Vector perturbations in bouncing cosmology

Nelson Pinto-Neto,[1,2,*] Júlio C. Fabris,[3,4,†] Júnior D. Toniato [ORCID],[3,‡] G. S. Vicente,[5,§] and Sandro D. P. Vitenti [ORCID][6,‖]

[1]*CBPF—Centro Brasileiro de Pesquisas Físicas, Xavier Sigaud st. 150,*
zip 22290-180 Rio de Janeiro, Brazil
[2]*PPGCosmo, Universidade Federal do Espírito Santo. Avenida Fernando Ferrari 514,*
zip 29075-910 Vitória-ES, Brazil
[3]*Núcleo de Astrofísica e Cosmologia (Cosmo-ufes), PPGCosmo & Departamento de Física, Universidade*
Federal do Espírito Santo. Avenida Fernando Ferrari 514, zip 29075-910 Vitória-ES, Brazil
[4]*National Research Nuclear University MEPhI, Kashirskoe shosse 31, Moscow 115409, Russia*
[5]*FAT—Faculdade de Tecnologia, UERJ—Universidade do Estado do Rio de Janeiro (Campus Resende),*
Rodovia Presidente Dutra, km 298, zip 27537-000 Rio de Janeiro, Brazil
[6]*Departamento de Física, Universidade Estadual de Londrina, Rodovia Celso Garcia Cid,*
Km 380, 86057-970 Londrina, Paraná, Brazil

An old question surrounding bouncing models concerns their stability under vector perturbations. Considering perfect fluids or scalar fields, vector perturbations evolve kinematically as a^{-2}, where a is the scale factor. Consequently, a definite answer concerning the bounce stability depends on an arbitrary constant, therefore, there is no definitive answer. In this paper, we consider a more general situation where the primeval material medium is a nonideal fluid, and its shear viscosity is capable of producing torque oscillations, which can create and dynamically sustain vector perturbations along cosmic evolution. In this framework, it can be defined that vector perturbations have a quantum mechanical origin, coming from quantum vacuum fluctuations in the far past of the bouncing model, as it is done with scalar and tensor perturbations. Under this prescription, one can calculate their evolution during the whole history of the bouncing model, and precisely infer the conditions under which they remain linear before the expanding phase. It is shown that such linearity conditions impose constraints on the free parameters of bouncing models, which are mild, although not trivial, allowing a large class of possibilities. Such conditions impose that vector perturbations are also not observationally relevant in the expanding phase. The conclusion is that bouncing models are generally stable under vector perturbations. As they are also stable under scalar and tensor perturbations, we conclude that bouncing models are generally stable under perturbations originated from quantum vacuum perturbations in the far past of their contracting phase.

I. INTRODUCTION

In standard hot big bang cosmology, classical primordial perturbations around a homogeneous background would never have been in causal contact and structure formation cannot be explained. Cosmic inflation solves this problem by generating primordial perturbations of quantum-mechanical origin, which are later stretched by expansion and explain the observed spectrum of perturbations [1]. However, in addition to the quantum production of perturbation modes from vacuum fluctuations, cosmic inflation is preceded by an initial singularity, at which quantum

effects are expected to be relevant. Therefore, it is natural to ask for a quantum description for both background and perturbations.

A quantum treatment of the early Universe enables the avoidance of the initial singularity. The absence of singularities allows the connection of the present expanding phase to a preceding contracting phase through a bounce [2–13]. The bounce physics depends on the quantization scheme. In the context of the Wheeler-DeWitt quantization of minisuperspace models using the de Broglie-Bohm quantum theory [14–16], the Bohmian evolution of the scale factor is free of singularities: they describe universes that contract classically from infinity, perform a quantum bounce, and are subsequently ejected into an expanding phase, where classical evolution, compatible with observations, is rapidly recovered [5–7,17].

The quantum theory of linear cosmological perturbations can be extended to such backgrounds [18–23]. Primordial

[*]nelson.pinto@pq.cnpq.br
[†]julio.fabris@cosmo-ufes.org
[‡]junior.toniato@ufes.br
[§]gustavo@fat.uerj.br
[‖]vitenti@uel.br

perturbations can naturally arise from quantum vacuum fluctuations in the far past of the contracting phase, where space-time is almost flat, and an adiabatic vacuum state can be prescribed. These perturbations are amplified during cosmic evolution, becoming the seeds of the large-scale structures of the Universe in the expanding phase. As well as in cosmic inflation, scalar and tensor perturbations of quantum mechanical origin can be shown to be almost scale invariant if the contracting phase is dominated by a dust fluid (maybe dark matter) at large scales. Furthermore, it can be shown that they never leave the linear regime up to the expanding phase, where they necessarily must become nonlinear in order to develop structures in the Universe [24–26].

In the references cited above, the matter content of the models are described by perfect fluids or scalar fields. In this case, vector perturbations, evolve like a^{-2}, as usual, where a is the scale factor. For big bang models with inflation, it is expected that such primordial vector perturbations become completely negligible after the inflationary phase. However, bouncing models contain a contracting phase, where these perturbations can increase, and one may wonder whether they can become nonlinear and destroy the homogeneity or the isotropy of the background while the Universe reaches the bounce. If one keeps restricted to perfect fluids and/or scalar fields, the vector modes do not have scale dependent dynamics, and consequently, the answer to this question will depend on an arbitrary constant, hence all answers are possible. However, if we enlarge the possibilities and consider the primeval material medium as a nonideal fluid, the shear viscosity is capable of producing torque oscillations, which can create and sustain vector perturbations along cosmic evolution. Furthermore, as for scalar and tensor perturbations, one can assume that vector perturbations also have a quantum mechanical origin, as described in Ref. [27]. For other approaches to vector perturbations, see Refs. [28,29].

The aim of this paper is to apply the framework developed in Ref. [27] for vector perturbations to the quantum bouncing models described above. A natural initial adiabatic quantum vacuum state for the vector perturbations can now be prescribed, which turns possible to evaluate the evolution of vector perturbations without any arbitrariness. Demanding that they stay linear during cosmic evaluation imposes constraints on the free parameters of the background model. We will see that these constraints are mild, although not trivial. Another important goal is to evaluate whether such vector perturbations can provide some signature of the collapsing phase, seed large-scale cosmic magnetic fields [30,31], and polarization of the cosmic microwave background (CMB) spectrum [32].

The paper is organized as follows. In Sec. II, the hydrodynamics of nonideal fluids in general relativity is described. In Sec. III, we set up the theory of cosmological perturbations for linear vector perturbations, taking into account the effects of shear viscosity, which is responsible for producing torque oscillations. Section IV introduces the quantum bouncing model. The formalism described in Sec. III is applied to it and the fundamental equations describing the evolution of vector perturbations are obtained. The consistency conditions for linearity are analyzed in Sec. V. In Sec. VI, after imposing adiabatic vacuum initial conditions for the vector perturbations, the analytical results are obtained, which are then confronted with more detailed numerical calculations. The constraints on the background model parameters, coming from the linearity conditions, are also obtained. Finally, in Sec. VII, we draw some general conclusions about our results.

II. NONIDEAL FLUID

In cosmology, it is usual to consider only perfect fluids as the matter content of the Universe. However, a realistic model must take into account dissipative phenomena, which are always present in the macroscopic description of a system. Bulk and shear viscosity, besides heat flow, are some examples of such dissipative processes. Applications of bulk viscosity in cosmological models have a very large amount of literature. These applications begun, to our knowledge, with the seminal work by Murphy [33], concerning the primordial universe, and it has been extended to the study of the dark sector of the Universe (see Ref. [34] and references therein). In the case of the primordial Universe, they may lead to the avoidance of the initial singularity; in the case of the dark sector of the Universe, bulk viscosity effects may imply negative pressure and contribute to the acceleration of the Universe.

Contrary to bulk viscosity, shear viscosity does not affect isotropic and homogeneous backgrounds. However, at the perturbative level, it has been shown that shear viscosity can be as important—or even more so—as bulk viscosity. These surprising results have been shown first in the context of warm inflation [35,36] and then the late Universe [37,38]. For the present Universe, dissipative effects may cure some problems connected with the excess of power in matter agglomeration at small scales, due to the zero pressure of cold dark matter.

The extra piece of the energy-momentum tensor containing bulk and shear viscosity, as proposed in Refs. [39,40], reads

$$\Delta T_{\mu\nu} = 2\lambda\sigma_{\mu\nu} + \zeta u^{\rho}_{;\rho}(g_{\mu\nu} - u_{\mu}u_{\nu}),$$

$$\sigma_{\mu\nu} \equiv u_{(\mu;\nu)} - u_{(\mu}u^{\rho}u_{\nu);\rho} - \frac{u^{\rho}_{;\rho}}{3}(g_{\mu\nu} - u_{\mu}u_{\nu}). \quad (1)$$

In this expression, λ is the shear viscosity coefficient, ζ the bulk viscosity coefficient, $g_{\mu\nu}$ the metric, ";" the covariant derivative compatible with the metric, u^{μ} is the normal vector orthogonal to the spatial hypersurfaces, and $\sigma_{\mu\nu}$ the shear. Round brackets in the indices indicate

symmetrization and we are working with a metric signature $(1, -1, -1, -1)$. The explicit form of λ and ζ, with their dependence on the physical parameters, depends on the physical system to be considered. This formulation is noncausal, in the sense that equilibrium is achieved instantaneously. In an isotropic and homogeneous cosmological background, these parameters normally depend only on the energy density. However, this is not the case when heat flux is present.

A causal formalism, taking into account a finite speed of sound, has been implemented by Israel and Stewart [41]. The general expressions, including bulk and shear viscosity, imply transport equations to compute the viscous pressure. In Ref. [34], the causal formulation of bulk viscosity has been investigated as a description of the dark sector of the Universe. The more important challenge in using the causal formulation is to have a suitable description of the relaxation time, and nonadiabatic sound speed. Strictly speaking, this implies having a microscopic model for the fluid content. In doing so, the hypothesis must be made on the nature of the fluids composing the dark sector of the primordial fields and matter in the Universe. Alternatively, a phenomenological hypothesis can be implemented. In Ref. [34] it has been shown that, in doing so, the results are quite similar to those obtained using the noncausal formalism. Based on these considerations, in what follows we will restrict the analysis to the Eckart-Landau formulation of dissipative phenomena, set down by Eq. (1).

The total dissipative energy-momentum tensor can be written as

$$\Delta T^{\mu\nu} = T^B_{\mu\nu} + T^S_{\mu\nu}. \qquad (2)$$

The bulk viscosity term alone is given by

$$T^B_{\mu\nu} = \zeta u^\rho_{;\rho}(g_{\mu\nu} - u_\mu u_\nu), \qquad (3)$$

depending essentially on the volume expansion given by $u^\rho_{;\rho}$. The trace part is given by

$$T^B = 3\zeta u^\rho_{;\rho}. \qquad (4)$$

The shear viscosity term alone is given by

$$T^S_{\mu\nu} = 2\lambda\sigma_{\mu\nu}. \qquad (5)$$

This term is zero for an isotropic expansion, as we can expect from a shear process. Naturally, the trace of the shear energy-momentum tensor is zero:

$$T^S = 0. \qquad (6)$$

Let us make a final remark concerning the Hamiltonian and Lagrangian formulations of gravitational systems in the presence of dissipative phenomena. The construction of the energy-momentum tensor including dissipative process involves thermodynamical arguments. Since, from the pure mechanical and macroscopic point of view, a dissipative process implies nonconservation of the mechanical energy, with mechanical energy dissipating through heat, the construction of a Lagrangian and Hamiltonian for dissipative systems is not always possible. In some cases, this difficulty can be overcome using the Rayleigh dissipative function [42], which can be done only when the dissipative process depends on the velocity, like in the air resistance phenomena. Indeed, in this case, it is possible to modify the Lagrange equations as

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = F(\dot{q}). \qquad (7)$$

In general, however, it is not possible to define the Lagrangian of dissipative systems just by introducing such dissipative functions. Fortunately, a full Hamiltonian/Lagrangian formulation of the problem we are investigating in the present article is not necessary, since we are only interested in the linear perturbative level, in which a straightforward Hamiltonian can be defined, as will be seen in the sequel.

III. COSMOLOGICAL VECTOR PERTURBATIONS

We want to investigate the behavior of small deviations of a given background cosmology. The geometry of spacetime is then given by

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}, \qquad (8)$$

where $g^{(0)}_{\mu\nu}$ is assumed to be the Friedmann-Robertson-Walker metric with a flat spatial section. We will work in the synchronous gauge, where $h_{0\mu} = 0$ and, once our interest is in vector perturbations, we can write

$$h_{ij} = a^2(\partial_j F_i + \partial_i F_j), \qquad (9)$$

with a being the background scale factor and F_i is an arbitrary vector field satisfying $\partial_i F^i = 0$. Latin indices run from 1 to 3, indicating spatial components. The perturbed line element is then written as

$$ds^2 = a(\eta)^2[d\eta^2 - (\delta_{ij} - \partial_j F_i - \partial_i F_j)dx^i dx^j], \qquad (10)$$

where η is the conformal time. Substituting this into Einstein's tensor and keeping first order terms only, one obtains

$$G^0{}_0 = \frac{3a'^2}{a^4} \qquad (11)$$

$$G^0{}_i = -\frac{\nabla^2 F'_i}{2a^2} \tag{12}$$

$$G^i{}_j = -\left(\frac{3a'^2}{a^4} - \frac{2a''}{a^3}\right)\delta^i_j$$
$$+ \left(\frac{\partial^i F''_j}{2a^2} + \frac{\partial_j F^{i''}}{2a^2} + \frac{a'\partial^i F'_j}{a^3} + \frac{a'\partial_j F^{i'}}{a^3}\right), \tag{13}$$

where the symbol $'$ indicates derivatives with respect to the conformal time η, and $\nabla^2 = \partial_i\partial^i$ the spatial conformal Laplacian.

The total energy-momentum tensor is written as

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} + T^S_{\mu\nu}, \tag{14}$$

where ρ is the fluid energy density, p its pressure, u^μ is the four velocity, and $T^{\mu\nu}_S$ is the shear viscosity component of the fluid, given in Eq. (5). At the background level the four velocity is given by $u^\mu = (1/a)\delta_0{}^\mu$, while its perturbation is described by a vector function, i.e., $\delta u^\mu = (1/a)v^j\delta_j{}^\mu$. Considering only vector perturbations, the components of the total energy-momentum tensor read

$$T^0{}_0 = \rho, \tag{15}$$

$$T^0{}_i = -(\rho + p)v_i, \tag{16}$$

$$T^i{}_j = -p\delta^i_j + \frac{\lambda}{a}(\partial^i v_j + \partial_j v^i - \partial^i F'_j - \partial_j F^{i'}). \tag{17}$$

With the expressions (13)–(17) of the Einstein and energy-momentum tensors, the field equations can be directly obtained. The background dynamics will be given by the usual Friedmann equations. The linear perturbations F_i and v^i can be decomposed in terms of eigenfunctions of the three-dimensional Laplace's operator, Q_i, satisfying the equations

$$\nabla^2 Q_i = -k^2 Q_i, \qquad \partial_i Q^i = 0. \tag{18}$$

We then write

$$F_i = F(\eta)Q_i \quad \text{and} \quad v_i = v(\eta)Q_i. \tag{19}$$

Note that there are two linearly independent vector fields satisfying Eq. (18). In practice this means that once quantized, we would have the equivalent to two uncoupled scalar quantum fields. Nonetheless, since we consider isotropic vacuum states, both modes have equally defined vacuum states. In practice, we account for these two modes by multiplying the vector power spectrum by a factor of 2.

Simplifying expressions with the following definitions,

$$h(\eta) = -kF(\eta), \tag{20}$$

$$\omega(\eta) = -a(\eta)^2(\rho + p)v(\eta), \tag{21}$$

$$\chi(\eta) = \lambda(\eta)a(\eta)k^2[F'(\eta) - v(\eta)], \tag{22}$$

Einstein's equations can be recast as

$$-\frac{kh'}{2} = \kappa\omega, \tag{23}$$

$$\omega' + \frac{2a'}{a}\omega + \chi = 0, \tag{24}$$

where $\kappa = 8\pi l_p^2$ is the gravitational coupling constant, with l_p being the Planck length. Equations (23) and (24) lead to

$$h'' + 2\frac{a'}{a}h' = \frac{2\kappa}{k}\chi. \tag{25}$$

Note that, without shear viscosity, $\lambda(\eta) = 0$, which implies that $\chi = 0$, and we get $h' \propto 1/a^2$, as usual.

From its definition, $\omega(\eta)$ can be understood as angular momentum. Following Ref. [27], in the limit of flat space-time, Eq. (25) represents Newton's second law in its angular version: torque is the rate of change of angular momentum. Hence, the χ function can be interpreted as torque force in the viscous fluid. As usual, we can take it to be proportional to the angular displacement of a given element of the fluid,

$$\chi(\eta) = k^2 b^2 \theta(\eta) \tag{26}$$

where $\omega = \theta'$, and $b^2 = v_t^2/c^2$, with v_t being the torsional velocity of sound. Note that originally we had 3 free functions, $\{F, v, \lambda\}$, or equivalently $\{h, \omega, \chi\}$, and only 2 dynamical equations. Thus, imposing (26) yields an extra condition which closes the system.

Equations (23) and (24) can now be decoupled, yielding

$$h'' + \frac{2a'}{a}h' + k^2 b^2 h = 0, \tag{27}$$

which can also be written as

$$\mu'' + \left(k^2 b^2 - \frac{a''}{a}\right)\mu = 0, \tag{28}$$

with $\mu = ah = -akF$.

Equation (27) [or Eq. (28)] describes the dynamical evolution of linear vector perturbations due to torque oscillations in the primordial fluid. These equations have the same form as the dynamical equations for tensor perturbations (primordial gravitational waves). The difference is that here

the constant b can vary between 0 and 1, while for gravitational waves $b = 1$.

IV. VECTOR PERTURBATIONS IN BOUNCING MODELS

The Wheeler-DeWitt quantization of minisuperspace models using the de Broglie-Bohm quantum theory introduces quantum corrections in the Friedmann equations which are able to remove the classical initial singularity of the standard cosmological model. For a general review on this subject, see Ref [17]. In the present paper, we will consider a particular bouncing solution which contains a dark matter and radiation fluids, see Ref. [43] for its quantum origin, and Ref. [44] for its connection to the observable Universe. The scale factor is obtained as a Bohmian trajectory, and it reads

$$Y(\eta_s) \equiv \frac{a(\eta_s)}{a_0} = \frac{\Omega_{m0}}{4} \eta_s^2 + \sqrt{\frac{1}{x_b^2} + \Omega_{r0}\eta_s^2}, \quad (29)$$

where a_0 is scale factor today, Ω_{m0} and Ω_{r0} are the usual dimensionless densities of presureless matter and radiation, respectively, and $x_b = a_0/a_b$, which arises as an integration constant after solving the equation for the Bohmian trajectories and with a_b being the value of the scale factor at the bounce. We are using the dimensionless conformal time variable, appropriated to numerical integrations, namely $\eta_s = (a_0/R_{H_0})\eta$, where $R_{H_0} = 1/H_0$ is the Hubble radius today. The scale factor in Eq. (29) describes a universe dominated by dust in the far past. As the universe contracts, radiation eventually dominates over dust and near the bounce quantum effects become relevant. The quantum bounce happens, and it is followed by another radiation and dust phases, which fits the standard cosmological model before nucleosynthesis.

The parameter x_b can be constrained by imposing that the curvature scale at the bounce, L_b, should be at least a few orders of magnitude bigger than the Planck length. This is because the quantum gravity approach we are using, the Wheeler-DeWitt quantization, must be understood as an approximation of a more involved theory of quantum gravity, which should be valid only at scales not so close to the Planck length. One has that

$$L_b \equiv \frac{1}{\sqrt{R}}\bigg|_{\eta_s=0} = \sqrt{\frac{a^3}{6a''}}\bigg|_{\eta_s=0}, \quad (30)$$

where R is the Ricci scalar. Using values of $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_{r0} \approx 8 \times 10^{-5}$, one can find the upper bound $x_b < 10^{31}$. Moreover, the bounce should take place at energy scales higher than the beginning of nucleosynthesis, which implies $x_b \gg 10^{11}$. Hence, one gets

$$10^{11} \ll x_b < 10^{31}. \quad (31)$$

Concerning the vector perturbations, taking into account the quantum formalism, we are interested in the evolution of the normal modes h_k's coming from the expansion in terms of creation and annihilation operators, which satisfy the equation of motion (27), i.e.,

$$h_k'' + \frac{2a'}{a}h_k' + k^2b^2h_k = 0. \quad (32)$$

The Hamiltonian yielding this dynamical equation reads

$$\mathcal{H} = \frac{\Pi_k^2}{2m} + \frac{m\nu^2 h_k^2}{2}, \quad (33)$$

where $m \propto a^2$ and $\nu = kb$. The constant of proportionality in the "mass" m can be inferred from the kinetic term of vector perturbations coming from the Einstein-Hilbert action. This is true since the Poisson algebra (and consequently the operator algebra) is defined by the kinetic term. It is given by (see Ref. [18], where it is also shown that the quantum effects leading to the bounce affect the perturbations only through the substitution of the classical scale factor by the Bohmian one)

$$S = \int d^4x \frac{a^2}{16\pi l_p^2} \frac{h'^2}{2}, \quad (34)$$

where we are using natural units $\hbar = c = 1$ and η has dimensions of length. Hence $m = a^2/(16\pi l_p^2)$.

Prescribing adiabatic vacuum initial conditions in the far past of the bouncing model yields, see Ref. [45],

$$|h_k| = \frac{1}{\sqrt{2m\nu}} = \frac{4l_p\sqrt{\pi}}{a\sqrt{2kb}}, \quad (35)$$

with h_k having physical dimensions of length$^{3/2}$, as it should be.

We now introduce new dimensionless variables, compatible with the expression (29):

$$k_s = \frac{kR_{H_0}}{a_0}, \quad |h_{k_s}| = \sqrt{\frac{a_0^3}{16\pi l_p^2 R_{H_0}}}|h_k| = \frac{1}{Y\sqrt{2k_sb}}, \quad (36)$$

where k_s is the wave number in Hubble radius units. This expression accounts for the adiabatic initial condition and must be evaluated where the adiabatic approximation is valid. The dynamical equation for the dimensionless normal modes preserves the form of Eqs. (27) and (28),

$$h_{k_s}'' + \frac{2Y'}{Y}h_{k_s}' + \nu_s^2 h_{k_s} = 0, \quad (37)$$

$$\mu_{k_s}'' + (\nu_s^2 - V)\mu_{k_s} = 0, \quad (38)$$

where $\mu_{k_s} = Yh_{k_s}$, we have defined an effective wave vector $\nu_s = k_s b$, $V = Y''/Y$ is the potential, and the upper prime now denotes a derivative with respect to η_s.

V. CONSISTENCY CONDITIONS

In this section we will determine the general conditions under which cosmological vector perturbations remain negligible with respect to the background structure.

We start by considering the perturbations in the metric structure. From the line element (10), one has that

$$|2\partial_{(i}F_{j)}| \ll |\delta_{ij}|. \tag{39}$$

Considering a unitary spatial vector field U^α, we can construct appropriated scalar quantities from the condition above. In fact, one can see that

$$U^iU^j\partial_{(i}F_{j)} = -\frac{h}{k}U^iU^j\partial_{(i}Q_{j)}, \tag{40}$$

after invoking the definitions (18) and (20). Note that, $U^iU^j\partial_{(i}Q_{j)}$ is proportional to $k\cos\phi\cos\psi$, where ϕ and ψ are the angles between the vector U^i and the vectors k^i and Q^i, respectively. As we noted before, F_i must be expanded in terms of two linearly independent eigenfunctions, $Q_i^{(1)}$ and $Q_i^{(2)}$; however, here it would only affect our results by a factor of 2. Since at this stage we are only interested in the order of magnitude, it is safe to ignore this detail in the present analysis. Thus, it is immediate to see that $|U^iU^j\partial_{(i}Q_{j)}| \lesssim |k|$. This leads to the scalar condition,

$$|h| \ll 1. \tag{41}$$

As the perturbation is quantized, the above condition implies that the mean value of the operator $\hat{h}^2$ should be less than unity, yielding

$$\langle h^2 \rangle = \frac{1}{(2\pi)^3}\int_{k_{\min}}^{k_{\max}}|h_k|^2 d^3k \ll 1, \tag{42}$$

where $k_{\min}$ and $k_{\max}$ are the ultraviolet and infrared limits, which we will discuss further. Introducing the dimensionless variables defined in Eq. (36), one gets

$$\langle h^2 \rangle = \frac{8}{\pi}\frac{l_p^2}{R_{H_0}^2}\int_{k_{s,\min}}^{k_{s,\max}}dk_s k_s^2|h_{k_s}|^2 \ll 1. \tag{43}$$

We now investigate the role of vector perturbations in Einstein's equations. Vector perturbations affect the dynamical Einstein's equation through the time derivative of the extrinsic curvature $K_i{}^j$, which contains a background and a first order part

$$\partial_t K_i{}^j = \partial_t H \delta_i{}^j - \frac{H\delta\sigma_i{}^j}{3} + \dots, \tag{44}$$

where t is cosmic time, H is the Hubble function, and the shear tensor reads

$$\delta\sigma_{ij} = K_{ij} - \frac{g^{ab}K_{ab}}{3}g_{ij}, \tag{45}$$

which is null in the background. Hence, from Eq. (44) one gets the second condition (see Ref. [24]),

$$|\delta\sigma_i{}^j| \ll \left|\frac{\partial_t H\delta_i{}^j}{3H}\right|. \tag{46}$$

Using the line element (10) and the decomposition (19), one sees that

$$\delta\sigma_{ij} = -\frac{a\partial_\eta h}{k}\partial_{(i}Q_{j)}. \tag{47}$$

Multiplying, as before, condition (46) by U^i in order to construct a consistency scalar relation, one gets,

$$\left|\frac{3H\partial_t h}{\partial_t H}\right| \ll 1 \Rightarrow \left|\frac{\sqrt{\rho}\partial_\eta h}{\sqrt{6\pi}l_p(\rho + p)}\right| \ll 1. \tag{48}$$

In order to obtain the second form of condition (48), we have used the classical Friedmann equation. Note, however, that quantum effects are important at the background level near the bounce, hence these two forms of the condition are not always equivalent. As the quantum corrections do not modify the matter equation of state relating the pressure and the energy density, $p = w\rho$, and their functional relation with the scale factor, the second form is the one which is valid at all times, including the bounce.

The quantum version of the classical condition (48) reads

$$\langle(\partial_\eta h)^2\rangle \ll \frac{6\pi l_p^2(\rho + p)^2}{\rho}. \tag{49}$$

Using the dimensionless variables, and the power spectrum of vector perturbations

$$P_v(k_s) = \frac{k_s}{2\pi^2}|h_{k_s}'|^2, \tag{50}$$

the final form of the second condition reads

$$\frac{4\pi Y^2 l_p^2}{\Omega_{r0}R_{H_0}^2}\int_{k_{s,\min}}^{k_{s,\max}}dk_s k_s P_v(k_s) \ll 1. \tag{51}$$

Equations (43) and (51) are the main results of this section. They are quite general, valid for many theoretical models beyond the one considered here.

The cosmological model we are considering in this work has two additional free parameters: x_b, related with the size of the bounce, and b, the torsional velocity of the sound. The domain of x_b is given in Eq. (31), while $0 < b < 1$. The consistency conditions (43) and (51) will impose further constraints on these parameters, which will be obtained in the following section.

VI. RESULTS

In this section we analyze the propagation equation of the quantum vector modes given in (37) and/or (38), with the scale factor (29). We will start by doing analytical calculations, which will be confirmed by the numerical results. From now on, we will omit the subindex s to the redefined variables discussed in the previous sections.

Working with the current values $\Omega_{r0} \sim 10^{-4}$ and $\Omega_{m0} \sim 0.274$, Eq. (29) becomes

$$Y = 6.85 \times 10^{-2} \eta^2 + \sqrt{\frac{1}{x_b^2} + \frac{\eta^2}{10^4}}. \qquad (52)$$

The production of vector perturbations depends on the influence of the potential V over the effective wave number ν in Eq. (38) of perturbation modes. In different conformal times, the potential will behave according with the dominant phase at that epoch. Far from the bounce, dust is the dominant component, when $|\eta| \gg 0.15$ and the potential assumes the form

$$V \approx \frac{2}{\eta^2} \quad \text{for } |\eta| \gg 0.15. \qquad (53)$$

For small values of the conformal time, radiation begins to dominate. This phase is divided in two parts. The first one is when the quantum effects are still subdominants, resulting in

$$V \approx \frac{13.7}{\eta} \quad \text{for } \frac{9}{x_b^{2/3}} \ll \eta \ll 0.15. \qquad (54)$$

After that, quantum effects become important and one has

$$V \approx \frac{10^4}{x_b^2 \eta^4} \quad \text{for } \frac{10^2}{x_b^1} < |\eta| < \frac{9}{x_b^{2/3}}. \qquad (55)$$

For even smaller values of $|\eta|$ the quantum effects are completely dominant during the bounce phase, with

$$V \approx \frac{x_b^2}{10^4} \quad \text{for } 0 \leq |\eta| < \frac{10^2}{x_b}. \qquad (56)$$

As mentioned in Sec. IV, we will prescribe adiabatic vacuum initial conditions, given in Eq. (35). In terms of μ they read

$$\mu_{ini} = \frac{e^{-i\nu\eta}}{\sqrt{2\nu}}, \qquad (57)$$

which should be imposed at the asymptotic past, far from the bounce, where dust dominates. Taking into account (53) and the initial condition above, the solution of (38) in the dust phase is recast to be

$$\mu_k(\eta) = -\frac{\sqrt{\pi\eta}}{2} H_{-3/2}(\nu\eta), \qquad (58)$$

with H_n being the Hankel function of type one. In the region where $\nu\eta \ll 1$, but still in the dust phase, where $Y \propto \eta^2$, we can expand formally the solution above in powers of ν. The Hankel function is a combination of two Bessel functions ($J_{3/2}$ and $J_{-3/2}$) and each Bessel function can be expanded in terms of a power law times a power series in its argument squared. Consequently, when expanding (58), we have two distinct power series, each multiplying a different power law. In terms of h_k, they read

$$h_k = \frac{\mu_k}{Y}$$
$$\approx \nu^{3/2}[A_1 + \mathcal{O}(\nu^2)] + \nu^{-3/2}\left[\frac{A_2}{\eta^3} + \mathcal{O}(\nu^2)\right], \qquad (59)$$

where A_1 and A_2 are constants.

The Hamiltonian leading to Eq. (37),

$$\mathcal{H} = \frac{\Pi_k^2}{2Y^2} + \frac{Y^2 \nu^2 h_k^2}{2}, \qquad (60)$$

yields the canonical equations

$$h_k' = \frac{\Pi_k}{Y^2}, \qquad \Pi_k' = -Y^2 \nu^2 h_k. \qquad (61)$$

For small values of ν, these equations can be solved in an iterative manner, reproducing the power series discussed above, with the leading order giving

$$h_k = C_1(\nu)[1 + \mathcal{O}(\nu^2)] + C_2(\nu)\left[\int \frac{d\eta}{Y^2} + \mathcal{O}(\nu^2)\right], \qquad (62)$$

$$\Pi_k = C_2(\nu)[1 + \mathcal{O}(\nu^2)] + C_1(\nu)\left[-\int Y^2 \nu^2 d\eta + \mathcal{O}(\nu^4)\right]. \qquad (63)$$

Specifying for the dust case ($Y \sim \eta^2$), the matching between (62) and (59) gives the ν dependence of the C's constants above, namely $C_1 \propto \nu^{3/2}$ and $C_2 \propto \nu^{-3/2}$.

The evolution of these perturbations is exactly the same as tensor perturbations. Therefore, their power spectrum and spectral index are already known (see, for instance, Ref. [18]) and they satisfy the relation

$$\nu^3 |h_k|^2 \propto \nu^{n_T} \quad \text{with} \quad n_T = \frac{12w}{1+3w}, \qquad (64)$$

where $w = p/\rho$ is the equation of state parameter of the fluid which is dominating the background when the mode is crossing the potential, $\nu^2 \approx V$. In the case of dust domination, $w = 0$, with $Y \propto \eta^2$, $\eta_T = 0$, one gets the growing modes when the Universe is contracting,

$$h_k \propto \frac{\nu^{-3/2}}{\eta^3}, \qquad \Pi_k \propto \nu^{-3/2}. \qquad (65)$$

When afterwards radiation dominates the background evolution, $Y \propto \eta$, one gets

$$h_k \propto \frac{\nu^{-3/2}}{\eta}, \qquad \Pi_k \propto \nu^{-3/2}. \qquad (66)$$

On the other hand, if the mode crosses the potential already in the radiation domination phase of the contraction, one has $\eta_T = 2$ and $Y \propto \eta$, yielding

$$h_k(\eta) \propto \frac{\nu^{-1/2}}{\eta}, \qquad \Pi_k \propto \nu^{-1/2}. \qquad (67)$$

During the bounce, the potential is almost constant and nothing happens.

In the expanding phase, the growing mode of h_k becomes a decaying mode and h_k saturates up to returning to the oscillatory phase, when $\nu^2 > V$. In the case of Π_k, there is a ν^2 growing mode correction which eventually dominates the constant mode, and Π_k grows up to the oscillatory regime, either with a $k^{1/2}$ spectrum in the case of dust entrance, or $k^{3/2}$ for radiation entrance. The behaviors obtained in (65), (66), and (67), can be verified through the numerical results presented in Fig. 1, as well as the conclusions relative to the expanding phase.

The behavior of the power spectrum can be directly inferred considering Eqs. (50) and (62). When the modes cross the potential in the dust dominated phase, one has

$$P_v \propto \frac{\nu^{-2}}{\eta^8}, \qquad (68)$$

and when they cross in the radiation era, one gets

$$P_v \propto \frac{1}{\eta^4}. \qquad (69)$$

These behaviors can also be verified numerically in Fig. 2.

As it can be seen from Figs. 1 and 2, the most critical region for conditions (43) and (51) to be satisfied is during the bounce, where the vector perturbations amplitudes reach their maximum value. Let us then evaluate these
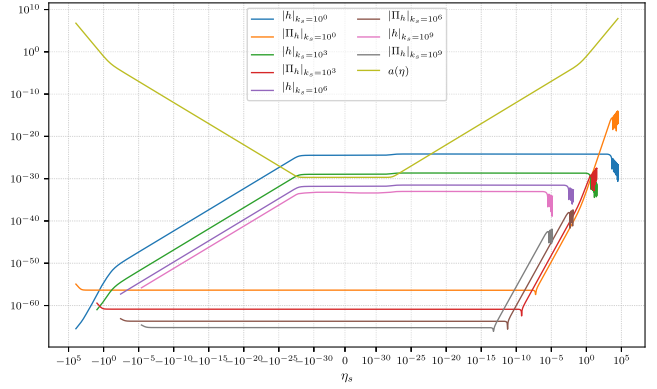

FIG. 1. Amplitudes of h_k and Π_k when the crossing of the potential occurs in two different phases. The relation (65) is verified for k_s equal to 10 and 10^3, when the modes cross the potential during the dust phase. For the higher values of k_s, 10^6, and 10^9, the crossing occurs during the radiation phase, satisfying relation (67). The scale factor is also shown in the figure to a better distinction of each phase of evolution. The values used for the free parameters are $x_b = 5 \times 10^{29}$ and $b = 10^{-3}$.

conditions at the bounce. We first need to know how these quantities are scaled with x_b, b, and k when $\eta = 0$.

The scalings of b and k of both h_k and P_v are embedded in relations (65)–(69). In the case of x_b, note that h_k grows as $1/|\eta|$ up to the bouncing phase, which begins in $\eta = -10^2/x_b$ [cf. Eq. (56)]. Hence, $|h_k| \propto x_b$. For the P_v scaling, note that it can be written as
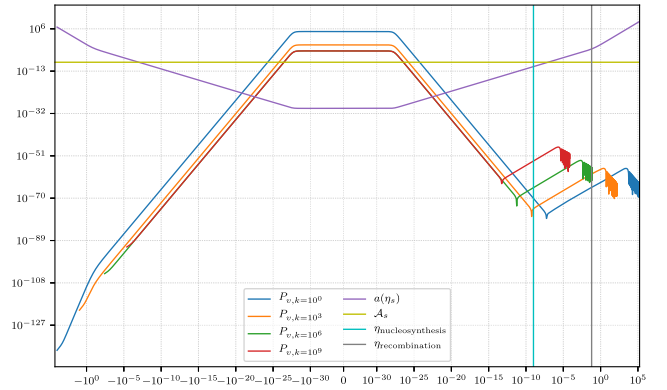
$$P_v = \frac{\Pi_k^2 k}{Y^4}. \qquad (70)$$

FIG. 2. Behavior of the power spectrum P_v for distinct values of k_s. For the two lowest values of k_s, 1 and 10^3, the vector modes are crossing the potential during dust domination phase and relation (68) is observed. The highest k_s values, 10^6 and 10^9, show the scale independence of P_v for modes crossing the potential during the radiation phase, in agreement with (69). The two vertical lines mark the nucleosynthesis and recombination era, while the $\mathcal{A}_s$ plot shows the approximated primordial amplitude for scalar modes. The values used for the free parameters are $x_b = 5 \times 10^{29}$ and $b = 10^{-3}$.

As Π_k is a constant in the contraction, this constant depends only on ν, and $Y \propto 1/x_b$, hence, $P_v \propto x_b^4$. These results can also be verified numerically, yielding:

(i) Modes crossing the potential at matter domination phase:

$$h_k \simeq \frac{10^2 x_b}{\nu^{3/2}}, \qquad P_v \simeq 10^{-124} \frac{x_b^4}{b\nu^2}. \qquad (71)$$

(ii) Modes crossing the potential at radiation domination phase:

$$h_k(\eta) \simeq \frac{x_b}{\nu^{1/2}}, \qquad P_v \simeq 10^{-126} \frac{x_b^4}{b}. \qquad (72)$$

Note there is always an extra b in P_v due to its definition, which is proportional to a k factor [cf. Eq. (50)].

We can now perform the integrations in the consistency relations (43) and (51). Let us begin with condition (43). The modes crossing the potential region during the dust phase have $\nu_c \sim \sqrt{2}/\eta_c$, where the index c refers to "crossing." Since the dust domination era ends when $\eta \sim 0.15$, we than split each integral in two parts, divided by $\sqrt{2}/0.15 \approx 10$. Thus, (43) yields

$$\frac{8}{\pi} \frac{l_p^2}{R_{H_0}^2 b^3} \int_{\nu_{\min}}^{\nu_{\max}} |h_k|^2 \nu^2 d\nu$$

$$= \frac{8}{\pi} \frac{l_p^2 x_b^2}{R_{H_0}^2 b^3} \left[10^2 \ln 10 - \ln \nu_{\min} + \frac{(\nu_{\max}^2 - 10^2)}{2} \right]. \qquad (73)$$

The solution presents an infrared and an ultraviolet divergence. The infrared divergence is logarithmic. As the number in front of the integral is very small, even assuming the minimum value of b ($b > 10^{-26}$, as we will see in the end of this section) leads to an infrared cutoff $L_{\text{infrared}} \approx \exp(10^{24}) R_{H_0}$, which is beyond any imaginable physical scale. In the case of the ultraviolet divergence, we use as $\nu_{\max}$ the value of the maximum of the potential V, which happens at the bounce, since modes with ν beyond this value will only oscillate without being enhanced. Thus, from (56), one has $\nu_{\max} = x_b^2/10^4$. Taking only the dominant term, the consistency condition (43) becomes

$$\frac{4}{10^4 \pi} \frac{l_p^2 x_b^4}{R_{H_0}^2 b^3} \ll 1. \qquad (74)$$

For the second condition, given in (51), the procedure is the same. However it will result in a much less restrictive constraint, as one can infer from the small values of P_v at the bounce, hence it is irrelevant.

Expression (74) reduces to

$$\frac{x_b^4}{b^3} \ll 10^{126}. \qquad (75)$$

Using the limits on x_b given in Eq. (31)], and $0 < b < 1$, the region in parameter space where vector perturbations
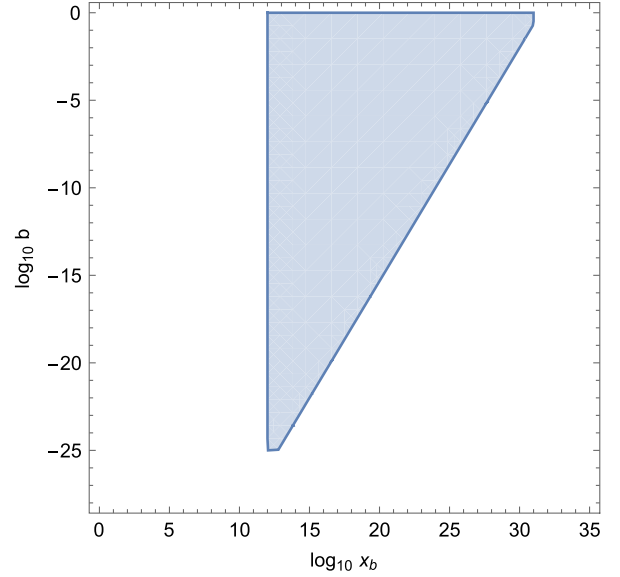

FIG. 3. Region of stability in parameter space for the bouncing models considered. The colored area represents the values of b and x_b which satisfies the consistency conditions, following Eq. (75).

remain controlled in such bouncing models are shown in Fig. 3. Note that the minimum value allowed for b is $b \approx 10^{-26}$.

VII. CONCLUSIONS

In this paper we set up the conditions under which linear vector perturbations remain controlled along the evolution of a general homogeneous and isotropic cosmological model. We considered a nonideal fluid, and its shear viscosity is capable of producing torque oscillations, which can create and dynamically sustain vector perturbations along cosmic evolution. In this framework, vector perturbations can be quantized. The resulting conditions (43) and (51) apply to any cosmological model ruled by Einstein's equations, and some particular quantum minisuperspace models.

One important application of the established conditions is to investigate whether bouncing models are stable under vector perturbations. In the case of a well-known quantum bounce, which fit cosmological observations at the background and linear perturbation level, it was shown that there is a large range of parameters in which the model is stable. However, as vector perturbations reach their largest values around the bounce itself, and as they decay afterwards, it seems to be impossible to detect their fingerprints in cosmological observations. Hence, vector perturbations, as modeled here, cannot be used to distinguish this particular bouncing model from inflationary models.

As bouncing models have already been shown to be stable under linear scalar and tensor perturbations, the present result indicates that bouncing models are stable under general linear perturbations as long as initially the

only departure from a homogeneous and isotropic geometry arises from quantum fluctuations.

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