$L_\mu - L_\tau$ effects to quarks and leptons from flavor unification

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In the family grand unification theory (GUT) models, we propose that gauge U(1)'s beyond the minimal GUT gauge group are family gauge symmetries. For the symmetry $L_\mu - L_\tau$, i.e., $Q_2 - Q_3$ in our case, to be useful for the LHC anomaly, we discuss an SU(9) family GUT and also present an example in Georgi's SU(11) family GUT.

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I. INTRODUCTION

The most interesting problem remaining in the Standard Model (SM) is the flavor or family problem. The family problem forces the symmetry of all of the massless chiral fields surviving below the grand unification theory (GUT) scale M_{GUT} or the Planck scale M_{P}. Chiral fields with quantum numbers consistent with the observed weak and electromagnetic phenomena were a crucial achievement of the SM. With electromagnetic and charged currents, leptons need representations which are a doublet or bigger. A left-handed lepton doublet (ν_e, e) alone is not free of gauge anomalies because the observed electromagnetic charges are not $\pm\frac{1}{2}$. The anomalies from the fractional electromagnetic charges of the u and d quarks add up to make the total anomaly from the first family vanish [1,2]. In view of the necessity to jointly use both leptons and quarks to cancel gauge anomalies in the SM, we can see that GUTs are fundamentally needed beyond other aesthetic viewpoints. It is very difficult to obtain another kind of chiral model free of gauge anomalies. If another chiral model is found consistently with some observed fact, that model should hold some truth. The same gauge structures of the first family, $\{\nu_e, e, u, d\}$, repeat two more times in the μ family and the τ family.

A correct treatment of flavors is necessary not only in the familiar fields of particle theory and high-energy physics but also in astronomy and, especially, cosmology. Big bang nucleosynthesis calculations can place upper limits on the number of active neutrinos. The mixings of six flavors of quark allow a CP violating phase which is successful in agreeing precisely with data on CP violation in K and B decays, yet the correct derivation of baryogenesis which itself needs CP violation and the tiny ratio $\eta = (\Delta n_B/n_\gamma) \simeq 9\times 10^{-11}$ remains challenging, especially as to whether the CP violation known in quark flavor mixing can suffice to explain the matter-antimatter asymmetry of the Universe. These are merely two examples of cosmological applications of flavor theory.

Recent phenomenological studies on the flavor problem have centered around two questions: (i) Why are there three families? (ii) What are the symmetries giving the observed Cabibbo-Kobayashi-Maskawa (CKM) and the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices?

For family unification, the first attempts were to use spinors of SO(10) in extended SO($4n+2$) [3,4], as a product of GUT unification of families [5], and in higher dimensions [6]. None of these examples belong to the chiral class of SO($4n+2$) gauge theories in four spacetime dimensions (4D): models in [3,4] decompose to vectorlike spinors of SO(10) in 4D, the model of [5] uses discrete factor groups in addition to the gauge symmetry, and Refs. [6,7] worked in higher dimensions. The chiral class in 4D was formulated in simple gauge groups 40 years ago by Georgi [8]. Along with the scheme of [8], some interesting models appeared in SO(14) [9] and in SU(N) gauge groups [10,11].

The second part was discussed recently in [12–15] in relation to the CKM and PMNS matrices. Until recently, no significant deviation from the CKM and PMNS matrices has appeared. If some deviation were to be observed, then it might predict beyond the SM (BSM) physics, probably in fourth family sterile neutrinos or in the important scalar interactions presented in this paper.

The obvious family dependences are in the masses and the CKM and PMNS matrices. Given these, the next level is to check lepton family universality in the decays of mesons. With hundreds of millions of B decays having already been found at the LHC, it is possible to check the universality of a ratio of the type

$$R_H = \frac{\mathcal{B}(B \to H\mu^+\mu^-)}{\mathcal{B}(B \to He^+e^-)}, \tag{1}$$

where H represents a hadron. In the last few years, observation of an anomaly in R_H with a 2.5σ level significance [16,17] (see also [18]) attracted a great deal of attention [19–25]. In the leptonic sector, the lepton family dependence was used to make the BNL $(g-2)_\mu$ observation draw near the SM prediction. This required an additional interaction for the muon family. If it is a gauge U(1) interaction, cancellation of anomalies necessitates contributions from other family members, for example, in the form of a quantum number such as $L_\mu - L_\tau$.

Within family GUT models, $L_\mu - L_\tau$ symmetry can affect the prediction of R_H phenomenology since the quantum number $L_\mu - L_\tau$ applies also to the quark members in the same family.

II. GRAND UNIFICATION OF FAMILIES

We use a grand unified theory with gauge group SU($N \geq 8$), focusing on SU(9) and SU(11). The conventional SU(5) will be naturally embedded into SU(N) in the sense that the defining representation is $N \supset 5 + (N-5)$ singlets. This is what was used in the models of [8–11]. Here we shall generalize the idea to include flavor lepton numbers L_e, L_μ, L_τ as the sixth, seventh, and eighth components of the defining representation of SU(N), while the further $(N-8)$ components will be treated as before. This will enable the special combination $(L_\mu - L_\tau)$ to play a special role. We name this type of model a family GUT, which is our main subject.

It will be necessary to introduce a few new states at the TeV scale and these will have a small effect on the logarithmic running of the gauge couplings. However, since this is a nonsupersymmetric family GUT the gauge coupling unification at the GUT scale is expected to be good but not precise. Precise unification has in our opinion been overemphasized because unless and until proton decay is discovered we cannot know what the GUT scale is, and precise unification by itself is insufficient unless the GUT scale fits with the proton lifetime. The effects of additional light states on the logarithmic running were studied in detail in [26], and from there one can deduce that in the present case there is only a small change and the precision of the unification will, in general, not be very different in family GUTs than in GUTs.

Let us use tensor notation such that the index $A = \{1, 2, \ldots, N\}$ is split into the GUT index $\alpha = \{1, 2, \ldots, 5\}$, family indices $I = \{6 = \text{electron family}, 7 = \text{muon family}, 8 = \text{tau family}\}$, and $\{9, \ldots, N\}$ for dummy numbers. We will use only the completely antisymmetric representations so that higher-dimensional quarks such as $\mathbf{6}, \mathbf{6}^*$ do not appear.

Consider the three family indices, $I = 6, 7, 8$, for U(3) representations, which are equivalently used as $I \to \{e \equiv 1, \mu \equiv 2, \tau \equiv 3\}$. The upper ($X^{IJ\cdots}$) and lower indices ($X_{IJ\ldots}$) are distinguished. The Levi-Civita symbols, ϵ^{IJK} and ϵ_{IJK} for the U(3) group, will be used to raise and lower the indices. Thus, $X^{(0,1,1)} = X_{(1,0,0)}$, etc. Note also that complex conjugation corresponds to taking the Hermitian conjugate, $X^{I*} = X_{-I}$.

Now we consider the following completely antisymmetric representations, which split into

$$\begin{aligned}
\Psi^A &= \psi^\alpha \oplus \mathbf{1}^{(1,0,0)} \oplus \mathbf{1}^{(0,1,0)} \oplus \mathbf{1}^{(0,0,1)} \oplus \mathbf{1}^{(0,0,0)} \oplus \cdots, \\
\Psi^{AB} &= \psi^{\alpha\beta(0,0,0)} \oplus \psi^{\alpha(1,0,0)} \oplus \psi^{\alpha(0,1,0)} \oplus \psi^{\alpha(0,0,1)} \oplus \psi^{\alpha(0,0,0)} \oplus \cdots \\
&\quad \oplus \mathbf{1}^{(1,1,0)} \oplus \mathbf{1}^{(1,0,1)} \oplus \mathbf{1}^{(0,1,1)} \oplus \mathbf{1}^{(1,0,0)} \oplus \mathbf{1}^{(0,1,0)} \oplus \mathbf{1}^{(0,0,1)} \oplus \mathbf{1}^{(0,0,0)} \oplus \cdots, \\
\Psi^{ABC} &= \psi^{\alpha\beta\gamma} \oplus \psi^{\alpha\beta(1,0,0)} \oplus \psi^{\alpha\beta(0,1,0)} \oplus \psi^{\alpha\beta(0,0,1)} \oplus \psi^{\alpha\beta(0,0,0)} \oplus \cdots \\
&\quad \oplus \psi^{\alpha(1,1,0)} \oplus \psi^{\alpha(1,0,1)} \oplus \psi^{\alpha(0,1,1)} \oplus \psi^{\alpha(1,0,0)} \oplus \psi^{\alpha(0,1,0)} \oplus \psi^{\alpha(0,0,1)} \oplus \psi^{\alpha(0,0,0)} \oplus \cdots \\
&\quad \oplus \mathbf{1}^{(1,1,1)} \oplus \mathbf{1}^{(1,1,0)} \oplus \mathbf{1}^{(1,0,1)} \oplus \mathbf{1}^{(1,0,0)} \oplus \mathbf{1}^{(0,1,0)} \oplus \mathbf{1}^{(0,0,1)} \oplus \mathbf{1}^{(0,0,0)} \oplus \cdots, \\
\Psi^{ABCD} &= \psi^{\alpha\beta\gamma\delta} \oplus \psi^{\alpha\beta\gamma(1,0,0)} \oplus \psi^{\alpha\beta\gamma(0,1,0)} \oplus \psi^{\alpha\beta\gamma(0,0,1)} \oplus \psi^{\alpha\beta\gamma(0,0,0)} \ldots \\
&\quad \oplus \psi^{\alpha\beta(1,1,0)} \oplus \psi^{\alpha\beta(1,0,1)} \oplus \psi^{\alpha\beta(0,1,1)} \oplus \psi^{\alpha\beta(1,0,0)} \oplus \psi^{\alpha\beta(0,1,0)} \oplus \psi^{\alpha\beta(0,0,1)} \oplus \psi^{\alpha\beta(0,0,0)} \oplus \cdots \\
&\quad \oplus \psi^{\alpha(1,1,1)} \oplus \psi^{\alpha(1,1,0)} \oplus \psi^{\alpha(1,0,1)} \oplus \psi^{\alpha(1,0,0)} \oplus \psi^{\alpha(0,1,0)} \oplus \psi^{\alpha(0,0,1)} \oplus \psi^{\alpha(0,0,0)} \\
&\quad \oplus \mathbf{1}^{(1,1,1)} \oplus \mathbf{1}^{(1,1,0)} \oplus \mathbf{1}^{(1,0,1)} \oplus \mathbf{1}^{(0,1,1)} \oplus \mathbf{1}^{(1,0,0)} \oplus \mathbf{1}^{(0,1,0)} \oplus \mathbf{1}^{(0,0,1)} \oplus \text{etc.}
\end{aligned} \tag{2}$$

The example for SU(9) given in [11] thus becomes, after removing vectorlike representations,

$$\begin{aligned}\Psi^{ABC} \oplus 9\Psi_A = {} & \psi_{\alpha\beta} \oplus \psi^{\alpha\beta(1,0,0)} \oplus \psi^{\alpha\beta(0,1,0)} \oplus \psi^{\alpha\beta(0,0,1)} \oplus \psi^{\alpha\beta(0,0,0)} \\ & \oplus \psi^{\alpha(1,1,0)} \oplus \psi^{\alpha(1,0,1)} \oplus \psi^{\alpha(0,1,1)} \oplus \psi^{\alpha(1,0,0)} \oplus \psi^{\alpha(0,1,0)} \oplus \psi^{\alpha(0,0,1)} \\ & \oplus \mathbf{1}^{(1,1,1)} \oplus \mathbf{1}^{(1,1,0)} \oplus \mathbf{1}^{(1,0,1)} \oplus \mathbf{1}^{(0,1,1)} \\ & \oplus 9\psi_\alpha \oplus 9\cdot \mathbf{1}_{(-1,0,0)} \oplus 9\cdot \mathbf{1}_{(0,-1,0)} \oplus 9\cdot \mathbf{1}_{(0,0,-1)} \oplus 9\cdot \mathbf{1}^{(0,0,0)} \\ = {} & \psi^{\alpha\beta(1,0,0)} \oplus \psi^{\alpha\beta(0,1,0)} \oplus \psi^{\alpha\beta(0,0,1)} \\ & \oplus \psi^{\alpha(1,1,0)} \oplus \psi^{\alpha(1,0,1)} \oplus \psi^{\alpha(0,1,1)} \oplus \psi^{\alpha(1,0,0)} \oplus \psi^{\alpha(0,1,0)} \oplus \psi^{\alpha(0,0,1)} \oplus 9\psi_{\alpha(0,0,0)} \\ & \oplus 8\cdot \mathbf{1}_{(-1,0,0)} \oplus 8\cdot \mathbf{1}_{(0,-1,0)} \oplus 8\cdot \mathbf{1}_{(0,0,-1)} \oplus 10\cdot \mathbf{1}^{(0,0,0)}. \end{aligned} \tag{3}$$

The remaining three lepton (doublet) families, out of the second line from the bottom of Eq. (3), will be $3\psi_{\alpha(0,0,0)}$, which do not carry $L_\mu - L_\tau$. To avoid confusion, we will always write quantum numbers of $L_\mu - L_\tau$ as subscripts within square brackets.

Georgi's family GUT model is for $N = 11$ [8],

$$\Psi^{ABCD} + \Psi_{ABC} + \Psi_{AB} + \Psi_A. \tag{4}$$

Defining the first three slots for

$$\mathrm{U}(1)_6 \times \mathrm{U}(1)_7 \times \mathrm{U}(1)_8,\ \text{or renamed as}\ \mathrm{U}(1)_1 \times \mathrm{U}(1)_2 \times \mathrm{U}(1)_3, \tag{5}$$

we obtain

$$\begin{aligned}\Psi^{ABCD} \to {} & \psi^{\alpha\beta\gamma\delta} \oplus \psi^{\alpha\beta\gamma(1,0,0)} \oplus \psi^{\alpha\beta\gamma(0,1,0)} \oplus \psi^{\alpha\beta\gamma(0,0,1)} \oplus 3\cdot \psi^{\alpha\beta\gamma(0,0,0)} \\ & \oplus \psi^{\alpha\beta(1,1,0)} \oplus \psi^{\alpha\beta(1,0,1)} \oplus \psi^{\alpha\beta(0,1,1)} \oplus 3\cdot \psi^{\alpha\beta(1,0,0)} \oplus 3\cdot \psi^{\alpha\beta(0,1,0)} \oplus 3\cdot \psi^{\alpha\beta(0,0,1)} \oplus 3\cdot \psi^{\alpha\beta}_{(0,0,0)} \\ & \oplus \psi^{\alpha(1,1,1)} \oplus 3\cdot \psi^{\alpha(1,1,0)} \oplus 3\cdot \psi^{\alpha(1,0,1)} \oplus 3\cdot \psi^{\alpha(0,1,1)} \oplus 3\cdot \psi^{\alpha(1,0,0)} \oplus 3\cdot \psi^{\alpha(0,1,0)} \oplus 3\cdot \psi^{\alpha}_{(0,0,1)} \oplus \psi^{\alpha(0,0,0)} \\ & \oplus 3\cdot \mathbf{1}^{(1,1,1)} \oplus 3\cdot \mathbf{1}^{(1,1,0)} \oplus 3\cdot \mathbf{1}^{(1,0,1)} \oplus 3\cdot \mathbf{1}^{(0,1,1)} \oplus \mathbf{1}^{(1,0,0)} \oplus \mathbf{1}^{(0,1,0)} \oplus \mathbf{1}^{(0,0,1)} \oplus \mathbf{1}^{(0,0,0)}, \\ \Psi^{ABC} \to {} & \psi^{\alpha\beta\gamma} \oplus \psi^{\alpha\beta(1,0,0)} \oplus \psi^{\alpha\beta(0,1,0)} \oplus \psi^{\alpha\beta(0,0,1)} \oplus 3\cdot \psi^{\alpha\beta(0,0,0)} \\ & \oplus \psi^{\alpha(1,1,0)} \oplus \psi^{\alpha(1,0,1)} \oplus \psi^{\alpha(0,1,1)} \oplus 3\cdot \psi^{\alpha(1,0,0)} \oplus 3\cdot \psi^{\alpha(0,1,0)} \oplus 3\cdot \psi^{\alpha(0,0,1)} \oplus 3\cdot \psi^{\alpha(0,0,0)} \\ & \oplus \mathbf{1}^{(1,1,1)} \oplus 3\cdot \mathbf{1}^{(1,1,0)} \oplus 3\cdot \mathbf{1}^{(1,0,1)} \oplus 3\cdot \mathbf{1}^{(0,1,1)} \oplus 3\cdot \mathbf{1}^{(1,0,0)} \oplus 3\cdot \mathbf{1}^{(0,1,0)} \oplus 3\cdot \mathbf{1}^{(0,0,1)} \oplus \mathbf{1}^{(0,0,0)}, \\ \Psi^{AB} \to {} & \psi^{\alpha\beta} \oplus \psi^{\alpha(1,0,0)} \oplus \psi^{\alpha(0,1,0)} \oplus \psi^{\alpha(0,0,1)} \oplus 3\cdot \psi^{\alpha(0,0,0)} \\ & \oplus \mathbf{1}^{(1,1,0)} \oplus \mathbf{1}^{(1,0,1)} \oplus \mathbf{1}^{(0,1,1)} \oplus 3\cdot \mathbf{1}^{(1,0,0)} \oplus 3\cdot \mathbf{1}^{(0,1,0)} \oplus 3\cdot \mathbf{1}^{(0,0,1)} \oplus 3\cdot \mathbf{1}^{(0,0,0)}, \\ \Psi^{A} \to {} & \psi^{\alpha} \oplus \mathbf{1}^{(1,0,0)} \oplus \mathbf{1}^{(0,1,0)} \oplus \mathbf{1}^{(0,0,1)} \oplus 3\cdot \mathbf{1}^{(0,0,0)}, \end{aligned} \tag{6}$$

which, removing vectorlike representations, leads to

$$\begin{aligned}\Psi^{ABCD} \oplus \Psi_{ABC} \oplus \Psi_{AB} \oplus \Psi_A = {} & 3\cdot \psi_{\alpha\beta(0,0,0)}(*) \oplus \psi^{(1,0,0)}_{\alpha\beta}(*) \oplus \psi^{(0,1,0)}_{\alpha\beta}(*) \oplus \psi^{(0,0,1)}_{\alpha\beta}(*) \oplus 2\cdot \psi^{\alpha\beta(1,0,0)}(**) \\ & \oplus 2\cdot \psi^{\alpha\beta(0,1,0)}(**) \oplus 2\cdot \psi^{\alpha\beta(0,0,1)}(**) \oplus \psi^{\alpha\beta(1,0,0)}(**) \oplus \psi^{\alpha\beta(0,1,0)}(**) \\ & \oplus \psi^{\alpha\beta(0,0,1)}(**) \oplus \psi^{(-1,0,0)}_{\alpha} \oplus \psi^{(0,-1,0)}_{\alpha} \\ & \oplus \psi^{(0,0,-1)}_{\alpha} \oplus 6\cdot \psi_{\alpha(0,0,0)} \oplus \psi_{\alpha(-1,0,0)} \oplus \psi_{\alpha(0,-1,0)} \oplus \psi_{\alpha(0,0,-1)} \\ & \oplus 3\cdot \psi^{\alpha(1,0,0)} \oplus 3\cdot \psi^{\alpha(0,1,0)} \oplus 3\cdot \psi^{\alpha(0,0,1)} \oplus 3\cdot \mathbf{1}^{(0,0,-1)} \\ & \oplus 3\cdot \mathbf{1}^{(0,-1,0)} \oplus 3\cdot \mathbf{1}^{(-1,0,0)} \oplus 4\cdot \mathbf{1}_{(-1,0,0)} \oplus 4\cdot \mathbf{1}_{(0,-1,0)} \oplus 4\cdot \mathbf{1}_{(0,0,-1)}. \end{aligned} \tag{7}$$

We want to define family numbers such that electron carries electron number, muon carries muon number, and tau carries tau number. Let S_i be SU(5)-singlet scalars.

A. Notation for $L_\mu - L_\tau$

Since ψ_α contains lepton doublets, it is better to define lepton family number by $\psi_{AB} \to \psi_{\alpha I}$. So, the lepton family number is defined by the subscript I: an electron doublet from $\psi_{\alpha 6}$, a muon doublet from $\psi_{\alpha 7}$, and a tau doublet from $\psi_{\alpha 8}$. For the $L^- \equiv L_\mu - L_\tau$ quantum numbers, we have $\psi_{\alpha[0]}$ for the electron family, $\psi_{\alpha[1]}$ for the muon family, and $\psi_{\alpha[-1]}$ for the tau family. We can use the U(3) Levi-Cevita symbol to raise or lower the family indices. For example, $\psi^{\alpha\beta IJ}$ is $\psi^{\alpha\beta}_K$, where K differs from I and J. Then $S_e^{(1,0,0)}$ is $S_{e[0]}$, etc. Let L^+ be the $L_\mu + L_\tau$ quantum number. $S_{e[0]}$ does not break L^+ and L^-. But the vacuum expectation value (VEV) of $S_1^{(0,1,1)}$ breaks $\mathrm{U}(1)_\mu \times \mathrm{U}(1)_\tau$ to $\mathrm{U}(1)_{\mu-\tau}$. With S_e and S_1, the following couplings combine some vectorlike pairs in the (**) lines of Eq. (7),

$$\psi_{\alpha\beta}^{(0,1,0)} S_e^{(1,0,0)} \psi^{\alpha\beta(0,0,1)}, \qquad \psi_{\alpha\beta}^{(0,0,1)} S_e^{(1,0,0)} \psi^{\alpha\beta(0,1,0)},$$
$$\psi_{\alpha\beta}^{(0,0,0)} S_1^{(0,1,1)} \psi^{\alpha\beta(1,0,0)}.$$

Therefore, from the (**) lines in Eq. (7) there remain, after the $\mathrm{U}(1)_{\mu-\tau}$ preserving VEVs of $\langle S_e^{(1,0,0)} \rangle$ and $\langle S_1^{(0,1,1)} \rangle$,

$$\psi_{\alpha\beta}^{(1,0,0)} \oplus 2\psi^{\alpha\beta(0,1,0)} \oplus 2\psi^{\alpha\beta(0,0,1)}, \tag{8}$$

which gives three families and one vectorlike pair. With the exact $L_\mu - L_\tau$ conservation, there is no way to remove the remaining vectorlike pair. We must break the gauge symmetry $\mathrm{U}(1)_{\mu-\tau}$ to remove the vectorlike pair. Let us introduce $S_\mu^{(0,1,0)}$ such that $\langle S_\mu \rangle \ll \langle S_e \rangle, \langle S_1 \rangle$. The $\mathrm{U}(1)_{\mu-\tau}$ breaking effects to low energy physics are subdominant compared to the $\mathrm{U}(1)_{\mu-\tau}$ preserving interactions. The coupling

$$\psi_{\alpha\beta}^{(1,0,0)} S_\mu^{(0,1,0)} \psi^{\alpha\beta(0,0,1)} \tag{9}$$

removes the vectorlike pair. Thus, we finally obtain the following three **10**'s[1]:

$$2\psi^{\alpha\beta(-1,2,-1)} \oplus \psi^{\alpha\beta(-1,-1,2)}. \tag{10}$$

[1]If we use $S_\tau^{(-1,-1,2)}$ instead of S_μ, we obtain $\psi^{\alpha\beta(-1,2,-1)} \oplus 2\psi^{\alpha\beta(-1,-1,2)}$. Here, we used the traceless family numbers as in Sec. III.

B. Changing chirality

The BSM contributions to the magnetic moments need a change of chirality [27]. The effective interaction for changing the chirality of the leptons is through $\psi_\alpha \leftrightarrow \psi^{\alpha\beta}$: $f^{(e)}\psi^{\alpha\beta}\psi_\alpha H_\beta^{(d)}$. In the SM, it gives masses to $Q_{\mathrm{em}} = -1$ leptons. If we consider only one Higgs doublet, there is no BSM contribution to the magnetic moments. We need more Higgs doublets. Let us denote this extra (inert) Higgs doublet without a VEV as H'_β, and its coupling to the muon family

$$h'_\mu \psi^{\alpha\beta(-1,2,-1)} \psi_\alpha^{(1,-2,1)} H'_\beta \tag{11}$$

introduces the BSM contribution to the magnetic moments. Note that $\psi_L^{\alpha\beta(0,1,0)}$ can be represented as $\psi_{\alpha\beta(0,-1,0),R}$ moving in the opposite direction. We used the muon quantum number to have an additional contribution to $(g-2)_\mu$. In Eq. (7), μ is in $\psi_\alpha^{(0,-1,0)}$ and μ^c is in $\psi^{\alpha\beta(0,1,0)} \equiv \psi_{\alpha\beta(0,-1,0)R}$, i.e., μ_L has muon number $+1$ and μ_R also has muon number $+1$. From Fig. 1, we estimate that

$$a'_\mu = \frac{(g-2)_\mu}{2} \propto e h'_{[22]} h'_{[23]}. \tag{12}$$

Here, m, M_1, and M_2 can be superheavy, but m_3 must be smaller than or at the electroweak scale.

The lhs and rhs contributions of Fig. 1 sandwiched between $\bar{u}(p')$ and $u(p)$ are

$$\mathrm{lhs} \propto m^2 m_3 e h'_{[22]} h'_{[23]} \int \frac{d^4k}{(2\pi)^4} \times \frac{(\not{k}' + M_1)\gamma_\mu(\not{k} + M_1)(\not{k} + M_2)}{(k'^2 - M_1^2)(k^2 - M_1^2)(k^2 - M_2^2)((p-k)^2 - m^2)^2}, \tag{13}$$

and we proceed similarly for the rhs. Here, we assumed that m^2 is the $H'_{\alpha[0]}$ mass and treated m_3 as the mass of $\psi_{\alpha\beta[+3]} \oplus \psi^\alpha_{[-3]}$, and

$$p = \text{momentum of } \mu_L, \qquad p' = \text{momentum of } \mu_L^c,$$
$$q = p' - p, \qquad k' = k + q. \tag{14}$$

Thus, from Fig. 1 the anomalous magnetic moment is estimated as

$$F_2(0) = \frac{e h'_{[22]} h'_{[23]} m^2 m_3}{8\pi^2 (M_2^2 - M_1^2)} \int_0^1 dx \int_0^x dy (y - x)\{\cdots\}, \tag{15}$$

where the ellipsis is

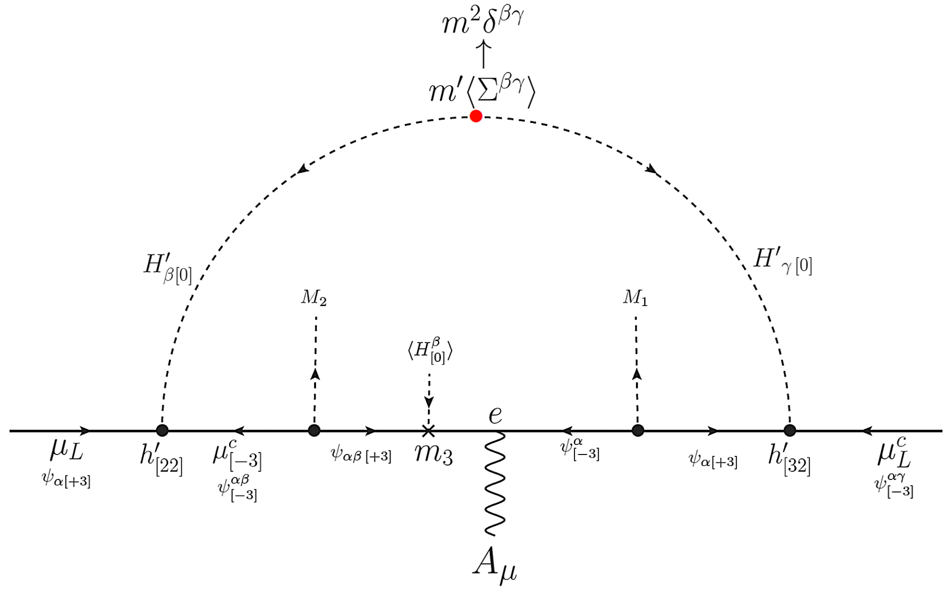

FIG. 1. A BSM contribution to $(g-2)_\mu$. m_3 can be placed on the lhs (as here) or rhs of the A_μ vertex, and $\langle H^\beta\rangle$ is a VEV of the SM Higgs doublet giving mass to μ. $\Sigma^{\beta\gamma}$ is an SU(5) singlet field $\propto \delta^{\beta\gamma}$.

$$\begin{aligned}\{\cdots\} = {}&(M_1M_2(x-y-1)-(1-3y)(x-y)m^2)\left(\frac{1}{\Delta_1}+\frac{1}{\Delta_2{}'}-\frac{1}{\Delta_2}-\frac{1}{\Delta_1{}'}\right)\\ &+\frac{(3y-2)(1-x)M_1^2+y(3y-1)M_2^2}{\Delta_1}-\frac{y(3y-1)M_1^2+(x-1)(2-3y)M_2 1^2}{\Delta_2}\\ &+\frac{[2(x+y-1)-3y(x-y)]M_2^2}{\Delta_2{}'}-\frac{[2(x+y-1)-3y(x-y)]M_1^2}{\Delta_1{}'}. \end{aligned}\tag{16}$$

The calculation through the Feynman parametrization is sketched in the Appendix A.

For $M_1, M_2 \ll m$, $\Delta_1 \simeq \Delta_2 \simeq \Delta_1' \simeq \Delta_2' \simeq (x-y)m^2$. In this case, the integral is simplified to

$$\begin{aligned}F_2(0) &= \frac{eh'_{[22]}h'_{[23]}m_3m_\mu}{8\pi^2m^2}\int_0^1 dx\int_0^x dy\,\frac{2y(1-3y)}{x-y}\\ &= -\frac{eh'_{[22]}h'_{[23]}m_3m_\mu}{8\pi^2m^2}\left(\frac{13}{6}+\ln\epsilon\right),\end{aligned}\tag{17}$$

where $\epsilon = \frac{\{M_1^2, M_2^2\}}{m^2}$.

For the other extreme, $M_1, M_2 \gg m$, $\Delta_1 \simeq (1-x)M_1^2 + (x-y)M_2^2$, $\Delta_2 \simeq yM_1^2 + (1-x)M_2^2$, $\Delta_1' \simeq (1-x+y)M_1^2$, $\Delta_2' \simeq (1-x+y)M_2^2$. In this case, the integral is simplified to give

$$F_2(0) = \frac{eh'_{[22]}h'_{[23]}m_3m_\mu}{16\pi^2M_1^2}\left(\frac{1+\xi+\xi^2}{\zeta^2\xi^3}\right),\tag{18}$$

where $\zeta = M_1/m$ and $\xi = M_2/M_1$. For the R_K phenomenology, we will need $\frac{m}{\sqrt{h'_{[22]}h'_{[23]}}} \approx O(0.7\times 10^6\ \text{GeV})$. So, suppose that $m = 10^6$ GeV, $M_2 \lesssim M_1$, $M_1 = 10^4$ GeV, $m_3 \approx 200$ GeV, and $h'_{[22]} \simeq h'_{[23]} \approx 1$, which gives $F_2(0) \approx 1.21\times 10^{-9}$.

The BNL value of $(g-2)_\mu$ minus the SM prediction is [28]

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 261(63)(48)\times 10^{-11}.\tag{19}$$

Thus, there is some region of the parameter space pulling the $(g-2)_\mu$ of the SM value to the BNL value with $m = O(10^6)$ GeV, with m_3 at the electroweak scale with heavy m, M_1, and M_2.

If we consider the symmetry $\mathrm{U}(1)_{\mu-\tau}$, the calculation is the same.

C. Models for R_{K,K^*}

For the family GUT interaction discussed in Sec. II B, let us consider what can be its effect on R_H with the symmetry

$\mathrm{U}(1)_{\mu-\tau}$. From Eq. (10), the $L_\mu - L_\tau$ quantum numbers of three families in the Georgi-Glashow model are

$$\psi^{\alpha\beta}_{[-3]} \oplus 2\psi^{\alpha\beta}_{[+3]} \oplus 2\psi_{\alpha[-3]} \oplus \psi_{\alpha[+3]}, \tag{20}$$

and the BSM Higgs field H' can be one from $(\psi^{\alpha}_{[0]} \oplus \psi_{\alpha[0]})$'s. To have the coupling (11), H' must be neutral in $L_\mu - L_\tau$. Thus, the H' couplings takes the form

$$h'_{[IJ]}\psi^{\alpha\beta}_{[I]}\psi_{\alpha[J]}H'_{\beta[0]}. \tag{21}$$

From Eq. (21), we note that b_L can decay to s_R,

$$b_L\left(\text{from}\,\psi^{\alpha\beta}_{[+3]}\right) \to s^c_L(\text{from}\,\psi_{\alpha[+3]}) + \text{neutral Higgs}\,(\text{from}\,H'_{[0]}\ \text{possible}). \tag{22}$$

In the next section, we will use the following interaction,

$$h'_{[23]}\bar{s}_R b_L H'^0 + h'_{[22]}\bar{\mu}_R\mu_L H'^0 + \text{H.c.}, \tag{23}$$

where the $h'_{[IJ]}$ couplings are set to real values by absorbing their phases to quark or lepton fields.

III. PHENOMENOLOGY OF $L_\mu - L_\tau$ FOR QUARKS VIA FAMILY GUT

From the allowed coupling (23) with the $L_\mu - L_\tau$ conservation, we will estimate R_K. The following contribution to R_K exists by the BSM neutral field H'^0:

$$\frac{h'_{[23]}h'_{[22]}}{m_0^2}\bar{\mu}_L\mu_R\bar{s}_R b_L + \text{H.c.} \tag{24}$$

We choose traceless combinations for the flavor indices. Thus, note that $\psi_{\alpha(-1,2,-1)}$ houses the muon doublet, $\psi_{\alpha(-1,2,-1)} = \psi_{\alpha[3]}$, where the number in brackets is the $L_\mu - L_\tau$ quantum number. The tau doublet belongs to $\psi_{\alpha(-1,-1,2)} = \psi_{\alpha[-3]}$. The quark doublet of the second family is in $\psi^{\alpha\beta(-1,2,-1)} = \psi^{\alpha\beta}_{(1,-2,1)} = \psi^{\alpha\beta}_{[-3]}$, which is in the L-handed notation. In terms of the right-handed notation, the $L_\mu - L_\tau$ quantum number is [3]. The quark doublet of the third family is in $\psi^{\alpha\beta(-1,-1,2)} = \psi^{\alpha\beta}_{(1,1,-2)} = \psi^{\alpha\beta}_{[3]}$, which houses b_L. μ^c_L is contained in $\psi^{\alpha\beta(-1,2,-1)} = \psi^{\alpha\beta}_{(1,-2,1)} = \psi^{\alpha\beta}_{[-3]}$. Thus, the fields participating in the b decay via the BSM field H'^0 are

$$\begin{aligned} &\mu_L : \psi_{\alpha[+3]}, \\ &b_L : \psi^{\alpha\beta}_{[+3]}, \\ &\mu_R : \psi^{\alpha\beta}_{[+3]}, \\ &s_R : \psi^{\alpha\beta}_{[+3]}. \end{aligned} \tag{25}$$

Thus, Eq. (24) preserves the $\mathrm{U}(1)_{\mu-\tau}$ gauge symmetry. Note also that the combination $h'_{[23]}h'_{[22]}$ appeared in the BSM contribution to $(g-2)_\mu$.

The lepton flavor universality from one-loop generated coupling in the SM is given by [29]

$$\frac{G_F\alpha_{\text{em}}V_{tb}V^*_{ts}C_9}{\sqrt{2}\pi}\bar{\ell}\gamma^\mu(1-\gamma_5)\ell\bar{s}_L\gamma_\mu b_L \tag{26}$$

where $\ell = e, \mu, \tau$ and $C_9 = (-\frac{1}{2} + \sin^2\theta_W)(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)$ times some factor arising from hadronic states.

The B^+ meson decay rate with lifetime $1.638(1.000 \pm 0.003) \times 10^{-12}$ s is [30]

$$\Gamma_{B^+} \simeq 4.02 \times 10^{-13}\ \text{GeV}, \tag{27}$$

while the branching ratio for $\bar{B} \to \bar{K}\ell^+\ell^-$ (for $\ell = e, \mu$) is [29]

$$\mathcal{B}(\bar{B} \to \bar{K}\ell^+\ell^-) = 1.04 \times 10^{-7}. \tag{28}$$

The family dependence is studied with a double ratio [16,17],

$$R_K = \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+J/\psi(\to \mu^+\mu^-))} \bigg/ \frac{\mathcal{B}(B^+ \to K^+e^+e^-)}{\mathcal{B}(B^+ \to K^+J/\psi(\to e^+e^-))}. \tag{29}$$

The first report on the family dependence is the Run 1 result of LHCb, $R_K = 0.745^{+0.090}_{-0.074} \pm 0.036$ in the q^2 interval of $q^2 = 1.1$–$6\ \text{GeV}^2$ [16], which gives a 2.6σ level anomaly from the SM prediction. The recent result from LHCb is $R_K = 0.846^{+0.060+0.016}_{-0.054-0.014}$ [17], which is a 2.5σ anomaly. However, the recent Belle report is consistent with the SM but with larger error bars, $R_K = 0.98^{+0.27}_{-0.23} \pm 0.06$ [18].

We will use the Run 1 result of LHCb since it covers a wide range of q^2,

$$R_K = 0.745^{+0.090}_{-0.074} \pm 0.036. \tag{30}$$

There is a claim that new physics by scalar mediation cannot explain the R_K phenomenology [31] (see also [32]).

But the assumptions deriving this conclusion do not include our scenario. First, they assumed only the SM gauge group, while our symmetry below the cutoff scale Λ is the SM gauge group times $\mathrm{U}(1)_{\mu-\tau}$. Second, the constraint, for example, Eq. (15) of [31], is from purely leptonic data. But our interaction, Eq. (24), is not related to the SM interaction and hence the parameters in Eq. (24) are restricted only by the $(g-2)_\mu$ phenomenology, whose allowed region will be given together with the R_K bound.

Incorporating the new operator $C_{bs\mu\mu}\bar{\mu}\mu\bar{s}P_{R(L)}b$, R_K is given by

$$R_K = \frac{\Gamma_\mu}{\Gamma_e} = 1 - \frac{\Gamma_\mu^{\rm NP}}{\Gamma_e^{\rm SM}}, \tag{31}$$

where

$$\Gamma_e^{\rm SM} \simeq \frac{1}{3}\int_{q_{\rm min}}^{q_{\rm max}} \Gamma_0 \beta_e \lambda^{3/2} \xi_P^2(q^2)(C_{9\rm eff}^2 + C_{10}^2)dq^2, \tag{32}$$

$$\Gamma_\mu^{\rm NP} \simeq \int_{q_{\rm min}}^{q_{\rm max}} \Gamma_0 \beta_\mu^3 \lambda^{1/2} \xi_P^2(q^2)|F_s|^2|C_s|^2 dq^2, \tag{33}$$

with

$$\Gamma_0 = \frac{G_F^2\alpha_{em}^2|V_{tb}V_{ts}^*|^2}{512\pi^5 M_B^3}, \qquad \beta_l = \sqrt{1-\frac{4m_l^2}{q^2}} \simeq 1, \tag{34}$$

$$\lambda = M_B^4 + M_K^4 + q^4 - 2(M_B^2 M_K^2 + (M_B^2 + M_K^2)q^2), \tag{35}$$

$$\xi_P(q^2) \simeq \frac{0.327}{(1-\frac{q^2}{M_B^2})^2}, \qquad F_S = \frac{M_B^2 - M_K^2}{m_b - m_s}\frac{(M_B^2 + M_K^2 - q^2)}{M_B^2}. \tag{36}$$

In our model, the contribution of $C_{bs\mu\mu}$ relative to the one in the SM is given by

$$C_s^2 \equiv \left(\frac{\sqrt{2}\pi}{G_F\alpha_{em}V_{tb}V_{ts}^*}\right)^2 \left(\frac{h'_{[23]}h'_{[22]}}{m_0^2}\right)^2. \tag{37}$$

From the experimental results, Eq. (30), we can determine the values of $\frac{h'_{[23]}h'_{[22]}}{m_0^2}$. For example, taking the central value of Eq. (30), we obtain

$$\left|\frac{\Gamma_\mu^{\rm NP}}{\Gamma_e^{\rm SM}}\right| = 1\text{–}0.745 \simeq 0.03|C_s|^2, \tag{38}$$

where we performed the q^2 integration in Eqs. (32) and (33). With the known SM parameters, $M_B, M_K, m_b, m_s =$ 5.2795, 0.89594, 4.8, 0.101 in GeV units, respectively, $|V_{tb}| = 0.999097$ and $|V_{ts}^*| = 0.04156$, we obtain $\frac{h'_{[23]}h'_{[22]}}{m_0^2} \simeq 2.5\times 10^{-9}\ \mathrm{GeV}^{-2}$. Determining $\frac{h'_{[23]}h'_{[22]}}{m_0^2}$ in this way, we can compare the $(g-2)_\mu$ shift by Fig. 1 with the measured value at the BNL. We note that there are more unknown parameters for the expression on the muon $(g-2)_\mu$: m_3, M_1, M_2 for a given $\frac{h'_{[23]}h'_{[22]}}{m_0^2}$.

To glimpse the behavior for $(g-2)_\mu$, we choose M_1^2/m_0^2 to be 10 and look for the allowed region of $\frac{h'_{[23]}h'_{[22]}}{m_0^2}$ up to the

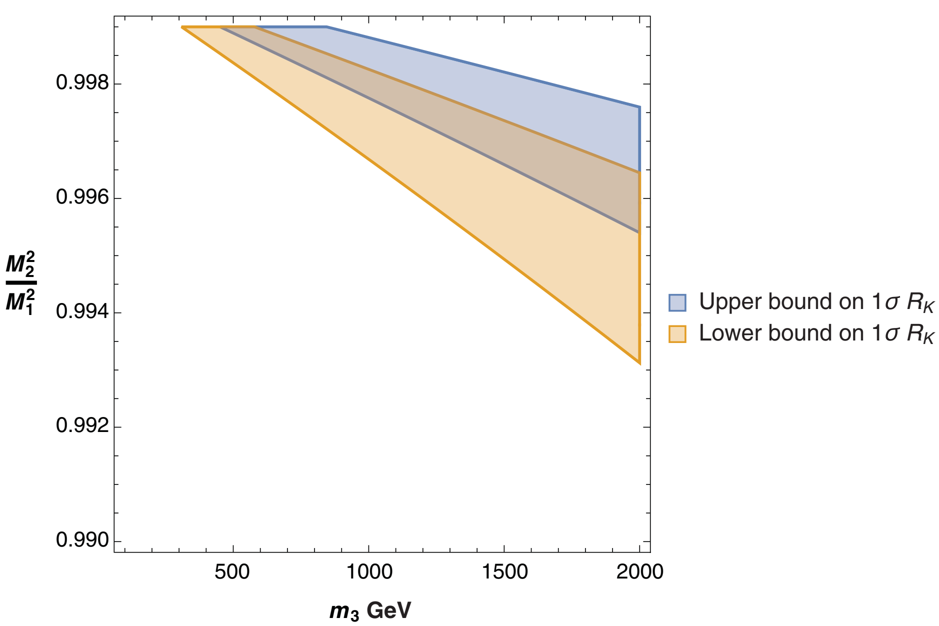

FIG. 2. Allowed region of parameter space $(\frac{M_2^2}{M_1^2}, m_3)$ from R_K and $(g-2)_\mu$ bounds for $M_1^2 = 10m_0^2$.

1σ region of experimental result of R_K. Then m_3 and $\frac{M_2^2}{M_1^2}$ are the only unknown parameters for the $(g-2)_\mu$ expression. Imposing the experimental result, Eq. (19), we can get the allowed region of parameter space $(\frac{M_2^2}{M_1^2}, m_3)$, which is shown in Fig. 2. In Fig. 2, the blue and orange regions correspond to the allowed regions of parameter space obtained by taking R_K to be the 1σ upper limit of R_K and the 1σ lower limit of R_K.

IV. CONCLUSION

In a family grand unification model, we related the BNL anomaly on the muon anomalous magnetic moment and the LHC anomaly on R_K via the symmetry $L_\mu - L_\tau$. As a family grand unification example, we used Georgi's SU(11).

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APPENDIX A: FEYNMAN PARAMETRIZATION INTEGRAL

The useful Feynman parameter integration is

$$\frac{1}{ab^2cd} = -\frac{\partial}{\partial b}\left(\frac{1}{abcd}\right) = -\frac{1}{(d-c)}\frac{\partial}{\partial b}\left(\frac{1}{abc} - \frac{1}{abd}\right), \tag{A1}$$

where

$$\frac{1}{abc} = -2!\int_0^1 dx \int_0^x dy \frac{1}{[a+(b-a)x+(c-b)y]^3}, \tag{A2}$$

$$\frac{1}{abd} = -2!\int_0^1 dx \int_0^x dy \frac{1}{[a+(b-a)x+(d-b)y]^3}. \tag{A3}$$

Let us choose, for (the lhs of) Fig. 1,

$$a = k'^2 - M_1^2, b = (p-k)^2 - m^2, c = k^2 - M_2^2, d = k^2 - M_1^2,$$
$$\ell = k + q(1-x) + p(y-x) = k' - qx + p(y-x). \tag{A4}$$

Equations (A2) and (A3) have the following denominators:

$$D_L = a + (b-a)x + (c-b)y = \ell^2 - \Delta_1,$$
$$D'_L = a + (b-a)x + (d-b)y = \ell^2 - \Delta'_1, \tag{A5}$$

where, neglecting $O(m_\mu^2)$,

$$\Delta_1 = M_1^2(1-x) + M_2^2 y + m^2(x-y),$$
$$\Delta'_1 = M_1^2(1-x+y) + m^2(x-y). \tag{A6}$$

A similar consideration of (the rhs of) Fig. 1 leads to, for D_R and D'_R,

$$\Delta_2 = M_2^2(1-x) + M_1^2 y + m^2(x-y),$$
$$\Delta'_2 = M_2^2(1-x+y) + m^2(x-y). \tag{A7}$$

[1] C. Bouchiat, J. Ilipoulos, and P. Meyer, An anomaly free version of Weinberg's model, Phys. Lett. **38B**, 519 (1972).

[2] D. J. Gross and R. Jackiw, Effect of anomalies on quasirenormalizable theories, Phys. Rev. D **6**, 477 (1972).

[3] M. Gell-Mann, P. Ramond, and R. Slansky, Complex spinors and unified theories, in *Supergravity Workshop*, edited by P. Van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979).

[4] F. Wilczek and A. Zee, Family from spinors, Phys. Rev. D **25**, 553 (1982).

[5] K. S. Babu, S. M. Barr, and I. Gogoladze, Family unification with SO(10), Phys. Lett. B **661**, 124 (2008).

[6] Y. Goto, Y. Kawamura, and T. Miura, Orbifold family unification on six dimensions, Phys. Rev. D **88**, 055016 (2013).

[7] J.-H. Huh, J. E. Kim, and B. Kyae, $SU(5)_{\rm flip} \times SU(5)'$ from $\mathbf{Z}_{12-I}$, Phys. Rev. D **80**, 115012 (2009).

[8] H. Georgi, Towards a grand unified theory of flavor, Nucl. Phys. **B156**, 126 (1979).

[9] J. E. Kim, Model of Flavor Unity, Phys. Rev. Lett. **45**, 1916 (1980).

[10] P. H. Frampton, SU(N) grand unification with several quark-lepton generations, Phys. Lett. **88B**, 299 (1979).

[11] P. H. Frampton, Unification of flavor, Phys. Lett. **89B**, 352 (1980).

[12] J. E. Kim, S.-J. Kim, S. Nam, and M. Shim, Jarlskog determinant and data on flavor matrices, arXiv:1907.12247 [Mod. Phys. Lett. A. (to be published)].

[13] P. H. Frampton and J. E. Kim, Anticorrelation of mass and mixing angle hierarchies, J. Korean Phys. Soc. **76**, 695 (2020).

[14] P. H. Frampton, J. E. Kim, S.-J. Kim, and S. Nam, Tetrahedral A_4 symmetry in anti-SU(5) GUT, Phys. Rev. D **101**, 055022 (2020).

[15] J. E. Kim and S.-J. Kim, On the progenitor quark mass matrix, J. Korean Phys. Soc. **77**, 10 (2020).

[16] R. Aaiz *et al.* (LHCb Collaboration), Test of Lepton Universality Using $B^+ \to K^+\ell^+\ell^-$ Decays, Phys. Rev. Lett. **113**, 151601 (2014).

[17] R. Aaiz *et al.* (LHCb Collaboration), Search for Lepton Universality Violation in $B^+ \to K^+\ell^+\ell^-$ Decays, Phys. Rev. Lett. **122**, 191801 (2019).

[18] A. Abdesselam *et al.* (Belle Collaboration), Test of Lepton Flavor Universality in $B^+ \to K^+\ell^+\ell^-$ Decays, Phys. Rev. Lett. **122**, 191801 (2019).

[19] A. Ali, P. Ball, L. T. Handoko, and G. Hiller, A comparative study of the decays $B \to (K, K^*)\ell^+\ell^-$ in Standard Model and supersymmetric theories, Phys. Rev. D **61**, 074024 (2000).

[20] M. Beneke, T. Feldman, and D. Seidel, Systematic approach to exclusive $B \to V\ell^+\ell^-$ decays, Nucl. Phys. **B612**, 25 (2001).

[21] C. H. Chen and C. Q. Geng, Exclusive decays of $B \to K^{(*)}\ell^+\ell^-$ in the PQCD, Phys. Rev. D **63**, 114025 (2001).

[22] A. Ali, G. Kramer, and G.-h. Zhu, $B \to K^+\ell^+\ell^-$ decay in soft-collinear effective theory, Eur. Phys. J. C **47**, 625 (2006).

[23] U. Egede, T. Hurth, J. Matias, M. Ramon, and W. Reece, New observables in the decay mode $\bar{B}_d \to \bar{K}^{*0}\ell^+\ell^-$, J. High Energy Phys. 11 (2008) 032.

[24] C. Bobeth, G. Hiller, and G. Piranishvili, *CP* asymmetries in $B \to \bar{K}^*(\to \bar{K}\pi)\bar{\ell}\ell$ and untagged $\bar{B}_s, B_s \to \phi(\to K^+K^-)\bar{\ell}\ell$ decays at NLO, J. High Energy Phys. 07 (2008) 106.

[25] W. Altmannshofer, P. Ball, A. Bharucha, A. J. Buras, and D. M. Straub, Symmetries and asymmetries of $B \to K^*\mu^+\mu^-$ decays in the Standard Model and beyond, J. High Energy Phys. 01 (2009) 019.

[26] U. Amaldi, H. De Boer, P. H. Frampton, H. Furstenau, and J. T. Liu, Consistency checks of grand unified theories, Phys. Lett. B **281**, 374 (1992).

[27] J. E. Kim, Neutrino magnetic moment, Phys. Rev. D **14**, 3000 (1976).

[28] See A. Hoecker and W. J. Marciano, Muon anomalous magnetic moment, Sec. 57 of M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).

[29] For example, see C. Bobeth, The decay $\bar{B} \to \bar{K}\ell^+\ell^-$ at low hadronic recoil and model-independent $\Delta B = 1$ constraints, J. High Energy Phys. 01 (2012) 107.

[30] For example, see Y. Amhis *et al.* (Heavy Flavor Averaging Group), Averages of b-hadron, c-hadron, and τ lepton properties as of 2018, arXiv:1909.12524, Eq. (38).

[31] R. Alonso, B. Grinstein, and J. Martin Camalich, $SU(2) \times U(1)$ Gauge Invariance and the Shape of New Physics in Rare B Decays, Phys. Rev. Lett. **113**, 241802 (2014).

[32] See also G. Hiller and M. Schmaltz, R_K and future $b \to s\ell\ell$ physics beyond the Standard Model opportunities, Phys. Rev. D **90**, 054014 (2014).