Hypergravity in five dimensions

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We show that a spin-5/2 field can be consistently coupled to gravitation without cosmological constant in five-dimensional spacetimes. The fermionic gauge "hypersymmetry" requires the presence of a finite number of additional fields, including a couple of $U(1)$ fields, a spinorial two-form, the dual of the graviton (of mixed (2,1) Young symmetry) and a spin-3 field. The gravitational sector of the action is described by the purely quadratic Gauss-Bonnet term, so that the field equations for the metric are of second order. The local gauge symmetries of the full action principle close, without the need of auxiliary fields, for a suitable extension of the "hyper-Poincaré" algebra. Apart from the Poincaré and spin-3/2 generators, it includes a generator of spin 2 and a $U(1)$ central extension. Noteworthy, the algebra admits an invariant trilinear form, and its generators allow us to precisely accommodate the entire field content within a single connection, so that the hypergravity action can be formulated as a gauge theory described by a Chern-Simons form in five dimensions.

I. INTRODUCTION

Soon after the advent of supersymmetry, it was natural to wonder about the possible superpartners of the graviton [1]. According to the basic rules of supersymmetry, there are just two possibilities. One of them corresponds to a massless spin-3/2 Rarita-Schwinger field, which can be consistently coupled to gravitation. The full theory is widely known as supergravity [2,3] and it has been extensively explored in different contexts (see e.g., [4–8]). One has barely heard about the remaining possibility, which involves a massless field of spin 5/2, and there is a good reason for that: in four spacetime dimensions the theory is inconsistent [9,10] (in agreement with the no-go theorems for interacting higher spin theories, see e.g., [11–13]). The obstructions to achieve a consistent coupling can be seen as follows. The action for a massless spin-5/2 field on flat space [14–18] possesses a fermionic gauge symmetry, that is not preserved once the spin-5/2 field is minimally coupled to gravity [9,10]. In few words, the variation of the fermionic action under the fermionic gauge

symmetry goes like the Riemann tensor, while the variation of the Einstein-Hilbert action becomes proportional to the Einstein tensor, leading no room for cancellation. Besides, the consistency of the fermionic field equation implies the vanishing of the Weyl tensor [9,19], which is certainly too stringent as a condition to be imposed on four-dimensional spacetimes. It has also been shown that neither the addition of cosmological constant nor including nonminimal couplings help in order to circumvent these obstructions [9,19,20] (for related discussions see e.g., [25–27], and references therein). A well-known proposal to surmount these obstacles at least at the cubic level is to turn on the cosmological constant [28,29] and proceed to constructing "formally consistent" classical equations of motion [30–32]. Other approaches to construct consistent interactions of higher spin fields include holographic reconstruction [33–35], light-front approach [36–38] and also examples in $3D$ [39–49].

In three spacetime dimensions the situation is radically different. Indeed, the Weyl tensor identically vanishes in $D = 3$, which allows to express the Riemann tensor in terms of the Ricci tensor and the Ricci scalar. Hence, the aforementioned obstructions to couple a spin-5/2 field to gravitation can be successfully bypassed.

Thus, in the next section we summarize the state of the art of hypergravity in three-dimensional spacetimes, aiming to identify some of its key features that can be extended to higher dimensions. Section III is then devoted to analyze some of the identified structures that can indeed be lifted to higher-dimensional spacetimes, paying special attention in

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a particular nonstandard formulation of supergravity in five dimensions that is endowed with further interesting structures for our purposes. The moral of the aforementioned results allows to formulate certain mild and reasonable hypotheses, so that in Sec. IV we show that, once relying on them, an action for hypergravity in five dimensions can be readily found through the standard Noether procedure. Remarkably, we obtain a local field theory described by a finite number of fields, of spin up to 3, on standard Riemann-Cartan geometry. The local gauge symmetries of the action principle are also unveiled, and then, we also show that the hypergravity action can be formulated as a gauge theory with standard fiber bundle structure, described by a Chern-Simons form in five dimensions. We conclude with some ending remarks in Sec. V, where we briefly analyze the spectrum of the theory around a braneworld-like background configuration, and also possible extensions of our results to higher odd dimensions.

II. HYPERGRAVITY IN THREE SPACETIME DIMENSIONS

The consistent coupling of a spin-5/2 field to general relativity in $D = 3$ was achieved by Aragone and Deser [50], who dubbed the theory as "hypergravity." It is the first and one of the simplest theories describing a higher spin field coupled to gravitation. The theory can be conveniently formulated in a local frame, so that the action reads

$$I = \frac{k}{4\pi} \int 2R^a e_a + i\bar{\psi}_a D\psi^a , \qquad (1)$$

where e^a stands for the dreibein, and $R^a = d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b\omega_c$ for the curvature two-form in terms of the dualized spin connection $\omega^a = \frac{1}{2}\epsilon^{abc}\omega_{bc}$. The irreducible spin-5/2 field $\psi^a = \psi^a_\mu dx^\mu$ is assumed to be "Γ-traceless," i.e., $\Gamma^a\psi_a = 0$, whose Lorentz covariant derivative is given by $D\psi^a = d\psi^a + \frac{1}{2}\omega^b\Gamma_b\psi^a + \epsilon^{abc}\omega_b\psi_c$ [51].

Following the "1.5 formalism" (see, e.g., [4]) it is simple to verify that the theory described by (1) is invariant under

$$\delta\psi^a = D\epsilon^a \quad \text{and} \quad \delta e^a = \frac{3}{2}i\bar{\epsilon}_b\Gamma^a\psi^b, \qquad (2)$$

where the spin-3/2 parameter $\epsilon^a = \epsilon^a(x^\mu)$ is also Γ-traceless. In analogy with supergravity [3], the spin connection was proposed to transform as [50]

$$\delta\omega^a = 3ie^{-1}\bar{\epsilon}^b\Gamma_c f^\nu_b\left(\frac{1}{2}e^c_\nu e^a - e^a_\nu e^c\right), \qquad (3)$$

with $f^\nu_b = \epsilon^{\nu\rho\lambda}D_\rho\psi_{\lambda b}$.

In [52], it was shown that hypergravity can be reformulated so that the field content corresponds to the components of a gauge field that takes values in the "hyper-Poincaré" algebra

$$A = e^a P_a + \omega^a J_a + \psi^\alpha_a Q^a_\alpha, \qquad (4)$$

where the fermionic generators Q^a_α are Γ-traceless, and its nonvanishing (anti)commutators read

$$[J_a, J_b] = \epsilon_{abc}J^c, \qquad [J_a, P_b] = \epsilon_{abc}P^c,$$

$$[J_a, Q_{\alpha b}] = \frac{1}{2}(\Gamma_a)^\beta_{\ \alpha}Q_{\beta b} + \epsilon_{abc}Q^c_\alpha,$$

$$\{Q^a_\alpha, Q^b_\beta\} = -\frac{2}{3}(C\Gamma^c)_{\alpha\beta}P_c\eta^{ab} + \frac{5}{6}\epsilon^{abc}C_{\alpha\beta}P_c$$

$$+ \frac{1}{3}(C\Gamma^{(a|})_{\alpha\beta}P^{|b)}. \qquad (5)$$

The hyper-Poincaré algebra (5) admits an invariant bilinear form, whose nonvanishing components are given by

$$\langle J_a, P_b\rangle = \eta_{ab}, \qquad \langle Q^a_\alpha, Q^b_\beta\rangle = \frac{2}{3}C_{\alpha\beta}\eta^{ab} - \frac{1}{3}\epsilon^{abc}(C\Gamma_c)_{\alpha\beta}, \qquad (6)$$

and hence the hypergravity action (1) can be written in terms of a Chern-Simons form

$$I = \frac{k}{4\pi}\int\left\langle AdA + \frac{2}{3}A^3\right\rangle, \qquad (7)$$

up to a boundary term. The local fermionic hypersymmetry is then spanned by $\delta A = d\lambda + [A, \lambda]$, with $\lambda = \epsilon^\alpha_a Q^a_\alpha$, so that

$$\delta\psi^a = D\epsilon^a, \qquad \delta\omega^a = 0, \qquad \delta e^a = \frac{3}{2}i\bar{\epsilon}^b\Gamma^a\psi_b, \quad (8)$$

which agree with (2), (3) only on-shell.

It is worth emphasizing that since the dynamical fields belong to the components of the connection (4), instead of a multiplet, the full set of transformation laws closes in the hyper-Poincaré algebra without the need of auxiliary fields.

Having formulated hypergravity in terms of a Chern-Simons action certainly helps in order to unveil the structure of the theory. Indeed, its uniqueness has been recently established by virtue of Becchi-Rouet-Stora-Tyutin-cohomological techniques [54]. Besides, it was also possible to analyze the asymptotic structure of the theory in [53]. The canonical generators of the asymptotic symmetries were shown to span a hypersymmetric nonlinear extension of the BMS$_3$ algebra (endowed with fermionic generators of conformal weight 5/2), with the same central extension as in the bosonic case [55]. This hyper-BMS$_3$ algebra was also shown to admit unitary representations [56–58]. Interestingly, as it occurs for $\mathcal{N} = 1$ supergravity [59], the anticommutator of the fermionic generators also allows to find (an infinite number of) BPS-like bounds for the bosonic charges, which in the hypersymmetric case turn out to be nonlinear. Circularly symmetric solutions describing both cosmological spacetimes [60–62]

and conical singularities [63,64] fulfill all of the bounds without saturating them (broken hypersymmetries), while configurations with conical surpluses (angular excess) do not satisfy the bounds. Hypersymmetry bounds saturate for configurations with unbroken hypersymmetries, possessing globally well-defined Killing vector-spinors that fulfill

$$de^a + \frac{1}{2}\omega^b\Gamma_b\epsilon^a + \epsilon^{abc}\omega_b\epsilon_c = 0. \tag{9}$$

In the case of fermions with antiperiodic boundary conditions, the BPS-like configuration that is maximally hypersymmetric corresponds to Minkowski spacetime; while for periodic boundary conditions it is given by the null orbifold [65], possessing a single unbroken hypersymmetry.

It is also worth noting that, as in the case of supergravity [59,66,67], the hypergravity theory can also be extended so as to include parity odd terms in the action [52,53], and consequently, the asymptotic hyper-BMS$_3$ algebra becomes endowed with an additional independent central extension along the Virasoro subalgebra.

Hypergravity in 3D also admits a negative cosmological constant [68–70] and it can be formulated as a Chern-Simons theory for $osp(1|4) \oplus osp(1|4)$, at the expense of including an additional spin-4 field and a Lorentz-like field of mixed symmetry in tangent space. The local Lorentz algebra is then enlarged from $sl(2,R)$ to $sp(4)$, so that the metric is not invariant under spin-4 gauge symmetries. The asymptotic symmetries are given by the direct sum of two copies of the $W_{(2,\frac{5}{2},4)}$ algebra, from which nonlinear bounds for the bosonic charges can also be established [70,71]. In the limit of vanishing cosmological constant the spin-4 and the Lorentz-like field can be consistently decoupled, so that hypergravity can be suitably formulated in terms of Riemann-Cartan geometry. Furthermore, as shown in [53], the hyper-BMS$_3$ algebra also arises from a suitable truncation of $W_{(2,\frac{5}{2},4)} \oplus W_{(2,\frac{5}{2},4)}$ in the flat limit.

III. LIFTING TO HIGHER DIMENSIONS

The formulation of hypergravity in 3D as a gauge theory of the hyper-Poincaré algebra has been shown to be fruitful, and it is then natural to capture some of its properties that could be lifted to higher dimensions.

In this sense, it is worth highlighting that the hyper-Poincaré algebra actually exists for any $D > 2$ dimensions [52], whose nonvanishing (anti)commutators read

$$[J_{ab}, J_{cd}] = J_{ad}\eta_{bc} - J_{bd}\eta_{ac} + J_{ca}\eta_{bd} - J_{cb}\eta_{ad}$$

$$[J_{ab}, P_c] = P_a\eta_{bc} - P_b\eta_{ac}$$

$$[J_{ab}, Q_c^\alpha] = -\frac{1}{2}(\Gamma_{ab})^\alpha{}_\beta Q_c^\beta + Q_a^\alpha\eta_{bc} - Q_b^\alpha\eta_{ac} \tag{10}$$

$$[J_{ab}, \bar{Q}_{\alpha c}] = \frac{1}{2}(\Gamma_{ab})^\beta{}_\alpha\bar{Q}_{\beta c} + \bar{Q}_{\alpha a}\eta_{bc} - \bar{Q}_{\alpha b}\eta_{ac}$$

$$\{Q^{\alpha a}, \bar{Q}_\beta^b\} = \frac{3(D-2)}{D^2}i\left[(D+1)(\Gamma^c)^\alpha{}_\beta P_c\eta^{ab}\right.$$
$$\left. - \frac{D+2}{D-2}(\Gamma^{abc})^\alpha{}_\beta P_c - 2(\Gamma^{(a|})^\alpha{}_\beta P^{|b)}\right] \tag{11}$$

where $\bar{Q}_a = Q_a^\dagger\Gamma^0$ generically stands for the Dirac conjugate [73].

Thus, the very existence of the algebra (10), (11) suggests the possibility that hypergravity in higher dimensions might be formulated as a gauge theory of some suitable extension of the hyper-Poincaré group. Indeed, if such extension admitted an invariant tensor of rank n, a non-Abelian Chern-Simons action in $D = 2n - 1$ dimensions might be a good candidate to explore.

Another interesting lesson that can be extracted from the three-dimensional case without cosmological constant is that, according to the action principle in (1), the spin-5/2 field couples to the geometry exclusively through the spin connection (the dreibein is not involved in the Lorentz covariant derivative). Hence, in our formulation, the fermionic field does not contribute to the stress-energy tensor, but it is noticed by the geometry due to the torsion $T^a = \frac{3}{4}i\bar{\psi}_b\Gamma^a\psi^b$. Such type of couplings are certainly excluded in $D = 4$ [19], but nonetheless, there is some evidence suggesting that this kind of couplings could be realized in higher odd dimensions. Indeed, a class of supergravity theories featuring fermions that are nonminimally coupled to the curvature, but not to the vielbein, is known to exist for any odd dimensions [74], from which one can also extract a helpful moral.

A. Nonstandard supergravity in five dimensions

For the sake of simplicity, let us consider the case of (nonstandard) supergravity in five dimensions, whose action reads

$$I = I_{\text{GB}} - 3I_b + I_\psi, \tag{12}$$

where

$$I_{\text{GB}} = \frac{1}{2}\int \varepsilon_{abcdf}R^{ab}R^{cd}e^f, \tag{13}$$

$$I_b = \int R^{ab}R_{ab}b, \tag{14}$$

$$I_\psi = 3\int \bar{\psi}R^{bc}\Gamma_{bc}D\psi + \text{H.c.} \tag{15}$$

Here, the gravitational sector is described by the pure Gauss-Bonnet action I_{GB} in (13). Since I_{GB} corresponds to a particular case of Lovelock theory, devoid of the

Einstein-Hilbert and cosmological terms, the field equations for the metric are of second order.

Invariance under local supersymmetry requires the presence of an additional bosonic term I_b (14), where $b = b_\mu dx^\mu$ stands for an Abelian 1-form.

The dynamics of the spin-3/2 field $\psi = \psi_\mu dx^\mu$, whose Lorentz covariant derivative is given by $D\psi = d\psi + \frac{1}{4}\omega^{ab}\Gamma_{ab}\psi$, is described by the fermionic term I_ψ in (15). Note that the fermionic field nonminimally couples to the spacetime geometry through the curvature two-form R^{ab} (instead of the vielbein).

The action (12) is invariant under the following local supersymmetry transformations

$$\delta\psi^a = D\epsilon^a, \qquad \delta\omega^{ab} = 0, \qquad (16)$$

$$\delta e^a = 3i\bar{\epsilon}\Gamma^a\psi + \text{H.c.}, \qquad \delta b = \bar{\epsilon}\psi + \text{H.c.} \quad (17)$$

It is worth noting that since I_{GB} in (13) is linear in the vielbein instead of the curvature, the local Lorentz symmetry is extended to the local Poincaré group in five dimensions. The full set of local symmetries also includes the gauge transformation associated to the bosonic Abelian field b, and hence, their algebra corresponds to super-Poincaré with a $U(1)$ central extension, so that the anticommutator of the fermionic generators reads

$$\{Q^\alpha, \bar{Q}_\beta\} = 3i(\Gamma^a)^\alpha{}_\beta P_a + \delta^\alpha_\beta K. \qquad (18)$$

The presence of the central extension plays a relevant role, since in this case the super-Poincaré algebra admits an invariant (anti)symmetric form of rank three, whose nonvanishing components are given by

$$\langle J_{ab}, J_{cd}, P_f \rangle = \frac{2}{3}\varepsilon_{abcdf}, \qquad (19)$$

$$\langle J_{ab}, J_{cd}, K \rangle = -4\eta_{a[c}\eta_{d]b},$$
$$\langle Q^\alpha, J_{ab}, \bar{Q}_\beta \rangle = -2(\Gamma_{ab})^\alpha{}_\beta, \qquad (20)$$

which is crucial in order to formulate this class of supergravity as a gauge theory. Indeed, the field content corresponds to the components of a connection for the centrally extended super-Poincaré algebra

$$A = e^a P_a + \frac{1}{2}\omega^{ab}J_{ab} + bK + \bar{\psi}Q - \bar{Q}\psi, \quad (21)$$

so that the local (super)symmetries are obtained from a gauge transformation $\delta A = d\lambda + [A, \lambda]$, where λ takes values on the super-Poincaré algebra with a $U(1)$ central extension. Thus, up to a boundary term, the supergravity action (12) can be written as a Chern-Simons form

$$I = \int \left\langle AF^2 - \frac{1}{2}A^3F + \frac{1}{10}A^5 \right\rangle, \qquad (22)$$

where $F = dA + A^2$.

In the next section we construct a hypergravity theory in five dimensions that shares some of these features.

IV. HYPERGRAVITY IN FIVE DIMENSIONS

Following the morals outlined in the previous sections, for a fermionic field $\psi^a = \psi^a_\mu dx^\mu$ that fulfills $\Gamma^a\psi_a = 0$, we look for a kinetic term whose coupling to the geometry does not involve the vielbein, being of the form

$$\bar{\psi}_a X^{ab} D\psi_b + \text{H.c.} \qquad (23)$$

Local Lorentz invariance then implies that X^{ab} must be necessarily proportional to the curvature two-form R^{cd}. It is then simple to verify that there are just five possibilities, so that X^{ab} is given by a combination of the following terms:

$$R^{ab}, \qquad \eta^{ab}R^{cd}\Gamma_{cd} \qquad (24)$$

$$R^a{}_c\Gamma^{cb}, \qquad \Gamma^a{}_c R^{cb}, \qquad \epsilon^{abcde}R_{cd}\Gamma_e \qquad (25)$$

Nonetheless, since the fermionic field is Γ-traceless, the last three possibilities in (25) become redundant, and the relevant ones then reduce to the remaining two in (24). Hence, X^{ab} must be of the form

$$X^a{}_b = \delta^a{}_b R^{cd}\Gamma_{cd} + \alpha R^a{}_b. \qquad (26)$$

We also assume that the local fermionic gauge symmetry (hypersymmetry) is spanned by a spin-3/2 parameter ϵ^a subject to the Γ-traceless condition ($\Gamma^a\epsilon_a = 0$), so that the transformation law of the fermionic field and the spin connection agree with those in the three-dimensional case, i.e.,

$$\delta\psi^a = D\epsilon^a, \qquad \delta\omega^{ab} = 0. \qquad (27)$$

As explained below, it is useful to fix the arbitrary constant in $X^a{}_b$ according to $\alpha = -4$, since it minimizes the number of additional bosonic fields, and in particular, it avoids the introduction of extra fields with mixed symmetry on tangent space.

A. Action principle and local (hyper)symmetries

According to the hypotheses outlined at the beginning of this section, the searched for action principle for hypergravity can then be readily obtained through the standard Noether procedure, starting from the fermionic kinetic term in (23), with $X^a{}_b$ given by (26) and $\alpha = -4$, and the local hypersymmetry transformation laws in (27).

The action is then given by

$$I = I_{\mathrm{GB}} + \frac{9}{5} I_b + I_{e^{ab}} + I_{\psi^a}, \qquad (28)$$

where I_{GB} stands for the pure Gauss-Bonnet action in (13), I_b is given by (14), and the remaining terms read

$$I_{e^{ab}} = -24 \int R^a{}_c R^{cb} e_{ab}, \qquad (29)$$

$$I_{\psi^a} = 3 \int \bar{\psi}_a (\delta^a_b R^{cd} \Gamma_{cd} - 4R^a{}_b) D\psi^b + \mathrm{H.c.} \quad (30)$$

Apart from the vielbein e^a, the spin connection ω^{ab} and the fermionic one-form field ψ^a, the theory includes additional bosonic fields described by an Abelian field $b = b_\mu dx^\mu$, and a one-form given by $e^{ab} = e^{ab}_\mu dx^\mu$, being symmetric and traceless in tangent space ($e^{ab} = e^{ba}$, $\eta_{ab} e^{ab} = 0$).

The "off-shell" spin can be readily identified through looking at the irreducible components of the fields under the pullback of the local Lorentz group. The fermionic one-form contains a spin-5/2 field $\psi_{(\mu\nu)}$ and a spinorial two-form $\psi_{[\mu\nu]}$, which are symmetric and antisymmetric under $\mu \leftrightarrow \nu$, respectively [75]. Analogously, the one-form e^{ab} decomposes as

$$e^{ab}_\mu : \{ e_{(\mu\nu\lambda)}, e^\nu{}_{\nu\mu}, e_{\mu\nu,\lambda} \}, \qquad (31)$$

so that its trace $e^\nu{}_{\nu\mu}$ corresponds to a spin-1 field, while the fully symmetric part $e_{(\mu\nu\lambda)}$ describes the highest spin field ($s = 3$). The remaining field $e_{\mu\nu,\lambda}$ is of mixed (2,1) Young symmetry, remarkably coinciding with the dual of the graviton in five dimensions [77] (see also [78]). It is worth highlighting that in our context, the field $e_{\mu\nu,\lambda}$ is strictly required by hypersymmetry, instead of duality at the linearized level.

We should also emphasize one of the advantages of keeping all of the irreducible components of the fields aforementioned. Indeed, if one wanted to project them out, one might follow the nowadays standard procedure of enlarging the Lorentz group by introducing additional fields of mixed symmetry in tangent space (see e.g., [18,25,40,76,79,80]). However, when the spacetime geometry becomes dynamical, the price to pay would be that the metric transforms in a nontrivial way under higher spin transformations, implying that Riemannian geometry should be extended in some suitable way, which is hitherto unknown. Hence, keeping all of the irreducible components aforementioned, allows us to formulate a consistent theory of hypergravity in five dimensions in a conservative way, described by a local field theory with a finite number of fields (of spin up to three) on standard Riemann-Cartan geometry.

The action is invariant under the following local hypersymmetry transformations

$$\delta\psi^a = D\epsilon^a, \qquad \delta\omega^{ab} = 0. \qquad (32)$$

$$\delta e^a = 3i\bar{\epsilon}^b \Gamma^a \psi_b + \mathrm{H.c.} \qquad (33)$$

$$\delta b = \bar{\epsilon}^a \psi_a + \mathrm{H.c.} \qquad (34)$$

$$\delta e^{ab} = \bar{\epsilon}^{(a} \psi^{b)} - \frac{1}{5} \eta^{ab} \bar{\epsilon}_c \psi^c + \mathrm{H.c.} \qquad (35)$$

Note that the transformations in (32), (33) agree with those in the three-dimensional case (extended for Dirac fermions).

Apart from the manifest local Lorentz invariance, the action (28) is also invariant under local translations, and $U(1)$ gauge transformations with parameters λ^a and λ, respectively. An additional local bosonic "spin 1-2-3" symmetry is spanned by a parameter λ^{ab}, that is symmetric and traceless. The transformation laws of the fields under these symmetries are given by

$$\delta e^a = D\lambda^a, \qquad \delta b = d\lambda, \qquad \delta e^{ab} = D\lambda^{ab}, \quad (36)$$

$$\delta\omega^{ab} = 0, \qquad \delta\psi^a = 0. \qquad (37)$$

The whole set of local gauge symmetries of the hypergravity action (28) turns out to close for an extension of the hyper-Poincaré algebra in (10), (11) in five dimensions, endowed with an additional $U(1)$ generator K and a symmetric and traceless spin-2 generator P_{ab}, whose non-vanishing (anti)commutators manifestly read

$$[J_{ab}, J_{cd}] = J_{ad}\eta_{bc} - J_{bd}\eta_{ac} + J_{ca}\eta_{bd} - J_{cb}\eta_{ad},$$

$$[J_{ab}, P_c] = P_a\eta_{bc} - P_b\eta_{ac},$$

$$[J_{ab}, P_{cd}] = P_{ad}\eta_{bc} - P_{bd}\eta_{ac} + P_{ca}\eta_{bd} - P_{cb}\eta_{ad},$$

$$[J_{ab}, Q_c^\alpha] = -\frac{1}{2}(\Gamma_{ab})^\alpha{}_\beta Q_c^\beta + Q_a^\alpha\eta_{bc} - Q_b^\alpha\eta_{ac},$$

$$[J_{ab}, \bar{Q}_{\alpha c}] = \frac{1}{2}(\Gamma_{ab})^\beta{}_\alpha \bar{Q}_{\beta c} + \bar{Q}_{\alpha a}\eta_{bc} - \bar{Q}_{\alpha b}\eta_{ac},$$

$$\{Q^{\alpha a}, \bar{Q}_\beta^b\} = \frac{54}{25} i(\Gamma^c)^\alpha{}_\beta P_c\eta^{ab} - \frac{21}{25} i(\Gamma^{abc})^\alpha{}_\beta P_c$$

$$- \frac{18}{25} i(\Gamma^{(a|})^\alpha{}_\beta P^{|b)} + \frac{1}{5}(4\eta^{ab}\delta^\alpha_\beta - (\Gamma^{ab})^\alpha{}_\beta) K$$

$$+ \frac{1}{10}(3\delta^\alpha_\beta P^{ab} + 2(\Gamma_c^{[a|})^\alpha{}_\beta P^{|b]c}). \qquad (38)$$

B. Hypergravity as a gauge theory for the extended hyper-Poincaré algebra

Interestingly, the local gauge symmetries of the hypergravity action close according to the extended

hyper-Poincaré algebra in (38) without the need of auxiliary fields. Indeed, this is a consequence of the fact that the field content precisely corresponds to the components of a single connection of the extended hyper-Poincaré algebra

$$A = e^a P_a + \frac{1}{2}\omega^{ab} J_{ab} + bK + \frac{1}{2}e^{ab} P_{ab} + \bar{\psi}^a Q_a - \bar{Q}_a \psi^a,$$

$$(39)$$

and hence, the local gauge symmetries are recovered from $\delta A = d\eta + [A, \eta]$, where η stands for an extended hyper-Poincaré-algebra-valued zero-form

$$\eta = \lambda^a P_a + \frac{1}{2}\sigma^{ab} J_{ab} + \lambda K + \frac{1}{2}\lambda^{ab} P_{ab} + \bar{\epsilon}^a Q_a - \bar{Q}_a \epsilon^a.$$

$$(40)$$

Besides, the additional bosonic generators K and P_{ab} extend the hyper-Poincaré algebra so that it admits an invariant (anti-)symmetric form of rank three. Their non-vanishing components are given by

$$\langle J_{ab}, J_{cd}, P_f \rangle = \frac{2}{3}\varepsilon_{abcdf},$$

$$\langle J_{ab}, J_{cd}, K \rangle = \frac{12}{5}\eta_{a[c}\eta_{d]b},$$

$$\langle J_{ab}, J_{cd}, P_{ef} \rangle = -64\left(\eta_{e[a}\eta_{b][c}\eta_{d]f} - \frac{1}{5}\eta_{c[a}\eta_{b]d}\eta_{ef}\right),$$

$$\langle Q_c^\alpha, J_{ab}, \bar{Q}_{\beta d} \rangle = 2[4\mathbb{P}_{c[a}\mathbb{P}_{b]d} - \mathbb{P}_{ce}\Gamma_{ab}\mathbb{P}^e{}_d]^\alpha{}_\beta,$$

$$(41)$$

where the Γ-traceless projector $(\mathbb{P}_{ab})^\alpha{}_\beta = \eta_{ab}\delta^\alpha{}_\beta - \frac{1}{5}(\Gamma_a\Gamma_b)^\alpha{}_\beta$, ensures Γ-traceless of the fermionic entries of the bracket.

Therefore, the hypergravity theory (28) can be formulated as a gauge theory with standard fiber bundle structure. Indeed, by virtue of (39) and (41), the action (28) can be written in terms of a Chern-Simons form in five dimensions

$$I = \int \left\langle AF^2 - \frac{1}{2}A^3 F + \frac{1}{10}A^5 \right\rangle,$$

$$(42)$$

up to a boundary term. Note that the field equations can then be compactly expressed in a manifestly covariant way under the extended hyper-Poincaré algebra, spanned by the set $G_I = \{J_{ab}, P_a, K, P_{ab}, Q_a, \bar{Q}_a\}$, according to

$$\langle F^2 G_I \rangle = 0.$$

$$(43)$$

Here, $F = dA + A^2$ stands for the field strength, whose components are given by

$$F = \tilde{T}^a P_a + \frac{1}{2}R^{ab} J_{ab} + \tilde{T}K + \frac{1}{2}\tilde{T}^{ab} P_{ab}$$

$$+ D\bar{\psi}^a Q_a - \bar{Q}_a D\psi^a,$$

$$(44)$$

where

$$\tilde{T}^a = T^a - 3i\bar{\psi}_b\Gamma^a\psi^b,$$

$$(45)$$

$$\tilde{T} = db - \bar{\psi}_a\psi^a,$$

$$(46)$$

$$\tilde{T}^{ab} = De^{ab} - \bar{\psi}^{(a}\psi^{b)} + \frac{1}{5}\eta^{ab}\bar{\psi}_c\psi^c,$$

$$(47)$$

and $T^a = De^a$ is the torsion.

V. ENDING REMARKS

We have proposed an action principle for hypergravity in five-dimensional spacetimes, that can be formulated as a field theory with standard fiber bundle structure. Noteworthy, the theory contains a finite number of higher spin fields (up to just $s = 3$), that can be seen as the components of a connection for the extended hyper-Poincaré group. Thus, as the fields are not arranged within the components of an irreducible multiplet, the possible inconsistencies associated to the Haag-Łopuszański–Sohnius theorem [81] can be successfully circumvented. In particular, bosonic and fermionic degrees of freedom are then not restricted to match.

The theory possesses a simple geometric structure, but as it is the case of generic non-Abelian Chern-Simons theories in five dimensions, the analysis of its dynamics is not so straightforward [82–86], since the constraint structure changes along phase space. A simple way of visualizing this is the following. The field equations (43) are trivially solved by Minkowski spacetime in vacuum, because in that case the field strength F in (44) vanishes. Nevertheless, since the field equations are purely quadratic in F, linear perturbations around this (maximally hypersymmetric) configuration possess a "linearization instability," implying that the analysis necessarily requires to go to higher order. As pointed out in [87,88], this feature appears to be welcome, since the theory naturally tends to explore different vacua. Note that solutions in vacuum, without torsion, fulfill the analogue of the Einstein equation

$$\varepsilon_{abcdf}R^{ab}R^{cd} = 0,$$

$$(48)$$

with an additional condition on the geometry that comes from the variation of the action (28) with respect to the bosonic fields b and e_{ab}, given by

$$R^a{}_c R^{cb} = 0.$$

$$(49)$$

Equations (48) and (49) also emerge from the consistency of the fermionic field equations. It is then worth

highlighting that the latter condition is not as stringent as requiring the vanishing of the Weyl tensor. Indeed, it is simple to verify that the spherically symmetric solution of (48) (see [89,90]) automatically fulfills (49), without imposing any additional condition on the geometry.

The field equations (48) and (49) can also be seen to admit braneworld-like solutions in vacuum, whose metric is given by [87]

$$ds^2 = e^{-2a|z|}(dz^2 + \tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu), \qquad (50)$$

where $\tilde{g}_{\mu\nu}$ stands for the metric of a maximally symmetric spacetime along the four-dimensional brane. Remarkably, a precise jump in the extrinsic curvature is allowed by the theory in vacuum, i.e., without the need of an induced stress-energy tensor on the brane. This effect can also be seen to arise from the fact that the analog of the Israel junction conditions in this case become quadratic in the extrinsic curvature, and hence, they admit nontrivial solutions even in vacuum (see e.g., [91]). As in [87], metric perturbations along the brane turn out to possess a well-defined propagator, precisely given by that of a Fierz-Pauli massless graviton, provided that the induced cosmological constant on the braneworld is strictly positive. In other words, perturbations of the metric along the brane reproduce linearized general relativity around a four-dimensional de Sitter spacetime with curvature radius given by a^{-2}.

It is also worth pointing out that perturbations of the fermionic fields on the braneworld-like metric (50) appear to reproduce a sort of partially massless version of the Sorokin-Vasiliev doublet [26] (see also [92–94]), on de Sitter spacetime [95].

It would also be interesting to study new dimensional reduction schemes that apply for the class of theories under discussion, as those recently proposed in [96].

Formulating hypergravity in presence of cosmological constant would also be worth to explore. Nonetheless, as in the three-dimensional case [68–70], the introduction of additional bosonic higher spin fields seems to be inevitable. Indeed, different couplings of higher spin fields to gravitation on AdS_5 along these lines have also been proposed in [97–99]. It might also be of interest to see whether any of the higher spin algebras already available in the literature could lead to sensible theories in higher odd dimensions.

Besides, the coupling of fermionic fields of half-integer spin with gravitation in three-dimensional spacetimes is also known to be consistent [50], and it has been shown that the theory possesses a suitable reformulation in terms of the hyper-Poincaré algebra with fermionic generators of spin $s = n + \frac{1}{2}$ [52,100]. Lifting these results to higher (odd) dimensions then certainly deserves consideration. Indeed, for fermionic generators of spin $s = 3/2$, preliminary results suggest that hypergravity theories with a finite number of bosonic fields can actually be formulated in odd spacetime dimensions. Interestingly, as in the five-dimensional case, the bosonic field with the highest spin is also given by $s = 3$ in $D = 5 \bmod 4$ dimensions; while for $D = 7 \bmod 4$, no bosonic higher spin fields are required ($s \leq 2$) [105].

ACKNOWLEDGMENTS

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