Deuteron EDM induced by *CP*-violating couplings of pseudoscalar mesons

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We analyze contributions to the electric dipole moment (EDM) and Schiff moment of deuteron induced by the CP-violating three-pseudoscalar meson couplings using phenomenological Lagrangian approach involving nucleons and pseudoscalar mesons $P = \pi, K, \eta, \eta'$. Deuteron is considered as a proton-neutron bound state and its properties are defined by one- and two-body forces. One-body forces correspond to a picture there; proton and neutron are quasifree constituents of deuteron and their contribution to the deuteron EDM (dEDM) is simply the sum of proton and neutron EDMs. Two-body forces in deuteron are induced by one-meson exchange between nucleons. They produce a contribution to the dEDM, which is estimated using corresponding potential approach. From numerical analysis of nucleon and deuteron EDMs, we derive stringent limits on CP-violating hadronic couplings and $\bar{\theta}$ parameter. We showed that proposed measurements of proton and deuteron EDMs at level of $\sim 10^{-29}$ by the Store Ring EDM and JEDI Collaborations will provide more stringent upper limits on the CP-violating parameters.

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I. INTRODUCTION

Study of nature of CP violation is one of the most important tasks in particle physics. Here the main puzzle consists in disagreement of predictions of Standard Model (SM) and existing data on CP-violating effects like, e.g., electric dipole moments (EDMs) of electron, nucleons, and more composite system like deuteron and nuclei. SM gives more stringent upper limits than experiments. It calls for search for a new physics (new particles or mechanisms) contributing to CP-violating effects. In particular, data bounds on the hadron and lepton EDMs are very useful for derivation of more stringent limits on parameters of new particles [1,2]. In QCD, the source of the CP violation is encoded in the so-called QCD vacuum angle $\bar{\theta}$, which according to current limits on EDMs is very small quantity ($\bar{\theta} \sim 10^{-10}$) and can be explained by the Peccei-Quinn mechanism [3]. As it was shown in QCD sum rules [4,5], this angle is related to the effective CP-violating hadronic couplings, which, e.g., define the EDMs of baryons. E.g., the expressions for the CP-violating $\eta(\eta')\pi\pi$ couplings derived in Refs. [4,5] read

$$f_{H\pi\pi} = -g_H \frac{\bar{\theta} M_\pi^2 R}{F_\pi M_H (1+R)^2}, \qquad H = \eta, \eta', \qquad (1)$$

where $g_\eta = \sqrt{1/3}$, $g_{\eta'} = \sqrt{2/3}$, $R = m_u/m_d$ is the ratio of the u and d current quark masses, $F_\pi = 92.4$ MeV is the pion decay constant, $M_\pi = 139.57$ MeV, $M_\eta = 547.862$ MeV, and $M_{\eta'} = 957.78$ MeV are the masses of the charged pion, η, and η' mesons, respectively.

In series of papers [6–8], we developed phenomenological Lagrangian approach involving nucleons, pseudoscalar mesons $P = \pi, \eta, \eta'$, and photon for analysis of nucleon EDM and deriving upper limits for the CP-violating couplings between hadrons and $\bar{\theta}$ angle. In particular, using existing upper limit on neutron EDM (nEDM) [9],

$$|d_n| < 2.9 \times 10^{-26}\ \mathrm{e \cdot cm}, \qquad (2)$$

which corresponds to the following boundary for the QCD angle $|\bar{\theta}| < 10^{-10}$, we derived more stringent upper limits for the CP-violating $\eta\pi\pi$ and $\eta'\pi\pi$ couplings

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$f_{\eta\pi\pi} < 4.4 \times 10^{-11}$ and $f_{\eta'\pi\pi} < 3.8 \times 10^{-11}$ than the ones deduced from experiment by the LHCb Collaboration [10]: $f_{\eta\pi\pi} < 6.7 \times 10^{-4}$ and $f_{\eta'\pi\pi} < 2.2 \times 10^{-4}$. Using limits for these coupling, one can estimate other hadronic EDMs where these couplings contribute. The proposed experiments for measurement of EDMs of charge particles (proton, deuteron, and possibly helium-3) with a sensitivity of 10^{-29} e · cm by several collaborations (the Storage Ring EDM at BNL [11], the JEDI at Jülich [12,13]) call for more accurate theoretical analysis of EDMs.

In this paper, we extend our analysis to the deuteron, which is considered as proton-neutron bound state. In addition to the deuteron EDM (dEDM), we estimate the slope of the EDM form factor, which is known as the Schiff moment. We will take into account the contributions of one- and two-body forces to the dEDM. One-body forces correspond to a picture where proton and neutron are quasifree constituents of deuteron and their contribution to the dEDM is simply the sum of proton and neutron EDMs. Two-body forces in deuteron are induced by one-meson (π, η, and η') exchange between nucleons. They produce a contribution to the dEDM, which is estimated using potential approach proposed in Ref. [14]. From numerical analysis of nucleon and deuteron EDMs, we derive stringent limits on the CP-violating hadronic couplings and $\bar{\theta}$ parameter. We show that proposed measurements of proton and deuteron EDMs at level of $\sim 10^{-29}$ by the Storage Ring EDM and JEDI Collaborations will provide more stringent upper limits on the CP-violation parameters.

The paper is organized as follows. In Sec. II, we briefly discuss our formalism and results for EDM and Schiff moments of nucleons. In Sec. III, we extend our formalism to the dEDM. In Sec. IV, we present our numerical results for the dEDM and discuss it in connection with planned experiments. In the Appendix, we present the results for the K-mesons contributions to the pseudoscalar meson and baryon CP-violating couplings relevant for the $|\Delta T| = 0, 1$ isospin transition.

II. FORMALISM

In this section, we briefly review our formalism, which is based on phenomenological Lagrangians formulated in terms of nucleons $N = (p, n)$, pseudoscalar mesons [pions $\pi = (\pi^{\pm}, \pi^0)$ and etas $H = (\eta, \eta')$], and photon A_μ (see details in Ref. [7]). The full Lagrangian needed for the analysis of nucleon EDMs is conventionally divided on free $\mathcal{L}_0$ and interaction $\mathcal{L}_{\rm int}$ parts [7]. In particular, the interaction part $\mathcal{L}_{\rm int}$ is given a sum of CP-even and CP-odd strong interactions terms $\mathcal{L}_S$ and $\mathcal{L}_S^{CP}$ and electromagnetic terms describing coupling of charged pions and nucleons with photon. In case of nucleon, we take into account minimal and nonminimal couplings (induced by anomalous magnetic moment k_N) with electromagnetic field.

In our approach, we use both versions of couplings of pseudoscalar mesons with baryons—pseudoscalar (PS) and pseudovector (PV). One can prove that both versions PS and PV are fully equivalent to each other when all required term in the underlying Lagrangian are taken into account [15,16]. In particular, in Ref. [15], the equivalence was proved in the framework of chiral quark models dealing with bare axial charge of the quark equal to 1. Using example of pion-nucleon scattering, it was shown that Lagrangian of the PS theory requires the inclusion of the seagull term $\mathcal{L}_{\rm seagull} = M_q/(2F^2)\bar{\psi}\vec{\pi}^2\psi$, which together with the s- and u-channel pole diagrams generates identically the same matrix element with PV coupling representing by the sum of two pole s- and u-channel diagram and derivative Weinberg-Tomozawa (WT) term. It is clear that working on hadronic level, where the nucleon charge g_A deviates from 1, one should include in the covariant derivative of the kinetic baryon term which is quadratic chiral fields and proportional to the factor $(g_A^2 - 1)$. The latter term generates the WT term proportional to the factor $(g_A^2 - 1)$ and sums up with the WT term proportional to the factor $(-g_A^2)$ generated by the sum of the PS-version pole diagrams after their transformation to the PV coupling. As result, the PS version generates the WT term with correct factor (-1).

Depending on particular process, it is convenient to use PS or PV version. In particular, in the calculation of the nucleon EDMs, we used the PS version of our approach to suppress the number of evaluated diagrams because the use of PV version requires taking into account of extra graph due gauging of derivative acting on charged pseudoscalar field. In the present paper, in calculation of the CP-violating meson-nucleon coupling, it is more easy to use the PV version of our approach. In particular, in the PV version, the WT term does not contribute to the CP-violating meson-nucleon coupling and we need to evaluate only the loop diagrams generated by the PV meson-baryon coupling. In the PS version, we need to calculate the loop diagrams generated by the PS coupling and seagull diagram (see the discussion below).

Another comment concerns a consistency of our Lagrangian to the chiral perturbation theory Lagrangian. Our main idea is to calculate the CP-violating meson-nucleon couplings in terms of strong CP-even meson-nucleon couplings and CP-odd mesonic couplings. It means that we identify the leading term (independent on meson mass) in our one-loop expression to the finite part of the three-level SU(3) chiral perturbation theory (ChPT) result [17]. Note that in ChPT such term is absorbed via redefinition of hadronic couplings to exclude double counting and violation of power counting. Now, we are on the position to display the interaction part of our model Lagrangian. In sector of (π, η, η') mesons and nucleon, the strong interaction Lagrangian in PV version reads (for simplicity, we drop the WT term because it does not contribute to the CP-violating meson-nucleon couplings)

$$\mathcal{L}_{\rm str}^{\rm PV} = \mathcal{L}_{\pi NN} + \mathcal{L}_{HNN} + \mathcal{L}_{H\pi\pi}^{CP}, \qquad (3)$$

where

$$\mathcal{L}_{\pi NN} = -\frac{g_{\pi NN}}{2M_N}\bar{N}\partial_\mu\vec{\pi}\,\vec{\tau}\gamma^\mu\gamma^5 N,$$
$$\mathcal{L}_{HNN} = -\frac{g_{HNN}}{2M_N}\bar{N}\partial_\mu H\gamma^\mu\gamma^5 N,$$
$$\mathcal{L}_{H\pi\pi}^{CP} = f_{H\pi\pi}M_H H\vec{\pi}^2, \qquad (4)$$

where $g_{\pi NN} = (g_A/F_\pi)m_N$, g_{HNN}, and $f_{H\pi\pi}$ are corresponding coupling constants, γ^μ, γ^5 are the Dirac matrices, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. Here $g_A = 1.275$ is the axial nucleon charge. For the constants $g_{\eta NN}$ and $g_{\eta' NN}$, we use the values deduced from recent analysis of photoproduction on nucleons in Ref. [18]: $g_{\eta NN} = g_{\eta' NN} = 0.9$.

Nucleon EDM is extracted from the electromagnetic vertex function, which is expanded in terms of four relativistic form factors $F_E(Q^2)$ (electric), $F_M(Q^2)$ (magnetic), $F_D(Q^2)$ (electric dipole), and $F_A(Q^2)$ (anapole) as [19,20]

$$M_{\rm inv} = \bar{u}_N(p_2)\Gamma^\mu(p_1, p_2)u_N(p_1), \qquad (5)$$

$$\Gamma^\mu(p_1, p_2) = \gamma^\mu F_E(Q^2) + \frac{i\sigma^{\mu\nu}}{2m_N}q_\nu F_M(Q^2) + \frac{\sigma^{\mu\nu}}{2m_N}q_\nu\gamma^5 F_D(Q^2) + \frac{1}{m_N^2}(\gamma^\mu q^2 - 2m_N q^\mu)\gamma^5 F_A(Q^2), \qquad (6)$$

where p_1 and p_2 are momenta of initial and final nucleon states, $Q^2 = (p_2 - p_1)^2$ is the transfer momentum squared. The nucleon EDM is defined as $d_N^E = -F_D(0)/(2m_N)$.

In preceding papers [6–8], we analyzed the nEDM, which is evaluated by taking into account the two-loop diagrams. E.g., in Figs. 1 and 2, we display the diagrams induced by minimal couplings of charged hadrons with photon (see details in Ref. [7]). The contributions to the nEDM induced by nonminimal coupling of proton and neutron to the electromagnetic field have been analyzed in Ref. [8]. The relevant interaction Lagrangian of charged pions and nucleons with photon reads

$$\mathcal{L}_{\rm em} = \mathcal{L}_{\gamma NN} + \mathcal{L}_{\gamma\pi\pi},$$
$$\mathcal{L}_{\gamma NN} = eA_\mu N\left(\gamma^\mu Q_N + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N}k_N\right)N, \qquad (7)$$

$$\mathcal{L}_{\gamma\pi\pi} = eA_\mu(\pi^- i\partial^\mu\pi^+ - \pi^+ i\partial^\mu\pi^-) + e^2 A_\mu A^\mu\pi^+\pi^-, \qquad (8)$$

where Q_N is the nucleon charge.

We showed that nonminimal contributions are of the same order of magnitude as the ones induced by minimal γ-proton coupling, but separate nonminimal contributions

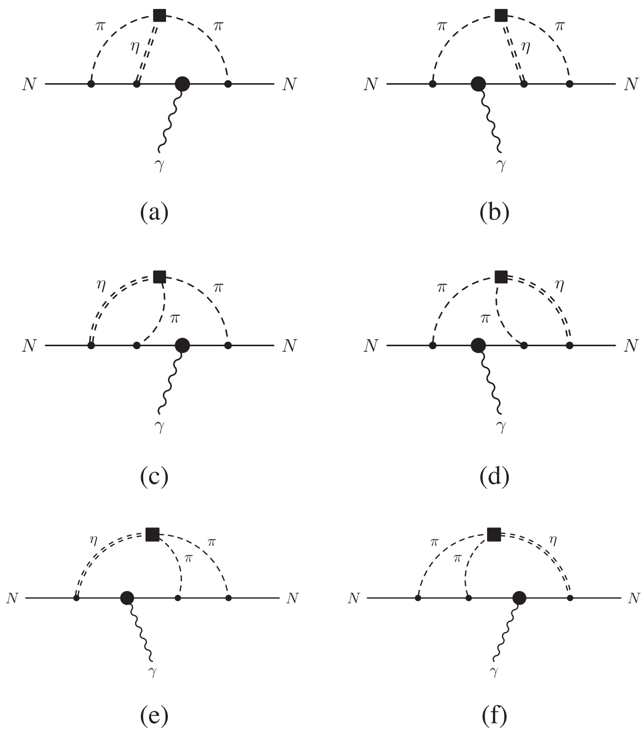

FIG. 1. Diagrams describing the nEDM induced by the minimal electric coupling of photon with charged baryon. Interaction between mesons and baryons is described in the framework of PS approach. The solid square denotes the CP-violating $\eta\pi^+\pi^-$ and $\eta\pi^0\pi^0$ vertices.

induced by anomalous magnetic moments of proton and neutron compensate each other due to their opposite sign. The total numerical contribution of the nonminimal couplings of the nucleon is relatively suppressed (by 1 order of magnitude) compared to the total contribution of the minimal coupling. Our final numerical results for the nEDM including minimal and nonminimal electromagnetic couplings of nucleons are [8]

$$d_n^E \simeq (6.62 f_{\eta\pi\pi} + 7.64 f_{\eta'\pi\pi}) \times 10^{-16}\ {\rm e\cdot cm}, \qquad (9)$$

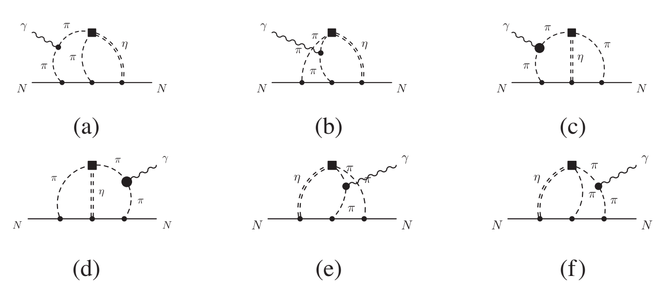

FIG. 2. Diagrams describing the nEDM induced by the minimal electric coupling of photon with charged pions. Interaction between mesons and baryons is described in the framework of PS approach. The solid square denotes the CP-violating $\eta\pi^+\pi^-$ vertex.

in terms of the CP-violating $\eta\pi\pi$ and $\eta'\pi\pi$ couplings and in terms of the QCD $\bar{\theta}$ angle,

$$|d_n^E| \simeq 0.64 \times 10^{-16}\bar{\theta}\ \mathrm{e\cdot cm}, \tag{10}$$

using the ratio of u- and d-quarks $R = 0.556$ from ChPT at 1 GeV scale [21] and

$$|d_n^E| \simeq 0.67 \times 10^{-16}\bar{\theta}\ \mathrm{e\cdot cm} \tag{11}$$

for $R = 0.468$ taken from lattice QCD at scale of 2 GeV [9]. Then using data on the nEDM, we deduced [8] the following upper limits on the QCD angle: $|\bar{\theta}| < 4.4 \times 10^{-10}$ (ChPT) and $|\bar{\theta}| < 4.7 \times 10^{-10}$ (lattice QCD).

In this paper, we first do an extension of our formalism to the proton EDM (pEDM), which is straightforward. Our numerical results for the pEDM in terms of the CP-violating $\eta\pi\pi$ and $\eta'\pi\pi$ couplings are

$$d_p^E \simeq (1.66 f_{\eta\pi\pi} + 1.77 f_{\eta'\pi\pi}) \times 10^{-16}\ \mathrm{e\cdot cm}. \tag{12}$$

Using relations of the $\eta\pi\pi$ and $\eta'\pi\pi$ couplings with $\bar{\theta}$, we express pEDM in terms of $\bar{\theta}$ as

$$|d_p^E| \simeq 0.15 \times 10^{-16}\bar{\theta}\ \mathrm{e\cdot cm} \tag{13}$$

for $R = 0.556$ from ChPT and same

$$|d_p^E| \simeq 0.16 \times 10^{-16}\bar{\theta}\ \mathrm{e\cdot cm} \tag{14}$$

for $R = 0.468$ from lattice QCD. The magnitude of pEDM is less than 1 for nEDM because in case of pEDM the contributions from diagrams in Figs. 1 and 2 have different sign in comparison with their contribution to the nEDM. Due to the leading diagrams to pEDM and nEDM are induced by the coupling of photon with charged pions with opposite charge, i.e., with π^- and π^+, respectively, the nucleon EDMs should have different signs.

After substitution of upper limits for the $\bar{\theta}$ derived in the neutron case, we get the following upper limits for the pEDM: $|d_p^E| \simeq 0.72 \times 10^{-26}$ e·cm (ChPT) and $|d_p^E| \simeq 0.68 \times 10^{-26}$ e·cm (lattice QCD). These limits are more stringent than existing limit $|d_p^E| < 2.5 \times 10^{-25}$ obtained in indirect way from analysis of the Hg atoms [22–24] and have the same order of magnitude as nEDM.

We go further and estimate the Schiff moments (SFMs) of nucleons. The nucleon SFM is defined as the slope of its EDM form factor [25],

$$S'_N = -\frac{d_N^E(Q^2)}{dQ^2}\bigg|_{Q^2=0}. \tag{15}$$

Our numerical results for the nucleon SFMs are

$$|S'_n| < (4.1 f_{\eta\pi\pi} + 4.4 f_{\eta'\pi\pi}) \times 10^{-3}\ \mathrm{e\cdot fm^3}, \tag{16}$$

$$|S'_p| < (3.7 f_{\eta\pi\pi} + 3.9 f_{\eta'\pi\pi}) \times 10^{-3}\ \mathrm{e\cdot fm^3} \tag{17}$$

in terms of the CP-violating $\eta\pi\pi$ and $\eta'\pi\pi$ couplings and in terms of the QCD vacuum angle

$$|S'_n| < 3.9 \times 10^{-4}\bar{\theta}\ \mathrm{e\cdot fm^3}, \tag{18}$$

$$|S'_p| < 3.6 \times 10^{-4}\bar{\theta}\ \mathrm{e\cdot fm^3} \tag{19}$$

for the ChPT set and

$$|S'_n| < 3.7 \times 10^{-4}\bar{\theta}\ \mathrm{e\cdot fm^3}, \tag{20}$$

$$|S'_p| < 3.4 \times 10^{-4}\bar{\theta}\ \mathrm{e\cdot fm^3} \tag{21}$$

for the lattice QCD set. Main contribution to the nucleon SFMs comes from the diagram describing the coupling of photon with charged pions (see Fig. 2), while the contribution of the graphs in Fig. 1 is suppressed. It is different from the nucleon EDMs, which are generated by both sets of diagrams in Figs. 1 and 2 with equal contribution on magnitude.

Note that our result for the neutron SFM is in good agreement with prediction of ChPT at the leading order in the chiral expansion [26]

$$|S'_n| = \frac{e g_{\pi NN} g_{\pi NN}^{CP}}{48\pi^2 M_\pi^2 m_N} < 4.4 \times 10^{-4}\bar{\theta}\ \mathrm{e\cdot fm^3} \tag{22}$$

and perturbative chiral quark model [19]

$$|S'_n| < 3.0 \times 10^{-4}\bar{\theta}\ \mathrm{e\cdot fm^3}. \tag{23}$$

III. DEUTERON EDM

The CP-violating HNN, $H = \eta, \eta'$ couplings have been calculated in ChPT in Ref. [6]. It is defined by pion-loop diagram and its value at the leading order in chiral expansion reads

$$\begin{aligned} g_{HNN}^{CP} &= -\frac{3 g_A^2 f_{H\pi\pi}}{16\pi^2 F_\pi^2} M_H m_N \\ &= -\frac{3 g_{\pi NN}^2 \tilde{f}_{H\pi\pi}}{16\pi^2}, \end{aligned} \tag{24}$$

which corresponds to the tree-level contribution in the SU(3) baryon ChPT [17] expressed in terms of the LECs b_D and b_F. Here, for convenience, the CP-violating constant $f_{H\pi\pi}$ was redefined to $\tilde{f}_{H\pi\pi}$ using relation $f_{H\pi\pi} = (m_N/m_H)\tilde{f}_{H\pi\pi}$.

Here, by analogy, we also calculate the CP-violating πNN and ηNN, $\eta' NN$ couplings, which are generated by similar loop diagram in Fig. 3(a),

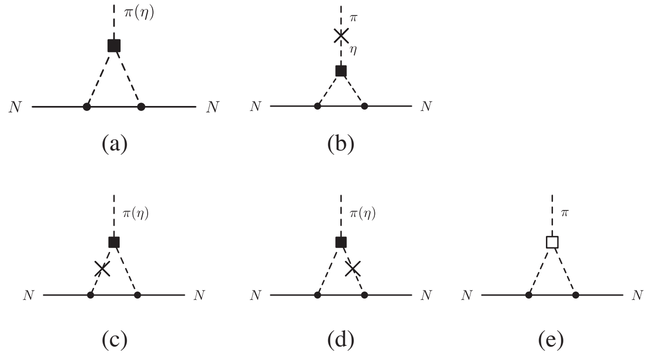

FIG. 3. *CP*-violating πNN and ηNN couplings. (a) Diagram induced by isospin-symmetric ($|\Delta T| = 0$) vertices; (b) and (c) diagrams induced by isospin-violating ($|\Delta T| = 1$) vertices induced by the internal $\pi^0 - \eta$ mixing; (d) diagram induced by isospin-violating ($|\Delta T| = 1$) pion-nucleon vertex and by the external $\pi^0 - \eta$ mixing. The cross symbol $\times$ denotes the $\pi^0 - \eta$ mixing; (e) diagram induced by *CP*-violating isospin-breaking coupling of three pseudoscalar mesons from Lagrangian (A5). The black box symbol denotes the *CP*-violating isospin-symmetric P^3 vertex. The white box symbol denotes the *CP*-violating isospin-breaking P^3 vertex.

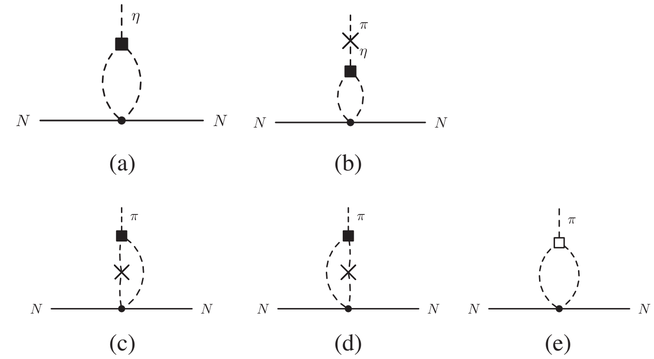

FIG. 4. *CP*-violating couplings due to seagull term (nonderivative two-meson-two-baryon coupling). (a) Diagram induced by isospin-symmetric ($|\Delta T| = 0$) vertex of ηNN and diagrams induced by isospin-violating ($|\Delta T| = 1$) πNN *CP*-violated couplings. (b),(c) Diagrams vertices induced by the internal $\pi^0 - \eta$ mixing. (d) Diagram induced by isospin-violating ($|\Delta T| = 1$) pion-nucleon vertex and by the external $\pi^0 - \eta$ mixing. The cross symbol $\times$ denotes the $\pi^0 - \eta$ mixing. (e) Diagram induced by *CP*-violating isospin-breaking coupling of three pseudoscalar mesons from Lagrangian (A5). The black box symbol denotes the *CP*-violating isospin-symmetric P^3 vertex. The white box symbol denotes the *CP*-violating isospin-breaking P^3 vertex.

$$g^{CP}_{\pi NN} = g^{CP}_{\pi NN}(M_\eta) + g^{CP}_{\pi NN}(M_{\eta'}), \tag{25}$$

$$g^{CP}_{\eta NN} = g^{CP}_{\eta NN}(M_\pi), \qquad g^{CP}_{\eta' NN} = g^{CP}_{\eta' NN}(M_\pi), \tag{26}$$

$$g^{CP}_{\pi NN}(M_H) = -\frac{g_{\pi NN} g_{HNN} \tilde{f}_{H\pi\pi}}{4\pi^2} \times \left[1 + \frac{A(M_H^2) - A(M_\pi^2)}{2m_N^2(M_H^2 - M_\pi^2)}\right], \tag{27}$$

$$g^{CP}_{HNN}(M) = -3\frac{g^2_{\pi NN}\tilde{f}_{H\pi\pi}}{16\pi^2}\left[1 + \frac{B(M_\pi^2)}{m_N^2}\right], \tag{28}$$

where

$$A(M^2) = M^4 \log\frac{m_N^2}{M^2} - M^3\sqrt{4m_N^2 - M^2}\,C(M),$$
$$B(M^2) = M^2 \log\frac{m_N^2}{M^2} - 2M\frac{3m_N^2 - M^2}{\sqrt{4m_N^2 - M^2}}C(M),$$
$$C(M) = \arctan\frac{2m_N^2 - M^2}{\sqrt{4m_N^2 M^2 - M^4}} + \arctan\frac{M}{\sqrt{4m_N^2 - M^2}}. \tag{29}$$

Remember that we do calculation in PV version of meson-baryon coupling. The use of PS coupling requires (as we stressed before) inclusion of additional diagram in Fig. 4(a) generated by the seagull nonderivative coupling of two pseudoscalar mesons with nucleons. As soon as such graph is included, the PS version for calculation of the *CP*-violating meson-nucleon couplings is fully equivalent to the PV version.

Our results for the *CP*-violating meson-nucleon coupling contains two main terms in square brackets: the first term is proportional to one, which is equivalent to the tree-level contribution and the second term contain chiral logarithms and arctangents, which represents the meson cloud contribution. We express both tree-level and meson cloud contributions in terms of strong meson-nucleon and *CP*-violating three-meson coupling in order to have opportunity to derive constraints on the $f_{H\pi\pi}$ couplings or rare $H \to \pi\pi$ decay rates from data on baryon and nuclei EDMs. Both tree-level results in the SU(3) ChPT and in our approach scales as m_q in chiral expansion. In our tree-level expression, the linear m_q dependence is hidden in the *CP*-violating three-pseudoscalar meson couplings.

As it is seen, that the *CP*-violating ηNN coupling dominates over the πNN one. It is why it makes sense to take into account the η exchange in the evaluation of the dEDM. Note that our *CP*-violating couplings have microscopic (loop) origin. Therefore, it is interesting to compare our loop results for the *CP*-violating meson-nucleon couplings with the results for these couplings derived using chiral techniques. In particular, the *CP*-violating πNN coupling at the leading order in chiral expansion was obtained in Ref. [4] in terms of current quark masses and $m_\Xi - m_\Sigma$ baryon mass difference,

$$g^{CP}_{\pi NN} = -\bar{\theta}\frac{(m_\Xi - m_\Sigma)R}{F_\pi(1+R)(2R_s - 1 - R)}, \tag{30}$$

where $R_s = m_s/m_d$, $m_\Xi = 1321.71$ MeV, and m_Σ are the masses of the $\Xi(1321)^-$ and $\Sigma(1189)^+$ and hyperons. This result later was confirmed in the SU(3) ChPT [17]. Both $g^{CP}_{\pi NN}$ and $g^{CP}_{\eta NN}$ couplings can be presented in terms of matrix elements of quark operators projected over nucleon states (nucleon condensates) [4],

$$g^{CP}_{\pi NN} = -\bar\theta \frac{m_d R}{F_\pi(1+R)} \langle N|\bar q \tau^3 q|N\rangle = -0.021\bar\theta, \quad (31)$$

$$g^{CP}_{\eta NN} = -\frac{\bar\theta}{\sqrt{3}} \frac{m_d R}{F_\pi(1+R)} \langle N|\bar q q|N\rangle = -0.132\bar\theta, \quad (32)$$

$$g^{CP}_{\eta' NN} = -\bar\theta \sqrt{\frac{2}{3}} \frac{m_d R}{F_\pi(1+R)} \langle N|\bar q q|N\rangle = -0.28\bar\theta \quad (33)$$

for R from lattice data at scale 2 GeV [9]. Matrix elements $\langle N|\bar q\tau^3 q|N\rangle$ and $\langle N|\bar q q|N\rangle$ can be related to the nucleon axial charge g_A (FLAG lattice average [27]; see also prediction of chiral quark model (PCQM) [28]) and pion-nucleon sigma-term $\sigma_{\pi N}$ [29,30],

$$\langle N|\bar q\tau^3 q|N\rangle = 1.022 \pm 0.080 \pm 0.060\,(\text{FLAG}), \quad (34)$$

$$\langle N|\bar q\tau^3 q|N\rangle = \frac{3}{5} g_A\,(\text{PCQM}), \quad (35)$$

$$\langle N|\bar q q|N\rangle = \frac{\sigma_{\pi N}}{\bar m} = 8.286, \quad (36)$$

where $\bar m = (m_u + m_d)/2 = 7$ MeV [21]. For $\sigma_{\pi N}$, we use the latest update 58 MeV derived in Ref. [31].

Our numerical results for the CP-violating constants in terms of the $\bar\theta$ parameter are given in Table I. For completeness, we display full result (up to one loop) and separately tree-level and meson cloud contributions.

One can see, that our prediction for the full $g^{CP}_{\pi NN}$ coupling is close to the prediction $0.027\bar\theta$ of Ref. [4] and in agreement with central value of Ref. [32]: $(-0.018 \pm 0.007)\bar\theta$. On the other hand, the CP-violating constants $g^{CP}_{\eta NN}$ and $g^{CP}_{\eta' NN}$ dominate over $g^{CP}_{\pi NN}$ by 1 order of magnitude. They are so-called isospin $|\Delta T| = 0$ CP-violating couplings [14,32].

In this section, we discuss calculation of the dEDM d^E_D, which is defined as the coupling of the external electric field $\vec E$ with deuteron spin $\vec S$: $H = -d^E_D(\vec S \cdot \vec E)$. The contributions to the dEDM comes from one-body forces (additive sum of the pEDM and nEDM) and from two-body forces due one-meson exchanges between nucleons. The two-body contribution to the dEDM is induced by the CP-violating meson-nucleon coupling. Therefore, the dEDM is defined as

$$d^E_D = d^E_p + d^E_n + d^{\pi NN}_D, \quad (37)$$

where $d^{\pi NN}_D$ is the two-body contribution due to pion meson exchange generated by $\pi^0 - \eta$ and $\pi^0 - \eta'$ mixing.

The two-body contributions can be estimated using potential approach [33] proposed and developed in Ref. [14]. In case of the pion exchange, it was shown that the dominant contribution comes due to the isospin triplet coupling [14],

$$d^{\pi NN}_D = -\frac{e g_{\pi NN} g^{CP(1)}_{\pi NN}}{12\pi M_\pi} \rho_\pi, \quad (38)$$

where

$$\rho_P = \frac{1+\xi_P}{(1+2\xi_P)^2}, \qquad \xi_P = \frac{\sqrt{m_N \epsilon_D}}{M_P}. \quad (39)$$

Here $P = \pi, \eta, \eta'$ and $\epsilon_D = 2.23$ MeV is the deuteron binding energy, $g^{CP(1)}_{\pi NN}$ is the CP-violating πNN isospin-breaking coupling constant [14,32,34] including the $\eta - \pi$ and $\eta' - \pi$ mixing,

$$g^{CP(1)}_{\pi NN} = g^{CP(\pi\eta)}_{\pi NN} + g^{CP(\pi\eta')}_{\pi NN}, \quad (40)$$

$$g^{CP(\pi\eta)}_{\pi NN} = \epsilon g^{CP}_\eta(M_\eta), \qquad g^{CP(\pi\eta')}_{\pi NN} = \epsilon' g^{CP}_{\eta'}(M'_\eta), \quad (41)$$

where

$$g^{CP}_H(M_H) = g^{CP,\text{Int}}_H(M_H) + g^{CP,\text{Ext}}_H(M_H), \quad (42)$$

$$g^{CP,\text{Int}}_H(M_H) = g^{CP}_{\pi NN} - \frac{4}{3} g^{CP}_{HNN}(M_H),$$
$$g^{CP,\text{Ext}}_H(M_H) = \left(\frac{\rho_H M_\pi}{\rho_\pi M_H} - 1\right)\left(g^{CP}_{HNN} - g^{CP}_{\pi NN}\frac{g_{HNN}}{g_{\pi NN}}\right), \quad (43)$$

where $SU_f(3)$ flavor breaking coefficients ϵ and ϵ' [35–37] are defined as

$$\epsilon = \epsilon_0 \chi \cos\varphi, \quad (44)$$

$$\epsilon' = -2\epsilon_0 (1/\chi) \sin\varphi, \quad (45)$$

with parameter ϵ_0 encoding the isospin breaking effects,

TABLE I. Contributions of $\Delta T = 0$ CP-violated couplings of meson and baryon interaction. All values have a factor $\bar\theta$.

Couplings	Tree level	Meson cloud	full
$g^{CP}_{\pi NN}$	−0.02	−0.001	−0.021
$g^{CP}_{\eta NN}$	−0.09	−0.003	−0.93
$g^{CP}_{\eta' NN}$	−0.127	+0.002	−0.125

$$\epsilon_0 = \frac{\sqrt{3}(1-R)}{2(2R_s - 1 - R)}, \tag{46}$$

and $\chi = 1 + (4M_K^2 - 3M_\eta^2 - M_\pi^2)/(M_{\eta'}^2 - M_\eta^2) \simeq 1.23$. Here $\varphi \simeq -21.6^0$ is mixing angle between η and η' mesons which is fixed from relation $\sin 2\varphi = -(4\sqrt{2}/3)(M_K^2 - M_\pi^2)/(M_{\eta'}^2 - M_\eta^2)$ [35–37]. Resulting values are $\epsilon = 0.017$ and $\epsilon' = 0.004$.

The first term in Eq. (42) is induced by the $\pi^0 - \eta$ mixing in the triangle loop diagrams in Figs. 3(c) and 3(d). The second term in Eq. (42) is induced by $\pi^0 - \eta$ mixing in the external meson leg [see Fig. 3(b)]. Therefore, one can denote two mechanisms of isospin violation due to the $\pi^0 - \eta$ mixing as *internal mechanism* [depicted in Figs. 3(c) and 3(d)] and as *external mechanism* [depicted in Fig. 3(b)]. One should note that the internal mechanism is strongly suppressed in comparison with external mechanism by a factor 10^{-2}. We get the following numerical results for the *CP*-violating $g_{\pi NN}^{CP(1)}$ coupling:

$$g_{\pi NN}^{CP(\pi\eta)} = 0.0016\bar{\theta} \tag{47}$$

due to $\pi - \eta$ mixing and

$$g_{\pi NN}^{CP(\pi\eta')} = 0.0005\bar{\theta} \tag{48}$$

due to $\pi - \eta'$ mixing without *K*-mesons contribution. As in case of $|\Delta T| = 0$ couplings in Fig. 4, we show additional "seagull" type diagrams needed in case of the use of PS version of the meson-baryon coupling. The total result with taking into account *K*-mesons contribution (see details in the Appendix) for the isospin breaking $|\Delta T| = 1$ *CP*-violating pion-nucleon coupling is $g_{\pi NN}^{CP(1)} = 0.0025\bar{\theta}$, which is in good agreement with prediction of Refs. [32,38]: $\bar{g}^1 = (0.003 \pm 0.002)\bar{\theta}$.

The ratio of the full *CP*-violating πNN coupling constants corresponding to the $|\Delta T| = 1$ and $|\Delta T| = 0$ is

$$R_{\pi NN} = \frac{g_{\pi NN}^{CP(1)}}{g_{\pi NN}^{CP}} = \frac{\bar{g}^1}{\bar{g}^0} = -0.12. \tag{49}$$

The latter expression also gives the prediction for the ratio of the couplings $\bar{g}^1$ and $\bar{g}^0$. One can see that our result for the $R_{\pi NN}$ is close to the lower boundary of the prediction of Ref. [32]: $R_{\pi NN} \sim -0.2 \pm 0.1$.

Finally, resulting contribution from one-meson exchange is

$$|d_D^{\pi NN}| = 0.28 \times 10^{-18}\bar{\theta}\ \mathrm{e\,cm}, \tag{50}$$

which is in good agreement with data (see Ref. [32]).

IV. DISCUSSION

The dEDM is contributed by the EDMs of constituent nucleons and correction due to one-meson exchange in the isospin channel $|\Delta T| = 1$. The latter is induced due to isospin breaking effects ($\eta - \pi$ and $\eta' - \pi$ mixing) and, therefore, it is relatively suppressed. Our final prediction for the dEDM in terms of the $\bar{\theta}$ angle reads

$$|d_D| = 0.482 \times 10^{-16}\bar{\theta}\ \mathrm{e\,cm}. \tag{51}$$

Next, using the upper limit for the $\bar{\theta}$ [8], we get

$$|d_D| < 2.2 \times 10^{-26}\ \mathrm{e\,cm}. \tag{52}$$

Here we take into account that proton and neutron EDMs have different signs.

In prospects of future experiments, an observation that the dEDM is proportional to the nucleon EDM and the other contributions are suppressed has big importance. A comparison between our theoretical prediction and sensitivity of future experimental measurements of the dEDM at the level of accuracy $\sim 10^{-29}$ from the EDM Collaboration [11] gives more stringent limit on the *CP*-violating parameter $\bar{\theta}$,

$$|\bar{\theta}| < 2 \times 10^{-13}. \tag{53}$$

The same order of magnitude for $\bar{\theta}$ has been obtained in framework of supersymmetric approach minimal super symmetry model [34]. These limits on the dEDM and $\bar{\theta}$ allow to derive new bounds on nucleon EDMs at level 10^{-29} and more stringent limits on the decay rates of the *CP*-violation processes $\eta \to \pi\pi$ and $\eta' \to \pi\pi$. In connection with planned EDM experiments, one can derive the limits for the branching ratios of these rare processes decays at level $\sim 10^{-21}$ and $\sim 10^{-23}$ for η and η' mesons, respectively. Direct observation of these decays at such level of accuracy is impossible. There is the same situation in case of future experiment on measurement of the proton EDM by the JEDI Collaboration [12,13]. We would like to stress that direct measurement of the decay rates of the *CP*-violation processes $\eta \to \pi\pi$ and $\eta' \to \pi\pi$ at level higher than limitations from data on EDMs could potentially signal about manifestation of new physics.

In conclusion, we derived limits on the proton EDM and nucleon SFMs from existing experimental data on neutron EDM. We calculated the dEDM with taking into account one- and two-body forces in deuteron. All these quantities were calculated using phenomenological Lagrangian approach involving the PS coupling between baryons and pseudoscalar mesons and the *CP*-violating couplings 3*P* couplings of pseudoscalar mesons. Note that these couplings are proportional to the QCD *CP*-violating parameter $\bar{\theta}$ and, therefore, encode a source of the strong *CP* violation in our formalism. Complementary we also derived the dependence of the dEDM on $\bar{\theta}$. In future, we plan to continue our study of the EDMs of baryons and nuclei induced by strong CP-violating effects, e.g., by taking into account of the CP-violation three-pseudoscalar meson vertices involving all nonet states (π, K, η, and η') and all isospin transitions $|\Delta T| = 0, 1, 2$.

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APPENDIX: SU(3) BARYON-MESON LAGRANGIAN AND CP-VIOLATING CONSTANS

The baryon-meson interaction Lagrangian involving nucleon, Λ, and Σ states in the framework of $SU(3)$ scheme reads [17,39]

$$\begin{aligned}\mathcal{L}_{BBM} = &-\frac{g_{\pi NN}}{2M_N}\bar{N}\partial_\mu\vec{\pi}\,\vec{\tau}\gamma^\mu\gamma_5 N \\ &-\frac{g_{\Lambda NK}}{M_N+M_\Lambda}(\bar{N}\gamma^\mu\gamma^5\Lambda\partial_\mu K + \text{H.c.}) \\ &-\frac{g_{\Sigma NK}}{M_N+M_\Sigma}(\bar{N}\gamma^\mu\gamma^5\vec{\Sigma}\,\vec{\tau}\,\partial_\mu K + \text{H.c.}), \qquad \text{(A1)}\end{aligned}$$

where the relations between meson-baryon couplings are

$$g_{\Lambda NK} = -\frac{g_{\pi NN}}{\sqrt{3}}\frac{m_\Lambda + m_N}{2m_N}(1+\alpha), \qquad \text{(A2)}$$

$$g_{\Sigma NK} = g_{\pi NN}\frac{m_\Sigma + m_N}{2m_N}(1-2\alpha), \qquad \text{(A3)}$$

and $\alpha = F/(F+D)$. We use the averaged values for $F = 0.47$ and $D = 0.8$ [6,17,40] fixed from data.

Effective Lagrangian for three meson couplings inducing the CP-violating processes with taking into account of isospin breaking effects reads [4,17]

$$\mathcal{L}_{CP} = -\bar{\theta}\frac{M_\pi^2}{6F_\pi}\frac{m_u m_d}{(m_u+m_d)^2}\text{Tr}(P^3), \qquad \text{(A4)}$$

where $P = P^a\lambda^a$ is the matrix of pseudoscalar fields. In terms of physical states, this Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_{CP} = &-\frac{M_\pi^2}{6F_\pi\bar{m}}m^*\bar{\theta}\sqrt{3}[\eta\vec{\pi}^2 + \sqrt{3}K^\dagger\vec{\pi}\vec{\tau}K - \eta K^\dagger K \\ &+\phi(\pi^0\vec{\pi}^2 - \pi^0 K^\dagger K - \sqrt{3}\eta K^\dagger\tau^3 K)] + \mathcal{O}(\phi^2), \qquad \text{(A5)}\end{aligned}$$

where ϕ is the $SU(3)$ breaking parameter

$$\phi = \frac{\sqrt{3}\bar{m}\,\tilde{\epsilon}}{2(m_s-\bar{m})}, \qquad \bar{m} = \frac{m_u+m_d}{2},$$
$$m^* = \frac{m_u m_d m_s}{m_s(m_u+m_d)+m_u m_d}, \qquad \tilde{\epsilon} = \frac{m_d - m_u}{m_d + m_u}. \qquad \text{(A6)}$$

This Lagrangian generates the CP-violating couplings involving π, K, η mesons and corresponds to the change of the isospin $|\Delta T| = 0$ and $|\Delta T| = 1$. Below we present the contribution of K mesons to the $g_{\pi NN}^{CP}$, $g_{\eta NN}^{CP}$, and $g_{\eta' NN}^{CP}$ couplings,

$$g_{\pi NN}^{CP,K} = g_{\pi NN}^{CP,\Lambda} + g_{\pi NN}^{CP,\Sigma}, \qquad \text{(A7)}$$

$$\begin{aligned}g_{\eta(\eta')NN}^{CP,K} &= -3(g_{\pi NN}^{CP,\Lambda} + g_{\pi NN}^{CP,\Sigma})\frac{f_{KK\eta(\eta')}}{f_{KK\pi}}, \\ g_{\pi NN}^{CP,B} &= -\frac{g_{BNK}^2 f_{KK\pi}(2m_N - m_B)}{16\pi^2 m_N^2} \\ &\quad\times[1 + A(m_B) - C(m_B)], \qquad \text{(A8)}\end{aligned}$$

where

$$\begin{aligned}A(m_B) &= \frac{m_B^2 - M_K^2}{m_N^2}\log\left(\frac{m_B^2}{M_K^2}\right), \\ C(m_B) &= \frac{(M_K^2 - m_B^2)^2 - m_N^2(M_K^2 + m_B^2)}{m_N^2\zeta}, \\ &\quad\times[\arctan((m_B^2 - M_K^2 - m_N^2)\zeta^{-1}) \\ &\quad - \arctan((m_B^2 - M_K^2 + m_N^2)\zeta^{-1})], \\ \zeta &= \sqrt{2M_K^2(m_B^2 + m_N^2) - (m_B^2 - m_N^2)^2 - M_K^4}, \qquad \text{(A9)}\end{aligned}$$

where $B = \Lambda, \Sigma$ denotes hyperons and $f_{P_1P_2P_3}$ is the CP-violating three-pseudoscalar meson transition couplings obtained from Eq. (A5).

The K-meson contribution to the $|\Delta T| = 1$ CP-violating coupling shown in Fig. 3(e) is

$$g_{\pi NN}^{CP,K} = \phi\left(\frac{2}{3}\frac{f_{\pi^0\pi^+\pi^-}}{f_{\eta\pi\pi}}g_{HNN}^{CP}(M_\pi) + 3g_{\pi NN}^{CP,K}\right). \qquad \text{(A10)}$$

This contribution has the same magnitude as the value generated by the internal mechanism from diagrams in Figs. 3(c) and 3(d) due to $f_{\eta\pi\pi}$ CP-violating coupling; the $g_{HNN}^{CP}(M_\pi)$ function was denoted before in Eq. (28). Main contribution to $|\Delta T| = 1$ CP-violated coupling of pion and nucleons due to K-mesons propagating in the loop also comes from the external mechanism [see Fig. 3(b)] which is given by the same structure integral,

$$\begin{aligned} g_{\pi NN}^{CP(1)} &= g_{\pi NN}^{CP,K(\pi\eta)} + g_{\pi NN}^{CP,K(\pi\eta')}, \\ g_{\pi NN}^{CP,K(\pi\eta)} &= \epsilon g_{\eta}^{CP}(M_\eta), g_{\pi NN}^{CP,K(\pi\eta')} = \epsilon' g_{\eta'}^{CP}(M_{\eta'}), \\ g_H^{CP,Ext}(M_H) &= \left(\frac{\rho_H M_\pi}{\rho_\pi M_H} - 1\right)\left(g_{HNN}^{CP,K} - g_{\pi NN}^{CP,K}\frac{g_{HNN}}{g_{\pi NN}}\right), \end{aligned} \tag{A11}$$

where $H = \eta, \eta'$. It contributes by amount of 15% to the CP-violating πNN coupling in case of the $|\Delta T| = 1$ isospin transition.

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