

***B*-meson light-cone distribution amplitude from Euclidean quantities**Wei Wang^{1,*}, Yu-Ming Wang^{2,†}, Ji Xu^{1,3,‡} and Shuai Zhao^{4,5,§}¹*INPAC, SKLPPC, MOE KLPPC, School of Physics and Astronomy,
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A new method for the model-independent determination of the light-cone distribution amplitude of the *B*-meson in heavy quark effective theory (HQET) is proposed by combining the large momentum effective theory and the numerical simulation technique on the Euclidean lattice. We demonstrate the autonomous scale dependence of the nonlocal quasi-HQET operator with the aid of the auxiliary field approach, and further determine the perturbative matching coefficient entering the hard-collinear factorization formula for the *B*-meson quasidistribution amplitude at the one-loop accuracy. These results will be crucial to explore the partonic structure of heavy-quark hadrons in the static limit and to improve the theory description of exclusive *B*-meson decay amplitudes based upon perturbative QCD factorization theorems.

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I. INTRODUCTION

B-meson light-cone distribution amplitude (LCDA) in heavy-quark effective theory (HQET) [1] serves as an indispensable ingredient for establishing QCD factorization theorems of exclusive *B*-meson decay amplitudes [2–6] and for constructing light-cone sum rules of numerous hadronic matrix elements, whose factorization properties are not yet completely explored at leading power in the heavy-quark expansion, from the vacuum-to-*B*-meson correlation functions [7–12]. Defined as the light-ray matrix elements of the composite HQET quark-gluon operators, they encode information of the nonperturbative strong interaction dynamics from the soft-scale fluctuation of the *B*-meson system and our limited knowledge of these distribution amplitudes has become the major stumbling block for precision calculations of the *B*-meson decay observables, which are crucial to disentangling the Standard Model (SM)

background contributions from the genuine new physics effects.

Model-independent properties of the leading-twist *B*-meson LCDA $\phi_B^+(\omega, \mu)$, including its renormalization group equation (RGE) and perturbative QCD constraints at large ω , have received increasing attention in recent years [13–19]. By contrast, nonperturbative determinations of $\phi_B^+(\omega, \mu)$ have been mainly performed in the framework of the QCD sum rules (QCDSR) invoking the quark-hadron duality ansatz [20], where both the perturbative corrections to the leading-power contribution and the subleading-power contributions from quark-gluon condensate operators were taken into account systematically. One main drawback of constructing the phenomenological models for the *B*-meson distribution amplitude $\phi_B^+(\omega, \mu)$ from the classical QCDSR method lies in the fact that the light-cone separation between the effective heavy-quark field and the light antiquark field needs to be sufficiently small (of order $1 \sim 3 \text{ GeV}^{-1}$) to guarantee the validity of the local operator product expansion (OPE) for the HQET correlation function under discussion. In addition, meaningful constraints on the first inverse moment $\lambda_B^{-1}(\mu)$ [21–24] can be obtained by measuring the integrated branching fractions of the radiative leptonic *B*-meson decays with a photon energy cut $E_\gamma \geq E_{\text{cut}}$ at the Belle II experiment. Keeping in mind that the exact RGE for the inverse moment $\lambda_B^{-1}(\mu)$ involves all the logarithmic moments $\sigma_B^{(n)}(\mu)$ in perturbation theory, the precise shape of the *B*-meson distribution amplitude, in particular its small-momentum behavior, cannot be controlled by a single nonperturbative parameter $\lambda_B^{-1}(\mu)$ to a good approximation. It is then

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evident that determining the momentum dependence of the B -meson LCDA $\phi_B^+(\omega, \mu)$ with model-independent techniques is of top priority for the precision descriptions of exclusive B -meson decays.

Performing the lattice QCD calculation of the leading-twist distribution amplitude $\phi_B^+(\omega, \mu)$ directly is known to be complicated by the appearance of the light-cone separated quark fields defining the very HQET matrix element for a long time. A promising approach to circumvent this long-standing problem has been recently proposed under the name of the large momentum effective theory (LaMET) by Ji [25,26] (see also [27–29] for a review). The essential strategy of this novel proposal consists in the construction of a time-independent quasiquantity which, on the one hand, can be readily computed on a Euclidean lattice and, on the other hand, approaches the original hadronic distribution amplitude on the light cone under Lorentz boost. In this paper, we implement Ji's proposal to extract the leading-twist LCDA $\phi_B^+(\omega, \mu)$ of the B -meson in HQET by demonstrating the multiplicative renormalization of the constructed quasi-HQET operator to all orders in perturbation theory, by determining the short-distance function appearing in the hard-collinear factorization formula of the B -meson quasidistribution amplitude, and by exploring future opportunities of lattice QCD calculations.

II. B -MESON (QUAS)DISTRIBUTION AMPLITUDES

The leading-twist LCDA $\tilde{\phi}_B^+(\eta, \mu)$ in coordinate space is defined by the renormalized HQET matrix element of a light-ray soft operator [13]

$$\begin{aligned} & \langle 0 | (\bar{q}_s Y_s)(\eta \bar{n}) \not{n} \gamma_5 (Y_s^\dagger h_v)(0) | \bar{B}(v) \rangle \\ &= i \tilde{f}_B(\mu) m_B \tilde{\phi}_B^+(\eta, \mu), \end{aligned} \quad (1)$$

where the soft light-cone ($\bar{n}^2 = 0$) Wilson line is given by

$$Y_s(\eta \bar{n}) = \text{P} \left\{ \text{Exp} \left[i g_s \int_{-\infty}^{\eta} dx \bar{n} \cdot A_s(x \bar{n}) \right] \right\}, \quad (2)$$

and the static decay constant $\tilde{f}_B(\mu)$ of the B -meson can be expressed in terms of f_B in QCD [30]. Applying the Fourier transformation for $\tilde{\phi}_B^+(\eta, \mu)$ leads to the momentum-space distribution function [20]

$$\phi_B^+(\omega, \mu) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\eta e^{i\bar{n} \cdot v \omega \eta} \tilde{\phi}_B^+(\eta - i\epsilon, \mu). \quad (3)$$

Following the construction presented in [25], we will employ the following B -meson quasi-distribution amplitude (see also [31])

$$\begin{aligned} & i \tilde{f}_B(\mu) m_B \varphi_B^+(\xi, \mu) \\ &= \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{i n_z \cdot v \xi \tau} \langle 0 | (\bar{q}_s Y_s)(\tau n_z) \not{n}_z \gamma_5 (Y_s^\dagger h_v)(0) | \bar{B}(v) \rangle, \end{aligned} \quad (4)$$

defined by the spatial correlation function of two collinear (effective) quark fields with $n_z = (0, 0, 0, 1)$. For the sake of demonstrating QCD factorization for the quasidistribution amplitude $\varphi_B^+(\xi, \mu)$, we will work in a Lorentz boosted frame of the B -meson with $\bar{n} \cdot v \gg n \cdot v$ and set $v_{\perp\mu} = 0$ without loss of generality. As a consequence, only the ultracollinear gluons couple with the boosted heavy quark in the low-energy effective theory and the soft Wilson lines $Y_s(\theta u)$ will be substituted by the ultracollinear Wilson lines $W_n(\theta u)$ in the boosted HQET (bHQET) accordingly [32].

III. MULTIPLICATIVE RENORMALIZATION

To facilitate the lattice QCD evaluation of the quasidistribution amplitude $\varphi_B^+(\xi, \mu)$, it is of vital importance to show that such quasiquantity will renormalize multiplicatively to all orders in perturbation theory applying the lattice regularization scheme. For this purpose, it has proven to be most convenient employing the one-dimensional auxiliary field formalism for the contour integrals introduced in [33]. The resulting Lagrangian for the ultracollinear gluon interactions with both the effective bottom-quark field h_v and the auxiliary field \mathcal{Q} can be written as

$$\mathcal{L} = \mathcal{L}_{\text{bHQET}} + \bar{\mathcal{Q}}(x)(i n_z \cdot D_n - \delta m)\mathcal{Q}(x), \quad (5)$$

where the ‘‘dynamical’’ mass term originates from the self-energy correction to the \mathcal{Q} field in the dimensionful cutoff scheme [34], in analogy to the scheme-dependent residual mass term in the HQET formalism [35–37], and the ultracollinear covariant derivative $D_n^\mu = \partial^\mu - i g_s T^a A_n^{a\mu}$. Alternatively, the ultraviolet (UV) linear divergences from the Wilson line corrections in (4) can be removed by introducing the proper subtraction term defined by a simpler matrix element but with the same power divergences [38–40]. It is straightforward to rewrite the nonlocal operator defining the B -meson quasidistribution amplitude as follows [41,42]:

$$\begin{aligned} \mathcal{O}(\tau n_z, 0) &= [\bar{\chi}_n(\tau n_z) \not{n}_z \gamma_5 \mathcal{Q}(\tau n_z)] [\bar{\mathcal{Q}}(0) h_v(0)] \\ &\equiv \mathcal{J}_{\chi\mathcal{Q}}(\tau n_z) \mathcal{J}_{\mathcal{Q}h_v}(0), \end{aligned} \quad (6)$$

where χ_n stands for the ultracollinear quark field.

Thanks to the heavy-quark spin symmetry and the light-quark chiral symmetry for the effective Lagrangian \mathcal{L} , both of the two currents $\mathcal{J}_{\chi\mathcal{Q}}$ and $\mathcal{J}_{\mathcal{Q}h_v}$ renormalize multiplicatively under radiative corrections [43]

$$\begin{aligned}\mathcal{J}_{\chi\mathcal{Q}}(\tau n_z) &= Z_{\chi\mathcal{Q}}^{(R)} \mathcal{J}_{\chi\mathcal{Q}}^{(R)}(\tau n_z, \mu), \\ \mathcal{J}_{\mathcal{Q}h_v}(0) &= Z_{\mathcal{Q}h_v}^{(R)} \mathcal{J}_{\mathcal{Q}h_v}^{(R)}(0, \mu),\end{aligned}\quad (7)$$

at all orders in α_s . We are therefore led to conclude the autonomous renormalization of the composite nonlocal operator $\mathcal{O}(\tau n_z, 0)$, namely

$$\mathcal{O}(\tau n_z, 0) = Z_{\chi\mathcal{Q}}^{(R)} Z_{\mathcal{Q}h_v}^{(R)} \mathcal{O}^{(R)}(\tau n_z, 0, \mu). \quad (8)$$

It needs to be stressed that such multiplicative-renormalization property holds in both dimensional regularization and lattice regularization schemes due to the reparametrization invariance of the heavy quark mass [35], which can be readily understood by introducing the generalized covariant derivative $i\mathcal{D}^\mu = iD^\mu + \delta m n_z^\mu$. In general, the renormalized nonlocal quasioperator $\mathcal{O}^{(R)}$ for a given regularization scheme violating translation invariance (including but not limited to the lattice regularization scheme) can be expressed as [42,44]

$$\mathcal{O}^{(R)}(\tau n_z, 0, \mu) = [Z_{\chi\mathcal{Q}}^{(R)} Z_{\mathcal{Q}h_v}^{(R)}]^{-1} e^{\overline{\delta m} \tau} \mathcal{O}(\tau n_z, 0), \quad (9)$$

with the imaginary mass $\delta m = i\overline{\delta m}$ due to the spacelike gauge vector n_z [45].

IV. HARD-COLLINEAR FACTORIZATION FORMULA:

We now proceed to determine the perturbative matching coefficient function entering the hard-collinear factorization formula for $\mathcal{O}^{(R)}(\tau n_z, 0)$ at $\tau \ll 1/\Lambda_{\text{QCD}}$

$$\mathcal{O}^{(R)}(\tau n_z, 0, \mu) = \int d\eta \tilde{H}(\tau, \eta, n_z \cdot v, \mu) P^{(R)}(\eta \bar{n}, 0, \mu),$$

$$P^{(R)}(\eta \bar{n}, 0, \mu) = [(\bar{\chi}_n W_n)(\eta \bar{n}) \not{n} \gamma_5 (W_n^\dagger h_v)(0)]^{(R)},$$

at leading power in $\Lambda_{\text{QCD}} \tau$, which can be Fourier-transformed into the momentum-space relation

$$\begin{aligned}\varphi_B^+(\xi, \mu) &= \int_0^\infty d\omega H(\xi, \omega, n_z \cdot v, \mu) \phi_B^+(\omega, \mu) \\ &+ O\left(\frac{\Lambda_{\text{QCD}}}{n_z \cdot v \xi}\right).\end{aligned}\quad (10)$$

Applying the default power counting scheme one can readily identify that the hard correction from the one-loop box diagram in Fig. 1 is power suppressed and it will therefore give rise to the vanishing contribution to the perturbative matching function H . We further verify explicitly that the collinear contribution to the quasisdistribution amplitude is precisely reproduced by the corresponding diagrams for the B -meson LCDA at one loop. The obtained hard function at $O(\alpha_s)$ reads

$$\begin{aligned}H(\xi, \omega, n_z \cdot v, \mu) &= \delta(\xi - \omega) + \frac{\alpha_s(\mu) C_F}{4\pi} \left\{ \frac{1}{\omega - \xi} \left[3 - 2 \ln\left(\frac{\mu}{2n_z \cdot v(\omega - \xi)}\right) - \frac{2\xi}{\omega} \ln\left(\frac{\xi}{\xi - \omega}\right) \right] \theta(-\xi) \theta(\omega) \right. \\ &+ \left. \left\{ \frac{1}{\omega - \xi} \left[3 - 2 \left(1 + \frac{2\xi}{\omega}\right) \ln\left(\frac{\mu}{2n_z \cdot v(\omega - \xi)}\right) - \frac{2\xi}{\omega} \left(\ln\left(\frac{\omega - \xi}{\xi}\right) + 1\right) \right] \right\}_\oplus \theta(\xi) \theta(\omega - \xi) \right. \\ &+ \left. \left\{ \frac{1}{\xi - \omega} \left[3 - 2 \ln\left(\frac{\mu}{2n_z \cdot v(\xi - \omega)}\right) - \frac{2\xi}{\omega} \ln\left(\frac{\xi}{\xi - \omega}\right) \right] \right\}_\oplus \theta(\omega) \theta(\xi - \omega) \right. \\ &+ \left. 2 \left[\ln^2 \frac{\mu}{n_z \cdot v \xi} - 3 \ln \frac{\mu}{n_z \cdot v \xi} + f(a) \right] \delta(\xi - \omega) \right\},\end{aligned}\quad (11)$$

where the plus distribution is defined by (with $a > 1$)

$$\{\mathcal{F}(\xi, \omega)\}_\oplus = \mathcal{F}(\xi, \omega) - \delta(\xi - \omega) \int_0^{a\xi} dt \mathcal{F}(\xi, t), \quad (12)$$

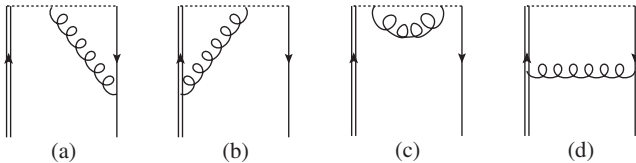


FIG. 1. One-loop corrections to the quasisdistribution amplitude of the B -meson $\varphi_B^+(\xi, \mu)$: the effective HQET bottom quark is represented by the double line, and the spacelike Wilson line is indicated by the dashed line.

and the subtraction-scheme-dependent term

$$\begin{aligned}f(a) &= \ln \frac{a^2}{4(a-1)^3} \ln \frac{\mu}{n_z \cdot v \xi} + \ln(a-1) \ln \frac{8(a-1)}{a} \\ &+ \text{Li}_2(1-a) + \ln a \ln\left(\frac{a}{4}\right) - \frac{1}{2} \ln(a-1) \\ &+ \ln(8a) + \ln^2 2 + \frac{\pi^2}{8} - 3\end{aligned}\quad (13)$$

will compensate the same scheme dependence of the newly introduced plus distribution for the convolution of the hard function H with a smooth test function. An advantage of introducing the above-mentioned plus function is that it

allows one to implement both the ultraviolet and infrared subtractions for the perturbative matching procedure simultaneously. Distinguishing the ultraviolet renormalization scale ν of the composite quasioperator $\mathcal{O}^{(R)}$ from the factorization scale μ of separating the hard and collinear strong interaction dynamics for this quantity, it is straightforward to demonstrate a complete cancellation of the μ dependence for the factorization formula of $\phi_B^+(\xi, \nu, \mu)$ at one loop, by employing the Lange-Neubert evolution equation of the B -meson distribution amplitude $\phi_B^+(\omega, \mu)$ [13].

V. PERSPECTIVES FOR LATTICE CALCULATIONS

An important step in obtaining the B -meson LCDA in bHQET based upon Ji's approach is to perform the lattice QCD simulation for the spatial correlation $\phi_B^+(\xi, \mu)$ in the moving B -meson frame with $n_z \cdot v \gg 1$. To this end, it will be instructive to understand the characteristic feature of $\phi_B^+(\xi, \mu)$ with distinct nonperturbative models of $\phi_B^+(\omega, \mu)$. Taking advantage of the two phenomenological models motivated by the HQET sum rule calculation at leading order [1] and at next-to-leading order [20]

$$\begin{aligned} \phi_{B,I}^+(\omega, \mu = 1.5 \text{ GeV}) &= \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0}, \\ \phi_{B,II}^+(\omega, \mu = 1.5 \text{ GeV}) &= \left[\frac{1}{k^2 + 1} - \frac{2(\sigma_B^{(1)} - 1)}{\pi^2} \ln k \right] \\ &\quad \times \frac{4}{\pi \omega_0} \frac{k}{k^2 + 1}, \\ k &= \frac{\omega}{1 \text{ GeV}}, \end{aligned} \quad (14)$$

the obtained QCD factorization formula (10) implies the shapes of $\phi_B^+(\xi, \mu)$ displayed in Fig. 2 at different values of $n_z \cdot v$, where the reference values of the logarithmic inverse moments $\omega_0 = 350 \text{ MeV}$ and $\sigma_B^{(1)} = 1.4$ are taken for illustration purposes. It is evident that $\phi_B^+(\xi, \mu)$ develops a radiative tail at large momentum ξ irrespective of the functional form of $\phi_B^+(\omega, \mu)$ and, in contrast to the quasiparton distribution function (PDF) [46], no peaks emerge in the momentum region $\xi \leq 0$. We also mention in passing that the predicted shapes of the leading-twist B -meson LCDA $\phi_B^+(\omega, \mu)$ at large ω from Ji's proposal can already confront with the perturbative QCD calculations carried out in [17–19] and it will thus provide interesting insight into the parton-hadron duality ansatz adopted in constructing perturbative QCD factorization theorems.

Implementing the lattice QCD computation of the spatial correlation $\phi_B^+(\xi, \mu)$ in practice will necessitate (a) reformulation of the hard-collinear factorization theorem (10) with either the lattice regularization scheme along the lines of [47] or the regularization-invariant momentum subtraction

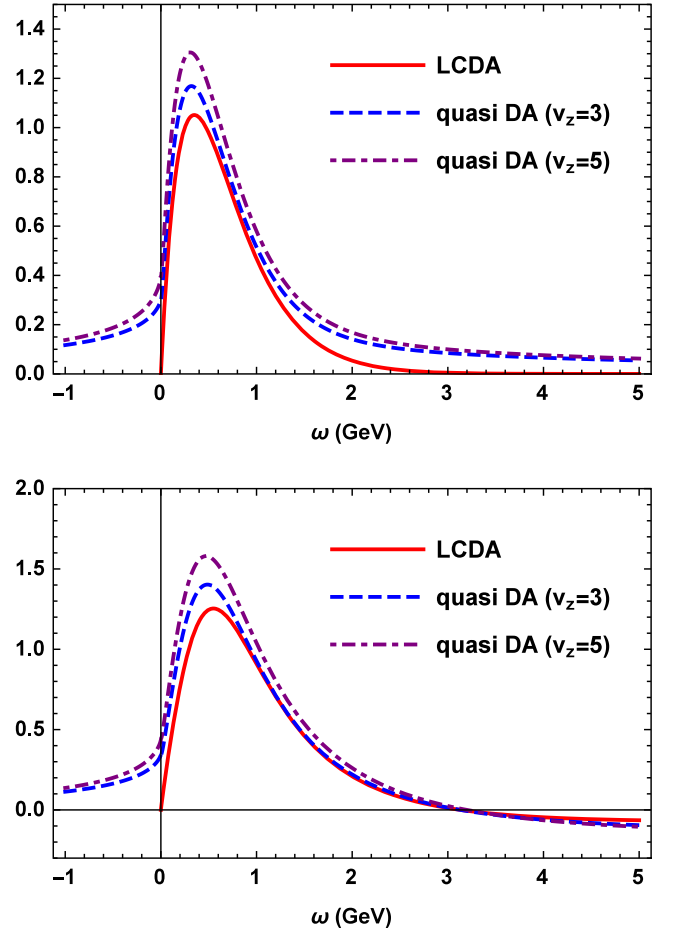


FIG. 2. Resulting ω shapes of the B -meson quasisdistribution amplitude $\phi_B^+(\xi = \omega, \mu = 1.5 \text{ GeV})$ in bHQET from the hard-collinear factorization theorem (10) and from the two non-perturbative models of $\phi_B^+(\omega, \mu = 1.5 \text{ GeV})$ presented in (14), with two different values of $n_z \cdot v$. The shadow region of $|\omega| \leq 200 \text{ MeV}$ is excluded due to inapplicability of the hard-collinear factorization formula (10) for $|n_z \cdot v \omega| \leq 1.0 \text{ GeV}$.

scheme [48] as already discussed in the context of the LaMET approach [46,49–52], and (b) improvement of various systematic uncertainties generated by the finite lattice spacing and the finite lattice box as well as by truncating the Fourier transformation from coordinate space with evaluations for a finite number of discrete τ 's to momentum space. In addition, computing the yet higher-order perturbative correction to the short-distance Wilson coefficient H and constructing the subleading-power factorization formula for the equal-time correlation function $\phi_B^+(\xi, \mu)$ will be also in high demand for precision determinations of the small-momentum behaviors of the B -meson LCDA $\phi_B^+(\omega, \mu)$.

It would be also of interest to construct a complementary method of determining the B -meson distribution amplitude $\phi_B^+(\omega, \mu)$ from the numerical simulation of the following Euclidean quantity in QCD:

$$\begin{aligned}
& i f_B m_B \psi_B^+(x, \mu) \\
& = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{i n_z \cdot p \tau} \langle 0 | (\bar{q} W_n)(\tau n_z) \not{n}_z \gamma_5 (W_n^\dagger b)(0) | \bar{B}(p) \rangle.
\end{aligned} \tag{15}$$

In this respect, the mostly nonperturbative renormalization method to renormalize the QCD heavy-light current presented in Refs. [53–56] can be applied for lattice calculations, while in HQET a concept for nonperturbative renormalization scheme in position space has been presented in Ref. [57]. From the perspectives of the continuum QCD, the newly introduced distribution amplitude $\psi_B^+(x, \mu)$ can be further matched onto the Euclidean HQET quantity $\varphi_B^+(\xi, \mu)$ by integrating out the short-distance fluctuations at the heavy-quark mass scale, in analogy to the hard-collinear factorization formula obtained in [58,59]. Furthermore, determining hadronic distribution functions on the light shell can be also achieved by constructing the spatial correlation functions of suitable local partonic currents and by establishing the desired QCD factorization formulas in coordinate space directly [60–64].

In comparison with the previous computation of $\phi_B^+(\omega, \mu)$ from the QCDSR method [20], the inapplicability of achieving a direct determination of the distribution amplitude locally in momentum space due to the singular behaviors of the nonperturbative quark-gluon condensate contributions has been resolved by combining the lattice simulation of the quasidistribution amplitude $\varphi_B^+(\xi, \mu)$ and the established hard-collinear factorization formula (10). Since our major objective is to explore the opportunity of accessing the light-cone dynamics of the leading-twist B -meson distribution amplitude $\phi_B^+(\omega, \mu)$ starting from the Euclidean space instead of carrying out a dedicated lattice calculation of the proposed quasidistribution amplitude $\varphi_B^+(\xi, \mu)$, it prevents us from drawing a definite conclusion on the theory precision of the obtained shape of $\phi_B^+(\omega, \mu)$. Actually, the numerical simulations of such quasidistribution amplitudes are still at an exploratory stage, even for the ones suitable for the determination of the light-meson distribution amplitude (DA). However, it might not be implausible to expect that the achieved accuracy of the obtained B -meson DA with our prescription can be comparable to the collinear pion DA (for the time being, the estimated errors of the shape parameters a_2^π and δ_2^π are at the level of (30–50)% as concluded in [60]) in the long run, provided that the desired methodologies to control both the statistical errors and the systematic uncertainties (see [27,29,65] for an elaborate discussion on lattice challenges) can be eventually constructed with the further development of new algorithms and computing techniques on the lattice (see, for instance [66]).

In particular, the fairly encouraging results from the state-of-art computations of the nucleon PDFs and the light-meson distribution amplitudes as summarized in

the comprehensive review [29] evidently demonstrate that the newly constructed LaMET formalism allows for a promising future to systematically compute a wide range of “parton observables” with the demanding computational resources and the tremendous development of new techniques and algorithms, which enable us to address the different sources of systematic uncertainties step by step in the future. Consequently, the obtained hard-collinear factorization formula for the B -meson quasidistribution amplitude in HQET will pave the way for the first-principle determination of the desperately desired LCDA $\phi_B^+(\omega, \mu)$, which is undoubtedly of the highest importance to improve the present theory precision for predicting any exclusive B -meson decay observable standing out constantly as the central focus of the ongoing experimental programs. In addition, inspecting the nontrivial relations between the two-particle and three-particle B -meson distribution amplitudes due to the QCD equations of motion [67–69] and the improved OPE constraints for $\phi_B^+(\omega, \mu)$ [18] with the updated values of λ_E and λ_H [70] by taking advantage of the forthcoming results obtained from the current strategy can be of substantial significance to boost our confidence on the robustness of the lattice HQET technique (see [71–74] for an alternative formalism).

VI. CONCLUSION

To summarize, we have proposed a novel approach to determine the momentum dependence of the leading-twist B -meson LCDA $\phi_B^+(\omega, \mu)$ in bHQET without introducing any approximation or assumption for its functional form. Applying the auxiliary heavy-quark field formalism, we have demonstrated explicitly the multiplicative renormalizability of the quasidistribution function $\varphi_B^+(\xi, \mu)$ at all orders in QCD. The perturbative matching function entering the hard-collinear factorization formula of the spatial correlation was further extracted with the OPE technique at $\mathcal{O}(\alpha_s)$. The present strategy of constructing the light-cone distribution functions in effective field theories can be also applied to the various B -meson shape functions relevant to the QCD description of $\bar{B} \rightarrow X_{d,s} \gamma$ and to the heavy-baryon distribution amplitudes appearing in the soft-collinear effective theory computation of $\Lambda_b \rightarrow \Lambda \ell \ell$. Our results are apparently of importance for exploring the delicate flavor structure of the SM and beyond at the LHCb and Belle II experiments.

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