Constant-roll brane inflation

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The scenario of constant-roll inflation in the frame of the Randall-Sundraum II (RS II) brane gravity model is considered. Based on the scenario, the smallness of the second slow-roll parameter is released and it is assumed as a constant which could be of the order of unity. Applying the Hamilton-Jacobi formalism, the constancy of the parameter gives a differential equation for the Hubble parameter which leads to an exact solution for the model. Reconsidering the perturbation equations, we show there are some modified terms appearing in the amplitude of the scalar perturbations and in turn in the scalar spectral index and tensor-to-scalar ratio. Comparing the theoretical results of the model with observational data, the free parameters of the model are determined. Then, the consistency of the model with the swampland criteria is investigated for the obtained values of the free parameters. As the final step, the attractor behavior of the model is considered.

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I. INTRODUCTION

Inflation is an exponential expansion of space in the very early Universe during an extremely short period of time. This scenario has received observational support [1–5] which makes it one of the cornerstones of physical cosmology, rendering any model for the evolution of the Universe incomplete without the inflation phase. The first realistic inflationary scenario was proposed four decades ago as a solution to two of the shortcomings of the hot big bang model [6–10], namely, the flatness and the horizon problems, and since then became the leading paradigm for the early Universe.

So far, many inflationary models have been introduced based on the slow-roll assumptions where the inflaton, a scalar field, slowly rolls down its potential. It is described by two dimensionless parameters, known as the slow-roll parameters, which their smallness during inflation guarantees an almost flat potential [11–14]. Example of such models include noncanonical inflation [15–23], tachyon inflation [24–27], Dirac-Born-Infeld (DBI) inflation [28–32], G-inflation [33–36], and warm inflation [37–44]. However, a different inflationary scenario has been proposed very recently which goes beyond the slow-roll approximation, where the second slow-roll parameter does not have to be smaller than unity and can be a constant [45,46]. This constant-roll inflation (CRI) scenario attracted a lot of interest among cosmologists as an alternative way for the inflation phase to take place [47–65].

Inspired by string theory, one can consider our observable Universe to be a $(3+1)$ four-dimensional hypersurface (the brane) embedded in a higher-dimensional spacetime (the bulk). We consider a five-dimensional space and assume that all standard model particles to be confined to the brane, and only gravity is allowed to propagate in the fifth dimension. The most popular are the Arkani-Hamed-Dimopoulos-Dvali (ADD) and the Randall-Sandrum (RS) models [66,67], which were proposed in an attempt to solve the hierarchy problem between the Planck scale and the electroweak scale. Furthermore, in some inflationary models in the context of the brane-world scenario, the inflaton potential arises naturally from higher-dimensional gravity [68–83] and yields interesting cosmological implications [84–88].

There has been a growing interest in applying the CRI scenario to many inflationary models which, depending on the details of the model, results in some modification of the Friedmann equation. In the brane-world scenario, the Friedmann equation will contain both quadratic and linear terms, which in the high-energy regime (i.e., $\rho \gg \lambda$) the linear term can be ignored. In this case, unlike the standard four-dimensional cosmology, the Hubble parameter behaves as $H \propto \rho$ rather than $H \propto \sqrt{\rho}$, a novel aspect of the CRI scenario in this context. It is expected that this modification affects relevant parameters and observable of the inflation phase, including the slow-roll parameters, the shape of the potential, end time of inflation, and the magnitude of the inflaton. Due to this novel feature of the Friedmann equation, it is important to consider the CRI

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scenario in the framework of the brane world and study the new features that may arise in this case.

Another motivation for considering the brane world comes from the swampland conjectures [89–91], which can be used as criteria to distinguish effective field theories (EFTs) that can be UV completed to a quantum theory of gravity. The first criterion requires the field range traversed by the fields to be bounded from above by a value of order 1, whereas the second criterion imposes a lower bound on the gradient of the potential. The latter bound is in direct tension with inflation where the first slow-roll parameter $\epsilon = M_p^2 V'^2/2V^2$ must be smaller than 1. Thus, some inflationary models are not compatible with these criteria and hence cannot be embedded into a consistent theory of quantum gravity. However, inflationary models in the brane-world scenario have the potential to evade the swampland constraints [92–99], and hence it will be interesting to investigate inflation in this framework and its implications.

The paper is organized as follows. In Sec. II, the main evolution equations of the model are given. The scenario of the constant-roll inflation is discussed in the frame of brane world in Sec. IV. The exact solutions for the model are obtained in Sec. III and the main dynamical parameters are obtained in terms of the scalar field. In Sec. IV, the cosmological density perturbations are considered, and the consistency of the model with the observational data and swampland criteria is investigated in Secs. V and VI, respectively. As the last step, the attractor behavior of the solution is studied in Sec. VII. The results are summarized in Sec. VIII.

II. THE MODEL

The action for the brane world is given by

$$S = \int d^5x\sqrt{-g}\left(\frac{M_5^3}{2}\mathcal{R} + \Lambda_5\right) + \int d^4x\sqrt{-h}(\mathcal{L} + \lambda), \quad (1)$$

where the first integral represents the action of the bulk and the second one corresponds to the brane, $\mathcal{R}$ is the Ricci scalar related to the five-dimensional metric g_{AB}, g and h denote the determinants of the metric on the five-dimensional space and the brane, respectively, Λ_5 the five-dimensional cosmological constant, $\mathcal{L}$ the Lagrangian of the matter fields, and λ the brane tension.

Taking variation of the action with respect to the metric yields the field equation

$$G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \left(\frac{8\pi}{M_4^2}\right)T_{\mu\nu} + \left(\frac{8\pi}{M_5^3}\right)^2\Pi_{\mu\nu} - E_{\mu\nu}. \quad (2)$$

Here, $T_{\mu\nu}$ is the energy-momentum tensor of the matter on the brane, $\Pi_{\mu\nu}$ a tensor that includes the terms quadratic in $T_{\mu\nu}$, and $E_{\mu\nu}$ represents the projection of Weyl tensor on the brane which portrays the effects of the bulk graviton on the dynamical evolution of the brane. Assuming the geometry of the Universe to be described by a five-dimensional Friedmann-Lemaitre-Robertson-Walker metric

$$ds_5^2 = -dt^2 + a^2\delta_{ij}dx^i dx^j + dy^2, \quad (3)$$

the Friedmann equation reads

$$H^2 = \frac{\Lambda_4}{3} + \left(\frac{8\pi}{3M_4^2}\right)\rho + \left(\frac{4\pi}{3M_5^3}\right)^2\rho^2 + \frac{\mathcal{C}}{a^4}, \quad (4)$$

with Λ_4 the cosmological constant of the brane, and $\mathcal{C}/a^4$ is known as the dark radiation.[1] The five- and four-dimensional Planck masses in the above equation are related as $M_4 = \sqrt{\frac{3}{4\pi\lambda}}M_5^3$.

During inflation, the dark radiation term gets diluted, and hence can be neglected. Also, here the RS fine-tuning is used to set the four-dimensional cosmological constant to zero. Thus, the Friedmann equation gets reduced to

$$H^2 = \frac{8\pi}{3M_4^2}\rho\left(1 + \frac{\rho}{2\lambda}\right). \quad (5)$$

Since all the matter fields are confined on the brane, the conservation of energy in this expanding Universe is the same as in standard cosmology, i.e.,

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (6)$$

Using the above equation and taking the time derivative of Eq. (5), we obtain the second Friedmann equation

$$\dot{H} = \frac{-4\pi}{M_4^2}\left(1 + \frac{\rho}{\lambda}\right)(\rho + p). \quad (7)$$

Inflation is driven by the inflaton, a scalar field ϕ, that is confined on the brane, and with energy density and pressure

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi), \qquad p = \frac{\dot{\phi}^2}{2} - V(\phi) \quad (8)$$

and obeys the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \quad (9)$$

It is widely common to consider the inflation at the energy scale where the energy density is larger than the tension of the brane, i.e., $\rho \gg \lambda$. Therefore, the above Friedmann equations are reduced to

$$H^2 = \left(\frac{4\pi}{3M_5^3}\right)^2\rho^2, \qquad \dot{H} = -3\left(\frac{4\pi}{3M_5^3}\right)H\dot{\phi}^2. \quad (10)$$

[1]This is because of its dependence on the scale factor is the same as the energy density of radiation.

III. CONSTANT-ROLL INFLATION

In slow-roll inflationary models, the inflaton rolls down its potential very slow which can be described in terms of dimensionless parameters

$$\epsilon = \frac{-\dot{H}}{H^2} \quad \text{and} \quad \eta = \frac{-\ddot{\phi}}{H\dot{\phi}} \tag{11}$$

which satisfy the conditions $\epsilon < 1$ and $\eta < 1$, known as slow-roll parameters (SRPs) [13]. Another scenario is the constant-roll inflation where the second slow-roll parameter is assumed to be constant and can be of order of unity,

$$\eta = \frac{-\ddot{\phi}}{H\dot{\phi}} = \beta = \text{constant}. \tag{12}$$

The fact that η is a constant result in a differential equation for the Hubble parameter that admits an exact solution for the model. For that, we first obtain the time derivative of the scalar field from the second Friedmann equation by taking the Hubble parameter as a function of the scalar field, i.e., $H \coloneqq H(\phi)$, and write

$$\dot{H} = \dot{\phi}H' \qquad \Rightarrow \qquad \dot{\phi} = \frac{-1}{3}\left(\frac{3M_5^3}{4\pi}\right)\frac{H'}{H}. \tag{13}$$

Then, it follows the following differential equation for the Hubble parameter:

$$HH'' - H'^2 - \tilde{\beta}H^3 = 0, \qquad \tilde{\beta} = \frac{4\pi}{3M_5^3}\beta \tag{14}$$

and has a solution given by

$$H(\phi) = \frac{-\alpha}{2\tilde{\beta}}\left[1 - \tanh\left(\frac{\sqrt{\alpha}}{2}(\phi + \phi_0)\right)\right], \tag{15}$$

where α and ϕ_0 are constants of integration. Note that since the Hubble parameter is positive and the term *tanh* is smaller than one, the constant α must be negative.

Now that we have the expression of $H(\phi)$, we can derive $\dot{\phi}(\phi)$ and $V(\phi)$, and we get

$$\dot{\phi} = \frac{M_5^3\sqrt{\alpha}}{4\pi}\tanh\left(\frac{\sqrt{\alpha}}{2}(\phi + \phi_0)\right), \tag{16}$$

$$V(\phi) = \left(\frac{M_5^3}{4\pi}\right)^2 \frac{\alpha}{2}\left[\frac{-3}{\beta} + \left(\frac{3}{\beta} - 1\right)\tanh^2\left(\frac{\sqrt{\alpha}}{2}(\phi + \phi_0)\right)\right]. \tag{17}$$

By integrating the equation of $\dot{\phi}$ above gives the time evolution of scalar field as

$$\phi(t) + \phi_0 = \frac{2}{\sqrt{\alpha}}\sinh\left[\exp\left(\frac{M_5^3\alpha}{8\pi}(t + t_0)\right)\right]. \tag{18}$$

A. Scalar field at the horizon crossing time

The inflationary phase will come to an end when the SRP $\epsilon(\phi)$ becomes equal to unity, i.e.,

$$\epsilon(\phi_e) \coloneqq 2\beta \frac{\tanh^2\left(\frac{\sqrt{\alpha}}{2}(\phi_e + \phi_0)\right)}{1 - \tanh^2\left(\frac{\sqrt{\alpha}}{2}(\phi_e + \phi_0)\right)} = 1, \tag{19}$$

where ϕ_e is the value of the field at the exit of inflation, which can be determined by solving the above algebraic equation. With this, we can quantify the amount of inflation the Universe underwent, corresponding to the number of e-folds from the beginning of inflation, the instant t_i, to the exit time t_e, and is given by

$$N = \int_{t_i}^{t_e} H dt = \int_{\phi(t_i)\equiv\phi_i}^{\phi(t_e)\equiv\phi_e} \frac{H}{\dot{\phi}} d\phi = -\frac{4\pi}{M_5^3}\int_{\phi_i}^{\phi_e} \frac{H^2}{H'} d\phi. \tag{20}$$

Substituting the solution we have obtained for the Hubble parameter, and after some manipulation, we obtain

$$\begin{aligned} N &= \frac{-4\pi}{M_5^3\tilde{\beta}}\ln\left(\tanh\left[\frac{\sqrt{\alpha}}{2}(\phi + \phi_0)\right]\right)\Bigg|_{\phi_i}^{\phi_e} \\ &= \frac{2\pi}{M_5^3\tilde{\beta}}\ln\left(\frac{\tanh^2[\frac{\sqrt{\alpha}}{2}(\phi_i + \phi_0)]}{\tanh^2[\frac{\sqrt{\alpha}}{2}(\phi_e + \phi_0)]}\right), \end{aligned} \tag{21}$$

or equivalently

$$\tanh^2\left[\frac{\sqrt{\alpha}}{2}(\phi_i + \phi_0)\right] = \frac{e^{2\beta N}}{1 - 2\beta}. \tag{22}$$

IV. COSMOLOGICAL PERTURBATIONS

In this section, we consider the impact on the quantum perturbations as one of the most important predictions of inflation which represents the main test that we have for verifying any inflationary model. The perturbations are usually divided into three types: scalar, vector, and tensor. Vector perturbations are usually ignored as they depend on the inverse of the scale factor and get diluted rapidly during inflation. Scalar perturbations are the seed for large scale structure formation in the Universe. The tensor perturbations describe the primordial gravitational waves which have not been detected yet and at present we have only an upper bound on the tensor-to-scalar ratio.

The study of the cosmological perturbation in constant-roll inflation is a little different than in the slow-roll scenario. Since the second SRP, η, might be of order unity, in calculating the scalar and tensor perturbations the terms η^2, $\epsilon\eta$, and $\epsilon\eta^2$ cannot be ignored. In this regard, the whole perturbation equations involving the second SRP should be reconsidered. In the following subsections, we are going to reconsider both scalar and tensor perturbations for any possible modification.

A. Scalar perturbations

To derive the perturbation parameters, we usually need to obtain the Mukhanov-Sasaki equation [12–14,100–104]. For this matter, the action is computed up to the second order of the perturbation parameter. Following [12,105], the spatially flat gauge is used in which, up to the leading order of ϵ, the fluctuations in the geometry of the action could be ignored. Since the scalar field lives on the brane, we have the same perturbation equation as we have in the standard four-dimensional cosmology, that is,

$$v_k''(\tau) + \left(k^2 - \frac{z''}{z}\right) v_k(\tau) = 0, \tag{23}$$

where again z has the same definition as $z^2 = a^2\dot{\phi}^2/H^2$. Therefore, after some algebraic manipulations, the term z''/z in the above equation can be expressed as

$$\frac{z''}{z} = \frac{1}{\tau^2}(2 + 6\epsilon - 3\beta - 9\epsilon\beta + \beta^2 + 2\epsilon\beta^2). \tag{24}$$

Making the change of variables $x = -k\tau$ and $f_k = v_k/\sqrt{-\tau}$, Eq. (23) becomes a Bessel differential equation as

$$\frac{d^2 f_k}{dx^2} + \frac{1}{x}\frac{df_k}{dx} + \left(1 - \frac{\nu^2}{x^2}\right) f_k = 0, \tag{25}$$

where we have used

$$\frac{z''}{z} = \frac{\nu^2 - \frac{1}{4}}{\tau^2} \quad \Rightarrow \quad \nu^2 = \frac{9}{4} + 6\epsilon - 3\beta - 9\epsilon\beta + \beta^2 + 2\epsilon\beta^2. \tag{26}$$

In general, the solutions to (25) are

$$f_k = c_1(k) H_\nu^{(1)}(-k\tau) + c_2(k) H_\nu^{(2)}(-k\tau). \tag{27}$$

Here $H_\nu^{(1)}$ and $H_\nu^{(2)}$ are the Hankel's functions of the first and second kind, respectively, and $c_1(k)$ and $c_2(k)$ are arbitrary constants. Comparing the asymptotic behavior of the general solution, with the solution of the equation in the subhorizon limit ($k\tau \ll 1$), the constants are determined, and finally one could obtain the amplitude of the scalar perturbations as

$$\mathcal{P}_s = A_s^2 \left(\frac{k}{aH}\right)^{3-2\nu}, \quad A_s^2 = \frac{1}{25\pi^2}\left(\frac{2^{\nu-3/2}\Gamma(\nu)}{\Gamma(3/2)}\right)^2 \left(\frac{H^2}{\dot{\phi}}\right)^2, \tag{28}$$

from which we deduce the scalar spectral index n_s as

$$n_s - 1 = 3 - 2\nu. \tag{29}$$

B. Tensor perturbations

The second SRP plays no role in the tensor perturbation equations, and hence the evolution equation for the tensor perturbation will have the same form as the scalar case. The amplitude of such perturbations has been calculated in the framework of the brane-world gravity and is given by [106,107]

$$A_T^2 = \frac{16\pi}{25M_p^2}\left(\frac{H}{2\pi}\right)^2 F^2(x), \tag{30}$$

where

$$F^2 = \left[\sqrt{1+x^2} - x^2 \sinh^{-1}\left(\frac{1}{x}\right)\right]^{-1}, \quad x \equiv HM_p\sqrt{\frac{3}{4\pi\lambda}}. \tag{31}$$

In high-energy regime, where $x \gg 1$, one arrives at $F(x) = 3x/2$ [107,108]. The tensor perturbations are measured indirectly through the parameter r, defined as the ratio of tensor perturbations to scalar perturbations, which can be determined using Eqs. (28) and (30) as

$$r = \frac{3}{2}\left(\frac{\Gamma(3/2)}{2^{\nu-3/2}\Gamma(\nu)}\right)^2 \epsilon. \tag{32}$$

Currently, the value of this parameter is not determined by the data, and only an upper bound $r < 0.064$ [3–5].

V. OBSERVATIONAL CONSTRAINTS ON THE MODEL

To determine the free parameters of the model, we compute the amplitude of the scalar perturbations, scalar spectral index, and tensor-to-scalar ratio at the time of horizon crossing and compare with the available observational data. First, by substituting the expression in Eq. (22) into Eq. 3, the slow-roll parameter ϵ can be written in terms of the number of e-folds as

$$\epsilon = \frac{-2\beta e^{2\beta N}}{1 - 2\beta - e^{2\beta N}}. \tag{33}$$

Note that [from Eqs. (26), (29), and (32)] the scalar spectral index and tensor-to-scalar ratio depend only on β and N at the time of horizon crossing. Comparing the theoretical results for n_s and r with allowed values of the spectral index and tensor-to-scalar ratio given by Planck Collaboration in the form of $r - n_s$ diagram, we extract the values of (N, β)

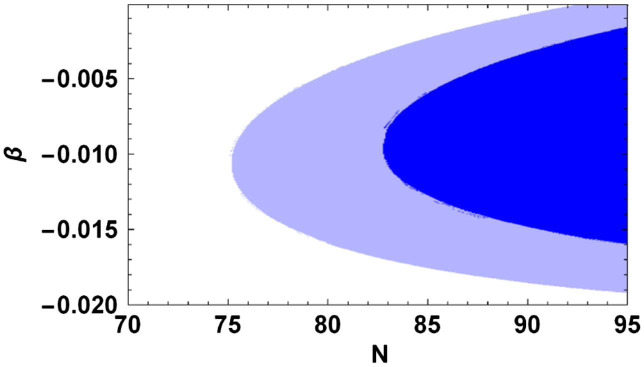

FIG. 1. The 68% (light blue) and 95% CL (dark blue) allowed region of the parameters β and N.

that are in agreement with the observational data. Using the 95% and 68% CL allowed regions of the parameters r and n_s from Planck TT, TE, EE + lowE + lensing + BK14 + BAO data sets [5], we show in Fig. 1 the corresponding model parameter space.

Using the amplitude of the scalar perturbations, the other constant of the model, i.e., α, is determined as

$$\alpha^3 = \left(\frac{\Gamma(3/2)}{2^{\nu-3/2}\Gamma(\nu)}\right)^2 \left(\frac{4\pi}{M_5^3}\right)^2 (4\pi(2\beta)^3 A_s \epsilon). \qquad (34)$$

To have numerical insight about the result of the model, Table I represents the values of α, scalar spectral index, tensor-to-scalar ratio, and the energy scale of inflation for different values of β and the number of e-folds, taken from Fig. 1.

Figure 2 portrays the behavior of the obtained potential versus the scalar field for different values of β and α. As it is illustrated, the potential rolls down from the top of the potential.

The crucial point for any inflationary model is to have a graceful exit from the inflation stage. Considering the behavior of the first SRP presents the required information about the inflationary times and its end. The evolution of ϵ versus the number of e-folds is depicted in Fig. 3, where it is realized that by approaching the end of inflation the parameter ϵ grows up and reaches 1.

TABLE I. Numerical results of the model.

β	N	α	n_s	r	$V^\star$
−0.011	76	4.92×10^{-33}	0.9580	0.0072	2.22×10^{53}
−0.014	80	4.95×10^{-33}	0.9589	0.0047	1.92×10^{53}
−0.007	80	4.16×10^{-33}	0.9594	0.0096	$2,45 \times 10^{53}$
−0.014	84	4.64×10^{-33}	0.9604	0.0041	1.85×10^{53}
−0.010	84	4.32×10^{-33}	0.9620	0.0065	2.15×10^{53}
−0.004	84	3.51×10^{-33}	0.9592	0.0119	2.62×10^{53}
−0.009	88	4.01×10^{-33}	0.9637	0.0066	2.16×10^{53}

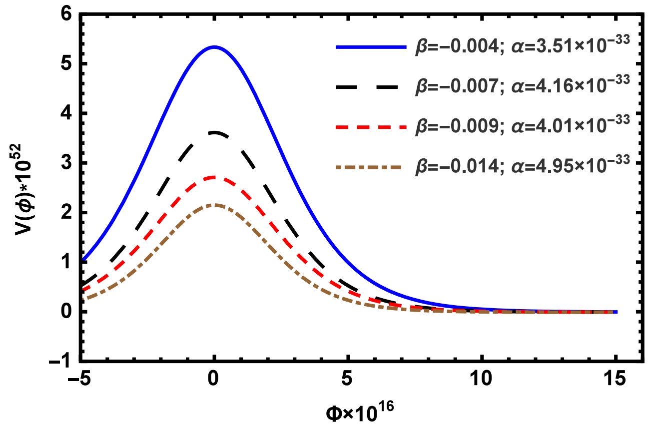

FIG. 2. Behavior of the potential of the scalar field.

VI. CONSISTENCY WITH THE SWAMPLAND CRITERIA

The recently proposed swampland criteria are a measure for separating the consistent EFT from the inconsistent EFT. The consistent EFTs can successfully be formulated in string theory, the best candidates of the quantum gravity. It is believed inflation occurred at the energy scale below the Planck energy and hence could be described by a low-energy effective field theory of string theory. Therefore, it is a natural desire to construct an inflationary model based on a consistent EFT, and for that we apply the swampland conjectures.

The first criterion concerns the distance conjecture which constraints on the range traversed by the scalar field as $\Delta\phi/M_p < c$ where c is of the order of unity. The evolution of the term $\Delta\phi/M_p$ for the model is presented in Fig. 4, where it is shown that $\Delta\phi/M_p$ is smaller than unity for the whole time of the inflation

The second criterion is a de Sitter conjecture which imposes a lower bound on the gradient of the potential. It states that $M_p|V'/V| > c'$ where c' is of the order of unity (further investigation determines that the constant could be

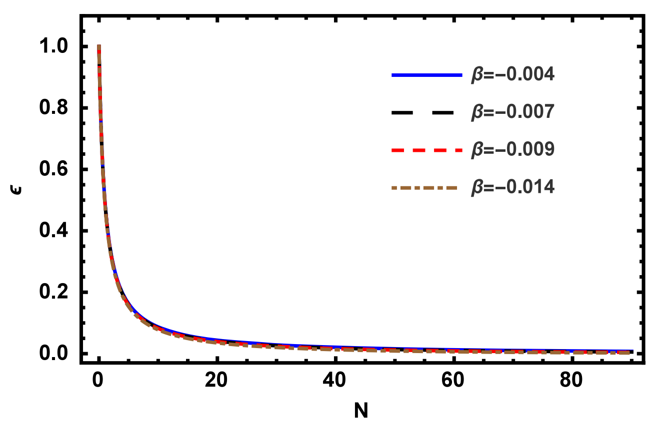

FIG. 3. Behavior of the first slow-roll parameter ϵ versus the number of e-folds.

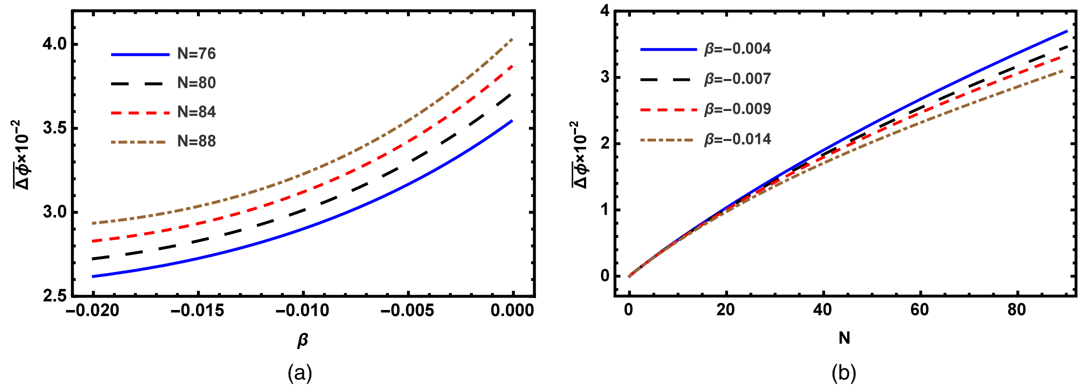

FIG. 4. Evolution of $\Delta\phi/M_p$ versus the number of e-folds for different values of β.

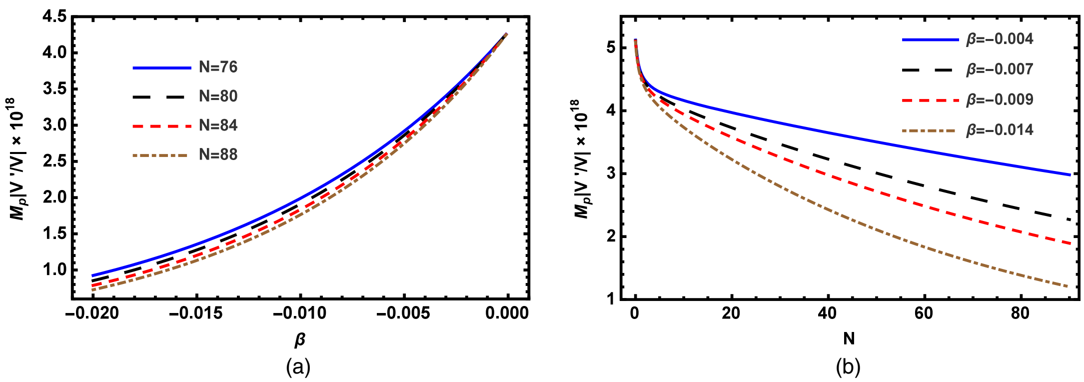

FIG. 5. Evolution of the gradient of the potential versus the number of e-folds for different values of β.

of the order of 0.1 [92]). In Fig. 5, we present the evolution of $M_p|V'/V|$, which shows that the magnitude of the gradient of the potential is bigger than one during the inflationary phase.

VII. ATTRACTOR BEHAVIOR

The last feature we are going to consider is the attractor behavior of the model. The solution of the model has been obtained in Sec. III, where we have used the Hamilton-Jacobi formalism [14,109–117]. This approach was first studied in [118], where the authors found exact solution for the large value of the parameter η. Considering the attractor behavior of the solution, it was claimed that constant-roll inflation presents a new class of attractor solution. This result has been reinvestigated in detail in [119,120] where it was shown that the solution and the perturbation equations are invariant under the transformation $\eta \to \bar{\eta} = 3 - \eta$, with two branches of solution that are symmetric under this transformation. The main result of [119] based on this duality transformation led the authors to the conclusion that the attractor behavior of the constant-roll inflation with large η is not a new class of attractor behavior which also has been claimed in [119]. It does not mean that the constant-roll has no attractor solution, but they are not a new class of attractor solution.

In our model, we found that the observational constraints on (r, n_s) require the parameter $\eta(=\beta)$ to be small. Consequently, the model certainly does not present a new class of the attractor solution and could be part of the slow-roll attractor. Therefore, the attractor behavior in our model could be investigated utilizing the same method as in the slow-roll scenario. In this regard, we follow a similar procedure as in [14,110] which is a common method for considering the attractor behavior of the inflationary models.

Assuming homogenous perturbation in the Hubble parameter, i.e., $H(\phi) = H_0 + \delta H(\phi)$, and substituting it into the Hamilton-Jacobi equation,

$$V(\phi) = \left(\frac{3M_5^3}{4\pi}\right) H(\phi) - \frac{1}{9}\left(\frac{3M_5^3}{4\pi}\right)^2 \frac{H'^2(\phi)}{H^2(\phi)}, \quad (35)$$

leads to the following differential equation:

$$\frac{\delta H'(\phi)}{\delta H(\phi)} = \left(1 + \frac{9}{2}\left(\frac{4\pi}{3M_5^3}\right)\frac{H_0^2}{H_0'^2}\right)\frac{H_0'}{H_0}, \quad (36)$$

where the equation has been obtained up to the first order of the perturbation term. Integration leads to

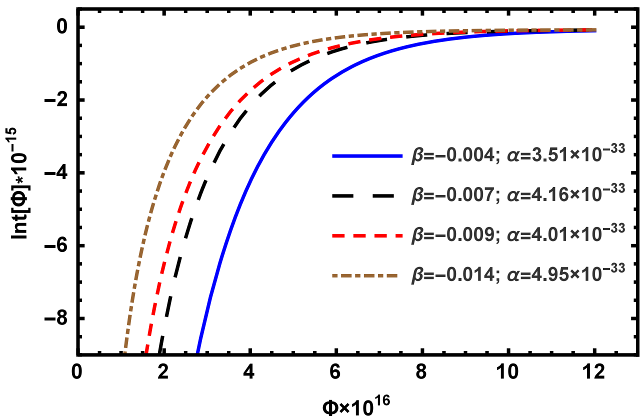

FIG. 6. The curves display the behavior of the integrand versus the scalar field during the inflationary time for different values of β and α.

$$\delta H(\phi) = \delta H_i \exp\left[\int_{\phi_i}^{\phi}\left(1+\frac{9}{2}\left(\frac{4\pi}{3M_5^3}\right)\frac{H_0^3(\phi)}{H_0'^2(\phi)}\right)\frac{H_0'(\phi)}{H_0(\phi)}d\phi\right]. \tag{37}$$

The integrand is illustrated in Fig. 6 versus the scalar field. The curves portray the behavior of the integrand versus the scalar field during the inflationary times. The area between the curve and the x-axis displays the actual value of the integral in the power of the exponential term in Eq. (37). Inflation begins for smaller field and it ends at bigger fields. Therefore, as the time passes and approaches the end of inflation, the area under the curve gets larger and larger and the integral becomes more and more negative. Then, the exponential term approaches to zero implying that the homogeneous perturbation $\delta H(\phi)$ dies away with time, and the model possesses attractor behavior.

VIII. CONCLUSION

The constant-roll inflation was investigated in the frame of the RSII brane gravity model. Based on this scenario, the Universe and all the matter fields, including the inflaton, are confined to a brane with positive tension, where the brane is embedded in five-dimensional space-time. The modified gravity model results in a modified Friedmann equation which contains both linear and quadratic terms of the energy density. In the high-energy limit, the quadratic term dominates, and consequently, the Hubble parameter becomes proportional to the energy density ρ, instead of $\sqrt{\rho}$. In this scenario, the inflaton rolls down its potential at a constant rate where the second slow-roll inflation parameter is taken to be constant which, in general, can be of order unity. Using the Hamilton-Jacobi approach, we derive a differential equation for the Hubble parameter. For our model, there is a nonlinear second-order differential equation that gives an exact solution for the model. Finding the Hubble parameter in terms of the scalar field, the other background parameters, such as the time derivative of the scalar field and the potential, were derived in terms of the scalar field. The slow-roll parameter ϵ was also obtained in terms of the scalar field, which is used to infer the scalar field at the end of inflation through the relation $\epsilon(\phi_e) = 1$. The scalar field at the beginning of inflation was acquired from the expression of the number of e-folds.

Another consequence of this scenario appears in the perturbation parameters where one could find the modified terms mainly in the amplitude of the scalar perturbations, scalar spectral index, and tensor-to-scalar ratio. Since the second slow-roll parameter might not be small, the scalar perturbation equations were reconsidered, and the modified scalar power spectrum was derived. The tensor power spectrum is the same as the slow roll in brane inflation because the second slow-roll inflation parameters play no role in tensor perturbation equations.

Computing the perturbation parameters at the time of horizon crossing, the scalar spectral index and tensor-to-scalar ratio are obtained only in terms of the constant β (i.e., the second slow-roll parameter) and the number of e-folds. Comparing the theoretical results of the model with the Planck data, a set of the (β, N) is found that for any point in this set, the model perfectly agrees with observational data. The other constant of the model, i.e., α, is determined from the amplitude of the scalar perturbation where there is an exact value for the parameter based on data. Using this result, a numerical result of the model about the main parameters including the energy scale of inflation is presented. In the next step, the consistency of the model with the recently proposed swampland criteria is considered. We tried to find whether the model with the obtained free parameter could satisfy the conjectures. Furthermore, the range of the scalar field values and the gradient of its potential appropriately satisfy both swampland criteria.

Finally, in the Hamilton-Jacobi formalism, we derived the differential equation (up to the first order) describing the behavior of a homogeneous perturbation for the Hubble parameter as a function of the inflaton field. We showed that the perturbation parameter reduces as the time approaches the end of inflation, which indicates that the solution of the model has the attractor behavior.

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