Chiral $SU_L(3) \times SU_R(3)$ symmetry of baryons with one charmed quark

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Single-charm baryons can be classified into two $SU(3)_F$ multiplets: (i) the $\bar{\mathbf{3}}_F$-plet and (ii) the $\mathbf{6}_F$-plet. Each $SU(3)_F$ multiplet of charmed baryons has at most four different chiral $SU_L(3) \times SU_R(3)$ symmetry multiplets associated with it, i.e., each physical state is a mixture of at most four "chiral" states. Chiral symmetry imposes conditions/constraints on the interaction Lagrangians involving different chiral multiplets. We construct effective chiral Lagrangians for hadronic interactions describing transitions, scatterings, and decays of a single-charm baryon with a single pseudoscalar meson belonging to the $SU(3)_F$ nonet (containing the π, K, η, η' mesons). For this purpose, we use chiral transformation properties of light diquarks in the linear realization of chiral symmetry. We discuss the $U_A(1)$ symmetry of these chiral interactions and show that this symmetry imposes selection rules that ought to have observable consequences, at least in principle.

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I. INTRODUCTION

Individual ground-state single-charm baryons have been known for more than 30 years, see Refs. [1–6], and recently there emerge many new data from the *BABAR*/Belle/BESIII/LHCb experiments [7–11]. By now all of the lowest-lying states have been observed, and a number of higher-lying states are known. In a (small) number of cases, pionic transitions/decays of excited states have been observed, as well [7–10].

Nagahiro *et al.* [12] have recently evaluated pionic decays of charmed baryons in a nonrelativistic constituent quark model with a harmonic oscillator confinement potential. They found that "the axial-vector-type coupling of the pion to the light quarks is essential, which is expected from chiral symmetry, to reproduce the decay widths especially of the low-lying Λ_c^* baryons." Of course, the nonrelativistic constituent quark model is not chirally symmetric, by its very construction, and, moreover, it leads to the well-known 33% overestimate of the nucleon axial current coupling constant g_A^N of the nucleon. This calls for a (more) careful treatment of chiral symmetry in the charmed baryons, preferably in a unified treatment with light-quark baryons, which we shall try and provide in the present paper.

We start from the original linear realization of chiral symmetry in QCD and classify the chiral properties of single-charm baryon fields. An alternative approach to the same problem can be found in Refs. [13–15]. Each $SU(3)_F$ multiplet of charmed baryons has four different chiral multiplets associated with it: (i) the $\bar{\mathbf{3}}_F$-plet is contained in the "direct" $\bar{\mathbf{3}}_F \in (\bar{\mathbf{3}}, \mathbf{1}) \oplus (\mathbf{1}, \bar{\mathbf{3}})$, the "mirror" $\bar{\mathbf{3}}_F \in (\mathbf{1}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{1})$, and in the seemingly "ambidextrous" $\bar{\mathbf{3}}_F \in \mathbf{B}^9 = (\mathbf{3}, \mathbf{3})$ chiral multiplet, as well as in its nonidentical mirror image $\bar{\mathbf{3}}_F \in \mathbf{B}^9_{[\mathrm{mir}]} = (\mathbf{3}, \mathbf{3})$ [16], whereas (ii) the $\mathbf{6}_F$-plet is contained in the direct $\mathbf{6}_F \in (\mathbf{6}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{6})$, or in the mirror $\mathbf{6}_F \in (\mathbf{1}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{1})$, as well as in both kinds of the ambidextrous chiral multiplet $\mathbf{6}_F \in \mathbf{B}^9 = (\mathbf{3}, \mathbf{3})$ and $\mathbf{6}_F \in \mathbf{B}^9_{[\mathrm{mir}]} = (\mathbf{3}, \mathbf{3})$. The mirror chiral multiplet might not appear in the ground state(s), that are described by local interpolating operators, which are therefore more restricted by the Pauli principle. Exited states are generally expected to contain all four chiral components.

Using these chiral properties, we construct all possible algebraically distinct $SU_L(3) \times SU_R(3)$ chirally invariant interactions of such baryons with one pseudoscalar (or scalar) meson. Some of our interaction Lagrangians violate the $U_A(1)$ symmetry, whereas others do not, which in fact implies selection rules. Now, the $U_A(1)$ chiral symmetry plays a special role in QCD: it is explicitly broken, above and beyond the usual current-quark mass terms, by topological effects, specifically by QCD instantons [17,18]. This explicit symmetry breaking is most visibly manifested in the η, η' meson masses, which are (much) larger than the pion's and/or kaon's masses, but there are precious few other places where it is manifested. In this paper, we show that charm hyperons interactions are one (new) place where $U_A(1)$ symmetry breaking is manifested.

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Our results imply, *inter alia*, that (i) some baryon-pseudoscalar-meson interactions are related to 't Hooft's instanton-induced interaction in QCD by way of crossing and/or Fierz relations; see e.g., Refs. [19,20] and (ii) their coupling constant(s) must be related to the η, η' masses, which may make them of different order of magnitude than the couplings of $U_A(1)$-conserving interactions. These two facts, besides being of some theoretical interest, ought to have practical implications: they imply bounds on some charmed baryons' pion interactions stemming from entirely different hadronic properties; that is, however, another topic, which will not be dealt with here.

In this paper, after some preliminary considerations in Sec. II that defines the one-heavy-two-light-quark baryon fields and their chiral transformation properties, in Sec. III we construct chiral Lagrangians for one-pseudoscalar-meson ("pionic") charmed baryon (i.e., light diquark) transitions/decays in the linear realization of chiral symmetry, at first with three light flavors $N_f = 3$ in Sec. III A, and then with two light flavors $N_f = 2$ in Sec. III B. That allows us to determine their $U_A(1)$ symmetry properties in Sec. III C. In Sec. IV, we discuss the phenomenology of chiral mixing. Finally, in Sec. V, we summarize and draw our conclusions. Technical aspects are relocated to the Appendix.

II. ONE-HEAVY-TWO-LIGHT-QUARK BARYONS

Baryons consisting of one heavy and two light quarks can be thought of as a bound state of the heavy quark and the light diquark, which can have different values of the total angular momentum $J = J_{\mathrm{diquark}} \otimes s_{\mathrm{heavy-quark}}$ and thus ought to be described by different fields, e.g., Dirac, Rarita-Schwinger [21–23], and Bargmann-Wigner [22,23] fields; see below.

In the present study, we shall only construct the Dirac fields of $J^P = 1/2^+$ (denoted as J) and the Rarita-Schwinger fields of $J^P = 3/2^+$ (denoted as J_μ), while those negative-parity fields can be easily obtained by adding one extra γ_5 matrix, i.e., $\gamma_5 J$ is a Dirac field of $J^P = 1/2^-$ and $\gamma_5 J_\mu$ is a Rarita-Schwinger field of $J^P = 3/2^-$. We note that the field J can couple to both the state X having the same (positive) parity as well as the state X' having the opposite (negative) parity [24–27], through

$$\langle 0|J|X\rangle = f_X u(p), \tag{1}$$

$$\langle 0|J|X'\rangle = f_{X'}\gamma_5 u'(p). \tag{2}$$

Or we can use its partner field $\gamma_5 J$,

$$\langle 0|\gamma_5 J|X\rangle = f_X \gamma_5 u(p), \tag{3}$$

$$\langle 0|\gamma_5 J|X'\rangle = f_{X'} u'(p). \tag{4}$$

So do the Rarita-Schwinger fields J_μ and $\gamma_5 J_\mu$. The Bargmann-Wigner fields of $J^P = 3/2^\pm$ (denoted as $J_{\mu\nu}$ with two antisymmetric Lorentz indices) are slightly different, because each of these fields already contains both positive- and negative-parity components. However, $J_{\mu\nu}$ still has its partner field $\gamma_5 J_{\mu\nu}$, and both of them can couple to the same states.

The (light-flavor) chiral properties of such baryon fields are entirely determined by the chiral properties of the light diquark, which have been classified in previously published papers: for $N_f = 2$ case, see Ref. [28], and for $N_f = 3$ case, see Ref. [29]. We could have followed the (far more) complicated route outlined in Ref. [29]: first take all bilocal three-quark interpolating fields, which allows for all possible chiral multiplets, the mirror ones included—the results are shown in Tables III–VII of Ref. [29].

One notices immediately the presence of both the direct and mirror chiral multiplets, as a consequence of the weaker influence of the Pauli principle on nonlocal fields. Taking the bilocal fields is necessary, as we are not considering only the ground states (which correspond to the local interpolator fields) of the single-charm baryons, but also the excited states, which are generally nonlocal. Then one may let the mass of one of the quarks grow beyond all bounds and thus obtain a one-heavy-two-light-quark baryon, which we are interested in.

A. Dirac fields with spin 1/2

1. Flavor $\bar{3}_F$ baryons

Let us start with writing down five baryon fields which contain a diquark formed by five sets of Dirac matrices, 1, γ_5, γ_μ, γ_μ, γ_5, and $\sigma_{\mu\nu}$,

$$\begin{aligned}
B^G_{\bar{3},1} &= \epsilon_{abc}\epsilon^{ABG}(q_A^{aT} C q_B^b)\gamma_5 c^c,\\
B^G_{\bar{3},2} &= \epsilon_{abc}\epsilon^{ABG}(q_A^{aT} C\gamma_5 q_B^b) c^c,\\
B^G_{\bar{3},3} &= \epsilon_{abc}\epsilon^{ABG}(q_A^{aT} C\gamma_\mu\gamma_5 q_B^b)\gamma^\mu c^c,\\
B^G_{\bar{3},4} &= \epsilon_{abc}\epsilon^{ABG}(q_A^{aT} C\gamma_\mu q_B^b)\gamma^\mu\gamma_5 c^c,\\
B^G_{\bar{3},5} &= \epsilon_{abc}\epsilon^{ABG}(q_A^{aT} C\sigma_{\mu\nu} q_B^b)\sigma_{\mu\nu}\gamma_5 c^c,
\end{aligned} \tag{5}$$

where a, b, c are color indices and the sum over repeated indices is taken; A, B, G are $SU(3)_F$ indices, so that $q_A = \{u, d, s\}$; ϵ^{ABG} is the totally antisymmetric matrix with $G = 1, 2, 3$, so that $B^G_{\bar{3},i}$ belongs to the $\bar{3}_F$-plet; c^c is the charm quark field with the color index c; C is the charge-conjugation matrix. The charm quark may also exist in the diquark, but one can always use the Fierz transformation to move it to the last position, and so write it as a combination of the five fields shown in Eq. (5).

Among these five fields, we can show that the fourth and the fifth ones vanish, $B^G_{\bar{3},4,5} = 0$, due to the Pauli principle. Therefore, in this case, there are altogether three independent fields, $B^G_{\bar{\mathbf{3}},1,2,3} \neq 0$, which can be associated with the three kinds of diquark fields,

$$\begin{aligned} B^G_{\bar{\mathbf{3}},1} &= \epsilon^{abc} P^G_{3ab} \gamma_5 c^c, \\ B^G_{\bar{\mathbf{3}},2} &= \epsilon_{abc} S^G_{3ab} c^c, \\ B^G_{\bar{\mathbf{3}},3} &= \epsilon_{abc} V^{G\mu}_{3ab} \gamma^\mu c^c. \end{aligned} \tag{6}$$

Here P^G_{3ab}, S^G_{3ab}, and $V^{G\mu}_{3ab}$ are the diquark fields of $J^P = 0^-$, 0^+, and 1^-, respectively.

2. Flavor $\mathbf{6}_F$ baryons

Among the five $\mathbf{6}_F$ baryon fields formed by the five different γ-matrices, only two are nonzero,

$$\begin{aligned} B^U_{\mathbf{6},4} &= \epsilon_{abc} S^U_{AB} (q^{aT}_A C \gamma_\mu q^b_B) \gamma^\mu \gamma_5 c^c, \\ B^U_{\mathbf{6},5} &= \epsilon_{abc} S^U_{AB} (q^{aT}_A C \sigma_{\mu\nu} q^b_B) \sigma_{\mu\nu} \gamma_5 c^c, \end{aligned} \tag{7}$$

where S^U_{AB} are the six totally symmetric matrices with $U = 1\ldots6$, so that $B^U_{\mathbf{6},i}$ belongs to the $\mathbf{6}_F$-plet. We can associate them with the two kinds of diquark fields,

$$\begin{aligned} B^U_{\bar{\mathbf{3}},4} &= \epsilon_{abc} A^{U\mu}_{6ab} \gamma^\mu \gamma_5 c^c, \\ B^U_{\bar{\mathbf{3}},5} &= \epsilon_{abc} T^{U\mu\nu}_{6ab} \sigma_{\mu\nu} \gamma_5 c^c, \end{aligned} \tag{8}$$

where $A^{U\mu}_{6ab}$ and $T^{U\mu\nu}_{6ab}$ are the diquark fields of $J^P = 1^+$ and $1^\pm$, respectively.

B. Rarita-Schwinger fields with spin 3/2

In this subsection, we study the properties of Rarita-Schwinger fields [21,22] in the form of

$$B_\mu(x) \sim \epsilon_{abc} (q^{aT}_A(x) C \Gamma_1 q^b_B(x)) \Gamma_2 q^c_C(x), \tag{9}$$

where there are eight possible pairs of Γ_1 and Γ_2,

$$\begin{aligned} (\Gamma_1, \Gamma_2) = {} & (\mathbf{1}, \gamma_\mu), (\gamma_5, \gamma_\mu\gamma_5), (\gamma_\mu\gamma_5, \gamma_5), \\ & (\gamma^\nu\gamma_5, \sigma_{\mu\nu}\gamma_5), (\gamma_\mu, \mathbf{1}), (\gamma^\nu, \sigma_{\mu\nu}), \\ & (\sigma_{\mu\nu}, \gamma^\nu), (\sigma_{\mu\nu}\gamma_5, \gamma^\nu\gamma_5). \end{aligned} \tag{10}$$

The fields formed by these (Γ_1, Γ_2) pairs are labeled by the subscript $i = (1, \ldots, 8)$ with the ordering of Eq. (10).

1. Flavor $\bar{\mathbf{3}}_F$ baryons

For flavor $\bar{\mathbf{3}}_F$ Rarita-Schwinger baryon fields, there are four apparently nonzero ones,

$$\begin{aligned} B^G_{\bar{\mathbf{3}},1\mu} &= \epsilon_{abc} \epsilon^{ABG} (q^{aT}_A C q^b_B) \gamma_\mu c^c, \\ B^G_{\bar{\mathbf{3}},2\mu} &= \epsilon_{abc} \epsilon^{ABG} (q^{aT}_A C \gamma_5 q^b_B) \gamma_\mu \gamma_5 c^c, \\ B^G_{\bar{\mathbf{3}},3\mu} &= \epsilon_{abc} \epsilon^{ABG} (q^{aT}_A C \gamma_\mu \gamma_5 q^b_B) \gamma_5 c^c, \\ B^G_{\bar{\mathbf{3}},4\mu} &= \epsilon_{abc} \epsilon^{ABG} (q^{aT}_A C \gamma^\nu \gamma_5 q^b_B) \sigma_{\mu\nu} \gamma_5 c^c, \end{aligned} \tag{11}$$

so there are altogether four independent fields in this case. Among these four freedoms, three can be related to the previous Dirac fields, and the only new field can be obtained by acting on the projection operator

$$P^{3/2}_{\mu\nu} = \left(g_{\mu\nu} - \frac{1}{4} \gamma_\mu \gamma_\nu \right) \tag{12}$$

on $B^G_{\bar{\mathbf{3}},3\mu}$ (or $B^G_{\bar{\mathbf{3}},4\mu}$),

$$\begin{aligned} B^G_{\bar{\mathbf{3}},\mu} &= P^{3/2}_{\mu\nu} B^G_{\bar{\mathbf{3}},3\nu} \\ &= \left(g_{\mu\nu} - \frac{1}{4} \gamma_\mu \gamma_\nu \right) \epsilon_{abc} \epsilon^{ABG} (q^{aT}_A C \gamma_\nu \gamma_5 q^b_B) \gamma_5 c^c \\ &= B^G_{\bar{\mathbf{3}},3\mu} + \frac{1}{4} \gamma_\mu \gamma_5 B^G_{\bar{\mathbf{3}},3}. \end{aligned} \tag{13}$$

2. Flavor $\mathbf{6}_F$ baryons

For flavor $\mathbf{6}_F$ Rarita-Schwinger baryon fields, we have four potentially nonzero ones,

$$\begin{aligned} B^U_{\mathbf{6},5\mu} &= \epsilon_{abc} S^U_{AB} (q^{aT}_A C \gamma_\mu q^b_B) c^c, \\ B^U_{\mathbf{6},6\mu} &= \epsilon_{abc} S^U_{AB} (q^{aT}_A C \gamma^\nu q^b_B) \sigma_{\mu\nu} c^c, \\ B^U_{\mathbf{6},7\mu} &= \epsilon_{abc} S^U_{AB} (q^{aT}_A C \sigma_{\mu\nu} q^b_B) \gamma^\nu c^c, \\ B^U_{\mathbf{6},8\mu} &= \epsilon_{abc} S^U_{AB} (q^{aT}_A C \sigma_{\mu\nu} \gamma_5 q^b_B) \gamma^\nu \gamma_5 c^c, \end{aligned} \tag{14}$$

so there are altogether four independent fields in this case. Among these four degrees of freedom, two can be related to the previous Dirac fields. The two new fields can be obtained using the projection operator

$$\begin{aligned} B^U_{\mathbf{6},\mu} &= P^{3/2}_{\mu\nu} B^U_{\mathbf{6},5\nu} \\ &= \left(g_{\mu\nu} - \frac{1}{4} \gamma_\mu \gamma_\nu \right) \epsilon_{abc} S^U_{AB} (q^{aT}_A C \gamma_\nu q^b_B) c^c \\ &= B^U_{\mathbf{6},5\mu} + \frac{1}{4} \gamma_\mu \gamma_5 B^U_{\mathbf{6},4}, \end{aligned} \tag{15}$$

$$
\begin{aligned}
B'^{U}_{\mathbf{6},\mu} &= P^{3/2}_{\mu\nu}(B^{U}_{\mathbf{6},7\nu} + B^{U}_{\mathbf{6},8\nu}) \\
&= \left(g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu\right)(B^{U}_{\mathbf{6},7\nu} + B^{U}_{\mathbf{6},8\nu}) \\
&= B^{U}_{\mathbf{6},7\nu} + B^{U}_{\mathbf{6},8\nu} + \frac{i}{2}\gamma_\mu\gamma_5 B^{U}_{\mathbf{6},5}. \qquad (16)
\end{aligned}
$$

C. Wigner-Bargmann (tensor) fields with spin 3/2

In this subsection, we study the antisymmetric Wigner-Bargmann (tensor) baryon fields $J_{\mu\nu}$ with $J_{\mu\nu} = -J_{\nu\mu}$ [23]. For the tensor fields, we can form 9 three-quark fields, where the possible pairs of Γ_1 and Γ_2 are

$$
\begin{aligned}
(\Gamma_1, \Gamma_2) = &(\gamma_\mu, \gamma_\nu\gamma_5) - \mu \leftrightarrow \nu, (\gamma_\mu\gamma_5, \gamma_\nu) - \mu \leftrightarrow \nu, \\
&\epsilon_{\mu\nu\rho\sigma}(\gamma^\rho, \gamma^\sigma), \epsilon_{\mu\nu\rho\sigma}(\gamma^\rho\gamma_5, \gamma^\sigma\gamma_5), \\
&(\mathbf{1}, \sigma_{\mu\nu}\gamma_5), (\gamma_5, \sigma_{\mu\nu}), (\sigma_{\mu\nu}, \gamma_5), \\
&(\sigma_{\mu\nu}\gamma_5, \mathbf{1}), \epsilon_{\mu\nu\rho\sigma}(\sigma_{\rho l}, \sigma_{\sigma l}). \qquad (17)
\end{aligned}
$$

The fields formed by these (Γ_1, Γ_2) pairs are labeled by the subscript $i = (1, \ldots, 9)$ with the ordering of Eq. (17).

1. Flavor $\bar{\mathbf{3}}_F$ baryons

The flavor $\bar{\mathbf{3}}_F$ tensor baryon fields have the following four potentially nonzero interpolators among nine fields:

$$
\begin{aligned}
B^{G}_{\bar{\mathbf{3}},2\mu\nu} &= \epsilon_{abc}\epsilon^{ABG}(q_A^{aT}C\gamma_\mu\gamma_5 q_B^b)\gamma_\nu c^c - (\mu \leftrightarrow \nu), \\
B^{G}_{\bar{\mathbf{3}},4\mu\nu} &= \epsilon_{abc}\epsilon^{ABG}\epsilon_{\mu\nu\rho\sigma}(q_A^{aT}C\gamma_\rho\gamma_5 q_B^b)\gamma_\sigma\gamma_5 c^c, \\
B^{G}_{\bar{\mathbf{3}},5\mu\nu} &= \epsilon_{abc}\epsilon^{ABG}(q_A^{aT}Cq_B^b)\sigma_{\mu\nu}\gamma_5 c^c, \\
B^{G}_{\bar{\mathbf{3}},6\mu\nu} &= \epsilon_{abc}\epsilon^{ABG}(q_A^{aT}C\gamma_5 q_B^b)\sigma_{\mu\nu} c^c. \qquad (18)
\end{aligned}
$$

Therefore, there are altogether four independent fields in this case. However, all of them can be related to the previously found Dirac and Rarita-Schwinger fields.

2. Flavor $\mathbf{6}_F$ baryons

The flavor $\mathbf{6}_F$ tensor baryon fields have the following five potentially nonzero interpolators:

$$
\begin{aligned}
B^{U}_{\mathbf{6},1\mu\nu} &= \epsilon_{abc}S^{U}_{AB}(q_A^{aT}C\gamma_\mu q_B^b)\gamma_\nu\gamma_5 c^c - (\mu \leftrightarrow \nu), \\
B^{U}_{\mathbf{6},3\mu\nu} &= \epsilon_{abc}S^{U}_{AB}\epsilon_{\mu\nu\rho\sigma}(q_A^{aT}C\gamma_\rho q_B^b)\gamma_\sigma c^c, \\
B^{U}_{\mathbf{6},7\mu\nu} &= \epsilon_{abc}S^{U}_{AB}(q_A^{aT}C\sigma_{\mu\nu} q_B^b)\gamma_5 c^c, \\
B^{U}_{\mathbf{6},8\mu\nu} &= \epsilon_{abc}S^{U}_{AB}(q_A^{aT}C\sigma_{\mu\nu}\gamma_5 q_B^b) c^c, \\
B^{U}_{\mathbf{6},9\mu\nu} &= \epsilon_{abc}S^{U}_{AB}\epsilon_{\mu\nu\rho\sigma}(q_A^{aT}C\sigma_{\rho l} q_B^b)\sigma_{\sigma l} c^c. \qquad (19)
\end{aligned}
$$

Therefore, there are altogether five independent fields in this case. Among these five freedoms, four can be related to the previously found Dirac and Rarita-Schwinger fields. The only new field can be obtained by acting the projection operator

$$
\Gamma^{\mu\nu\alpha\beta} = \left(g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}g^{\nu\beta}\gamma^\mu\gamma^\alpha + \frac{1}{2}g^{\mu\beta}\gamma^\nu\gamma^\alpha + \frac{1}{6}\sigma^{\mu\nu}\sigma^{\alpha\beta}\right),
$$

on $B^{U}_{\mathbf{6},7\mu\nu} + B^{U}_{\mathbf{6},8\mu\nu}$,

$$
\begin{aligned}
B^{U}_{\mathbf{6},\mu\nu} &= \Gamma^{\mu\nu\alpha\beta}(B^{U}_{\mathbf{6},7\alpha\beta} + B^{U}_{\mathbf{6},8\alpha\beta}) \\
&= B^{U}_{\mathbf{6},7\mu\nu} + B^{U}_{\mathbf{6},8\mu\nu} - \frac{1}{2}\gamma_\mu\gamma_5(B^{U}_{\mathbf{6},7\nu} + B^{U}_{\mathbf{6},8\nu}) \\
&\quad + \frac{1}{2}\gamma_\nu\gamma_5(B^{U}_{\mathbf{6},7\mu} + B^{U}_{\mathbf{6},8\mu}) + \frac{1}{3}\sigma^{\mu\nu}B^{U}_{\mathbf{6},5}. \qquad (20)
\end{aligned}
$$

D. Chiral transformation properties

Summarizing the above three subsections, we obtain four independent flavor $\bar{\mathbf{3}}_F$ baryon fields and five independent flavor $\mathbf{6}_F$ ones,

$$
\begin{aligned}
&B^{G}_{\bar{\mathbf{3}},1}, B^{G}_{\bar{\mathbf{3}},2}, B^{G}_{\bar{\mathbf{3}},3}, B^{G}_{\bar{\mathbf{3}},\mu}, \\
&B^{U}_{\mathbf{6},5}, B^{U}_{\mathbf{6},4}, B^{U}_{\mathbf{6},\mu}, B'^{U}_{\mathbf{6},\mu}, B^{U}_{\mathbf{6},\mu\nu}. \qquad (21)
\end{aligned}
$$

In this subsection, we study their chiral transformation properties, where the chiral transformation of quarks is given by the following equations:

$$
\begin{aligned}
U(1)_V &: q \to \exp\left(i\frac{\lambda^0}{2}a_0\right)q = q + \delta q, \\
SU(3)_V &: q \to \exp\left(i\frac{\vec{\lambda}}{2}\cdot\vec{a}\right)q = q + \delta^{\vec{a}}q, \\
U(1)_A &: q \to \exp\left(i\gamma_5\frac{\lambda^0}{2}b_0\right)q = q + \delta_5 q, \\
SU(3)_A &: q \to \exp\left(i\gamma_5\frac{\vec{\lambda}}{2}\cdot\vec{b}\right)q = q + \delta_5^{\vec{b}}q. \qquad (22)
\end{aligned}
$$

Here $\lambda^0 = \sqrt{2/3}\,\mathbf{1}_{3\times3}$ and $\vec{\lambda}$ are the eight Gell-Mann matrices; a^0 is an infinitesimal parameter for the $U(1)_V$ transformation, $\vec{a}$ the octet of $SU(3)_V$ group parameters, b^0 an infinitesimal parameter for the $U(1)_A$ transformation, and $\vec{b}$ the octet of the chiral transformations.

Following Ref. [30], we can extract the chiral transformation properties of $B^{G}_{\bar{\mathbf{3}},1}$ and $B^{G}_{\bar{\mathbf{3}},2}$ to be

$$
\begin{aligned}
\delta_5^{\vec{b}}(B^{G}_{\bar{\mathbf{3}},1} - B^{G}_{\bar{\mathbf{3}},2}) &= \frac{i}{2}\gamma_5 b^N\lambda^N_{FG}(B^{F}_{\bar{\mathbf{3}},1} - B^{F}_{\bar{\mathbf{3}},2}), \\
\delta_5^{\vec{b}}(B^{G}_{\bar{\mathbf{3}},1} + B^{G}_{\bar{\mathbf{3}},2}) &= -\frac{i}{2}\gamma_5 b^N\lambda^N_{FG}(B^{F}_{\bar{\mathbf{3}},1} + B^{F}_{\bar{\mathbf{3}},2}), \qquad (23)
\end{aligned}
$$

those of $B^{G}_{\bar{\mathbf{3}},3}$, $B^{U}_{\mathbf{6},4}$, $B^{G}_{\bar{\mathbf{3}},\mu}$, and $B^{U}_{\mathbf{6},\mu}$ to be

TABLE I. The Abelian $U_A(1)$, the Lorentz group $SO(3,1)$, and the non-Abelian $SU_L(3) \times SU_R(3)$ chiral transformation properties/axial charges of Dirac, Rarita-Schwinger, and Bargmann-Wigner baryon fields with $N_f = 3$. In the sixth column, we show the sign under transposition of the two quarks in the color state. In the last column, we show the symbols $\mathbf{B}^\alpha$ and $\mathbf{H}^\alpha$ used to construct chiral invariant Lagrangians.

	$U_A(1)$	$SO(3,1)$	$SU_L(3) \times SU_R(3)$	$SU(3)_F$	$SU(3)_C$	Symbol
$B^G_{\bar{\mathbf{3}},1} - B^G_{\bar{\mathbf{3}},2}$	−2	$(\frac{1}{2},0) \oplus (0,\frac{1}{2})$	$(\bar{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{1},\bar{\mathbf{3}})$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{B}^3$
$B^G_{\bar{\mathbf{3}},1} + B^G_{\bar{\mathbf{3}},2}$	+2		$(\mathbf{1},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{1})$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{B}^3_{[\mathrm{mir}]}$
$B^G_{\bar{\mathbf{3}},3}$	0		$(\mathbf{3},\mathbf{3})_{[\mathrm{mir}]}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{B}^9_{[\mathrm{mir}]}$
$B^U_{\mathbf{6},4}$	0			$\mathbf{6}$	$\mathbf{1}$	
$B^U_{\mathbf{6},5}$	+2		$(\mathbf{1},\mathbf{6}) \oplus (\mathbf{6},\mathbf{1})$	$\mathbf{6}$	$\mathbf{1}$	$\mathbf{B}^6_{[\mathrm{mir}]}$
$B^G_{\bar{\mathbf{3}},\mu}$	0	$(1,\frac{1}{2}) \oplus (\frac{1}{2},1)$	$(\mathbf{3},\mathbf{3})$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{B}^9$
$B^U_{\mathbf{6},\mu}$	0			$\mathbf{6}$	$\mathbf{1}$	
$B'^U_{\mathbf{6},\mu}$	−2		$(\mathbf{6},\mathbf{1}) \oplus (\mathbf{1},\mathbf{6})$	$\mathbf{6}$	$\mathbf{1}$	$\mathbf{B}^6$
$B^U_{\mathbf{6},\mu\nu}$	+2	$(\frac{3}{2},0) \oplus (0,\frac{3}{2})$	$(\mathbf{1},\mathbf{6}) \oplus (\mathbf{6},\mathbf{1})$	$\mathbf{6}$	$\mathbf{1}$	$\mathbf{B}^6_{[\mathrm{mir}]}$

$$\delta_5^{\vec{b}} B^G_{\bar{\mathbf{3}},3} = -i\gamma_5 b^N \mathbf{T}^N_{GU} B^U_{\mathbf{6},4},$$
$$\delta_5^{\vec{b}} B^U_{\mathbf{6},4} = -\frac{i}{2}\gamma_5 b^N \mathbf{T}^{\dagger N}_{UG} B^G_{\bar{\mathbf{3}},3},$$
$$\delta_5^{\vec{b}} B^G_{\bar{\mathbf{3}},\mu} = i\gamma_5 b^N \mathbf{T}^N_{GU} B^U_{\mathbf{6},\mu},$$
$$\delta_5^{\vec{b}} B^U_{\mathbf{6},\mu} = \frac{i}{2}\gamma_5 b^N \mathbf{T}^{\dagger N}_{UG} B^G_{\bar{\mathbf{3}},\mu}, \tag{24}$$

and those of $B^U_{\mathbf{6},5}$, $B'^U_{\mathbf{6},\mu}$, and $B^U_{\mathbf{6},\mu\nu}$ to be

$$\delta_5^{\vec{b}} B^U_{\mathbf{6},5} = i\gamma_5 b^N \mathbf{F}^N_{\mathrm{UV}} B^V_{\mathbf{6},5},$$
$$\delta_5^{\vec{b}} B'^U_{\mathbf{6},\mu} = -i\gamma_5 b^N \mathbf{F}^N_{\mathrm{UV}} B'^V_{\mathbf{6},\mu},$$
$$\delta_5^{\vec{b}} B^U_{\mathbf{6},\mu\nu} = i\gamma_5 b^N \mathbf{F}^N_{\mathrm{UV}} B^V_{\mathbf{6},\mu\nu}, \tag{25}$$

where the matrices $\mathbf{T}^N$ and $\mathbf{F}^N$ are given in the Appendix.

Altogether we obtain one $(\bar{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{1},\bar{\mathbf{3}})$, one $(\mathbf{1},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{1})$, two (3,3), one $(\mathbf{6},\mathbf{1}) \oplus (\mathbf{1},\mathbf{6})$, and two $(\mathbf{1},\mathbf{6}) \oplus (\mathbf{6},\mathbf{1})$ chiral multiplets,

$$B^G_{\bar{\mathbf{3}},1} - B^G_{\bar{\mathbf{3}},2} \in (\bar{\mathbf{3}},\mathbf{1}) \oplus (\mathbf{1},\bar{\mathbf{3}}),$$
$$B^G_{\bar{\mathbf{3}},1} + B^G_{\bar{\mathbf{3}},2} \in (\mathbf{1},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{1}),$$
$$(B^G_{\bar{\mathbf{3}},3}, B^U_{\mathbf{6},4}) \in (\mathbf{3},\mathbf{3})_{[\mathrm{mir}]},$$
$$B^U_{\mathbf{6},5} \in (\mathbf{1},\mathbf{6}) \oplus (\mathbf{6},\mathbf{1}),$$
$$(B^G_{\bar{\mathbf{3}},\mu}, B^U_{\mathbf{6},\mu}) \in (\mathbf{3},\mathbf{3}),$$
$$B'^U_{\mathbf{6},\mu} \in (\mathbf{6},\mathbf{1}) \oplus (\mathbf{1},\mathbf{6}),$$
$$B^U_{\mathbf{6},\mu\nu} \in (\mathbf{1},\mathbf{6}) \oplus (\mathbf{6},\mathbf{1}). \tag{26}$$

TABLE II. The Abelian $U_A(1)$, the Lorentz group $SO(3,1)$, and the non-Abelian $SU_L(2) \times SU_R(2)$ chiral transformation properties/axial charges of Dirac, Rarita-Schwinger, and Bargmann-Wigner baryon fields with $N_f = 2$.

	$U_A(1)$	$SO(3,1)$	$SU_L(2) \times SU_R(2)$	$SU(2)_F$	$SU(3)_C$	Symbol
$H^G_{\mathbf{0},1} - H^G_{\mathbf{0},2}$	−2	$(\frac{1}{2},0) \oplus (0,\frac{1}{2})$	$(\mathbf{0},\mathbf{0})$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{H}^1$
$H^G_{\mathbf{0},1} + H^G_{\mathbf{0},2}$	+2		$(\mathbf{0},\mathbf{0})$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{H}^1$
$H^G_{\mathbf{0},3}$	0		$(\frac{\mathbf{1}}{\mathbf{2}},\frac{\mathbf{1}}{\mathbf{2}})_{[\mathrm{mir}]}$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{H}^4_{[\mathrm{mir}]}$
$H^U_{\mathbf{1},4}$	0			$\mathbf{1}$	$\mathbf{1}$	
$H^U_{\mathbf{1},5}$	+2		$(\mathbf{0},\mathbf{1}) \oplus (\mathbf{1},\mathbf{0})$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{H}^3_{[\mathrm{mir}]}$
$H^G_{\mathbf{0},\mu}$	0	$(1,\frac{1}{2}) \oplus (\frac{1}{2},1)$	$(\frac{\mathbf{1}}{\mathbf{2}},\frac{\mathbf{1}}{\mathbf{2}})$	$\mathbf{0}$	$\mathbf{1}$	$\mathbf{H}^4$
$H^U_{\mathbf{1},\mu}$	0			$\mathbf{1}$	$\mathbf{1}$	
$H'^U_{\mathbf{1},\mu}$	−2		$(\mathbf{1},\mathbf{0}) \oplus (\mathbf{0},\mathbf{1})$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{H}^3$
$H^U_{\mathbf{1},\mu\nu}$	+2	$(\frac{3}{2},0) \oplus (0,\frac{3}{2})$	$(\mathbf{0},\mathbf{1}) \oplus (\mathbf{1},\mathbf{0})$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{H}^3_{[\mathrm{mir}]}$

Note that once the chiral representation of the meson field $M^b = \sigma^b + i\gamma_5\pi^b$ is fixed as $(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})$, the chiral representations of the above baryon fields are also fixed, i.e., either as direct or as the mirror.

We summarize their chiral properties in Table I, which are primarily determined by the chiral properties of the light diquarks involved, but also by the Dirac structure of the whole baryon field. Of course, they also depend on the number of light flavor N_f. Table I is for the $N_f = 3$ case. For completeness, we also show the $N_f = 2$ case in Table II, where the symbol $H^{G/U}_{SU(2)_F,i(\mu\nu)}$ denotes the corresponding baryon fields in the $N_f = 2$ case. A significant difference is that both $H^G_{\mathbf{0},1} - H^G_{\mathbf{0},2}$ and $H^G_{\mathbf{0},1} + H^G_{\mathbf{0},2}$ are chiral singlets, so their mirror fields are just themselves.

In the next section, we shall use these baryon fields to construct chiral invariant Lagrangians together with pseudoscalar mesons belonging to the $SU(3)_F$ nonet. To do this, only their chiral properties are relevant, so we shall use the symbols $\mathbf{B}^\alpha$ and $\mathbf{H}^\alpha$ to denote the corresponding chiral multiplets, where the "exponent index" α indicates how many components these multiplets contain, as shown in Tables I and II.

III. BARYON-MESON INTERACTIONS

The one-heavy-two-light-quark baryon interpolating fields can be found in Sec. II. Their chiral transformation properties are essentially determined by those of the light diquarks that they contain. It is then no surprise that their interactions are also determined by the diquark-meson interactions.

Yet, there are (many) more baryon fields than there are diquarks, the main difference being the additional Lorentz indices μ, ν that have to be "absorbed" somehow. That is usually by one of three ways: (i) by operating with derivatives on the spinless meson fields; or (ii) by contractions with the free Lorentz indices on the vector and axial-vector meson fields; or (iii) by contractions with free Lorentz index on another baryon field.

Thus, certain differences appear in the interaction Lagrangians that depend on the total spin of the baryon in question: the spin-$\frac{1}{2}$ Dirac fields couple differently among themselves from the corresponding coupling to the spin-$\frac{3}{2}$ Rarita-Schwinger and the Bargmann-Wigner fields. Manifestly, a large number of off-diagonal terms can be constructed with ease, so we shall not dwell on that issue, but shall keep it in the back of our mind.

A. Baryon-meson interactions with $N_f = 3$

We use $M^b = \sigma^b + i\gamma_5\pi^b$ to denote the scalar σ^b and pseudoscalar π^b meson field nonets belonging to the $(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})$ chiral multiplet, and $M^{\dagger b} = \sigma^b - i\gamma_5\pi^b$ defining its mirror conjugate,

$$\delta_5^{\vec{b}}(\sigma^b + i\gamma_5\pi^b) = -i\gamma_5 b^a d_{abc}(\sigma^c + i\gamma_5\pi^c),$$
$$\delta_5^{\vec{b}}(\sigma^b - i\gamma_5\pi^b) = i\gamma_5 b^a d_{abc}(\sigma^c - i\gamma_5\pi^c). \tag{27}$$

The chirally invariant interactions are listed in Table III. There are altogether five distinct terms (as well as five mirror ones),

$$\mathcal{L}_1 = g_1 \bar{\mathbf{B}}^3 \sum_{a=0}^{8} M^a \mathbf{G}_1^a \mathbf{B}^3, \tag{28}$$

$$\mathcal{L}_2 = g_2 \bar{\mathbf{B}}^3_{[\mathrm{mir}]} \sum_{a=0}^{8} M^a \mathbf{G}_2^a \mathbf{B}^9, \tag{29}$$

$$\mathcal{L}_3 = g_3 \bar{\mathbf{B}}^3 \sum_{a=0}^{8} M^{\dagger a} \mathbf{G}_3^a \mathbf{B}^9, \tag{30}$$

$$\mathcal{L}_4 = g_4 \bar{\mathbf{B}}^6_{[\mathrm{mir}]} \sum_{a=0}^{8} M^a \mathbf{G}_4^a \mathbf{B}^9, \tag{31}$$

$$\mathcal{L}_5 = g_5 \bar{\mathbf{B}}^6 \sum_{a=0}^{8} M^{\dagger a} \mathbf{G}_5^a \mathbf{B}^9, \tag{32}$$

where the matrices $\mathbf{G}^a_{1,2,3,4,5}$ with $a = 0\ldots8$ are as follows:

$$\mathbf{G}_1^0 = (\lambda^0)^T = \sqrt{\frac{2}{3}}\mathbf{1}_{3\times3}, \qquad \mathbf{G}_1^N = (\lambda^N)^T,$$
$$\mathbf{G}_2^0 = \left(-\sqrt{\frac{8}{3}}\mathbf{1}_{3\times3}, \mathbf{0}_{3\times6}\right), \qquad \mathbf{G}_2^N = ((\lambda^N)^T, -\sqrt{2}\mathbf{T}^N),$$
$$\mathbf{G}_3^0 = \left(-\sqrt{\frac{8}{3}}\mathbf{1}_{3\times3}, \mathbf{0}_{3\times6}\right), \qquad \mathbf{G}_3^N = ((\lambda^N)^T, \sqrt{2}\mathbf{T}^N),$$
$$\mathbf{G}_4^0 = \left(\mathbf{0}_{6\times3}, \frac{2}{\sqrt{3}}\mathbf{1}_{6\times6}\right), \qquad \mathbf{G}_4^N = (\mathbf{T}^{\dagger N}, \sqrt{2}\mathbf{F}^N),$$
$$\mathbf{G}_5^0 = \left(\mathbf{0}_{6\times3}, -\frac{2}{\sqrt{3}}\mathbf{1}_{6\times6}\right), \qquad \mathbf{G}_5^N = (\mathbf{T}^{\dagger N}, -\sqrt{2}\mathbf{F}^N).$$

The matrices $\mathbf{T}^N$ and $\mathbf{F}^N$ in the above expressions can be found in the Appendix.

One can see in Table III that there are no chirally invariant interactions between $\bar{\mathbf{B}}^6$ and $\mathbf{B}^6$ baryon fields, but there is a standard mirror-field chiral invariant mass (constant—no meson interaction) term between the $\bar{\mathbf{B}}^6_{[\mathrm{mir}]}$ and $\mathbf{B}^6$ fields. Similar behaviors can be found for the $\bar{\mathbf{B}}^9$, $\mathbf{B}^9$, and $\bar{\mathbf{B}}^9_{[\mathrm{mir}]}$ baryon fields. Only the $\bar{\mathbf{B}}^3$ and $\mathbf{B}^3$ baryon fields allow a diagonal interaction with $\mathrm{M} \in (\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})$ mesons, so all the rest are off-diagonal. Therefore, only the $\mathbf{B}^3$ baryon may acquire an effective mass term from the spontaneous symmetry breaking in the case with three

TABLE III. The non-Abelian ($SU_L(3) \times SU_R(3)$) chiral invariant interactions of baryons $\mathbf{B}$, antibaryons $\bar{\mathbf{B}}$ belonging to various chiral multiplets, and meson fields $\mathrm{M} \in (\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$. $\cdots$ means that there is no chiral invariant interaction, and $\mathbf{1}$ means that the "mirror mass" term $\bar{\mathbf{B}}_{[\mathrm{mir}]}\mathbf{B}$ itself (without interaction with mesons) is chirally invariant. This table is for the $N_f = 3$ case, and there are five more interactions containing $\mathbf{B}^9_{[\mathrm{mir}]}$ and $\bar{\mathbf{B}}^9_{[\mathrm{mir}]}$.

	$\mathrm{B}^3 \in (\bar{\mathbf{3}}, \mathbf{1}) \oplus (\mathbf{1}, \bar{\mathbf{3}})$	$\mathrm{B}^3_{[\mathrm{mir}]} \in (\mathbf{1}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{1})$	$\mathrm{B}^6 \in (\mathbf{6}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{6})$	$\mathrm{B}^6_{[\mathrm{mir}]} \in (\mathbf{1}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{1})$	$\mathrm{B}^9 \in (\mathbf{3}, \mathbf{3})$
$\bar{\mathrm{B}}^3 \in (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{1})$	$\mathrm{M} \in (\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$	$\mathbf{1}$	$\cdots$	$\cdots$	$\mathrm{M}^\dagger \in (\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$
$\bar{\mathrm{B}}^3_{[\mathrm{mir}]} \in (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3})$	$\mathbf{1}$	$\mathrm{M}^\dagger \in (\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	$\cdots$	$\cdots$	$\mathrm{M} \in (\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$
$\bar{\mathrm{B}}^6 \in (\mathbf{1}, \bar{\mathbf{6}}) \oplus (\bar{\mathbf{6}}, \mathbf{1})$	$\cdots$	$\cdots$	$\cdots$	$\mathbf{1}$	$\mathrm{M}^\dagger \in (\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$
$\bar{\mathrm{B}}^6_{[\mathrm{mir}]} \in (\bar{\mathbf{6}}, \mathbf{1}) \oplus (\mathbf{1}, \bar{\mathbf{6}})$	$\cdots$	$\cdots$	$\mathbf{1}$	$\cdots$	$\mathrm{M} \in (\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$
$\bar{\mathrm{B}}^9 \in (\bar{\mathbf{3}}, \bar{\mathbf{3}})$	$\mathrm{M}^\dagger \in (\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	$\mathrm{M} \in (\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$	$\mathrm{M}^\dagger \in (\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	$\mathrm{M} \in (\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$	$\cdots$

flavors. Even this one disappears in the case with two flavors, as we shall see in the next subsection.

We shall see in Sec. III C that the $SU_L(3) \times SU_R(3)$ chirally invariant interactions involving pseudoscalar mesons (π, K, η, η') necessarily violate the $U(1)_A$ symmetry. This fact translates into a statement about the strength of these effective interactions, as all of them have to be related to the η', η masses, and not to the usual pion-quark coupling, as one might expect. However, we shall also see that the $SU_L(2) \times SU_R(2)$ chirally invariant interactions involving pseudoscalar mesons (π, η^*) do not necessarily violate the $U(1)_A$ symmetry, i.e., they may respect it.

B. Baryon-meson interactions with $N_f = 2$

Before we begin, we recall that the $N_f = 2$ scalar-pseudoscalar meson chiral multiplet M^a comes in two varieties, A^a, B^a, defined in Eqs. (24) and (25) of Ref. [31] as

$$A = \sum_{a=0}^{3} A^a \tau^a = \sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}, \tag{33}$$

$$B = \sum_{a=0}^{3} B^a \tau^a = \boldsymbol{\tau} \cdot \boldsymbol{\sigma} + i\gamma_5 \eta. \tag{34}$$

Both chiral multiplets $A \pm B \in (\frac{1}{2}, \frac{1}{2})$, but they transform into each other under $U_A(1)$ chiral transformations,

$$\delta_5(A + B) = -2i\gamma_5 a(A + B), \tag{35}$$

$$\delta_5(A - B) = +2i\gamma_5 a(A - B). \tag{36}$$

Therefore, the $-$ linear combination has a positive $U_A(1)$ chiral charge, whereas the $+$ linear combination has a negative $U_A(1)$ chiral charge, which allows us to build $U_A(1)$ chirally conserving and breaking interactions with equal ease.

The two-flavor baryon-meson interactions have the same form as the three-quark interactions in Eqs. (29)–(32), but the one corresponding to Eq. (28) does not exist anymore. Thus, there are 4 two flavor analogous of Eqs. (29)–(32) (as well as four mirror ones),

$$\mathcal{L}'_2 = g'_2 \bar{\mathbf{H}}^1 \sum_{a=0}^{3} M^a \mathbf{G}'^a_2 \mathbf{H}^4, \tag{37}$$

$$\mathcal{L}'_3 = g'_3 \bar{\mathbf{H}}^1 \sum_{a=0}^{3} M^{\dagger a} \mathbf{G}'^a_3 \mathbf{H}^4, \tag{38}$$

$$\mathcal{L}'_4 = g'_4 \bar{\mathbf{H}}^3_{[\mathrm{mir}]} \sum_{a=0}^{3} M^a \mathbf{G}'^a_4 \mathbf{H}^4, \tag{39}$$

$$\mathcal{L}'_5 = g'_5 \bar{\mathbf{H}}^3 \sum_{a=0}^{3} M^{\dagger a} \mathbf{G}'^a_5 \mathbf{H}^4, \tag{40}$$

where the matrices $\mathbf{G}'^a_{2,3,4,5}$ with $a = 0\ldots3$ are as follows:

$$\begin{aligned}
\mathbf{G}'^0_2 &= (-\sqrt{6}\mathbf{1}_{1\times1}, \mathbf{0}_{1\times3}), & \mathbf{G}'^N_2 &= (\mathbf{0}_{1\times1}, -\sqrt{2}\mathbf{T}'^N), \\
\mathbf{G}'^0_3 &= (-\sqrt{6}\mathbf{1}_{1\times1}, \mathbf{0}_{1\times3}), & \mathbf{G}'^N_3 &= (\mathbf{0}_{1\times1}, \sqrt{2}\mathbf{T}'^N), \\
\mathbf{G}'^0_4 &= (\mathbf{0}_{3\times1}, \sqrt{3}\mathbf{1}_{3\times3}), & \mathbf{G}'^N_4 &= (\mathbf{T}'^{\dagger N}, \sqrt{2}\mathbf{F}'^N), \\
\mathbf{G}'^0_5 &= (\mathbf{0}_{3\times1}, -\sqrt{3}\mathbf{1}_{3\times3}), & \mathbf{G}'^N_5 &= (\mathbf{T}'^{\dagger N}, -\sqrt{2}\mathbf{F}'^N).
\end{aligned}$$

The matrices $\mathbf{T}'^N$ and $\mathbf{F}'^N$ in the above expressions can be found in the Appendix.

Besides, there are other two baryon-meson interactions in the $N_f = 2$ case (as well as two mirror ones),

$$\mathcal{L}'_6 = g'_6 \bar{\mathbf{H}}^3_{[\mathrm{mir}]} \sum_{a=0}^{3} M^{\dagger a} \mathbf{G}'^a_6 \mathbf{H}^4, \tag{41}$$

$$\mathcal{L}'_7 = g'_7 \bar{\mathbf{H}}^3 \sum_{a=0}^{3} M^a \mathbf{G}'^a_7 \mathbf{H}^4, \tag{42}$$

TABLE IV. The non-Abelian ($SU_L(2)\times SU_R(2)$) chiral invariant interactions of baryons $\mathbf{H}$, antibaryons $\bar{\mathbf{H}}$ belonging to various chiral multiplets, and meson fields $\mathrm{M}\in(\frac{1}{2},\frac{1}{2})$. This table is for the $N_f=2$ case, and there are six more interactions containing $\mathbf{H}^4_{[\mathrm{mir}]}$ and $\bar{\mathbf{H}}^4_{[\mathrm{mir}]}$.

	$\mathbf{H}^1\in(\mathbf{0},\mathbf{0})$	$\mathbf{H}^3\in(\mathbf{1},\mathbf{0})\oplus(\mathbf{0},\mathbf{1})$	$\mathbf{H}^3_{[\mathrm{mir}]}\in(\mathbf{0},\mathbf{1})\oplus(\mathbf{1},\mathbf{0})$	$\mathbf{H}^4\in(\frac{1}{2},\frac{1}{2})$
$\bar{\mathbf{H}}^1\in(\mathbf{0},\mathbf{0})$	**1**	⋯	⋯	$\mathrm{M}\in(\frac{1}{2},\frac{1}{2})/\mathrm{M}^\dagger\in(\frac{1}{2},\frac{1}{2})$
$\bar{\mathbf{H}}^3\in(\mathbf{0},\mathbf{1})\oplus(\mathbf{1},\mathbf{0})$	⋯	⋯	**1**	$\mathrm{M}\in(\frac{1}{2},\frac{1}{2})/\mathrm{M}^\dagger\in(\frac{1}{2},\frac{1}{2})$
$\bar{\mathbf{H}}^3_{[\mathrm{mir}]}\in(\mathbf{1},\mathbf{0})\oplus(\mathbf{0},\mathbf{1})$	⋯	**1**	⋯	$\mathrm{M}\in(\frac{1}{2},\frac{1}{2})/\mathrm{M}^\dagger\in(\frac{1}{2},\frac{1}{2})$
$\bar{\mathbf{H}}^4\in(\frac{1}{2},\frac{1}{2})$	$\mathrm{M}\in(\frac{1}{2},\frac{1}{2})/\mathrm{M}^\dagger\in(\frac{1}{2},\frac{1}{2})$	$\mathrm{M}\in(\frac{1}{2},\frac{1}{2})/\mathrm{M}^\dagger\in(\frac{1}{2},\frac{1}{2})$	$\mathrm{M}\in(\frac{1}{2},\frac{1}{2})/\mathrm{M}^\dagger\in(\frac{1}{2},\frac{1}{2})$	⋯

where

$$\mathbf{G}_6^{\prime 0}=(\mathbf{0}_{3\times1},-\sqrt{3}\,\mathbf{1}_{3\times3}),\qquad \mathbf{G}_6^{\prime N}=(\mathbf{T}^{\prime\dagger N},\sqrt{2}\,\mathbf{F}^{\prime N}),$$
$$\mathbf{G}_7^{\prime 0}=(\mathbf{0}_{3\times1},\sqrt{3}\,\mathbf{1}_{3\times3}),\qquad \mathbf{G}_7^{\prime N}=(\mathbf{T}^{\prime\dagger N},-\sqrt{2}\,\mathbf{F}^{\prime N}).$$

Note that the above forms are the same for both ground-state and excited diquarks with identical chiral transformation properties, as only the chiral properties count here, which do not depend on spatial variables.

C. $U_A(1)$ chiral symmetry

The $N_f=3$ chiral-invariant diagonal interaction (in Table III) of the $\mathbf{B}^3\in(\bar{\mathbf{3}},\mathbf{1})\oplus(\mathbf{1},\bar{\mathbf{3}})$ and $\bar{\mathbf{B}}^3\in(\mathbf{1},\mathbf{3})\oplus(\mathbf{3},\mathbf{1})$ multiplets violates the Abelian $U_A(1)$ chiral symmetry. Recall that the scalar and pseudoscalar meson fields M and $M^\dagger$ carry $g_A^{(0)}=-2$ and $+2$, respectively. Therefore, by using the result of Table III, we have the net $U_A(1)$ charge as

$$g_A^{(0)}=+2-2+2=2\neq0, \tag{43}$$

for the diagonal interaction Eq. (28). The off-diagonal interactions (29)–(32), on the other hand, are $U_A(1)$ invariant, because they involve the $\mathbf{B}^9\in(\mathbf{3},\mathbf{3})$, which carries $g_A^{(0)}=0$ axial baryon number.

As all of these effective diquark-meson interactions are six-quark/antiquark operators, one might conclude that all of them must be related to the $U_A(1)$ symmetry-breaking 't Hooft instanton-induced interaction,

$$\mathcal{L}_{\mathrm{tH}}^{(6)}=K[\det{}_f(\bar{\psi}(1+\gamma_5)\psi)+\det{}_f(\bar{\psi}(1-\gamma_5)\psi)], \tag{44}$$

by means of crossing and Fierz relations, as Eq. (44) is the only $SU_L(3)\times SU_R(3)$ chirally invariant and Lorentz-invariant parity-conserving six-quark/antiquark operator. Therefore, all such terms ought to be $U_A(1)$ symmetry breaking themselves.

That conclusion would be too hasty, however, as the $(\mathbf{3},\mathbf{3})$-multiplet diquarks are Lorentz vector (or axial-vector) mesons, which means that no Lorentz-invariant interaction with Lorentz pseudoscalar (or scalar) mesons and diquark is allowed. Rather, the said interaction is a Lorentz vector (or axial-vector) with one free Lorentz index that must be contracted with a γ_μ matrix. Consequently, such interactions are not Fierz related to the 't Hooft interaction and preserve $U_A(1)$ symmetry. Note, moreover, that all $SU_L(3)\times SU_R(3)$- and $U_A(1)$-conserving interactions are off-diagonal.

In the case of $N_f=2$, as discussed above in Sec. II D, both $H^G_{\mathbf{0},1}-H^G_{\mathbf{0},2}$ and $H^G_{\mathbf{0},1}+H^G_{\mathbf{0},2}$ are chiral singlets, so they are $SU_L(2)\times SU_R(2)$ chiral invariant. However, they are not $U_A(1)$ chiral invariant, as shown in Table II. Hence, one can see in Table IV that for each interaction term containing $\mathbf{H}^1$ without Abelian $U_A(1)$ chiral symmetry breaking, there is one interaction term with Abelian $U_A(1)$ chiral symmetry breaking, i.e., interactions given in Eqs. (37) and (38) as well as their mirror ones. Plainly put, the non-Abelian chiral symmetry does not restrict the Abelian $U_A(1)$ chiral symmetry breaking, in the $N_f=2$ case, in contrast to the $N_f=3$ case. This is in agreement with the conclusions of Refs. [14,31].

D. Lorentz structure

So far, we have not included the Lorentz indices in the interaction Lagrangians, which we shall do that next. As shown in Table II, the baryon fields have Lorentz group indices associated with them: (i) the Dirac fields have only the Dirac index (ranging from 1 to 4); (ii) the Rarita-Schwinger fields have one Dirac index and a Lorentz one μ; and (iii) the Bargmann-Wigner fields have two antisymmetric Lorentz indices μ, ν. Therefore, the interaction Lagrangians as shown in Eqs. (28)–(32) and Eqs. (37)–(42) may have one or more unsaturated Lorentz indices.

One way to saturation of Lorentz indices is to have a derivative four-vector ∂_μ acting only on the scalar-pseudoscalar meson fields, but never on the Lorentz indices of baryon fields. In this way, one avoids having to apply the auxiliary conditions $\partial_\mu\mathbf{B}^\mu=0$ for the Rarita-Schwinger and Wigner-Bargmann fields.

Thus, a new set of derivative-coupled interaction Lagrangians appears even in the linear realization of chiral symmetry,

$$
\begin{aligned}
\mathcal{L}_1^{\rm I} &= g_1^{\rm I}\bar{\mathbf{B}}^3(\partial^\mu M)^a\mathbf{G}_1^a\gamma_\mu\gamma_5\mathbf{B}^3,\\
\mathcal{L}_1^{\rm II} &= g_1^{\rm II}\bar{\mathbf{B}}^3_{[\rm mir]}(\partial^\mu M)^{\dagger a}\mathbf{G}_1^a\gamma_\mu\gamma_5\mathbf{B}^3_{[\rm mir]},\\
\mathcal{L}_2^{\rm I} &= g_2^{\rm I}\bar{\mathbf{B}}^3_{[\rm mir]}(\partial^\mu M)^a\mathbf{G}_2^a\mathbf{B}^9_\mu,\\
\mathcal{L}_2^{\rm II} &= g_2^{\rm II}\bar{\mathbf{B}}^3(\partial^\mu M)^{\dagger a}\mathbf{G}_2^a\gamma_\mu\gamma_5\mathbf{B}^9_{[\rm mir]},\\
\mathcal{L}_3^{\rm I} &= g_3^{\rm I}\bar{\mathbf{B}}^3(\partial^\mu M)^{\dagger a}\mathbf{G}_3^a\mathbf{B}^9_\mu,\\
\mathcal{L}_3^{\rm II} &= g_3^{\rm II}\bar{\mathbf{B}}^3_{[\rm mir]}(\partial^\mu M)^a\mathbf{G}_3^a\gamma_\mu\gamma_5\mathbf{B}^9_{[\rm mir]},\\
\mathcal{L}_4^{\rm I} &= g_4^{\rm I}\bar{\mathbf{B}}^6_{[\rm mir]}(\partial^\mu M)^a\mathbf{G}_4^a\mathbf{B}^9_\mu,\\
\mathcal{L}_4^{\rm II} &= g_4^{\rm II}\bar{\mathbf{B}}^6_{[\rm mir],\mu\nu}(\partial^\mu M)^a\mathbf{G}_4^a\mathbf{B}^{9,\nu},\\
\mathcal{L}_4^{\rm III} &= g_4^{\rm III}\bar{\mathbf{B}}^6_\mu(\partial^\mu M)^{\dagger a}\mathbf{G}_4^a\mathbf{B}^9_{[\rm mir]},\\
\mathcal{L}_5^{\rm I} &= g_5^{\rm I}\bar{\mathbf{B}}^6_\mu(\partial^\nu M)^{\dagger a}\mathbf{G}_5^a\gamma_\nu\gamma_5\mathbf{B}^{9,\mu},\\
\mathcal{L}_5^{\rm II} &= g_5^{\rm II}\bar{\mathbf{B}}^6_{[\rm mir]}(\partial^\mu M)^a\mathbf{G}_5^a\gamma_\mu\gamma_5\mathbf{B}^9_{[\rm mir]},
\end{aligned}
\tag{45}
$$

where we have explicitly added the Lorentz indices in the symbol $\mathbf{B}^\alpha$ (see Table I),

$$
\begin{aligned}
\mathbf{B}^3 &= B^G_{\bar{\mathbf{3}},1} - B^G_{\bar{\mathbf{3}},2},\\
\mathbf{B}^3_{[\rm mir]} &= B^G_{\bar{\mathbf{3}},1} + B^G_{\bar{\mathbf{3}},2},\\
\mathbf{B}^9_{[\rm mir]} &= (B^G_{\bar{\mathbf{3}},3}, B^U_{\mathbf{6},4}),\\
\mathbf{B}^6_{[\rm mir]} &= B^U_{\mathbf{6},5},\\
\mathbf{B}^9_\mu &= (B^G_{\bar{\mathbf{3}},\mu}, B^U_{\mathbf{6},\mu}),\\
\mathbf{B}^6_\mu &= B'^U_{\mathbf{6},\mu},\\
\mathbf{B}^6_{[\rm mir],\mu\nu} &= B^U_{\mathbf{6},\mu\nu}.
\end{aligned}
\tag{46}
$$

These interactions are particularly interesting because they are the terms that lead to derivative-coupled interactions in the nonlinear realization of chiral symmetry, quite unlike the ordinary nucleon case, where the kinetic energy produces such derivative terms.

E. Lorentz and chiral mixing

First, let us note that the Lorentz group indices (p,k) are not good quantum numbers—rather, only the (total) angular momentum $j \in |p-k|, \ldots, (p+k)$ of a free particle is conserved. Similarly, the left- and right-handed isospins (I_L, I_R) are not good quantum numbers—rather, only the isospin $I \in |I_L - I_R|, \ldots, (I_L + I_R)$ is conserved.

This means that one may have identical spin fields/particles from different representations of the Lorentz group. For example, spin-3/2 baryons can be described by both the Rarita-Schwinger fields $\mathbf{H}_\mu$ and the Bargmann-Wigner ones $\mathbf{H}_{\mu\nu}$. Consequently, physical baryons may be admixtures of these two, depending on the interactions, which ought to preserve both the Lorentz and chiral symmetries. Similarly, the spin-1/2 baryons can be described by both the Dirac fields $\mathbf{H}$ and by the spin-1/2 complement to the Rarita-Schwinger field $\mathbf{H}_\mu$. As these three types of fields (may) have different chiral properties, see Table II, we must expect to have chiral mixing as well. All of this has been known for some time in the case of three-light-quark baryons, as documented in Refs. [32,33]. An analogous study of spin-1/2 and spin-3/2 charmed baryons would demand a (far) deeper knowledge of experimental baryons' pion decay rates and/or axial couplings than presently available.

A general feature of the linear realization of chiral symmetry is that physical baryon states are linear superpositions of bare baryon fields, i.e., there is (chiral) mixing of different chiral multiplets with identical overall quantum numbers, such as the spin J, and flavor SU(3). For example, the baryon fields with the same spin and isospin, e.g., $\mathbf{H}^1$ and $\mathbf{H}^4$, mix as a consequence of spontaneous and explicit chiral symmetry breaking. Unfortunately, the diagonal and off-diagonal mass terms alone are insufficient to determine the mixing angles; see Sec. IV A.

Another way to determine some functions of the mixing angles is to use the hyperon's axial couplings. They are generally unknown, unless one accepts a lattice QCD calculation [34] as experimental input; see Sec. IV B. The knowledge of off-diagonal pion coupling constants, e.g., as in Refs. [32,33], is another possible input, but, at the present time, it is insufficient to fix the mixing parameters.

IV. CHIRAL MIXING AND EXPERIMENT

In order to fix the free parameters of this model, one needs experimental input, just as it was needed in the case of three-light-quark baryons [29–31,35]. There are, however, several important differences: (i) there are fewer (4) chiral multiplets here than in the three-light-quark case (6) and (ii) there are fewer experimental data available here: two masses of $\Lambda_c(2595)$ and $\Lambda_c(2625)$ + (possibly) one axial coupling, than in the three-light-quark case (where we had four masses + two axial couplings [31,32,35]), which are not in "conflict" with each other.

A. Chiral mixing and hyperon masses

The baryon fields with the same spin and isospin, e.g., $\mathbf{H}^1$ and $\mathbf{H}^4$, mix as a consequence of spontaneous and explicit chiral symmetry breaking. Generally speaking, the masses of baryons are determined by the non-Abelian $(SU_L(2) \times SU_R(2))$ chiral invariant interactions, which are shown in Sec. III B.

For every flavor $SU(3)$ multiplet, there are two different "naive" chiral fields, plus two "mirror fields," see Tables I, V, and VI, thus leading to four independent operators, and therefore to (at most) four-operator mixing. Now, a 4×4 orthogonal mixing matrix has $4 \times 3/2 = 6$ independent matrix elements, which, in turn, can be parametrized with six mixing angles. In order to fix these

TABLE V. The Abelian and the non-Abelian axial charges (+ sign indicates naive, − sign mirror transformation properties) of the non-Abelian chiral multiplets containing a flavor $\bar{\mathbf{3}}$-plet.

Case	Field	$g_A^{(0)}$	$g_A^{(\bar{3})}$	$SU_L(3)\times SU_R(3)$
I	$\mathbf{B}^3$	−2	+1	$(\bar{\mathbf{3}},\mathbf{1})\oplus(\mathbf{1},\bar{\mathbf{3}})$
II	$\mathbf{B}^3_{[\mathrm{mir}]}$	+2	−1	$(\mathbf{1},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{1})$
III	$\mathbf{B}^9$	0	+1	$(\mathbf{3},\mathbf{3})$
IV	$\mathbf{B}^9_{[\mathrm{mir}]}$	0	−1	$(\mathbf{3},\mathbf{3})$

mixing angles, one needs, in principle at least, six independent observables, such as masses, axial couplings, etc. One can go through a calculation analogous to the one in Sec. IV B of Ref. [29], but we shall see that, unfortunately, the diagonal and off-diagonal mass terms stemming from our interactions alone are insufficient to determine the mixing angles.

One can see in Table IV that there are no chiral-invariant interactions between $\bar{\mathbf{H}}^4$ and $\mathbf{H}^4$ on one hand, nor between $\bar{\mathbf{H}}^4_{[\mathrm{mir}]}$ and $\mathbf{H}^4_{[\mathrm{mir}]}$ on the other. A similar statement holds for the $\bar{\mathbf{H}}^3$ and $\mathbf{H}^3$ baryons, and the $\bar{\mathbf{H}}^1$ and $\mathbf{H}^1$ baryons, which do not allow a diagonal interaction with $\mathrm{M}\in(\frac{\mathbf{1}}{\mathbf{2}},\frac{\mathbf{1}}{\mathbf{2}})$ mesons. The remaining nonzero interaction terms are all off-diagonal.

What is more, there are no diagonal chirally invariant interactions involving the isotriplet hyperons, neither. Consequently, the Σ-hyperon masses are also independent of spontaneous symmetry breaking. Therefore, none of the $\mathbf{H}^1$, $\mathbf{H}^3$, $\mathbf{H}^4$ baryons acquires an effective mass term from the spontaneous symmetry breaking, and the "traditional" way, such as in Refs. [31,32,35], of determining chiral mixing angles/parameters from the masses does not work here, as yet. Perhaps, with the increase of the number of observed hyperons, this may change. Indeed, a similar analysis in Ref. [13] has identified (only) pairs of "chiral partners" in each flavor channel, whereas we are looking for up to four such states in each flavor channel.

But, even if one could identify all four hyperons and reproduce their masses, at the present moment, four (diagonal) masses leave two free parameters short of the six necessary ones. That will have to be dealt with by means of two independent axial couplings.

TABLE VI. The Abelian and the non-Abelian axial charges (+ sign indicates naive, − sign mirror transformation properties) of the non-Abelian chiral multiplets containing a flavor 6-plet.

Case	Field	$g_A^{(0)}$	$g_A^{(\mathbf{6})}$	$SU_L(3)\times SU_R(3)$
I	$\mathbf{B}^6$	−2	+1	$(\mathbf{6},\mathbf{1})\oplus(\mathbf{1},\mathbf{6})$
II	$\mathbf{B}^6_{[\mathrm{mir}]}$	+2	−1	$(\mathbf{1},\mathbf{6})\oplus(\mathbf{6},\mathbf{1})$
III	$\mathbf{B}^9$	0	+1	$(\mathbf{3},\mathbf{3})$
IV	$\mathbf{B}^9_{[\mathrm{mir}]}$	0	−1	$(\mathbf{3},\mathbf{3})$

B. Axial couplings

Another complementary way to determine (some of) the mixing angles is to use the hyperon's (isovector) axial coupling. Each charmed hyperon has two independent axial couplings: (i) the flavor singlet $g_A^{(0)}$ and (ii) the flavor nonsinglet $g_A^{(N)}$, where N can be either $\bar{\mathbf{3}}$-plet or **6**-plet.

Bare hyperonfields belonging to different chiral multiplets have different values of these two independent axial coupling constants, see Tables I, V, and VI, so that the physical (observed) axial couplings are chiral admixtures (linear superpositions) of the individual axial couplings.

The hyperon interpolating fields in QCD have well-defined $U_A(1)$ chiral transformation properties, see Tables I, V, and VI, that can be used to calculate the physical flavor-singlet/isoscalar axial coupling $g^{(0)}_{\mathrm{A\,mix}}$ of a chiral binary mixture,

$$g^{(0)}_{\mathrm{A\,mix}} = g^{(0)}_{A(I)}\cos^2\theta + g^{(0)}_{A(II)}\sin^2\theta, \tag{47}$$

and similarly for the non-Abelian axial coupling,

$$g^{(N)}_{\mathrm{A\,mix}} = g^{(N)}_{A(I)}\cos^2\theta + g^{(N)}_{A(II)}\sin^2\theta. \tag{48}$$

Note, however, that due to the different (bare) non-Abelian $g_A^{(N)}$ and Abelian $g_A^{(0)}$ axial couplings, see Tables V and VI, the mixing formula Eq. (47) may yield substantially different predictions from one case to another. Thus, for each known physical value of the axial coupling, one can eliminate one of the mixing angles/parameters.

The two axial couplings are generally unknown, with the possible exception of a lattice QCD calculation [34], leading to $g_A^{(\bar{3})}\sim 0.75$ for the $\Lambda-\Sigma$ transition (isovector) axial coupling, which is consistent with a phenomenological estimate from Ref. [12,36]. Such a reduced value implies the presence of mirror fields in chiral mixing scenarios, which generally reduce this number down from unity. Note, however, that the knowledge of the isovector axial coupling does not help us choose between the two possible naive-mirror-field pairs: as one can see in Table V that both the $(\bar{\mathbf{3}},\mathbf{1})$ multiplet and the (3,3) multiplet have (bare) isovector axial couplings equal to ± 1.

As we have seen above, a full complement of four (different) charmed hyperons with identical spin, isospin, and flavor $SU(3)$ content does not exist at this point in time—therefore, we may have to eliminate one naive and/or perhaps even one mirror field from consideration.

C. Three-field mixing

When we have three-state mixing, there are three mixing angles that could be fixed by three masses alone, or alternatively, one of the masses can be replaced by the knowledge of one axial coupling of the ground state hyperon. This is not likely to happen in the foreseeable future. An alternative would be to have both isovector and isoscalar axial couplings, perhaps again by way of lattice QCD calculation. That would allow one to analyze three-field mixtures, as follows.

For ternary chiral mixtures, we have

$$g_{A\,\mathrm{mix}}^{(0)} = g_{A(I)}^{(0)} \cos^2\theta + \sin^2\theta \times (g_{A(\mathrm{III})}^{(0)} \cos^2\varphi + g_{A(\mathrm{II})}^{(0)} \sin^2\varphi), \tag{49}$$

$$g_{A\,\mathrm{mix}}^{(N)} = g_{A(I)}^{(N)} \cos^2\theta + \sin^2\theta \times (g_{A(\mathrm{III})}^{(N)} \cos^2\varphi + g_{A(\mathrm{II})}^{(N)} \sin^2\varphi). \tag{50}$$

Note that the above "experimental" value of the non-Abelian axial coupling constant $g_A^{(N)} \sim 0.75$ eliminates only one pair of mixed fields (II-IV), due to the (negative) sign of the axial coupling, and leaves five allowed pairs. An inkling of the Abelian axial coupling constant $g_A^{(0)}$, which has been measured for three-light-quark baryons [28,31] but not for charm hyperons, would take us a great deal forward.

Experimental knowledge of (other) pion-hyperon coupling constants, as studied in Ref. [12], is insufficient to fix the mixing parameters at the present moment as they were fixed, e.g., in Refs. [32,33].

D. Discussion

There are two calculations in the literature that deserve discussion/comparison with (i) Migura *et al.* [37] and (ii) Harada *et al.* [14].

(i) Migura *et al.* [37] have used the 't Hooft interaction (in addition to a confining potential) in a (semi)relativistic quark model to calculate the mass spectrum of charmed baryons (but not their pion decays), and they predicted a spectrum that agrees fairly with the observed one. That suggests, though it is not a proof, that the 't Hooft interaction ought to lead to similarly good results in our approach, as well. Unfortunately, Migura *et al.* [37] have used the (nonrelativistic) $SU_{FS}(8)$ flavor-spin group, and not the Lorentz and chiral groups, as we did here, to classify the hyperon states. This fact makes the comparison of our two approaches difficult.

It remains to be seen if the observed value(s) of charmed baryon pion coupling constants are consistent with Migura *et al.*'s predictions—for that purpose, it will be necessary to calculate the charmed-baryon-pseudoscalar-meson coupling constants as functions of the 't Hooft interaction coupling constant, which is fairly known.

(ii) Harada *et al.* [14] have recently studied $U_A(1)$ symmetry breaking in the interactions of $\bar{\mathbf{3}}$-plet diquarks and pseudoscalar mesons, within the setting of $SU_L(3) \times SU_R(3)$ chiral symmetry. They found only one such interaction, their Eq. (16), which is the same as our Eq. (28). Note that their calculation is instructive, as it shows the connection with the 't Hooft interaction.

V. SUMMARY AND CONCLUSIONS

In this paper, we have considered chiral transformation properties of one-heavy-two-light-quark baryons. We constructed all local single-charm baryon interpolating fields and determined their chiral transformation properties. Based on these properties, we constructed all the chirally invariant interactions of such baryons with one pseudoscalar (or scalar) meson.

We found that in the good $SU_L(3) \times SU_R(3)$ limit one chirally invariant interaction Lagrangian violates the $U_A(1)$ symmetry. This means (i) that interaction must be related to 't Hooft's instanton-induced interaction in QCD and (ii) its coupling constant must be related to the η, η' masses [17,18]. Some, perhaps limited progress in this direction has been reported in Ref. [14].

These facts, in addition to being of theoretical interest, have practical implications: they imply selection rules (in the chiral limit) on the baryons' pion couplings and decay widths, stemming from two different QCD effects: (i) $U_A(1)$ symmetry-breaking effects and (ii) current-quark mass. That is, however, another topic which will not be dealt with here.

Finally, we note that chiral interaction Lagrangians for pion transitions between two charmed baryons have been formulated in terms of effective Lagrangians satisfying the heavy-quark and chiral symmetries. Yasui [38], in particular, extended these early ideas and studied the excited states of one-heavy-quark baryons with *arbitrary* angular momentum of the "brown muck" in the $\mathcal{O}(1/M)$ expansion and with chiral Lagrangians. Nevertheless, these nonlinear Lagrangians do not allow insight into their QCD origins and in particular not into their $U_A(1)$ symmetry properties. In the future, one may attempt a "chiral boost," a la Weinberg, of our linear Lagrangians, so as to compare their predictions with the nonlinear ones. As for the $\mathcal{O}(1/M)$ expansion of the brown muck, we are inclined to a pessimistic disposition, in view of its failure in the case of $D_{s0}(2317)$ meson.

We hope to return to the question of crossing and Fierz relations between apparently independent interaction Lagrangians reported above.

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APPENDIX: $SU_F(3)$ SYMMETRY MATRICES

The matrices $\mathbf{T}^N$ are

$$\mathbf{T}^1 = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \tag{A1}$$

$$\mathbf{T}^2 = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 \\ i & 0 & i & 0 & 0 & 0 \end{pmatrix}, \tag{A2}$$

$$\mathbf{T}^3 = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & -\sqrt{2} & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A3}$$

$$\mathbf{T}^4 = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}, \tag{A4}$$

$$\mathbf{T}^5 = \begin{pmatrix} 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 \end{pmatrix}, \tag{A5}$$

$$\mathbf{T}^6 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \tag{A6}$$

$$\mathbf{T}^7 = \begin{pmatrix} 0 & 0 & i & 0 & 0 & i \\ 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \tag{A7}$$

$$\mathbf{T}^8 = \begin{pmatrix} 0 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}} & 0 \\ 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A8}$$

The matrices $\mathbf{F}^N$ are

$$\mathbf{F}^1 = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A9}$$

$$\mathbf{F}^2 = \begin{pmatrix} 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A10}$$

$$\mathbf{F}^3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A11}$$

$$\mathbf{F}^4 = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \tag{A12}$$

$$\mathbf{F}^5 = \begin{pmatrix} 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \tag{A13}$$

$$\mathbf{F}^6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}, \tag{A14}$$

$$\mathbf{F}^7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 \end{pmatrix}, \tag{A15}$$

$$\mathbf{F}^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}. \tag{A16}$$

The matrices $\mathbf{T}'^N$ are

$$\mathbf{T}'^1 = (1 \quad 0 \quad -1), \tag{A17}$$

$$\mathbf{T}'^2 = (i \quad 0 \quad i), \tag{A18}$$

$$\mathbf{T}'^3 = (0 \quad -\sqrt{2} \quad 0). \tag{A19}$$

The matrices $\mathbf{F}'^N$ are

$$\mathbf{F}'^1 = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}, \tag{A20}$$

$$\mathbf{F}'^2 = \begin{pmatrix} 0 & -\frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & \frac{i}{\sqrt{2}} & 0 \end{pmatrix}, \tag{A21}$$

$$\mathbf{F}'^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{A22}$$

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