Anisotropic transport properties of a hadron resonance gas in a magnetic field

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An intense transient magnetic field is produced in high energy heavy-ion collisions mostly due to the spectator protons inside the two colliding nuclei. The magnetic field introduces anisotropy in the medium, and hence the isotropic scalar transport coefficients become anisotropic and split into multiple components. Here, we calculate the anisotropic transport coefficients' shear, bulk viscosity, and electrical conductivity, and the thermal diffusion coefficients for a multicomponent hadron resonance gas (HRG) model for a nonzero magnetic field by using the Boltzmann transport equation in a relaxation time approximation (RTA). The anisotropic transport coefficient component along the magnetic field remains unaffected by the magnetic field, while perpendicular dissipation is governed by the interplay of the collisional relaxation time and the magnetic time scale, which is inverse of the cyclotron frequency. We calculate the anisotropic transport coefficients as a function of temperature and magnetic field using the HRG model. The neutral hadrons are unaffected by the Lorentz force and do not contribute to the anisotropic transports, we estimate within the HRG model the relative contribution of isotropic and anisotropic transports as a function of magnetic field and temperature. We also give an estimation of these anisotropic transport coefficients for the hadronic gas at finite baryon chemical potential (μ_B).

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I. INTRODUCTION

In the initial stage of heavy ion collisions an intense transient magnetic field $eB \sim (1\text{–}10)m_\pi^2$ (for $\sqrt{s_{\rm NN}} =$ 200 GeV collisions) is expected to be produced [1–5]. Theoretically, it was also shown that the magnitude of the magnetic field almost linearly rises with center of mass energy collisions [2,3].

A general consensus is that the initial large magnetic field will decay quickly (within a few fm) and become so weak that its effect may be negligible in any bulk observables. However, the initial hot and dense phase of quark gluon plasma (QGP) and later time hadronic phase both have finite electrical conductivities, this finite conducting medium will definitely modify the decay of the magnetic field according to the laws of magnetohydrodynamics (MHD) [6–10] or through a transport simulation [11], a matter which is still under investigation [3,12,13]. Usually the transport coefficients such as shear, bulk viscosity, and electrical conductivity are taken as an input to dynamical models such as relativistic MHD. Hence, it is important to calculate these transport coefficients in presence of magnetic field. The calculation of transport coefficients in quark and hadronic matter in the presence of a magnetic field were carried out in recent Refs. [14–41], where shear viscosity [14–20], electrical conductivity [18–32], and bulk viscosity [33–37] were calculated in the presence of a magnetic field. The dynamics of heavy quark in the presence of a magnetic field within the framework of the Fokker-Planck equation was studied in [39,40]. In the present work, we carry out a similar investigation where we consider a multicomponent hadron resonance gas and evaluate the shear viscosity and electrical conductivity in the presence of a magnetic field. In principle, one can calculate these transport coefficients in the presence of a magnetic field by solving QCD on a space-time lattice, but due to the current computational limitation and some technical difficulties it is unlikely to obtain the accurate result of these quantities in the low-temperature regime. However, it is well known that the hadron resonance gas (HRG) model successfully reproduces lattice data just below the crossover temperature (T_c) [42], and it is expected that at much lower temperatures HRG, as an effective model, can be reliably used to calculate transport coefficients of hadronic matter. Since the magnetic field is nonzero in the hadronic phase it motivates us to calculate the transport coefficients in the presence of the magnetic field. In Refs. [43,44], thermodynamical properties of hadron resonance gas in the presence of the magnetic field has been investigated.

Here, we would like to mention that recently in Refs. [20,30,31] transport coefficients (electrical conductivity and shear viscosity) for a HRG were studied in the presence of the magnetic field using the relaxation time approximation. The relaxation time was obtained from the constant cross section of hadrons. One of the crucial differences between the present work and the previous work [20] is that we give a general framework of using projection tensors [45] consisting of magnetic and hydrodynamical tensor degrees of freedom along which the viscous correction to a single particle distribution function can be systematically expanded in a Chapman-Enskog (CE) series. This is unlike the heuristic basis [46] used in the previous works. Hence, the present formalism can be used to systematically derive second and higher order nonresistive MHD equations in the lines of Ref. [38] but using a general CE series expansion. Apart from this important technical difference, in the present study we have calculated all the transport coefficients, which are available in a Landau frame, i.e., shear viscosity, bulk viscosity, and baryon diffusion (as well as electrical conductivity) for hadronic matter. Additionally, we do not estimate the relaxation time from the hadronic size but rather treat this as a free parameter. In the present work, we have separately explored the contributions of neutral and electrical charged hadrons to shear viscosity, which might be important phenomenologically. Due to the Lorentz force, the transport coefficients for electrically charged hadrons becomes anisotropic, whereas, the neutral hadrons only contribute to the isotropic transport processes. We give some estimate of the relative contribution of such anisotropic and the isotropic transport coefficients within the HRG model for zero and nonzero μ_B.

The article is organized as follows: in Sec. II, we briefly discuss the thermodynamics of the HRG model. In Sec. III, we introduce the Boltzmann transport equation in relaxation time approximation and the ansatz for the off-equilibrium distribution function required to calculate the transport coefficients. In the same section we discuss the transport coefficients obtained from relaxation time approximation with and without the magnetic field. Next, in Sec. IV we discuss numerical results obtained for HRG. We give a summary of our work in Sec. V. At the end, detailed derivation of various transport coefficients are given in the Appendixes. Throughout the paper we use the natural unit, the four vectors are denoted by the greek indices and the three vectors are denoted by the latin indices unless stated otherwise.

II. FORMALISM

A. Thermodynamics

Here, we start with a brief discussion of the HRG model to define the thermodynamical quantities like entropy density s, enthalpy per particle h, etc., which are used for the calculations of different transport coefficients. All thermodynamic quantities are derived from the grand canonical partition function Z of the hadronic matter with volume V at temperature T and chemical potential of ith species μ_i:

$$\ln Z = V\sum_i \int \frac{d^3\vec{p}_i}{(2\pi)^3} g_i r_i \ln[1 + r_i e^{\beta(p_i^0-\mu_i)}], \tag{1}$$

where $\mu_i = B_i\mu_B$ with B_i as the baryon number of the hadronic species and μ_B as the baryon chemical potential. Note that g_i, $p_i^0 = \{\vec{p}_i^{\,2} + m_i^2\}^{1/2}$ are degeneracy factors and energy of the hadrons of species i with mass m_i; $r_i = \pm$ stands for fermion or bosons, respectively. The total degeneracy factor of a particular species of hadron is obtained as $g_i = g_i^s \times g_i^I$, where g_i^s, g_i^I are the spin and iso-spin degeneracy factors, respectively.

Once the partition function is defined, the thermodynamic quantities pressure (P), energy density (ϵ), and net baryon density (ρ) are calculated from the following standard definitions:

$$P = \frac{T}{V}\ln Z,$$
$$\epsilon = \frac{T^2}{V}\frac{\partial}{\partial T}\ln Z,$$
$$\rho = \frac{T}{V}\frac{\partial}{\partial \mu}\ln Z. \tag{2}$$

Using Eq. (2), we can further define the entropy density s and the enthalpy per particle h by using the relations

$$s = \sum_i (\epsilon + P - \mu_i\rho_i)/T,$$
$$h = (\epsilon + P)/\rho, \tag{3}$$

where ρ_i is the baryon density of hadron species i.

III. BOLTZMANN TRANSPORT EQUATION

The calculation of all the transport coefficients considered here are based on relaxation time approximation of the collision kernel of the Boltzmann equation, hence, it is worthwhile to discuss the method for the sake of completeness. The general form of the Boltzmann equation in the presence of external fields in the relaxation time approximation is given by [16,18,19,46],

$$p^\mu\partial_\mu f_i + qF^{\mu\nu}p_\nu\frac{\partial f_i}{\partial p^\mu} = -\frac{U\cdot p}{\tau_c}\delta f_i, \tag{4}$$

where $F^{\mu\nu}$ is the electromagnetic field strength tensor. For our case, only the magnetic field is present, hence $F^{\mu\nu} = -B^{\mu\nu}$ with $B^{\mu\nu} = \epsilon^{\mu\nu\rho\alpha}B_\rho U_\alpha$. Note that B is the magnetic field strength and b^μ is the unit four vector defined as

$b^\mu = \frac{B^\mu}{B}$. So, for a small deviation of the distribution function from the equilibrium, Eq. (4) can be written as follows:

$$p^\mu \partial_\mu f_{i0} = \left(-\frac{U\cdot p}{\tau_c}\right)\left[1 - \frac{qB\tau_c}{U\cdot p} b^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu}\right]\delta f_i. \quad (5)$$

The equilibrium distribution function for i^{th} hadron species is $f_{i0} = (e^{\beta(U\cdot p - \mu_i)} + r)^{-1}$, where $r = \pm 1$ depending on the statistics. In all proceeding calculations, hydrodynamic four-velocity u^μ is defined in the Landau frame such that $u_\nu T^{\mu\nu} = \sum_i \int d^3p\, p_i^\mu f_i = \epsilon u^\mu$, where $T^{\mu\nu}$ is the energy-momentum tensor, and ϵ is the energy density.

Here, we construct δf_i as a linear combination of the thermodynamic forces times appropriate tensorial coefficients so that δf_i turns out to be a Lorentz scalar,

$$\delta f_i = A_i X + B_i^\mu X_\mu + C_i^{\mu\nu} X_{\mu\nu}, \quad (6)$$

where $X_{\mu\nu\ldots}$ represents the thermodynamic forces. Replacing the above form of δf_i in the Boltzmann transport equation and comparing the coefficients of the thermodynamic forces, we get the unknown coefficients A_i, B_i^μ, and $C_i^{\mu\nu}$ in the expression for δf_i. Using the δf_i in the thermodynamic flows, we obtain the transport coefficients as discussed in detail in Appendix B.

Subsequently, the dissipative quantities like current density (J_D^μ), stress tensor $(\pi^{\mu\nu})$, bulk viscous pressure (Π), and particle diffusion current (n^μ) can be written as follows:

$$\begin{aligned} J_D^\mu &= \sigma^{\mu\nu} E_\nu \\ \pi^{\mu\nu} &= \eta^{\mu\nu\alpha\beta} V_{\alpha\beta} \\ \Pi &= \zeta^{\mu\nu}\partial_\mu u_\nu \\ n^\mu &= \kappa^{\mu\nu}\nabla_\nu(\mu/T), \end{aligned} \quad (7)$$

where the tensor coefficients $\sigma_{\mu\nu}$ is given in Eq. (A11) and the rest can be written as

$$\begin{aligned} \eta^{\mu\nu\alpha\beta} &= \frac{1}{15}\sum_i g_i \int \frac{d^3p_i(\vec{p}_i)^4}{(2\pi)^3 p_i^0} C_i^{(n)\mu\nu\alpha\beta} \\ \zeta^{\mu\nu} &= \frac{1}{3}\sum_i g_i \int \frac{d^3p_i(\vec{p}_i)^2}{(2\pi)^3 p_i^0} C_i^{(n)\mu\nu}, \\ \kappa^{\mu\nu} &= -\frac{1}{3}\sum_i^{\text{baryons}} g_i \int \frac{d^3p_i(\vec{p}_i)^2}{(2\pi)^3 p_i^0} K_i^{(n)\mu\nu}, \end{aligned} \quad (8)$$

where the coefficients $C^{(n)\mu\nu\alpha\beta}$, $C^{(n)\mu\nu}$ and $K^{(n)\mu\nu}$ are given in Eqs. (B6), (B26), and (B37), respectively. Note that here diffusion current refers to baryon diffusion, and hence the sum is over all baryons (antibaryons).

A. Transport coefficients without a magnetic field

After the short discussion on the thermodynamical quantities, we discuss here about the transport coefficients of a relativistic system of particles in the absence of any external magnetic fields. The electrical conductivity (σ), shear viscosity (η), bulk viscosity (ζ), and the diffusion coefficient (κ) for a HRG are given in terms of the temperature and the relaxation time of hadrons,

$$\begin{aligned} \sigma &= \sum_i g_i^s q_i^2 \frac{1}{3T}\int \frac{d^3p_i}{(2\pi)^3}\frac{|\vec{p}_i|^2}{(p_i^0)^2}\tau_c f_{i0}(1 - r_i f_{i0}) \\ \eta &= \sum_i \frac{g_i}{15T}\int \frac{d^3\vec{p}_i}{(2\pi)^3}\frac{|\vec{p}_i|^4}{(p_i^0)^2}\tau_c f_{i0}(1 - r_i f_{i0}) \\ \zeta &= \sum_i \frac{g_i}{T}\int \frac{d^3\vec{p}_i}{(2\pi)^3 (p_i^0)^2} Q_i^2 \tau_c f_{i0}(1 - r_i f_{i0}) \\ \kappa &= \sum_i \frac{g_i}{3h}\int \frac{d^3\vec{p}_i}{(2\pi)^3}\frac{|\vec{p}_i|^2}{(p_i^0)^2}\tau_c (h - p_i^0) f_{i0}(1 - r_i f_{i0}), \end{aligned} \quad (9)$$

where q_i stands for the electric charge of hadrons type i, τ_c is the relaxation time of hadrons, which is taken to be the same for all hadrons for the sake of simplicity. The Q_i is a function of the speed of sound along with other thermodynamic quantities, the details of which are given in Appendix B. The derivation of the transport coefficients given in Eq. (9) can be found in Refs. [47,48] as well as in Appendix B. Similar expressions can also be obtained in Kubo relation [49,50].

In the present article, we aim to calculate the transport coefficients of HRG in the presence of a magnetic field; the values of these coefficients without the magnetic fields are obtained by taking the limit of a vanishing magnetic field. The expression for the transport coefficients in the presence of magnetic fields are given in the next few subsections, and the corresponding detailed derivation for the same is given in Appendix B.

B. Electrical conductivity in a magnetic field

In the presence of a magnetic field, the transport coefficients involve another time scale, cyclotron time $\tau_{iB} = p_i^0/(eB)$, along with the usual relaxation time τ_c, which usually depends on the rate of contact collisions between the constituents. The index i refers to a type of hadronic species.

The nonzero Lorentz force, due to the magnetic fields, gives rise to an anisotropic transport phenomenon (as the force along the magnetic field is zero and nonzero in other directions). It is obvious that if the collision time τ_c is much smaller than the cyclotron time τ_{iB} the effect of the magnetic field is negligible, i.e., the system is almost isotropic when $\tau_c/\tau_{iB} \ll 1$, and it becomes anisotropic when $\tau_c/\tau_{iB} \sim 1$ or greater. We also note that along the magnetic field the Lorentz force does not work, so the parallel component of any transport coefficient (denoted by $\|$) remains the same as without the magnetic field, given in Eq. (9). Here, we need a little bit more clarification. In linear theory, any thermodynamic fluxes are proportional to the corresponding thermodynamic forces, and the proportionality constants are known as transport coefficients. If the system is isotropic, the transport coefficients are scalar, but for an anisotropic medium, the transport coefficients are components of a tensor. The decompositions of the transport coefficient tensor in terms of the available basis ($u^\mu, g^{\mu\nu}, b^\mu, b^{\mu\nu}$) are not unique, and we choose here a particular combination such that the decomposition has a component parallel to the magnetic field, which is denoted with a subscript $\|$. Whereas, the remaining components can have two or more components usually denoted with a subscript $\perp$ and $\times$. The $\times$-component is basically a Hall component, which was absent for $B = 0$ while $\perp$-component at $B = 0$ will still exist, and it will be exactly equal to $\|$-component, which restores the isotropic property of the medium at $B = 0$. For electrical conductivity, the expressions of parallel ($\sigma_\|$), perpendicular ($\sigma_\perp$), and cross ($\sigma_\times$) components for hadron resonance gas are given below:

$$\sigma_\| = \sum_i g_i^s q_i^2 \frac{\beta}{3} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^2}{(p_i^0)^2} \tau_c f_{i0}(1 - r_i f_{i0}),$$
$$\sigma_\perp = \sum_i g_i q_i^2 \frac{\beta}{3} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^2}{(p_i^0)^2} \frac{\tau_c}{1 + (\tau_c/\tau_{iB})^2} f_{i0}(1 - r_i f_{i0}),$$
$$\sigma_\times = \sum_i g_i q_i^2 \frac{\beta}{3} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^2}{(p_i^0)^2} \frac{(\tau_c^2/\tau_{iB})}{1 + (\tau_c/\tau_{iB})^2} f_{i0}(1 - r_i f_{i0}). \tag{10}$$

As mentioned earlier, the detailed derivation is given in Appendix A. To compare our results for electrical conductivities (10) to some of the earlier findings [18,19,21] where the conductivities are denoted with $\sigma_{0,1,2}$, we found the following relations hold:

$$\sigma_\| = \sigma_0 + \sigma_2,$$
$$\sigma_\perp = \sigma_0,$$
$$\sigma_\times = \sigma_1. \tag{11}$$

C. Shear viscosity in a magnetic field

The most general form of the δf_i in the presence of a magnetic field where only shear stress is present is given by

$$\delta f_i = \sum_{n=0}^{4} c_n C^{(n)}_{\mu\nu\alpha\beta} p_i^\mu p_i^\nu V^{\alpha\beta} \tag{12}$$

$$= [c_0 P^0_{\langle\mu\nu\rangle\alpha\beta} + c_1(P^1_{\langle\mu\nu\rangle\alpha\beta} + P^{-1}_{\langle\mu\nu\rangle\alpha\beta}) + ic_2(P^1_{\langle\mu\nu\rangle\alpha\beta} - P^{-1}_{\langle\mu\nu\rangle\alpha\beta}) + c_3(P^2_{\langle\mu\nu\rangle\alpha\beta} + P^{-2}_{\langle\mu\nu\rangle\alpha\beta}) + ic_4(P^2_{\langle\mu\nu\rangle\alpha\beta} - P^{-2}_{\langle\mu\nu\rangle\alpha\beta})] p_i^\mu p_i^\nu V^{\alpha\beta}, \tag{13}$$

where $V_{\alpha\beta} = \frac{1}{2}(\frac{\partial U_\alpha}{\partial x_\beta} + \frac{\partial U_\beta}{\partial x_\alpha})$; the form of projectors $P^n_{\langle\mu\nu\rangle\alpha\beta}$ will be given in Appendix B for $n = -2, -1, 0, 1, 2$. Using this expression for δf_i, the shear viscous coefficients turnout to be

$$\eta_\| = \sum_i \frac{g_i}{15T} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^4}{p_{i0}^2} \tau_c f_{i0}(1 - r_i f_{i0})$$
$$\eta_\perp = \sum_i \frac{g_i}{15T} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^4}{p_{i0}^2} \frac{\tau_c}{1 + (\tau_c/\tau_{iB})^2} f_{i0}(1 - r_i f_{i0})$$
$$\eta'_\perp = \sum_i \frac{g_i}{15T} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^4}{p_{i0}^2} \frac{\tau_c}{1 + (2\tau_c/\tau_{iB})^2} f_{i0}(1 - r_i f_{i0})$$
$$\eta_\times = \sum_i \frac{g_i}{15T} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^4}{p_{i0}^2} \frac{\tau_c^2/\tau_{iB}}{1 + (\tau_c/\tau_{iB})^2} f_{i0}(1 - r_i f_{i0})$$
$$\eta'_\times = \sum_i \frac{g_i}{15T} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^4}{p_{i0}^2} \frac{\tau_c^2/\tau_{iB}}{\frac{1}{2} + 2(\tau_c/\tau_{iB})^2} f_{i0}(1 - r_i f_{i0}). \tag{14}$$

The coefficients $\eta_\|$, $\eta_\perp$, $\eta'_\perp$ are even functions of magnetic field B. The two coefficients $\eta_\times$, $\eta'_\times$ may have either sign, and they are odd functions of B. The later two coefficients are also called transverse viscosity coefficients [45]. We note the expressions for shear viscosities given in Eq. (14) are identical to those given in Refs. [16,18,19,46].

D. Bulk viscosity in a magnetic field

Similarly, for bulk viscosity we restrict ourselves to only the divergence of the fluid four velocity and neglect the other thermodynamic forces,

$$\delta f_i = \sum_{n=1}^{3} c_n C_n^{\mu\nu} \partial_\mu U_\nu. \tag{15}$$

Using this δfi, the bulk viscous coefficients turn out to be

$$\zeta_\| = \zeta_\perp = \sum_i \frac{g_i \tau_c}{T} \int \frac{d^3\vec{p}_i}{(2\pi)^3 p_{i0}^2} Q_i^2 f_{i0}(1 - r_i f_{i0}), \tag{16}$$

$$\zeta_\times = 0. \tag{17}$$

The bulk viscous coefficients remain unchanged under the influence of the magnetic field as was also shown in Ref. [38] using Grad's 14 moment approximation. The detailed derivation of Eq. (17) is given in Appendix B.

E. Net baryon diffusion coefficient in a magnetic field

For the case of diffusion, we keep only the term containing the spacial derivative of μ/T in the expression for δf_i,

$$\delta f_i = K^{\mu\nu} p_{i\mu} \partial_\nu (\mu_i/T). \tag{18}$$

Using this δf_i the diffusion coefficients turn out to be

$$\begin{aligned}
\kappa_{\parallel} &= \sum_i^{\text{baryons}} \frac{g_i}{3h} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^2}{(p_i^0)^2} \tau_c (h - B_i p_{i0}) f_{i0}(1 - r_i f_{i0}) \\
\kappa_{\perp} &= \sum_i^{\text{baryons}} \frac{g_i}{3h} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^2}{(p_i^0)^2} \frac{\tau_c (h - B_i p_{i0})}{1 + (\frac{\tau_c}{\tau_{iB}})^2} f_{i0}(1 - r_i f_{i0}) \\
\kappa_{\times} &= \sum_i^{\text{baryons}} \frac{g_i}{3h} \int \frac{d^3\vec{p}_i}{(2\pi)^3} \frac{|\vec{p}_i|^2}{(p_i^0)^2} \frac{\tau_c (\frac{\tau_c}{\tau_{iB}})(h - B_i p_{i0})}{1 + (\frac{\tau_c}{\tau_{iB}})^2} f_{i0}(1 - r_i f_{i0}),
\end{aligned} \tag{19}$$

where h is the enthalpy density as defined in Eq. (3), and the sum runs over baryons only. Due to the anisotropy induced by the magnetic field, we have three diffusion coefficients. Here again, the details can be found in Appendix B.

IV. RESULTS

In the formalism section, we have summarized the analytic expressions for the anisotropic components of the shear viscosity, bulk viscosity, thermal diffusion, and the electrical conductivity for a finite magnetic field. In this section, we will explore the temperature and magnetic field dependence of these transport coefficients for HRG model calculations.

Before discussing the results for HRG with physical masses of hadrons, let us first consider the simpler massless case for QGP. Here, we also compare the result obtained from our numerical implementation of the HRG model to that of a lattice QCD (LQCD) result for a sanity check. In the massless limit (also known as the Stefan-Boltzmann (SB) limit) the thermodynamical quantities like pressure (P) and energy density (ϵ), varies as T^4, and the entropy density (s) varies as T^3, more explicitly,

$$\begin{aligned}
P_{SB} &= g \frac{\zeta(4)}{\pi^2} T^4, \\
\epsilon_{SB} &= g \frac{3\zeta(4)}{\pi^2} T^4, \\
s_{SB} &= g \frac{4\zeta(4)}{\pi^2} T^3,
\end{aligned} \tag{20}$$

where $\zeta(4)$ stands for zeta function. Here, the subscript SB stands for the SB limit. In this limit the interaction measure

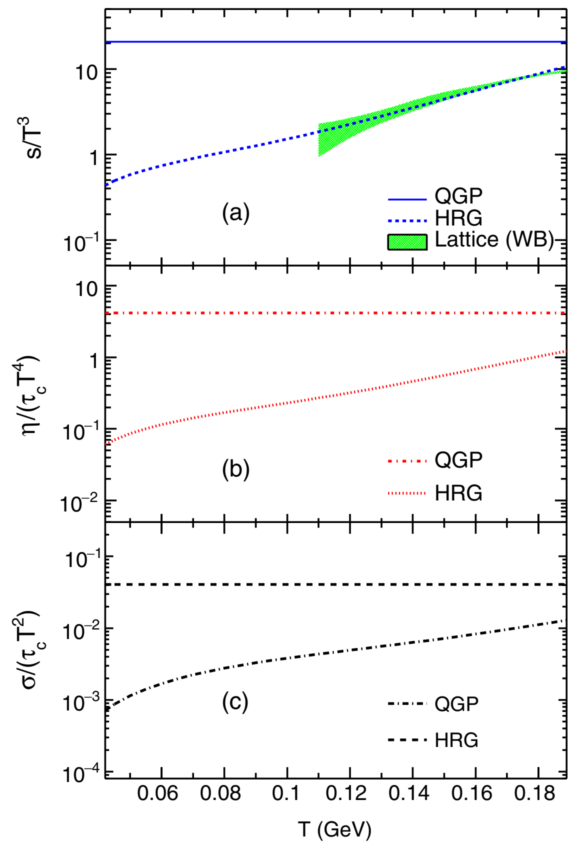

FIG. 1. (a) Normalized entropy density s/T^3, (b) shear viscosity $\eta/(\tau_c T^4)$, and (c) electrical conductivity $\sigma/(\tau_c T^2)$ as functions of T for massless QGP (horizontal lines) and HRG.

$(\epsilon - 3P)/T^4$ becomes zero, and we consider the HRG to be a noninteracting gas. It is clear that in the SB limit $P/T^4, \epsilon/T^4$, and s/T^3 are constants for a given degeneracy. For example, a 3 flavor quark gluon plasma with the degeneracy factor $g = 16 + \frac{7}{8}(24 + 12) = 47.5$ yields $P/T^4 = 5.2$, $\epsilon/T^4 = 15.6$, and $s/T^3 = 20.8$. However, for the physical masses of hadrons all these thermodynamic quantities have a smaller value than their corresponding SB values and approach SB values from below as $m/T \to \infty$. This is shown in the top panel of Fig. 1(a) for the normalized entropy density where the result obtained from the HRG model is shown by the blue dotted line; the corresponding s_{SB}/T^3 is shown by the blue horizontal solid line. For comparison, we also show the LQCD (shown by the green band) result from Ref. [51] in the temperature range of 120–180 MeV. It is clear from Fig. 1(a) that the normalized entropy density obtained from the Lattice QCD calculation and HRG matches very well in the temperature range considered here, also both results approach SB values as temperature increases.

Now, let us discuss the shear viscosity and the electrical conductivity of a massless gas without any magnetic field as given in Eq. (9). In the massless limit the corresponding expressions are [19]

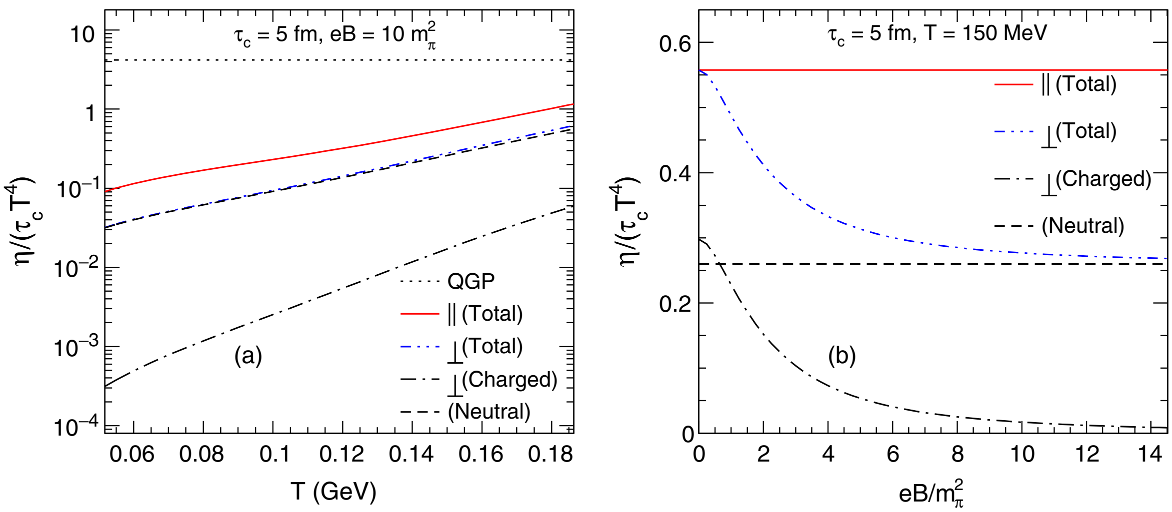

FIG. 2. The anisotropic component of the shear viscosities $\eta_\perp, \eta_\parallel$ for HRG and isotropic value for massless QGP are plotted against the axes of (a) temperature (T) of the medium and (b) external magnetic field (B).

$$\eta = g\frac{4\zeta(4)}{5\pi^2}\tau_c T^4,$$
$$\sigma = g_q q^2\frac{\zeta(2)}{3\pi^2}\tau_c T^2, \tag{21}$$

where $g_q q^2 = 12 \times (\frac{4e^2}{9} + \frac{e^2}{9} + \frac{e^2}{9}) = 8e^2$ for 3 flavor QGP. We note that similar to the thermodynamic quantities in the SB limit the normalized shear viscosity and electrical conductivity $\eta/(\tau_c T^4)$ and $\sigma/(\tau_c T^2)$ are constants. These normalized SB values $\eta_{SB}/(\tau_c T^4)$ and $\sigma_{SB}/(\tau_c T^2)$ are shown by the red dash-dotted and black dash horizontal lines in Figs. 1(b) and 1(c), respectively. For a HRG, both $\eta/(\tau_c T^4)$ and $\sigma/(\tau_c T^2)$ have smaller values compared to their corresponding SB values and approach SB values from below in the large temperature limit as shown by the red dot and black dash-dotted lines in Figs. 1(b) and 1(c), respectively.

The striking similarity between the temperature dependence of thermodynamic quantity s/T^3 and the transport coefficients like $\eta/(\tau_c T^4)$, $\sigma/(\tau_c T^2)$, clearly shows that we may gain information about the degrees of freedom of the system under consideration. Alternatively, we might get information about relaxation time τ_c if the temperature dependence of η and σ are known from other means.

Next, we explore the role of B and T on shear viscosity as shown in Fig. (2). For reference, we have also shown the values of $\eta/(\tau_c T^4)$ for a massless QGP (black dotted line) and that of HRG with $B = 0$ (shown by the red solid line). The $\eta_\perp/\tau_c T^4$ of charged hadrons for $eB = 10m_\pi^2$ and $\tau_c =$ 5 fm is shown by the dash-dotted line in Fig. 2(a). Since HRG is composed of both charged and neutral hadrons, it is interesting to study the relative contribution of the charged and uncharged hadrons to the total shear viscosity. Neutral hadrons only contribute to isotropic shear viscosity since, for neutral hadrons, η has a single component, which is essentially $\eta = \eta_\parallel$. It is clear from Fig. 2(a) that the anisotropic shear viscous coefficients from the charged hadrons contribution is quite smaller than that of the isotropic shear viscosity, which also contains contributions from the neutral hadrons. However, the above fact is only true for large magnetic fields [in Fig. 2(a) $B = 10m_\pi^2$]. For smaller magnetic fields, the $\eta_\perp/\tau_c T^4$ becomes comparable or even larger than the isotropic $\eta/\tau_c T^4$ as shown in Fig. 2(b). The $\parallel$ (red solid line) and $\perp$ (blue dash-double-dotted line) components of shear viscosity are plotted against B-axis in Fig. 2(b). The neutral hadrons contribution, which is independent of B, is shown by a dashed line while the charged hadrons contribution is shown by a dash-dotted line. The blue dash-double-dotted line is basically a summation of the dash (neutral hadrons) and dash-dotted (charge hadrons) lines. To get some numerical estimate, we note that for $B = 0$ the charged hadron contribution in the viscosity is more than 50% than the neutral hadrons. As B increases, the charge hadron contribution decreases and for $eB \geq 10m_\pi^2$, this contribution reduces to ~4%–8%.

Let us now consider the electrical conductivity, where gluons in the QGP phase and the neutral hadrons in the HRG phase play no role due to the charge neutrality. The results for the electrical conductivity as a function of T and B are plotted in Figs. 3(a) and 3(b). For comparison, here also we show the massless SB limit for QGP (horizontal black dotted line) and HRG (red solid line) for $B = 0$. We found that the T and B dependence of the electrical conductivity and the shear viscosity are very similar in nature. They mostly differ due to the different contribution from the neutral hadrons. For example, the neutral hadrons do not contribute to the electrical conductivity but play a role in the transport phenomenon related to the shear viscosity. At this point, we would like to add a few comments: (i) we note that both $\eta/(\tau_c T^4)$ and $\sigma/(\tau_c T^2)$

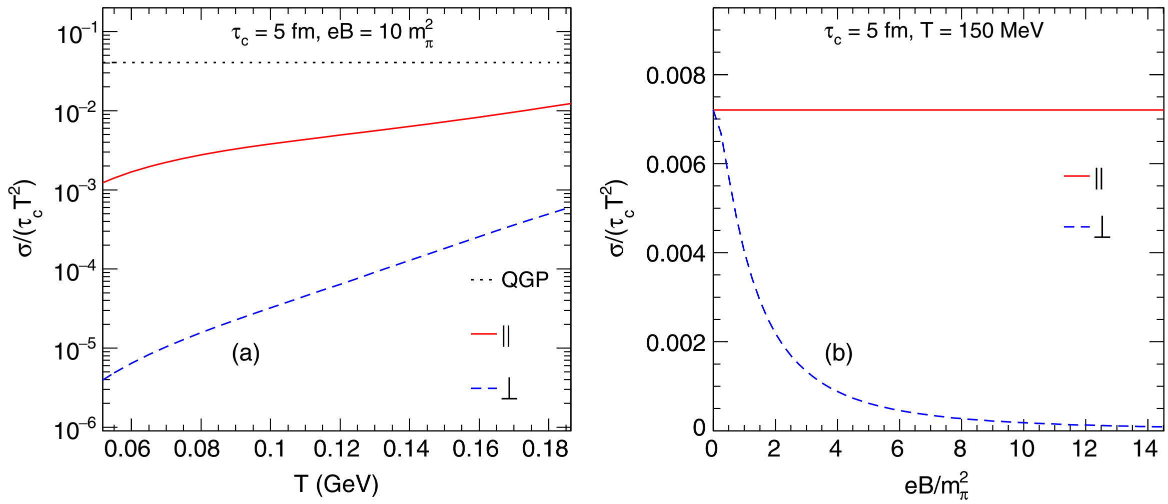

FIG. 3. Anisotropic component of the electrical conductivity ($\sigma_\perp$) for $eB = 10m_\pi^2$ and its isotropic value ($\sigma_\parallel$) for $B = 0$ are plotted as functions of (a)temperature (T) and (b) the external magnetic field (B).

have the largest values for massless QGP, (ii) in the presence of the magnetic field, the transport coefficient becomes anisotropic and among the various components the $\parallel$ component is the largest and equal to the corresponding isotropic value of the transport coefficient (i.e., for $B = 0$), and (iii) there is a small difference in the temperature dependence of the isotropic and the anisotropic transport coefficients.

Finally, we discuss the diffusion coefficient κ. Similar to the electrical conductivity in the presence of a magnetic field, the thermal diffusion coefficient also has three components: $\kappa_\parallel$, $\kappa_\perp$, and $\kappa_\times$. As usual, the $\kappa_\parallel$ by construction is independent of the magnetic field, but $\kappa_\perp$ and $\kappa_\times$ are functions of the magnetic field. In Fig. 4, we show the diffusion coefficients as a function of temperature for $B = 10m_\pi^2$ and $\mu_B = 300$. From Fig. 4, we see that $\kappa_\perp$ and $\kappa_\times$ are always smaller than $\kappa_\parallel$ for the temperature range considered here. A nonzero Hall diffusion coefficient $\kappa_\times$ can be attributed to the nonzero μ_B because for finite μ_B the particles and the antiparticles flow, due to the Hall effect, do not cancel out. Similarly, one can get nonzero Hall shear viscosities $\eta_\times$, $\eta'_\times$ and the Hall electrical conductivity for nonvanishing μ_B. All of these Hall-like transport coefficients vanish for a net-baryon free medium because the contribution from the particles and the antiparticles are exactly equal and opposite. Figure 5 demonstrate this μ_B dependent Hall viscosity ($\eta_\times$), Hall conductivity ($\sigma_\times$), and the Hall diffusion ($\kappa_\times$) for $T = 150$ MeV, $eB = 10m_\pi^2$, and $\tau_c = 5$ fm. It is clearly seen that both $\eta_\times$ (black dashed line), $\sigma_\times$ (blue dash-dotted line), and $\kappa_\times$ (blue dash-double-dotted line) increase

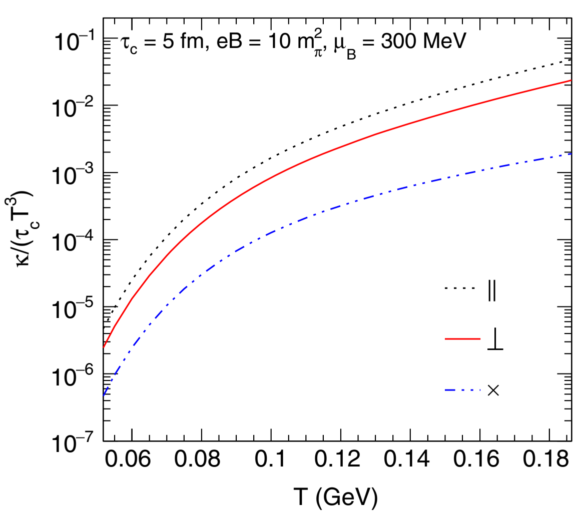

FIG. 4. Temperature dependence of the diffusion coefficients $\kappa_{\parallel,\perp,\times}$ in the presence of the magnetic field.

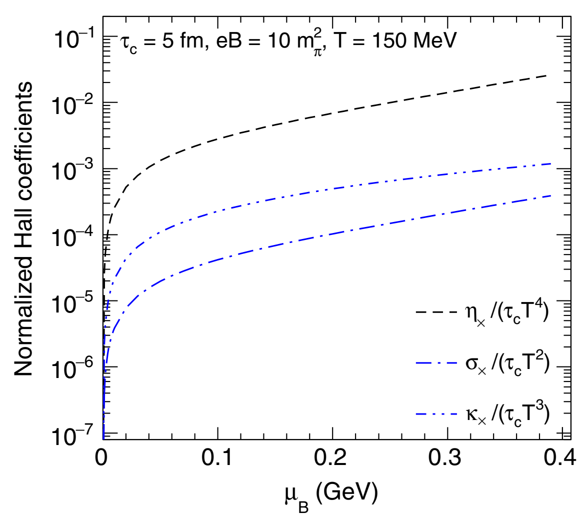

FIG. 5. Baryon chemical potential (μ_B) dependence of (normalized) Hall viscosity ($\eta_\times$) [black dashed line], conductivity ($\sigma_\times$)[blue dash-dotted line], and the diffusion coefficients $\kappa_\times$ (blue dash-double-dotted line).

monotonically from zero at $\mu_B = 0$. The growing tendency can be understood from the μ_B dependent of the net baryon density of the HRG system, which is roughly proportional to $\sinh(\mu/T)$ for the Maxwell-Boltzmann distribution, which at high temperatures fairly well describes the Fermi-Dirac or Bose-Einstein distribution functions.

The present methodology is semiclassical (as we consider the quantum statistical distribution function) in nature and does not include the Landau quantization- a quantum aspect, which is visible in the strong magnetic field. This effect is separately addressed in Ref. [52], but the complete understanding is still missing and we need further theoretical research in this direction. The physics of the anisotropic dissipation of the relativistic fluid in a magnetic field is also applicable for nonrelativistic fluid, such as different condensed matter and biological systems.

V. SUMMARY

In high energy heavy-ion collisions, large transient magnetic fields are produced predominantly in the perpendicular direction to the reaction plane. This magnetic field breaks the isotropy of the system and, as a result, the transport coefficients become anisotropic. We evaluate the anisotropic transport coefficients of the HRG and massless QGP by using the relaxation time approximation method. We use a unique tensorial decomposition of the anisotropic thermodynamic forces, which reduces the computational complexity for evaluating anisotropic transport coefficients. Along with the usual relaxation time, which appears in the collision kernel of the Boltzmann equation and controls the rate of reaching equilibrium for systems that are initially away from the equilibrium in magnetic fields, we have another timescale equal to the inverse of the cyclotron frequency. The measure of anisotropy turned out to be a function of the ratio of these two time scales. It is not surprising that we found the anisotropy increases with the magnetic field, and due to the specific choice of tensorial decomposition the $\parallel$ components of the anisotropic transport coefficients turned out to be the same with the isotropic case (i.e., for $B = 0$). We estimate the relative contribution of electrically charged and neutral hadrons to the various transport coefficients using the HRG model. Since the neutral hadrons are unaffected by the Lorentz force, they do not contribute in the anisotropic transport phenomenon. We have shown that the charged hadron contribution in the viscosity is more than 50% than the neutral hadrons. As B increases, the charged hadron contribution decreases and for $eB \geq 10m_\pi^2$ this contribution reduces to 4%–8%. In case of diffusion constant, we need to consider a medium with finite μ_B. In this study we show the result for $\mu_B = 300$ MeV. We also find that nondissipative Hall-like shear viscosity and conductivity increases monotonically with μ_B from zero at $\mu_B = 0$. It turned out that there are three diffusion coefficients in nonzero magnetic fields and among them the $\parallel$ component is the largest one. It is interesting to note that in calculating the diffusion coefficients we do not explicitly take into account the electric charge of the hadrons, but we observe the anisotropic diffusion coefficients due to the imbalance of particle and antiparticle numbers. We also sketch chemical potential dependence of Hall transport coefficients- how they grow from their vanishing values for (net) baryon free matter? These anisotropic pictures of dissipations might have a broad implication in other research fields where relevant impositions of systems might have to be considered.

ACKNOWLEDGMENTS

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APPENDIX A: ELECTRICAL CONDUCTIVITY IN THE PRESENCE OF A MAGNETIC FIELD

Electrical conductivity in the absence of the magnetic field for a quasiparticle system having degeneracy g, electric charge q, and four momentum $p^\mu \equiv (p^0, \vec{p})$ is [21,46,50],

$$\sigma = gq^2 \frac{\beta}{3} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{p^2}{(p^0)^2} \tau_c f_0 (1 - r f_0), \qquad \text{(A1)}$$

where $r = \pm$ stands for the fermion/boson, and τ_c is the thermal relaxation time. In this section we are dealing with only one hadron species.

For deriving the expression of the electrical conductivity in the presence of a magnetic field, let us start with Ohm's law,

$$J^i = \sigma^{ij} E_j. \qquad \text{(A2)}$$

Here, $J^i = J_0^i + J_D^i$ with J_0^i, J_D^i are the ideal and the dissipative parts of the three electric current density, respectively. Note that σ^{ij} is the electrical conductivity tensor, E^j's are the electric field components in the jth direction, and i, j runs from 1 to 3.

Now, the dissipative part of the current density according to the microscopic definition can be expressed as

$$J_D^i = gq \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{p^i}{p^0} \delta f. \qquad \text{(A3)}$$

Here, δf is a deviation of the distribution function f from its equilibrium part $f_0 = \frac{1}{e^{\beta(p^0-\mu)} + r}$.

Comparing the Ohm's law and the microscopic definition of the dissipative current density we get

$$\sigma^{ij}E_j = J^i_D = gq\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{p^i}{p^0}\delta f. \tag{A4}$$

To find the δf, we use relativistic Boltzmann equation (RBE) [18,19,21,46],

$$\frac{\partial f}{\partial t} + \frac{p^j}{p^0}\frac{\partial f}{\partial x^j} + \frac{dp_j}{dt}\frac{\partial f}{\partial p^j} = I[\delta f], \tag{A5}$$

where $I[\delta f]$ is the linearized collision integral. Use of the relaxation time approximation (RTA) corresponds to $I[\delta f] = -\frac{\delta f}{\tau_c}$, and we also note that the term $\frac{dp_j}{dt}$ on the lhs of the above equation represents the force due to the electric $\vec{E}$ and the magnetic field $\vec{B}$. So, Eq. (A5) can be written as (assuming vanishing $\frac{\partial f}{\partial t}$ and $\frac{\partial f}{\partial x^j}$)

$$\begin{aligned}
-q\left(\vec{E} + \frac{\vec{p}}{p^0}\times\vec{B}\right)\frac{\partial f}{\partial \vec{p}} &= -\frac{\delta f}{\tau_c},\\
\Rightarrow q\vec{E}\frac{\partial f}{\partial \vec{p}} + \left(\frac{\vec{p}}{p^0}\times\vec{B}\right)\frac{\partial f}{\partial \vec{p}} &= \frac{\delta f}{\tau_c},\\
\Rightarrow q\vec{E}\frac{\vec{p}}{p^0}\frac{\partial f_0}{\partial p^0} + \left(\frac{\vec{p}}{p^0}\times\vec{B}\right)\frac{\partial(\delta f)}{\partial \vec{p}} &= \frac{\delta f}{\tau_c}.
\end{aligned} \tag{A6}$$

Since the second term of lhs is $(\frac{\vec{p}}{p^0}\times\vec{B})\frac{\vec{p}}{p^0}\frac{\partial f_0}{\partial p^0} = 0$, so we have considered the δf term.

Now, we assume $\delta f = -\phi\frac{\partial f_0}{\partial p^0}$ where $\phi = \vec{p}\cdot\vec{F}$ with $\vec{F} = (l\hat{e} + m\hat{b} + n(\hat{e}\times\hat{b})$ where $\hat{e}$ and $\hat{b}$ are unit vectors along $\vec{E}$ and $\vec{B}$.

So, Eq. (A6) becomes

$$\begin{aligned}
&\frac{1}{p^0}[-qE\hat{e} + qB\hat{b}\times(l\hat{e} + m\hat{b} + n(\hat{e}\times\hat{b}))]\\
&\quad = (l\hat{e} + m\hat{b} + n(\hat{e}\times\hat{b}))/\tau_c.
\end{aligned} \tag{A7}$$

Now, comparing coefficients of $\hat{e}$, $\hat{b}$, and $(\hat{e}\times\hat{b})$ and solving for l, m, and n we get

$$\begin{aligned}
l &= \left(\frac{-qE\tau_c}{p^0}\right)\frac{1}{1+(\tau_c/\tau_B)^2}\\
m &= \left(\frac{-qE\tau_c}{p^0}\right)\frac{(\tau_c/\tau_B)^2}{1+(\tau_c/\tau_B)^2}(\hat{e}\cdot\hat{b})\\
n &= \left(\frac{-qE\tau_c}{p^0}\right)\frac{(\tau_c/\tau_B)}{1+(\tau_c/\tau_B)^2},
\end{aligned} \tag{A8}$$

where $\tau_B = p_0/(eB)$ is inverse of cyclotron frequency.

Hence, ϕ can be expressed as

$$\phi = \frac{q\tau_c}{1+(\tau_c/\tau_B)^2}\frac{p_i}{p^0}\{\delta_{ij} - (\tau_c/\tau_B)\epsilon_{ijk}h_k + (\tau_c/\tau_B)^2 b_i b_j\}E_j, \tag{A9}$$

and

$$\begin{aligned}
\delta f &= -\phi\frac{\partial f_0}{\partial p^0} = \phi\beta f_0(1-f_0)\\
\Rightarrow \delta f &= \frac{q\tau_c}{1+(\tau_c/\tau_B)^2}\frac{p_i}{p^0}\{\delta_{ij} - (\tau_c/\tau_B)\epsilon_{ijk}b_k\\
&\quad + (\tau_c/\tau_B)^2 b_i b_j\}E_j\beta f_0(1-f_0).
\end{aligned} \tag{A10}$$

Now, using the above expression of δf in Eq. (A4), we get

$$\begin{aligned}
\sigma^{ij} &= gq^2\beta\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{p^i p^j}{(p^0)^2}\frac{\tau_c}{1+(\tau_c/\tau_B)^2}\{\delta_{ij} - (\tau_c/\tau_B)\epsilon_{ijk}b_k\\
&\quad + (\tau_c/\tau_B)^2 b_i b_j\}f_0(1-f_0)\\
&= \delta_{ij}\sigma_0 - \epsilon_{ijk}b_k\sigma_1 + b_i b_j\sigma_2,
\end{aligned} \tag{A11}$$

where,

$$\sigma_n = gq^2\frac{\beta}{3}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{|\vec{p}|^2}{(p^0)^2}\frac{\tau_c(\tau_c/\tau_B)^n}{1+(\tau_c/\tau_B)^2}f_0(1-f_0), \tag{A12}$$

and $n = 0, 1, 2$. One can identify $\parallel$, $\perp$ and $\times$ components from σ^n by using relations [18,19,21,46],

$$\begin{aligned}
\sigma_\parallel &= \sigma_0 + \sigma_2 = gq^2\frac{\beta}{3}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{|\vec{p}|^2}{(p^0)^2}\tau_c f_0(1-rf_0)\\
\sigma_\perp &= \sigma_0 = gq^2\frac{\beta}{3}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{|\vec{p}|^2}{(p^0)^2}\frac{\tau_c}{1+(\tau_c/\tau_B)^2}f_0(1-rf_0)\\
\sigma_\times &= \sigma_1 = gq^2\frac{\beta}{3}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{|\vec{p}|^2}{(p^0)^2}\frac{(\tau_c^2/\tau_B)}{1+(\tau_c/\tau_B)^2}f_0(1-rf_0).
\end{aligned} \tag{A13}$$

APPENDIX B: STRUCTURE OF RBE IN RTA

In the presence of a magnetic field, RBE with RTA can be written as [16,18,19,46],

$$p^\mu\partial_\mu f + qF^{\mu\nu}p_\nu\frac{\partial f}{\partial p^\mu} = -\frac{U\cdot p}{\tau_c}\delta f, \tag{B1}$$

where $F^{\mu\nu}$ is a field strength tensor, which carry only magnetic field term $F^{\mu\nu} = -Bb^{\mu\nu}$ with $B^{\mu\nu} = \epsilon^{\mu\nu\rho\alpha}B_\rho U_\alpha$. Note that B is the magnetic field strength, and b^μ is the unit four vector. So, for a small deviation of the distribution function from the equilibrium, Eq. (B1) can be written as follows:

$$p^\mu\partial_\mu f_0 = \left(-\frac{U\cdot p}{\tau_c}\right)\left[1-\frac{qB\tau_c}{U\cdot p}b^{\mu\nu}p_\nu\frac{\partial}{\partial p^\mu}\right]\delta f. \quad \text{(B2)}$$

Equilibrium distribution function is $f_0 = \frac{1}{e^{\beta(U\cdot p-\mu)}+r}$ where chemical potential μ has space-time dependency.

So, the left hand side of the above equation can be written as,

$$\begin{aligned}p^\mu\partial_\mu f_0 &= p^\mu U_\mu D f_0 + p^\mu\nabla_\mu f_0\\ &= \frac{\partial f_0}{\partial T}((U\cdot p)DT + p^\mu\nabla_\mu T)\\ &\quad+\frac{\partial f_0}{\partial(\mu/T)}\left((U\cdot p)D\left(\frac{\mu}{T}\right)+p^\mu\nabla_\mu\left(\frac{\mu}{T}\right)\right)\\ &\quad+\frac{\partial f_0}{\partial U^\nu}((U\cdot p)DU^\nu + p^\mu\nabla_\mu U^\nu),\end{aligned} \quad \text{(B3)}$$

where U^μ is four velocity of particle $D\equiv U^\mu\partial_\mu$, $\nabla^\mu\equiv\Delta^{\mu\nu}\partial_\mu$ with $\Delta^{\mu\nu}=g^{\mu\nu}-U^\mu U^\nu$, $g^{\mu\nu}\equiv\mathrm{diag}(1,-1,-1,-1)$. Now, using the energy-momentum conservation ($\partial_\mu T_0^{\mu\nu}=0$), current conservation ($\partial_\mu N_0^\mu=0$) equations, and the Gibbs-Duhem relation we get

$$\begin{aligned}p^\mu\partial_\mu f_0 =& \frac{f_0(1-rf_0)}{T}\left\{Q\nabla_\sigma U^\sigma - p^\mu p^\nu\left[\nabla_\mu U_\nu - \frac{1}{3}\Delta_{\mu\nu}\nabla_\sigma U^\sigma\right]\right.\\ &\left.+\left[1-\frac{(U\cdot p)}{h}\right]p^\mu T\nabla_\mu\left(\frac{\mu}{T}\right)\right\},\end{aligned} \quad \text{(B4)}$$

where $Q=(U\cdot p)^2(\frac{4}{3}-\gamma')+(U\cdot p)[(\gamma''-1)h-\gamma'''T]-\frac{1}{3}m^2$ and $h=mS_3^1/S_2^1$. The expressions for γ', γ'', γ''', and S_n^α are

$$\begin{aligned}\gamma' &= \frac{(S_2^0/S_2^1)^2-(S_3^0/S_2^1)^2+4z^{-1}S_2^0S_3^1/(S_2^1)^2+z^{-1}S_3^0/S_2^1}{(S_2^0/S_2^1)^2-(S_3^0/S_2^1)^2+3z^{-1}S_2^0S_3^1/(S_2^1)^2+2z^{-1}S_3^0/S_2^1-z^{-2}}\\ \gamma'' &= 1+\frac{z^{-2}}{(S_2^0/S_2^1)^2-(S_3^0/S_2^1)^2+3z^{-1}S_2^0S_3^1/(S_2^1)^2+2z^{-1}S_3^0/S_2^1-z^{-2}}\\ \gamma''' &= \frac{S_2^0/S_2^1+5z^{-1}S_3^1/S_2^1-S_3^0S_3^1/(S_2^1)^2}{(S_2^0/S_2^1)^2-(S_3^0/S_2^1)^2+3z^{-1}S_2^0S_3^1/(S_2^1)^2+2z^{-1}S_3^0/S_2^1-z^{-2}},\end{aligned} \quad \text{(B5)}$$

where $z=m/T$ and $S_n^\alpha(z)=\sum_{k=1}^\infty(-r)^{k-1}e^{k\mu/T}k^{-\alpha}K_n(kz)$, $K_n(x)$ denoting the modified Bessel function of order n.

1. Shear Viscosity

In presence of a magnetic field, the general expression of δf for shear viscosity is considered as

$$\delta f = \sum_{n=0}^{4} c_n C_{(n)\mu\nu\alpha\beta}p^\mu p^\nu V^{\alpha\beta} \quad \text{(B6)}$$

$$\begin{aligned}&= [c_0 P^0_{\langle\mu\nu\rangle\alpha\beta} + c_1(P^1_{\langle\mu\nu\rangle\alpha\beta}+P^{-1}_{\langle\mu\nu\rangle\alpha\beta})\\ &\quad + ic_2(P^1_{\langle\mu\nu\rangle\alpha\beta}-P^{-1}_{\langle\mu\nu\rangle\alpha\beta}) + c_3(P^2_{\langle\mu\nu\rangle\alpha\beta}+P^{-2}_{\langle\mu\nu\rangle\alpha\beta})\\ &\quad + ic_4(P^2_{\langle\mu\nu\rangle\alpha\beta}-P^{-2}_{\langle\mu\nu\rangle\alpha\beta})]p^\mu p^\nu V^{\alpha\beta},\end{aligned} \quad \text{(B7)}$$

where $V_{\alpha\beta}=\frac{1}{2}(\frac{\partial U_\alpha}{\partial x_\beta}+\frac{\partial U_\beta}{\partial x_\alpha})$ and $P^{(\mathrm{m})}_{\langle\mu\nu\rangle\alpha\beta}=P^{(\mathrm{m})}_{\mu\nu\alpha\beta}+P^{(\mathrm{m})}_{\nu\mu\alpha\beta}$. The fourth rank projection tensor is defined in terms of the second rank projection tensor as [45],

$$P^{(\mathrm{m})}_{\mu\nu,\mu'\nu'} = \sum_{\mathrm{m}_1=-1}^{1}\sum_{\mathrm{m}_2=-1}^{1}P^{(\mathrm{m}_1)}_{\mu\mu'}P^{(\mathrm{m}_2)}_{\nu\nu'}\delta(m,m_1+m_2), \quad \text{(B8)}$$

and the second rank projection tensor is defined as

$$\begin{aligned}P^0_{\mu\nu} &= b_\mu b_\nu,\\ P^1_{\mu\nu} &= \frac{1}{2}(\Delta_{\mu\nu}-b_\mu b_\nu+ib_{\mu\nu}),\\ P^{-1}_{\mu\nu} &= \frac{1}{2}(\Delta_{\mu\nu}-b_\mu b_\nu-ib_{\mu\nu}),\end{aligned}$$

where $\Delta^{\mu\nu}=g^{\mu\nu}-u^\mu u^\nu$. The second rank projection tensor satisfies the following properties,

$$P^{(\mathrm{m})}_{\mu\kappa}P^{(\mathrm{m}')}_{\kappa\nu} = \delta_{\mathrm{mm}'}P^{(\mathrm{m})}_{\mu\nu}, \quad \text{(B9)}$$

$$(P^{(\mathrm{m})}_{\mu\nu})^\dagger = P^{(-\mathrm{m})}_{\mu\nu} = P^{(\mathrm{m})}_{\nu\mu}, \quad \text{(B10)}$$

$$\sum_{\mathrm{m}=-1}^{1}P^{(\mathrm{m})}_{\mu\nu} = \delta_{\mu\nu}, \qquad P^{(\mathrm{m})}_{\mu\mu} = 1. \quad \text{(B11)}$$

Substituting the above expression on the right hand side of the Boltzmann transport Eq. (B2), we get

$$\begin{aligned}&\left(-\frac{U\cdot p}{\tau_C}\right)\left[1-\frac{qB\tau_C}{U\cdot p}b^{\mu\nu}p_\nu\frac{\partial}{\partial p^\mu}\right]\delta f\\ &= \left(-\frac{U\cdot p}{\tau_C}\right)\left[1-\frac{qB\tau_C}{U\cdot p}b^{\mu\nu}p_\nu\frac{\partial}{\partial p^\mu}\right]\sum_{n=0}^{4}c_n C_{(n)\alpha\beta\rho\sigma}p^\alpha p^\beta V^{\rho\sigma}\\ &= \left(-\frac{U\cdot p}{\tau_C}\right)\left[p^\alpha p^\beta V^{\rho\sigma}\sum_{n=0}^{4}c_n C_{(n)\alpha\beta\rho\sigma}\right.\end{aligned} \quad \text{(B12)}$$

$$-\frac{qB\tau_C}{U\cdot p}b^{\mu\nu}p_\nu(\Delta^\alpha_\mu p^\beta+\Delta^\beta_\mu p^\alpha)V^{\rho\sigma}\sum_{n=0}^{4}c_nC_{(n)\alpha\beta\rho\sigma}\Bigg]=T_1+T_2, \tag{B13}$$

where

$$T_1=\left(-\frac{U\cdot p}{\tau_C}\right)\left[p^\alpha p^\beta V^{\rho\sigma}\sum_{n=0}^{4}c_nC_{(n)\alpha\beta\rho\sigma}\right]\quad\text{and,}\quad T_2=qBb^{\mu\nu}p_\nu(\Delta^\alpha_\mu p^\beta+\Delta^\beta_\mu p^\alpha)V^{\rho\sigma}\sum_{n=0}^{4}c_nC_{(n)\alpha\beta\rho\sigma}. \tag{B14}$$

Now,

$$T_1=\left(-\frac{U\cdot p}{\tau_C}\right)p^\alpha p^\beta V^{\rho\sigma}[c_0P^0_{\langle\alpha\beta\rangle\rho\sigma}+c_1(P^1_{\langle\alpha\beta\rangle\rho\sigma}+P^{-1}_{\langle\alpha\beta\rangle\rho\sigma})+ic_2(P^1_{\langle\alpha\beta\rangle\rho\sigma}-P^{-1}_{\langle\alpha\beta\rangle\rho\sigma})+c_3(P^2_{\langle\alpha\beta\rangle\rho\sigma}+P^{-2}_{\langle\alpha\beta\rangle\rho\sigma})$$
$$+ic_4(P^2_{\langle\alpha\beta\rangle\rho\sigma}-P^{-2}_{\langle\alpha\beta\rangle\rho\sigma})], \tag{B15}$$

and

$$T_2=qBb^{\mu\nu}p_\nu(\Delta^\alpha_\mu p^\beta+\Delta^\beta_\mu p^\alpha)V^{\rho\sigma}\sum_{n=0}^{4}c_nC_{(n)\alpha\beta\rho\sigma}=2qBb^{\mu\nu}p_\nu\Delta^\alpha_\mu p^\beta V^{\rho\sigma}\sum_{n=0}^{4}c_nC_{(n)\alpha\beta\rho\sigma}. \tag{B16}$$

Since, $C_{(n)\alpha\beta\rho\sigma}=C_{(n)\beta\alpha\rho\sigma}$.

So,

$$T_2=2qBb^{\mu\nu}p_\nu\Delta^\alpha_\mu p^\beta V^{\rho\sigma}[c_0P^0_{\langle\alpha\beta\rangle\rho\sigma}+c_1(P^1_{\langle\alpha\beta\rangle\rho\sigma}+P^{-1}_{\langle\alpha\beta\rangle\rho\sigma})+ic_2(P^1_{\langle\alpha\beta\rangle\rho\sigma}-P^{-1}_{\langle\alpha\beta\rangle\rho\sigma})+c_3(P^2_{\langle\alpha\beta\rangle\rho\sigma}+P^{-2}_{\langle\alpha\beta\rangle\rho\sigma})$$
$$+ic_4(P^2_{\langle\alpha\beta\rangle\rho\sigma}-P^{-2}_{\langle\alpha\beta\rangle\rho\sigma})]. \tag{B17}$$

$$T_2=2qBV^{\rho\sigma}p_\mu p_\nu[i(P^{2\langle\mu\nu\rangle\alpha\beta}-P^{-2\langle\mu\nu\rangle\alpha\beta})+\frac{i}{2}(P^{1\langle\mu\nu\rangle\alpha\beta}-P^{-1\langle\mu\nu\rangle\alpha\beta})][c_0P^0_{\langle\alpha\beta\rangle\rho\sigma}+c_1(P^1_{\langle\alpha\beta\rangle\rho\sigma}+P^{-1}_{\langle\alpha\beta\rangle\rho\sigma})$$
$$+ic_2(P^1_{\langle\alpha\beta\rangle\rho\sigma}-P^{-1}_{\langle\alpha\beta\rangle\rho\sigma})+c_3(P^2_{\langle\alpha\beta\rangle\rho\sigma}+P^{-2}_{\langle\alpha\beta\rangle\rho\sigma})+ic_4(P^2_{\langle\alpha\beta\rangle\rho\sigma}-P^{-2}_{\langle\alpha\beta\rangle\rho\sigma})].$$
$$=2qBV^{\rho\sigma}p_\mu p_\nu[c_0\cdot 0+\frac{i}{2}c_1(P^{1\langle\mu\nu\rangle}_{\rho\sigma}-P^{-1\langle\mu\nu\rangle}_{\rho\sigma})-\frac{1}{2}c_2(P^{1\langle\mu\nu\rangle}_{\rho\sigma}+P^{-1\langle\mu\nu\rangle}_{\rho\sigma})+c_3(P^{2\langle\mu\nu\rangle}_{\rho\sigma}-P^{-2\langle\mu\nu\rangle}_{\rho\sigma})-c_4(P^{2\langle\mu\nu\rangle}_{\rho\sigma}+P^{-2\langle\mu\nu\rangle}_{\rho\sigma})]$$
$$=2qBV^{\rho\sigma}p_\mu p_\nu[P^{1\langle\mu\nu\rangle}_{\rho\sigma}\left(\frac{i}{2}c_1-\frac{1}{2}c_2\right)+P^{1\langle\mu\nu\rangle}_{\rho\sigma}\left(-\frac{i}{2}c_1-\frac{1}{2}c_2\right)+P^{2\langle\mu\nu\rangle}_{\rho\sigma}(ic_3-c_4)+P^{-2\langle\mu\nu\rangle}_{\rho\sigma}(-ic_3-c_4)]. \tag{B18}$$

The left hand side of the RBE equation, neglecting the terms that include the spatial gradients of temperature and chemical potential in terms of the projection operator $P^n_{\langle\mu\nu\rangle\alpha\beta}$, turns out to be

$$T_1+T_2=-\frac{f_0(1-rf_0)}{T}p^\mu p^\nu V^{\rho\sigma}[P^0_{\langle\mu\nu\rangle\alpha\beta}+P^1_{\langle\mu\nu\rangle\alpha\beta}+P^{-1}_{\langle\mu\nu\rangle\alpha\beta}+P^2_{\langle\mu\nu\rangle\alpha\beta}+P^{-2}_{\langle\mu\nu\rangle\alpha\beta}]. \tag{B19}$$

Now, equating the right hand side with the left hand side of the relativistic Boltzmann Eq. (B2) with the help of Eqs. (B18), (B17), and (B3) we get

$$c_0=\frac{1}{2}\frac{f_0(1-rf_0)\tau_c}{T(U\cdot p)},$$
$$c_1=\frac{1}{2}\frac{(U\cdot p)f_0(1-rf_0)\tau_c}{T[(U\cdot p)^2+(qB\tau_c)^2]},$$
$$c_2=\frac{1}{2}\frac{(qB)f_0(1-rf_0)\tau_c^2}{T[(U\cdot p)^2+(qB\tau_c)^2]},$$
$$c_3=\frac{1}{2}\frac{(U\cdot p)f_0(1-rf_0)\tau_c}{T[(U\cdot p)^2+(2qB\tau_c)^2]},$$
$$c_4=\frac{(qB)f_0(1-rf_0)\tau_c^2}{T[(U\cdot p)^2+(2qB\tau_c)^2]}. \tag{B20}$$

Using the above expressions, the shear viscosities turns out to be

$$\eta_\parallel=\frac{2}{15}\int\frac{d^3\vec{p}}{(2\pi)^3p_0}|\vec{p}|^4c_0, \tag{B21}$$

$$\eta_\perp=\frac{2}{15}\int\frac{d^3\vec{p}}{(2\pi)^3p_0}|\vec{p}|^4c_1, \tag{B22}$$

$$\eta_\times=\frac{2}{15}\int\frac{d^3\vec{p}}{(2\pi)^3p_0}|\vec{p}|^4c_2, \tag{B23}$$

$$\eta'_\perp=\frac{2}{15}\int\frac{d^3\vec{p}}{(2\pi)^3p_0}|\vec{p}|^4c_3, \tag{B24}$$

$$\eta'_\times=\frac{2}{15}\int\frac{d^3\vec{p}}{(2\pi)^3p_0}|\vec{p}|^4c_4. \tag{B25}$$

2. Bulk viscosity

As mentioned earlier in the text, in the presence of a magnetic field there are three components of the bulk viscosity and the form of δf which corresponds to them is

$$\delta f = \sum_{n=1}^{3} c_n C_{(n)\mu\nu} \partial^\mu U^\nu$$
$$= (c_1 P^0_{\mu\nu} + c_2(P^1_{\mu\nu} + P^{-1}_{\mu\nu}) + c_3(P^1_{\mu\nu} - P^{-1}_{\mu\nu}))\partial^\mu U^\nu. \tag{B26}$$

So, the right hand side of RBE becomes

$$-\frac{U\cdot p}{\tau_c}\left[1-\frac{qB\tau_c}{(U\cdot p)}b^{\mu\nu}p_\nu\frac{\partial}{\partial p^\mu}\right]\delta f$$
$$=-\frac{U\cdot p}{\tau_c}\left[1-\frac{qB\tau_c}{(U\cdot p)}b^{\mu\nu}p_\nu\frac{\partial}{\partial p^\mu}\right]$$
$$\times\{c_1(b^\mu b^\nu)+c_2(\Delta^{\mu\nu}-b^\mu b^\nu)+ic_3 b^{\mu\nu}\}\partial_\mu U_\nu$$
$$=-\frac{U\cdot p}{\tau_c}\{c_1(b^\mu b^\nu)+c_2(\Delta^{\mu\nu}-b^\mu b^\nu)+ic_3 b^{\mu\nu}\}\partial_\mu U_\nu$$
$$=-\frac{U\cdot p}{\tau_c}\{c_2(\partial^\mu U_\mu)+(c_1-c_2)b^\mu b^\nu\partial_\mu U_\nu+ic_3 b^{\mu\nu}\partial_\mu U_\nu\}. \tag{B27}$$

Equating the coefficients of $\partial^\mu U_\mu$, $b^\mu b^\nu \partial_\mu U_\nu$ and $b^{\mu\nu}\partial_\mu U_\nu$ from Eqs. (B27) and (B3) we get

$$c_1 = \frac{\tau_C Q}{(U\cdot p)}\frac{f_0(1-rf_0)}{T} \tag{B28}$$

$$c_2 = \frac{\tau_C Q}{(U\cdot p)}\frac{f_0(1-rf_0)}{T} \tag{B29}$$

$$c_3 = 0. \tag{B30}$$

Thus, the bulk viscosity can be derived from the relation

$$\Pi^{\mu\nu} = \Pi\Delta^{\mu\nu} = \int \frac{d^3\vec{p}}{(2\pi)^3 p^0} p^\mu p^\nu \delta f. \tag{B31}$$

Note that Π is known as bulk pressure. Therefore,

$$\Pi = \frac{1}{3}\int \frac{d^3\vec{p}}{(2\pi)^3 p^0}\Delta_{\mu\nu}p^\mu p^\nu\{c_1(b^\alpha b^\beta)$$
$$+ c_2(\Delta^{\alpha\beta} - b^\alpha b^\beta) + c_3 b^{\alpha\beta}\}\partial_\alpha U_\beta. \tag{B32}$$

So, the components of bulk viscosity in the presence of a magnetic field are

$$\zeta_\parallel = \zeta_\perp = \frac{\tau_c}{T}\int \frac{d^3\vec{p}}{(2\pi)^3(p^0)^2}Q^2 f_0(1-rf_0) \tag{B33}$$

$$\zeta_\times = 0, \tag{B34}$$

where Q is already addressed in an earlier subsection. Since without magnetization, there will be no magnetic field dependent component of bulk viscosity, so its numerical results have not been explored.

3. Diffusion coefficient

The δf for the thermal diffusion components in presence of the magnetic field can be written as

$$\delta f = K^{\mu\nu} p_\mu \partial_\nu \alpha_0, \tag{B35}$$

where $\alpha_0 = \frac{\mu}{T}$.

The second order tensor $K^{\mu\nu}$ can be broken down into the new projectors:

$$P^\parallel_{\mu\nu} = P^0_{\mu\nu} = b_\mu b_\nu,$$
$$P^\perp_{\mu\nu} = (P^1_{\mu\nu} + P^{-1}_{\mu\nu}) = (\Delta_{\mu\nu} - b_\mu b_\nu),$$
$$P^\times_{\mu\nu} = (P^1_{\mu\nu} - P^{-1}_{\mu\nu}) = ib_{\mu\nu}. \tag{B36}$$

Using these projectors the δf becomes

$$\delta f = [K_\parallel P^0_{\mu\nu} + K_\perp P^\perp_{\mu\nu} + K_\times P^\times_{\mu\nu}]p^\mu\partial^\nu\alpha_0$$
$$= [K_\parallel b_\mu b_\nu + K_\perp(\Delta_{\mu\nu} - b_\mu b_\nu) + K_\times(ib_{\mu\nu})]p^\mu\partial^\nu\alpha_0. \tag{B37}$$

Now, with this δf the right hand side of the Boltzmann transport equation becomes

$$-\frac{U\cdot p}{\tau_C}\left[1-\frac{qB\tau_C}{(U\cdot p)}b^{\mu\nu}p_\nu\frac{\partial}{\partial p^\mu}\right]\delta f$$
$$=-\frac{U\cdot p}{\tau_C}\left[1-\frac{qB\tau_C}{(U\cdot p)}b^{\mu\nu}p_\nu\frac{\partial}{\partial p^\mu}\right]$$
$$\times[K_\parallel b_\alpha b_\beta + K_\perp(\Delta_{\alpha\beta}-b_\alpha b_\beta)+K_\times(ib_{\alpha\beta})]p^\alpha\partial^\beta\alpha_0$$
$$=-\frac{U\cdot p}{\tau_C}\left[p^\alpha-\frac{qB\tau_C}{(U\cdot p)}b^{\mu\nu}p_\nu\delta^\alpha_\mu\right]$$
$$\times[K_\parallel b_\alpha b_\beta + K_\perp(\Delta_{\alpha\beta}-b_\alpha b_\beta)+K_\times(ib_{\alpha\beta})]\partial^\beta\alpha_0$$
$$=-\frac{U\cdot p}{\tau_C}[K_\parallel b_\alpha b_\beta + K_\perp(\Delta_{\alpha\beta}-b_\alpha b_\beta)+K_\times(ib_{\alpha\beta})]p^\alpha\partial^\beta\alpha_0$$
$$+qBp_\nu[b^{\alpha\nu}K_\parallel b_\alpha b_\beta + b^{\alpha\nu}K_\perp(\Delta_{\alpha\beta}-b_\alpha b_\beta)$$
$$+b^{\alpha\nu}K_\times(ib_{\alpha\beta})]p^\alpha\partial^\beta\alpha_0. \tag{B38}$$

Using relation Eq. (B9), we have

$$b^{\alpha\nu}P^\parallel_{\alpha\beta} = -iP^{\times\alpha\nu}P^\parallel_{\alpha\beta} = -i[P^{1\alpha\nu} - P^{-1\alpha\nu}]P^0_{\alpha\beta} = 0; \tag{B39}$$

$$b^{\alpha\nu}P^{\perp}_{\alpha\beta} = -iP^{\times\alpha\nu}P^{\perp}_{\alpha\beta} = -i[P^{1\alpha\nu} - P^{-1\alpha\nu}][P^{1}_{\alpha\beta} + P^{-1}_{\alpha\beta}] = -i[P^{1\nu}_{\beta} - P^{2\nu}_{\beta}] = -iP^{\times\nu}_{\beta}; \tag{B40}$$

$$b^{\alpha\nu}P^{\times}_{\alpha\beta} = -iP^{\times\alpha\nu}P^{\times}_{\alpha\beta} = -i[P^{1\alpha\nu} - P^{-1\alpha\nu}][P^{1}_{\alpha\beta} - P^{-1}_{\alpha\beta}] = -i[P^{1\nu}_{\beta} + P^{-1\nu}_{\beta}] = -iP^{\perp\nu}_{\beta}. \tag{B41}$$

Using the above expressions in Eq. (B38), the rhs of RBE becomes

$$\begin{aligned}
-\frac{U\cdot p}{\tau_C}\left[1-\frac{qB\tau_C}{(U\cdot p)}b^{\mu\nu}p_\nu\frac{\partial}{\partial p^\mu}\right]\delta f &= -\frac{U\cdot p}{\tau_C}[K_\parallel P^{\parallel}_{\nu\beta}p^\nu + K_\perp P^{\perp}_{\nu\beta}p^\nu + K_\times P^{\times}_{\nu\beta}p^\nu]\partial^\beta\alpha_0 + qB[K_\parallel 0\cdot p^\nu - iK_\times P^{\perp}_{\nu\beta}p^\nu - iK_\perp P^{\times}_{\nu\beta}p^\nu]\partial^\beta\alpha_0\\
&= \partial^\beta\alpha_0\left\{K_\parallel\left(-\frac{U\cdot p}{\tau_c}\right)P^{\parallel}_{\nu\beta}p^\nu - \left[\frac{U\cdot p}{\tau_c}K_\perp + iqBK_\times\right]P^{\perp}_{\nu\beta}p^\nu - \left[\frac{U\cdot p}{\tau_c}K_\times + iqBK_\perp\right]P^{\times}_{\nu\beta}p^\nu\right\}\\
&= \partial^\beta\alpha_0\left\{K_\parallel\left(-\frac{U\cdot p}{\tau_c}\right)b_\nu b_\beta p^\nu - \left[\frac{U\cdot p}{\tau_c}K_\perp + iqBK_\times\right](\Delta_{\nu\beta} - b_\nu b_\beta)p^\nu\right.\\
&\quad\left. - \left[\frac{U\cdot p}{\tau_c}K_\times + iqBK_\perp\right]ib_{\nu\beta}p^\nu\right\}\\
&= \partial^\beta\alpha_0\left\{\left[K_\parallel\left(-\frac{U\cdot p}{\tau_c}\right) + \left(\frac{U\cdot p}{\tau_c}K_\perp + iqBK_\times\right)\right]b_\nu b_\beta p^\nu - \left[\frac{U\cdot p}{\tau_c}K_\perp + iqBK_\times\right]\Delta_{\nu\beta}p^\nu\right.\\
&\quad\left. - \left[\frac{U\cdot p}{\tau_c}K_\times + iqBK_\perp\right]ib_{\nu\beta}p^\nu\right\}.
\end{aligned} \tag{B42}$$

So, from Eqs. (B42) and (B3) the RBE becomes

$$\begin{aligned}
f_0(1-rf_0)\left[1-\frac{(U\cdot p)}{h}\right]p^\mu\nabla_\mu\alpha_0 &= \partial^\beta\alpha_0\left\{\left[K_\parallel\left(-\frac{U\cdot p}{\tau_c}\right) + \left(\frac{U\cdot p}{\tau_c}K_\perp + iqBK_\times\right)\right]b_\nu b_\beta p^\nu\right.\\
&\quad\left. - \left[\frac{U\cdot p}{\tau_c}K_\times + iqBK_\perp\right]ib_{\nu\beta}p^\nu\right\} - \left[\frac{U\cdot p}{\tau_c}K_\perp + iqBK_\times\right]p^\mu\nabla_\mu\alpha_0.
\end{aligned} \tag{B43}$$

Equating the coefficients for different tensorial terms, we get

$$\begin{aligned}
\left[\frac{U\cdot p}{\tau_c}K_\perp + iqBK_\times\right] &= f_0(1-rf_0)\left[1-\frac{(U\cdot p)}{h}\right],\\
-\frac{U\cdot p}{\tau_c}K_\parallel + \frac{U\cdot p}{\tau_c}K_\perp + iqBK_\times &= 0,\\
\frac{U\cdot p}{\tau_c}K_\times + iqBK_\perp &= 0.
\end{aligned} \tag{B44}$$

Equating the above three equations, we get

$$K_\parallel = -\frac{\tau_C f_0(1-rf_0)}{U\cdot p}\left[1-\frac{(U\cdot p)}{h}\right], \tag{B45}$$

$$K_\perp = -\frac{\tau_C(U\cdot p)f_0(1-rf_0)}{(U\cdot p)^2 + (qB\tau_C)^2}\left[1-\frac{(U\cdot p)}{h}\right], \tag{B46}$$

$$K_\times = -\frac{qB\tau_C^2 f_0(1-rf_0)}{(U\cdot p)^2 + (qB\tau_C)^2}\left[1-\frac{(U\cdot p)}{h}\right]. \tag{B47}$$

So, the thermal diffusion coefficient κ's become

$$\begin{aligned}
\kappa_\parallel &= -\frac{1}{3}\int\frac{d^3\vec{p}}{(2\pi)^3 p_0}|\vec{p}|^2K_\parallel,\\
&= \frac{1}{3h}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{|\vec{p}|^2}{p_0^2}\tau_c(h-p_0)f_0(1-rf_0)\\
\kappa_\perp &= -\frac{1}{3}\int\frac{d^3\vec{p}}{(2\pi)^3 p_0}|\vec{p}|^2K_\perp,\\
&= \frac{1}{3h}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{|\vec{p}|^2}{p_0^2}\frac{\tau_c(h-p_0)}{1+(\frac{\tau_c}{\tau_B})^2}f_0(1-rf_0)\\
\kappa_\times &= -\frac{1}{3}\int\frac{d^3\vec{p}}{(2\pi)^3 p_0}|\vec{p}|^2K_\times\\
&= \frac{1}{3h}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{|\vec{p}|^2}{p_0^2}\frac{\tau_c(\frac{\tau_c}{\tau_B})(h-p_0)}{1+(\frac{\tau_c}{\tau_B})^2}f_0(1-rf_0).
\end{aligned} \tag{B48}$$

[1] A. Bzdak and V. Skokov, Event-by-event fluctuations of magnetic and electric fields in heavy ion collisions, Phys. Lett. B **710**, 171 (2012).
[2] K. Tuchin, Particle production in strong electromagnetic fields in relativistic heavy-ion collisions, Adv. High Energy Phys. **2013**, 490495 (2013).
[3] W. T. Deng and X. G. Huang, Event-by-event generation of electromagnetic fields in heavy-ion collisions, Phys. Rev. C **85**, 044907 (2012).
[4] H. Li, X. L. Sheng, and Q. Wang, Electromagnetic fields with electric and chiral magnetic conductivities in heavy ion collisions, Phys. Rev. C **94**, 044903 (2016).
[5] V. Roy and S. Pu, Event-by-event distribution of magnetic field energy over initial fluid energy density in $\sqrt{s_{\mathrm{NN}}} = 200$ GeV Au-Au collisions, Phys. Rev. C **92**, 064902 (2015).
[6] V. Roy, S. Pu, L. Rezzolla, and D. Rischke, Analytic Bjorken flow in one-dimensional relativistic magnetohydrodynamics, Phys. Lett. B **750**, 45 (2015).
[7] S. Pu, V. Roy, L. Rezzolla, and D. H. Rischke, Bjorken flow in one-dimensional relativistic magnetohydrodynamics with magnetization, Phys. Rev. D **93**, 074022 (2016).
[8] M. Hongo, Y. Hirono, and T. Hirano, Anomalous-hydrodynamic analysis of charge-dependent elliptic flow in heavy-ion collisions, Phys. Lett. B **775**, 266 (2017).
[9] G. Inghirami, L. Del Zanna, A. Beraudo, M. H. Moghaddam, F. Becattini, and M. Bleicher, Numerical magneto-hydrodynamics for relativistic nuclear collisions, Eur. Phys. J. C **76**, 659 (2016).
[10] G. Inghirami, M. Mace, Y. Hirono, L. Del Zanna, D. E. Kharzeev, and M. Bleicher, Magnetic fields in heavy ion collisions: flow and charge transport, Eur. Phys. J. C **80**, 293 (2020).
[11] S. K. Das, S. Plumari, S. Chatterjee, J. Alam, F. Scardina, and V. Greco, Directed flow of charm quarks as a witness of the initial strong magnetic field in ultra-relativistic heavy ion collisions, Phys. Lett. B **768**, 260 (2017).
[12] D. Satow, Nonlinear electromagnetic response in quark-gluon plasma, Phys. Rev. D **90**, 034018 (2014).
[13] V. Skokov, A. Illarionov, and V. Toneev, Estimate of the magnetic field strength in heavy-ion collisions, Int. J. Mod. Phys. A **24**, 5925 (2009).
[14] K. Tuchin, On viscous flow and azimuthal anisotropy of the quark–gluon plasma in a strong magnetic field, J. Phys. G **39**, 025010 (2012).
[15] S. Li and H.-U. Yee, Shear viscosity of quark-gluon plasma in weak magnetic field in perturbative QCD: Leading log, Phys. Rev. D **97**, 056024 (2018).
[16] P. Mohanty, A. Dash, and V. Roy, One particle distribution function and shear viscosity in magnetic field: A relaxation time approach, Eur. Phys. J. A **55**, 35 (2019).
[17] S. Ghosh, B. Chatterjee, P. Mohanty, A. Mukharjee, and H. Mishra, Impact of magnetic field on shear viscosity of quark matter in Nambu–Jona-Lasinio model, Phys. Rev. D **100**, 034024 (2019).
[18] J. Dey, S. Satapathy, A. Mishra, S. Paul, and S. Ghosh, From non-interacting to interacting picture of quark gluon plasma in presence of magnetic field and its fluid property, arXiv:1908.04335.
[19] J. Dey, S. Satapathy, P. Murmu, and S. Ghosh, Shear viscosity and electrical conductivity of relativistic fluid in presence of magnetic field: A massless case, arXiv:1907.11164.
[20] A. Das, H. Mishra, and R. K. Mohapatra, Transport coefficients of hot and dense hadron gas in a magnetic field: A relaxation time approach, Phys. Rev. D **100**, 114004 (2019).
[21] A. Harutyunyan and A. Sedrakian, Electrical conductivity of a warm neutron star crust in magnetic fields, Phys. Rev. C **94**, 025805 (2016).
[22] B. O. Kerbikov and M. A. Andreichikov, Electrical conductivity of dense quark matter with fluctuations and magnetic field included, Phys. Rev. D **91**, 074010 (2015).
[23] S. I. Nam, Electrical conductivity of quark matter at finite T under external magnetic field, Phys. Rev. D **86**, 033014 (2012).
[24] X. G. Huang, A. Sedrakian, and D. H. Rischke, Kubo formulas for relativistic fluids in strong magnetic fields, Ann. Phys. (Amsterdam) **326**, 3075 (2011).
[25] K. Hattori, S. Li, D. Satow, and H. U. Yee, Longitudinal conductivity in strong magnetic field in perturbative QCD: Complete leading order, Phys. Rev. D **95**, 076008 (2017).
[26] M. Kurian, S. Mitra, S. Ghosh, and V. Chandra, Transport coefficients of hot magnetized QCD matter beyond the lowest Landau level approximation, Eur. Phys. J. C **79**, 134 (2019).
[27] M. Kurian and V. Chandra, Effective description of hot QCD medium in strong magnetic field and longitudinal conductivity, Phys. Rev. D **96**, 114026 (2017).
[28] B. Feng, Electric conductivity and Hall conductivity of the QGP in a magnetic field, Phys. Rev. D **96**, 036009 (2017).
[29] K. Fukushima and Y. Hidaka, Electric Conductivity of hot and Dense Quark Matter in a Magnetic Field with Landau Level Resummation via Kinetic Equations, Phys. Rev. Lett. **120**, 162301 (2018).
[30] A. Das, H. Mishra, and R. K. Mohapatra, Electrical conductivity and Hall conductivity of a hot and dense hadron gas in a magnetic field: A relaxation time approach, Phys. Rev. D **99**, 094031 (2019).
[31] A. Das, H. Mishra, and R. K. Mohapatra Electrical conductivity and Hall conductivity of hot and dense quark gluon plasma in a magnetic field: A quasi particle approach, Phys. Rev. D **101**, 034027 (2020).
[32] S. Ghosh, A. Bandyopadhyay, R. L. S. Farias, J. Dey, and G. Krein, Anisotropic electrical conductivity of magnetized hot quark matter, arXiv:1911.10005.
[33] K. Hattori, X. G. Huang, D. H. Rischke, and D. Satow, Bulk viscosity of quark-gluon plasma in strong magnetic fields, Phys. Rev. D **96**, 094009 (2017).
[34] X.-G. Huang, M. Huang, D. H. Rischke, and A. Sedrakian, Anisotropic hydrodynamics, bulk viscosities, and r-modes of strange quark stars with strong magnetic fields, Phys. Rev. D **81**, 045015 (2010).
[35] N. O. Agasian, Bulk viscosity of quark-gluon matter in a magnetic field, Phys. At. Nucl. **76**, 1382 (2013).
[36] N. O. Agasian, Low-energy theorems of QCD and bulk viscosity at finite temperature and baryon density in a magnetic field, JETP Lett. **95**, 171 (2012).

[37] M. Kurian and V. Chandra, Bulk viscosity of a hot QCD medium in a strong magnetic field within the relaxation-time approximation, Phys. Rev. D **97**, 116008 (2018).

[38] G. S. Denicol, X. G. Huang, E. Molnár, G. M. Monteiro, H. Niemi, J. Noronha, D. H. Rischke, and Q. Wang, Nonresistive dissipative magnetohydrodynamics from the Boltzmann equation in the 14-moment approximation, Phys. Rev. D **98**, 076009 (2018).

[39] M. Kurian, S. K. Das, and V. Chandra Heavy quark dynamics in a hot magnetized QCD medium, Phys. Rev. D **100**, 074003 (2019).

[40] B. Singh, L. Thakur, and H. Mishra, Heavy quark complex potential in a strongly magnetized hot QGP medium, Phys. Rev. D **97**, 096011 (2018).

[41] B. Singh, S. Mazumder, and H. Mishra, HQ Collisional energy loss in a magnetized medium, J. High Energy Phys. 05 (2020) 068.

[42] P. Braun-Munzinger, K. Redlich, and J. Stachel, *Quark Gluon Plasma*, edited by R. C. Hwa and X. N. Wang (World Scientific Publishing, Singapore, 2004), Vol. 3.

[43] G. Endrodi, QCD equation of state at nonzero magnetic fields in the Hadron Resonance Gas model, J. High Energy Phys. 04 (2013) 023.

[44] A. Bhattacharyya, S. K. Ghosh, R. Ray, and S Samanta, Exploring effects of magnetic field on the hadron resonance gas, Europhys. Lett. **115**, 62003 (2016).

[45] S. Hess, *Tensors for physics*, Undergraduate Lecture Notes in Physics (Springer, Cham, 2015), https://doi.org/10.1007/978-3-319-12787-3.

[46] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon Press, UK, 1987).

[47] S. Gavin, Transport coefficients in ultrarelativistic heavy ion collisions, Nucl. Phys. **A 435**, 826 (1985).

[48] P. Chakraborty and J. I. Kapusta, Quasiparticle theory of shear and bulk viscosities of hadronic matter, Phys. Rev. C **83**, 014906 (2011).

[49] S. Ghosh, A real-time thermal field theoretical analysis of Kubo-type shear viscosity: Numerical understanding with simple examples, Int. J. Mod. Phys. A **29**, 1450054 (2014).

[50] S. Ghosh, Electrical conductivity of hadronic matter from different possible mesonic and baryonic loops, Phys. Rev. D **95**, 036018 (2017).

[51] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Full result for the QCD equation of state with $2+1$ flavors, Phys. Lett. B **730**, 99 (2014).

[52] S. Samanta, J. Dey, S. Satapathy, and S. Ghosh, Quantum expression of electrical conductivity from massless quark matter to hadron resonance gas in presence of magnetic field, arXiv:2002.04434.