac{23\omega^6}{35840} + \frac{131\omega^4}{215040} + \frac{\omega^2}{1920} + \frac{11}{107520} \\ &\quad - \left(\frac{3\omega^2}{256} + \frac{1}{512} \right) \xi^3 + \left(\frac{\omega^4}{64} + \frac{9\omega^2}{512} + \frac{3}{1024} \right) \xi^2 \\ &\quad - \left(\frac{3\omega^6}{1280} + \frac{49\omega^4}{7680} + \frac{43\omega^2}{7680} + \frac{1}{1024} \right) \xi \end{aligned} \quad (16)$$

and

$$\begin{aligned} \mathfrak{T}_\theta^\theta &= \mathfrak{T}_\phi^\phi = \frac{5\omega^6}{3584} + \frac{7\omega^4}{2048} + \frac{173\omega^2}{43008} + \frac{283}{215040} \\ &\quad - \left(\frac{21\omega^2}{512} + \frac{23}{1024} \right) \xi^3 + \left(\frac{3\omega^4}{32} + \frac{121\omega^2}{1024} + \frac{87}{2048} \right) \xi^2 \\ &\quad - \left(\frac{3\omega^6}{512} + \frac{19\omega^4}{512} + \frac{659\omega^2}{15360} + \frac{147}{10240} \right) \xi. \end{aligned} \quad (17)$$

The stress-energy tensor of the massive spinor field is still of the form (13) with the coefficients $\beta_a^{(k)b}$ depending solely on ω . The components of \mathfrak{T}_a^b tensor can be written in the form

$$\mathfrak{T}_t^t = -\frac{3\omega^6}{3584} + \frac{233\omega^4}{71680} + \frac{47\omega^2}{17920} + \frac{269}{430080}, \quad (18)$$

$$\mathfrak{T}_r^r = \frac{3\omega^6}{4480} - \frac{19\omega^4}{107520} + \frac{\omega^2}{15360} + \frac{17}{430080} \quad (19)$$

and

$$\mathfrak{T}_\theta^\theta = \mathfrak{T}_\phi^\phi = \frac{33\omega^6}{35840} - \frac{11\omega^4}{10240} - \frac{317\omega^2}{430080} - \frac{143}{860160}. \quad (20)$$

Similarly, for the components of the stress-energy tensor of the massive vector field, one has

$$\mathfrak{T}_t^t = -\frac{53\omega^6}{35840} + \frac{7019\omega^4}{215040} + \frac{201\omega^2}{8960} + \frac{179}{35840}, \quad (21)$$

$$\mathfrak{T}_r^r = \frac{83\omega^6}{35840} - \frac{181\omega^4}{215040} + \frac{\omega^2}{1280} + \frac{11}{35840} \quad (22)$$

and

$$\mathfrak{T}_\theta^\theta = \mathfrak{T}_\phi^\phi = \frac{19\omega^6}{17920} - \frac{51\omega^4}{10240} - \frac{79\omega^2}{14336} - \frac{19}{14336}. \quad (23)$$

Before starting our discussion of the Morris-Thorne conditions let us count the general properties of the obtained stress-energy tensors. By construction, they are regular and covariantly conserved. Inspection of (13) shows that the components of T_a^b change sign a few times in the region close to r_0 and rapidly go to 0 as $r \rightarrow \infty$. The energy density at the throat (and by continuity in its vicinity) is negative for $\omega^2 < 4.6$ and $\omega^2 < 22.7458$ for the spinor and the vector field, respectively. A different picture emerges from the analysis of the minimally coupled and the conformally coupled scalar fields. Indeed, in both cases the energy density is always negative for $\omega^2 > 0$. In summary, in all considered cases there is a pocket of negative energy in the vicinity of the throat for small values of ω (say $\omega^2 < 1$).

III. DISCUSSION

Although the stress-energy tensor of the quantized fields is interesting in its own right, now we shall examine whether the quantum fields it describes are of sufficient exoticity to support the Damour-Solodukhin wormhole. In our context, the energy density and the radial tension are defined as

$$\rho = -T_t^t \quad (24)$$

and

$$\tau = -T_r^r. \quad (25)$$

In what follows we shall not impose any additional restrictions on the parameter ω .

Let us start with the massive spinor fields. Inspection of the components of the stress-energy tensor at the throat of the wormhole shows that the first Morris-Thorne condition (6) is satisfied for $\omega < 1.7044$. On the other hand, the radial tension is always negative and consequently the components of the stress-energy of the massive spinor field will not help to support the Damour-Solodukhin wormhole. Similarly, the first condition for the massive vector field is satisfied for $\omega < 3.0727$ whereas the radial tension is always negative. Since the second condition cannot be satisfied, the stress-energy tensor does not have the form required to support the wormhole.

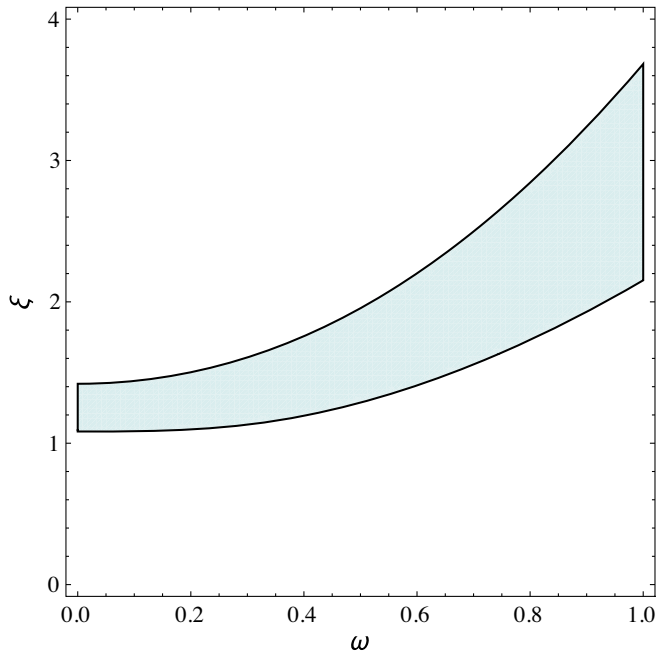


FIG. 1. The shaded region of the (ω, ξ) -plane represents values of the parameters for which the Morris-Thorne conditions are satisfied. Both conformally and minimally coupled fields lie outside the shaded region.

Now, let us consider the massive scalar field with the general curvature coupling and examine if the Morris-Thorne conditions can be satisfied simultaneously in the two-dimensional parameter space (ω, ξ) . The results of the numerical calculations are illustrated graphically. Inspection of Fig. 1 shows that there is a region in the (ω, ξ) -plane in which the stress-energy tensor has the form required to support the wormhole. It is quite important result as it shows that within the framework of the Schwinger-DeWitt approximation it is possible to construct the stress-energy tensor with the desired properties. Of the twenty-one cases analyzed so far (with the result of this paper included), only in the four cases the stress-energy

tensor has the required form to sustain the wormhole throat. It should be noted however, that if the physical values of the coupling parameter are chosen, i.e., either the minimal, $\xi = 0$, or the conformal, $\xi = 1/6$, coupling, then the energy density and the radial tension tend to destroy all the wormholes. The general picture that emerges from the analysis is that the scalar case is preferable as it (in our formulation of the theory) allows for an arbitrary value of the coupling parameter. Moreover, if the wormhole geometry depends (except its mass) on at least one additional parameter then the space of parameters can be sufficiently rich and can contain the region(s) in which the Morris-Thorne conditions are satisfied.

This somewhat pessimistic conclusion does not mean that the stress-energy tensor of other fields is useless in the context of wormholes. Indeed, since the stress-energy tensor constructed within the Schwinger-DeWitt framework depends functionally on a general metric, one could try to solve the semiclassical Einstein field equations self-consistently. Consider, for simplicity, the general static and spherically-symmetric spacetime described by the line element

$$ds^2 = -F(l)dt^2 + dl^2 + r^2(l)(d\theta^2 + \sin^2\theta d\phi^2), \quad (26)$$

where $F(l)$ and $r(l)$ are two functions of the proper distance l . For this line element the semiclassical Einstein field equations

$$R_a^b[g] - \frac{1}{2}R[g]\delta_a^b = 8\pi T_a^b[g] \quad (27)$$

reduce to the system of the two independent ordinary differential equations. Imposing the appropriate boundary conditions one can attempt to solve the equations numerically and decide if the thus obtained solution describes traversable wormhole. This work is in progress and the results will be published elsewhere.

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