Semileptonic decays $D \to \pi^+\pi^-e^+\nu_e$ and $D_s \to \pi^+\pi^-e^+\nu_e$ as the probe of constituent quark-antiquark pairs in the light scalar mesons

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Decays $D \to \pi^+\pi^-e^+\nu_e$ and $D_s \to \pi^+\pi^-e^+\nu_e$ serve as probes that check the existence of constituent $q\bar{q}$ components in the wave functions of scalar mesons decaying into $\pi^+\pi^-$. There exists a great deal of concrete evidence in favor of the exotic four-quark nature of light scalars. At the same time, the further expansion of the area of the $q^2\bar{q}^2$ model validity for light scalars on ever new processes seems extremely interesting and important. We analyze the BESIII and CLEO data on the decays $D^+ \to \pi^+\pi^-e^+\nu_e$ and $D_s^+ \to \pi^+\pi^-e^+\nu_e$ and show that the results of these experiments together can be interpreted in favor of the four-quark nature of light scalar mesons $\sigma(500)$ and $f_0(980)$. Our approach can also be applied to the description of other similar decays involving light scalars.

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I. INTRODUCTION

In the works [1,2], a program was proposed for studying the $\sigma(500)$, $f_0(980)$, and $a_0(980)$ resonances in semileptonic decays of D and B mesons. These decays provide direct probe of constituent two-quark components in the wave functions of light scalars [1,2]. So for the decays of D_s^+, D^0, and D^+ mesons we have: $D_s^+ \to s\bar{s}e^+\nu_e \to [\sigma(500) + f_0(980)]e^+\nu_e \to \pi^+\pi^-e^+\nu_e$, $D^0 \to d\bar{u}e^+\nu_e \to a_0^-(980)e^+\nu_e \to \pi^-\eta e^+\nu_e$, $D^+ \to d\bar{d}e^+\nu_e \to a_0^0(980)e^+\nu_e \to \pi^0\eta e^+\nu_e$, and $D^+ \to d\bar{d}e^+\nu_e \to [\sigma(500) + f_0(980)]e^+\nu_e \to \pi^+\pi^-e^+\nu_e$. The development of this program [1–4] resulted in evidences in favor of the exotic nature of light scalar mesons. Certainly, there are many theoretical works in which the semileptonic decays of D mesons are explored from many different aspects, see, for example, Refs. [5–9] and references herein.

The available data on the branching fractions of the semileptonic decays $D_s^+ \to \pi^+\pi^-e^+\nu_e$ and $D^+ \to \pi^+\pi^-e^+\nu_e$ involving light scalar mesons [10–12] are collected in Table I. The CLEO and BESIII collaborations also presented data on the shapes of the $\pi^+\pi^-$ S-wave mass spectra in these decays [10,11]. In this paper, in the light of the program [1,2], we analyze the recent BESIII data [11] on the decay $D^+ \to \pi^+\pi^-e^+\nu_e$ together with the CLEO data [10] on the decay $D_s^+ \to \pi^+\pi^-e^+\nu_e$. We show that the results of these experiments on the $\pi^+\pi^-$ mass spectra can be interpreted in favor of the four-quark nature of light scalar mesons.

This paper is organized as follows. In Sec. II we present the general formulas for the semileptonic decay widths of D_s^+ and D^+ mesons into light scalars. In Sec. III we consider the production of the mixed $\sigma(500) - f_0(980)$ resonance complex which proceeds via direct couplings of σ and f_0 with $q\bar{q}$ pairs created in semileptonic decays of D^+ and D_s^+ mesons. We find a sharp contradiction of this production mechanism with the data on the $\pi^+\pi^-$ mass spectra in the $D^+ \to \pi^+\pi^-e^+\nu_e$ decay. Section IV is devoted to an analysis of the four-quark production mechanism of the σ and f_0 states. Within the existing data, this mechanism seems to be the most real. This section also contains an important remark about the dip/peak manifestation of the $f_0(980)$ resonance.

II. SEMILEPTONIC DECAY WIDTHS

First of all, we write the differential width for the D^+ and D_s^+ decays into $\pi^+\pi^-e^+\nu_e$ in the form

$$\frac{d^2\Gamma_{D^+_{c\bar{q}} \to (S\to\pi^+\pi^-)e^+\nu_e}(s,q^2)}{d\sqrt{s}dq^2} = \frac{G_F^2|V_{cq}|^2}{24\pi^3} p^3_{\pi^+\pi^-}(m_{D^+_{c\bar{q}}}, q^2, s)|f_+^{D^+_{c\bar{q}}}(q^2)|^2 \frac{2\sqrt{s}}{\pi} \times |F^{D^+_{c\bar{q}}}_{q\bar{q}\to S\to\pi^+\pi^-}(s)|^2 \rho_{\pi^+\pi^-}(s), \quad (1)$$

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TABLE I. Branching fractions ($\mathcal{B}$) and widths ($\Gamma = \mathcal{B}/\tau_D$, where τ_D is the D lifetime [12]) of semileptonic decays of the D_s^+ and D^+ mesons.

Decay	$\mathcal{B}$ ($\times 10^{-4}$)	Collaboration	Γ ($\times 10^8$ s^{-1})
$D_s^+ \to f_0(980)e^+\nu_e,\ f_0(980) \to \pi^+\pi^-$	$20 \pm 3 \pm 1$	CLEO [10]	39.7 ± 6.3
$D^+ \to \sigma(500)e^+\nu_e,\ \sigma(500) \to \pi^+\pi^-$	$6.30 \pm 0.43 \pm 0.32$	BESIII [11]	6.06 ± 0.51
$D^+ \to f_0(980)e^+\nu_e,\ f_0(980) \to \pi^+\pi^-$	<0.28	BESIII [11]	<0.27

where the index $q(\bar q) = d(\bar d), s(\bar s)$; $D^+_{c\bar d} \equiv D^+$, $D^+_{c\bar s} \equiv D^+_s$, next we use the notation that is convenient; s and q^2 are the invariant mass squared of the virtual scalar state S (or the $\pi^+\pi^-$ system) and the $e^+\nu_e$ system, respectively; G_F is the Fermi constant, $|V_{cq}|$ is a Cabibbo-Kobayshi-Maskawa matrix element (note that $|V_{cs}|/|V_{cd}| \simeq 20.92$ [12]); $p_{\pi^+\pi^-}$ is the magnitude of the three-momentum of the $\pi^+\pi^-$ system in the D meson rest frame,

$$p_{\pi^+\pi^-}(m_{D^+_{c\bar q}}, q^2, s) = \sqrt{[(m_{D^+_{c\bar q}} - \sqrt{s})^2 - q^2][(m_{D^+_{c\bar q}} + \sqrt{s})^2 - q^2]}/(2m_{D^+_{c\bar q}}), \quad (2)$$

and $\rho_{\pi^+\pi^-}(s) = (1 - 4m^2_{\pi^+}/s)^{1/2}$. In a simplest pole approximation, the form factor $f_+^{D^+_{c\bar q}}(q^2)$ has the form

$$f_+^{D^+_{c\bar q}}(q^2) = \frac{f_+^{D^+_{c\bar q}}(0)}{1 - q^2/m_A^2}, \quad (3)$$

where m_A, in principle, can be extracted from the data by fitting [10]. The amplitude $F^{D^+_{c\bar q}}_{q\bar q \to S \to \pi^+\pi^-}(s)$ describes

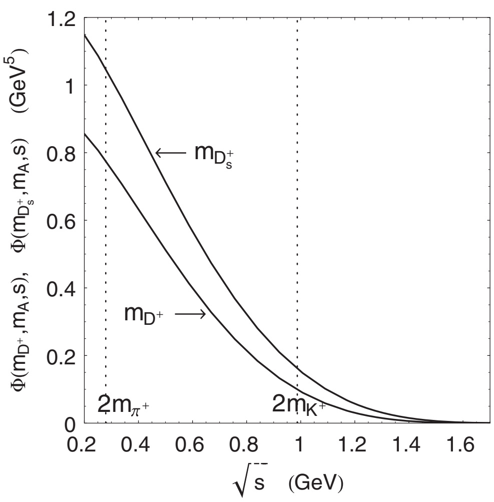

FIG. 1. The solid curves show the functions $\Phi(m_{D^+}, m_A, s)$ at $m_A = m_{D_1^+} = 2.42$ GeV and $\Phi(m_{D_s^+}, m_A, s)$ at $m_A = m_{D_{s1}^+} = 2.46$ GeV. The vertical dotted lines indicate the $\pi^+\pi^-$ and the K^+K^- threshold positions.

the formation and $\pi^+\pi^-$ decay of the virtual scalar state S produced in the $D^+_{c\bar q} \to \pi^+\pi^- e^+\nu_e$ decay. For example, in case of direct production of a single scalar resonance, $|F^{D^+_{c\bar q}}_{q\bar q \to S \to \pi^+\pi^-}(s)|^2 \rho_{\pi^+\pi^-}(s) = \sqrt{s}\Gamma_{S \to \pi^+\pi^-}(s)/|D_S(s)|^2$, where $\Gamma_{S\to\pi^+\pi^-}(s)$ is the $S \to \pi^+\pi^-$ decay width, $1/D_S(s)$ is the propagator of S, and the amplitude normalization (in this case) is hidden in $f_+^{D^+_{c\bar q}}(0)$. The $\pi^+\pi^-$ invariant mass distribution is given by

$$\frac{d\Gamma_{D^+_{c\bar q} \to (S \to \pi^+\pi^-)e^+\nu_e}(s)}{d\sqrt{s}} = \frac{G_F^2 |V_{cq}|^2}{24\pi^3} |f_+^{D^+_{c\bar q}}(0)|^2 \Phi(m_{D^+_{c\bar q}}, m_A, s) \frac{2\sqrt{s}}{\pi} \times |F^{D^+_{c\bar q}}_{q\bar q \to S \to \pi^+\pi^-}(s)|^2 \rho_{\pi^+\pi^-}(s), \quad (4)$$

where

$$\Phi(m_{D^+_{c\bar q}}, m_A, s) = \int_0^{(m_{D^+_{c\bar q}} - \sqrt{s})^2} \frac{p^3_{\pi^+\pi^-}(m_{D^+_{c\bar q}}, q^2, s)}{|1 - q^2/m_A^2|^2} dq^2. \quad (5)$$

Figure 1 illustrates the energy dependence of $\Phi(m_{D^+_{c\bar q}}, m_A, s)$ for D^+ and D_s^+ decays. Note that this function notably enhances the $\pi^+\pi^-$ mass spectrum as $\sqrt{s}$ decreases.

III. $q\bar q$-PROBE IN OPERATION

We now consider the production of the mixed $\sigma(500) - f_0(980)$ resonance complex (briefly σ and f_0) which proceeds via direct couplings of σ and f_0 with $q\bar q$ pairs created in semileptonic decays of D^+ and D_s^+ mesons (see Fig. 2). This mechanism is the probe that verifies the existence of the corresponding constituent $q\bar q$ component in the wave function of a scalar meson. There exists a great deal of concrete evidence in favor of the exotic four-quark nature of light scalars [13], see also Ref. [14]. Reviews of the current situation can be found, for example, in Refs. [3,4,15,16]. At the same time, the further expansion of the area of the $q^2\bar q^2$ model validity for light scalars on ever new processes seems to us extremely interesting and important.

The transition amplitude $q\bar q \to S \to \pi^+\pi^-$ corresponding to the indicated mechanism is denoted by $F^{D^+_{c\bar q},\mathrm{direct}}_{q\bar q \to S \to \pi^+\pi^-}(s)$ and write it in the form

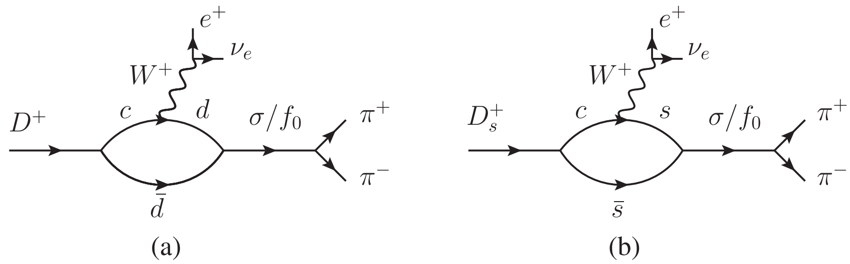

FIG. 2. Model of the $D^+ \to (\sigma/f_0 \to \pi^+\pi^-)e^+\nu_e$ and $D_s^+ \to (\sigma/f_0 \to \pi^+\pi^-)e^+\nu_e$ decays.

$$F^{D^+_{c\bar{q}},\text{direct}}_{q\bar{q}\to S\to\pi^+\pi^-}(s) = e^{i\delta_B^{\pi\pi}(s)}\sum_{r,r'} g_{q\bar{q}r}G^{-1}_{rr'}g_{r'\pi^+\pi^-} = e^{i\delta_B^{\pi\pi}(s)}(g_{q\bar{q}\sigma},\ \ g_{q\bar{q}f_0})\begin{pmatrix} D_\sigma & -\Pi_{\sigma f_0} \\ -\Pi_{f_0\sigma} & D_{f_0}\end{pmatrix}^{-1}\begin{pmatrix} g_{\sigma\pi^+\pi^-} \\ g_{f_0\pi^+\pi^-}\end{pmatrix}, \quad (6)$$

where $r(r') = \sigma, f_0$; $g_{q\bar{q}r}$ and $g_{r\pi^+\pi^-}$ are the coupling constants, D_r is the inverse propagator of the unmixed scalar resonance r with the mass m_r, and $\Pi_{rr'} = \Pi_{r'r}$ is a non-diagonal element of the polarization operator. D_r has the form

$$D_r \equiv D_r(s) = m_r^2 - s + \sum_{ab}[\text{Re}\Pi_r^{ab}(m_r^2) - \Pi_r^{ab}(s)], \quad (7)$$

where $\Pi_r^{ab}(s)$ stands for the diagonal matrix element of the polarization operator of the resonance r corresponding to the contribution of the ab intermediate state ($\pi^+\pi^-, \pi^0\pi^0, K^+K^-, K^0\bar{K}^0$, etc). $\text{Re}\Pi_r^{ab}(s)$ is defined by the singly subtracted at $s = 0$ dispersion integral of

$$\text{Im}\Pi_r^{ab}(s) = \sqrt{s}\Gamma_{r\to ab}(s) = \eta_{ab}\frac{g^2_{rab}}{16\pi}\rho_{ab}(s), \quad (8)$$

where g_{rab} is the coupling constant of r with ab, $\rho_{ab}(s) = \sqrt{s - m_{ab}^{(+)2}}\sqrt{s - m_{ab}^{(-)2}}/s$, $m_{ab}^{(\pm)} = m_a \pm m_b$ [here $s > m_{ab}^{(+)2}$], and $\eta_{ab} = 1$ (1/2) for different (identical) decay particles ab, respectively. We also have

$$\Pi_{rr'} \equiv \Pi_{rr'}(s) = C_{rr'} + \sum_{ab}\frac{g_{r'ab}}{g_{rab}}\Pi_r^{ab}(s), \quad (9)$$

where $C_{rr'}$ being the resonance mixing parameter. The determinant of $G_{rr'}$ is $\Delta = D_\sigma D_{f_0} - \Pi^2_{\sigma f_0}$. Thus the amplitudes for the D^+ and D_s^+ decays have the form:

$$F^{D^+,\text{direct}}_{d\bar{d}\to S\to\pi^+\pi^-}(s) = \frac{e^{i\delta_B^{\pi\pi}(s)}}{\Delta(s)}\{g_{d\bar{d}\sigma}[D_{f_0}(s)g_{\sigma\pi^+\pi^-} + \Pi_{\sigma f_0}(s)g_{f_0\pi^+\pi^-}] + g_{d\bar{d}f_0}[D_\sigma(s)g_{f_0\pi^+\pi^-} + \Pi_{\sigma f_0}(s)g_{\sigma\pi^+\pi^-}]\}. \quad (10)$$

$$F^{D_s^+,\text{direct}}_{s\bar{s}\to S\to\pi^+\pi^-}(s) = \frac{e^{i\delta_B^{\pi\pi}(s)}}{\Delta(s)}\{g_{s\bar{s}\sigma}[D_{f_0}(s)g_{\sigma\pi^+\pi^-} + \Pi_{\sigma f_0}(s)g_{f_0\pi^+\pi^-}] + g_{s\bar{s}f_0}[D_\sigma(s)g_{f_0\pi^+\pi^-} + \Pi_{\sigma f_0}(s)g_{\sigma\pi^+\pi^-}]\}. \quad (11)$$

Here, we use the expressions and numbers from Ref. [17] (corresponding to fitting variant 1 from Table I therein) for propagators $1/D_\sigma(s)$ and $1/D_{f_0}(s)$ of $\sigma(500)$ and $f_0(980)$ resonances, the polarization operator matrix element $\Pi_{\sigma f_0}(s)$, the $\delta_B^{\pi\pi}(s)$ phase of the elastic background in the S-wave $\pi\pi$ scattering, $g_{\sigma\pi^+\pi^-}$ and $g_{f_0\pi^+\pi^-}$ coupling constants, etc.

Note that our principal conclusions are independent of a concrete fitting variants presented in Refs. [17–19], containing the excellent simultaneous descriptions of the phase shifts, inelasticity, and mass distributions in the reactions $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$, and $\phi \to \pi^0\pi^0\gamma$. Also note that the expressions in square brackets in Eqs. (10) and (11) are real for $\sqrt{s}$ below the K^+K^- threshold.

Consider the variant corresponding to the following simple choice of direct coupling constants σ and f_0 with $q\bar{q}$:

$$g_{s\bar{s}\sigma} = 0, \qquad g_{d\bar{d}f_0} = 0, \qquad g_{d\bar{d}\sigma} = g_0/\sqrt{2}, \qquad g_{s\bar{s}f_0} = g_0. \quad (12)$$

Further, without loss of generality, we put $g_0 = 1$. The normalization constants $f_+^{D_s^+}(0)$ and $f_+^{D^+}(0)$ in (3) are assumed to be equal. Then, substituting (10) and (11) into (4) and integrating over the intervals $2m_\pi < \sqrt{s} <$ 1.4 GeV and 0.6 GeV $< \sqrt{s} <$ 1.2 GeV, respectively, we get the ratio of the widths

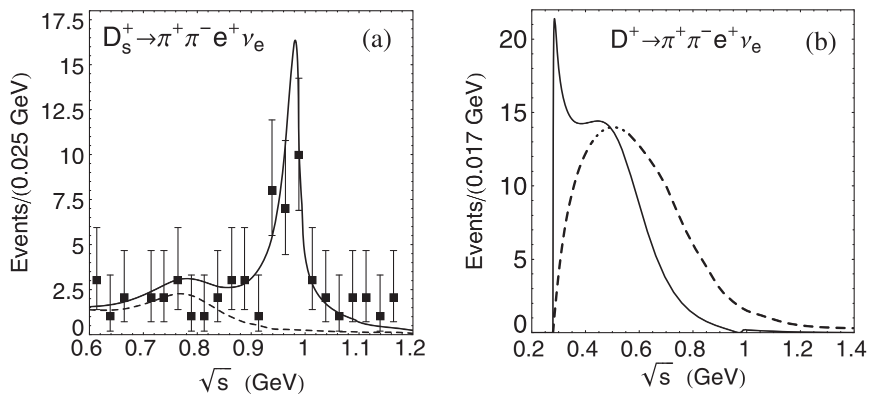

FIG. 3. (a) The points with the error bars are the CLEO data [10] on the $\pi^+\pi^-$ invariant mass distribution in the decay $D_s^+ \to \pi^+\pi^- e^+\nu_e$ dominated by the $f_0(980)$ resonance production. The dashed curve shows the total contribution from three noncoherent background processes estimated by CLEO [10]. (b) The dashed curve represents the smoothed BESIII histogram with 0.017-GeV-wide-step for the $\pi^+\pi^-$ S-wave distribution extracted by BESIII from the treatment of $D^+ \to \pi^+\pi^- e^+\nu_e$ events [11]. Uncertainties in the BESIII data can range from 10% to 20%. The K_S^0 veto region around 0.5 GeV [11] is shown by the dotted curve. The solid curves in (a) and (b) correspond to the model described by Eqs. (10)–(12).

$$\frac{\Gamma_{D_s^+\to\pi^+\pi^-e^+\nu_e}}{\Gamma_{D^+\to\pi^+\pi^-e^+\nu_e}} \approx 5.62. \qquad (13)$$

Thus, we have satisfactory agreement with the data given in Table I, according to which this ratio is equal to 6.55 ± 1.18. However, Fig. 3 indicates that the joint description of the $\pi^+\pi^-$ mass spectra in $D_s^+ \to \pi^+\pi^- e^+\nu_e$ and $D^+ \to \pi^+\pi^- e^+\nu_e$ decays sharply contradicts the BESIII [11] data at $\sqrt{s} < 1$ GeV. These data demonstrate a smooth and wide $\pi^+\pi^-$ spectrum in the decay $D^+ \to \pi^+\pi^- e^+\nu_e$ [see Fig. 3(b)], due to, according to the authors of Ref. [11], the $\sigma(500)$ resonance production. It is interesting that this contradiction is caused by the small mass and large width of the unshielded σ resonance [12,17–21], i.e., its main features. The factor $\Phi(m_{D^+_{c\bar{q}}}, m_A, s)$ in (4) more enhances the $\pi^+\pi^-$ mass spectrum in the near-threshold region (see Fig. 1). Note that the fundamental role of the chiral shielding in the fate of the $\sigma(500)$ meson was demonstrated in the linear σ model [22] (which turned out to be a nontrivial realization of QCD in the low-energy region) using examples of the reactions $\pi\pi \to \pi\pi$ and $\gamma\gamma \to \pi\pi$ [20,21].

But what is the sensitivity of the mass spectra shown in Fig. 3 to possible deviations of $g_{s\bar{s}\sigma}$ and $g_{d\bar{d}f_0}$ from zero? Let the values of these constants are in the intervals:

$$-0.2 < g_{s\bar{s}\sigma} < 0.2, \qquad -0.2 < g_{d\bar{d}f_0} < 0.2$$

[compare with Eq. (12) at $g_0 = 1$]. Then the ratio $\Gamma_{D_s^+\to\pi^+\pi^-e^+\nu_e}/\Gamma_{D^+\to\pi^+\pi^-e^+\nu_e}$ will be in the range from 5 to 7. From Eqs. (10) and (11) it can be seen that the difference of $g_{s\bar{s}\sigma}$ from zero affects only the amplitude $F^{D_s^+,\text{direct}}_{s\bar{s}\to S\to\pi^+\pi^-}(s)$ and the difference of $g_{d\bar{d}f_0}$ from zero affects only the amplitude $F^{D^+,\text{direct}}_{d\bar{d}\to S\to\pi^+\pi^-}(s)$. As a result, it turns out that the mass spectrum in Fig. 3(b) varies slightly only in the $f_0(980)$ region. In most cases, the expected small peak from $f_0(980)$ resonance appears in it. Thus, a contradiction with the data presented in Fig. 3(b) remains completely throughout the entire region $\sqrt{s} < 1$ GeV. Difference of $g_{s\bar{s}\sigma}$ from zero worsens the description of the $\pi^+\pi^-$ mass spectrum in the decay $D_s^+ \to \pi^+\pi^- e^+\nu_e$ in the $f_0(980)$ region shown in Fig. 3(a). Worsening is associated with a noticeable rise of the left wing of the $f_0(980)$ resonance. But a particularly significant effect of $\sigma(500)$ arises near the $\pi^+\pi^-$ threshold when the $g_{s\bar{s}\sigma} \approx -0.2$. The $\pi^+\pi^-$ mass spectrum in the decay $D_s^+ \to \pi^+\pi^- e^+\nu_e$ at $\sqrt{s} < 0.5$ GeV turns out to be similar to one in the decay $D^+ \to \pi^+\pi^- e^+\nu_e$ in the same region of $\sqrt{s}$ [see Fig. 3(b)]. Such a manifestation of the $\sigma(500)$ resonance in $D_s^+ \to \pi^+\pi^- e^+\nu_e$ is extremely improbable.

So, we discard the above-described model of the creation of σ and f_0 states due to the presence of $d\bar{d}$ and $s\bar{s}$ components in their wave functions, respectively. Figuratively, we can say that the $q\bar{q}$ probe existing in semileptonic $(D^+, D_s^+) \to \pi^+\pi^- e^+\nu_e$ decays does not find, to a first approximation, the corresponding $q\bar{q}$ components.

It was directly shown in Ref. [1] that the transition $s\bar{s} \to \sigma(500)$ is negligible compared to the transition $s\bar{s} \to f_0(980)$. In the work [1], it was also shown that the intensity of the $s\bar{s} \to f_0(980)$ transition is about thirty percent of the intensity of the $s\bar{s} \to \eta_s$ (where $\eta_s = s\bar{s}$), $g^2_{s\bar{s}f_0}/g^2_{s\bar{s}\eta_s} \approx 0.3$, contrary expected equality of these intensities in the chiral-symmetric models like the Nambu-Jona-Lasinio one. The above analysis obviously supports the conclusion made in Ref. [1] that the decay $D_s^+ \to \pi^+\pi^- e^+\nu_e$ testifies to the previous conclusions about the dominant role of the four-quark components in $\sigma(500)$ and $f_0(980)$ mesons.

IV. FOUR-QUARK PRODUCTION MECHANISM

Let us now consider the four-quark $\sigma(500) = u\bar{u}d\bar{d}$ and $f_0(980) = s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$ meson production which is

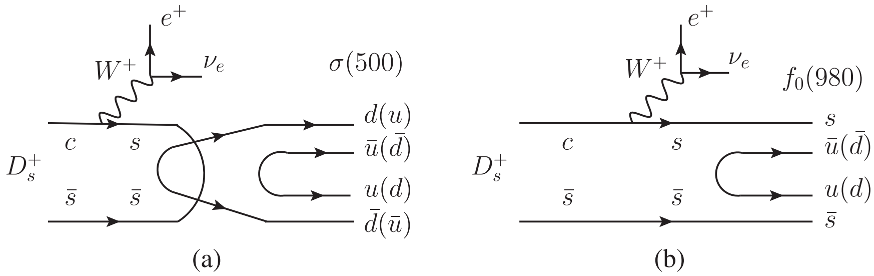

FIG. 4. Production of the four-quark $\sigma(500)$ and $f_0(980)$ mesons in D_s^+ decays.

symbolically depicted in the diagrams of Figs. 4 and 5. (We emphasize that in the processing of the data we use, of course, the resonance complex of the mixed states σ and f_0 states [17–19].) These are ideal $q^2\bar{q}^2$ states of the MIT bag with superallowed decays $\sigma \to \pi\pi$ and $f_0 \to K\bar{K}$ [13]. On the contrary, the decays $\sigma \to K\bar{K}$ and $f_0 \to \pi\pi$ are suppressed for these states by the Okubo-Zweig-Iizuka (OZI) rule [23–27]. Due to the small mass of σ, the OZI suppressed decay $\sigma \to K\bar{K}$ does not play any role at all. At the same time, the main decay of $f_0(980)$ under the $K\bar{K}$ threshold is precisely the decay $f_0(980) \to \pi\pi$ due to a small $\sigma - f_0$ mixing. Thus, the decay $D_s^+ \to \pi^+\pi^- e^+\nu_e$, owing to the OZI-suppression of the σ resonance creation [see Fig. 4(a)], is dominated by the $f_0(980)$ resonance production [see Fig. 4(b)] followed by its decay into $\pi^+\pi^-$: $D_s^+ \to f_0(980)e^+\nu_e \to \pi^+\pi^- e^+\nu_e$.

In the decay $D^+ \to \pi^+\pi^- e^+\nu_e$, production of the four-quark states $\sigma(500)$ and $f_0(980)$ is not suppressed by the OZI rule, see Fig. 5, and it would seem that both states should manifest themselves as enhancements in the $\pi^+\pi^-$ mass spectrum. However, the remarkable fact confirmed in many reactions is that when there are no valence $s\bar{s}$ pairs in the generating channel, the $f_0(980)$ resonance manifests itself (each time) in the $\pi\pi$ mass spectrum not in the form of a peak, but in the form of a sharp dip or sharp ledge, or a completely insignificant fluctuation. The reason for this is the destructive interference of the $f_0(980)$ contribution with a large and smooth background, which is present in the $\pi\pi$ decay channel and has a phase of $\approx 90°$. Striking examples here are the data on the reactions $\pi\pi \to \pi\pi$ [28,29], $pp \to p(\pi\pi)p$ [30], $J/\psi \to \omega\pi^+\pi^-$ [31], $\Upsilon(10860) \to \Upsilon(1S)\pi^+\pi^-$ [32], and, of course, the discussed new BESIII data on $D^+ \to \pi^+\pi^- e^+\nu_e$ [11] (see also in this connection a comment in Ref. [33]).

And vice versa, when valence $s\bar{s}$ pairs are present in the generating channel, such as in the reactions $K^-p \to \pi^+\pi^-(\Lambda, \Sigma^0)$ [34], $J/\psi \to \phi\pi^+\pi^-$ [35], $D_s^+ \to \pi^+\pi^+\pi^-$ [36], and $D_s^+ \to \pi^+\pi^- e^+\nu_e$ [10], then a sharp peak is observed in the $f_0(980)$ resonance region.

The described picture of the creation of four-quark resonances in the $D^+ \to \pi^+\pi^- e^+\nu_e$ and $D_s^+ \to \pi^+\pi^- e^+\nu_e$ decays can be effectively realized in the language of hadronic states, see Figs. 6 and 7. The mechanisms indicated in Figs. 6 and 7 imply that the S-wave $\pi^+\pi^-$ system can be produced via seed four-quark fluctuations $d\bar{d} \to \pi\pi$, $d\bar{d} \to K\bar{K}$, and $s\bar{s} \to K\bar{K}$, which are then dressed by strong interactions in the final state. According to Figs. 6 and 7, we write the amplitudes $F^{D^+}_{d\bar{d}\to S\to\pi^+\pi^-}(s)$ and $F^{D_s^+}_{s\bar{s}\to S\to\pi^+\pi^-}(s)$ from Eq. (4) in the form

$$F^{D^+}_{d\bar{d}\to S\to\pi^+\pi^-}(s) = \lambda_{d\bar{d}\pi^+\pi^-}[1 + I_{\pi^+\pi^-}(s)T_0^0(s)] + \lambda_{d\bar{d}K^0\bar{K}^0} I_{K^0\bar{K}^0}(s) T_{K^0\bar{K}^0\to\pi^+\pi^-}(s), \quad (14)$$

$$F^{D^+s}_{s\bar{s}\to S\to\pi^+\pi^-}(s) = \lambda_{s\bar{s}K^0\bar{K}^0}[I_{K^+K^-}(s) + I_{K^0\bar{K}^0}(s)] \times T_{K^0\bar{K}^0\to\pi^+\pi^-}(s), \quad (15)$$

where $T_0^0(s) = T_{\pi^+\pi^-\to\pi^+\pi^-}(s) + \frac{1}{2}T_{\pi^0\pi^0\to\pi^+\pi^-}(s)$ is the S-wave amplitude of the reaction $\pi\pi \to \pi\pi$ in the channel with isospin $I = 0$ composed of the amplitudes related to individual charge channels; $T_0^0(s) = [\eta_0^0(s)\exp(2i\delta_0^0(s)) - 1]/(2i\rho_{\pi^+\pi^-}(s))$, where $\eta_0^0(s)$ and $\delta_0^0(s)$ are the corresponding inelasticity and phase of $\pi\pi$ scattering; $T_{K^0\bar{K}^0\to\pi^+\pi^-}(s)$ is

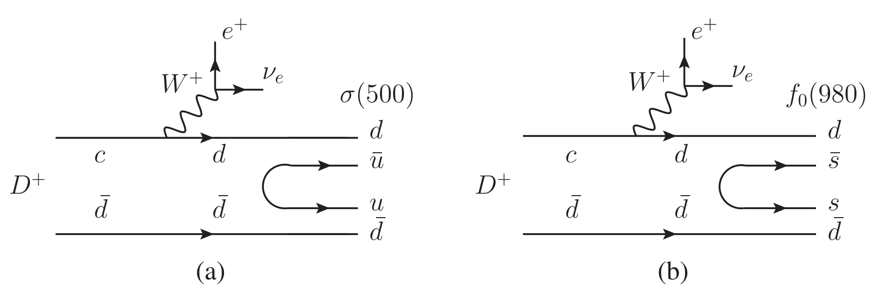

FIG. 5. Production of the four-quark $\sigma(500)$ and $f_0(980)$ mesons in D^+ decays.

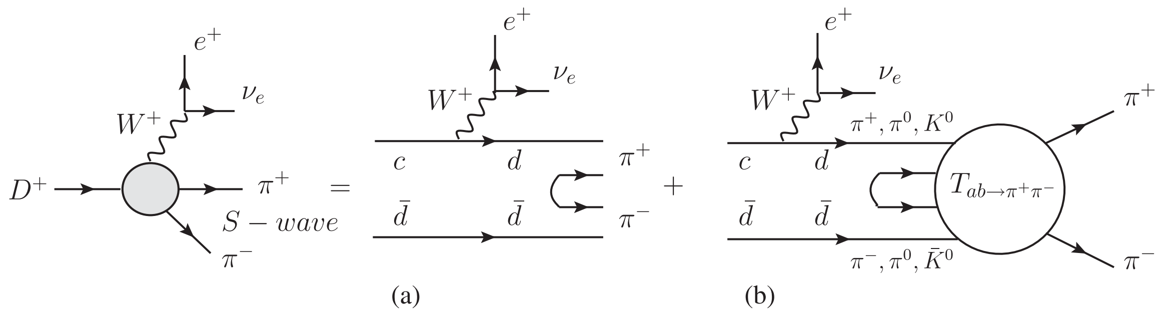

FIG. 6. The semileptonic decay $D^+ \to \pi^+\pi^- e^+\nu_e$.

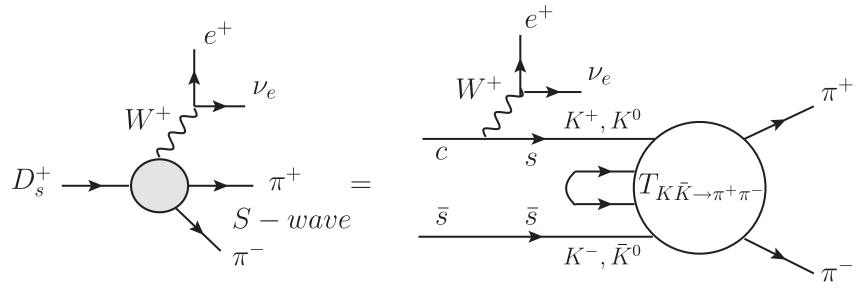

FIG. 7. The semileptonic decay $D_s^+ \to \pi^+\pi^- e^+\nu_e$.

the amplitude of the S-wave transition $K^0\bar{K}^0 \to \pi^+\pi^-$; $T_{K^+K^-\to\pi^+\pi^-}(s) = T_{K^0\bar{K}^0\to\pi^+\pi^-}(s)$ [17–19,37,38]. Functions $I_{a\bar{a}}(s)$ (where $a\bar{a} = \pi^+\pi^-, K^+K^-, K^0\bar{K}^0$) are the amplitudes of the one-loop two-point diagrams describing $a\bar{a} \to a\bar{a} \to$ (*the scalar state with a mass equaling* $\sqrt{s}$) transitions in which initial $a\bar{a}$ pairs are produced by $q\bar{q}$ sources described by coupling constants $\lambda_{q\bar{q}a\bar{a}}$. Above the $a\bar{a}$ threshold, $I_{a\bar{a}}(s)$ has the form [17]

$$I_{a\bar{a}}(s) = \tilde{C}_{a\bar{a}} + \rho_{a\bar{a}}(s)\left(i - \frac{1}{\pi}\ln\frac{1+\rho_{a\bar{a}}(s)}{1-\rho_{a\bar{a}}(s)}\right), \qquad (16)$$

where $\rho_{a\bar{a}}(s) = \sqrt{1 - 4m_a^2/s}$ (we put $m_{\pi^0} = m_{\pi^+}$ and take into account the mass difference of K^+ and K^0); if $\sqrt{s} < 2m_K$, then $\rho_{K\bar{K}}(s) \to i|\rho_{K\bar{K}}(s)|$; $\tilde{C}_{\pi^+\pi^-}$ and $\tilde{C}_{K^+K^-} = \tilde{C}_{K^0\bar{K}^0}$ are subtraction constants in the loops.

For reasons of $SU(3)$ symmetry, we will assume that all seed coupling constants in Eqs. (14) and (15) are the same: $\lambda_{d\bar{d}\pi^+\pi^-} = \lambda_{s\bar{s}K^0\bar{K}^0} = \lambda_{s\bar{s}K^+K^-} = \lambda_{d\bar{d}K^0\bar{K}^0}$. For reasons of $SU(4)$ symmetry, $f_+^{D_s^+}(0) = f_+^{D^+}(0)$. Then, for example, the product $f_+^{D_s^+}(0)\lambda_{s\bar{s}K^0\bar{K}^0}$ will determine the absolute normalization of the widths $\Gamma_{D_s^+\to\pi^+\pi^-e^+\nu_e}$ and

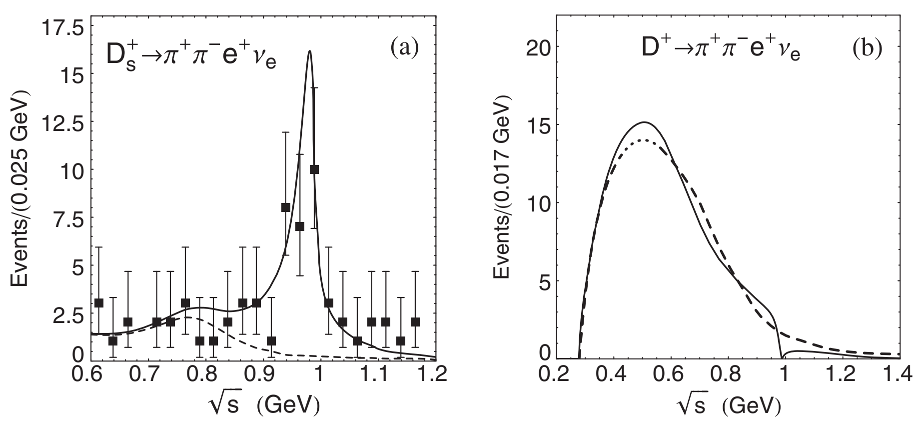

FIG. 8. The same as in plots (a) and (b) in Fig. 3, but the solid theoretical curves correspond to the model describable by Eqs. (14)–(16).

$\Gamma_{D^+\to\pi^+\pi^- e^+\nu_e}$. But the ratio $\Gamma_{D_s^+\to\pi^+\pi^- e^+\nu_e}/\Gamma_{D^+\to\pi^+\pi^- e^+\nu_e}$ does not depend on this parameter.

Since the amplitudes $T_0^0(s)$ and $T_{K^0\bar{K}^0\to\pi^+\pi^-}(s)$ are known [17–19] from the analysis of the data on the reactions $\pi\pi\to\pi\pi$, $\pi\pi\to K\bar{K}$, and $\phi\to\pi^0\pi^0\gamma$, then we have only two parameters $\tilde{C}_{\pi^+\pi^-}$ and $\tilde{C}_{K^+K^-}$ to describe the $\pi^+\pi^-$ mass spectra in the decays $D^+\to\pi^+\pi^- e^+\nu_e$ and $D_s^+\to\pi^+\pi^- e^+\nu_e$ as well as the value of the ration $\Gamma_{D_s^+\to\pi^+\pi^- e^+\nu_e}/\Gamma_{D^+\to\pi^+\pi^- e^+\nu_e}$ in agreement with experiment.

The choice of $\tilde{C}_{\pi^+\pi^-}=1.8$ and $\tilde{C}_{K^+K^-}=1.0$ provides a good simultaneous description of the $\pi^+\pi^-$ mass spectra in the decays $D^+\to\pi^+\pi^- e^+\nu_e$ and $D_s^+\to\pi^+\pi^- e^+\nu_e$, see Fig. 8, and gives the ratio $\Gamma_{D_s^+\to\pi^+\pi^- e^+\nu_e}/\Gamma_{D^+\to\pi^+\pi^- e^+\nu_e}\simeq 6.55$, which is in excellent agreement with the data. Let us note that Fig. 3(b) demonstrates a sharp contradiction with the BESIII data in all region of $\sqrt{s}$ for the $q\bar{q}$ production mechanism, which is discussed immediately below Eq. (13). In contrast, Fig. 8(b) shows a good agreement with the data in the case of the creation of four-quark resonances.

In summary, in the light of the program [1,2], we have analyzed the recent BESIII data [11] on the decay $D^+\to\pi^+\pi^- e^+\nu_e$ together with the CLEO data [10] on the decay $D_s^+\to\pi^+\pi^- e^+\nu_e$ and showed that the results on the $\pi^+\pi^-$ mass spectra of these experiments together can be interpreted in favor of the four-quark nature of light scalar mesons $\sigma(500)$ and $f_0(980)$. Our approach can also be applied to the description of other similar decays involving light scalars.

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$$R=\frac{\mathcal{B}(D^+\to f_0(980)l^+\nu)+\mathcal{B}(D^+\to f_0(600)l^+\nu)}{\mathcal{B}(D^+\to a_0^0(980)l^+\nu)}=\begin{cases}1 & \text{for two-quark scalars,}\\ 3 & \text{for four-quark scalars.}\end{cases}$$

However, the BESIII experiment [11] suggests that $\mathcal{B}(D^+\to f_0(980)e^+\nu_e\to\pi^+\pi^- e^+\nu_e)\approx 0$. Thus, it becomes clear that the naive quark counting rules cannot be applied in

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Correction: Equation (16) contained a sign error and has been fixed. Minor modifications to the captions of Figures 6 and 7 have been made.