Polarization in $\chi_{cJ} \to \Lambda\bar{\Lambda}$ decays

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We perform an analysis on the χ_{cJ} polarization in the ψ' radiative transition and propose to use it to probe the mechanism of χ_{cJ} decay to the baryon-antibaryon pairs in experiment. To start with the unpolarized e^+e^- collisions, we follow the polarization transfer to the χ_{cJ} states, and estimate the degree of χ_{cJ} polarization. We show that the $\Lambda\bar{\Lambda}$ pair has rich spin configurations, which are beneficial for us to study its decay asymmetry parameter and measure the helicity amplitudes. The experimental observables to reveal the χ_{cJ} and $\Lambda\bar{\Lambda}$ spin polarization are presented.

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I. INTRODUCTION

With a huge statistics of ψ' sample available at e^+e^- collisions, the baryonic decays of $\chi_{cJ}(J = 0, 1, 2)$ states are extensively studied at experiments with a motivation to test χ_{cJ} decay mechanisms [1–4], and their branching fractions are measured at a few percentage precision [5]. For the decays to the octet baryon final states, all measurements have established the fact that χ_{c0} state has larger branching fraction than other two states. This indicates a significant deviation from the expectation of helicity selection rule (HSR) [6], which predicts the vanishing branching fraction for χ_{c0} decay to baryon antibaryon pairs if one neglects the quark mass bounded in the baryons.

The measurements put a great challenge to the theoretical understanding of χ_{cJ} property and decay mechanisms. The earlier investigation on the color singlet contribution was based on the perturbative QCD (pQCD) theory, and predicted nonvanishing branching fractions for the χ_{c1}, $\chi_{c2} \to \Lambda\bar{\Lambda}$ decays, but the values are too small as compared with the measured ones [7]. The contribution from the high order of Fock states has been investigated in the pQCD framework. It was argued that the contribution from the color octet component is not suppressed as compared to the color singlet contribution [8]. Nonetheless, the sum of color singlet and octet contributions only accounts for partial branching fraction. For example, it predicted about 3% and 18% amount of measured branching fractions for χ_{c1} and χ_{c2} decaying to $\Lambda\bar{\Lambda}$ final states [8], respectively. To explain the HSR evasion in χ_{c0} decays, the long distance contribution has also been estimated based on the charm hadron loop [9,10], and other phenomenological model, such as the quark creation model and exchange of intermediate states, were also investigated [11].

In order to test these decay models, we propose to use the polarization information as a probe to figure out the χ_{cJ} decay mechanism. The possible polarization components of χ_{cJ} states can be described with a set of multipole parameters in their spin density matrix, with the highest rank equal to $2J$, here J is the spin of χ_{cJ} states. Except for the χ_{c0} state, the χ_{c1} and χ_{c2} provide us with both the even and odd polarization. The degree of polarization carries the dynamical information of χ_{cJ} decaying to the baryon antibaryon pairs. Especially, the subsequent hyperon weak decay, $\Lambda \to p\pi^-(\bar{\Lambda} \to \bar{p}\pi^+)$, naturally serves as spin polarimetry, which can be used to express the χ_{cJ} polarization.

The χ_{cJ} decays to hyperon antihyperon pair provides a complementary laboratory to study the hyperon properties. Compared to the hyperon production from J/ψ decays, the branching fraction is suppressed by one order of magnitude in the χ_{cJ} decays. But χ_{cJ} decay offers more spin configurations so that the hyperon pair has a rich degree of freedom of polarization. Employing the polarization transfer in χ_{cJ} decays is beneficial to studies of hyperon properties, such as the measurement of hyperon decay asymmetry parameters and test of the spin quantum correlations in hyperon decays [12,13].

We start with construction of a spin density matrix for ψ' in Sec. II, whose form is well known due to the fact that the particle ψ' couples to the virtual photon. But we would like to recapitulate it for the sake of self-contained description. A polarization analysis for the involved particles is performed in Sec. III. We formulate the $\Lambda\bar{\Lambda}$ polarimetry in Sec. IV, and some polarization observables are given in Sec. V. The last section is devoted to some applications, such as the measurements

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TABLE I. Definition of helicity angles and amplitude for each decay, and $\lambda_i (i = 1, ..., 6)$ denotes the helicity of a given particle ahead it.

Decays	Angles	Amplitudes
$\psi' \to \chi_{cJ}(\lambda_1)\gamma(\lambda_2)$	$\Omega_0 = (\theta_0, \phi_0)$	$A^{(J)}_{\lambda_1,\lambda_2}$
$\chi_{cJ} \to \Lambda(\lambda_3)\bar{\Lambda}(\lambda_4)$	$\Omega_1 = (\theta_1, \phi_1)$	$B^{(J)}_{\lambda_3,\lambda_4}$
$\Lambda \to p(\lambda_5)\pi^-$	$\Omega_2 = (\theta_2, \phi_2)$	F_{λ_5}
$\bar{\Lambda} \to \bar{p}(\lambda_6)\pi^+$	$\Omega_3 = (\theta_3, \phi_3)$	G_{λ_5}

of $\Lambda\bar{\Lambda}$ asymmetry parameters and the helicity amplitudes for χ_{cJ} decays.

II. SPIN DENSITY MATRIX

We consider the χ_{cJ} particles produced from the ψ' radiative transition, $\psi' \to \chi_{cJ}\gamma$, and we restrict ourself to the case of ψ' production from the unpolarized e^+e^- collisions. The subsequential decay of $\chi_{cJ} \to \Lambda\bar{\Lambda}$ will serves as a polarimetry to measure the χ_{cJ} polarization due to the weak decay of $\Lambda \to p\pi^-(\bar{\Lambda} \to \bar{p}\pi^+)$, here hyperon is characterized with the asymmetry angular distribution $I(\theta) \propto 1 + P\alpha_\Lambda \cos\theta$, here θ is the polar angle for proton in the Λ helicity system, and the asymmetry parameter $\alpha_\Lambda(\alpha_{\bar{\Lambda}})$ is measured with high precision recently [14], and P measures the $\Lambda(\bar{\Lambda})$ polarization.

Studies of spin transfer in the charmonium sequential decays were once performed with the method of covariant tensor formalism by using the spin folding law [15–18]. It is also convenient to employ the method of helicity amplitude to perform the polarization analysis [19,20], especially for the decays involving photons. In this way, the decay amplitude for each step decay can be described with the helicity angles $\Omega_i(\theta_i, \phi_i)$ and helicity amplitude as shown in Table I. The helicity angles are defined in the helicity system, which is shown in Fig. 1:

(i) $e^+e^- \to \psi' \to \chi_{cJ}\gamma$: The helicity system of z-axis for this decay is taken along the e^+ flying direction, and angles (θ_0, ϕ_0) are taken as the χ_{cJ} particle moving direction in the e^+e^- center-of-mass (CM) system.

(ii) $\chi_{cJ} \to \Lambda\bar{\Lambda}$: The azimuthal angle ϕ_1 is the angle between the χ_{cJ} decay plane and χ_{cJ} production plane. If the $\Lambda\bar{\Lambda}$ momenta are boosted to the χ_{cJ} rest frame, they are still located in the χ_{cJ} decay plane, and then we take θ_1 as the angle between the Λ momentum in the χ_{cJ} rest frame and the χ_{cJ} momentum.

(iii) $\Lambda \to p\pi^-$: The azimuthal angle ϕ_2 is the angle between the Λ production and decay planes. The polar angle θ_2 is taken as the angle between the proton momentum in the Λ rest frame and the Λ momentum in the χ_{cJ} rest frame.

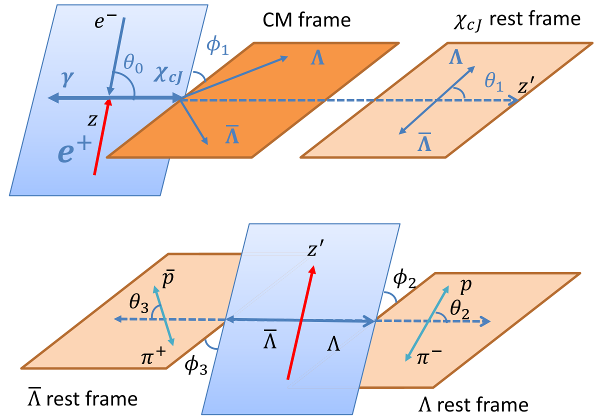

FIG. 1. Definition of helicity system and helicity angles for each decay in the sequential decay $\psi' \to \chi_{cJ}\gamma$, $\chi_{cJ} \to \Lambda\bar{\Lambda} \to p\bar{p}\pi^+\pi^-$.

(iv) $\bar{\Lambda} \to \bar{p}\pi^+$: The azimuthal angle ϕ_3 is the angle between the $\bar{\Lambda}$ production and decay planes. The polar angle θ_3 is taken as the angle between the antiproton momentum in the $\bar{\Lambda}$ rest frame and the $\bar{\Lambda}$ momentum in the χ_{cJ} rest frame.

The spin density matrix (SDM) encodes complete polarization information of a particle. Let us consider construction of a SDM for the final particles of a given decay, e.g., $A(Jm) \to B(\vec{p}, \lambda_b) + C(-\vec{p}, \lambda_c)$. The dynamical information is characterized by the decay matrix elements,

$$M_{\lambda_b,\lambda_c} = \langle \vec{p}\lambda_b, -\vec{p}\lambda_c | \mathcal{T} | Jm \rangle, \quad (1)$$

where J is the spin of parent particle with magnetic quantum number m, and λ_b, λ_c denote helicities of two final particles, and the decay matrix M is defined with the transition operator $\mathcal{T}$ between the final and initial states. The SDM of final states is defined in the helicity space $|\lambda_b\rangle \otimes |\lambda_c\rangle$ as $\rho_f = |\lambda_b\lambda_c\rangle\langle\lambda_b\lambda_c|$. It relates to the SDM of initial particle, ρ_i, by the decay matrix as [21]

$$\rho_f = M\rho_i M^\dagger. \quad (2)$$

Here $\rho_{f/i}$ is normalized to the unpolarized decay rate, and the decay matrix is expressed with the helicity amplitude

$$M = N_J D^{J*}_{m,\lambda_b-\lambda_c}(\phi, \theta, 0) H_{\lambda_b,\lambda_c}, \quad (3)$$

where N_J is a normalization factor, $H_{\lambda_b,\lambda_c} = 4\pi\sqrt{\frac{w}{p}}\langle Jm\lambda_b\lambda_c|\mathcal{T}|Jm\rangle$ is helicity amplitude with mass w for the parent particle A, and $D^J_{i,j}(\phi, \theta, 0)$ is the Wigner D-function with helcity angles θ and ϕ for final state B or C. Here we take the convention of two arguments, since rotation of helicity system only needs two successive rotations for the initial particle to overlap with that for the final state particles. Some symmetry constraints, such as the parity conservation and identical particle symmetry, will be imposed on the independent helicity amplitude in the following analysis.

A. ψ' particle

We consider the ψ' particle produced from the unpolarized e^+e^- collisions, so the SDM for e^+e^- pair is reduced to an unity matrix, and the quantization axis is chosen as the z-axis of e^+e^- CM system, so the helicity angles for ψ' are fixed to $\Omega = (0,0)$. The e^+e^- pair couples to the ψ' particle via a virtual photon, which conserves the e^+e^- helicities. This requires the helicity $\lambda_-(\lambda_+)$ for $e^-(e^+)$ satisfying $\lambda = \lambda_+ - \lambda_- = \pm 1$, and contribution from helicity amplitude with $\lambda = 0$ is negligible. The element of ψ' SDM is written as

$$\rho_{\lambda,\lambda'}(\psi') = \frac{1}{2}\sum_{m=\pm 1} D^{1*}_{m,\lambda}(0,0,0) D^{1}_{m,\lambda'}(0,0,0) = \frac{1}{2}\mathrm{diag}\{1,0,1\}. \tag{4}$$

It is interesting to note here that the SDM deviates from the unit matrix, which implies some degrees of polarization for ψ' particles produced from the unpolarized e^+e^- collisions.

In experiments for massive spin-1 particles, it is sometimes convenient to use the spin operator with the 3×3 traceless matrix $S_i (i = x, y, z)$ in Cartesian system to define the vector polarization $\vec{\mathcal{P}} = \langle \vec{S} \rangle$ and the tensor polarization

$$T_{ij} = \frac{1}{2}\sqrt{\frac{3}{2}}\left(\langle S_i S_j + S_j S_i \rangle - \frac{4}{3}\delta_{ij}\right), \tag{5}$$

where $\langle \ldots \rangle$ denotes taking average in the ψ' spin space.

Using the ψ' SDM as given by Eq. (4), one has $\mathcal{P}_x = \mathcal{P}_y = \mathcal{P}_z = 0$, $T_{xx} = T_{yy} = -\frac{1}{2\sqrt{6}}$, and $T_{zz} = \frac{1}{\sqrt{6}}$. This suggests that the ψ' produced in e^+e^- collisions be only the tensor polarized, with polarization degree $T = \sqrt{T_{xx}^2 + T_{yy}^2 + T_{zz}^2} = 1/2$. Of the most interesting is that this polarization can be transferred to the ψ' (or J/ψ) daughter particles, as having been observed, the $\Lambda(\bar{\Lambda})$ transverse polarization in the $\psi \to \Lambda\bar{\Lambda}$ decay [14].

B. χ_{cJ} states

Spin density matrix of χ_{cJ} states describing their production from the $\psi'(m) \to \chi_{cJ}(\lambda_1)\gamma(\lambda_2)$ decay can be calculated in a straightforward way using Eq. (2), which is written as

$$\rho_{\lambda_1,\lambda_1'}(\chi_{cJ}) \propto \sum_{m,m',\lambda_2} \rho_{m,m'}(\psi') D^{1*}_{m,\lambda_1-\lambda_2}(\phi_0,\theta_0,0) \times D^{1}_{m',\lambda_1'-\lambda_2}(\phi_0,\theta_0,0) A^{(J)}_{\lambda_1,\lambda_2} A^{(J)*}_{\lambda_1',\lambda_2}, \tag{6}$$

with $J = 0$, 1 and 2 for three χ_{cJ} states, respectively. Here the helicity amplitude $A^{(J)}_{\lambda_1,\lambda_2}$ contains the dynamical information for the transition $\psi' \to \chi_{cJ}\gamma$. It was argued that the electric dipole dominates this transition, and it was confirmed by the BESIII measurement [22]. Hence, the helicity amplitudes $A^{(J)}_{\lambda_1,\lambda_2}$ are chosen to satisfy the $E1$ transition relations [23], such as

$$A^{(1)}_{1,1} = A^{(1)}_{0,1} \quad \text{for } \chi_{c1},$$
$$A^{(2)}_{2,1} = \sqrt{2}A^{(2)}_{1,1} = \sqrt{6}A^{(2)}_{0,1} \quad \text{for } \chi_{c2}. \tag{7}$$

This charmonium transition conserves the parity, and helicity amplitudes satisfy the relations $A^{(J)}_{-\lambda_1,-\lambda_2} = (-1)^J A^{(J)}_{\lambda_1,\lambda_2}$. So we have $A^{(0)}_{0,-1} = A^{(0)}_{0,1}$ for the χ_{c0} final state, and $A^{(1)}_{-1,-1} = -A^{(1)}_{1,1}$, $A^{(1)}_{0,-1} = -A^{(1)}_{0,1}$ for χ_{c1}, and $A^{(2)}_{-2,-1} = A^{(2)}_{2,1}$, $A^{(2)}_{-1,-1} = A^{(2)}_{1,1}$, $A^{(2)}_{0,-1} = A^{(2)}_{0,1}$ for χ_{c2} final state.

One can see that the χ_{cJ} SDM is independent of ϕ_0 angle [see Eq. (6)], which follows from the fact that the ψ' SDM restricts the ψ' spin projection to two values, i.e., $m = m' = \pm 1$. Thus if one observes the χ_{cJ} azimuthal angular distribution, it should be flat over full 2π space. An important result following from this feature is that all elements of χ_{cJ} SDM are real numbers.

If the polarization of χ_{cJ} states is not measured, summation of its helicities is equal to taking trace over the $\rho(\chi_{cJ})$, and thus gives the χ_{cJ} angular distribution. With all above relations for helicity amplitudes, the angular distribution parameter, α, can be determined in detecting the χ_{cJ} angular distribution in the ψ' rest frame, e.g.,

$$\frac{dN}{d\cos\theta_0} \propto 1 + \alpha\cos^2\theta_0, \tag{8}$$

with

$$\alpha = \begin{cases} 1 & \text{for } \chi_{c0}, \\ \dfrac{|A^{(1)}_{0,1}|^2 - 2|A^{(1)}_{1,1}|^2}{|A^{(1)}_{0,1}|^2 + 2|A^{(1)}_{1,1}|^2} & \text{for } \chi_{c1}, \\ \dfrac{|A^{(2)}_{0,1}|^2 - 2|A^{(2)}_{1,1}|^2 + |A^{(2)}_{2,1}|^2}{|A^{(2)}_{0,1}|^2 + 2|A^{(2)}_{1,1}|^2 + |A^{(2)}_{2,1}|^2} & \text{for } \chi_{c2}. \end{cases} \tag{9}$$

If the $E1$ transition relations [see Eq. (7)] are used, one has $\alpha = 1, -1/3$, and $1/13$ for χ_{c0}, χ_{c1}, and χ_{c2} states, respectively.

C. $\Lambda\bar{\Lambda}$ pairs

To describe the combined system of $\Lambda\bar{\Lambda}$ produced from χ_{cJ} decays, the joint spin density matrix, $\rho^{\Lambda\bar{\Lambda}}$, can be constructed from the individual SDM as $\rho^{\Lambda} \otimes \rho^{\bar{\Lambda}}$. It can be expressed with a 4×4 matrix, and its elements can be calculated as

$$\rho^{\Lambda\bar{\Lambda}}_{\lambda_3\lambda_4,\lambda_3'\lambda_4'} \propto \sum_{\lambda_1,\lambda_1'} \rho_{\lambda_1,\lambda_1'}(\chi_{cJ}) D^{J*}_{\lambda_1,\lambda_3-\lambda_4}(\phi_1,\theta_1,0) \times D^{J}_{\lambda_1',\lambda_3'-\lambda_4'}(\phi_1,\theta_1,0) B^{(J)}_{\lambda_3,\lambda_4} B^{(J)*}_{\lambda_3',\lambda_4'}, \quad (10)$$

where $\rho(\chi_{cJ})(J=0,1,2)$ is the χ_{cJ} SDM, and $B^{(J)}_{\lambda_3,\lambda_4}$ the helicity amplitudes. The χ_{cJ} decays conserve the parity, so we have the relation $B^{(J)}_{-\lambda_3,-\lambda_4} = (-1)^J B^{(J)}_{\lambda_3,\lambda_4}$.

A simple case is the $\chi_{c0}\to\Lambda\bar{\Lambda}$ decay, for which the SDM of χ_{c0} is reduced to $\rho_{\lambda_1,\lambda_1'}(\chi_{c0}) = \delta_{\lambda_1 0}\delta_{\lambda_1' 0}$, and this leads to $B^{(0)}_{++} = B^{(0)}_{--}$ ($\pm$ is short for $\pm 1/2$), and other element, $B^{(0)}_{\lambda_3,\lambda_4}$, will vanish if $\lambda_3\neq\lambda_4$. Thus we get the joint SDM for the $\Lambda\bar{\Lambda}$ system as $\rho^{\Lambda\bar{\Lambda}}_{\lambda_3\lambda_4,\lambda_3'\lambda_4'} = \frac{1}{2}\delta_{\lambda_3\lambda_4}\delta_{\lambda_3'\lambda_4'}$, where the factor $1/2$ comes from the normalization requirement.

III. POLARIZATION ANALYSIS

A. χ_{cJ} states

With the obtained SDM for χ_{cJ} states, the polarization is characterized by the real multipole parameter, r^L_M, which is determined by

$$r^L_M = \mathrm{Tr}[\rho(\chi_{cJ}) Q^L_M], \quad (11)$$

where Q^L_M is a set of Hermitian basis matrices as given in Ref. [24]. The L-rank index ranges from 1 to $2J$, and M is taken as successive integers within the interval $[-L, L]$. Then the degree of polarization is given by

$$d = \sqrt{\sum_{L=1}^{2J}\sum_{M=-L}^{L}(r^L_M)^2}. \quad (12)$$

On the other hand, the spin density matrix can be rewritten generally in terms of the real parameter r^L_M for a spin-J particle, e.g.,

$$\rho(\chi_{cJ}) = \frac{r^0_0}{2J+1}\left(I + 2J\sum_{L=1}^{2J}\sum_{M=-L}^{L} r^L_M Q^L_M\right), \quad (13)$$

where I denotes unity matrix with dimension $2J+1$, and r^0_0 corresponds to the unpolarized decay rate with $r^0_0 = \mathrm{Tr}\rho$. Namely we take the convention to normalize the spin density matrix to r^0_0.

For χ_{c1} and χ_{c2} states produced from the $\psi'\to\chi_{cJ}(\lambda_1)\gamma(\lambda_2)$ decay, the rank of their SDM should not be larger than the rank of ψ' SDM, which is known as the rank condition [25] which follows from Eq. (2). The ψ' SDM has rank 2, so some elements of χ_{cJ} SDM will be vanishing, i.e., $\rho_{\lambda_1,\lambda_1'} = 0$, if $|\lambda_1 - \lambda_1'| > J$ with $J=1$ and 2 for χ_{c1} and χ_{c2} states, respectively. We concern only with the χ_{cJ} SDM and the photon helicity λ_2 is summed out with $\lambda_2=\pm 1$. Thus the helicity of final states is required within the ranges $|\lambda_1-\lambda_2|\leq 1$ and $|\lambda_1'-\lambda_2|\leq 1$. Either $\lambda_2 = 1$ or -1 will lead to the λ_1 and λ_1' taking the values with the requirement $|\lambda_1-\lambda_1'|\leq J$.

The independent elements of $\rho_{\lambda_1,\lambda_1'}(\chi_{cJ})$ can be largely reduced if we take consideration of the general properties for the SDM. For example, the SDM is Hermitian matrix, and all elements of χ_{cJ} SDM are real numbers as we argued in the previous section, so we have $\rho_{\lambda_1,\lambda_1'} = \rho_{\lambda_1',\lambda_1}$. Another important property follows from the symmetry relations of spherical tensor for the parity conserving decays, and it imposes a strong constraint on the SDM as

$$\rho_{\lambda_1,\lambda_1'}(\chi_{cJ}) = (-1)^{\lambda_1-\lambda_1'}\rho_{-\lambda_1,-\lambda_1'}(\chi_{cJ}). \quad (14)$$

For the spin-zero of χ_{c0} particle, its SDM is reduced to the unpolarized decay rate r^0_0, i.e., it is isotropic from the ψ' transition.

For χ_{c1} state, the elements $\rho_{1,-1} = \rho_{-1,1} = 0$ due to the requirement of rank condition in the decay. One has to choose 5 independent elements to form the real number of Hermitian matrix. The parity conservation requires that $\rho_{1,1} = \rho_{-1,-1}$, $\rho_{1,0} = -\rho_{0,1}$. Thus the χ_{c1} SDM can be expressed with three independent elements or parameters as

$$\rho(\chi_{c1}) = r^0_0 \begin{bmatrix} \frac{1}{3}(1+r^2_0) & \frac{r^2_1}{\sqrt{6}} & 0 \\ \frac{r^2_1}{\sqrt{6}} & \frac{1}{3}(1-2r^2_0) & -\frac{r^2_1}{\sqrt{6}}, \\ 0 & -\frac{r^2_1}{\sqrt{6}} & \frac{1}{3}(1+r^2_0) \end{bmatrix}, \quad (15)$$

with

$$r^0_0 = (3-\cos^2\theta_0)a^2_{0,1},$$
$$r^0_0 r^2_0 = -2a^2_{0,1}\cos^2\theta_0,$$
$$r^0_0 r^2_1 = \frac{\sqrt{3}a^2_{0,1}}{2}\sin(2\theta_0), \quad (16)$$

where $a_{0,1}$ is the modulus of helicity amplitude $A_{0,1}$.

The degree of χ_{c1} polarization is solely determined by the real parameters r^2_0 and r^2_1, which are independent of the amplitude $a_{0,1}$ as shown in the above equations. According to Eq. (12), one has

$$d_1 = \frac{|x|\sqrt{x^2+3}}{3-x^2}, \quad \text{with} \quad x = \cos\theta_0. \quad (17)$$

The integration over the full space yields an estimation of χ_{c1} polarization to be about 76%. In Cartesian system, the polarization of spin-1 particle is classified into the linear and tensor polarization. The linear polarization is determined by the multipole parameters with rank-1 indices, which are vanishing in the χ_{c1} SDM. This results in the vanishing of linear polarization. Thus the χ_{c1} polarization is fully coming from the tensor polarization. Due to the radiative photon in the $\psi' \to \chi_{c1}\gamma$ transition is tensor polarized, which induces some net degree of χ_{c1} polarization.

The same analysis can be performed to the χ_{c2} SDM. Elements with $|\lambda_1 - \lambda_1'| > 2$ are vanishing, following from the rank condition of the decay, i.e., $\rho_{2,-1} = \rho_{2,-2} = \rho_{1,-2} = \rho_{-1,2} = \rho_{-2,2} = \rho_{-2,1} = 0$. One needs 12 independent elements to express the Hermitian matrix with all real elements. Further consideration of the parity conservation, one has relations such as $\rho_{2,2} = \rho_{-2,-2}$, $\rho_{1,1} = \rho_{-1,-1}$, $\rho_{2,1} = -\rho_{-1,-2}$, $\rho_{2,0} = \rho_{0,-2}$, and $\rho_{1,0} = -\rho_{0,-1}$. Thus the number of independent elements is reduced to 7, and the χ_{c2} SDM is written as,

$$\rho(\chi_{c2}) = \begin{bmatrix} \rho_{22} & \rho_{21} & \rho_{20} & 0 & 0 \\ \rho_{21} & \rho_{11} & \rho_{10} & \rho_{1-1} & 0 \\ \rho_{20} & \rho_{10} & \rho_{00} & -\rho_{10} & \rho_{20} \\ 0 & \rho_{1-1} & \rho_{10} & \rho_{11} & -\rho_{21} \\ 0 & 0 & \rho_{20} & -\rho_{21} & \rho_{22} \end{bmatrix}, \quad (18)$$

with

$$\rho_{22} = \frac{1}{35} r_0^0 (2\sqrt{70} r_0^2 + \sqrt{14} r_0^4 + 7),$$
$$\rho_{21} = \frac{r_0^0(\sqrt{6} r_1^2 + r_1^4)}{\sqrt{35}}, \qquad \rho_{20} = \frac{r_0^0(2r_2^2 - \sqrt{3} r_2^4)}{\sqrt{35}},$$
$$\rho_{11} = -\frac{1}{35} r_0^0 (\sqrt{70} r_0^2 + 4\sqrt{14} r_0^4 - 7),$$
$$\rho_{10} = \frac{r_0^0(r_1^2 - \sqrt{6} r_1^4)}{\sqrt{35}}, \qquad \rho_{1-1} = \sqrt{\frac{2}{35}} r_0^0 (\sqrt{3} r_2^2 - 2r_2^4),$$
$$\rho_{00} = \frac{1}{35} r_0^0 (-2\sqrt{70} r_0^2 + 6\sqrt{14} r_0^4 + 7). \quad (19)$$

Here the real multipole parameters, r_M^L, are calculated to be

$$r_0^0 = \frac{1}{12} a_{2,1}^2 [\cos(2\theta_0) + 27],$$
$$r_0^0 r_0^2 = \frac{1}{3}\sqrt{\frac{5}{14}} a_{2,1}^2 [2\cos(2\theta_0) + 3],$$
$$r_0^0 r_1^2 = -\frac{5}{2}\sqrt{\frac{5}{42}} a_{2,1}^2 \sin(\theta_0)\cos(\theta_0),$$
$$r_0^0 r_2^2 = \sqrt{\frac{5}{42}} a_{2,1}^2 \sin^2(\theta_0), \qquad r_0^0 r_0^4 = \frac{a_{2,1}^2[3\cos(2\theta_0) + 1]}{2\sqrt{14}},$$
$$r_0^0 r_1^4 = -\sqrt{\frac{5}{7}} a_{2,1}^2 \sin(\theta_0)\cos(\theta_0),$$
$$r_0^0 r_2^4 = \frac{1}{2}\sqrt{\frac{5}{14}} a_{2,1}^2 \sin^2(\theta_0), \quad (20)$$

where $a_{2,1}$ is the modulus of the helicity amplitude $A_{2,1}$.

One can see that the χ_{c2} is even polarization, since the χ_{c2} SDM involves only the even rank of real multipole parameters. Thus the linear polarization is vanishing, and the polarization is fully coming from the tensor polarization with rank equal to 2 and 4. The degree of χ_{c2} polarization is entirely determined by the parameters r_M^2 and r_M^4 with $M = 0, 1, 2$. According to Eq. (12), one has

$$d_2 = \frac{\sqrt{2x^4 + 67x^2 + 23}}{\sqrt{2}(x^2 + 13)} \quad \text{with} \quad x = \cos\theta_0. \quad (21)$$

The integration over the full space yields an estimation of χ_{c2} polarization to be 70%, in which the ψ' transverse polarization is responsible for most parts in the polarization transfer.

B. $\Lambda\bar{\Lambda}$ pair

For the spin-1/2 particle of Λ or $\bar{\Lambda}$, the SDM can be expressed with a 2×2 matrix, therefore, we need a 4×4 matrix to express the SDM for the Λ and $\bar{\Lambda}$ system, which can be decomposed as a direct product of two Pauli matrices for the Λ and $\bar{\Lambda}$ spin representation as

$$\rho^{\Lambda\bar{\Lambda}} = \frac{C_{00}}{4}\left[1 + \sum_{\overline{i,j=0}}^{4} C_{ij}\sigma_i^{\Lambda} \otimes \sigma_j^{\bar{\Lambda}}\right], \quad (22)$$

where 1 denotes a unit matrix with dimension 4×4, $\overline{i,j=0}$ denotes the summation with lower bound i, $j = 0$ but excluding $(i,j) = (0,0)$. Here taking $j = 1$, 2 and 3 corresponds to the Pauli matrices σ_x, σ_y and σ_z, and if i, $j = 0$, σ_0 is an unit matrix with dimension 2×2. Hence, C_{00} corresponds to the unpolarized decay rate for $\chi_{cJ} \to \Lambda\bar{\Lambda}$ with $C_{00} = \mathrm{Tr}\rho^{\Lambda\bar{\Lambda}}$. With the obtained $\rho^{\Lambda\bar{\Lambda}}$ matrix, the C_{ij} parameters are calculated with $C_{00}C_{ij} = \mathrm{Tr}[\sigma_i \otimes \sigma_j \cdot \rho^{\Lambda\bar{\Lambda}}]$.

For the decay $\chi_{c0} \to \Lambda\bar{\Lambda}$, we have $C_{ij} = \delta_{ij}$. This indicates that the Λ and $\bar{\Lambda}$ have the same component of helicity correlations, and their helicities are parallel to the Λ flying direction in the χ_{c0} rest frame.

For the decay $\chi_{c1} \to \Lambda\bar{\Lambda}$, the unpolarized decay rate is calculated to be

$$\begin{aligned} C_{00} = \frac{r_0^0}{24}\{ & b_{\frac{1}{2},-\frac{1}{2}}^2 [4\sqrt{3} r_1^2 \sin(\theta_1)\cos(\theta_1)\cos(\phi_1) \\ & + 3r_0^2\cos(2\theta_1) + r_0^2 + 4] \\ & - 2b_{\frac{1}{2},\frac{1}{2}}^2 [4\sqrt{3} r_1^2 \sin(\theta_1)\cos(\theta_1)\cos(\phi_1) \\ & + 3r_0^2\cos(2\theta_1) + r_0^2 - 2]\}, \end{aligned} \quad (23)$$

where b_{λ_3,λ_4} is the modulus of helicity amplitude B_{λ_3,λ_4}. The degree of Λ linear polarization is related to the C_{ij} parameters as $\mathcal{P}_x = C_{10}$, $\mathcal{P}_y = C_{20}$ and the longitudinal polarization $\mathcal{P}_z = C_{30} = 0$, since the χ_{cJ} decays conserve the parity, which is produced from the unpolarized ψ' particle. The transverse polarization is calculated to be

$$\mathcal{P}_x^{\Lambda} = -\mathcal{P}_x^{\bar{\Lambda}} = \frac{r_1^2 r_0^0 b_{\frac{1}{2},-\frac{1}{2}} b_{\frac{1}{2},\frac{1}{2}} \sin(\Delta_1)\cos(\theta_1)\sin(\phi_1)}{\sqrt{6}C_{00}},$$

$$\mathcal{P}_y^{\Lambda} = \mathcal{P}_y^{\bar{\Lambda}} = \frac{r_0^0 b_{\frac{1}{2},-\frac{1}{2}} b_{\frac{1}{2},\frac{1}{2}} \sin(\Delta_1)}{3\sqrt{2}C_{00}}[\sqrt{3}r_1^2\cos(2\theta_1)\cos(\phi_1) - 3r_0^2\sin(\theta_1)\cos(\theta_1)], \quad (24)$$

where Δ_1 is the phase angle difference between the amplitudes $B_{-1/2,1/2}$ and $B_{1/2,1/2}$. Other parameters C_{ij} with $i, j \neq 0$ measure the spin correlation between Λ and $\bar{\Lambda}$, e.g., $\mathcal{P}_{xy} = C_{12}$, which is calculated to be

$$\mathcal{P}_{xy} = -\mathcal{P}_{yx} = \frac{r_1^2 r_0^0 b_{\frac{1}{2},-\frac{1}{2}}^2 \sin(\theta_1)\sin(\phi_1)}{2\sqrt{3}C_{00}}. \quad (25)$$

Other correlation parameters C_{ij} are given in the Appendix A.

For the $\chi_{c2} \to \Lambda\bar{\Lambda}$ decay, the unpolarized decay rate is calculated to be

$$C_{00} = r_0^0\left[a_{00} + \sum_{i=0}^{2}(a_{2i}r_i^2 + a_{4i}r_i^4)\right], \quad (26)$$

with parameters a_{00}, a_{2i}, a_{4i} $(i = 0, 1, 2)$ given in Appendix A.

The unpolarized ψ' decay conserves the parity, so the particles Λ and $\bar{\Lambda}$ are not polarized longitudinally. But they could be transversely polarized, with degree of polarization being related the C_{10} or C_{20} parameters as

$$\mathcal{P}_x^{\Lambda} = \mathcal{P}_x^{\bar{\Lambda}} = C_{10}, \quad (27)$$

$$\mathcal{P}_y^{\Lambda} = -\mathcal{P}_y^{\bar{\Lambda}} = C_{20}. \quad (28)$$

The full formulas for $\mathcal{P}_x^{\Lambda}$ and $\mathcal{P}_y^{\Lambda}$ are given in Appendix A. While quantities to measure the polarization correlation are given by

$$\mathcal{P}_{ij} = C_{ij},$$

with $i, j = x, y, z$, and C_{ij} parameters are given in Appendix A.

IV. $\Lambda\bar{\Lambda}$ POLARIMETRY

Detection of $\Lambda\bar{\Lambda} \to (p\pi^-)(\bar{p}\pi^+)$ decay is particular interesting. In contradiction to the strong decays or radiative transitions, the weak decays of $\Lambda\bar{\Lambda}$ can be used as the polarimetry, which is able to analyze both even and odd polarization in the angular distributions. The joint angular distribution for the decay $\Lambda\bar{\Lambda} \to p\bar{p}\pi^+\pi^-$ can be written in terms of the $\Lambda\bar{\Lambda}$ SDM, $\rho^{\Lambda\bar{\Lambda}}$, determined in a given process. Then we have

$$I(\Omega_2, \Omega_3) = \sum_{\lambda_i,\lambda_i'} \rho^{\Lambda\bar{\Lambda}}_{\lambda_3\lambda_4,\lambda_3'\lambda_4'} D^{1/2*}_{\lambda_3,\lambda_5}(\Omega_2) D^{1/2}_{\lambda_3',\lambda_5}(\Omega_2) \times D^{1/2*}_{\lambda_4,\lambda_6}(\Omega_3) D^{1/2}_{\lambda_4',\lambda_6}(\Omega_3)|F_{\lambda_5}|^2|G_{\lambda_6}|^2, \quad (29)$$

where the summation is taken over all involved helicities λ_i and λ_i' $(i = 3, 4, 5, 6)$, and $\Lambda(\bar{\Lambda})$ helicity amplitude, $F_{\lambda_5}(G_{\lambda_6})$ can be related to the decay asymmetry parameters as

$$\alpha_{\Lambda} = \frac{|F_{1/2}|^2 - |F_{-1/2}|^2}{|F_{1/2}|^2 + |F_{-1/2}|^2}, \quad \alpha_{\bar{\Lambda}} = \frac{|G_{1/2}|^2 - |G_{-1/2}|^2}{|G_{1/2}|^2 + |G_{-1/2}|^2}. \quad (30)$$

The angular distribution can be written in a compact matrix form as

$$I(\Omega_2, \Omega_3) = \rho^{\Lambda\bar{\Lambda}} \cdot (M^{\Lambda} \otimes M^{\bar{\Lambda}})^T, \quad (31)$$

where the Λ and $\bar{\Lambda}$ decay matrices are

$$M^{\Lambda} = \frac{1}{2}\begin{bmatrix} 1 + \alpha_{\Lambda}\cos\theta_2 & e^{i\phi_2}\alpha_{\Lambda}\sin\theta_2 \\ e^{-i\phi_2}\alpha_{\Lambda}\sin\theta_2 & 1 - \alpha_{\Lambda}\cos\theta_2 \end{bmatrix}, \quad (32)$$

$$M^{\bar{\Lambda}} = \frac{1}{2}\begin{bmatrix} 1 + \alpha_{\bar{\Lambda}}\cos\theta_3 & e^{i\phi_3}\alpha_{\bar{\Lambda}}\sin\theta_3 \\ e^{-i\phi_3}\alpha_{\bar{\Lambda}}\sin\theta_3 & 1 - \alpha_{\bar{\Lambda}}\cos\theta_3 \end{bmatrix}. \quad (33)$$

After some algebra, we have

$$I(\Omega_2, \Omega_3) = \frac{1}{4}(C_{00} + T_1\alpha_{\Lambda} + \bar{T}_1\alpha_{\bar{\Lambda}} + T_2\alpha_{\Lambda}\alpha_{\bar{\Lambda}}), \quad (34)$$

where C_{00} is the unpolarized decay rate for $\chi_{cJ} \to \Lambda\bar{\Lambda}$, and T_1 and $\bar{T}_1$ measure the transverse polarization for Λ and $\bar{\Lambda}$, respectively. T_2 measures the $\Lambda\bar{\Lambda}$ spin correlations. They are

$$\begin{aligned} T_1 &= \sin\theta_2\sin\phi_2 C_{20} + \sin\theta_2\cos\phi_2 C_{10} + \cos\theta_2 C_{30}, \\ \bar{T}_1 &= \sin\theta_3\sin\phi_3 C_{02} + \sin\theta_3\cos\phi_3 C_{01} + \cos\theta_3 C_{03}, \\ T_2 &= \sin\theta_2[\sin\theta_3(\sin\phi_2\sin\phi_3 C_{22} + \cos\phi_2\cos\phi_3 C_{11} \\ &\quad + \sin\phi_2\cos\phi_3 C_{21} + \cos\phi_2\sin\phi_3 C_{12}) \\ &\quad + \cos\theta_3(\sin\phi_2 C_{23} + \cos\phi_2 C_{13})] \\ &\quad + \cos\theta_2[\sin\theta_3(\sin\phi_3 C_{32} + \cos\phi_3 C_{31}) + \cos\theta_3 C_{33}], \end{aligned} \quad (35)$$

where C_{ij} measures the $\Lambda\bar{\Lambda}$ polarization or their spin correlations as given in the previous section.

A simple case is for the $\chi_{c0} \to \Lambda\bar{\Lambda}$ decay, in which the C_{ij} parameter of $\Lambda\bar{\Lambda}$ SDM is reduced to Kronecker delta function. Thus the angular distribution is reduced to

$$I(\Omega_2, \Omega_3) = \frac{1}{4}\{1 + [\sin\theta_2 \sin\theta_3 \cos(\phi_2 - \phi_3) + \cos\theta_2 \cos\theta_3]\alpha_\Lambda \alpha_{\bar{\Lambda}}\}. \quad (36)$$

For $\chi_{c1}, \chi_{c2} \to \Lambda\bar{\Lambda}$ decays, the longitudinal polarization for particles Λ and $\bar{\Lambda}$ is vanishing, i.e., $C_{03} = C_{30} = 0$, other C_{ij} parameters are given in Appendix A. Then calculation of the joint angular distribution $I(\Omega_2, \Omega_3)$ is straightforward.

V. POLARIZATION OBSERVABLE

In experiments, the degree of polarization for a given particle cannot be always accessible in the modern detector, except for the dedicated polarization measurement. Analyzing polarization of a particle is dependent on the study of its decaying to two or three-body final states. In this situation, the analysis of the angular distribution serves as the polarimetry. Generally speaking, the strong, weak, or radiative decay can be used for this purpose in experiment.

Both χ_{c1} and χ_{c2} particles are of tensor polarization produced from the ψ' radiative transition, which is characterized by the real multipole parameters r^L_M. The helicity angles in the $\chi_{cJ} \to \Lambda\bar{\Lambda}$ can be selected to form a polarization observable to represent its multipole parameters. In experiment, a simple way is to look at the first moments of Wigner D-function if one has sufficient data statistics. Let $W(\theta_1, \phi_1)$ be the unpolarization decay rate for $\chi_{cJ} \to \Lambda\bar{\Lambda}$, the first moments of observable $\hat{O}(\theta_1, \phi_1)$ is defined by

$$\langle \hat{O} \rangle = \frac{\int W(\theta_1, \phi_1)\hat{O}(\theta_1, \phi_1) d\cos\theta_1 d\phi_1}{\int W(\theta_1, \phi_1) d\cos\theta_1 d\phi_1}. \quad (37)$$

For $\chi_{c1} \to \Lambda\bar{\Lambda}$ decay, its tensor polarization associated with r^2_0 and r^2_1 is revealed with observables

$$\langle \mathrm{Re}(D^2_{0,0}(\theta_1, \phi_1)) \rangle = \frac{3b - 2}{5} r^2_0, \quad (38)$$

$$\langle \mathrm{Re}(D^2_{1,0}(\theta_1, \phi_1)) \rangle = \frac{3b - 1}{5\sqrt{2}} r^2_1, \quad (39)$$

with $b = b^2_{\frac{1}{2},-\frac{1}{2}}/(b^2_{\frac{1}{2},-\frac{1}{2}} + b^2_{\frac{1}{2},\frac{1}{2}})$.

For $\chi_{c2} \to \Lambda\bar{\Lambda}$ decay, the six real parameters, r^2_M, $r^4_M (M = 0, 1, 2)$, are related to the moments as

$$\langle \mathrm{Re}(D^2_{2,0}(\theta_1, \phi_1)) \rangle = -\frac{1 + b}{\sqrt{35}} r^2_2,$$
$$\langle \mathrm{Re}(D^2_{1,0}(\theta_1, \phi_1)) \rangle = \frac{1 + b}{\sqrt{35}} r^2_1,$$
$$\langle \mathrm{Re}(D^2_{0,0}(\theta_1, \phi_1)) \rangle = -\sqrt{\frac{2}{35}}(1 + b) r^2_0,$$
$$\langle \mathrm{Re}(D^4_{2,0}(\theta_1, \phi_1)) \rangle = -\frac{2}{9\sqrt{7}}(2 - 5b) r^4_2,$$
$$\langle \mathrm{Re}(D^4_{1,0}(\theta_1, \phi_1)) \rangle = \frac{2}{9\sqrt{7}}(2 - 5b) r^4_1,$$
$$\langle \mathrm{Re}(D^4_{0,0}(\theta_1, \phi_1)) \rangle = -\frac{2}{9}\sqrt{\frac{2}{7}}(2 - 5b) r^4_0. \quad (40)$$

The Λ transverse polarization can be represented with the following moments:

$$\langle \sin\theta_2 \cos\phi_2 \rangle = \frac{\alpha_\Lambda}{3} \mathcal{P}^\Lambda_x, \quad (41)$$

$$\langle \sin\theta_2 \sin\phi_2 \rangle = \frac{\alpha_\Lambda}{3} \mathcal{P}^\Lambda_y. \quad (42)$$

Similar relations can be obtained for the $\bar{\Lambda}$ transverse polarization with the replacement of $(\theta_2, \phi_2) \to (\theta_3, \phi_3)$. The spin correlation between Λ and $\bar{\Lambda}$ can be related to the moments as follows

$$\langle \cos\theta_2 \cos\theta_3 \rangle = \frac{\alpha_\Lambda \alpha_{\bar{\Lambda}}}{9} \mathcal{P}_{zz}, \quad (43)$$

$$\langle \sin\theta_2 \sin\theta_3 \cos\phi_2 \cos\phi_3 \rangle = \frac{\alpha_\Lambda \alpha_{\bar{\Lambda}}}{9} \mathcal{P}_{xx}, \quad (44)$$

$$\langle \sin\theta_2 \sin\theta_3 \sin\phi_2 \sin\phi_3 \rangle = \frac{\alpha_\Lambda \alpha_{\bar{\Lambda}}}{9} \mathcal{P}_{yy}, \quad (45)$$

$$\langle \sin\theta_2 \sin\theta_3 \sin\phi_2 \cos\phi_3 \rangle = \frac{\alpha_\Lambda \alpha_{\bar{\Lambda}}}{9} \mathcal{P}_{yx}, \quad (46)$$

$$\langle \sin\theta_2 \sin\theta_3 \cos\phi_2 \sin\phi_3 \rangle = \frac{\alpha_\Lambda \alpha_{\bar{\Lambda}}}{9} \mathcal{P}_{xy}, \quad (47)$$

$$\langle \sin\theta_2 \cos\theta_3 \sin\phi_2 \rangle = \frac{\alpha_\Lambda \alpha_{\bar{\Lambda}}}{9} \mathcal{P}_{yz}, \quad (48)$$

$$\langle \sin\theta_2 \cos\theta_3 \cos\phi_2 \rangle = \frac{\alpha_\Lambda \alpha_{\bar{\Lambda}}}{9} \mathcal{P}_{xz}. \quad (49)$$

VI. APPLICATION

A. Measurement of $\alpha_\Lambda \alpha_{\bar{\Lambda}}$

The $\psi' \to \gamma\chi_{cJ}$ decays are an ideal laboratory to study the χ_{cJ} properties. These radiative transitions have a relatively large branching fractions, about 9.5–9.7%. Especially at e^+e^- colliders, the χ_{cJ} states are produced with a very clean background compared with other experiments, e.g., at the

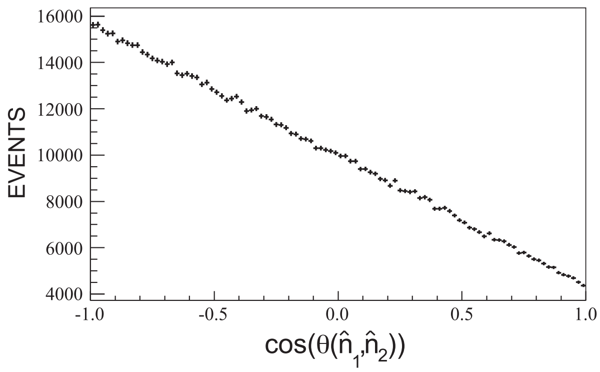

FIG. 2. Distribution of $\cos(\theta(\hat{n}_1, \hat{n}_2))$, where $\hat{n}_1(\hat{n}_2)$ is the unit momentum of proton (antiproton) defined in the $\Lambda(\bar{\Lambda})$ rest frame. In MC simulation, we set $\alpha_\Lambda = -\alpha_{\bar{\Lambda}} = 0.75$ [5].

hadron colliders. If there will be 3.2 billion ψ' events accumulated at BESIII [26], one will have about 300 million events for each χ_{cJ} state.

Efforts for the precision measurements of the Λ and $\bar{\Lambda}$ decay parameters are motivated by the search for evidence of CP violation in the baryon sector. Recently, the most precise measurement on α_Λ and $\alpha_{\bar{\Lambda}}$ came from the J/ψ decays to $\Lambda\bar{\Lambda}$ with a total uncertainty of about 1.6%. It gave a surprising result that deviates significantly from the PDG value [5] with a significance larger than 5σ. Measurement of these parameters in the χ_{cJ} decays to $\Lambda\bar{\Lambda}$ will provide an independent test to confirm the new results.

Decay of $\chi_{c0} \to \Lambda\bar{\Lambda}$ provides a simple and intuitive way to measure the product of $\alpha_\Lambda\alpha_{\bar{\Lambda}}$ parameters. Since the degrees of polarization are fully correlated in the $\Lambda\bar{\Lambda}$ pair in the x, y, z-directions, i.e., $\mathcal{P}_{xx} = \mathcal{P}_{yy} = \mathcal{P}_{zz} = 1$, the joint angular distribution for proton and antiproton is dependent on the $\alpha_\Lambda\alpha_{\bar{\Lambda}}$ product as given in Eq. (36). In this situation, only the product of the parameter, $\alpha_\Lambda\alpha_{\bar{\Lambda}}$, can be measured. If we assume the CP symmetry in the $\Lambda\bar{\Lambda}$ decays to the final state $p\bar{p}\pi^+\pi^-$, then we can get the Λ decay parameter as $\alpha_\Lambda = |\alpha_{\bar{\Lambda}}|$. In experiments, one can observe the distribution of $\hat{O} = \sin\theta_2 \sin\theta_3 \cos(\phi_2 - \phi_3) + \cos\theta_2 \cos\theta_3$, which should distribute as a line with slope equal to $\alpha_\Lambda\alpha_{\bar{\Lambda}}$ according Eq. (36). Actually, the observable $\hat{O}$ is equal to the cosine angle, $\cos(\theta(\hat{n}_1, \hat{n}_2))$, spanned by the unit momenta of proton ($\hat{n}_1$) and antiproton ($\hat{n}_2$), which are defined in the Λ and $\bar{\Lambda}$ rest frames, respectively. A Monte-Carlo (MC) simulation shows the $\cos(\theta(\hat{n}_1, \hat{n}_2))$ distribution as displayed in Fig. 2, and a linear fit to this distribution will yield the slope equal to $-\alpha_\Lambda^2$, with a sensitivity $\frac{\delta\alpha_\Lambda}{\alpha_\Lambda} \propto \frac{1}{\sqrt{N}}$, where N is the statistics of the MC events.

In contrast to the χ_{c0} decay, the χ_{c1} particle is tensor polarized and this leads to the Λ and $\bar{\Lambda}$ being transversely polarized in the $\chi_{c1} \to \Lambda\bar{\Lambda}$ decay. This is essential in the measurements of the decay asymmetry parameters for Λ and $\bar{\Lambda}$ simultaneously. From Eq. (34), one can see that the

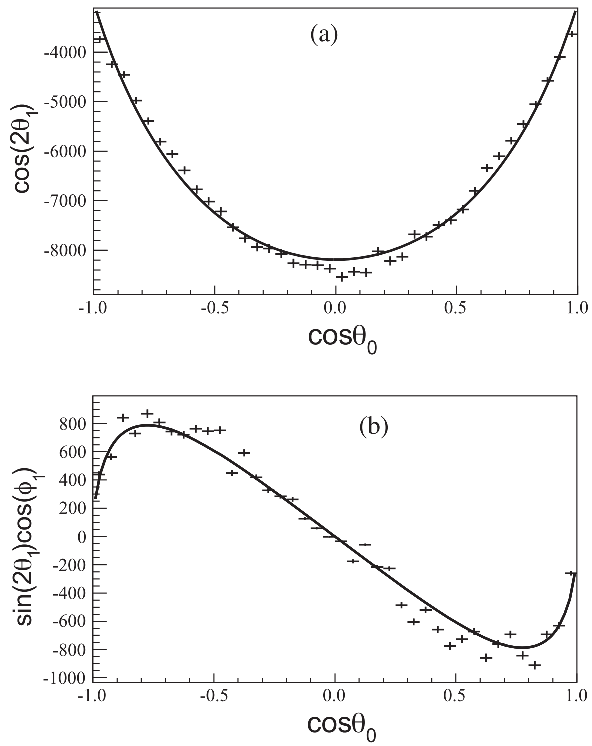

FIG. 3. Moments for showing χ_{c1} tensor polarizations. (a) $\cos(2\theta_1)$, (b) $\sin(2\theta_1)\cos(\phi_1)$. Here, the dots with error bars are MC events, and the curves are the expected tensor polarizations.

analyzing power of extraction of the $\alpha_\Lambda(\alpha_{\bar{\Lambda}})$ parameter is dependent on the factor $T_1(\bar{T}_1)$, which are related to the $\Lambda(\bar{\Lambda})$ transverse polarization, as given by Eq. (24). Observation of χ_{c1} transverse polarization in experiment is equivalent to displaying the first moments distribution of $\langle\cos(2\theta_1)\rangle$ for r_0^2 and $\langle\sin(2\theta_1)\cos(\phi_1)\rangle$ for r_1^2. We perform a MC simulation of χ_{c1} decay. Without loss of generality, we naively set the helicity amplitude of χ_{c1} decay as $b_{1/2,1/2} = b_{1/2,-1/2} = 1$. The two moments for illustration of the χ_{c1} tensor polarization are shown in Fig. 3. One can see that the MC events (dots with error bars) are well distributed according the χ_{c1} tensor polarization (curve) as given by Eq. (16).

To reveal the $\Lambda(\bar{\Lambda})$ transverse polarization, one can use the helicity angles θ_1 and ϕ_1 to form the moments as given by Eq. (38). It follows from Eq. (24) that the nonvanishing transverse polarization left is $\mathcal{P}_y$ in the overall 2π-azimuthal space, which lies along the normal to the $\chi_{c1} \to \Lambda\bar{\Lambda}$ decay plane. This is the consequence of the parity conservation in the decay. The moments of $\langle\sin\theta_2 \sin\phi_2\rangle$ and $\langle\sin\theta_3 \sin\phi_3\rangle$ are shown in Fig. 4; they show significantly the Λ and $\bar{\Lambda}$ transverse polarization of $\mathcal{P}_y$ component.

In experiments, extraction of the $\Lambda\bar{\Lambda}$ decay asymmetry parameters in the χ_{c1} decay can make use of the global experimental information by using the maximum likelihood method to fit the joint angular distribution to the

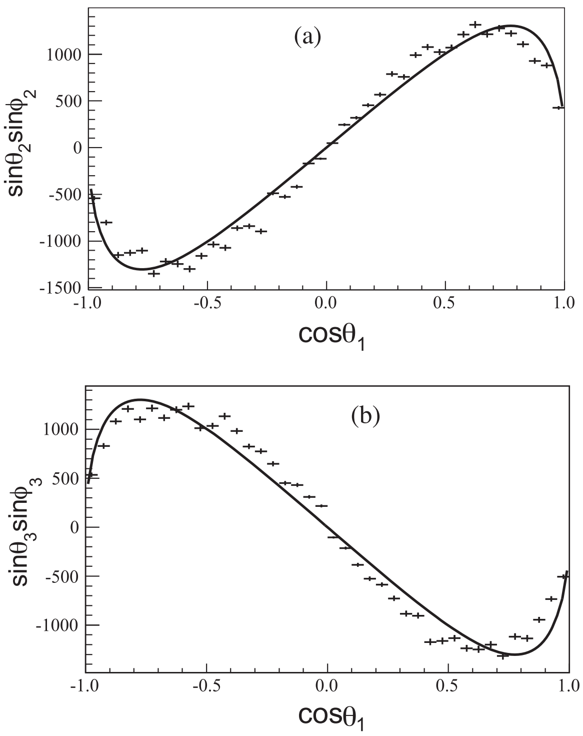

FIG. 4. Moments $\sin\theta_2 \sin\phi_2$ for showing Λ transverse polarizations $\mathcal{P}_y$ (a), and $\sin\theta_3 \sin\phi_3$ for $\bar{\Lambda}$ transverse polarization (b), where the dots with error bars are MC events, and the curves are the expected $\mathcal{P}_y$ distribution.

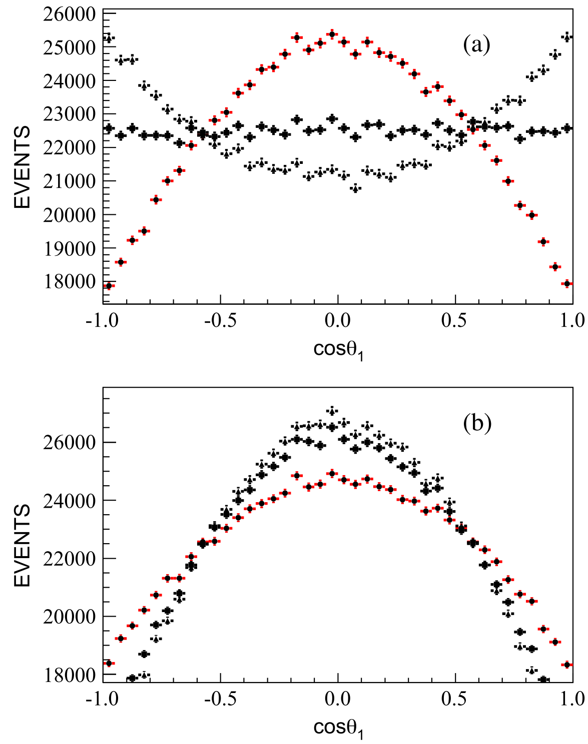

FIG. 5. Angular distributions of Λ particle in the χ_{c1} (a) and χ_{c2} (b) decays. In plots, the dots with full circle, square, and triangle markers represent the MC simulations with setting of ratio b_{++}/b_{+-} to 0, 0.7 and 1, respectively.

$p\bar{p}\pi^+\pi^-$ final states. The existence of the $\Lambda(\bar{\Lambda})$ transverse polarization allows to access the α_Λ and $\alpha_{\bar{\Lambda}}$ simultaneously. This method continues to hold for the χ_{c2} decay.

B. Measurement of helicity amplitudes

The helicity amplitudes are accessible by examining the helicity angular distribution of Λ or $\bar{\Lambda}$ particle in the χ_{cJ} decays. But this method works only for χ_{c1} and χ_{c2} decays, since χ_{c0} is a spin-0 particle, and its decay yields a flat angular distribution. For $\chi_{c1} \to \Lambda\bar{\Lambda}$ decay, the Λ angular distribution reads

$$\frac{dN}{d\cos\theta_1} \propto 1 + \alpha_1 \cos^2\theta_1, \tag{50}$$

with the angular distribution parameter $\alpha_1 = \frac{3r_0^2(b_{+-}^2 - 2b_{++}^2)}{2(1+r_0^2)b_{++}^2 + (2-r_0^2)b_{+-}^2}$, where the sign $+(-)$ is short for the helicity value $+\frac{1}{2}(-\frac{1}{2})$. The tensor polarization r_0^2 is well known for the $E1$ transition $\psi' \to \gamma\chi_{c1}$, thus the parameter α_1 is solely determined by the ratio of b_{++}/b_{+-}. In the viewpoint of perturbative QCD theory, the helicity b_{++} is suppressed due to the helicity conservation of strong decays. To show the dependence of angular distribution on the ratio b_{++}/b_{+-}, we perform a MC simulation by setting the amplitude ratio as 0, 0.7, and 1. As shown in Fig. 5, the angular distribution is sensitive to this ratio.

For $\chi_{c2} \to \Lambda\bar{\Lambda}$ decay, it receives contribution of rank-4 tensor polarization besides the r_0^2 component. So the angular distribution shows up the $\cos^4\theta_1$ term. It reads

$$\frac{dN}{d\cos\theta_1} \propto 1 + \alpha_2 \cos^2\theta_1 + \beta\cos^4\theta_1, \tag{51}$$

with $\alpha_2 = C_2/C_0$ and $\beta = C_4/C_0$, where

$$\begin{aligned} C_0 &= 224(1+x) + 16\sqrt{70}r_0^2(1+2x) + 24\sqrt{14}(3xr_0^4 - 2)r_0^4, \\ C_2 &= \sqrt{70}r_0^2(-48 - 96x) + 240\sqrt{14}r_0^4(2-3x), \\ C_4 &= 280\sqrt{14}r_0^4(3x-2), \end{aligned} \tag{52}$$

with $x = b_{++}^2/b_{+-}^2$. The tensor polarization r_0^2 and r_0^4 is well determined in the $E1$ transition, and then both of α_2 and β parameters are determined with the single parameter x. To check the dependence of these two parameters on the helicity ratio, we perform a MC simulation with a naive setting $\sqrt{x} = 0$, 0.7, and 1. From the pattern of angular distribution as shown in Fig. 5, one can see that either setting $\sqrt{s} = 0.7$ or 1, it yields almost the same pattern. The parameters α_2 and β are insensitive to the helicity ratio in $\chi_{c2} \to \Lambda\bar{\Lambda}$ decay.

VII. SUMMARY AND CONCLUSION

We perform an analysis of χ_{cJ} polarization in the ψ' radiative transition and propose to use it to probe the mechanism of χ_{cJ} decays to the baryon-antibaryon pairs in experiment. We have shown an interesting phenomenon that unpolarized e^+e^- collisions can yield the tensor polarized ψ' resonance, and further transfer the polarization to χ_{cJ} states. The analysis shows that the χ_{cJ} states are even-tensor polarized. The degrees of polarization are determined to be about 76% and 70% for χ_{c1} and χ_{c2}, respectively. In the decays $\chi_{cJ} \to \Lambda\bar{\Lambda}$, the baryons are transversely polarized along the direction normal to the χ_{cJ} decay plane, and they also get the correlated polarizations, which are essential to make the α_Λ and $\alpha_{\bar{\Lambda}}$ parameters accessible simultaneously. The experimental observables to reveal the χ_{cJ} and $\Lambda\bar{\Lambda}$ polarization are presented.

We performed a MC simulation to show that this polarization can be used to measure the $\Lambda\bar{\Lambda}$ decay asymmetry parameters $\alpha_\Lambda\alpha_{\bar{\Lambda}}$. In the χ_{c0} decay, this parameter product can be measured as the slope of the linear distribution of observable $\hat{O} = \sin\theta_2 \sin\theta_3 \cos(\phi_2 - \phi_3) + \cos\theta_2 \cos\theta_3$, with the precision comparable to that measured in the $J/\psi \to \Lambda\bar{\Lambda}$ with the same statistics. Using the tensor polarization in the χ_{c1} and χ_{c2} decays, the asymmetry parameters for the individual Λ and $\bar{\Lambda}$ particles can be measured by performing a maximum likelihood fit to the joint angular distribution of the data events. A simple way to measure the helicity amplitude ratio for the χ_{c1} and $\chi_{c2} \to \Lambda\bar{\Lambda}$ decays is also demonstrated with a MC simulation.

Although the analysis is performed based on the decays $\chi_{cJ} \to \Lambda\bar{\Lambda}$, all formalisms are applicable to the case in which the χ_{cJ} decays to the octet baryon antibaryon pairs, e.g., $\chi_{cJ} \to p\bar{p}$, $\Sigma\bar{\Sigma}$ and $\Xi\bar{\Xi}$.

ACKNOWLEDGMENTS

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APPENDIX: REAL MULTIPOLE PARAMETERS C_{ij} OF $\Lambda\bar{\Lambda}$ PAIRS IN THE $\chi_{c1}, \chi_{c2} \to \Lambda\bar{\Lambda}$ DECAY

For the decay $\chi_{c1} \to \Lambda\bar{\Lambda}$, the nonvanishing C_{ij} parameters are calculated to be

$$
\begin{aligned}
C_{00} &= \frac{r_0^0}{24}\{b^2_{\frac{1}{2},-\frac{1}{2}}[4\sqrt{3}r_1^2\sin(\theta_1)\cos(\theta_1)\cos(\phi_1) + 3r_0^2\cos(2\theta_1) + r_0^2 + 4] \\
&\quad - 2b^2_{\frac{1}{2},\frac{1}{2}}[4\sqrt{3}r_1^2\sin(\theta_1)\cos(\theta_1)\cos(\phi_1) + 3r_0^2\cos(2\theta_1) + r_0^2 - 2]\}, \\
C_{00}C_{11} &= \frac{r_0^0}{12}\{b^2_{\frac{1}{2},-\frac{1}{2}}[\sqrt{3}r_1^2\sin(2\theta_1)\cos(\phi_1) - 3r_0^2\sin^2(\theta_1)] \\
&\quad + b^2_{\frac{1}{2},\frac{1}{2}}[4\sqrt{3}r_1^2\sin(\theta_1)\cos(\theta_1)\cos(\phi_1) + 3r_0^2\cos(2\theta_1) + r_0^2 - 2]\}, \\
C_{00}C_{12} &= -C_{00}C_{21} = \frac{r_1^2 r_0^0 b^2_{\frac{1}{2},-\frac{1}{2}}\sin(\theta_1)\sin(\phi_1)}{2\sqrt{3}}, \\
C_{00}C_{13} &= C_{00}C_{31} = \frac{r_0^0 b_{\frac{1}{2},-\frac{1}{2}} b_{\frac{1}{2},\frac{1}{2}}\cos(\Delta_1)[\sqrt{3}r_1^2\cos(2\theta_1)\cos(\phi_1) - 3r_0^2\sin(\theta_1)\cos(\theta_1)]}{3\sqrt{2}}, \\
C_{00}C_{22} &= \frac{r_0^0}{12}\{b^2_{\frac{1}{2},-\frac{1}{2}}[\sqrt{3}r_1^2\sin(2\theta_1)\cos(\phi_1) - 3r_0^2\sin^2(\theta_1)] \\
&\quad - b^2_{\frac{1}{2},\frac{1}{2}}[4\sqrt{3}r_1^2\sin(\theta_1)\cos(\theta_1)\cos(\phi_1) + 3r_0^2\cos(2\theta_1) + r_0^2 - 2]\}, \\
C_{00}C_{23} &= -C_{00}C_{32} = -\frac{r_1^2 r_0^0 b_{\frac{1}{2},-\frac{1}{2}} b_{\frac{1}{2},\frac{1}{2}}\cos(\Delta_1)\cos(\theta_1)\sin(\phi_1)}{\sqrt{6}}, \\
C_{00}C_{33} &= -\frac{r_0^0}{24}\{b^2_{\frac{1}{2},-\frac{1}{2}}(4\sqrt{3}r_1^2\sin(\theta_1)\cos(\theta_1)\cos(\phi_1) + 3r_0^2\cos(2\theta_1) + r_0^2 + 4), \\
&\quad + 2b^2_{\frac{1}{2},\frac{1}{2}}[4\sqrt{3}r_1^2\sin(\theta_1)\cos(\theta_1)\cos(\phi_1) + 3r_0^2\cos(2\theta_1) + r_0^2 - 2]\},
\end{aligned} \tag{A1}
$$

where b_{λ_3,λ_4} is the modulus of the amplitude B_{λ_3,λ_4} for $\chi_{c1} \to \Lambda\bar{\Lambda}$, and Δ_1 is the phase angle difference between the amplitudes $B_{-1/2,1/2}$ and $B_{1/2,1/2}$.

For $\chi_{c2} \to \Lambda\bar{\Lambda}$, the unpolarized decay rate, C_{00}, is given by Eq. (26) with nonzero a_{ij} parameters as

$$
\begin{aligned}
a_{00} &= \frac{1}{10}(b^2_{\frac{1}{2},-\frac{1}{2}} + b^2_{\frac{1}{2},\frac{1}{2}}),\\
a_{20} &= -\frac{1}{4\sqrt{70}}(b^2_{\frac{1}{2},-\frac{1}{2}} + 2b^2_{\frac{1}{2},\frac{1}{2}})(3\cos(2\theta_1)+1),\\
a_{21} &= -\sqrt{\frac{3}{70}}(b^2_{\frac{1}{2},-\frac{1}{2}} + 2b^2_{\frac{1}{2},\frac{1}{2}})\sin(\theta_1)\cos(\theta_1)\cos(\phi_1),\\
a_{22} &= -\frac{1}{2}\sqrt{\frac{3}{70}}(b^2_{\frac{1}{2},-\frac{1}{2}} + 2b^2_{\frac{1}{2},\frac{1}{2}})\sin^2(\theta_1)\cos(2\phi_1),\\
a_{40} &= -\frac{2b^2_{\frac{1}{2},-\frac{1}{2}} - 3b^2_{\frac{1}{2},\frac{1}{2}}}{160\sqrt{14}}(20\cos(2\theta_1)+35\cos(4\theta_1)+9),\\
a_{41} &= -\frac{2b^2_{\frac{1}{2},-\frac{1}{2}} - 3b^2_{\frac{1}{2},\frac{1}{2}}}{16\sqrt{35}}(2\sin(2\theta_1)+7\sin(4\theta_1))\cos(\phi_1),\\
a_{42} &= -\frac{2b^2_{\frac{1}{2},-\frac{1}{2}} - 3b^2_{\frac{1}{2},\frac{1}{2}}}{4\sqrt{70}}\sin^2(\theta_1)(7\cos(2\theta_1)+5)\cos(2\phi_1).
\end{aligned}
\tag{A2}
$$

$$
\mathcal{P}^{\Lambda}_x = \frac{b_{\frac{1}{2},-\frac{1}{2}}b_{\frac{1}{2},\frac{1}{2}}r^0_0}{8\sqrt{35}C_{00}}\sin(\Delta_2)\sin(\phi_1)\{4\sin(\theta_1)\cos(\phi_1)[\sqrt{3}r^4_2(7\cos(2\theta_1)+5)-4r^2_2]+\cos(\theta_1)[\sqrt{6}r^4_1(7\cos(2\theta_1)+1)-8r^2_1]\},
\tag{A3}
$$

$$
\begin{aligned}
\mathcal{P}^{\Lambda}_y = {}& \frac{b_{\frac{1}{2},-\frac{1}{2}}b_{\frac{1}{2},\frac{1}{2}}r^0_0\sin(\Delta_2)}{40\sqrt{7}C_{00}}\{\sqrt{5}\cos(\phi_1)[\sqrt{6}r^4_1(\cos(2\theta_1)+7\cos(4\theta_1))-8r^2_1\cos(2\theta_1)]\\
&-2\sqrt{5}\sin(2\theta_1)\cos(2\phi_1)[\sqrt{3}r^4_2(1-7\cos(2\theta_1))+2r^2_2]+\sqrt{3}\sin(2\theta_1)[4\sqrt{5}r^2_0-5r^4_0(7\cos(2\theta_1)+1)]\}.
\end{aligned}
\tag{A4}
$$

To get a compact expression, we integrate out the angle ϕ_1, and then the nonvanishing C_{ij} parameters read

$$
\begin{aligned}
C_{00} = {}& \frac{\pi r^0_0}{1120}\{-2b^2_{\frac{1}{2},-\frac{1}{2}}[4\sqrt{70}r^2_0 3\cos(2\theta_1)+1)+20\sqrt{14}r^4_0\cos(2\theta_1)+35\sqrt{14}r^4_0\cos(4\theta_1)+9\sqrt{14}r^4_0-112]\\
&+\sqrt{14}b^2_{\frac{1}{2},\frac{1}{2}}(3r^4_0(20\cos(2\theta_1)+35\cos(4\theta_1)+9)-16\sqrt{5}r^2_0(3\cos(2\theta_1)+1))+224b^2_{\frac{1}{2},\frac{1}{2}}\},\\
C_{00}C_{02} = {}& -C_{00}C_{20} = \frac{1}{20}\sqrt{\frac{3}{7}}\pi b_{\frac{1}{2},-\frac{1}{2}}b_{\frac{1}{2},\frac{1}{2}}r^0_0\sin(\Delta_2)\sin(2\theta_1)[5r^4_0(7\cos(2\theta_1)+1)-4\sqrt{5}r^2_0],\\
C_{00}C_{11} = {}& \frac{\pi r^0_0}{1120}\{\sqrt{14}[b^2_{\frac{1}{2},\frac{1}{2}}(3r^4_0(20\cos(2\theta_1)+35\cos(4\theta_1)+9)-16\sqrt{5}r^2_0(3\cos(2\theta_1)+1))\\
&-8b^2_{\frac{1}{2},-\frac{1}{2}}\sin^2(\theta_1)(5r^4_0(7\cos(2\theta_1)+5)-6\sqrt{5}r^2_0)]+224b^2_{\frac{1}{2},\frac{1}{2}}\},\\
C_{00}C_{13} = {}& -C_{00}C_{31} = \frac{1}{20}\sqrt{\frac{3}{7}}\pi b_{\frac{1}{2},-\frac{1}{2}}b_{\frac{1}{2},\frac{1}{2}}r^0_0\cos(\Delta_2)\sin(2\theta_1)[4\sqrt{5}r^2_0-5r^4_0(7\cos(2\theta_1)+1)],\\
C_{00}C_{22} = {}& \frac{\pi r^0_0}{1120}\{\sqrt{14}[b^2_{\frac{1}{2},\frac{1}{2}}(16\sqrt{5}r^2_0(3\cos(2\theta_1)+1)-3r^4_0(20\cos(2\theta_1)+35\cos(4\theta_1)+9))\\
&-8b^2_{\frac{1}{2},-\frac{1}{2}}\sin^2(\theta_1)(5r^4_0(7\cos(2\theta_1)+5)-6\sqrt{5}r^2_0)]-224b^2_{\frac{1}{2},\frac{1}{2}}\},\\
C_{00}C_{33} = {}& \frac{\pi r^0_0}{1120}\{2b^2_{\frac{1}{2},-\frac{1}{2}}[4\sqrt{70}r^2_0(3\cos(2\theta_1)+1)+20\sqrt{14}r^4_0\cos(2\theta_1)+35\sqrt{14}r^4_0\cos(4\theta_1)+9\sqrt{14}r^4_0-112],\\
&+\sqrt{14}b^2_{\frac{1}{2},\frac{1}{2}}[3r^4_0(20\cos(2\theta_1)+35\cos(4\theta_1)+9)-16\sqrt{5}r^2_0(3\cos(2\theta_1)+1)]+224,b^2_{\frac{1}{2},\frac{1}{2}}\},
\end{aligned}
\tag{A5}
$$

where b_{λ_3,λ_4} is the modulus of the amplitude B_{λ_3,λ_4} for $\chi_{c2} \to \Lambda\bar{\Lambda}$, and Δ_2 is the phase angle difference between the amplitude $B_{-1/2,1/2}$ and $B_{1/2,1/2}$.

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