Energy conditions in $f(Q)$ gravity

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A complete theory of gravity impels us to go beyond Einstein's general relativity. One promising approach lies in a new class of teleparallel theory of gravity named $f(Q)$, where the nonmetricity Q is responsible for the gravitational interaction. The important roles any of these alternative theories should obey are the energy condition constraints. Such constraints establish the compatibility of a given theory with the causal and geodesic structure of space-time. In this work, we present a complete test of energy conditions for $f(Q)$ gravity models. The energy conditions allowed us to fix our free parameters, restricting the families of $f(Q)$ models compatible with the accelerated expansion our Universe passes through. Our results strengthen the viability of $f(Q)$ theory, leading us close to the dawn of a complete theory for gravitation.

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I. INTRODUCTION

The dark sector of the Universe is one of the most challenging problems that science has been facing. The fact that approximately 96% of the matter and energy contents of the Universe is unknown suggests that our standard description of gravity is incomplete [1]. The standard description is based on Einstein-Hilbert field equations for general relativity (GR), and, despite this issue on the dark sector, such a theory has not been successfully tested since 1919, when it predicted the perihelion advance of Mercury. Recently, the observations of gravitational waves performed by LIGO and Virgo [2], besides the captured images of a black hole from the Event Horizon Telescope [3], confirmed the success of Einstein's general relativity as a classical theory of gravity. However, standard general relativity also fails as a fundamental theory to explain gravity interaction at a quantum level.

Therefore, there are several motivations to explore theories beyond the standard formulation of gravity. Among these efforts, we highlight models based on the so-called symmetric teleparallel gravity or $f(Q)$ gravity, introduced by Jiménez, Heisenberg, and Koivisto [4], where the nonmetricity Q is responsible for the gravitational interaction. Investigations on $f(Q)$ gravity have been rapidly developed as well as observational constraints to confront it against standard GR formulation.

An interesting set of constraints on $f(Q)$ gravity were done by Lazkoz *et al.* [5], where the $f(Q)$ Lagrangian is written as a polynomial function of the redshift z. The constraints for these models were successfully derived using data from the expansion rate, type Ia supernovae, quasars, gamma-ray bursts, baryon acoustic oscillation data, and cosmic microwave background distance. Another relevant analysis about $f(Q)$ gravity consists in understanding its behavior under different energy conditions.

As is known, the energy conditions represent paths to implement the positiveness of the stress-energy tensor in the presence of matter. Moreover, they can be used to describe the attractive nature of gravity, besides assigning the fundamental causal and the geodesic structure of space-time [6]. In this paper, we studied the strong, the weak, the null, and the dominant energy conditions for $f(Q)$ gravity, working with a perfect fluid matter distribution. The actual accelerating phase our Universe passes through has the constraint that the strong energy condition should be violated. This constraint, together with the actual values of Hubble and deceleration parameters, allowed us to test the viability of different forms of $f(Q)$ gravity.

The ideas presented in this paper are organized in the following way: In Sec. II, we present the generalities on $f(Q)$ gravity and the field equations as well as the energy conservation equation for a perfect fluid. In Sec. III, we show the explicit forms of the energy conditions derived from the Raychaudhury equations. The two scenarios for $f(Q)$ gravity and their constraints are carefully analyzed through Sec. IV. We also verified the deviations between

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our scenarios and the ΛCDM cosmological model in Sec. V. Our final remarks and perspectives are discussed in Sec. VI.

II. MOTION EQUATIONS IN $f(Q)$ GRAVITY

Let us consider the action for $f(Q)$ gravity given by [4]

$$\mathcal{S} = \int \frac{1}{2} f(Q)\sqrt{-g}d^4x + \int \mathcal{L}_m \sqrt{-g}d^4x, \tag{1}$$

where $f(Q)$ is a general function of the Q, $\mathcal{L}_m$ is the matter Lagrangian density, and g is the determinant of the metric $g_{\mu\nu}$.

The nonmetricity tensor and its traces are such that

$$Q_{\gamma\mu\nu} = \nabla_\gamma g_{\mu\nu}, \tag{2}$$

$$Q_\gamma = Q_\gamma{}^\mu{}_\mu, \qquad \tilde{Q}_\gamma = Q^\mu{}_{\gamma\mu}. \tag{3}$$

Moreover, the superpotential as a function of the nonmetricity tensor is given by

$$4P^\gamma{}_{\mu\nu} = -Q^\gamma{}_{\mu\nu} + 2Q_{(\mu^\gamma\nu)} - Q^\gamma g_{\mu\nu} - \tilde{Q}^\gamma g_{\mu\nu} - \delta^\gamma_{(\gamma^Q\nu)}, \tag{4}$$

where the trace of the nonmetricity tensor [4] has the form

$$Q = -Q_{\gamma\mu\nu}P^{\gamma\mu\nu}. \tag{5}$$

Another relevant ingredient for our approach is the energy-momentum tensor for the matter, whose definition is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}. \tag{6}$$

Taking the variation of action (1) with respect to the metric tensor, one can find the field equations

$$\frac{2}{\sqrt{-g}}\nabla_\gamma(\sqrt{-g}f_Q P^\gamma{}_{\mu\nu}) + \frac{1}{2}g_{\mu\nu}f + f_Q(P_{\mu\gamma i}Q_\nu{}^{\gamma i} - 2Q_{\gamma i\mu}P^{\gamma i}{}_\nu) = -T_{\mu\nu}, \tag{7}$$

where $f_Q = \frac{df}{dQ}$. Besides, we can also take the variation of (1) with respect to the connection, yielding

$$\nabla_\mu\nabla_\gamma(\sqrt{-g}f_Q P^\gamma{}_{\mu\nu}) = 0. \tag{8}$$

Here we are going to consider the standard Friedmann-Lemaître-Robertson-Walker (FLRW) line element, which is explicitly written as

$$ds^2 = -dt^2 + a^2(t)\delta_{\mu\nu}dx^\mu dx^\nu, \tag{9}$$

where $a(t)$ is the scale factor of the Universe. The previous line element enables us to write the trace of the nonmetricity tensor as

$$Q = 6H^2.$$

Now, let us take the energy-momentum tensor for a perfect fluid, or

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu + pg_{\mu\nu}, \tag{10}$$

where p represents the pressure and ρ represents the energy density. Therefore, by substituting (9) and (10) in (7), one can find

$$3H^2 = \frac{1}{2f_Q}\left(-\rho + \frac{f}{2}\right), \tag{11}$$

$$\dot{H} + 3H^2 + \frac{\dot{f}_Q}{f_Q}H = \frac{1}{2f_Q}\left(p + \frac{f}{2}\right), \tag{12}$$

as the modified Friedmann equations for $f(Q)$ gravity. Here the dot $(\cdot)$ represents one derivative with respect to time. The modified Friedmann equations enable us to write the density and the pressure for the Universe, respectively, as

$$\rho = \frac{f}{2} - 6H^2 f_Q, \tag{13}$$

$$p = \left(\dot{H} + 3H^2 + \frac{\dot{f}_Q}{f_Q}H\right)2f_Q - \frac{f}{2}. \tag{14}$$

In analogy with GR, we can rewrite Eqs. (11) and (12) as

$$3H^2 = -\frac{1}{2}\tilde{\rho}, \tag{15}$$

$$\dot{H} + 3H^2 = \frac{\tilde{p}}{2}, \tag{16}$$

respectively, where

$$\tilde{\rho} = \frac{1}{f_Q}\left(\rho - \frac{f}{2}\right), \tag{17}$$

$$\tilde{p} = -2\frac{\dot{f}_Q}{f_Q}H + \frac{1}{f_Q}\left(p + \frac{f}{2}\right). \tag{18}$$

The previous equations are going to be components of a modified energy-momentum tensor $\tilde{T}_{\mu\nu}$, embedding the dependence on the trace of the nonmetricity tensor.

III. ENERGY CONDITIONS

The energy conditions (ECs) are the essential tools to understand the geodesics of the Universe. Such conditions can be derived from the well-known Raychaudhury equations, whose forms are [7]

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu, \quad (19)$$

$$\frac{d\theta}{d\tau} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}n^\mu n^\nu, \quad (20)$$

where θ is the expansion factor, n^μ is the null vector, and $\sigma^{\mu\nu}$ and $\omega_{\mu\nu}$ are, respectively, the shear and the rotation associated with the vector field u^μ. For attractive gravity, Eqs. (19) and (20) satisfy the following conditions:

$$R_{\mu\nu}u^\mu u^\nu \geq 0, \quad (21)$$

$$R_{\mu\nu}n^\mu n^\nu \geq 0. \quad (22)$$

Therefore, if we are working with a perfect fluid matter distribution, the energy conditions recovered from standard GR are

(i) strong energy conditions (SECs) if $\tilde{\rho} + 3\tilde{p} \geq 0$;
(ii) weak energy conditions (WECs) if $\tilde{\rho} \geq 0$, $\tilde{\rho} + \tilde{p} \geq 0$;
(iii) null energy condition (NEC) if $\tilde{\rho} + \tilde{p} \geq 0$; and
(iv) dominant energy conditions (DECs) if $\tilde{\rho} \geq 0$, $|\tilde{p}| \leq \rho$.

Taking Eqs. (17) and (18) into WEC, NEC, and DEC constraints, we are able to prove that

(i) WECs if $\rho \geq 0, \rho + p \geq 0$;
(ii) NEC if $\rho + p \geq 0$; and
(iii) DECs if $\rho \geq 0, |p| \leq \rho$,

corroborating with the work from Capozziello *et al.* [6]. In the case of the SEC, we yield to the constraint

$$\rho + 3p - 6\dot{f}_Q H + f \geq 0. \quad (23)$$

Moreover, let us consider the Hubble, deceleration, jerk, and snap parameters, whose forms are, respectively,

$$H = \frac{\dot{a}}{a}, \qquad q = -\frac{1}{H^2}\frac{\ddot{a}}{a},$$
$$j = \frac{1}{H^3}\frac{\dddot{a}}{a}, \qquad s = \frac{1}{H^4}\frac{\ddddot{a}}{a}. \quad (24)$$

Such parameters enable us to represent the time derivatives of H as

$$\dot{H} = -H^2(1+q), \quad (25)$$

$$\ddot{H} = H^3(j + 3q + 2), \quad (26)$$

$$\dddot{H} = H^4(s - 2j - 5q - 3). \quad (27)$$

So, by using Eqs. (25)–(27), we can rewrite (13) and (14) as

$$\rho = \frac{f}{2} - 6H^2 f_Q, \quad (28)$$

$$p = \left(-H^2(1+q) + 3H^2 + \frac{\dot{f}_Q}{f_Q}H\right)2f_Q - \frac{f}{2}, \quad (29)$$

respectively, which are the density and the pressure for the $f(Q)$ gravity. Therefore, the previous equations establish the following constraints for the energy conditions:

$$\text{SEC:}\ \rho + 3p - 6\dot{f}_Q H + f = \frac{f}{2} - 6H^2 f_Q + 3\left(-H^2(1+q) + 3H^2 + \frac{\dot{f}_Q}{f_Q}H\right)2f_Q - 3\frac{f}{2} - 6\dot{f}_Q H + f \geq 0, \quad (30)$$

$$\text{NEC:}\ \rho + p = -6H^2 f_Q + \left(-H^2(1+q) + 3H^2 + \frac{\dot{f}_Q}{f_Q}H\right)2f_Q \geq 0, \quad (31)$$

$$\text{WEC:}\ \rho = \frac{f}{2} - 6H^2 f_Q \geq 0, \rho + p = -6H^2 f_Q + \left(-H^2(1+q) + 3H^2 + \frac{\dot{f}_Q}{f_Q}H\right)2f_Q \geq 0, \quad (32)$$

$$\text{DEC:}\ \rho = \frac{f}{2} - 6H^2 f_Q \geq 0, \rho \pm p = \frac{f}{2} - 6H^2 f_Q \pm 3\left(-H^2(1+q) + 3H^2 + \frac{\dot{f}_Q}{f_Q}H\right)2f_Q - 3\frac{f}{2} \geq 0. \quad (33)$$

IV. CONSTRAINING $f(Q)$ THEORIES

In this section, we discuss the viability of the functional form of $f(Q)$ in FLRW space-time. In order to do so, we take the present values for the Hubble and the decelerating parameters as $H_0 = 67.9$ and $q_0 = -0.503$, respectively [8,9]. Moreover, several observations confirm that the Universe is going through an accelerated phase [10], carried by a negative pressure regime. Such a scenario imposes that SEC needs to be violated [8].

There are several approaches in the literature deriving energy conditions beyond Einstein's GR; we can see for instance, EC constraints in $f(R)$ theory [11,12], $f(G)$ theory [13,14], $f(T)$ theory [15], $f(\mathcal{G}, T)$ theory [16],

$f(R,T,R_{\mu\nu}T^{\mu\nu})$ theory [17], $f(R,\mathcal{G})$ theory [18], $f(R,\Box R,T)$ gravity [19], $f(R,T)$ theory [20], etc. However, the previous studies are mainly focused on the WEC, whereas our intent in this paper is to study the constraint of all the ECs in $f(Q)$ theory. To investigate the ECs with the present values of the geometrical parameters in $f(Q)$ theory, we need to fix the functional form of $f(Q)$. Once this form fixed, it will be easy to investigate the cosmological scenarios. In their beautiful work, Harko *et al.* [21] discussed the coupling matter in modified Q gravity by assuming a power law and an exponential form of $f(Q)$. This investigation motivated us to work with a polynomial form for $f(Q)$ gravity. Moreover, we also introduce a logarithmic dependence of $f(Q)$, which we are going to analyze carefully.

A. $f(Q) = Q + mQ^n$

In this subsection, we presume the $f(Q)$ as an algebraic polynomial function of Q with free parameters m and n. The previous function establishes that the ECs need to satisfy the following conditions:

$$\begin{aligned}\text{SEC:}\ & 3H_0^2 - m6^n\{2^{-1}(2n-1)-1\}H_0^{2n} + \frac{1}{2}[H_0^2(6-12q_0) - m6^n(2n-1)H_0^{2n}\{n(q_0+1)-3\}] \\ & + 2^{1+n}3^nH_0^{2n}m(-1+n)n(1+q_0) \geq 0,\end{aligned} \tag{34}$$

$$\text{NEC:}\ -3H_0^2 - m2^{-1}6^n(2n-1)H_0^{2n} + \frac{1}{6}[H_0^2(6-12q_0) - m6^n(2n-1)H_0^{2n}\{n(q_0+1)-3\}] \geq 0, \tag{35}$$

$$\begin{aligned}\text{WEC:}\ & -3H_0^2 - m2^{-1}6^n(2n-1)H_0^{2n} \geq 0 \\ \text{and}\ & -3H_0^2 - m2^{-1}6^n(2n-1)H_0^{2n} + \frac{1}{6}[H_0^2(6-12q_0) - m6^n(2n-1)H_0^{2n}\{n(q_0+1)-3\}] \geq 0,\end{aligned} \tag{36}$$

$$\begin{aligned}\text{DEC:}\ & -3H_0^2 - m2^{-1}6^n(2n-1)H_0^{2n} \geq 0 \\ \text{and}\ & -3H_0^2 - m2^{-1}6^n(2n-1)H_0^{2n} \pm \frac{1}{6}[H_0^2(6-12q_0) - m6^n(2n-1)H_0^{2n}\{n(q_0+1)-3\}] \geq 0.\end{aligned} \tag{37}$$

From (34)–(37), one can easily observe that the ECs directly depend on the free parameters m and n. Nevertheless, one cannot take the values of m and n arbitrarily, which may violate the ECs as well as the current scenario of the Universe dominated by the dark energy. Therefore, using (34)–(37), we found some restrictions on the model parameters m and n. Using WEC, we found that m should be less than or equal to -1 ($m \leq -1$) and n should be greater than or equal to 0.9 ($n \geq 0.9$). Also, we found that (34), (35), and (37) reduce the range of the model parameter to $0.9 \leq n \leq 2$, in order to proper describe SEC, NEC, and DEC. Finally, we conclude that, for $m \leq -1$ and $0.9 \leq n \leq 2$, this model represents the current stage of the Universe. In addition to this, we showed the profile of all energy conditions for some range of model parameters m and n. From Fig. 1, one can observe that WEC, NEC, and DEC are satisfied, while SEC is violated, corroborating with an accelerated expansion [22,23].

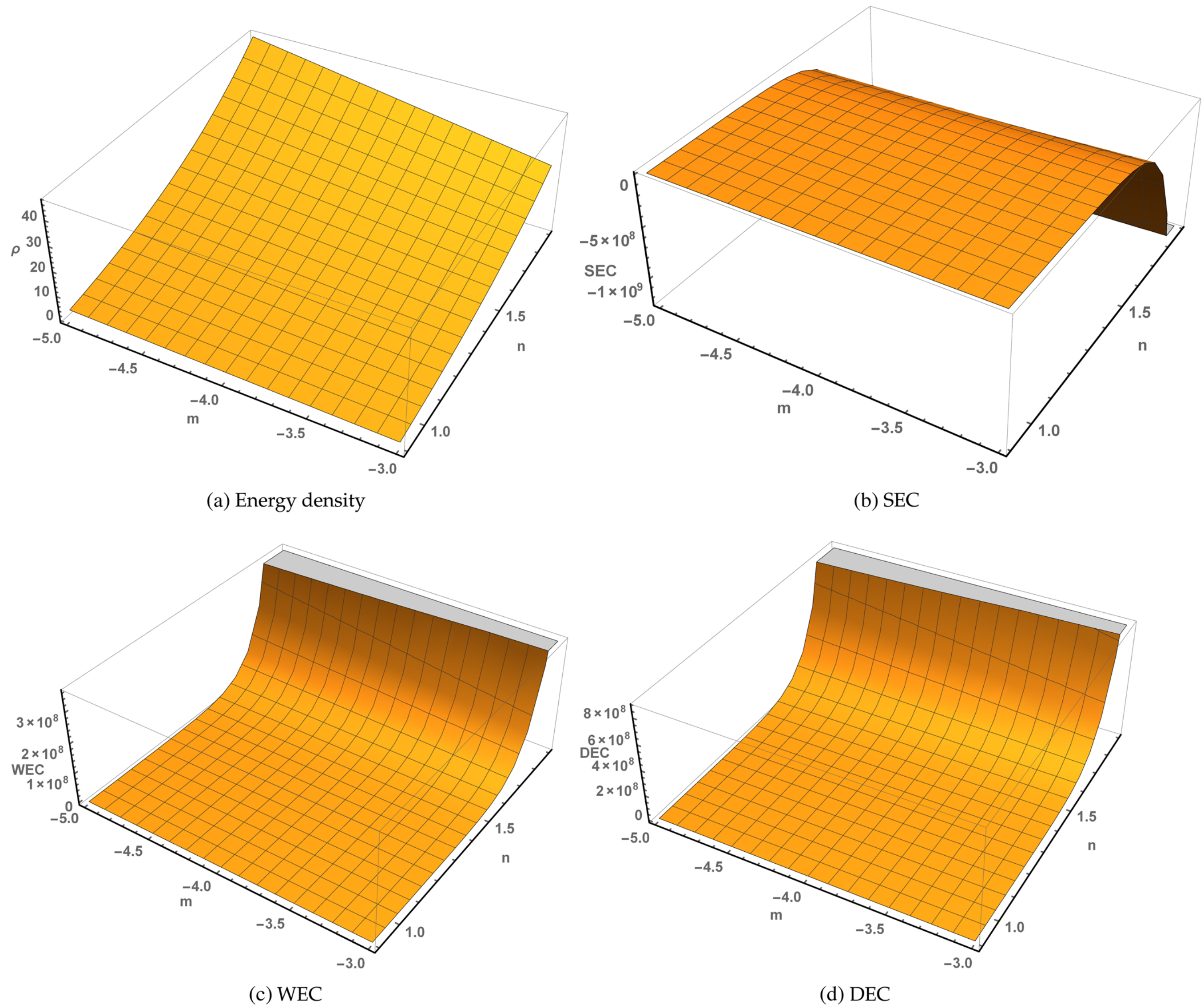
B. $f(Q) = \alpha + \beta \log Q$

Here, we introduce $f(Q)$ as a logarithmic function of the nonmetricity with free parameters α and β. Therefore, the ECs are impelled to obey the constraints

$$\text{SEC:}\ -\beta - 2\beta(q_0+1) + \frac{3}{2}[\alpha + \beta\log(6H_0^2)] - \frac{3H_0^2[\alpha - 2\beta + \beta\log(6H_0^2)] - 2\beta H_0^2(q_0+1)}{2H_0^2} \geq 0, \tag{38}$$

$$\text{NEC:}\ -\beta + \frac{1}{2}[\alpha + \beta\log(6H_0^2)] - \frac{3H_0^2[\alpha - 2\beta + \beta\log(6H_0^2)] - 2\beta H_0^2(q_0+1)}{6H_0^2} \geq 0, \tag{39}$$

$$\begin{aligned}\text{WEC:}\ & -\beta + \frac{1}{2}[\alpha + \beta\log(6H_0^2)] \geq 0 \\ \text{and}\ & -\beta + \frac{1}{2}[\alpha + \beta\log(6H_0^2)] - \frac{3H_0^2[\alpha - 2\beta + \beta\log(6H_0^2)] - 2\beta H_0^2(q_0+1)}{6H_0^2} \geq 0,\end{aligned} \tag{40}$$



(a) Energy density (b) SEC

(c) WEC (d) DEC

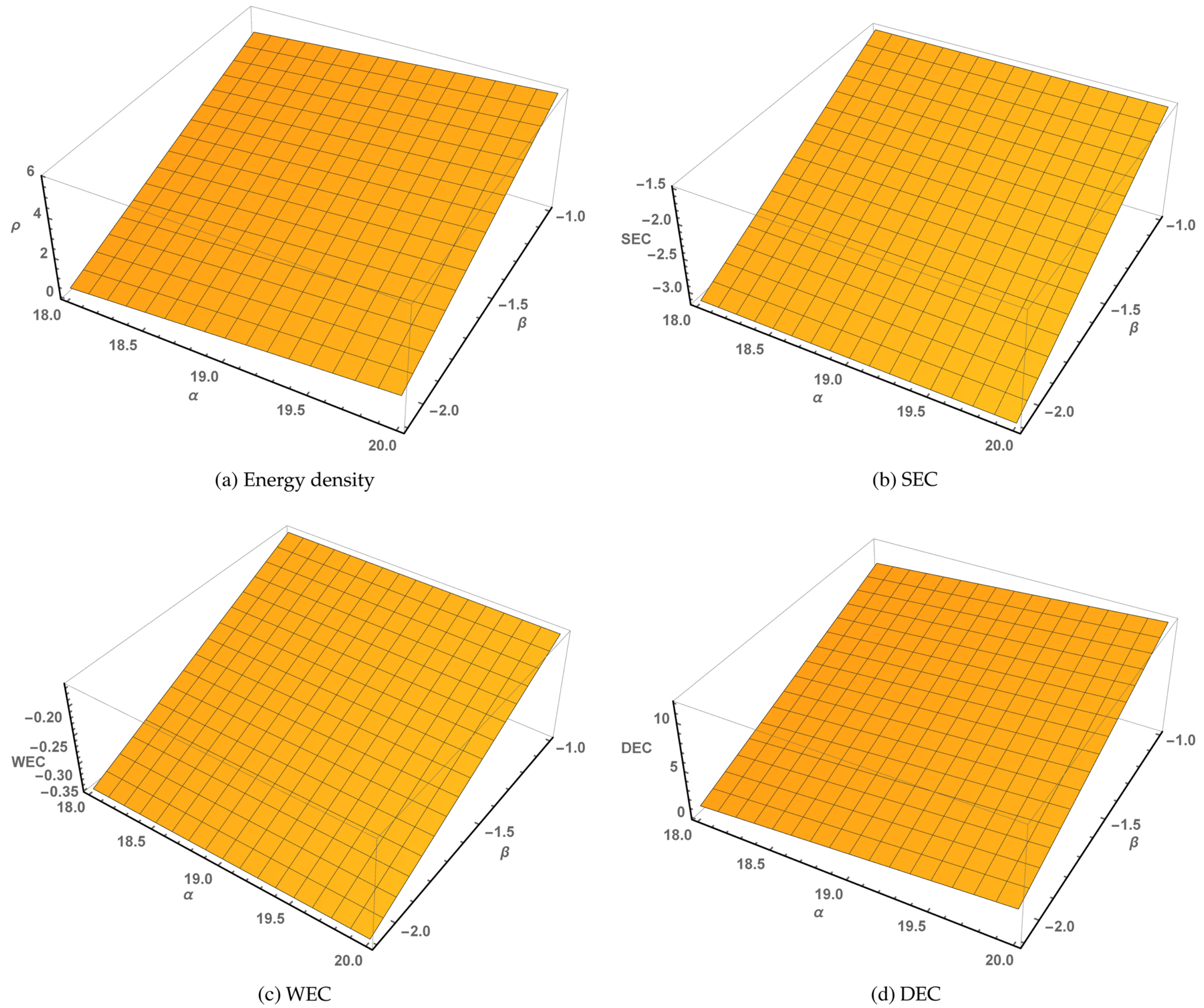
FIG. 1. Energy conditions for $f(Q) = Q + mQ^n$ derived with the present values of H_0 and q_0 parameters.

$$\text{DEC}: \quad -\beta + \frac{1}{2}[\alpha + \beta \log(6H_0^2)] \geq 0$$
$$\text{and} \quad -\beta + \frac{1}{2}[\alpha + \beta \log(6H_0^2)] \pm \frac{3H_0^2[\alpha - 2\beta + \beta \log(6H_0^2)] - 2\beta H_0^2(q_0 + 1)}{6H_0^2} \geq 0. \tag{41}$$

The ECs shown in Eqs. (38)–(41) unveil their direct dependence on free parameters α and β. The previous equations also established that we cannot choose arbitrary values for these free parameters, as observed in our previous model. Through Eqs. (38)–(41), we found that SEC is violated and WEC is partially satisfied ($\rho > 0$) if $\alpha \geq -9\beta$, ($\beta \leq -1$), besides NEC and DEC are still obeyed. This violation of WEC with positive density notably makes this $f(Q)$ theory behaves like scalar-tensor gravity models [24], and such a violation can be interpreted as natural contributions from quantum effects to classical gravity [25]. The features of these conditions can be appreciated in Fig. 2, where the graphics were depicted considering a specific range of values for parameters α and β.

V. DEVIATION FROM THE STANDARD ΛCDM MODEL

So far, the ΛCDM is the most successful model used to describe the actual observations of the Universe. Such a model has been broadly tested by several different surveys along the past few years, such as WMAP, Planck, and The



(a) Energy density (b) SEC

(c) WEC (d) DEC

FIG. 2. Energy conditions for $f(Q) = \alpha + \beta \log Q$ derived with the present values for H_0 and q_0 parameters.

Dark Energy Survey. As pointed out by Lazkoz *et al.* [5], the $f(Q)$ model can mimic the ΛCDM one by taking $f(Q) = f_\Lambda(Q) = -Q$. Therefore, considering this specific mapping for f_Λ, we find the following energy conditions:

(i) SEC: $6H^2 q \geq 0$,
(ii) NEC: $2H^2(1+q) \geq 0$,
(iii) WEC: $3H^2 \geq 0$ and $2H^2(1+q) \geq 0$, and
(iv) DEC: $3H^2 \geq 0$ and $2H^2(1+q) \geq 0$ or $-2H^2(-2+q) \geq 0$.

By taking the actual values of H_0 and q_0 in the above conditions, one can prove that WEC, NEC, and DEC are satisfied; however, SEC is violated. This is the expected behavior for a standard accelerated phase for the Universe. Moreover, we realize an analogous description with respect to energy conditions between our first model for $f(Q)$ gravity and the ΛCDM.

Besides, the recent observations from the Planck Collaboration, as well as the ΛCDM model, confirm that the equation of state parameter is $\omega \simeq -1$ [8]. This behavior corresponds to a negative pressure regime for the Universe, which configures its current accelerated phase. Therefore, ω parameter presents as a suitable candidate to compare our models with ΛCDM. Once the equation of state parameter is defined as

$$\omega = \frac{p}{\rho}, \tag{42}$$

our previous model yields to the following forms of ω:

$$\omega = -1 + \frac{2(q_0+1)\{m6^n n(2n-1)H_0^{2n} + 6H_0^2\}}{3m6^n(2n-1)H_0^{2n} + 18H_0^2} \tag{43}$$

for $f(Q) = Q + mQ^n$ gravity and

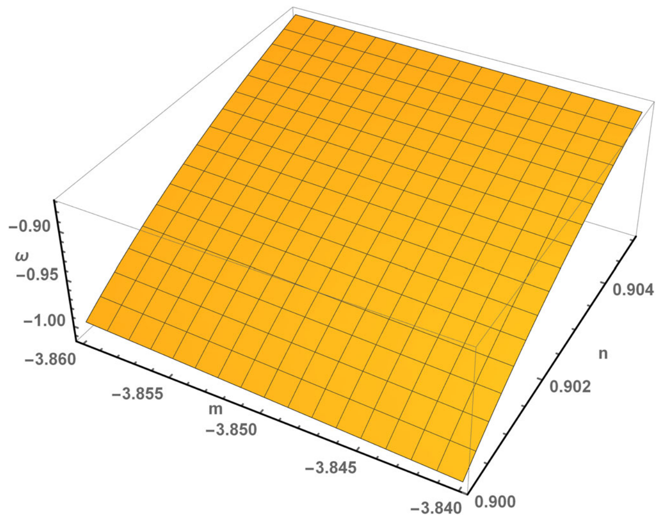


FIG. 3. Equation of state parameter ω for $f(Q) = Q + mQ^n$ derived with the present values for H_0 and q_0 parameters.

$$\omega = -1 + \frac{2\beta(q_0 + 1)}{3\alpha - 6\beta + 3\beta \log(6H_0^2)} \tag{44}$$

for $f(Q) = \alpha + \beta \log Q$. In Figs. 3 and 4, we have shown the profiles of the equation of state parameter for both $f(Q)$ models here introduced. The graphics were depicted considering the energy condition constraints for free parameters m, n, α, and β. From these figures, one can observe that the values of ω are very close to -1, which is in agreement with the recent observational data. Consequently, our models fit the equation of state parameter as good as ΛCDM, corroborating the violation of SEC, and confirming their viability to describe an accelerated Universe.

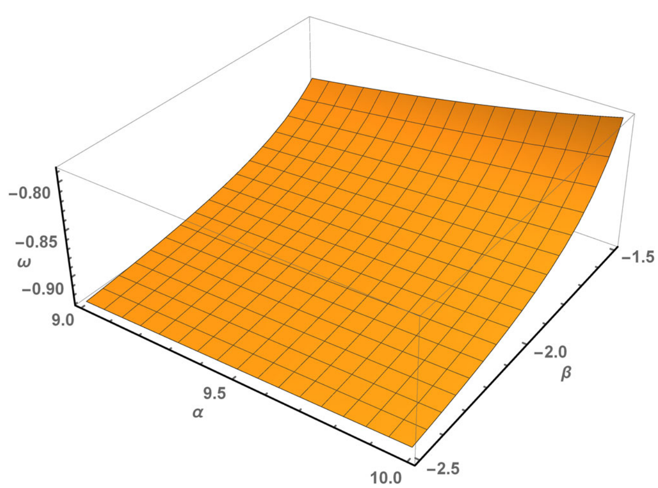


FIG. 4. Equation of state parameter ω for $f(Q) = \alpha + \beta \log Q$ derived with the present values for H_0 and q_0 parameters.

VI. CONCLUSION

There are several theories of gravity beyond Einstein's GR; however, one critical role to define their self-consistencies is the energy condition. The physical motivation for a new theory of gravity is related to its compatibility with the causal and geodesic structure of space-time, which can be addressed through different sets of energy conditions. In the present study, we derived the strong, the weak, the null, and the dominant energy conditions for two different $f(Q)$ gravity models. Inspired by the work of Harko *et al.* [21], our first model was a polynomial function of the nonmetricity Q and has two free parameters m and n. The energy conditions established $m \leq -1$ and $0.9 \leq n \leq 2$ as constraints to describe an accelerated expansion of the Universe.

As a second approach, we introduced a gravity model with a logarithmic dependence on the nonmetricity. Such a model means that the $f(Q)$ smoothly tends to the Einstein-Hilbert model $[f(Q) \propto Q]$ and had two free parameters named α and β. The graphics presented in Fig. 2 unveil a desired accelerating Universe for $18 \leq \alpha \leq 20$ and $-2 \leq \beta \leq -1$. Moreover, such a theory violates both SEC and WEC with positive density, exhibiting a behavior analogous to scalar-tensor field gravity models [24].

As a matter of completeness, we compared our energy constraints with those from the ΛCDM model. In the ΛCDM gravity, all energy conditions are satisfied except SEC. This behavior is compatible with our first proposed model, where $f(Q) = Q + mQ^n$, strengthening its potential as a promising new description for gravity.

Moreover, the equation of state parameters, derived from our two $f(Q)$ approaches, are compatible with a current phase of negative pressure, presenting values close to -1. This behavior also corroborates the ΛCDM description for dark energy, as well as current experimental observations [8].

These previous results allowed us to verify the viability of different families of $f(Q)$ gravity models, lighting new routes for a complete description of gravity compatible with the dark energy era. Another interesting fact is that there is plenty of freedom for our free parameters, enabling several testable scenarios for $f(Q)$ gravity. Such tests could include the absence of ghost modes, gravitational wave constraints, and cosmological parameters derived from the cosmic microwave background. Besides, it would be interesting to study carefully the coupling of $f(Q)$ with inflaton fields, looking for possible analytic models or for cosmological parameter constraints. We intend to address some of these investigations in the near future and hope to report on them.

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