

Shadow of a naked singularity without photon sphere

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It is generally believed that the shadows of either a black hole or naked singularity arise due to photon spheres developing in these spacetimes. Here we propose a new spherically symmetric naked singularity spacetime which has no photon sphere, and we show that the singularity casts a shadow in the absence of the photon sphere. We discuss some novel features of this shadow and the lightlike geodesics in this spacetime. We compare the shadow of the naked singularity here with shadows cast by Schwarzschild black hole and the first type of Joshi-Malafarina-Narayan (JMN1) naked singularity, where for the last two spacetimes the shadow is formed due to the presence of a photon sphere. It is seen, in particular, that the size of shadow of the singularity is considerably smaller than that of a black hole. Our analysis shows that the shadow of this naked singularity is distinguishable from the shadow of a Schwarzschild black hole and the JMN1 naked singularity. These results are useful and important in the context of recent observations of shadow of the M87 Galactic center.

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I. INTRODUCTION

It is well known that when massive matter clouds undergo a catastrophic continual gravitational collapse, the total mass collapses into a spacetime singularity. At such a singularity, the density, pressures, and spacetime curvatures diverge [1,2]. To understand the final state of such a collapse better, one could consider a collapsing spherically symmetric compact object within a dynamical spacetime configuration, where the internal dynamical metric seeded by the collapsing matter is matched externally to a Schwarzschild solution.

When the collapsing matter is homogeneous and dust-like, the final state of the gravitational collapse is necessarily a Schwarzschild black hole. It then has a central spacetime singularity which is covered by a null hypersurface [3], known as the event horizon. Since this singularity is covered by a null surface, so no information from the same can travel to any other spacetime events or faraway observers. As this singularity is not causally connected to any other spacetime points, it is a spacelike singularity.

On the other hand, there can be other types of singularity which are causally connected with other spacetime points. Such causally connected singularities are also known as naked singularities. There are two types of naked singularities: nulllike and timelike singularities. One can show that both these singularities can be formed as the final state

of gravitational collapse of an inhomogeneous matter cloud [4–12]. There are many well-known spacetimes, e.g., Joshi-Malafarina-Narayan (JMN) spacetimes [8,9], Janis-Newman-Winicour (JNW) spacetime [13], which have central timelike singularities. It can also be shown that the timelike and null singularities formed during a gravitational collapse of massive matter cloud can be gravitationally strong [4–12]. Around a strong curvature naked singularity, the quantum gravitational effects should be very large and powerful. This may indicate and imply a possible quantum gravity resolution of strong spacetime singularity that may happen around the extreme curvature regions of a spacetime [14–16]. Therefore, as timelike and null strong singularities are causally connected to the other external spacetime points, the quantum gravity effects can be locally or globally observable.

In [8,9], it is shown that JMN spacetimes with central timelike singularities are obtained as the asymptotic equilibrium state of a quasistatic gravitational collapse. Also, it is important to investigate the observational signatures of timelike singularities in the context of recent observation of shadow of the M87 Galactic center by the Event Horizon Telescope (EHT) Collaboration [17]. As for our own galaxy, the trajectories of “S” stars around our Galactic center as observed by the GRAVITY and SINFONI Collaborations [18–20] are providing very useful data. Recently, many studies have been made [10,21–39], where the behavior of timelike and null geodesics around timelike singularities and other compact objects has been investigated. In [30,32], the shadows cast by JMN spacetimes are investigated and it is shown that the shadow cast by first type of JMN spacetime (JMN1) can be similar to the

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shadow cast by the Schwarzschild black hole. JNW naked singularity spacetime also can cast a shadow, and the properties of the shadow can be similar to the Schwarzschild black hole shadow [32,33].

It is worth noting, however, that for JNW and JMN1 spacetimes, it is the existence of photon sphere that causes the shadow. On the other hand, in [40], it is shown that a thin matter shell can also cast a shadow in the absence of a photon sphere in the spacetime. These results emphasize that the occurrence of a shadow is not a property of black holes only, but it also can be cast by other compact objects in the presence of a photon sphere or thin shell of matter [30,32,40].

In this paper, we present another new naked singularity spacetime which resembles with a Schwarzschild spacetime at large distances. We show that this naked singularity spacetime can cast and admit a shadow though it has no photon sphere, or any thin shell of matter. The interesting fact that emerges is that the shadow is created by the singularity itself. This is an intriguing feature, because just like a black hole or other compact objects, the singularity itself, when not covered by an event horizon, behaves like an “object” in its own right, even to the extent of casting a shadow. Our analysis here shows that the size of shadow of the singularity is considerably smaller than that of a black hole. It is seen that the shadow of this naked singularity is distinguishable from shadow of a Schwarzschild black hole and JMN1 naked singularity. Since the shadow is cast by the singularity, one may possibly speculate that the effect of quantum gravitational resolution of the singularity could be likely identified from the observed shadow and the various detailed physical features it may exhibit. Also, this novel feature of the spacetime singularity could be important in

the context of the recent observations of shadow of the Galactic center M87.

The plan of the paper is as follows. In Sec. II, we discuss the JMN1 spacetime and the new asymptotically flat naked singularity spacetime is presented. In Sec. III, we discuss the nature of light trajectories around the Schwarzschild black hole, JMN1 naked singularity, and the new naked singularity solution presented here. In that section, we also discuss the shadow of naked singularity, and compare the same with that of the black hole. Finally, in Sec. IV, we discuss our results. Throughout the paper, we take $G = c = 1$.

II. NAKED SINGULARITY SPACETIMES

In this section, we present a new spherically symmetric, asymptotically flat naked singularity spacetime. We first discuss the basic spacetime properties of this naked singularity spacetime, and then we briefly discuss JMN1 spacetime.

A. The naked singularity spacetime

The line element of the proposed spherically symmetric, static naked singularity spacetime can be written as

$$ds^2 = -\frac{dt^2}{\left(1 + \frac{M}{r}\right)^2} + \left(1 + \frac{M}{r}\right)^2 dr^2 + r^2 d\Omega^2, \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and M is a positive constant, where we will prove that M is the Arnowitt-Deser-Miser (ADM) mass of the above spacetime. The expressions of the Kretschmann scalar and Ricci scalar for this spacetime are

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \frac{4M^2((M - 2r)^2 r^4 + 4(M + r)^2 r^4 + (M + r)^4 (M + 2r)^2)}{r^4 (M + r)^8}, \quad (2)$$

$$R = \frac{2M^3(M + 4r)}{r^2(M + r)^4}. \quad (3)$$

From the above expressions of the Kretschmann scalar and Ricci scalar, it can be seen that the spacetime has a strong curvature singularity at the center $r = 0$. No null surfaces such as an event horizon exist around the singularity in this spacetime. Therefore, the singularity is visible to the outside observer. As we know, the ADM mass of a spacetime can be written as [3,41]

$$M_{\text{ADM}} = -\frac{1}{8\pi} \lim_{S \rightarrow \infty} \oint_S (\mathbb{R} - \mathbb{R}_0) \sqrt{\sigma} d^2\theta, \quad (4)$$

where S is the bounded two-surface. \mathbb{R} is the extrinsic curvature of the two-surface S embedded in spacelike

hypersurface Σ , \mathbb{R}_0 is the extrinsic curvature of S embedded in flat space, and the infinitesimal area of the two-surface is written as $\sqrt{\sigma} d^2\theta = r^2 \sin\theta d\theta d\phi$, where σ is the determinant of the induced metric on the two-surface. The limit $S \rightarrow \infty$ implies that the radius of the two-surface S tends to infinity. For the above spacetime [Eq. (1)], the unit normal on the S can be written as

$$n_\alpha = \sqrt{g_{rr}} \partial_\alpha r = \left(1 + \frac{M}{r}\right) \partial_\alpha r. \quad (5)$$

The extrinsic curvature of S embedded in Σ can be calculated as

$$\mathbb{R} = n^\alpha_{;\alpha} = \frac{2}{r\left(1 + \frac{M}{r}\right)}, \quad (6)$$

and the extrinsic curvature of S embedded in flat space is $\mathbb{R}_0 = \frac{2}{r}$. Therefore, using Eq. (4), the ADM mass of the new naked singularity spacetime [Eq. (1)] can be written as

$$M_{\text{ADM}} = -\frac{1}{2} \lim_{r \rightarrow \infty} r^2 \left[\frac{2}{r(1 + \frac{M}{r})} - \frac{2}{r} \right] = M. \quad (7)$$

Hence, the ADM mass of the proposed naked singularity spacetime is M . The above metric [Eq. (1)] resembles with Schwarzschild metric at a large distance. Therefore, at a sufficient distance from the naked singularity, the behavior of timelike and null geodesics is similar to what is seen in the Schwarzschild geometry. However, near the singularity, the causal structure of this spacetime becomes different from the causal structure of Schwarzschild spacetime. Distinguishable behavior of null geodesics and the shadow cast by the naked singularity in this spacetime are discussed in the next section.

Using Einstein field equations, we can write the energy density and pressures of this naked singularity spacetime as

$$-T'_t = \rho = \frac{M^2(M + 3r)}{\kappa r^2(M + r)^3}, \quad (8)$$

$$T'_r = p_r = -\frac{M^2(M + 3r)}{\kappa r^2(M + r)^3}, \quad (9)$$

$$T^\theta_\theta = T^\phi_\phi = p_\theta = p_\phi = \frac{3M^2}{\kappa r^4} \left(1 + \frac{M}{r}\right)^{-4}, \quad (10)$$

where $\kappa = \frac{8\pi G}{c^2} = 8\pi$, as in this paper, we consider $G = c = 1$. To satisfy strong, weak, and null energy conditions, we need

$$\rho = \frac{M^2(M + 3r)}{\kappa r^2(M + r)^3} \geq 0, \quad (11)$$

$$\rho + p_r \geq 0, \quad (12)$$

$$\rho + p_\theta = \frac{M^2(M^2 + 4Mr + 6r^2)}{\kappa r^2(M + r)^4} \geq 0, \quad (13)$$

$$\rho + p_r + 2p_\theta = \frac{6M^2}{\kappa r^4} \left(1 + \frac{M}{r}\right)^{-4} \geq 0, \quad (14)$$

and it can be easily verified that all these conditions are fulfilled for this spacetime. As it can be seen, this spacetime is seeded by an anisotropic fluid, where the anisotropy in the pressures can be written as

$$p_r - p_\theta = -\frac{M^2(M^2 + 4Mr + 6r^2)}{\kappa r^2(M + r)^4}. \quad (15)$$

The equation of state (α) for an anisotropic fluid can be written as [42]

$$\alpha = \frac{2p_\theta + p_r}{3\rho}. \quad (16)$$

Therefore, using Eqs. (8)–(10), we can write the equation of state for this spacetime as

$$\alpha = \frac{2}{(3 + \frac{M}{r})(1 + \frac{M}{r})} - \frac{1}{3}, \quad (17)$$

where we can clearly see that as r tends to zero, the equation of state becomes $-1/3$. On the other hand, as r tends to infinity, the equation of state becomes $+1/3$. It is important to know the source of the anisotropic fluid which seeds such a particular spacetime. As we know, to be a minimally coupled static scalar field solution [42] of Einstein equations, a spacetime should satisfy

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \kappa \left(g^{\mu\nu}L + 2\frac{\partial L}{\partial g_{\mu\nu}} \right), \quad (18)$$

where $L = (-\frac{1}{2}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi - V(\Phi))$ and Φ is scalar field which satisfies $\square\Phi(r) = V'(\Phi(r))$, where $V(\Phi)$ is the scalar field potential. Using the above equations, one can check that for minimally coupled scalar field solution, we need $\rho + p_\theta = 0$. From Eqs. (8)–(10), it is easy to verify that this naked singularity spacetime is not a minimally coupled scalar field solution of Einstein equations.

B. JMN1 spacetime

The line element of the JMN1 spacetime can be written as

$$ds^2 = -(1 - M_0) \left(\frac{r}{R_b} \right)^{\frac{M_0}{1-M_0}} dt^2 + \frac{dr^2}{1 - M_0} + r^2 d\Omega^2, \quad (19)$$

where the dimensionless quantity M_0 should be $0 < M_0 < 1$ and at R_b , JMN1 spacetime matches with the external Schwarzschild spacetime. In [8], it is shown that in asymptotic time, JMN1 spacetime can be formed as an end state of the gravitational collapse of anisotropic matter fluid. This spacetime has a timelike strong singularity at the center [8]. Using the parameters of JMN1 spacetime, the external Schwarzschild spacetime can be written as

$$ds^2 = -\left(1 - \frac{M_0 R_b}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{M_0 R_b}{r}\right)} + r^2 d\Omega^2.$$

One can prove that this spacetime smoothly matches with external Schwarzschild metric. JMN1 spacetime is an anisotropic fluid solution of Einstein equations, where the energy density and pressures can be written as

$$\rho = \frac{M_0}{\kappa r^2}, \quad P_r = 0, \quad P_\theta = \frac{M_0^2}{4\kappa(1-M_0)r^2}. \quad (20)$$

Therefore, the equation of state for this spacetime can be written as $\alpha = \frac{M_0}{6(1-M_0)}$. There are many literature where particle trajectories in JMN1 spacetime and the shadow cast by the same are discussed, and it is shown that those results are very important to understand the distinguishable observational signatures of black holes and naked singularities.

In the next section, we discuss about the properties of lightlike geodesics and the shadow of naked singularity mentioned in Eq. (1).

III. SHADOWS OF NAKED SINGULARITY AND BLACK HOLE SPACETIMES

A spherically symmetric, static spacetime can be written as

$$ds^2 = -g_{tt}dt^2 + g_{rr}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (21)$$

where g_{tt} , g_{rr} are the functions of r only and the azimuthal part of the spacetime shows the spherical symmetry. For the lightlike geodesics in the above spacetime, in the $\theta = \frac{\pi}{2}$ plane, we can write

$$\frac{1}{b^2} = \frac{g_{tt}g_{rr}}{h^2} \left(\frac{dr}{d\lambda} \right)^2 + V_{\text{eff}}, \quad (22)$$

where the effective potential $V_{\text{eff}} = g_{tt}/r^2$ and the impact parameter $b = \frac{h}{\gamma}$, where γ and h are conserved energy and conserved angular momentum per lightlike particle, respectively. In the above equation, we use $k_\mu k^\mu = 0$, where k^μ is the null four velocity. From the nature of effective potential of photons, one can investigate the stable and unstable orbits of photons. As we know, when this effective potential has a maximum value at some radius, we get unstable circular lightlike geodesics. The sphere corresponding to the radius of these unstable circular lightlike geodesics is known as the photon sphere. At the photon sphere radius (r_{ph}), we can write $V_{\text{eff}}(r_{ph}) = \frac{\gamma^2}{h^2}$, $V'_{\text{eff}}(r_{ph}) = 0$, and $V''_{\text{eff}}(r_{ph}) < 0$.

When these conditions are satisfied in a spacetime, we can say that there would exist a photon sphere at r_{ph} . For Schwarzschild spacetime, photon sphere exists at $r_{ph} = 3M_T$, where M_T is the Schwarzschild mass. The turning point (r_{tp}) of lightlike geodesics can be found when $V_{\text{eff}}(r_{tp}) = \frac{\gamma^2}{h^2} = \frac{1}{b_{tp}^2}$. Therefore, from the expression of V_{eff} , at the turning point, we can write the impact parameter b_{tp} as

$$b_{tp} = \frac{r_{tp}}{\sqrt{g_{tt}(r_{tp})}}. \quad (23)$$

In a spacetime, when only one extremum value of effective potential of null geodesics exists and if the extremum value is the maximum value of the potential, then the minimum impact parameter of turning point of light geodesics is $b_{tp} = b_{ph}$, where b_{ph} is the impact parameter corresponding to the photon sphere. Lightlike geodesics, coming from faraway source, cannot reach to the faraway observer with the impact parameter $b < b_{ph}$. They will be trapped inside the photon sphere. Therefore, the light geodesics coming from behind the lens with impact parameter $b < b_{ph}$ cannot reach to the outside observer and this creates a shadow of radius b_{ph} in the observer's sky. Hence, one can conclude that in this case, it is the photon sphere's shadow which can be seen in the observer's sky. On the other hand, when the photon sphere does not exist in a spacetime and the effective potential diverges at origin, then such a shadow would not form in that spacetime.

In Fig. 1, we show the trajectory of photons which are coming out from the far away source and deflected due to the curvature in JMN1, Schwarzschild, and the new naked singularity spacetimes. In JMN1 spacetime, photon sphere exists when $M_0 > \frac{2}{3}$ and for $M_0 < \frac{2}{3}$, photon sphere does not exist. For $M_0 > \frac{2}{3}$, JMN1 spacetime matches with external Schwarzschild metric at $R_b < 3M_T$, where M_T is the Schwarzschild mass. Therefore, effectively, the spacetime configuration has a photon sphere which is in the external Schwarzschild spacetime. However, inside the photon sphere, at the center, there exists a spacetime singularity which is not covered by any event horizon. Therefore, around the photon sphere lightlike geodesics behave in the similar way what can be seen around the photon sphere of a black hole. The effective potential and the behavior of lightlike geodesics in JMN1 spacetime are shown in Figs. 1(a) and 1(b), respectively, where we take Schwarzschild mass $M_T = 1$ and $M_0 = 0.7$. Therefore, the JMN1 spacetime matches with external Schwarzschild spacetime at $R_b = 2.857$ and the radius of the photon sphere corresponding to the external Schwarzschild spacetime is $r_{ph} = 3$. Hence, for $M_0 > \frac{2}{3}$, shadow is effectively cast by the photon sphere of the external Schwarzschild spacetime. In Figs. 1(c) and 1(d), we show the effective potential and nature of light geodesics, respectively, in the Schwarzschild spacetime with Schwarzschild mass $M_T = 1$. The brown circles in Figs 1(b) and 1(d) show the size of photon sphere ($r_{ph} = 3$) in JMN1 spacetime configuration and in Schwarzschild spacetime, respectively, and the red circles in these two figures show the size of the impact parameter (b_{ph}) corresponding to the radius of the photon sphere. From Eq. (23), one can calculate that the shadow radius or the impact parameter corresponding to the photon sphere radius is $b_{ph} = 3\sqrt{3}M_T = 3\sqrt{3}$.

For the new naked singularity spacetime, the minimum turning point radius is $r_{tp} = 0$. In Fig. 1(e), it is shown that

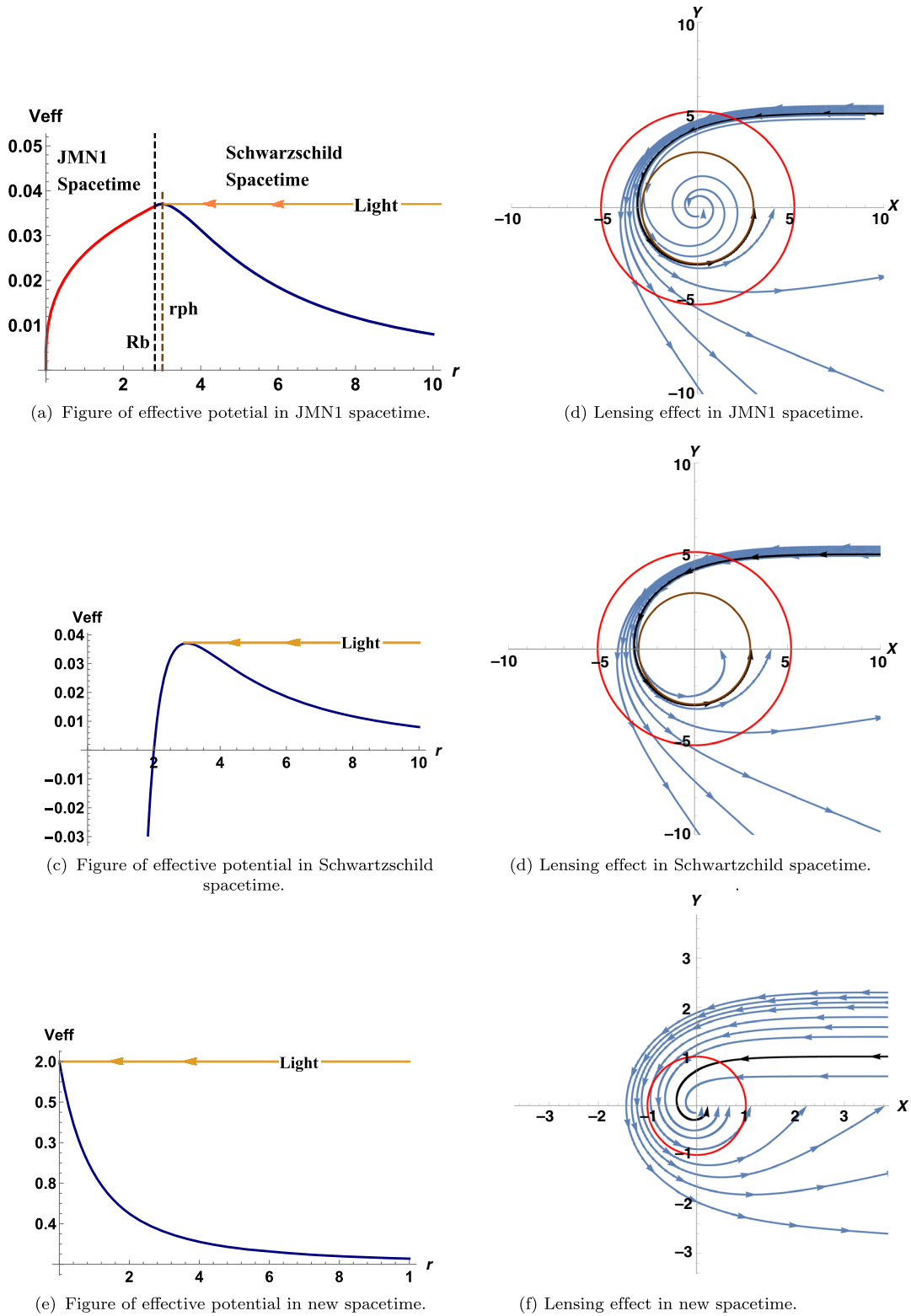


FIG. 1. In Figs. 1(a), 1(c), and 1(e), we show the nature of effective potentials of the null geodesics in JMN1, Schwarzschild, and the naked singularity spacetimes given here, respectively. In Figs. 1(b), 1(d), and 1(f), the light trajectories in these spacetimes are shown. In Figs. 1(b), 1(d), and 1(f), the blue lines are the null geodesics. Effective potential of lightlike geodesics in JMN1 spacetime configuration is shown in Fig. 1(a). As we can see, the effective potential has a maximum value in JMN1 spacetime configuration and in Schwarzschild spacetime. Therefore, for these two spacetimes, photon spheres exist. The brown circles in Figs. 1(b) and 1(d) show the size of the photon sphere. The red circles show the size of the shadow. A detailed analysis of these figures is given in the text.

the effective potential of lightlike geodesics has a finite value. Therefore, photons with finite energy can reach very close to the central naked singularity. Photons having energy $E_{ph} \geq V_{\text{eff}}(0)$ can reach to the central singularity. Now, the value of effective potential and the impact parameter corresponding to the minimum value of the radius of the turning point (i.e., $r_{tp} = 0$) are

$$V_{\text{eff}}(r_{tp})|_{r_{tp}=0} = \frac{1}{(r_{tp} + M)^2} \Big|_{r_{tp}=0} = \frac{1}{M^2}, \quad (24)$$

$$b_{tp}|_{r_{tp}=0} = (r_{tp} + M)|_{r_{tp}=0} = M. \quad (25)$$

Therefore, we can get finite value of the impact parameter corresponding to the zero value of minimum turning point radius and that finite value of the impact parameter $b_{tp}(r_{tp} = 0) = M$, where M is the ADM mass of the naked singularity spacetime given here. Hence, though the value of the minimum turning point radius is zero, due to the finite value of corresponding impact parameter, in this case also, the lightlike geodesics coming from a distant source with impact parameter $b < M$ must be trapped closer to the singularity. Therefore, in this naked singularity spacetime, the curvature of the spacetime itself around the singularity can cast a shadow in the observer sky, even when there exists no photon sphere in this case. As the minimum turning point of the photons is at the singularity, one can conclude that it is the shadow of the singularity itself. In Fig. 1(f), we show the nature of lightlike geodesics in this naked singularity spacetime. In that figure, the lightlike geodesic highlighted in black corresponds to the critical impact parameter $b_{tp} = M$. From Figs. 1(b), 1(d), and 1(f), the distinguishable nature of lightlike geodesics in the new naked singularity spacetime, and in JMN1 and Schwarzschild spacetimes, can be clearly seen. We note that to visualize the shadow of a central massive object which can be seen in a physical situation, one needs to calculate the intensity of light coming out from the accreting matter around the massive central object.

In this paper, for simplicity, we have considered spherically symmetric, radially freely falling, thin accreting matter which radiates monochromatic radiation, where the emissivity (in the emitter frame) per unit volume falls as

$$j(\nu_e) \propto \frac{\delta(\nu_e - \nu_*)}{r^2}, \quad (26)$$

where ν_e is the emitted photon frequency as measured in the rest frame of the emitter. The observed specific intensity (I_{ν_o}) of the light coming out from the accreting matter can be written as the function of X, Y which are the coordinates of the asymptotic observer's sky [39],

$$I_{\nu_o}(X, Y) = \int_{\gamma} g^3 j(\nu_e) dl_{\text{prop}}, \quad (27)$$

where $g = \nu_o/\nu_e$ is the redshift factor and dl_{prop} is the infinitesimal proper length in the rest frame of emitter with $dl_{\text{prop}} = -k_{\alpha} u_e^{\alpha} d\lambda$, where k^{μ} is the null four velocity and u_e^{α} is the timelike four velocity of the emitter. Here λ is the affine parameter and the integration is done along the lightlike trajectory γ . The redshift factor can be written as

$$g = \frac{k_{\alpha} u_o^{\alpha}}{k_{\beta} u_e^{\beta}}, \quad (28)$$

where $u_o^{\mu} = (1, 0, 0, 0)$ is the four velocity of the static distant observer. As we mentioned before, in this paper, we consider spherically symmetric accreting matter which is radially freely falling. The components of four velocity of radially freely falling particle in a general spherically symmetric, static spacetime [Eq. (21)] can be written as

$$u_e^t = \frac{1}{g_{tt}}, \quad u_e^r = -\sqrt{\frac{(1 - g_{tt})}{g_{tt}g_{rr}}}, \quad u_e^{\theta} = u_e^{\phi} = 0. \quad (29)$$

Using the components of four velocity of radially freely falling matter, we can write down the redshift factor as

$$g = \frac{1}{\frac{1}{g_{tt}} - \frac{k_r}{k_t} \sqrt{\frac{(1 - g_{tt})}{g_{tt}g_{rr}}}}, \quad (30)$$

where

$$\frac{k^r}{k^t} = \sqrt{\frac{g_{tt}}{g_{rr}} \left(1 - \frac{g_{tt} b^2}{r^2} \right)}. \quad (31)$$

Now, integrating Eq. (27) over all observed frequencies, we get [39]

$$I_o(X, Y) \propto - \int_{\gamma} \frac{g^3 k_t dr}{r^2 k^r}, \quad (32)$$

where $I_o(X, Y)$ is the intensity distribution in the (X, Y) plane of the observer's sky, where $X^2 + Y^2 = b^2$. Using Eq. (32), one can simulate the shadow. In Figs 2(a), 2(c), and 2(e), we show how intensity varies with the impact parameter b in the Schwarzschild, JMN1, and the naked singularity spacetimes given here, respectively. In Figs. 2(b), 2(d), and 2(f), we simulate the shadows cast by Schwarzschild, JMN1, and the new naked singularity spacetimes, respectively.

From Fig. 2, one can see that due to the presence of photon sphere in the Schwarzschild spacetime and in the external Schwarzschild spacetime of the JMN1 space-time configuration (with $M_0 > \frac{2}{3}$), the shadow radius is

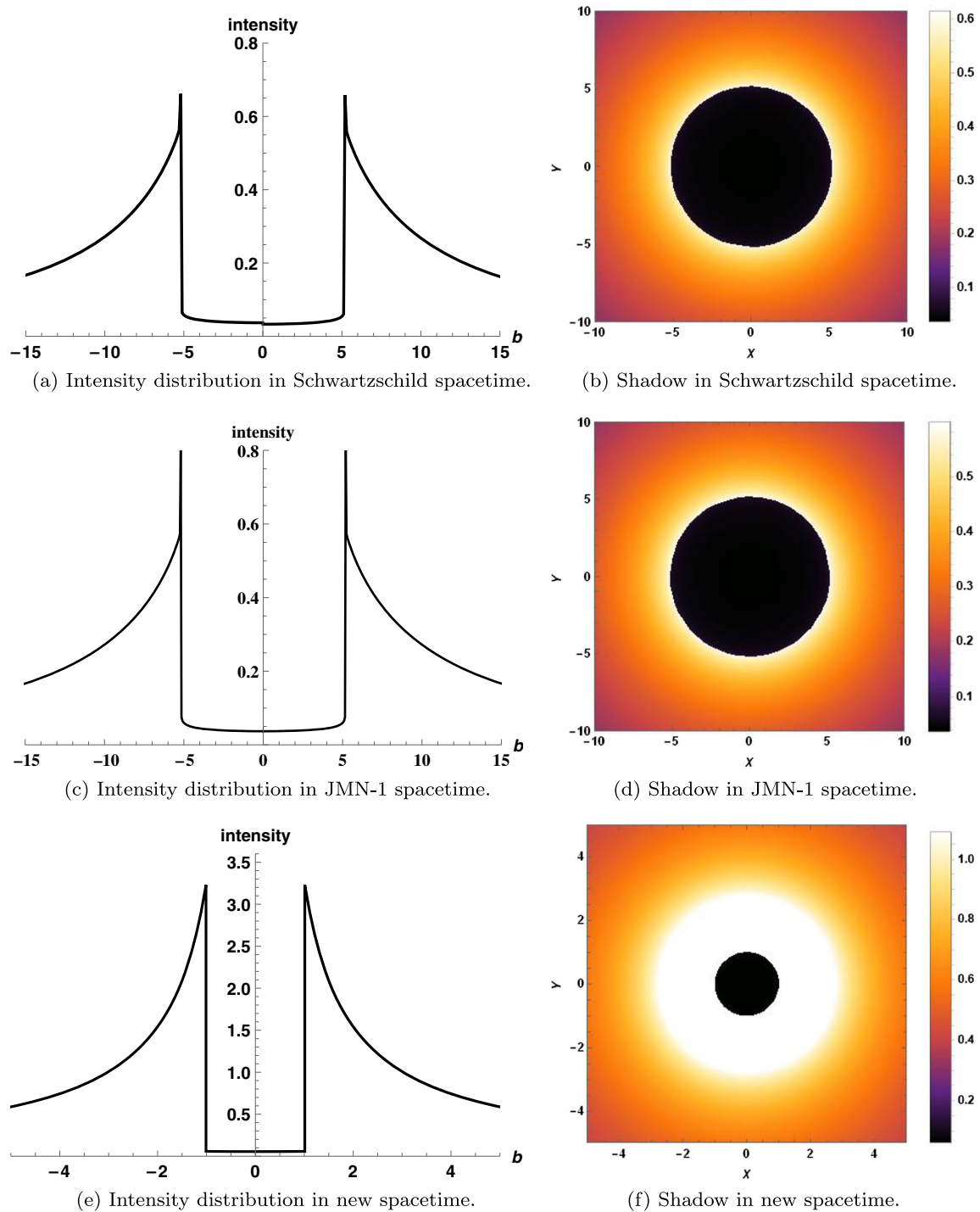


FIG. 2. In this figure, the intensity map in observer sky (left column figures) and the shadow of the central object (right column figures) are shown for Schwarzschild spacetime, JMN1 spacetime configuration (with $M_0 = 0.7$), and the naked singularity spacetime given here. The radius of the shadow of the Schwarzschild black hole [Fig. 2(b)] and JMN1 naked singularity [Fig. 2(d)] is $3\sqrt{3}$, where the Schwarzschild mass is $M_T = 1$. The shadow shown in the right bottom corner is the shadow of the new naked singularity spacetime and the size of the shadow is $M = 1$, where M is the ADM mass.

$b_{ph} = 3\sqrt{3}M_T = 3\sqrt{3}$. The shadow cast by the JMN1 spacetime configuration with $M_0 > \frac{2}{3}$ is not distinguishable from the shadow cast by a Schwarzschild black hole [30]. On the other hand, the radius of the shadow cast by the new

naked singularity spacetime is M , where M is the ADM mass of the spacetime. If we consider the ADM mass to be $M = 1$, then the shadow radius will be $b_{1p}|_{r_{1p}=0} = M = 1$. Therefore, the shadow cast by Schwarzschild black hole

TABLE I. Mass of sagittarius A* and its shadow size in naked singularity and black hole spacetimes.

Object	Mass (M_{\odot})	Shadow (diameter) (μarcsec)
Black hole	$(4.31 \pm 0.38) \times 10^6$	56 ± 8
JMN naked singularity	$(4.31 \pm 0.38) \times 10^6$	56 ± 8
New naked singularity	$(4.31 \pm 0.38) \times 10^6$	10 ± 8

and JMN1 naked singularity spacetime is $3\sqrt{3}$ times the size of the shadow cast by the naked singularity considered here. It can be seen that in each of the shadow images of Fig. 2, there exists a color bar which shows that the intensity is not zero inside the critical b_{lp} (i.e., $3\sqrt{3}$ for Schwarzschild and JMN1 spacetimes and 1 for new naked singularity spacetime), though it looks like absolutely dark.

As we know, in the near future, the EHT group will release the shadow of the Sagittarius A* (Sgr-A*). Therefore, it is very important to theoretically predict the size of the shadow. From the S stars motions around our galaxy center and from other physical phenomenon, it is estimated that the mass of the central object of our Milky Way Galaxy is around $4.31 \times 10^6 M_{\odot}$ with an error $\pm 0.38 M_{\odot}$ [39]. If the central object is a black hole or JMN1 naked singularity, then it is not possible to distinguish the black hole shadow from a naked singularity shadow. In Table I, it is shown that the angular size of the shadow of the Schwarzschild black hole and the JMN1 naked singularity is $56 \pm 8 \mu\text{arcsec}$.

However, when such a spacetime exists where a naked singularity can form the shadow (e.g., the one we have analyzed here), then the shadow size can be quite different for the same value of the ADM mass. In Table I, we show that the naked singularity spacetime we discussed, with ADM mass $(4.31 \pm 0.38) \times 10^6 M_{\odot}$, would cast a shadow of diameter $10 \pm 8 \mu\text{arcsec}$.

IV. CONCLUSION

In this paper, we study the nature of light geodesics and shadow in a new naked singularity spacetime, and compare the results with the light geodesics and shadow in the Schwarzschild and JMN1 (with $M_0 > \frac{2}{3}$) spacetimes. We show that the nature of light geodesics in JMN1 and Schwarzschild spacetimes is quite different from that of light geodesics in the proposed naked singularity spacetime. We also show that the shadow cast by the photon sphere (Schwarzschild and JMN1 with $M_0 > \frac{2}{3}$ case) is $3\sqrt{3}$ times bigger than the shadow cast by the singularity,

that happens for the new naked singularity spacetime, when we take same ADM mass for all spacetimes.

The main purpose of this work is to show that the shadow is not only the property of a black hole or a photon sphere, but a singularity by itself can also cast a shadow, as demonstrated explicitly by the example spacetime we have considered and proposed here. An important point that follows is one of the observable signatures of a naked singularity is the shadow that it could cast. Thus, such a shadow could be an important property of a naked singularity. Further, the important characteristic of such a shadow of singularity is, it would be considerably smaller in size, as compared to the shadow of a black hole. This would provide an important characteristic and observable difference that might distinguish a singularity from the black hole.

Near the naked singularity, the quantum gravity effects should be dominant, and therefore, such quantum gravity effects might be manifested or can be observed in the shadow cast by a naked singularity. This will require a detailed analysis of the various features encoded in such shadows. Hence, this novel feature of a naked singularity may be important as well in the context of recent observations of the shadow of the Galactic center M87 [17] and for the upcoming image of the center of Milky Way Galactic center (Sgr-A*). Of course, what we have given here is a case study, and more detailed analysis will be needed to know if shadows are a general feature associated with singularities.

It is intriguing to note that when proposed initially, black holes were considered to be highly exotic objects, and their existence was even doubted sometimes. The past few decades have, however, seen them to emerge as more realistic astrophysical objects, used to explain several cosmic phenomena. Similarly, spacetime singularities and the naked singularities are seen as probable exotic entities in the cosmos. But, as shown here, if they manifest observable features such as casting and creating shadows in their own right, this could be an interesting and useful step to bring them in an observable domain and in the realm of physical reality.

- [1] P.M. Chesler, R. Narayan, and E. Curiel, [arXiv:1902.08323](#).
- [2] P.R. Brady, *Prog. Theor. Phys. Suppl.* **136**, 29 (1999).
- [3] E. Poisson, *A Relativist's Toolkit: The Mathematics of Black-Hole Mechanics* (Cambridge University Press, Cambridge, United Kingdom, 2004).
- [4] P.S. Joshi and I.H. Dwivedi, *Phys. Rev. D* **47**, 5357 (1993).
- [5] R. Goswami, P.S. Joshi, C. Vaz, and L. Witten, *Phys. Rev. D* **70**, 084038 (2004).
- [6] H. Kudoh, T. Harada, and H. Iguchi, *Phys. Rev. D* **62**, 104016 (2000).
- [7] K. Mosani, D. Dey, and Pankaj S. Joshi, *Phys. Rev. D* **101**, 044052 (2020).
- [8] P.S. Joshi, D. Malafarina, and R. Narayan, *Classical Quantum Gravity* **28**, 235018 (2011).
- [9] P.S. Joshi, D. Malafarina, and R. Narayan, *Classical Quantum Gravity* **31**, 015002 (2014).
- [10] K. Bhattacharya, D. Dey, A. Mazumdar, and T. Sarkar, *Phys. Rev. D* **101**, 044052 (2020).
- [11] D. Dey, P. Kocherlakota, and P.S. Joshi, [arXiv:1907.12792](#).
- [12] U. Banik, D. Dey, K. Bhattacharya, and T. Sarkar, *Gen. Relativ. Gravit.* **49**, 116 (2017).
- [13] A. I. Janis, E. T. Newman, and J. Winicour, *Phys. Rev. Lett.* **20**, 878 (1968).
- [14] R. Goswami, P.S. Joshi, and P. Singh, *Phys. Rev. Lett.* **96**, 031302 (2006).
- [15] L. Szulc, *Classical Quantum Gravity* **24**, 6191 (2007).
- [16] A. Ashtekar, T. Pawłowski, P. Singh, and K. Vandersloot, *Phys. Rev. D* **75**, 024035 (2007).
- [17] K. Akiyama *et al.* (Event Horizon Telescope Collaboration), *Astrophys. J.* **875**, L5 (2019).
- [18] R. Abuter *et al.* (GRAVITY Collaboration), *Astron. Astrophys.* **618**, L10 (2018).
- [19] F. Eisenhauer *et al.*, *Astrophys. J.* **628**, 246 (2005).
- [20] S. Gillessen *et al.*, *Astrophys. J.* **837**, 30 (2017).
- [21] S. Zhou, R. Zhang, J. Chen, and Y. Wang, *Int. J. Theor. Phys.* **54**, 2905 (2015).
- [22] D. Dey, K. Bhattacharya, and T. Sarkar, *Phys. Rev. D* **88**, 083532 (2013).
- [23] D. Dey, K. Bhattacharya, and T. Sarkar, *Phys. Rev. D* **87**, 103505 (2013).
- [24] V. Perlick, *Classical Quantum Gravity* **9**, 1009 (1992).
- [25] P. Bambhaniya, A. B. Joshi, D. Dey, and P. S. Joshi, *Phys. Rev. D* **100**, 124020 (2019).
- [26] D. Dey, P. S. Joshi, A. B. Joshi, and P. Bambhaniya, *Int. J. Mod. Phys. D* **28**, 1930024 (2019).
- [27] S. Sahu, M. Patil, D. Narasimha, and Pankaj S. Joshi, *Phys. Rev. D* **86**, 063010 (2012).
- [28] S. Sahu, M. Patil, D. Narasimha, and Pankaj S. Joshi, *Phys. Rev. D* **88**, 103002 (2013).
- [29] R. Shaikh, P. Banerjee, S. Paul, and T. Sarkar, *Phys. Rev. D* **99**, 104040 (2019).
- [30] R. Shaikh, P. Kocherlakota, R. Narayan, and P.S. Joshi, *Mon. Not. R. Astron. Soc.* **482**, 52 (2019).
- [31] R. Shaikh, *Phys. Rev. D* **100**, 024028 (2019).
- [32] R. Shaikh and P. S. Joshi, *J. Cosmol. Astropart. Phys.* **10** (2019) 064.
- [33] G. Gyulchev, P. Nedkova, T. Vetsov, and S. Yazadjiev, *Phys. Rev. D* **100**, 024055 (2019).
- [34] N. Ortiz, O. Sarbach, and T. Zannias, *Phys. Rev. D* **92**, 044035 (2015).
- [35] K.-i. Nakao, N. Kobayashi, and H. Ishihara, *Phys. Rev. D* **67**, 084002 (2003).
- [36] K.-i. Nakao, C.-M. Yoo, and T. Harada, *Phys. Rev. D* **99**, 044027 (2019).
- [37] N. Sakai, H. Saida, and T. Tamaki, *Phys. Rev. D* **90**, 104013 (2014).
- [38] R. Narayan, M. D. Johnson, and C. F. Gammie, *Astrophys. J. Lett.* **885**, L33 (2019).
- [39] C. Bambi, *Phys. Rev. D* **87**, 107501 (2013).
- [40] D. Dey, R. Shaikh, and P. S. Joshi, [arXiv:2003.06810](#).
- [41] L. Brewin, *Gen. Relativ. Gravit.* **39**, 521 (2007).
- [42] D. Dey, K. Bhattacharya, and T. Sarkar, *Gen. Relativ. Gravit.* **47**, 103 (2015).