Anisotropic fluid cosmology: An alternative to dark matter?

M. Cadoni,[1,2,*] A. P. Sanna,[1,†] and M. Tuveri[2,‡]

[1]*Dipartimento di Fisica, Università di Cagliari Cittadella Universitaria, 09042 Monserrato, Italy*

[2]*I.N.F.N, Sezione di Cagliari, Cittadella Universitaria, 09042 Monserrato, Italy*

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We use anisotropic fluid cosmology to describe the present, dark energy-dominated Universe without assuming the presence of dark matter. The resulting anisotropic fluid spacetime naturally generates inhomogeneities at small scales, triggered by an anisotropic stress, that could therefore be responsible for structure formation at these scales. We show that the dynamics of the scale factor a is described by the usual Friedmann-Lemaître-Robertson-Walker cosmology and decouples completely from that describing inhomogeneities. Assuming that the fluid inherits the equation of state from galactic dynamics, we show that, in the large scale regime, it can be described as a generalized Chaplygin gas. We find that our model fits well the distance modulus experimental data of type Ia supernovae, thus correctly modeling the observed accelerated expansion of the Universe. Conversely, in the small scale regime, we use cosmological perturbation theory to derive the power spectrum $P(k)$ for mass density distribution. At short wavelengths, we find a $1/k^4$ behavior, in good accordance with the observed correlation function for matter distribution at small scales.

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I. INTRODUCTION

Our present understanding of cosmology, large scale structure of our Universe and galactic dynamics is based on the ΛCDM model [1]. The model is based on the cosmological principle, which states that at sufficiently large scales, our Universe appears to be homogeneous and isotropic. The ΛCDM model shows good agreement with observational data; however, it is not completely satisfactory from a conceptual point of view. It postulates that about 95% of the matter contained in our Universe is of an exotic nature. At the galactic level, it fails to explain the Tully-Fisher (TF) relation. Moreover, there is also some tension between the ΛCDM model and observations at the level of galaxies, galaxy clusters [2], and the determination of the Hubble parameter [3,4].

Although the Universe is homogeneous and isotropic at large scales, the existence of structures such as galaxies, stars, or planets implies that it is inhomogeneous and anisotropic on smaller scales. For what concerns inhomogeneity, the transition scale is about $\mathcal{R} = 100$–$300\ \mathrm{Mpc}/h$ [5–8], where h parametrizes the Hubble constant, i.e., $H_0 = 100h\ \mathrm{km\,s^{-1}\,Mpc}$.

Thus, if one focuses only on the present, dark-energy dominated era, the simplest description of our Universe should be that of an inhomogeneous cosmological model, in which inhomogeneities disappear when averaged at scales larger than $\mathcal{R}$. In the ΛCDM framework, inhomogeneities are explained in terms of the gravitational growth due to dark matter of a primeval scale invariant and Gaussian perturbations generated during inflation [9,10]. The early Universe is homogeneous and isotropic, the departures from homogeneity at high redshift are well described by perturbation theory, which results in a scale-invariant power spectrum $P(k)$ for the mass distribution at long wavelengths [11,12],

$$P(k) \sim k, \tag{1}$$

where k represents the wave number. At small scales, a method to describe the statistical distribution of cosmic structures is given by the two-point correlation function, $\xi(r)$, which is the Fourier transform of $P(k)$. Observations show that, at physical scales ranging from 100 kpc/h to 10 Mpc/h, $\xi(r)$ is well-described by a simple power law [13–16],

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}, \tag{2}$$

where $r_0 \sim 5\ \mathrm{Mpc}/h$ is the so-called "correlation length" and γ is experimentally determined. Observations suggest that $\gamma \in [1.8, 2]$ [13–16]. Using Eq. (2) with $\gamma = 2$, we get, computing the Fourier transform,

$$P(k) \sim k^{-4}. \tag{3}$$

*mariano.cadoni@ca.infn.it
†asanna9564@yahoo.it
‡matteo.tuveri@ca.infn.it

The inhomogeneous cosmological model has been extensively studied in cosmology; see, e.g., [17–24]. However, it is not easy to consider them as a full alternative to the ΛCDM model. They cannot be proposed as a model describing the full history of our Universe due to the gravitational instability of mass distributions in the early Universe in Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology.

On the other hand, an inhomogeneous cosmological model can be used as an effective description of the present, dark energy-dominated era of our Universe, i.e., for redshift $z \lesssim 1$ [1]. In particular, we expect this inhomogeneous model to determine the short wavelength behavior (3) since the latter is valid on scales which are smaller than the homogeneity transition scale mentioned at the beginning of this section.

At galactic scales, gravity sourced by an anisotropic fluid can give an effective description of the additional force commonly attributed to dark matter [25]. Thus, it is also the most natural candidate for the source in our inhomogeneous cosmology model. Indeed, motivated mainly by the conceptual difficulties at the level of galactic dynamics, recently, several alternative proposals have been put forward to explain the galactic phenomenology commonly attributed to dark matter [25–29]. Typically, these alternative approaches use infrared modifications of general relativity (GR), where the additional force at the galactic level is generated by the interaction between baryonic matter and dark energy (DE). A common feature of these attempts is the fact that, in the weak field regime, they all reproduce Milgrom's modified Newtonian dynamics (MOND) [30,31], which gives a simple explanation of the Tully-Fisher relation.

So far, the previously mentioned attempts have been mainly confined to galactic dynamics. However, there are several reasons that strongly motivate their extension to cosmology. Firstly, dark matter plays a crucial role not only in galactic dynamics, but also in structure formation [9]. Any alternative to dark matter should therefore not only explain anomalous galactic rotation curves but also structure formation. Secondly, the threshold acceleration parameter a_0, appearing in the TF relation, has the same order of magnitude of the Hubble constant H_0, indicating the existence of a deep connection between galactic dynamics and cosmology. Last but not least, in the emergent gravity scenario, the additional force beyond the Newtonian one is a "dark force" that originated from the response of dark energy to the presence of baryonic matter, linking again the physics at galactic scales to cosmology.

It is therefore tempting to look for a unified description encompassing different regimes of gravity: Newtonian, galactic, cosmological. This paper is devoted to the attempt of building such a cosmological model, motivated by the emergent gravity description of galactic dynamics, without assuming the presence of dark matter. It is known that, at the galactic level, dark force effects allow for an effective description in terms of GR sourced by an anisotropic fluid [25]. We will therefore use anisotropic fluid cosmology as an effective description of the cosmological effects of DE-baryonic matter interaction in a dark energy-dominated Universe.

The structure of this paper is as follows. In Sec. II, we discuss how an anisotropic fluid spacetime can be used to describe cosmic structures at small scales. We set up our cosmological model sourced by an anisotropic fluid and solve the resulting cosmological equations. The equation of state (EoS) for our anisotropic fluid is described in Sec. III. In Sec. IV, we investigate the large scale regime of our cosmological model, show that the predictions of our model fits very well the distance modulus data of type Ia supernovae, and discuss its relationship in terms of a generalized Chapligyn gas. In Sec. V, we discuss cosmological perturbations. We first consider isotropic perturbations, which describe the behavior of the power spectrum $P(k)$ for a mass density distribution at large wavelengths. Thereafter, we consider perturbations described by an anisotropic stress tensor. This allows us to find $P(k) \sim 1/k^4$, within a good accordance with the observed correlation function for matter distribution at small scales. Finally, in Sec. VI, we present our conclusions.

II. ANISOTROPIC FLUID COSMOLOGICAL MODEL

An anisotropic fluid can be used as a description of a two-fluid model [32]. Moreover, as elucidated in the previous section, it is a promising candidate for describing an universe made of baryonic matter and dark energy (and their interaction), and the transition from an inhomogeneous universe at short scales to a homogeneous one at large scales, during the dark energy-dominated epoch.

We set up a cosmological model in which the various forms of matter, sourcing cosmological evolution and structure formation, are described by an anisotropic fluid with an energy-momentum tensor given by [33,34]

$$T_{\mu\nu} = T^{(pf)}_{\mu\nu} + \pi_{\mu\nu}, \tag{4}$$

where $T^{(pf)}$ is the stress-energy tensor of the perfect fluid. The anisotropic stress tensor $\pi_{\mu\nu}$ is[1]

$$\pi_{\mu\nu} = \sqrt{3}\mathcal{S}\left[w_\mu w_\nu - \frac{1}{3}(u_\mu u_\nu + g_{\mu\nu})\right], \tag{5}$$

where the fluid velocity u_μ and the spacelike vector w_ν satisfy $u^\nu u_\nu = -1$, $w^\nu w_\nu = 1$, and $u^\mu w_\mu = 0$. The energy density is given by ρ and the pressure components $p_\perp$, $p_\parallel$ are perpendicular and parallel to the spacelike vector w_ν,

[1]Throughout this paper, we will use natural units $c = \hbar = 1$.

respectively. The quantity $\mathcal{S}$ quantifies the degree of anisotropy, $\mathcal{S} = 3^{-1/2}(p_\parallel - p_\perp)$.

One can easily check that $\pi_{\mu\nu}$ satisfies the usual relations for a gauge-invariant anisotropic stress tensor, $\pi^{\mu\nu}u_\nu = \pi^\mu_\mu = 0$ and $\pi_{00} = \pi_{0i} = 0$.[2]

In the comoving frame, the only nonvanishing components of $\pi^{\mu\nu}u_\nu$ are the spatial ones which can be identified with the radial and transverse components of the anisotropic fluid pressure,

$$T^1_1 = p_\parallel = P + \frac{2\mathcal{S}}{\sqrt{3}}; \quad T^2_2 = T^3_3 = p_\perp = P - \frac{\mathcal{S}}{\sqrt{3}}. \tag{6}$$

When $\mathcal{S} = 0$, we have $p_\parallel = p_\perp = P$; the fluid is perfect, homogeneous, and isotropic. Conversely, $\mathcal{S} \neq 0$ implies $p_\parallel \neq p_\perp$, signalizing anisotropic departure from a perfect fluid.

If we take $p_\perp = p_\parallel$ and assume a spatially homogeneous and isotropic universe, we get the usual FLRW cosmological model with p, ρ, and the scale factor of the metric depending on the cosmological time T only.

The simplest way to achieve $p_\perp \neq p_\parallel$ is to allow for a dependence of $p_\perp$, $p_\parallel$, and ρ from the radial coordinate r. The spacetime is not homogenous anymore, but remains isotropic, the only manifestation of anisotropy being $p_\perp \neq p_\parallel$, which therefore becomes the source of the inhomogeneities. This is consistent with the cosmological principle only if at large scales, i.e., $r \to \infty$, $p_\perp - p_\parallel \to 0$, reinstating the homogeneity and isotropy of the solution.

A convenient parametrization of the spacetime metric is

$$\begin{aligned} ds^2 &= a^2(t)\left[-f(r)e^{\gamma(r)}dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2\right], \\ d\Omega^2 &= d\theta^2 + \sin^2\theta d\phi^2, \end{aligned} \tag{7}$$

where t is the conformal time, a the scale factor, and f, γ are metric functions. Choosing an appropriate frame, the fluid velocity vectors are given by $u^\nu = (a^{-1}f^{-1/2}e^{-\gamma/2}, 0, 0, 0)$, $w^\nu = (0, a^{-1}f^{1/2}, 0, 0,)$. Einstein's equations $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = G_{\mu\nu} = 8\pi G T_{\mu\nu}$ give three independent equations,

$$\begin{aligned} 3\left(\frac{\dot a}{a}\right)^2 - \frac{e^\gamma f}{r^2}(-1 + f + rf') &= 8\pi G a^2 \rho f e^\gamma, \\ \frac{\dot a}{af}(f' + f\gamma') &= 0, \end{aligned} \tag{8}$$

$$\begin{aligned} &\frac{e^{-\gamma}}{r^2a^2f^2}[r^2\dot a^2 + e^\gamma a^2 f(-1 + f + rf' + rf\gamma') - 2r^2 a\ddot a] \\ &\quad = 8\pi G p_\parallel \frac{a^2}{f}, \end{aligned} \tag{9}$$

where the dot and the prime denote derivatives with respect to t and r, respectively. Covariant conservation of the stress-energy tensor gives two more equations,

$$\begin{aligned} \dot\rho + \frac{\dot a}{a}(3\rho + p_\parallel + 2p_\perp) &= 0, \\ p'_\parallel + \frac{2}{r}(p_\parallel - p_\perp) + \frac{1}{2}(\rho + p_\parallel)\left(\gamma' + \frac{f'}{f}\right) &= 0. \end{aligned} \tag{10}$$

The form of the spacetime metric (7), together with Eq. (4), describes, as particular cases, the various regimes of gravity sourced by an (an)isotropic fluid: Newtonian, galactic, cosmological. When $\dot a = 0$ (we set $a = 1$), our model reproduces a static, spherically symmetric, anisotropic fluid space-time, which has been used for several applications [33–37]. In particular, it has been used to explain galactic dynamics without assuming the presence of dark matter [25]. In this latter case, the radial pressure $p_\parallel$ gives an additional component to the acceleration (dark force) at galactic scales. This is what we call the MOND regime of gravity, because it reproduces the MOND theory in the weak-field approximation. If, in addition to $\dot a = 0$, we also impose $p_\parallel = p_\perp$, we obtain GR sourced by a static, spherically symmetric perfect fluid (the Newtonian regime of gravity). On the other hand, if $\dot a \neq 0$, our model describes a nonhomogeneous cosmological model, which interpolates between the MOND regime at galactic scales and the usual FLRW cosmology at $r \to \infty$.

A. Decoupling of cosmological degrees of freedom from inhomogeneities

When $\dot a \neq 0$, the second equation in (8) and the second equation in (10) can be solved, respectively, for γ and $p_\perp$,

$$e^{-\gamma} = f, \qquad p_\perp = p_\parallel + \frac{r}{2}p'_\parallel. \tag{11}$$

The remaining equations give then,

$$\begin{aligned} 3\left(\frac{\dot a}{a}\right)^2 + \frac{1 - f - rf'}{r^2} &= 8\pi G a^2 \rho, \\ \left(\frac{\dot a}{a}\right)^2 - 2\frac{\ddot a}{a} + \frac{f - 1}{r^2} &= 8\pi G a^2 p_\parallel, \\ \dot\rho + \frac{\dot a}{a}(3\rho + 3p_\parallel + rp'_\parallel) &= 0. \end{aligned} \tag{12}$$

As usual in cosmology, the system has to be closed, imposing an equation of state for the fluid, $p_\parallel = p_\parallel(\rho)$.

[2] Latin indices indicate the spatial component of the tensor.

Using the first two equations, the third equation in (12) can be easily integrated to give

$$a^2\rho(r,t)=a^2(t)\hat{\rho}(t)+\mathcal{E}(r), \quad a^2\hat{\rho}(t)=\frac{3}{8\pi G}\mathcal{H}^2, \tag{13}$$

where $\mathcal{H}\equiv \dot{a}/a$ is the conformal Hubble parameter and $\mathcal{E}(r)$ is an integration function, which depends on the radial coordinate r only. Physically, it represents the inhomogeneities in the baryonic matter density distribution and should therefore be negligible at scales larger than $\mathcal{R}$ as mentioned in the Introduction. We can now separate the r-dependent and the t-dependent parts in the first equation (12). The former determines the metric function f,

$$f=1-\frac{2Gm_B(r)}{r}-\frac{2GM}{r}, \qquad m_B(r)=4\pi\int drr^2\mathcal{E}(r), \tag{14}$$

where $m_B(r)$ is the Misner-Sharp mass associated with the inhomogeneities in the baryonic matter and M is an integration constant with the dimensions of a mass. Using Eq. (14) into the second equation (12), we get

$$\begin{aligned} a^2p_{\parallel}(r,t)&=a^2(t)\hat{p}(t)+\mathcal{P}(r),\\ a^2\hat{p}(t)&=\frac{1}{8\pi G}\left[\left(\frac{\dot{a}}{a}\right)^2-2\frac{\ddot{a}}{a}\right],\\ \mathcal{P}(r)&=-\frac{m_B(r)+M}{4\pi r^3}. \end{aligned} \tag{15}$$

Equations (13) and (15) clearly show that the contributions of inhomogeneities (r-dependent terms) to matter density and pressure can be separated from the homogeneous (t-dependent) cosmological contributions. This, in turn, allows us to separate the dynamics of cosmological evolution, which determines $a, \hat{\rho}, \hat{p}$, from the effect of inhomogeneities. In fact, Eq. (12) are completely equivalent to the FLRW equations for $a, \hat{\rho}, \hat{p}$,

$$\begin{aligned} &a^2\hat{\rho}=\frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2, \qquad a^2\hat{p}=\frac{1}{8\pi G}\left[\left(\frac{\dot{a}}{a}\right)^2-2\frac{\ddot{a}}{a}\right],\\ &\dot{\hat{\rho}}+\frac{\dot{a}}{a}(3\hat{\rho}+3\hat{p})=0, \end{aligned} \tag{16}$$

together with $\mathcal{P}(r)$ of Eq. (15), which determines the pressure from $m_B(r)$ given by Eq. (14). The perpendicular component of the pressure $p_\perp$ is then determined from $p_\parallel$ using Eq. (11).

This is a quite interesting result: cosmological degrees of freedom decouple from inhomegenities. This implies that the scale factor a is completely determined by the homogeneous and isotropic component of density and pressure $\hat{\rho}, \hat{p}$ through the usual FLRW equations (16), whereas the only effect of inhomogeneities is to produce a nonvanishing, r-dependent, pressure $\mathcal{P}(r)$.

Also taking into account Eq. (14), the physical interpretation of the latter is quite simple. The term proportional to M gives a Schwarzschild-like contribution, i.e., an inhomogeneity generated by a pointlike source located at $r=0$. Its presence is not compatible with observations; we have therefore to set the integration constant $M=0$.

The term proportional to $m_B(r)$ gives instead the contribution of spherically symmetric inhomogeneities distributed with density $\mathcal{E}(r)$. Since we want to recover usual FLRW cosmology at a large distance ($r\to\infty$), we have to assume $m_B(r)\sim-\frac{1}{2G}\mathcal{K}r^3+\frac{c_1}{r}$ with $\mathcal{K}=0,\pm1$. As we shall see in detail in the next section, the first term gives the spatial curvature of the spacetime, whereas the second one gives a contribution to f and $p_\parallel(r,t)$ that vanishes in the $r\to\infty$ limit. The physical effect of the $\mathcal{P}(r)$ term in Eq. (15) can be explained as a Newtonian contribution to the pressure, $P_N=\frac{1}{4\pi}\frac{m_B}{r^3}$, which produces the radial acceleration $a^r=4\pi GrP_N$ [25].

B. FLRW cosmology

Usual FLRW cosmology can be obtained as a limiting case of our anisotropic fluid cosmology in two different, albeit related, ways. In the first way, standard cosmology is obtained in the large scale limit $r\to\infty$. In fact, in this limit, both $\mathcal{P}(r)$ and $\mathcal{E}(r)$ go to zero, $p_\parallel=p_\perp=\hat{p}$, $\rho=\hat{\rho}$, and Eq. (16) become the FLRW equations written in terms of $p_\parallel$ and ρ. The same equations can be obtained by setting the integration function $\mathcal{E}(r)$ identically to zero, so that we identically get $p_\parallel=p_\perp=\hat{p}$ and $\rho=\hat{\rho}$.

It is quite interesting to notice that the derivation of the FLRW equations as a limiting case of anisotropic fluid cosmology allows us to generate the constant spatial curvature term in those equations from a constant contribution to the density function $\mathcal{E}(r)$. In fact, setting $\mathcal{E}(r)=-\frac{3}{8\pi G}\mathcal{K}$, we get $m_B=-\frac{\mathcal{K}}{2}r^3$, $f=1+\mathcal{K}r^2$, and Eq. (16) become

$$\begin{aligned} &3\left(\frac{\dot{a}}{a}\right)^2-3\mathcal{K}=8\pi Ga^2\rho\left(\frac{\dot{a}}{a}\right)^2, \quad -2\frac{\ddot{a}}{a}+\mathcal{K}=8\pi Ga^2p,\\ &\dot{\rho}+\frac{\dot{a}}{a}(3\rho+3p)=0, \end{aligned} \tag{17}$$

where, for notation simplicity, we set $p_\parallel=p$.

Although our model works also for a 3D space with constant positive, negative or zero curvature, in the following we will consider, consistently with observations, only FLRW cosmologies with $\mathcal{K}=0$.

In this paper, we use the simplest description of dark energy, i.e., that of a cosmological constant Λ, which corresponds to constant energy density $\rho=\Lambda/8\pi G$ and equation of state $p=-\rho$. As it is well-known, in this case, Eq. (17) (with $\mathcal{K}=0$) give as a solution the dS spacetime, i.e., $a=\sqrt{3/\Lambda}t^{-1}$.

III. THE EQUATION OF STATE

A crucial issue of our cosmological model is the determination of the EoS for the anisotropic fluid in the large scale regime we are considering in this paper. The equation of state is not only important to determine the background solution but also to describe the latter density perturbations around it. In our approach, the anisotropic fluid is meant to give an effective description of dark energy, baryonic matter, and their interaction. Although we know very well the EoS for (perfect) fluids describing pure, noninteracting DE[3] or pure, noninteracting, stiff baryonic matter (with $p = 0$), presently, we do not have a direct way to derive the EoS for the anisotropic fluid describing the interacting case. One possibility is to use hints coming from the anisotropic fluid description at galactic scales to infer information about the EoS of the fluid at a cosmological level [25,27,28].

At galactic scales, the interaction between dark energy and baryonic matter is described by a dark force, which manifests itself through the radial component of the pressure of the anisotropic fluid [25],

$$p_\parallel = p = \frac{1}{4\pi r^2}\sqrt{\frac{m_B(r)}{GL}}, \tag{18}$$

where $m_B(r)$ is the baryonic matter distribution in the Galaxy and $L \sim H_0^{-1}$ is the size of the cosmological horizon. Although this equation does not represent a barotropic equation of state for the fluid, i.e., a relation between the pressure and the density of the fluid, it can be used as a hint to infer the EoS for our anisotropic fluid in the large scale cosmological regime. That Eq. (18) cannot be directly used in the context of a DE-dominated universe is also evident from another simple argument. In this cosmological regime, we expect the contribution of baryonic matter to be completely negligible. The mass appearing under the square root has to be therefore considered as the total effective mass $m_E(r)$, which is the sum of the baryonic mass, $m_B(r)$, the DE contribution $m_\Lambda(r)$, and an interaction term $m_I(r)$: $m_E(r) = m_B(r) + m_\Lambda(r) + m_I(r)$. At galactic scales, m_Λ and m_I can be neglected, we have $m_E(r) \sim m_B(r)$, and we get Eq. (18). Conversely, in the limit $r \sim L$, both baryonic matter and its interaction with DE can be neglected. From $\rho_\Lambda \sim \frac{1}{GL^2}$ we get $m_\Lambda \sim \frac{L}{G}$, so that Eq. (18) gives $p \sim -\rho_\Lambda$, where we have taken into account that the pressure in Eq. (18) is negative. We will therefore promote Eq. (18) to an effective equation relating the radial pressure of our anisotropic fluid with the effective matter density ρ_E generating the effective mass m_E,

$$p_\parallel = \frac{1}{\sqrt{4\pi} r^2}\sqrt{\frac{\int d^3x \rho_E(r)}{GL}}. \tag{19}$$

Notice that in our description the effective matter density ρ_E is expected to mimic the effects of both dark and baryonic matter in the ΛCDM model.

Equation (19) still does not have the form of an equation of state. In order to bring it into this form, we consider the large scale regime of our cosmological model, when the interaction between DE and baryonic matter cannot be neglected; hence, the EoS is expected to deviate from the simple form $p \sim -\rho$. In the large scale limit $r \to \infty$, the contribution of inhomogeneities to the density ρ and to pressure $p_\parallel$ dies out. Cosmological evolution is therefore described by the FLRW equations (17) with $\mathcal{K} = 0$. On the other hand, although the Universe is dominated by dark energy, the interaction of the latter with baryonic matter cannot be completely neglected. Being $m_E(r) = \frac{4\pi a^3}{3}\rho_E(t) r^3$ and taking into account that, at large distances, we have $r \sim L$, Eq. (18) takes the form of an effective equation of state,

$$p = \frac{1}{\sqrt{12\pi GL}}\sqrt{\rho} a^{3/2}. \tag{20}$$

In Sec. V B, we will consider inhomogeneities as density perturbations of the dS background that describe the short wavelength behavior of the power spectrum (3). In order to do this, we will consider in Eq. (19) both p and ρ_E as small perturbations of the (constant) pressure and energy density sourcing the dS spacetime.

IV. LARGE SCALE COSMOLOGICAL REGIME AND GENERALIZED CHAPLYGIN GAS MODEL

We consider the large scale regime of our cosmological model. We have seen in the previous section that, in the large scale limit $r \to \infty$, cosmological evolution is described by the FLRW equations (16) with an effective EoS given by (20). Thus, the cosmological equations (16) can be written as

$$\dot{a}^2 - 2\ddot{a}a = \frac{\sqrt{2}}{L}\dot{a}a^{7/2}, \tag{21}$$

which can be easily integrated by defining the new variable $K = \frac{\dot{a}}{\sqrt{a}}$ and the new time $\tau = \int a^{5/2} dt$. The solution of Eq. (21) can be written in an implicit form in terms of the conformal time t as

$$t(a) = -\frac{\sqrt{2}c_1^5}{H_0}\left\{\ln\left(\frac{1-c_1\sqrt{a}}{1+c_1\sqrt{a}}\right) + \frac{1}{2}\ln\left(\frac{1-c_1\sqrt{a}+c_1^2 a}{1+c_1\sqrt{a}+c_1^2 a}\right) + \sqrt{3}\left[\mathrm{arctg}\left(\frac{1-2c_1\sqrt{a}}{\sqrt{3}}\right) - \mathrm{arctg}\left(\frac{1+2c_1\sqrt{a}}{\sqrt{3}}\right)\right]\right\}, \tag{22}$$

where c_1 is an integration constant.

[3]The reader should remember that we are modeling DE as a cosmological constant with $p = -\rho$.

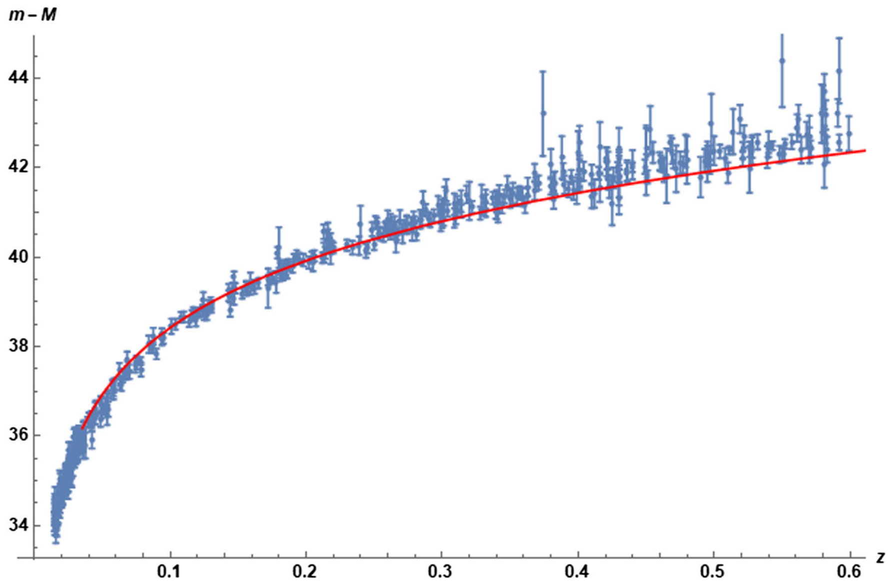

FIG. 1. Theoretical prediction of our cosmological model for the distance modulus as a function of the redshift (red line) vs SNIa observational data, taken from Ref. [38]. For the Hubble constant, we adopt the value $H_0 = 67.6\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, obtained combining the Planck and BAO data [1].

From the metric in Eq. (7), one can easily derive the conformal time t in terms of the cosmological time T and the luminous distance D_L,

$$t = \int \frac{dT}{a}, \qquad D_L = \frac{1}{a}\int_T^{T_0} \frac{dT}{a} = \frac{1}{a} t(a)\bigg|_a^{a=1}, \tag{23}$$

respectively, where T_0 is the present cosmological time and the scale factor at present cosmological time is normalized to 1, $a(T_0) = 1$.

D_L can be calculated using Eq. (22) and substituted into the distance modulus,

$$m - M = 25 + 5\log_{10}\left(\frac{D_L}{\mathrm{Mpc}}\right), \tag{24}$$

where m and M are the apparent and absolute magnitude, respectively.

We can now compare the theoretical prediction for the distance modulus of our cosmological model with EoS (20), as function of the redshift z, with the observational data for the type Ia supernovae, taken from the Supernova Cosmology Project (SCP) Union 2.1 Compilation [38]. The scale factor (22) contains an integration constant c_1, which enters in the relationship between conformal time t and cosmological time T. It can be fixed by fitting the prediction of our model with the observational data. The fit gives the result $c_1 = 0.769$.

In Fig. 1, we show the comparison between the theoretical prediction of our model for the distance modulus with the observations of [38], finding a good agreement. Notice that, in Fig. 1, we have only considered observational data with $z \lesssim 0.6$. This corresponds to the range of validity of our cosmological model, which is meant to describe our late-time, dark energy-dominated Universe.

A. Connection with generalized Chaplygin gas model

It is interesting to notice that our cosmological model based on the EoS (20) belongs to a class of models known as "new generalized Chaplygin gas" (NGCG) [39–41]. This does not come as a surprise, because these models are meant to give a unified description of dark energy and dark matter. This is alike to what we have achieved by means of our anisotropic fluid cosmology, but with an important difference: in our description, there is no dark matter but only dark energy, baryonic matter, and their interaction, whose effect at galactic scales and in cosmology should replace that of DM.

The EoS for NGCG has the general form $p_{\mathrm{NGCG}} = \mathcal{A}(a)\rho_{\mathrm{NGCG}}^{-\zeta}$. More precisely, it can be written as

$$p_{\mathrm{NGCG}} = \frac{\zeta A a^{-3(1+\zeta)(1+\eta)}}{\rho_{\mathrm{NGCG}}^{\zeta}}, \tag{25}$$

where ζ, η are some parameters and A is a positive constant.

Comparing Eq. (25) with our EoS (20), we can determine the parameters ζ, η, A,

$$\zeta = -1/2, \qquad \eta = -2, \qquad A = \frac{1}{3L}\sqrt{\frac{3}{\pi G}} = \frac{H_0}{3}\sqrt{\frac{3}{\pi G}}. \tag{26}$$

V. COSMOLOGICAL PERTURBATIONS

One of the main goals of our anisotropic fluid cosmology is to describe structures at small scales and to derive the matter power spectrum (3). There are two different approaches for doing that. The first one is phenomenological: one just assumes the validity of our model and then finds the inhomogeneity function $\mathcal{E}(r)$ by fitting observational data about the mass density distribution. Obviously, this approach has very low predictive power.

Alternatively, one can consider $\mathcal{E}(r)$ as a small perturbation of a FLRW universe dominated by dark energy. Using the simplest description for dark energy, that of a cosmological constant, we need to consider perturbations near the dS cosmological solution generated by an anisotropic fluid. We will first consider generic perturbations around a given cosmological background $g^{(0)}_{\mu\nu}$ given by Eq. (7) (we set $\gamma = 0$ and $f = 1$; i.e., we consider a spatially flat universe) in the linear regime. We will then specialize our calculations to the dS background.

We start from the usual form for the perturbed metric,

$$g_{\mu\nu}(t,x) = g^{(0)}_{\mu\nu}(t) + h_{\mu\nu}(t,x), \tag{27}$$

where the background metric depends only on the conformal time t, whereas $h_{\mu\nu}(t,x)$ depends both on t and on the spatial coordinates x^i.

It is well-known that, for cosmological perturbations, the split (27) depends on the choice of coordinates, i.e., on the gauge choice. We have two possible choices: either we fix the gauge, or we can choose to work with manifest gauge invariant quantities, like, e.g., the Bardeen potentials. In this paper, we choose the first approach and use the Newtonian conformal gauge,

$$ds^2 = a^2[-(1+2\phi)d\eta^2 + (1+2\psi)\delta_{ij}dx^i dx^j]. \tag{28}$$

We are assuming that our cosmological background is sourced by a perfect fluid of density ρ and pressure P. We can therefore treat the anisotropic stress $\pi_{\mu\nu}$ as a perturbation of the stress-energy tensor. Consistently, we will also treat $\mathcal{S} = 3^{-1/2}(p_\parallel - p_\perp)$ as a small perturbation; i.e., we will take $|(p_\parallel - p_\perp)/P| \ll 1$ and consider only terms of order 1 in the perturbative expansion.

The background components for the stress-energy tensor are those pertaining to a perfect fluid: $T^0_0 = -\rho, T^0_i = T^i_0 = 0, T^i_j = P\delta^i_j$. For the perturbations, we have instead

$$\delta T^0_0 = -\delta\rho, \quad \delta T^0_i = -\delta T^i_0 = v_i(\rho+P), \quad \delta T^i_j = \delta P\delta^i_j + \pi^i_j, \tag{29}$$

where $v_i = a\delta u_i$ parametrize fluid velocity perturbations.

The perturbed Einstein equations $\delta G^\mu_\nu = 8\pi G\delta T^\mu_\nu$ therefore are

$$\begin{aligned} 3\mathcal{H}(\mathcal{H}\phi - \dot\psi) + \nabla^2\psi &= -4\pi G a^2\delta\rho, \\ \partial_i(\dot\psi - \mathcal{H}\phi) &= 4\pi G a^2(\rho+P)v_i, \end{aligned} \tag{30a}$$

$$\begin{aligned} &\partial^i\partial_j(\psi+\phi) = -8\pi G a^2\pi^i_j, \\ &(\mathcal{H}^2 + 2\dot{\mathcal{H}})\phi + \mathcal{H}\dot\phi - \ddot\psi - 2\mathcal{H}\dot\psi + \frac{1}{3}\nabla^2(\psi+\phi) \\ &\quad = 4\pi G a^2\delta P. \end{aligned} \tag{30b}$$

The covariant conservation equation for the stress-energy tensor $\nabla_\mu\delta T^\mu_\nu = 0$ gives two more equations,

$$\dot{\delta\rho} + 3\mathcal{H}(\delta\rho + \delta P) + (\rho+P)(\partial_i v^i + 3\dot\psi) = 0; \tag{31a}$$

$$(\rho+P)(4\mathcal{H}v_i + \dot v_i + \partial_i\phi) + v_i(\dot\rho + \dot P) + \partial_i\delta P + \partial_j\pi^j_i = 0. \tag{31b}$$

A. dS background: Isotropic perturbations

Let us now consider cosmological perturbations of the dS background solution, i.e., a background whose EoS is $P = -\rho$. We first consider the case of isotropic perturbations; i.e., we set the anisotropic stress-tensor $\pi_{ij} = 0$ (equivalently, $\mathcal{S} = 0$).

Using polar coordinates for the 3D spatial sections of the 4D metric and passing to the corresponding Fourier space, labeled by wave vector modulus k, Eqs. (30) and (31) give

$$\begin{aligned} 3\mathcal{H}(\mathcal{H}\phi_k - \dot\psi_k) - k^2\psi_k &= -4\pi G a^2\delta\rho_k, \\ ik_i(\dot\psi_k - \mathcal{H}\phi_k) &= 0, \\ -k_i k_j(\psi_k + \phi_k) &= 0, \end{aligned} \tag{32a}$$

$$\begin{aligned} &(\mathcal{H}^2 + 2\dot{\mathcal{H}})\phi_k + \mathcal{H}\dot\phi_k - \ddot\psi_k - 2\mathcal{H}\dot\psi_k - \frac{1}{3}k^2(\psi_k + \phi_k) \\ &\quad = 4\pi G a^2\delta P_k; \end{aligned} \tag{32b}$$

$$\dot{\delta\rho}_k + 3\mathcal{H}(\delta\rho_k + \delta P_k) = 0, \qquad ik_i\delta P_k = 0, \tag{32c}$$

where quantities with a lower index k, $\phi_k = \phi_k(k,t)$, $\psi_k = \psi_k(k,t)$, $\delta\rho_k = \delta\rho_k(k,t)$, $\delta P_k = \delta P_k(k,t)$ represent the 3D Fourier transform of the corresponding quantities. Because Fourier modes evolve independently, in the following, for the sake of simplicity, we will drop the lower index k in the Fourier transforms.

The previous equations can be easily solved to give

$$\delta P = 0, \quad \delta\rho = \mathcal{F}(k)a^{-3}, \quad \phi = -\psi = \mathcal{B}(k)a^{-1}, \tag{33}$$

where $\mathcal{F}(k)$ and $\mathcal{B}(k)$ are arbitrary functions of k, with

$$|\mathcal{F}(k)| = \frac{k^2|\mathcal{B}(k)|}{4\pi G}. \tag{34}$$

This is the well-known relation between the matter density and gravitational potential power spectra,

$$\langle|\mathcal{F}(k)|^2\rangle \propto k^4\langle|\mathcal{B}(k)|^2\rangle. \tag{35}$$

Our solution depends on an arbitrary function $\mathcal{F}(k)$ of the wave vector k; therefore, it determines only the relation between the matter density and the gravitational potential. This result does not come unexpected. In fact, Eq. (32c) implies that isotropic perturbations behave as incoherent, stiff, matter: $\delta P = 0$. Therefore, the dynamics of the perturbations fixes the EoS, preventing the possibility to impose it from outside. Thus, in the large-scale regime of a DE-dominated universe we are considering, the mass distribution at long wavelengths (1), cannot be determined by the dynamics of perturbations. Physically, this expresses the fact that the large-scale distribution of matter cannot be determined by the interaction between dark energy and baryonic matter, the latter being relevant for the distribution at small scales only. Thus, in our approach, the observed long-wavelength power spectrum (1) has to be used to determine the arbitrary function $\mathcal{F}(k)$. Assuming the validity of Eq. (1), we get $\langle|\mathcal{B}(k)|^2\rangle \sim k^{-3}$.

B. dS background: Anisotropic perturbations

Let us now pass to consider anisotropic perturbations of the dS background, i.e., the case $\pi_{ij} \neq 0$. Taking into account the considerations of Sec. II and those at the beginning of the present one, this boils down to the consideration of perturbations generated by an anisotropic fluid with $p_\parallel \neq p_\perp$. We are now dealing with the small-scale regime of our cosmological model, for which we expect the spatial distribution of inhomogeneities to be determined by the dynamics of perturbations. Thus, the distribution of small-scale structures in our Universe is determined by our effective anisotropic fluid, which encodes the interaction between dark energy and baryonic matter.

Equations (30) and (31) now give

$$3\mathcal{H}(\mathcal{H}\phi - \dot{\psi}) + \nabla^2\psi = -4\pi G a^2 \delta\rho, \qquad \partial_i(\dot{\psi} - \mathcal{H}\phi) = 0,$$
$$\partial_i\partial_j(\psi + \phi) = -8\pi G a^2 \pi_{ij}; \tag{36a}$$

$$(\mathcal{H}^2 + 2\dot{\mathcal{H}})\phi + \mathcal{H}\dot{\phi} - \ddot{\psi} - 2\mathcal{H}\dot{\psi} + \frac{1}{3}\nabla^2(\psi + \phi) = 4\pi G a^2 \delta P; \tag{36b}$$

$$\dot{\delta\rho} + 3\mathcal{H}(\delta\rho + \delta P) = 0, \qquad \partial_i \delta P + \partial_j \pi^j_i = 0. \tag{36c}$$

We consider anisotropic perturbations that can be derived by a scalar potential Π. Being π_{ij} traceless, we can write

$$\pi_{ij} \equiv \partial_i\partial_j \Pi - \frac{1}{3}\delta_{ij}\nabla^2\Pi. \tag{37}$$

This allows us to simplify drastically our system of equations. Passing to Fourier space and dropping the lower index in the Fourier transforms in order to simplify the notation, from Eqs. (36a), (36b), and (36c), we get four independent equations,

$$\phi + \psi = -8\pi G a^2 \Pi, \qquad k^2\psi = 4\pi G a^2 \delta\rho,$$
$$\dot{\delta\rho} + 3\mathcal{H}(\delta\rho + \delta P) = 0, \qquad \delta P = \frac{2}{3}k^2\Pi. \tag{38}$$

The last equation above is fully consistent with the fact that the anisotropy in the perturbation is linked directly to $\delta P \neq 0$, as we have seen at the beginning of this section. As usual, we need an equation of state for the perturbations in order to close the system. As discussed in Sec. III, this information is encoded in Eq. (19), which is inherited from the galactic dynamics.

Since we are considering small perturbations of the dS spacetime due to an anisotropic fluid, the pressure perturbation δP in Eq. (38) can be identified with the dark force (19), i.e., $\delta P = p_\parallel$. This identification is evident from Eq. (6), which allows us to write $p_\parallel$ as a background pressure plus the anisotropic stress contribution. Furthermore, in Eq. (19), the effective matter density ρ_E is the source of the dark force. We can therefore set $\delta\rho = \rho_E$ in Eq. (38). Equation (19) determines only the spatial profile of δP once $\delta\rho$ is known, whereas it is insensitive to their dependence on the conformal time. This is consistent with the fact that Eq. (19) originated in galactic dynamics.

In order to solve the system, we therefore need a factorization of δP and $\delta\rho$ in space—and time—profiles, and we also need an EoS consistent with this factorization,

$$\delta P = w\delta\rho, \quad \delta\rho(r,t) = \delta\hat{\rho}(t)\delta\rho(r), \quad \delta P(r,t) = \delta\hat{P}(t)\delta P(r), \tag{39}$$

with w constant. Notice that we are using a perfect fluid equation of state for the perturbation. Since Eq. (19) is written in terms of the radial coordinate r, we will also solve the third equation in (38) in coordinate space.

Differentiating (19) with respect to r and using Eq. (39), we get

$$\dot{\delta\hat{\rho}} + 3\mathcal{H}(1+w)\delta\hat{\rho} = 0, \qquad \frac{d}{dr}[r^2\delta\rho] = \frac{H_0}{16\pi G w^2}. \tag{40}$$

It is quite interesting to notice that our ansatz (39) allows us to perform, at the perturbative level, the same decoupling of cosmological degrees of freedom from inhomogeneities that we have described in Sec. II A in our anisotropic fluid cosmology. The EoS, $\delta P = w\delta\rho$, determines, through the first equation in (40), the time dependence of the homogenous part of the matter density, whereas Eq. (19) determines the inhomogeneity profile trough the second equation in (40).

The general solution of the second equation in (40) contains a term proportional to $1/r$ and a term β/r^2, with the β integration constant. Equation (19) requires $\beta = 0$ so that the solution of (40) is

$$\delta\hat{\rho}(t) \sim a^{-3(1+w)}, \qquad \delta\rho(r) = \frac{H_0}{16\pi G w^2}\frac{1}{r}. \tag{41}$$

The cosmological evolution of the homogeneous part of the perturbation is that pertaining to a perfect fluid, whereas the profile for inhomogeneities is given by an harmonic function in 3D.

The Fourier transform of the spatial profile of ρ gives $\delta\rho_k \sim \frac{H_0}{16\pi G w^2}\frac{1}{k^2}$, and the power spectrum is

$$P(k) \propto \langle \delta\rho_k^2 \rangle = \frac{\int d^3k \delta\rho_k^2}{\int d^3k} \sim \left(\frac{H_0}{16\pi G w^2}\right)^2 \frac{1}{k^4}. \tag{42}$$

This is the result of our cosmological model for the power spectrum of mass distribution at short wavelengths. It gives a theoretical determination of Eq. (3).

Since our model deals with the late-time cosmology, describing the phenomenology of dark energy, baryonic matter, and their interaction, it does not come as a surprise we are only able to reproduce the power spectrum (3), predicted by the galaxy two-point correlation function. On the other hand, it fails to reproduce the power spectrum at the equivalence epoch $P(k) \sim k^{-3}$, which depends on the physics governing matter radiation at the equivalence epoch [42–44].

VI. CONCLUSIONS

In this paper, we have proposed an anisotropic fluid cosmological model for describing our present, dark energy-dominated Universe. The model does not assume the presence of dark matter. Dark energy, baryonic matter, and their possible effective interaction are codified in a peculiar EoS for the anisotropic fluid. This EoS is inherited from the relation between pressure and mass used to explain galactic dynamics without assuming the presence of dark matter [25].

The strength of our model, as compared with the usual ΛCDM scenario, is that it allows us to generate inhomogeneities at small scales in a natural way, through anisotropy in the fluid pressure, described by an anisotropic stress tensor. It can be therefore used to explain mass distribution, i.e., the matter density power spectrum, at short wavelengths. Cosmological dynamics, i.e., time evolution of the scale factor and the homogeneous component of matter density, completely decouples from inhomogeneities. The former is ruled by usual FLRW cosmology, whereas the latter are determined by the relation between pressure and density of the anisotropic fluid. Using perturbation theory, we have been able to derive the power spectrum $P(k)$ for the mass density distribution at short wavelengths. We have found that $P(k)$ behaves as $1/k^4$, in good accordance with the observed 2-point galaxy correlation function for the matter distribution at small scales.

We have also found that the predictions of our model concerning the accelerated expansion of the Universe and mass distribution at small scales are in accordance with observations. In the large distances regime, our model is well-described by a generalized Chaplygin gas and fits observational data from type IA supernovae (for observational data with $z \lesssim 0.6$), used to probe the present accelerated expansion of our Universe.

Let us conclude with the drawbacks of our approach. In the present form, our anisotropic fluid cosmological model can be only used to describe a dark energy-dominated universe but not to describe the early-time cosmology and the radiation/baryonic matter dominated eras. This is because any mass distribution is intrinsically unstable in FLRW cosmology. Therefore, any inhomogeneous, FLRW-based cosmological model can only be used to describe late-time cosmology and cosmic structures at small scales but not the evolution of perturbations from early-time cosmology. Thus, large scale structures can only be explained in the framework of usual FLRW cosmology, in terms of the growing of small perturbations of the early Universe described by linear perturbation theory.

This is fully consistent with the results of our paper. We have seen in Sec. V A that the behavior of the power spectrum $P(k)$ for the mass density distribution, at large wavelengths, is not determined by perturbations in the present dark energy-dominated Universe. The observed linear scaling $P(k) \sim k$ has to be explained in terms of small fluctuations in the early Universe. Conversely, small scale structures find a natural explanation in our model as inhomogeneities triggered by an anisotropic stress tensor. This is in turn consistent with the fact that the presence of dark matter is crucial for structure formation in the ΛCDM model. As expected, in our cosmological model, the anisotropic stress tensor, generated by the assumed dark energy/baryonic matter interaction, plays the same role that dark matter plays in the ΛCDM model for small scale structure formations.

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