$SU(3)_C \times SU(3)_L \times U(1)_X$ model from $SU(6)$

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We propose the $SU(3)_C \times SU(3)_L \times U(1)_X$ model arising from $SU(6)$ breaking. One family of the Standard Model (SM) fermions arises from two $\bar{6}$ representations and one 15 representation of $SU(6)$ gauge symmetry. To break the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry down to the SM, we introduce three $SU(3)_L$ triplet Higgs fields, where two of them come from the $\bar{6}$ representation while the other one from the 15 representation. We study the gauge boson masses and Higgs boson mass in detail, and find that the vacuum expectation value (VEV) of the Higgs field for $SU(3)_L \times U(1)_X$ gauge symmetry breaking is around 10 TeV. The neutrino masses and mixing can be generated via the littlest inverse seesaw mechanism. In particular, we have normal hierarchy for neutrino masses and the lightest active neutrino is massless. Also, we consider constraints from the charged lepton flavor changing decays as well. Furthermore, introducing two $SU(3)_L$ adjoint fermions, one $SU(3)_C$ adjoint scalar, and one $SU(3)_L$ triplet scalar, we can achieve gauge coupling unification within 1%. These extra particles can provide a dark matter candidate as well.

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I. INTRODUCTION

The Standard Model (SM) has made a great achievement in explaining the experimental result. However, many significant problems remain to be answered. Two of the most import issues are the fermion generation and the $U(1)_Y$ hypercharge. Since the SM did not explain the origin of the hypercharge, one may expect that the quantum number comes from a bigger group, for example, the grand unified theory (GUT). In the traditional $SU(3)_C \times SU(3)_L \times U(1)_X$ (331) model, it successfully explained why there are three generations by tactfully eliminating $SU(3)_L$ gauge anomalies. However, the $U(1)_X$ number is given by hand just like $U(1)_Y$ in the SM, which is not satisfying and inspires us to embed the 331 model into a bigger group to understand the $U(1)_X$ number more naturally. In this paper, we shall propose a 331 model generated from a $SU(6)$ model, where the $U(1)_X$ charge is determined from the $SU(6)$ breaking.

In the traditional 331 models [1–21], the left-handed lepton and one left-handed quark triplet are in the $\bar{3}$ antifundamental representation of $SU(3)_L$, while two left-handed quark triplets are in the 3 fundamental representation. Thus, we must have three generations of leptons in order to cancel gauge anomalies. The electric charge operator could be calculated from the diagonal generators of $SU(3)_L \times U(1)_X$ as follows

$$Q = T_3 + \beta T_8 + X. \tag{1.1}$$

Previous models can be classified via the β value. For models with $\beta = \frac{1}{\sqrt{3}}$ [9,21–25], there are at least three scalars [see the following Eq. (2.12)] in Higgs sector in order to break $SU(3)_L$ to $U(1)_{\rm EM}$ and generate all the SM fermion and gauge vector masses at tree level. In these models, according to Eq. (1.1), $Q = \pm{\rm diag}[\frac{2}{3} + X, -\frac{1}{3} + X, -\frac{1}{3} + X]$ (there could be a minus sign for $\bar{3}$ multiplets), all the representations must contain two particles with the same charge. For Higgs fields which contain two zero-charged particles, there must be two of them in the same representation.

For models with $\beta = \sqrt{3}$ [9–11], it is obvious that all the three scalar triplets are all in different representations,

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because $Q = \pm\mathrm{diag}[1+X, X, -1+X]$ for particles in (anti)fundamental representation. Moreover, to generate all charged fermion masses in the tree level, we need three scalar triplets and one scalar sextet. Such models also contain exotic charged particles such as double charged Higgs and quarks with charge $\pm\frac{5}{3}$ and $\pm\frac{4}{3}$. In particular, there exists the Landau pole problem for $U(1)_X$ not far from the TeV scale.

We propose the $SU(3)_C \times SU(3)_L \times U(1)_X$ model, which can be obtained from the $SU(6)$ breaking. Such kind of models have been studied previously [26,27]. One family of the SM fermions arises from two $\bar{6}$ representations and one 15 representation of $SU(6)$ gauge symmetry. To break the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry down to the SM gauge symmetry, we introduce three $SU(3)_L$ triplet Higgs fields, where two of them arise from $\bar{6}$ representation while the other one from 15 representation. We discuss the gauge boson masses and Higgs boson mass in details, and show that the vacuum expectation value (VEV) of the Higgs field for $SU(3)_L \times U(1)_X$ gauge symmetry breaking is around 10 TeV. We explain the neutrino masses and mixing via the littlest inverse seesaw mechanism. Especially, the normal hierarchy for neutrino masses is realized and the lightest active neutrino is massless. Moreover, we study constraints from the charged lepton flavor changing decays as well. Furthermore, introducing two $SU(3)_L$ adjoint fermions, one $SU(3)_C$ adjoint scalar, and one $SU(3)_L$ triplet scalar, we can achieve gauge coupling unification within 1%. These extra particles can give us a dark matter candidate as well.

The paper is organized as follows. In Sec. II, we present the models and Yukawa terms. The gauge sector and Higgs sector are studied in Sec. III and Sec. IV, respectively. We discuss the neutrino masses and mixing, as well as the charged lepton flavor changing decays in Sec. V. In Sec. VI, we consider gauge coupling unification and dark matter candidate. Our conclusion is in Sec. VII.

II. THE $SU(3)_C \times SU(3)_L \times U(1)_X$ MODEL

In our 3-3-1 model, the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge group arises from a large $SU(6)$ gauge group. The $U(1)_X$ charge operator for the 6 representation of the $SU(6)$ group is

$$T_{U(1)_X} = \frac{1}{2\sqrt{3}}\mathrm{diag}[-1,-1,-1,1,1,1]. \tag{2.1}$$

The following representations of the $SU(6)$ group can be decomposed into representations of the $SU(3)_C \times SU(3)_L \times U(1)_X$ group as below

$$6 \to \left(3,1,\frac{-1}{2\sqrt{3}}\right) \bigoplus \left(1,3,\frac{1}{2\sqrt{3}}\right), \tag{2.2}$$

$$\bar{6} \to \left(\bar{3},1,\frac{1}{2\sqrt{3}}\right) \bigoplus \left(1,\bar{3},\frac{-1}{2\sqrt{3}}\right), \tag{2.3}$$

$$15 \to \left(\bar{3},1,\frac{-1}{\sqrt{3}}\right) \bigoplus \left(1,\bar{3},\frac{1}{\sqrt{3}}\right) \bigoplus (3,3,0). \tag{2.4}$$

One family of the SM fermions and extra fermions in our model is

$$\bar{6} \to \left(1,\bar{3},\frac{-1}{2\sqrt{3}}\right) \bigoplus \left(\bar{3},1,\frac{1}{2\sqrt{3}}\right) \tag{2.5}$$

$$\hookrightarrow f_i = (e_{Li}, -\nu_{Li}, N_i) \bigoplus d^c_{Ri}, \tag{2.6}$$

$$\bar{6}' \to \left(1,\bar{3},\frac{-1}{2\sqrt{3}}\right) \bigoplus \left(\bar{3},1,\frac{1}{2\sqrt{3}}\right) \tag{2.7}$$

$$\hookrightarrow f'_i = (e'_{Li}, -\nu'_{Li}, N'_i) \bigoplus D^c_{Ri}, \tag{2.8}$$

$$15 \to (3,3,0) \bigoplus \left(1,\bar{3},\frac{1}{\sqrt{3}}\right) \bigoplus \left(\bar{3},1,\frac{-1}{\sqrt{3}}\right) \tag{2.9}$$

$$\hookrightarrow F_i = (u_{Li}, d_{Li}, D_{Li}) \bigoplus Xf^c_i = (\nu'^c_{Ri}, e'^c_{Ri}, e^c_{Ri}) \bigoplus u^c_{Ri}. \tag{2.10}$$

Besides, we have fermions transforming as singlet under the $SU(3)_C \times SU(3)_L \times U(1)_X$ group, which are N_{si} and N'_{si}. For all the fermions above, $i = 1, 2, 3$ stands for fermion generation.

In $SU(6)$ model, two $\bar{6}$ antifundamental representations and one 15 antisymmetric representation of the fermions are anomaly free. Thus, our model is anomaly free. To be concrete, we can verify it easily as well. According to [28,29], first, for $U(1)_X$, we have

$$\sum_{\psi i} X_{\psi i} = \sum_{\psi i} X^3_{\psi i} = 0, \tag{2.11}$$

which makes $U(1)_X$ gauge structure anomaly free. For gauge structure of $SU(3)_L/SU(3)_C$, since the number of fermion multiplets in 3 representation equals to the number of fermion multiplets in $\bar{3}$ representation for every generation, it is also anomaly free.

Our model has 3 scalar multiplets coming from two $\bar{6}$ and one 15 representations of the $SU(6)$ group, which are

$$15 \to \left(1,\bar{3},\frac{1}{\sqrt{3}}\right):\ T_u = \frac{1}{\sqrt{2}}\begin{pmatrix} v_u + \rho_1 + i\sigma_1 \\ \sqrt{2}\chi_1^+ \\ \sqrt{2}\chi_2^+ \end{pmatrix},$$

$$\langle T_u \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_u \\ 0 \\ 0 \end{pmatrix}, \tag{2.12}$$

$$\bar{6} \to \left(1, \bar{3}, \frac{-1}{2\sqrt{3}}\right):\ T_d = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\xi_2^- \\ v_d + \rho_2 + i\sigma_2 \\ \rho_3 + i\sigma_3 \end{pmatrix},$$

$$\langle T_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_d \\ 0 \end{pmatrix}, \tag{2.13}$$

$$\bar{6} \to \left(1, \bar{3}, \frac{-1}{2\sqrt{3}}\right):\ T = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\xi_1^- \\ \rho_4 + i\sigma_4 \\ v_t + \rho_5 + i\sigma_5 \end{pmatrix},$$

$$\langle T \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_t \end{pmatrix}. \tag{2.14}$$

We use

$$\tan\theta = \frac{v_u}{v_d}, \tag{2.15}$$

$$k = v_t / \sqrt{v_d^2 + v_u^2}, \tag{2.16}$$

to parametrize the 3 VEVs, which break the $SU(3)_L \times U(1)_X$ gauge group down to the $U(1)_{\mathrm{EM}}$ gauge group. We write the $U(1)_{\mathrm{EM}}$ charge operator as

$$Q = c_1 T_{8L} + c_2 T_{3L} + c_3 X I. \tag{2.17}$$

Then the condition, which only neutral states of the scalar multiplets can get VEVs, gives

$$c_1 = \frac{c_2}{\sqrt{3}} = \frac{1}{2} c_3. \tag{2.18}$$

To make SM particles have the same electric charges as in the SM, we find

$$c_3 = \frac{2}{\sqrt{3}}, \tag{2.19}$$

leading to

$$Q = \frac{1}{\sqrt{3}} T_{8L} + T_{3L} + \frac{2}{\sqrt{3}} X I. \tag{2.20}$$

The Yukawa terms and Majorana mass terms of our model are

$$\begin{aligned} -\mathcal{L}_{\mathrm{qua}} &= y_{ij}^u F_i u_{Rj}^c T_u + y_{ij}^d F_i d_{Rj}^c T_d + y_{ij}^D F_i D_{Rj}^c T + \mathrm{H.c}, \\ -\mathcal{L}_{\mathrm{lep}} &= y_{ij}^{\nu} f_i f_j T_u + y_{ij}^e f_i X f_j^c T_d + y_{ij}^{L'} f_i' X f_j^c T + y_{ij}^N f_i \bar{T} N_{sj} + y_{ij}^{N'} f_i' \bar{T} N_{sj}' + \mathrm{H.c}, \\ -\mathcal{L}_{\mathrm{neu}}^{\mathrm{maj}} &= \frac{1}{2} (N_s N_s') \begin{Bmatrix} M_s & M_{ss'} \\ M_{ss'}^T & M_s' \end{Bmatrix} \begin{pmatrix} N_s \\ N_s' \end{pmatrix} + \mathrm{H.c}, \end{aligned} \tag{2.21}$$

where M_s, M_s' and $M_{ss'}$ are 3×3 matrix. For simplicity, we do not include all the gauge invariant terms in Eq (2.21).

III. GAUGE BOSONS

We write $W_a (a = 1, 2, \ldots, 8)$, which is in the adjoint representation of $SU(3)_L$ in the form of

$$W_a T_a = \frac{1}{2} \begin{bmatrix} W_3 + \frac{1}{\sqrt{3}} W_8 & W_1 - iW_2 & W_4 - iW_5 \\ W_1 + iW_2 & -W_3 + \frac{1}{\sqrt{3}} W_8 & W_6 - iW_7 \\ W_4 + iW_5 & W_6 + iW_7 & -\frac{2}{\sqrt{3}} W_8 \end{bmatrix}. \tag{3.1}$$

For the adjoint representation of the $SU(3)_L$ group, the electric charge operator is

$$Q = \frac{1}{\sqrt{3}} T_{8L} + T_{3L} = \frac{1}{3} \mathrm{diag}[2, -1, 1], \tag{3.2}$$

giving

$$[Q, W_a T_a] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \frac{W_1 - iW_2}{\sqrt{2}} & \frac{W_4 - iW_5}{\sqrt{2}} \\ -\frac{W_1 + iW_2}{\sqrt{2}} & 0 & 0 \\ -\frac{W_4 + iW_5}{\sqrt{2}} & 0 & 0 \end{bmatrix}. \tag{3.3}$$

We thus define $W^{\pm} \equiv \frac{W_1 \mp iW_2}{\sqrt{2}}$, $W'^{\pm} \equiv \frac{W_4 \mp iW_5}{\sqrt{2}}$, $V \equiv \frac{W_6 - iW_7}{\sqrt{2}}$, and $V^* \equiv \frac{W_6 + iW_7}{\sqrt{2}}$. $W^{\pm}$ and $W'^{\pm}$ are charged, while V is neutral. Thus, we do not have the double-charged gauge bosons in our model, which is a significant phenomenological difference from traditional 331 models.

With

$$D_\mu = \partial_\mu - i g_L W_\mu^a T_a - i g_X X B_\mu, \tag{3.4}$$

we get

$$(D^\mu\langle T\rangle)^\dagger(D_\mu\langle T\rangle)+(D^\mu\langle T_d\rangle)^\dagger(D_\mu\langle T_d\rangle)+(D^\mu\langle T_u\rangle)^\dagger(D_\mu\langle T_u\rangle)$$
$$=\left(\frac{g_L}{2}\sqrt{v_u^2+v_d^2}\right)^2W_\mu^+W^{-\mu}+\left(\frac{g_L}{2}\sqrt{v_u^2+v_t^2}\right)^2W_\mu'^+W'^{-\mu}+\left(\frac{g_L}{2}\sqrt{v_d^2+v_t^2}\right)^2V_\mu V^{*\mu}+\frac{1}{2}(BW_3W_8)M_{\rm mix}^2\begin{pmatrix}B\\W_3\\W_8\end{pmatrix}, \tag{3.5}$$

$$M_{\rm mix}^2=\begin{Bmatrix}\frac{g_X^2}{12}(4v_u^2+v_d^2+v_t^2) & -\frac{\sqrt{3}g_Lg_X}{12}(2v_u^2+v_d^2) & -\frac{g_Lg_X}{12}(2v_u^2-v_d^2+2v_t^2)\\ -\frac{\sqrt{3}g_Lg_X}{12}(2v_u^2+v_d^2) & \frac{g_L^2}{4}(v_u^2+v_d^2) & \frac{g_L^2}{4\sqrt{3}}(v_u^2-v_d^2)\\ -\frac{g_Lg_X}{12}(2v_u^2-v_d^2+2v_t^2) & \frac{g_L^2}{4\sqrt{3}}(v_u^2-v_d^2) & \frac{g_L^2}{12}(v_u^2+v_d^2+4v_t^2)\end{Bmatrix}. \tag{3.6}$$

And we get

$$M_W=\frac{g_L}{2}\sqrt{v_u^2+v_d^2}, \tag{3.7}$$

$$M_{W'}=\frac{g_L}{2}\sqrt{v_u^2+v_t^2}, \tag{3.8}$$

$$M_V=\frac{g_L}{2}\sqrt{v_d^2+v_t^2}. \tag{3.9}$$

To make $W^\pm$, which is the familiar $W^\pm$ gauge boson in the SM, have the right mass, we have

$$\sqrt{v_u^2+v_d^2}=246\ \text{GeV}. \tag{3.10}$$

Also, by diagonalizing $M_{\rm mix}^2$, we get

$$M_A=0, \tag{3.11}$$

$$M_Z^2=m_1^2(1-\sqrt{1-\rho}), \tag{3.12}$$

$$M_{Z'}^2=m_1^2(1+\sqrt{1-\rho}), \tag{3.13}$$

with

$$m_1^2=\frac{1}{6}\left(g_L^2(v_u^2+v_d^2+v_t^2)+\frac{g_X^2}{4}(4v_u^2+v_d^2+v_t^2)\right), \tag{3.14}$$

$$\rho=\frac{3g_L^2(g_L^2+g_X^2)(v_t^2v_u^2+v_t^2v_d^2+v_u^2v_d^2)}{\left(g_L^2(v_u^2+v_d^2+v_t^2)+\frac{g_X^2}{4}(4v_u^2+v_d^2+v_t^2)\right)^2}, \tag{3.15}$$

where A, Z, and Z' are the eigenstates of the mixing of B, W_3. and W_8. A and Z are the photon and the Z gauge boson in the SM, respectively.

We also find

$$B=\frac{g_L}{\sqrt{g_L^2+g_X^2}}A+\cdots, \tag{3.16}$$

which means $g_X=\frac{2g_Lg_Y}{\sqrt{3g_L^2-g_Y^2}}$.

With the condition that $|k|\gg 1$, we have

$$M_Z\approx\frac{g_L}{2\cos\theta_W}\sqrt{v_u^2+v_d^2}, \tag{3.17}$$

$$M_{Z'}\approx\frac{g_L}{\sqrt{3-\tan^2\theta_W}}|v_t|, \tag{3.18}$$

and

$$A=\sqrt{\cos^2\theta_W-\frac{\sin^2\theta_W}{3}}B+\sin\theta_WW_3+\frac{\sin\theta_W}{\sqrt{3}}W_8, \tag{3.19}$$

$$Z\approx-\sin\theta_W\sqrt{1-\frac{\tan^2\theta_W}{3}}B+\cos\theta_WW_3-\frac{\sin\theta_W\tan\theta_W}{\sqrt{3}}W_8, \tag{3.20}$$

$$Z'\approx\frac{\tan\theta_W}{\sqrt{3}}B-\sqrt{1-\frac{\tan^2\theta_W}{3}}W_8, \tag{3.21}$$

where θ_W is the Weinberg angle.

According to [30] $M_{Z'}$ larger than 4.5 TeV, $|v_t|$ needs to be larger than 10 TeV.

IV. HIGGS SECTOR

The most general Higgs potential in our model is

$$\begin{aligned} V_{\mathrm{Higgs}} = & -m_1^2|T|^2 - m_2^2|T_d|^2 - m_3^2|T_u|^2 + l_1|T|^4 + l_2|T_d|^4 + l_3|T_u|^4 + l_{13}|T|^2|T_u|^2 + l_{12}|T|^2|T_d|^2 + l_{23}|T_u|^2|T_d|^2 \\ & + l'_{12}|T^\dagger T_d|^2 + l'_{13}|T^\dagger T_u|^2 + l'_{23}|T_u^\dagger T_d|^2 + (y_1 T^\dagger T_d|T|^2 + y_2 T^\dagger T_d|T_d|^2 + y_3 T^\dagger T_d|T_u|^2 + \mathrm{H.c.}) \\ & + (-BT^\dagger T_d + AT_u T_d T + y_{12}T^\dagger T_d T^\dagger T_d + y_{123}T_u^\dagger T_d T_u T^\dagger + \mathrm{H.c.}). \end{aligned} \tag{4.1}$$

Since $\langle \frac{\partial V_{\mathrm{Higgs}}}{\partial \rho_i} \rangle = 0 (i = 1, 2, \ldots, 5)$, we get 4 independent relations, which are

$$m_1^2 = \frac{l_{12}v_d^2 v_t + 2l_1 v_t^3 + \sqrt{2}Av_d v_u + l_{13}v_t v_u^2}{2v_t}, \tag{4.2}$$

$$m_2^2 = \frac{l_{12}v_t^2 v_d + 2l_2 v_d^3 + \sqrt{2}Av_t v_u + l_{23}v_d v_u^2}{2v_d}, \tag{4.3}$$

$$m_3^2 = \frac{l_{23}v_d^2 v_u + 2l_3 v_u^3 + \sqrt{2}Av_t v_d + l_{13}v_u v_t^2}{2v_u}, \tag{4.4}$$

$$B = \frac{y_1 v_t^2 + y_2 v_d^2 + y_3 v_u^2}{2}. \tag{4.5}$$

A. Mixing of $\xi^\pm_{1,2}, \chi^\pm_{1,2}$

From the Higgs potential V_{Higgs}, we get

$$V_{Higgs} \ni (\chi_1^+ \xi_2^+ \chi_2^+ \xi_1^+) M_c^2 (\chi_1^- \xi_2^- \chi_2^- \xi_1^-)^{\mathrm{T}}, \tag{4.6}$$

$$M_c^2 = \begin{Bmatrix} -\frac{Av_d v_t}{\sqrt{2}v_u} + \frac{1}{2}l'_{23}v_d^2 & -\frac{Av_t}{\sqrt{2}} + \frac{1}{2}l'_{23}v_d v_u & \frac{1}{2}y_{123}v_d v_t & \frac{1}{2}y_{123}v_d v_u \\ -\frac{Av_t}{\sqrt{2}} + \frac{1}{2}l'_{23}v_d v_u & -\frac{Av_u v_t}{\sqrt{2}v_d} + \frac{1}{2}l'_{23}v_u^2 & \frac{1}{2}y_{123}v_u v_t & \frac{1}{2}y_{123}v_u^2 \\ \frac{1}{2}y_{123}v_d v_t & \frac{1}{2}y_{123}v_u v_t & -\frac{Av_t v_d}{\sqrt{2}v_u} + \frac{1}{2}l'_{13}v_t^2 & -\frac{Av_d}{\sqrt{2}} + \frac{1}{2}l'_{13}v_t v_u \\ \frac{1}{2}y_{123}v_d v_u & \frac{1}{2}y_{123}v_u^2 & -\frac{Av_d}{\sqrt{2}} + \frac{1}{2}l'_{13}v_t v_u & -\frac{Av_d v_u}{\sqrt{2}v_t} + \frac{1}{2}l'_{13}v_u^2 \end{Bmatrix}. \tag{4.7}$$

Eigenstates from the mixing of $\xi^\pm_{1,2}, \chi^\pm_{1,2}$ are

$$\eta_1^\pm = -\frac{v_u}{\sqrt{v_u^2 + v_t^2}}\chi_2^\pm + \frac{v_t}{\sqrt{v_u^2 + v_t^2}}\xi_1^\pm, \qquad m^2_{\eta_1} = 0, \tag{4.8}$$

$$\eta_2^\pm = -\frac{v_u}{\sqrt{v_u^2 + v_d^2}}\chi_1^\pm + \frac{v_d}{\sqrt{v_u^2 + v_d^2}}\xi_2^\pm, \qquad m^2_{\eta_2} = 0, \tag{4.9}$$

and massive eigenstates $\eta_3^\pm, \eta_4^\pm$. The expressions and masses of $\eta_3^\pm$ and $\eta_4^\pm$ are not given here because they are tedious and easy to get. Apparently, $\eta_1^\pm$ and $\eta_2^\pm$ are Goldstone bosons.

B. Mixing of σ_i

We have

$$V_{\mathrm{Higgs}} \ni \frac{1}{2}(\sigma_1\sigma_2\sigma_5\sigma_3\sigma_4) \begin{Bmatrix} -\frac{Av_d v_t}{\sqrt{2}v_u} & -\frac{Av_t}{\sqrt{2}} & -\frac{Av_d}{\sqrt{2}} & 0 & 0 \\ -\frac{Av_t}{\sqrt{2}} & -\frac{Av_u v_t}{\sqrt{2}v_d} & -\frac{Av_u}{\sqrt{2}} & 0 & 0 \\ -\frac{Av_d}{\sqrt{2}} & -\frac{Av_u}{\sqrt{2}} & -\frac{Av_d v_u}{\sqrt{2}v_t} & 0 & 0 \\ 0 & 0 & 0 & -\frac{v_t}{v_d}m_{34}^2 & m_{34}^2 \\ 0 & 0 & 0 & m_{34}^2 & -\frac{v_t}{v_d}m_{34}^2 \end{Bmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_5 \\ \sigma_3 \\ \sigma_4 \end{pmatrix}, \tag{4.10}$$

where

$$m_{34}^2 = -\frac{1}{2} l'_{12} v_d v_t + \frac{A v_u}{\sqrt{2}} + y_{12} v_d v_t.$$

The eigenstates are

$$a_1 = \frac{v_u}{\sqrt{v_u^2 + v_t^2}} \sigma_1 - \frac{v_t}{\sqrt{v_u^2 + v_t^2}} \sigma_5, \tag{4.11}$$

$$\begin{aligned} a_2 =& \frac{v_t^2 v_u}{\sqrt{(v_u^2 + v_t^2)(v_t^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2)}} \sigma_1 \\ &- \frac{v_d (v_u^2 + v_t^2)}{\sqrt{(v_u^2 + v_t^2)(v_t^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2)}} \sigma_2 \\ &+ \frac{v_u^2 v_t}{\sqrt{(v_u^2 + v_t^2)(v_t^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2)}} \sigma_5, \end{aligned} \tag{4.12}$$

$$a_3 = \frac{v_d}{\sqrt{v_d^2 + v_t^2}} \sigma_3 + \frac{v_t}{\sqrt{v_d^2 + v_t^2}} \sigma_4, \tag{4.13}$$

$$a_4 = -\frac{v_t}{\sqrt{v_d^2 + v_t^2}} \sigma_3 + \frac{v_d}{\sqrt{v_d^2 + v_t^2}} \sigma_4, \tag{4.14}$$

$$\begin{aligned} a_5 =& \frac{v_t v_d}{\sqrt{v_t^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2}} \sigma_1 + \frac{v_t v_u}{\sqrt{v_t^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2}} \sigma_2 \\ &+ \frac{v_u v_d}{\sqrt{v_t^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2}} \sigma_5, \end{aligned} \tag{4.15}$$

and their masses satisfy

$$m_{a_1}^2 = m_{a_2}^2 = m_{a_3}^2 = 0, \tag{4.16}$$

$$m_{a_4}^2 = -\frac{v_d^2 + v_t^2}{v_d v_t} m_{34}^2, \tag{4.17}$$

$$m_{a_5}^2 = -\frac{A(v_t^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2)}{\sqrt{2} v_d v_u v_t}. \tag{4.18}$$

a_1, a_2 and a_3 are Goldstone bosons.

C. Mixing of ρ_i

From the Higgs potential V_{Higgs}, we have

$$V_{\mathrm{Higgs}} \ni \frac{1}{2} \rho_i [M_\rho^2]_{i,j} \rho_j, \tag{4.19}$$

$$M_\rho^2 = \begin{pmatrix} -\frac{A v_d v_t}{\sqrt{2} v_u} + 2 l_3 v_u^2 & \frac{A v_t}{\sqrt{2}} + l_{23} v_d v_u & \frac{A v_d}{\sqrt{2}} + l_{13} v_u v_t & y_3 v_u v_t & y_3 v_u v_d \\ \frac{A v_t}{\sqrt{2}} + l_{23} v_d v_u & -\frac{A v_u v_t}{\sqrt{2} v_d} + 2 l_2 v_d^2 & \frac{A v_u}{\sqrt{2}} + l_{12} v_d v_t & y_2 v_d v_t & y_2 v_d^2 \\ \frac{A v_d}{\sqrt{2}} + l_{13} v_u v_t & \frac{A v_u}{\sqrt{2}} + l_{12} v_d v_t & -\frac{A v_d v_u}{\sqrt{2} v_t} + 2 l_1 v_t^2 & y_1 v_t^2 & y_1 v_d v_t \\ y_3 v_u v_t & y_2 v_d v_t & y_1 v_t^2 & -\frac{v_t}{v_d} m_{34}'^2 & m_{34}'^2 \\ y_3 v_u v_d & y_2 v_d^2 & y_1 v_d v_t & m_{34}'^2 & -\frac{v_d}{v_t} m_{34}'^2 \end{pmatrix}, \tag{4.20}$$

$$m_{34}'^2 = -\frac{1}{2} l'_{12} v_d v_t + \frac{A v_u}{\sqrt{2}} - y_{12} v_d v_t. \tag{4.21}$$

The lightest eigenstate,

$$h_1 = -\frac{v_d}{\sqrt{v_d^2 + v_t^2}} \rho_3 + \frac{v_t}{\sqrt{v_d^2 + v_t^2}} \rho_4, \tag{4.22}$$

is massless, which is a Goldsten boson.

The next to the lightest eigenstate is the SM Higgs boson, whose mass M_H should be 125 GeV. The independent parameters in the Higgs potential affecting M_H are $\tan\theta$, k, l_1, l_2, l_3, l_{12}, l_{13}, l_{23}, l'_{12}, y_1, y_2, y_3, y_{12}, and A. All these parameters except A are dimensionless. For simplicity, in Fig. 1, we show the dependence of M_H on (A, l_3) and $(\tan\theta, k)$ respectively while fixing other parameters.

V. NEUTRINO MASS, MIXING, AND FCNC

From Eq. (2.21), the neutrino mass matrix in the basis $(\nu_L, \nu'_L, \nu_R'^c, N, N_s, N'_s, N')$ is

$$M = \begin{bmatrix} 0 & 0 & 0 & \frac{(y^{\nu T} - y^\nu) v_u}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & \frac{y^{L'} v_t}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & \frac{y^{L'T} v_t}{\sqrt{2}} & 0 & \frac{y^{eT} v_d}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{(y^\nu - y^{\nu T}) v_u}{\sqrt{2}} & 0 & \frac{y^e v_d}{\sqrt{2}} & 0 & \frac{y^N v_t}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & \frac{y^{NT} v_t}{\sqrt{2}} & M_s & M_{ss'} & 0 \\ 0 & 0 & 0 & 0 & M_{ss'}^T & M'_s & \frac{y^{N'T} v_t}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & \frac{y^{N'} v_t}{\sqrt{2}} & 0 \end{bmatrix}. \tag{5.1}$$

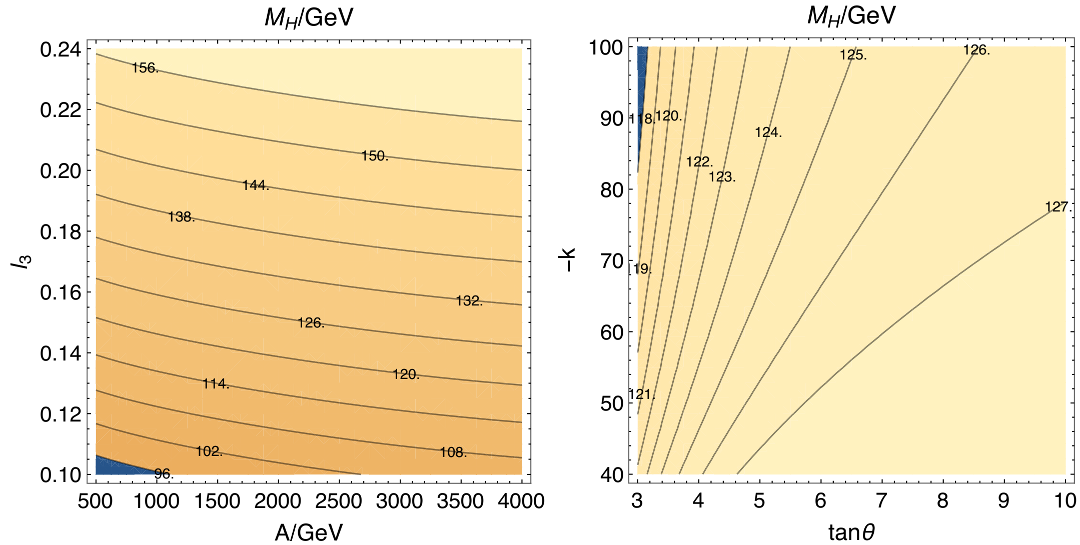

FIG. 1. Higgs boson mass. On the $A - l_3$ plane, we choose that $\tan\theta = 6$, $k = -60$, and all other dimensionless parameters in $V_{\rm Higgs}$ are 0.1. On the $\tan\theta - k$ plane, we choose that $A = 1$ TeV, $l_3 = 0.16$, and all other dimensionless parameters in $V_{\rm Higgs}$ are 0.1.

Every element in M is a 3×3 matrix. Because $(y^\nu - y^{\nu T})$ is an antisymmetric matrix, we have

$$\det[M] = 0, \tag{5.2}$$

which means the lightest neutrino eigenstate is massless.

For simplicity, we choose $M_{ss'}$ to be a zero matrix. In the limits of $\tan\theta \gg 1$ and $|k| \gg 1$, we approximately get that (ν_L, N, N_s) are only mixing with themselves and the mass matrix is

$$M' = \begin{bmatrix} 0 & \frac{(y^{\nu T} - y^\nu) v_u}{\sqrt{2}} & 0 \\ \frac{(y^\nu - y^{\nu T}) v_u}{\sqrt{2}} & 0 & \frac{y^N v_t}{\sqrt{2}} \\ 0 & \frac{y^{N^T} v_t}{\sqrt{2}} & M_s \end{bmatrix}. \tag{5.3}$$

We define $M_D = \frac{(y^{\nu T} - y^\nu) v_u}{\sqrt{2}}$ and $M_N = \frac{y^N v_t}{\sqrt{2}}$. Notice that the situation here looks very similar to the littlest inverse seesaw (LIS) model [31,32], in which the elements of M_s are very small to generate the very small neutrino masses. Since $\det[M_D]$ is zero, the lightest eigenstate of the mixing of ν_L, N and N_s is massless.

The three light eigenvalues of $M'^\dagger M'$ forms the SM neutrino mass squares, which are constrained by neutrino oscillation experiments. According to [31], in the case that $M_D, M_s \ll M_N$, the three light neutrino mass squares are eigenvalues of $M_\nu^\dagger M_\nu$ with

$$M_\nu = M_D (M_N^T)^{-1} M_s M_N^{-1} M_D^T. \tag{5.4}$$

For simplicity, we set M_N and M_s to be diagonal, which are

$$M_N = v_t \mathrm{diag}[c_N, c_N, c_N], \tag{5.5}$$

$$M_s = \mathrm{diag}[k_1, k_2, k_3], \tag{5.6}$$

Since M_D is antisymmetric, it can be written as

$$M_D = v_u \begin{bmatrix} 0 & d_1 & d_2 \\ -d_1 & 0 & d_3 \\ -d_2 & -d_3 & 0 \end{bmatrix}. \tag{5.7}$$

So we have

$$M_\nu = \frac{v_u^2}{v_t^2 c_N^2} \begin{bmatrix} d_1^2 k_2 + d_2^2 k_3 & d_2 d_3 k_3 & -d_1 d_3 k_2 \\ d_2 d_3 d_3 & d_1^2 d_1 + d_3^2 k_3 & d_1 d_2 k_1 \\ -d_1 d_3 k_2 & d_1 d_2 k_1 & d_1^2 k_1 + d_3^2 k_3 \end{bmatrix}. \tag{5.8}$$

Suppose eigenvalues of $M_\nu^\dagger M_\nu$ are $m_1^2 = 0$, m_2^2, and m_3^2. However, we can always rescale d_i ($i = 1$, 2, 3) and k_j ($j = 1$, 2, 3) to $10^{-R_D} d_i$ and $R_s k_j$ without changing the neutrino mixing pattern and $\frac{m_3}{m_2}$. But the masses will be changed to $10^{-2R_D} R_s m_i (i =, 1, 2, 3)$.

Because the lightest neutrino in our model is massless, we should choose appropriate values of a_i, k_j, c_N, $\tan\theta$, and k to give

$$U_\nu^\dagger M_\nu^\dagger M_\nu U_\nu = \mathrm{diag}[0, m_2^2 = \Delta m_{21}^2, m_3^2 = \Delta m_{31}^2], \tag{5.9}$$

where U_ν is parametrized by θ_{12}, θ_{13}, θ_{23}, and δ, i.e., the normal hierarchy (NH) for neutrino masses. We choose

$$(d_1, d_2, d_3) = 10^{-R_D}(0.49, 0.29, 0.82), \tag{5.10}$$

$$(k_1, k_2, k_3) = R_s(0.33, 0.038 e^{0.36\pi i}, -0.027 e^{0.13\pi i}), \tag{5.11}$$

TABLE I. Model and experimental values of the light active neutrino masses, leptonic mixing angles, and CP violating phase for the scenario of the NH neutrino masses [33,34].

Observable	Model	bpf $\pm 1\sigma$	bpf $\pm 1\sigma$
$\Delta m^2_{21}(10^{-5}\ \mathrm{eV}^2)$	7.36	$7.55^{+0.20}_{-0.16}$	$7.39^{+0.21}_{-0.20}$
$\Delta m^2_{31}(10^{-3}\ \mathrm{eV}^2)$	2.53	2.50 ± 0.03	$2.525^{+0.033}_{-0.031}$
$\theta^{(l)}_{12}(°)$	33.83	$34.5^{+1.2}_{-1.0}$	$33.82^{+0.78}_{-0.76}$
$\theta^{(l)}_{13}(°)$	8.57	$8.45^{+0.16}_{-0.14}$	$8.61^{+0.12}_{-0.13}$
$\theta^{(l)}_{23}(°)$	49.82	$47.9^{+1.0}_{-1.7}$	$49.7^{+0.9}_{-1.1}$
$\delta^{(l)}_{CP}(°)$	-142.05	-142^{+38}_{-27}	217^{+40}_{-28}

where R_s is determined by $\tan\theta$, k, c_N, and R_D to give the right neutrino masses. For example, when $\tan\theta = 6$, $k = -60$, $c_N = -1$ and $R_D = 1$, R_s needs to be 1.8×10^{-4} GeV, giving us the three mixing angles, CP violating phase δ and neutrino masses in Table I.

Next, we shall discuss the implication of the $3-3-1$ model in the charged lepton flavor changing decays. There are in total three processes, which are $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$. The branch ratio of lepton e_i decaying to lepton e_j is

$$\mathrm{BR}(e_i \to e_j) = \frac{\alpha_W^3 m_{e_i}^5 s_W^2}{256\pi^2 \Gamma_i}\left|\sum_{k=1}^{k=9}\left(U^\dagger_{j,k}U_{k,i}G\left(\frac{m^2_{N_k}}{M^2_W}\right)\right.\right.$$
$$\left.\left.\times\frac{1}{M_W^2} + U^\dagger_{j+3,k}U_{k,i+3}\sum_{k=1}^{k=9}G\left(\frac{m^2_{N_k}}{M^2_{W'}}\right)\frac{1}{M^2_{W'}}\right)\right|^2, \tag{5.12}$$

where $U^\dagger M'^\dagger M' U = \mathrm{diag}[m^2_{N_1}, m^2_{N_2}, \ldots, m^2_{N_9}]$. Experimental results ask us that the branch ratio of charged lepton decay should satisfy

$$\mathrm{BR}(\mu \to e\gamma) \le 4.2 \times 10^{-13}, \tag{5.13}$$

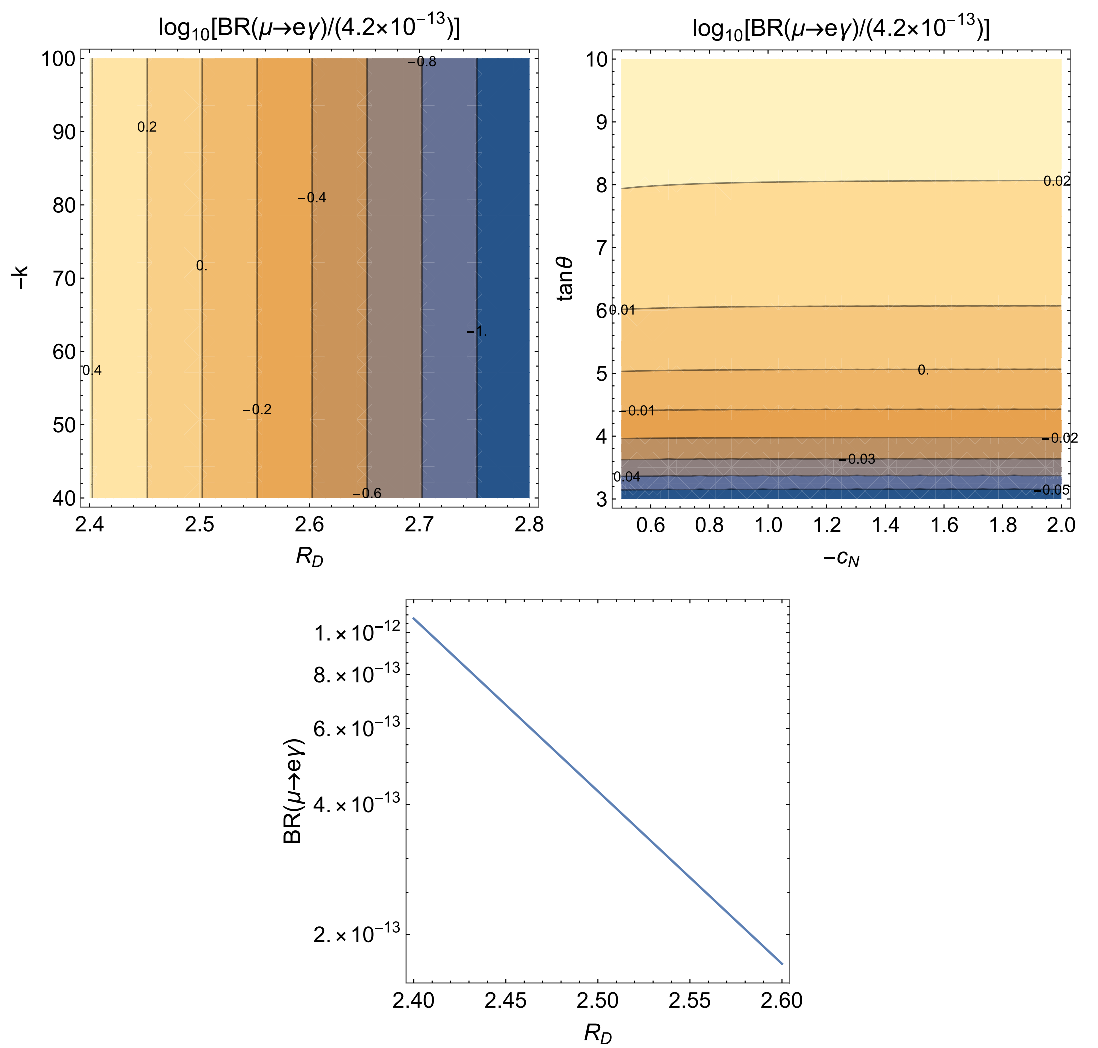

FIG. 2. $\mathrm{BR}(\mu \to e\gamma)$. On the $R_D - k$ plane, we choose that $c_N = -1$, $\tan\theta = 6$. On the $c_N - \tan\theta$ plane, we choose that $k = -60$, $R_D = 2.5$. The curve of $\mathrm{BR}(\mu \to e\gamma)$ is got when $\tan\theta = 6$, $k = -60$, $c_N = -1$.

$$\mathrm{BR}(\tau \to \mu\gamma) \leq 4.4 \times 10^{-8}, \tag{5.14}$$

$$\mathrm{BR}(\tau \to e\gamma) \leq 3.3 \times 10^{-8}. \tag{5.15}$$

Independent parameters influencing $\mathrm{BR}(e_i \to e_j\gamma)$ are $\tan\theta$, k, R_D, and c_N, while R_s is determined by other parameters to give the right neutrino masses. In Fig. 2, we show the dependence of $\mathrm{BR}(\mu \to e\gamma)$ on these parameters. We find that $\mathrm{BR}(\mu \to e\gamma)$ mainly depends on R_D. To make that $\mathrm{BR}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$, R_D needs to be larger than 2.5, which means that $(d_1, d_2, d_3) < (1.55, 0.92, 2.81) \times 10^{-3}$. In the case that $R_D \sim 2.5$, $\mathrm{BR}(\tau \to \mu\gamma)$, and $\mathrm{BR}(\tau \to e\gamma)$ are around 10^{-14} and 10^{-13} respectively. Our model has more parameters in the neutrino mass matrix M than traditional 331 models [1–21], and then it is easier to satisfy these constraints from lepton flavor changing decays.

VI. UNIFICATION OF GAUGE COUPLINGS

The renormalization group equation (RGE) for gauge coupling is

$$\mu \frac{dg_i}{d\mu} = \sum_n \frac{1}{(16\pi^2)^n} \beta_i^{(n)}, \tag{6.1}$$

where i stands for the i-loop correction in RGE running. In this section, we consider two-loop correction. Equations of 1-loop and 2-loop corrections are

$$\beta_g^{(1)} = b_i g_i^3, \tag{6.2}$$

$$\beta_g^{(2)} = B_{ij} g_j^2 + \sum_\alpha d_i^\alpha \mathrm{Tr}[y^\alpha y^{\alpha\dagger}], \tag{6.3}$$

where $\alpha = d, u, D, \nu, e, L', N, N'$.

In our model, we get

$$b = \left[\frac{13}{2}, -\frac{9}{2}, -5\right], \tag{6.4}$$

$$B = \begin{bmatrix} 6 & 20 & 12 \\ \frac{5}{2} & 23 & 12 \\ \frac{3}{2} & 12 & 12 \end{bmatrix}, \tag{6.5}$$

$$d = \begin{bmatrix} -\frac{3}{4} & -3 & -\frac{3}{4} & -2 & -\frac{5}{2} & -\frac{5}{2} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{3}{2} & -\frac{3}{2} & -\frac{3}{2} & -4 & -2 & -2 & -\frac{1}{2} & -\frac{1}{2} \\ -3 & -3 & -3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{6.6}$$

To make gauge couplings unify at the GUT scale, we add two fermion multiplets, FA and FA', as well as two scalar multiplets, SA and T' in high scale. The details are

$$(1,8,0)\colon FA, \quad \Delta b = (0,2,0), \quad \Delta B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 48 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{6.7}$$

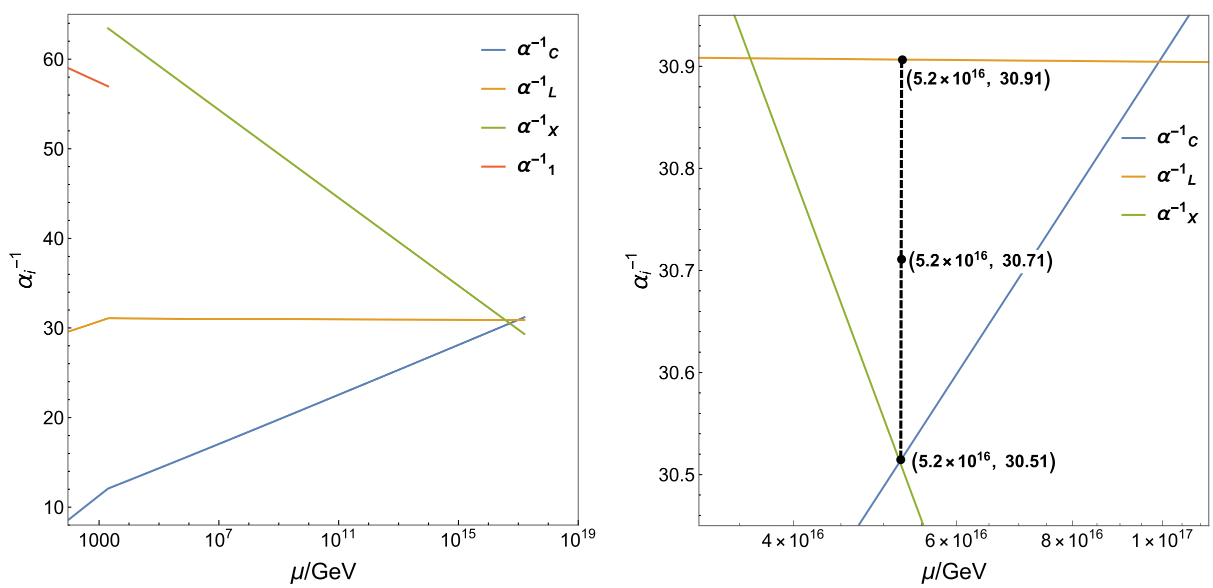

FIG. 3. Gauge coupling unification, where $\alpha_i = \frac{g_i^2}{4\pi}$ and $g_1^2 = \frac{5}{3} g_Y^2$.

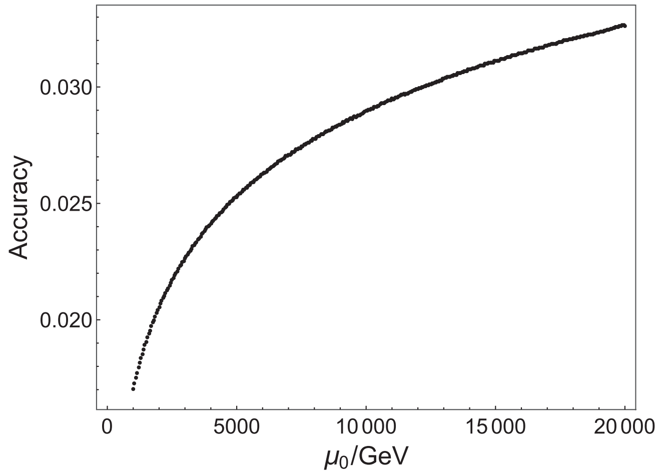

FIG. 4. Accuracy of gauge coupling unification. We assume that all the new particles beyond the SM have universal mass around the energy scale of μ_0.

$$(1,8,0): FA', \quad \Delta b = (0,2,0), \quad \Delta B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 48 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{6.8}$$

$$(8,1,0): SA, \quad \Delta b = (0,0,1), \quad \Delta B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 42 \end{bmatrix}, \tag{6.9}$$

$$\left(1,\bar{3},\frac{-1}{2\sqrt{3}}\right): T', \quad \Delta b = \left(\frac{1}{12},\frac{1}{6},0\right), \quad \Delta B = \begin{bmatrix} \frac{1}{12} & \frac{4}{3} & 0 \\ \frac{1}{6} & \frac{11}{3} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{6.10}$$

FA and FA' can decay via the Yukawa coupling terms $FAf_i(T')^*$, $FA'f_i(T')^*$, $FAf'_i(T')^*$, and $FA'f'_i(T')^*$. In principle, we can introduce the Z_2 symmetry where FA, FA', and $(T')^*$ are odd while all the other particles are even. Thus, the lightest particle of FA, FA', and $(T')^*$ can be a dark matter candidate. In addition, SA can decay into the SM quarks only at nonrenormalizable level, for example, $SAF_iu^c_{Rj}T_u/M_*$, $SAF_id^c_{Rj}T_d/M_*$, and $SAF_iD^c_{Rj}T/M_*$. Thus, we have two cases. First, SA can be a dark matter candidate if Z_2 symmetry is imposed to forbid SA decaying to quarks. We will leave this part of work in the future. For simplicity, we make all the particles beyond the SM take part in the RGE running at the energy scale of 2 TeV, then the gauge coupling unification can be satisfied with accuracy of 0.65% at the energy scale of 5.2×10^{16} GeV, which is shown in Fig. 3. We define the accuracy of gauge coupling unification as $|\alpha_X^{-1}(\mu') - \alpha_C^{-1}(\mu')|/\alpha_C^{-1}(\mu')$ with μ' satisfying $\alpha_X^{-1}(\mu') = \alpha_L^{-1}(\mu')$, which is different from our choice of the accuracy of unification of gauge couplings in Fig. 3 and Fig. 5. Assuming all the new particles beyond the SM have universal mass around the energy scale of μ_0, we present the relation of the accuracy of gauge coupling unification and μ_0 in Fig. 4. μ_0 needs to be smaller than 12 TeV to achieve the gauge coupling unification with an accuracy better than 3%, which implies that the mass of the dark matter candidate needs to be smaller than 12 TeV.

Alternatively, to make SA decay, we can add two fermion multiplets in 6 and $\bar{6}$ representation of the $SU(6)$ gauge group respectively, then the gauge coupling unification can be satisfied with accuracy of 0.68% at the energy scale of 6.2×10^{16} GeV, which is shown in Fig. 5. Also, we make all the particles beyond the SM take part in the RGE running at the energy scale of 2 TeV.

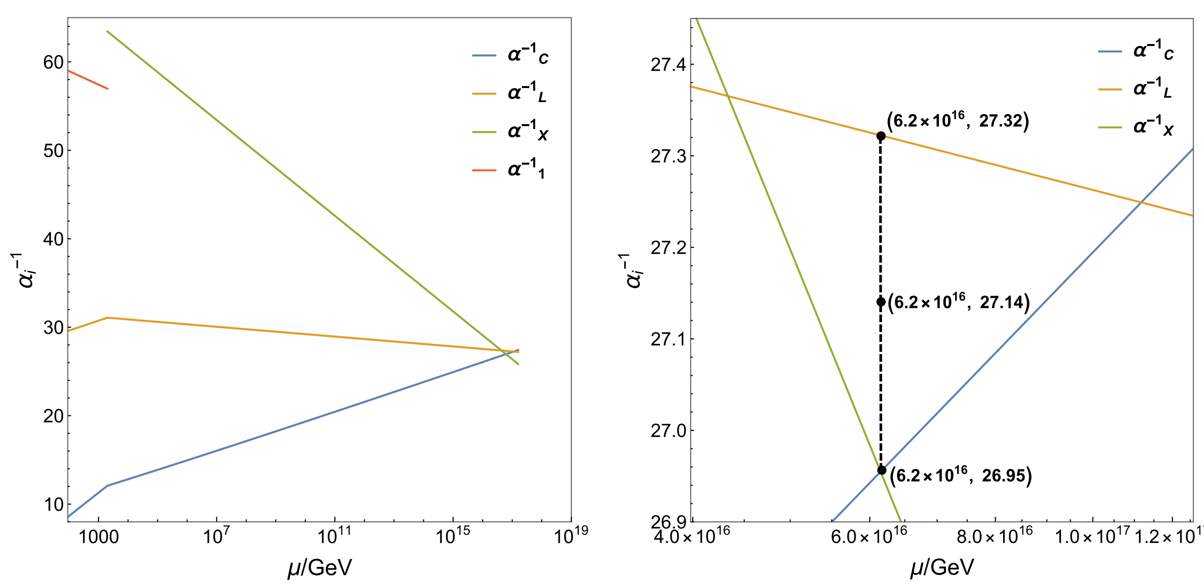

FIG. 5. Gauge coupling unification, where $\alpha_i = \frac{g_i^2}{4\pi}$ and $g_1^2 = \frac{5}{3} g_Y^2$.

VII. CONCLUSIONS

We have proposed a new $SU(3)_C \times SU(3)_L \times U(1)_X$ model, in which gauge symmetry can be realized from $SU(6)$ breaking. The SM fermions in each of the three generations come from two $\bar{6}$ representations and one 15 representation of the $SU(6)$ gauge group besides two singlets of the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge group. There are three scalar multiplets, where two come from $\bar{6}$ representations of $SU(6)$ and one from 15 representation. And their VEVs are v_u, v_d, and v_t, respectively. There are additional gauge bosons, $W^{\pm\prime}$, Z', and V/V^*, in our model besides the SM gauge bosons. v_t needs to be larger than 10 TeV to make the mass of Z' larger than 4.5 TeV. It is easy to give the 125 GeV Higgs boson mass when we set all the dimensionless parameters in the Higgs potential to be ~ 0.1 and A to be ~ 1 TeV. When $M_{ss'}$ are set to be a zero matrix and in the limits of $\tan\theta \gg 1$ and $|k| \gg 1$, the mixing of ν_L, N and N_s is the same as in the littlest inverse seesaw model. The lightest neutrino in our model is massless. With parameters in y^ν, y^N and M_s set to be appropriate values, we obtained the light active neutrino masses, leptonic mixing angles, and CP violating phase highly consistent with the experimental data for the scenario of NH neutrino mass. To make $\mathrm{BR}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$, parameters in y^ν needs to be smaller than $\sim 10^{-3}$, and in this case $\mathrm{BR}(\tau \to \mu\gamma)$ and $\mathrm{BR}(\tau \to e\gamma)$ are around 10^{-14} and 10^{-13}, respectively. With additional two fermion multiplets, FA and FA', as well as two scalar multiplets, SA and T', the gauge coupling unification can be realized with accuracy of 0.65% at the energy scale of 5.2×10^{16} GeV. SA can be a dark matter candidate if Z_2 symmetry is imposed. Alternatively, we can add two fermionic multiplets in 6 and $\bar{6}$ representations of the $SU(6)$ gauge group to make SA decay, then the gauge coupling unification can be satisfied with accuracy of 0.68% at the energy scale of 6.2×10^{16} GeV.

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