P-wave bottom baryons of the $SU(3)$ flavor $\mathbf{6}_F$

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We investigate P-wave bottom baryons of the $SU(3)$ flavor $\mathbf{6}_F$, and systematically study their D-wave decays into ground-state bottom baryons and pseudoscalar mesons. Together with [H. X. Chen *et al.*, Phys. Rev. D **91**, 054034 (2015); Q. Mao *et al.*, Phys. Rev. D **92**, 114007 (2015); H. X. Chen *et al.*, Phys. Rev. D **95**, 094008 (2017); H. M. Yang *et al.*,Eur. Phys. J. C **80**, 80 (2020)], a rather complete study is performed on both mass spectra and decay properties of P-wave bottom baryons, using the method of QCD sum rules and light-cone sum rules within the framework of heavy quark effective theory. Among all the possibilities, we find four Σ_b, four $\Xi_b^\prime$, and six Ω_b baryons, with limited widths and so capable of being observed. Their masses, mass splittings within the same multiplets, and decay properties are extracted (summarized in Table VI) for future experimental searches.

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I. INTRODUCTION

The strong interaction holds quarks and gluons together inside a single hadron. It is similar to the electromagnetic interaction in some aspects, which holds electrons and protons together inside a single atom. The latter leads to the well-known fine structure of line spectra, and it is interesting to investigate whether the former also leads to some fine structure of hadron spectra [1–4]. An ideal platform to study this is the singly bottom baryon system [5–8]: light quarks together with gluons circle around the nearly static bottom quark, and the whole system behaves as the QCD analog of the hydrogen.

In recent years important experimental progress has been made in the field of singly bottom baryons. Until three years ago there were only two excited bottom baryons, $\Lambda_b(5912)^0$ and $\Lambda_b(5920)^0$, which were observed by LHCb and CDF in 2012 [9,10]. However, in the past three years the LHCb and CMS collaborations discovered as many as nine excited bottom baryons:

(i) In 2018 the LHCb collaboration observed the $\Sigma_b(6097)^\pm$ in the $\Lambda_b^0\pi^\pm$ invariant mass spectrum, and the $\Xi_b(6227)^-$ in the $\Lambda_b^0K^-$ and $\Xi_b^0\pi^-$ invariant mass spectra [11,12]:

$$\Xi_b(6227)^-:\ M = 6226.9 \pm 2.0 \pm 0.3 \pm 0.2\ \text{MeV},\quad \Gamma = 18.1 \pm 5.4 \pm 1.8\ \text{MeV}, \tag{1}$$

$$\Sigma_b(6097)^+:\ M = 6095.8 \pm 1.7 \pm 0.4\ \text{MeV},\quad \Gamma = 31 \pm 5.5 \pm 0.7\ \text{MeV}, \tag{2}$$

$$\Sigma_b(6097)^-:\ M = 6098.0 \pm 1.7 \pm 0.5\ \text{MeV},\quad \Gamma = 28.9 \pm 4.2 \pm 0.9\ \text{MeV}. \tag{3}$$

(ii) In 2020 the LHCb collaboration observed the $\Omega_b(6316)^-$, $\Omega_b(6330)^-$, $\Omega_b(6340)^-$, and $\Omega_b(6350)^-$ in the $\Xi_b^0K^-$ invariant mass spectrum [13]:

$$\Omega_b(6316)^-:\ M=6315.64\pm0.31\pm0.07\pm0.50\,\text{MeV},\quad \Gamma<2.8\,\text{MeV}, \tag{4}$$

$$\Omega_b(6330)^-:\ M=6330.30\pm0.28\pm0.07\pm0.50\,\text{MeV},\quad \Gamma<3.1\,\text{MeV}, \tag{5}$$

$$\Omega_b(6340)^-:\ M=6339.71\pm0.26\pm0.05\pm0.50\,\text{MeV},\quad \Gamma<1.5\,\text{MeV}, \tag{6}$$

$$\Omega_b(6350)^-:\ M=6349.88\pm0.35\pm0.05\pm0.50\,\text{MeV},\quad \Gamma=1.4^{+1.0}_{-0.8}\pm0.1\,\text{MeV}. \tag{7}$$

(iii) In 2019 the LHCb collaboration observed the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ in the $\Lambda_b^0\pi^+\pi^-$ invariant mass distribution [14]. Later in 2020 the CMS collaboration confirmed them, and further observed

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a broad excess of events in the $\Lambda_b^0\pi^+\pi^-$ mass distribution in the region of 6040–6100 MeV [15], whose mass and width were later measured by LHCb to be [16]

$$\Lambda_b(6072)^0\colon\ M = 6072.3 \pm 2.9 \pm 0.6 \pm 0.2\ \text{MeV},\quad \Gamma = 72 \pm 11 \pm 2\ \text{MeV}. \tag{8}$$

Various theoretical methods and models have been applied to study singly bottom baryons in the past thirty years, such as various quark models [17–26], various molecular models [27–38], the quark pair creation model [39–42], the chiral perturbation theory [43–45], QCD sum rules [46–48], lattice QCD [49–52], etc. More theoretical studies can be found in Refs. [53–57], and we refer to recent reviews for detailed discussions [1,5–8,58,59].

Especially, the $\Lambda_b(5912)^0$ and $\Lambda_b(5920)^0$ were studied by Capstick and Isgur in 1986 as P-wave bottom baryons [60], and their predicted masses are in very good agreement with the LHCb and CDF measurements [9,10]. The $\Sigma_b(6097)^\pm$ and $\Xi_b(6227)^-$ are also good candidates of P-wave bottom baryons [23,39–41,46,47,55–57], while there exists the molecular interpretation for the $\Xi_b(6227)^-$ [37,38]. Besides, the four excited Ω_b baryons, $\Omega_b(6316)^-$, $\Omega_b(6330)^-$, $\Omega_b(6340)^-$, and $\Omega_b(6350)^-$, are all good candidates of P-wave bottom baryons [26,42,48]. Other than this, the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ can be well interpreted as D-wave bottom baryons [61–67].

In this paper we shall investigate P-wave bottom baryons of the $SU(3)$ flavor $\mathbf{6}_F$, and systematically study their D-wave decays into ground-state bottom baryons and pseudoscalar mesons. Previously in Refs. [68–71], we have systematically studied their mass spectra and S-wave decay properties using the method of QCD sum rules [72,73] and light-cone sum rules [74–78] within the heavy quark effective theory (HQET) [79–81]. These results will be reanalyzed in the present study, so that a rather complete study can be performed on both mass spectra and decay properties of P-wave bottom baryons. Similar methods applied to investigate singly heavy mesons and baryons can be found in Refs. [82–96].

This paper is organized as follows. In Sec. II, we briefly introduce our notations for P-wave bottom baryons of the $SU(3)$ flavor $\mathbf{6}_F$, and categorize them into four bottom baryon multiplets $[\mathbf{6}_F, 1, 0, \rho]$, $[\mathbf{6}_F, 0, 1, \lambda]$, $[\mathbf{6}_F, 1, 1, \lambda]$, and $[\mathbf{6}_F, 2, 1, \lambda]$. Then in Sec. III we study their D-wave decays into ground-state bottom baryons and pseudoscalar mesons (π or K), separately for these four multiplets. In Sec. IV we discuss the results and conclude this paper.

II. P-WAVE BOTTOM BARYONS

At the beginning we briefly introduce our notations. A singly bottom baryon is composed by one *bottom* quark and two light *up*/*down*/*strange* quarks, with the following internal structures:

(i) According to the Pauli principle, the total symmetry of the two light quarks is antisymmetric.
(ii) The color structure of the two light quarks is antisymmetric ($\bar{\mathbf{3}}_C$).
(iii) The flavor structure of the two light quarks is either symmetric [$SU(3)$ flavor $\mathbf{6}_F$] or antisymmetric [$SU(3)$ flavor $\bar{\mathbf{3}}_F$].
(iv) The spin structure of the two light quarks is either symmetric ($s_l \equiv s_{qq} = 1$) or antisymmetric ($s_l = 0$).
(v) The orbital structure of the two light quarks is either symmetric (λ-type with $l_\rho = 0$ and $l_\lambda = 1$, meaning that the orbital excitation is between the bottom quark and the two-light-quark system) or antisymmetric (ρ-type with $l_\rho = 1$ and $l_\lambda = 0$, meaning that the orbital excitation is between the two light quarks).

Accordingly, we categorize P-wave bottom baryons into eight baryon multiplets, four of which belong to the $SU(3)$ flavor $\mathbf{6}_F$ representation, as shown in Fig. 1. We use $[F(\text{flavor}), j_l, s_l, \rho/\lambda]$ to denote them, where j_l is the total angular momentum of the light components, satisfying $j_l = l_\lambda \otimes l_\rho \otimes s_l$. Each multiplet contains one or two

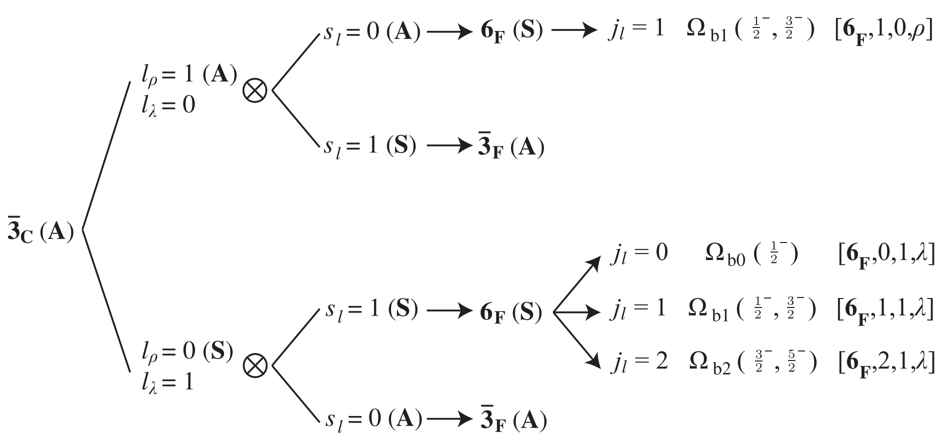


FIG. 1. Categorization of P-wave bottom baryons belonging to the $SU(3)$ flavor $\mathbf{6}_F$ representation.

TABLE I. Parameters of the *P*-wave bottom baryons belonging to the $SU(3)$ flavor $\mathbf{6}_F$ representation. See Refs. [68,69] for detailed discussions. In the last column we list the decay constant f, where isospin factors are explicitly taken into account, satisfying $f_{\Sigma_b^+} = f_{\Sigma_b^-} = \sqrt{2} f_{\Sigma_b^0}$ and $f_{\Xi_b^{\prime 0}} = f_{\Xi_b^{\prime -}}$.

Multiplets	B	ω_c (GeV)	Working region (GeV)	$\bar{\Lambda}$ (GeV)	Baryons (j^P)	Mass (GeV)	Difference (MeV)	f (GeV^4)
$[\mathbf{6}_F, 1, 0, \rho]$	Σ_b	1.83	$0.27 < T < 0.34$	1.31 ± 0.11	$\Sigma_b(1/2^-)$	6.05 ± 0.12	3 ± 1	$0.079 \pm 0.019(\Sigma_b^-(1/2^-))$
					$\Sigma_b(3/2^-)$	6.05 ± 0.12		$0.037 \pm 0.009(\Sigma_b^-(3/2^-))$
	Ξ_b'	1.98	$0.26 < T < 0.36$	1.45 ± 0.11	$\Xi_b'(1/2^-)$	6.18 ± 0.12	3 ± 1	$0.072 \pm 0.016(\Xi_b'^-(1/2^-))$
					$\Xi_b'(3/2^-)$	6.19 ± 0.11		$0.034 \pm 0.008(\Xi_b'^-(3/2^-))$
	Ω_b	2.13	$0.26 < T < 0.37$	1.58 ± 0.09	$\Omega_b(1/2^-)$	6.32 ± 0.11	2 ± 1	$0.133 \pm 0.028(\Omega_b^-(1/2^-))$
					$\Omega_b(3/2^-)$	6.32 ± 0.11		$0.063 \pm 0.013(\Omega_b^-(3/2^-))$
$[\mathbf{6}_F, 0, 1, \lambda]$	Σ_b	1.70	$0.26 < T < 0.32$	1.25 ± 0.10	$\Sigma_b(1/2^-)$	6.05 ± 0.11	$\cdots$	$0.077 \pm 0.018(\Sigma_b^-(1/2^-))$
	Ξ_b'	1.85	$0.27 < T < 0.33$	1.40 ± 0.09	$\Xi_b'(1/2^-)$	6.20 ± 0.11	$\cdots$	$0.069 \pm 0.015(\Xi_b'^-(1/2^-))$
	Ω_b	2.00	$0.27 < T < 0.34$	1.54 ± 0.09	$\Omega_b(1/2^-)$	6.34 ± 0.11	$\cdots$	$0.127 \pm 0.028(\Omega_b^-(1/2^-))$
$[\mathbf{6}_F, 1, 1, \lambda]$	Σ_b	1.94	$0.29 < T < 0.36$	1.25 ± 0.11	$\Sigma_b(1/2^-)$	6.06 ± 0.13	6 ± 3	$0.075 \pm 0.016(\Sigma_b^-(1/2^-))$
					$\Sigma_b(3/2^-)$	6.07 ± 0.13		$0.035 \pm 0.008(\Sigma_b^-(3/2^-))$
	Ξ_b'	1.97	$0.35 < T < 0.38$	1.38 ± 0.09	$\Xi_b'(1/2^-)$	6.21 ± 0.11	7 ± 2	$0.069 \pm 0.012(\Xi_b'^-(1/2^-))$
					$\Xi_b'(3/2^-)$	6.22 ± 0.11		$0.032 \pm 0.006(\Xi_b'^-(3/2^-))$
	Ω_b	2.00	$0.38 < T < 0.39$	1.48 ± 0.07	$\Omega_b(1/2^-)$	6.34 ± 0.10	6 ± 2	$0.122 \pm 0.019(\Omega_b^-(1/2^-))$
					$\Omega_b(3/2^-)$	6.34 ± 0.09		$0.058 \pm 0.009(\Omega_b^-(3/2^-))$
$[\mathbf{6}_F, 2, 1, \lambda]$	Σ_b	1.84	$0.27 < T < 0.34$	1.30 ± 0.13	$\Sigma_b(3/2^-)$	6.11 ± 0.16	12 ± 5	$0.102 \pm 0.028(\Sigma_b^-(3/2^-))$
					$\Sigma_b(5/2^-)$	6.12 ± 0.15		$0.043 \pm 0.012(\Sigma_b^-(5/2^-))$
	Ξ_b'	1.96	$0.26 < T < 0.35$	1.41 ± 0.12	$\Xi_b'(3/2^-)$	6.23 ± 0.15	11 ± 5	$0.091 \pm 0.023(\Xi_b'^-(3/2^-))$
					$\Xi_b'(5/2^-)$	6.24 ± 0.14		$0.038 \pm 0.010(\Xi_b'^-(5/2^-))$
	Ω_b	2.08	$0.26 < T < 0.37$	1.53 ± 0.10	$\Omega_b(3/2^-)$	6.35 ± 0.13	10 ± 4	$0.162 \pm 0.035(\Omega_b^-(3/2^-))$
					$\Omega_b(5/2^-)$	6.36 ± 0.12		$0.069 \pm 0.015(\Omega_b^-(5/2^-))$

bottom baryons with the total angular momenta $j = j_l \otimes s_b = |j_l \pm 1/2|$, which have similar masses according to the heavy quark effective theory.

In Refs. [68,69] we have systematically studied the mass spectrum of *P*-wave bottom baryons, and the results are reanalyzed in the present study, as summarized in Table I. Some of them are used as input parameters when studying decay properties of *P*-wave bottom baryons. Especially, we use the following mass values when calculating their decay widths:

(i) In Ref. [97] we found that the $\Omega_b(6316)^-$ can be explained as a *P*-wave Ω_b baryon of either $J^P = 1/2^-$ or $3/2^-$, belonging to the $[\mathbf{6}_F, 1, 0, \rho]$ doublet. Hence, we use the following mass values for this doublet, taken from the LHCb experiment [13] as well as their mass sum rules:

$$
\begin{aligned}
M_{[\Sigma_b(1/2^-),1,0,\rho]} &= 6.05 \text{ GeV},\\
M_{[\Sigma_b(3/2^-),1,0,\rho]} &= 6.05 \text{ GeV},\\
M_{[\Xi_b'(1/2^-),1,0,\rho]} &= 6.18 \text{ GeV},\\
M_{[\Xi_b'(3/2^-),1,0,\rho]} &= 6.19 \text{ GeV},\\
M_{[\Omega_b(1/2^-),1,0,\rho]} &= 6315.64 \text{ MeV},\\
M_{[\Omega_b(3/2^-),1,0,\rho]} &= 6315.64 \text{ MeV}.
\end{aligned} \tag{9}
$$

(ii) For the $[\mathbf{6}_F, 0, 1, \lambda]$ singlet, we use the following mass values taken from their mass sum rules:

$$
\begin{aligned}
M_{[\Sigma_b(1/2^-),0,1,\lambda]} &= 6.05 \text{ GeV},\\
M_{[\Xi_b'(1/2^-),0,1,\lambda]} &= 6.20 \text{ GeV},\\
M_{[\Omega_b(1/2^-),0,1,\lambda]} &= 6.34 \text{ GeV}.
\end{aligned} \tag{10}
$$

(iii) In Ref. [97] we found that the $\Omega_b(6330)^-$ and $\Omega_b(6340)^-$ can be explained as *P*-wave Ω_b baryons of $J^P = 1/2^-$ and $3/2^-$ respectively, belonging to the $[\mathbf{6}_F, 1, 1, \lambda]$ doublet. Hence, we use the following mass values for this doublet, taken from the LHCb experiment [13] as well as their mass sum rules:

$$
\begin{aligned}
M_{[\Sigma_b(1/2^-),1,1,\lambda]} &= 6.06 \text{ GeV},\\
M_{[\Sigma_b(3/2^-),1,1,\lambda]} &= 6.07 \text{ GeV},\\
M_{[\Xi_b'(1/2^-),1,1,\lambda]} &= 6.21 \text{ GeV},\\
M_{[\Xi_b'(3/2^-),1,1,\lambda]} &= 6.22 \text{ GeV},\\
M_{[\Omega_b(1/2^-),1,1,\lambda]} &= 6330.30 \text{ MeV},\\
M_{[\Omega_b(3/2^-),1,1,\lambda]} &= 6339.71 \text{ MeV}.
\end{aligned} \tag{11}
$$

(iv) In Refs. [97,98] we found that the $\Sigma_b(6097)^\pm$, $\Xi_b(6227)^-$, and $\Omega_b(6350)^-$ can be explained as P-wave bottom baryons of $J^P = 3/2^-$, belonging to the $[\mathbf{6}_F, 2, 1, \lambda]$ doublet. Hence, we use the following mass values for this doublet, taken from the LHCb experiments [11–13] as well as their mass sum rules:

$$
\begin{aligned}
M_{[\Sigma_b(3/2^-),2,1,\lambda]} &= 6096.9 \text{ MeV},\\
M_{[\Sigma_b(5/2^-),2,1,\lambda]} - M_{[\Sigma_b(3/2^-),2,1,\lambda]} &= 12 \text{ MeV},\\
M_{[\Xi'_b(3/2^-),2,1,\lambda]} &= 6226.9 \text{ MeV},\\
M_{[\Xi'_b(5/2^-),2,1,\lambda]} - M_{[\Xi'_b(3/2^-),2,1,\lambda]} &= 11 \text{ MeV},\\
M_{[\Omega_b(3/2^-),2,1,\lambda]} &= 6349.88 \text{ MeV},\\
M_{[\Omega_b(5/2^-),2,1,\lambda]} - M_{[\Omega_b(3/2^-),2,1,\lambda]} &= 10 \text{ MeV}. \qquad (12)
\end{aligned}
$$

Note that the above interpretations are just possible assignments, and there exist many other possibilities for the $\Sigma_b(6097)^\pm$, $\Xi_b(6227)^-$, $\Omega_b(6316)^-$, $\Omega_b(6330)^-$, $\Omega_b(6340)^-$, and $\Omega_b(6350)^-$.

We use the following parameters for ground-state bottom baryons, pseudoscalar and vector mesons [4]:

$$
\begin{aligned}
\Lambda_b(1/2^+)&:\ m = 5619.60 \text{ MeV},\\
\Xi_b(1/2^+)&:\ m = 5793.20 \text{ MeV},\\
\Sigma_b(1/2^+)&:\ m = 5813.4 \text{ MeV},\\
\Sigma_b^*(3/2^+)&:\ m = 5833.6 \text{ MeV},\\
\Xi'_b(1/2^+)&:\ m = 5935.02 \text{ MeV},\\
\Xi_b^*(3/2^+)&:\ m = 5952.6 \text{ MeV},\\
\Omega_b(1/2^+)&:\ m = 6046.1 \text{ MeV},\\
\Omega_b^*(3/2^+)&:\ m = 6063 \text{ MeV},\\
\pi(0^-)&:\ m = 138.04 \text{ MeV},\\
K(0^-)&:\ m = 495.65 \text{ MeV},\\
\rho(1^-)&:\ m = 775.21 \text{ MeV},\\
&\quad \Gamma = 148.2 \text{ MeV}, \qquad g_{\rho\pi\pi} = 5.94,\\
K^*(1^-)&:\ m = 893.57 \text{ MeV},\\
&\quad \Gamma = 49.1 \text{ MeV}, \qquad g_{K^*K\pi} = 3.20, \qquad (13)
\end{aligned}
$$

with the following Lagrangians:

$$
\begin{aligned}
\mathcal{L}_{\rho\pi\pi} &= g_{\rho\pi\pi} \times (\rho^0_\mu \pi^+ \partial^\mu \pi^- - \rho^0_\mu \pi^- \partial^\mu \pi^+) + \cdots,\\
\mathcal{L}_{K^*K\pi} &= g_{K^*K\pi} K^{*+}_\mu \times (K^- \partial^\mu \pi^0 - \partial^\mu K^- \pi^0) + \cdots. \qquad (14)
\end{aligned}
$$

III. D-WAVE DECAY PROPERTIES

In the present study we shall investigate P-wave bottom baryons of the $SU(3)$ flavor $\mathbf{6}_F$, and study their D-wave decays into ground-state bottom baryons together with pseudoscalar mesons (π or K). Their S-wave decay properties have been systematically studied in Refs. [70,71], and we shall use the same method of light-cone sum rules within the heavy quark effective theory to investigate the following decay channels (the coefficients at right-hand sides are isospin factors):

$$(a1)\ \mathbf{\Gamma}[\Sigma_b[1/2^-] \to \Sigma_b^* + \pi] = 2 \times \mathbf{\Gamma}[\Sigma_b^-[1/2^-] \to \Sigma_b^{*0} + \pi^-], \qquad (15)$$

$$(b1)\ \mathbf{\Gamma}[\Sigma_b[3/2^-] \to \Lambda_b + \pi] = \mathbf{\Gamma}[\Sigma_b^-[3/2^-] \to \Lambda_b^0 + \pi^-], \qquad (16)$$

$$(b2)\ \mathbf{\Gamma}[\Sigma_b[3/2^-] \to \Sigma_b + \pi] = 2 \times \mathbf{\Gamma}[\Sigma_b^-[3/2^-] \to \Sigma_b^0 + \pi^-], \qquad (17)$$

$$(b3)\ \mathbf{\Gamma}[\Sigma_b[3/2^-] \to \Sigma_b^* + \pi] = 2 \times \mathbf{\Gamma}[\Sigma_b^-[3/2^-] \to \Sigma_b^{*0} + \pi^-], \qquad (18)$$

$$(c1)\ \mathbf{\Gamma}[\Sigma_b[5/2^-] \to \Lambda_b + \pi] = \mathbf{\Gamma}[\Sigma_b^-[5/2^-] \to \Lambda_b^0 + \pi^-], \qquad (19)$$

$$(c2)\ \mathbf{\Gamma}[\Sigma_b[5/2^-] \to \Sigma_b + \pi] = 2 \times \mathbf{\Gamma}[\Sigma_b^-[5/2^-] \to \Sigma_b^0 + \pi^-], \qquad (20)$$

$$(c3)\ \mathbf{\Gamma}[\Sigma_b[5/2^-] \to \Sigma_b^* + \pi] = 2 \times \mathbf{\Gamma}[\Sigma_b^-[5/2^-] \to \Sigma_b^{*0} + \pi^-], \qquad (21)$$

$$(d1)\ \mathbf{\Gamma}[\Xi'_b[1/2^-] \to \Xi_b^* + \pi] = \frac{3}{2} \times \mathbf{\Gamma}[\Xi_b^{\prime -}[1/2^-] \to \Xi_b^{*0} + \pi^-], \qquad (22)$$

$$(d2)\ \mathbf{\Gamma}[\Xi'_b[1/2^-] \to \Sigma_b^* + K] = 3 \times \mathbf{\Gamma}[\Xi_b^{\prime -}[1/2^-] \to \Sigma_b^{*0} + K^-], \qquad (23)$$

$$(e1)\ \mathbf{\Gamma}[\Xi'_b[3/2^-] \to \Xi_b + \pi] = \frac{3}{2} \times \mathbf{\Gamma}[\Xi_b^{\prime -}[3/2^-] \to \Xi_b^0 + \pi^-], \qquad (24)$$

$$(e2)\ \mathbf{\Gamma}[\Xi'_b[3/2^-] \to \Lambda_b + K] = \mathbf{\Gamma}[\Xi_b^{\prime -}[3/2^-] \to \Lambda_b^0 + K^-], \qquad (25)$$

$$(e3)\ \mathbf{\Gamma}[\Xi'_b[3/2^-] \to \Xi'_b + \pi] = \frac{3}{2} \times \mathbf{\Gamma}[\Xi_b^{\prime -}[1/2^-] \to \Xi_b^{\prime 0} + \pi^-], \qquad (26)$$

$$(e4)\ \mathbf{\Gamma}[\Xi'_b[3/2^-] \to \Sigma_b + K] = 3 \times \mathbf{\Gamma}[\Xi_b^{\prime -}[3/2^-] \to \Sigma_b^0 + K^-], \qquad (27)$$

$$(e5)\,\boldsymbol{\Gamma}[\Xi_b'[3/2^-]\to\Xi_b^*+\pi]=\frac{3}{2}\times\boldsymbol{\Gamma}[\Xi_b'^-[3/2^-]\to\Xi_b^{*0}+\pi^-], \tag{28}$$

$$(e6)\,\boldsymbol{\Gamma}[\Xi_b'[3/2^-]\to\Sigma_b^*+K]=3\times\boldsymbol{\Gamma}[\Xi_b'^-[3/2^-]\to\Sigma_b^{*0}+K^-], \tag{29}$$

$$(f1)\,\boldsymbol{\Gamma}[\Xi_b'[5/2^-]\to\Xi_b+\pi]=\frac{3}{2}\times\boldsymbol{\Gamma}[\Xi_b'^-[5/2^-]\to\Xi_b^{0}+\pi^-], \tag{30}$$

$$(f2)\,\boldsymbol{\Gamma}[\Xi_b'[5/2^-]\to\Lambda_b+K]=\boldsymbol{\Gamma}[\Xi_b'^-[5/2^-]\to\Lambda_b^{0}+K^-], \tag{31}$$

$$(f3)\,\boldsymbol{\Gamma}[\Xi_b'[5/2^-]\to\Xi_b'+\pi]=\frac{3}{2}\times\boldsymbol{\Gamma}[\Xi_b'^-[1/2^-]\to\Xi_b'^{0}+\pi^-], \tag{32}$$

$$(f4)\,\boldsymbol{\Gamma}[\Xi_b'[5/2^-]\to\Sigma_b+K]=3\times\boldsymbol{\Gamma}[\Xi_b'^-[5/2^-]\to\Sigma_b^{0}+K^-], \tag{33}$$

$$(f5)\,\boldsymbol{\Gamma}[\Xi_b'[5/2^-]\to\Xi_b^*+\pi]=\frac{3}{2}\times\boldsymbol{\Gamma}[\Xi_b'^-[5/2^-]\to\Xi_b^{*0}+\pi^-], \tag{34}$$

$$(f6)\,\boldsymbol{\Gamma}[\Xi_b'[5/2^-]\to\Sigma_b^*+K]=3\times\boldsymbol{\Gamma}[\Xi_b'^-[5/2^-]\to\Sigma_b^{*0}+K^-], \tag{35}$$

$$(g1)\,\boldsymbol{\Gamma}[\Omega_b[1/2^-]\to\Xi_b^*+K]=2\times\boldsymbol{\Gamma}[\Omega_b^-[1/2^-]\to\Xi_b^{*0}+K^-], \tag{36}$$

$$(h1)\,\boldsymbol{\Gamma}[\Omega_b[3/2^-]\to\Xi_b+K]=2\times\boldsymbol{\Gamma}[\Omega_b^-[3/2^-]\to\Xi_b^{0}+k^-], \tag{37}$$

$$(h2)\,\boldsymbol{\Gamma}[\Omega_b[3/2^-]\to\Xi_b'+K]=2\times\boldsymbol{\Gamma}[\Omega_b^-[3/2^-]\to\Xi_b'^{0}+K^-], \tag{38}$$

$$(h3)\,\boldsymbol{\Gamma}[\Omega_b[3/2^-]\to\Xi_b^*+K]=2\times\boldsymbol{\Gamma}[\Omega_b^-[3/2^-]\to\Xi_b^{*0}+K^-], \tag{39}$$

$$(i1)\,\boldsymbol{\Gamma}[\Omega_b[5/2^-]\to\Xi_b+K]=2\times\boldsymbol{\Gamma}[\Omega_b^-[5/2^-]\to\Xi_b^{0}+k^-], \tag{40}$$

$$(i2)\,\boldsymbol{\Gamma}[\Omega_b[5/2^-]\to\Xi_b'+K]=2\times\boldsymbol{\Gamma}[\Omega_b^-[5/2^-]\to\Xi_b'^{0}+K^-], \tag{41}$$

$$(i3)\,\boldsymbol{\Gamma}[\Omega_b[5/2^-]\to\Xi_b^*+K]=2\times\boldsymbol{\Gamma}[\Omega_b^-[5/2^-]\to\Xi_b^{*0}+K^-]. \tag{42}$$

We shall calculate their decay widths through

$$\mathcal{L}_{X_b(1/2^-)\to Y_b(3/2^+)P}=g\bar{X}_b(1/2^-)\gamma_\mu\gamma_5 Y_{b\nu}(3/2^+)\partial^\mu\partial^\nu P, \tag{43}$$

$$\mathcal{L}_{X_b(3/2^-)\to Y_b(1/2^+)P}=g\bar{X}_{b\mu}(3/2^-)\gamma_\nu\gamma_5 Y_{b}(1/2^+)\partial^\mu\partial^\nu P, \tag{44}$$

$$\mathcal{L}_{X_b(3/2^-)\to Y_b(3/2^+)P}=g\bar{X}_{b\mu}(3/2^-)Y_{b\nu}(3/2^+)\partial^\mu\partial^\nu P, \tag{45}$$

$$\mathcal{L}_{X_b(5/2^-)\to Y_b(1/2^+)P}=g\bar{X}_{b\mu\nu}(5/2^-)Y_{b}(1/2^+)\partial^\mu\partial^\nu P, \tag{46}$$

$$\begin{aligned}\mathcal{L}&_{X_b(5/2^-)\to Y_b(3/2^+)P}\\ &=g\bar{X}_{b\mu\nu}(5/2^-)\gamma_\rho\gamma_5 Y_b^\mu(3/2^+)\partial^\nu\partial^\rho P\\ &\quad+g\bar{X}_{b\mu\nu}(5/2^-)\gamma_\rho\gamma_5 Y_b^\nu(3/2^+)\partial^\mu\partial^\rho P,\end{aligned} \tag{47}$$

where $X_b^{(\mu\nu)}$, $Y_b^{(\mu)}$, and P denote the P-wave bottom baryon, ground-state bottom baryon, and pseudoscalar meson, respectively.

As an example, we shall study the D-wave decay of $\Omega_b^-(3/2^-)$ belonging to $[\mathbf{6}_F,2,1,\lambda]$ into $\Xi_b^0(1/2^+)$ and $K^-(0^-)$ in the next subsection. The four bottom baryon multiplets $[\mathbf{6}_F,1,0,\rho]$, $[\mathbf{6}_F,0,1,\lambda]$, $[\mathbf{6}_F,1,1,\lambda]$, and $[\mathbf{6}_F,2,1,\lambda]$ will be separately investigated in the following subsections.

A. $\Omega_b^-(3/2^-)$ of $[\mathbf{6}_F,2,1,\lambda]$ to D-wave $\Xi_b^0K^-$

In this subsection we study the D-wave decay of the $\Omega_b^-(3/2^-)$ belonging to $[\mathbf{6}_F,2,1,\lambda]$ into $\Xi_b^0(1/2^+)$ and $K^-(0^-)$. To do this we need to calculate the three-point correlation function,

$$\begin{aligned}\Pi^\alpha&(\omega,\omega')\\ &=\int d^4x e^{-ik\cdot x}\langle 0|J^\alpha_{3/2,-,\Omega_b^-,2,1,\lambda}(0)\bar{J}_{\Xi_b^0}(x)|K^-(q)\rangle\\ &=\frac{1+\not{v}}{2}G^\alpha_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-}(\omega,\omega'),\end{aligned} \tag{48}$$

at both hadron and quark-gluon levels. In this expression $k'=k+q$, $\omega=v\cdot k$, and $\omega'=v\cdot k'$. The two interpolating fields $J^\alpha_{3/2,-,\Omega_b^-,2,1,\lambda}$ and $J_{\Xi_b^0}$ have been constructed in Refs. [68,94]:

$$\begin{aligned}J&^\alpha_{3/2,-,\Omega_b^-,2,1,\lambda}\\ &=i\epsilon_{abc}([\mathcal{D}_t^\mu s^{aT}]C\gamma_t^\nu s^b+s^{aT}C\gamma_t^\nu[\mathcal{D}_t^\mu s^b])\\ &\quad\times\left(g_t^{\alpha\mu}\gamma_t^\nu\gamma_5+g_t^{\alpha\nu}\gamma_t^\mu\gamma_5-\frac{2}{3}g_t^{\mu\nu}\gamma_t^\alpha\gamma_5\right)h_v^c,\end{aligned} \tag{49}$$

$$J_{\Xi_b^0}=\epsilon_{abc}[d^{aT}C\gamma_5 s^b]h_v^c, \tag{50}$$

which couple to $\Omega_b^-(3/2^-)$ and $\Xi_b^0(1/2^+)$, respectively.

At the hadron level, we write $G^{\alpha}_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-}$ as

$$G^{\alpha}_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-}(\omega,\omega') = g_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-} \times \frac{f_{\Omega_b^-[\frac{3}{2}^-]} f_{\Xi_b^0}}{(\bar{\Lambda}_{\Omega_b^-[\frac{3}{2}^-]} - \omega')(\bar{\Lambda}_{\Xi_b^0} - \omega)} \gamma\cdot q\gamma_5 q^{\alpha} + \cdots, \tag{51}$$

where $\cdots$ denotes other possible decay amplitudes.

At the quark-gluon level, we calculate $G^{\alpha}_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-}$ using the method of operator product expansion:

$$\begin{aligned}
&G^{\alpha}_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-}(\omega,\omega') \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 8 \times \bigg(\frac{f_K m_s u}{4\pi^2 t^2}\phi_{2;K}(u) + \frac{f_K m_s^2 u}{12(m_u+m_s)\pi^2 t^2}\phi^{\sigma}_{3;K}(u) \\
&+ \frac{f_K m_s^2 m_K^2 u}{48(m_u+m_s)\pi^2}\phi^{\sigma}_{3;K}(u) + \frac{f_K m_s u}{64\pi^2}\phi_{4;K}(u) + \frac{f_K u}{12}\langle\bar{s}s\rangle\phi_{2;K}(u) + \frac{f_K m_s m_K^2 u t^2}{288(m_u+m_s)}\langle ss\rangle\phi^{\sigma}_{3;K}(u) \\
&+ \frac{f_K u t^2}{192}\langle ss\rangle\phi_{4;K}(u) + \frac{f_K u t^2}{192}\langle g_s\bar{s}\sigma G s\rangle\phi_{2;K}(u) + \frac{f_K u t^4}{3072}\langle g_s\bar{s}\sigma G s\rangle\phi_{4;K}(u)\bigg) \times \gamma\cdot q\gamma_5 q^{\alpha} \\
&- \int_0^\infty dt \int_0^1 du \int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)} e^{i\omega t(1-\alpha_2-u\alpha_3)} \times \bigg(\frac{f_{3K} u}{2\pi^2 t^2}\Phi_{3;K}(\underline{\alpha}) - \frac{f_{3K}}{2\pi^2 t^2}\Phi_{3;K}(\underline{\alpha}) \\
&+ \frac{i f_{3K} u^2 \alpha_3}{2\pi^2 t v\cdot q}\Phi_{3;K}(\underline{\alpha}) + \frac{i f_{3K} u\alpha_2}{2\pi^2 t v\cdot q}\Phi_{3;K}(\underline{\alpha}) - \frac{i f_{3K} u}{2\pi^2 t v\cdot q}\Phi_{3;K}(\underline{\alpha})\bigg) \times \gamma\cdot q\gamma_5 q^{\alpha} + \cdots.
\end{aligned} \tag{52}$$

Then we perform double Borel transformations to both Eqs. (51) and (52), and obtain

$$\begin{aligned}
&g_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-} f_{\Omega_b^-[\frac{3}{2}^-]} f_{\Xi_b^0} e^{-\frac{\bar{\Lambda}_{\Omega_b^-[\frac{3}{2}^-]}}{T_1}} e^{-\frac{\bar{\Lambda}_{\Xi_b^0}}{T_2}} \\
&= 8 \times \bigg(-\frac{i f_K m_s u_0}{4\pi^2} T^3 f_2\Big(\frac{\omega_c}{T}\Big)\phi_{2;K}(u_0) - \frac{i f_K m_K^2 u_0}{12(m_u+m_s)\pi^2} T^3 f_2\Big(\frac{\omega_c}{T}\Big)\phi^{\sigma}_{3;K}(u_0) + \frac{i f_K m_s u_0}{64\pi^2} T f_0\Big(\frac{\omega_c}{T}\Big)\phi_{4;K}(u_0) \\
&+ \frac{i f_K u_0}{12}\langle\bar{s}s\rangle T f_0\Big(\frac{\omega_c}{T}\Big)\phi_{2;K}(u_0) - \frac{i f_K m_s u_0}{288(m_u+m_s)}\langle\bar{s}s\rangle\frac{1}{T}\phi^{\sigma}_{3;K}(u_0) - \frac{i f_K u_0}{192}\langle\bar{s}s\rangle\frac{1}{T}\phi_{4;K}(u_0) \\
&- \frac{i f_K u_0}{192}\langle g_s\bar{s}\sigma G s\rangle\frac{1}{T}\phi_{2;K}(u_0) + \frac{i f_K u_0}{3072}\langle g_s\bar{s}\sigma G s\rangle\frac{1}{T^3}\phi_{4;K}(u_0)\bigg) \\
&- \bigg(-\frac{i f_{3K}}{2\pi^2} T^3 f_2\Big(\frac{\omega_c}{T}\Big) \int_0^{\frac{1}{2}} d\alpha_2 \int_{\frac{1}{2}-\alpha_2}^{1-\alpha_2} d\alpha_3 \Big(\frac{u_0}{\alpha_3}\Phi_{3;K}(\underline{\alpha}) - \frac{1}{\alpha_3}\Phi_{3;K}(\underline{\alpha})\Big) \\
&+ \frac{i f_{3K}}{2\pi^2} T^3 f_2\Big(\frac{\omega_c}{T}\Big) \int_0^{\frac{1}{2}} d\alpha_2 \int_{\frac{1}{2}-\alpha_2}^{1-\alpha_2} d\alpha_3 \frac{1}{\alpha_3}\frac{\partial}{\partial\alpha_3}(\alpha_3 u_0\Phi_{3;K}(\underline{\alpha}) + \alpha_2\Phi_{3;K}(\underline{\alpha}) - \Phi_{3;K}(\underline{\alpha}))\bigg).
\end{aligned} \tag{53}$$

In the above expression, ω and ω' have been transformed to T_1 and T_2; we work at the symmetric point $T_1 = T_2 = 2T$ so that $u_0 = \frac{T_1}{T_1+T_2} = \frac{1}{2}$; $f_n(x) \equiv 1 - e^{-x}\sum_{k=0}^{n}\frac{x^k}{k!}$. We refer to Refs. [77,78,99–104] for explicit forms of the light-cone distribution amplitudes contained in the above sum rule equations, and more examples can be found in the Appendix.

In the present study we use the following values for various quark and gluon parameters at the renormalization scale 2 GeV [4,105–112]:

$$
\begin{aligned}
\langle \bar{q}q \rangle &= -(0.24 \pm 0.01\ \text{GeV})^3, \\
\langle \bar{s}s \rangle &= (0.8 \pm 0.1) \times \langle \bar{q}q \rangle, \\
\langle g_s \bar{q}\sigma G q \rangle &= M_0^2 \times \langle \bar{q}q \rangle, \\
\langle g_s \bar{s}\sigma G s \rangle &= M_0^2 \times \langle \bar{s}s \rangle, \\
M_0^2 &= 0.8\ \text{GeV}^2, \\
\langle g_s^2 GG \rangle &= (0.48 \pm 0.14)\ \text{GeV}^4. \qquad (54)
\end{aligned}
$$

After fixing $\omega_c = 1.665$ GeV to be the average of the threshold values of the $\Omega_b^-(3/2^-)$ and Ξ_b^0 mass sum rules, we calculate the coupling constant $g_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-}$ from Eq. (53) to be

$$
\begin{aligned}
g_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-} &= 7.27^{+1.10}_{-0.68}{}^{+2.22}_{-1.67}{}^{+2.34}_{-1.66}{}^{+1.96}_{-1.42}\ \text{GeV}^{-2} \\
&= 7.27^{+3.92}_{-2.83}\ \text{GeV}^{-2}. \qquad (55)
\end{aligned}
$$

Here the uncertainties are due to the Borel mass, the parameters of Ξ_b^0, the parameters of $\Omega_b^-(3/2^-)$, and various quark and gluon parameters listed in Eq. (54), respectively. Some of these parameters can be found in Sec. II. The variation of $g_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-}$ is shown in Fig. 4(d) as a function of the Borel mass T, where we find that its Borel mass dependence is moderate and acceptable.

The D-wave decay of $\Omega_b^-(3/2^-)$ into $\Xi_b^0K^-$ is kinematically allowed, and its amplitude is

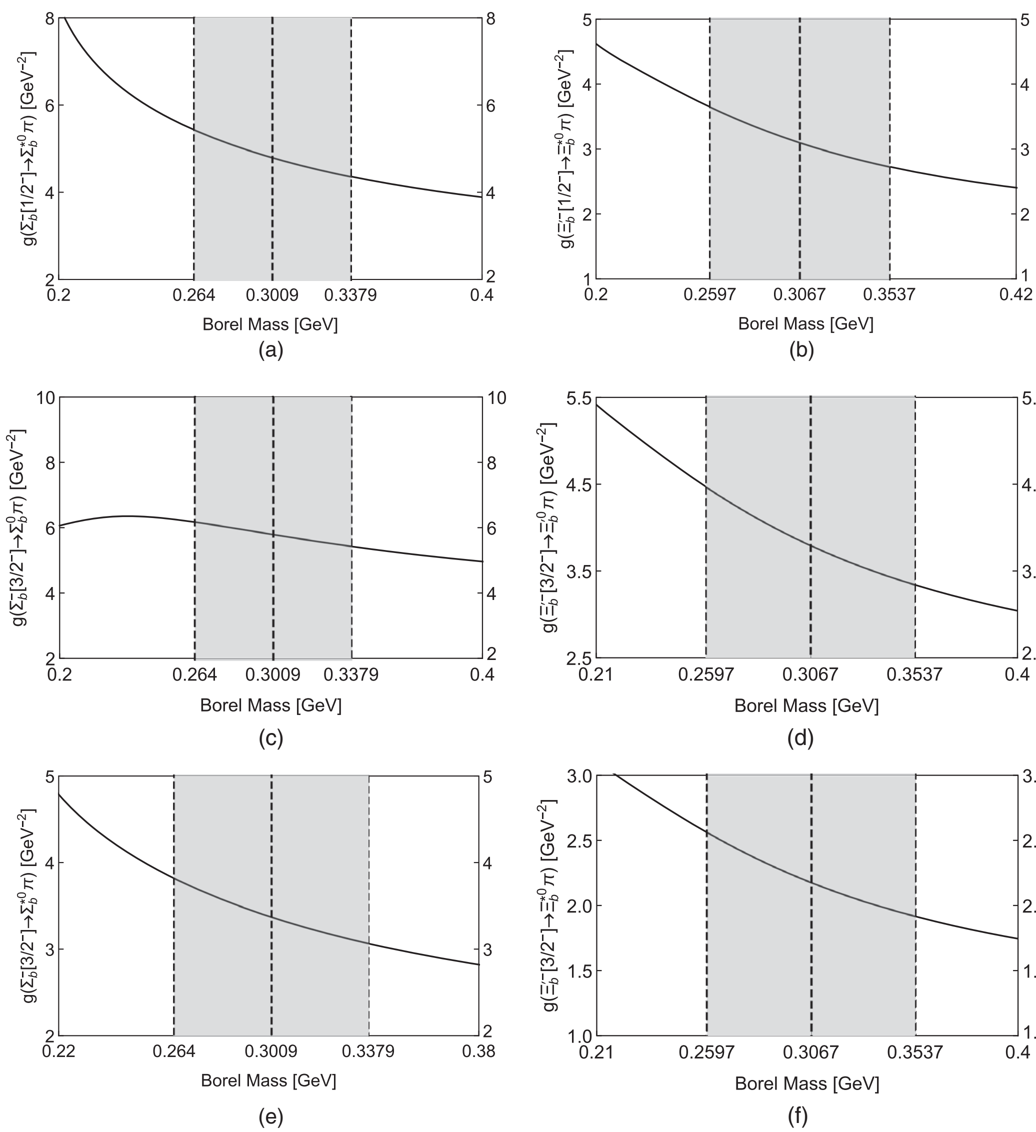


FIG. 2. The coupling constants as functions of the Borel mass T: (a) $g_{\Sigma_b^-[\frac{1}{2}^-]\to\Sigma_b^{*0}\pi^-}$, (b) $g_{\Xi_b'^-[\frac{1}{2}^-]\to\Xi_b^{*0}\pi^-}$, (c) $g_{\Sigma_b^-[\frac{3}{2}^-]\to\Sigma_b^{0}\pi^-}$, (d) $g_{\Xi_b'^-[\frac{3}{2}^-]\to\Xi_b'^{0}\pi^-}$, (e) $g_{\Sigma_b^-[\frac{3}{2}^-]\to\Sigma_b^{*0}\pi^-}$, and (f) $g_{\Xi_b'^-[\frac{3}{2}^-]\to\Xi_b^{*0}\pi^-}$. Here the bottom baryon doublet $[\bar{\mathbf{6}}_F, 1, 0, \rho]$ is investigated.

$$\mathcal{M}(\Omega_b^-(3/2^-) \to \Xi_b^0 + K^-) = g_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0 K^-} \bar{u}_0^\mu \gamma^\nu \gamma_5 u_1 p_{2,\mu} p_{2,\nu}. \quad (56)$$

This amplitude can be used to further calculate its width through

$$\Gamma(\Omega_b^-(3/2^-) \to \Xi_b^0 + K^-) = \frac{|\vec{p}_2|}{32\pi^2 m_0^2} \times g^2_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0 K^-} \times p_{2,\mu} p_{2,\nu} p_{2,\rho} p_{2,\sigma} \times \mathrm{Tr}\Big[\gamma^\nu \gamma_5 (\not{p}_1 + m_1) \gamma^\sigma \gamma_5 \times \Big(g^{\rho\mu} - \frac{\gamma^\rho\gamma^\mu}{3} - \frac{p_0^\rho \gamma^\mu - p_0^\mu \gamma^\rho}{3 m_0} - \frac{2 p_0^\rho p_0^\mu}{3 m_0^2}\Big)(\not{p}_0 + m_0)\Big], \quad (57)$$

from which we obtain

$$\Gamma_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0 K^-} = 4.6^{+3.3}_{-1.9} \text{ MeV}. \quad (58)$$

In the following subsections we shall follow the same procedures to separately investigate the four bottom baryon multiplets $[\mathbf{6}_F, 1, 0, \rho]$, $[\mathbf{6}_F, 0, 1, \lambda]$, $[\mathbf{6}_F, 1, 1, \lambda]$, and $[\mathbf{6}_F, 2, 1, \lambda]$.

B. The $[\mathbf{6}_F,1,0,\rho]$ doublet

There are six bottom baryons contained in the $[\mathbf{6}_F, 1, 0, \rho]$ doublet, which are $\Sigma_b(\frac{1}{2}^-/\frac{3}{2}^-)$, $\Xi_b'(\frac{1}{2}^-/\frac{3}{2}^-)$, and $\Omega_b(\frac{1}{2}^-/\frac{3}{2}^-)$. We study their D-wave decays into ground-state bottom baryons and pseudoscalar mesons, and find twelve nonzero coupling constants:

$$\begin{aligned}
(a1)\ & g_{\Sigma_b[\frac{1}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]\pi} = 4.80 \text{ GeV}^{-2},\\
(b2)\ & g_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b[\frac{1}{2}^+]\pi} = 5.79 \text{ GeV}^{-2},\\
(b3)\ & g_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]\pi} = 3.37 \text{ GeV}^{-2},\\
(d1)\ & g_{\Xi_b'[\frac{1}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]\pi} = 3.10 \text{ GeV}^{-2},\\
(d2)\ & g_{\Xi_b'[\frac{1}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]K} = 3.63 \text{ GeV}^{-2},\\
(e3)\ & g_{\Xi_b'[\frac{3}{2}^-]\to\Xi_b'[\frac{1}{2}^+]\pi} = 3.79 \text{ GeV}^{-2},\\
(e4)\ & g_{\Xi_b'[\frac{3}{2}^-]\to\Sigma_b[\frac{1}{2}^+]K} = 4.45 \text{ GeV}^{-2},\\
(e5)\ & g_{\Xi_b'[\frac{3}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]\pi} = 2.18 \text{ GeV}^{-2},\\
(e6)\ & g_{\Xi_b'[\frac{3}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]K} = 2.55 \text{ GeV}^{-2},\\
(g1)\ & g_{\Omega_b[\frac{1}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]K} = 4.54 \text{ GeV}^{-2},\\
(h2)\ & g_{\Omega_b[\frac{3}{2}^-]\to\Xi_b'[\frac{1}{2}^+]K} = 5.56 \text{ GeV}^{-2},\\
(h3)\ & g_{\Omega_b[\frac{3}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]K} = 3.19 \text{ GeV}^{-2}.
\end{aligned} \quad (59)$$

We show some of these coupling constants as functions of the Borel mass T in Fig. 2. Based on them, we further find six D-wave decay channels that are kinematically allowed:

TABLE II. Decay properties of the P-wave bottom baryons belonging to the $[\mathbf{6}_F, 1, 0, \rho]$ doublet. In Refs. [68,69] we studied the mass spectrum of P-wave bottom baryons, and the results are reanalyzed and summarized in the second and third columns. In Refs. [70,71] we studied the S-wave decay properties of P-wave bottom baryons into ground-state bottom baryons together with pseudoscalar mesons or vector mesons, and the results are reanalyzed and summarized in the fifth column. The sixth column "D-wave width" is added in the present study. Possible experimental candidates are given in the last column for comparison.

Baryon (j^P)	Mass (GeV)	Difference (MeV)	Decay channels	S-wave width (MeV)	D-wave width (MeV)	Total width (MeV)	Candidate
$\Sigma_b(\frac{1}{2}^-)$	6.05 ± 0.12	3 ± 1	$\Sigma_b(\frac{1}{2}^-) \to \Sigma_b\pi$	710	$\cdots$	710	$\cdots$
			$\Sigma_b(\frac{1}{2}^-) \to \Sigma_b^*\pi$	$\cdots$	0.62		
			$\Sigma_b(\frac{1}{2}^-) \to \Lambda_b\rho \to \Lambda_b\pi\pi$	4.3×10^{-3}			
$\Sigma_b(\frac{3}{2}^-)$	6.05 ± 0.12		$\Sigma_b(\frac{3}{2}^-) \to \Sigma_b\pi$	$\cdots$	0.84	410	$\cdots$
			$\Sigma_b(\frac{3}{2}^-) \to \Sigma_b^*\pi$	410	0.098		
			$\Sigma_b(\frac{3}{2}^-) \to \Lambda_b\rho \to \Lambda_b\pi\pi$	5.1×10^{-3}			
$\Xi_b'(\frac{1}{2}^-)$	6.18 ± 0.12	3 ± 1	$\Xi_b'(\frac{1}{2}) \to \Xi_b'\pi$	250	$\cdots$	250	$\cdots$
			$\Xi_b'(\frac{1}{2}^-) \to \Xi_b^*\pi$	$\cdots$	0.29		
			$\Xi_b'(\frac{1}{2}^-) \to \Xi_b\rho \to \Xi_b\pi\pi$	1.2×10^{-5}			
$\Xi_b'(\frac{3}{2}^-)$	6.19 ± 0.11		$\Xi_b'(\frac{3}{2}^-) \to \Xi_b'\pi$	$\cdots$	0.47	160	$\cdots$
			$\Xi_b'(\frac{3}{2}^-) \to \Xi_b^*\pi$	160	0.064		
			$\Xi_b'(\frac{3}{2}^-) \to \Xi_b\rho \to \Xi_b\pi\pi$	8.0×10^{-5}			
$\Omega_b(\frac{1}{2}^-)$	6.32 ± 0.11	2 ± 1		$\cdots$		~ 0	$\Omega_b(6316)^-$ [13]
$\Omega_b(\frac{3}{2}^-)$	6.32 ± 0.11			$\cdots$		~ 0	

TABLE III. Decay properties of the *P*-wave bottom baryons belonging to the $[\mathbf{6}_F, 0, 1, \lambda]$ singlet. See the caption of Table II for detailed explanations.

Baryon (j^P)	Mass (GeV)	Difference (MeV)	Decay channels	*S*-wave width (MeV)	*D*-wave width (MeV)	Total width (MeV)	Candidate
$\Sigma_b(\frac{1}{2}^-)$	6.05 ± 0.11	⋯	$\Sigma_b(\frac{1}{2}^-) \to \Lambda_b \pi$	1300	⋯	1300	⋯
$\Xi'_b(\frac{1}{2}^-)$	6.20 ± 0.11	⋯	$\Xi'_b(\frac{1}{2}^-) \to \Xi_b \pi$	990	⋯	1900	⋯
			$\Xi'_b(\frac{1}{2}^-) \to \Lambda_b K$	910	⋯		
$\Omega_b(\frac{1}{2}^-)$	6.34 ± 0.11	⋯	$\Omega_b(\frac{1}{2}^-) \to \Xi_b K$	2700	⋯	2700	⋯

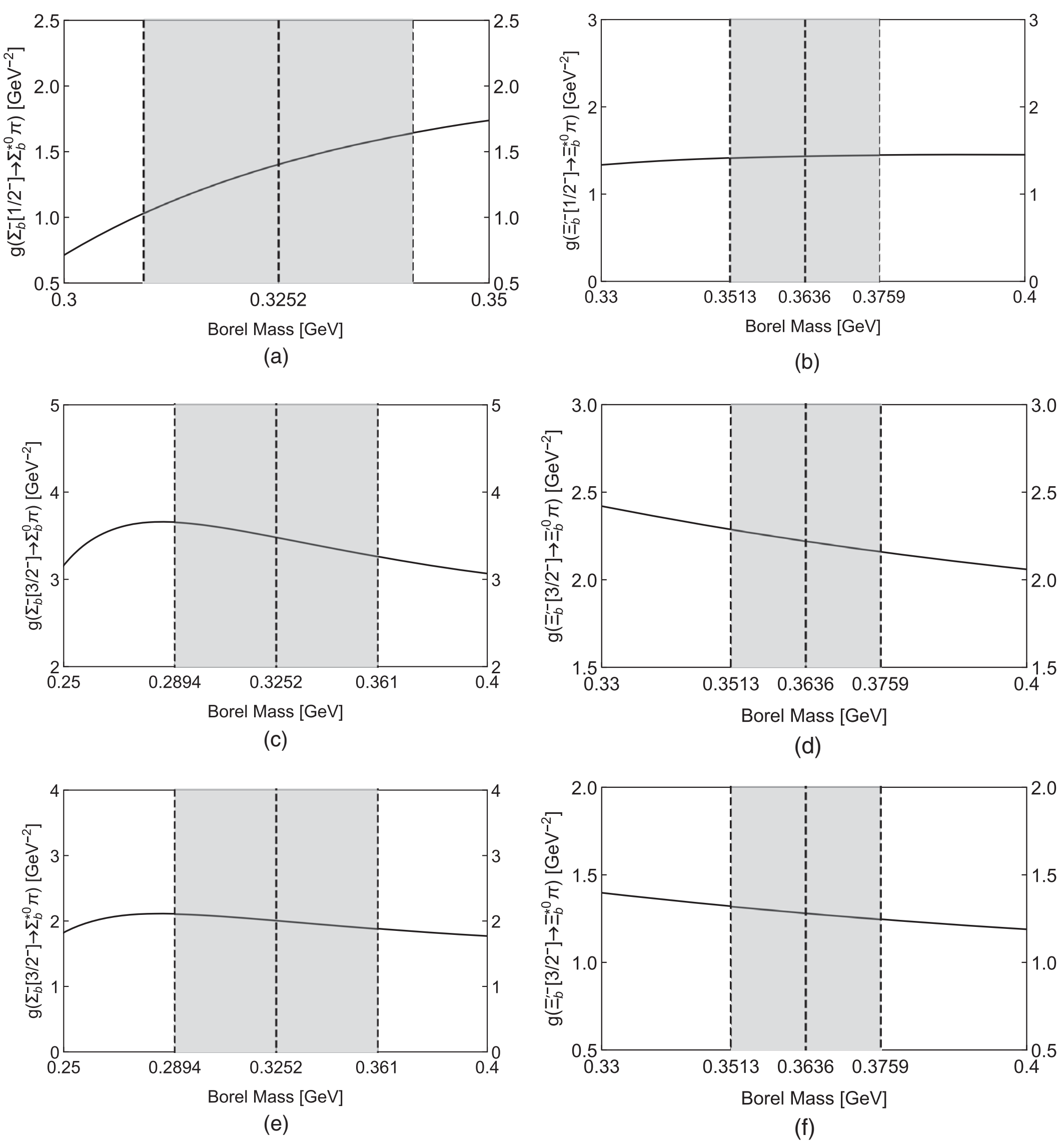


FIG. 3. The coupling constants as functions of the Borel mass T: (a) $g_{\Sigma_b^-[\frac{1}{2}^-]\to\Sigma_b^{*0}\pi^-}$, (b) $g_{\Xi_b^{\prime-}[\frac{1}{2}^-]\to\Xi_b^{*0}\pi^-}$, (c) $g_{\Sigma_b^-[\frac{3}{2}^-]\to\Sigma_b^{0}\pi^-}$, (d) $g_{\Xi_b^{\prime-}[\frac{3}{2}^-]\to\Xi_b^{\prime0}\pi^-}$, (e) $g_{\Sigma_b^-[\frac{3}{2}^-]\to\Sigma_b^{*0}\pi^-}$, and (f) $g_{\Xi_b^{\prime-}[\frac{3}{2}^-]\to\Xi_b^{*0}\pi^-}$. Here the bottom baryon doublet $[\mathbf{6}_F, 1, 1, \lambda]$ is investigated.

$$(a1)\ \Gamma_{\Sigma_b[\frac{1}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]\pi} = 0.62\ \text{MeV},$$
$$(b2)\ \Gamma_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b[\frac{1}{2}^+]\pi} = 0.84\ \text{MeV},$$
$$(b3)\ \Gamma_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]\pi} = 0.098\ \text{MeV},$$
$$(d1)\ \Gamma_{\Xi_b'[\frac{1}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]\pi} = 0.29\ \text{MeV},$$
$$(e3)\ \Gamma_{\Xi_b'[\frac{3}{2}^-]\to\Xi_b'[\frac{1}{2}^+]\pi} = 0.47\ \text{MeV},$$
$$(e5)\ \Gamma_{\Xi_b'[\frac{3}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]\pi} = 0.064\ \text{MeV}. \qquad (60)$$

We summarize these D-wave decay widths in Table II, where possible experimental candidates are given for comparisons. In Refs. [68,69] we have studied the mass spectrum of P-wave bottom baryons, and the results are reanalyzed and summarized in this Table. In Refs. [70,71] we have studied S-wave decay properties of P-wave bottom baryons into ground-state bottom baryons together with pseudoscalar mesons or vector mesons, and the results are also reanalyzed and summarized in this Table.

C. The $[\mathbf{6}_F,0,1,\lambda]$ singlet

There are three bottom baryons contained in the $[\mathbf{6}_F,0,1,\lambda]$ singlet, which are $\Sigma_b(\frac{1}{2}^-)$, $\Xi_b'(\frac{1}{2}^-)$, and $\Omega_b(\frac{1}{2}^-)$. We study their D-wave decays into ground-state bottom baryons and pseudoscalar mesons, but find all the coupling constants to be zero. For completeness, we summarize these results in Table III, together with their mass spectrum, S-wave decay properties, and possible experimental candidates.

D. The $[\mathbf{6}_F,1,1,\lambda]$ doublet

There are six bottom baryons contained in the $[\mathbf{6}_F,1,1,\lambda]$ doublet, which are $\Sigma_b(\frac{1}{2}^-/\frac{3}{2}^-)$, $\Xi_b'(\frac{1}{2}^-/\frac{3}{2}^-)$, and $\Omega_b(\frac{1}{2}^-/\frac{3}{2}^-)$. We study their D-wave decays into ground-state bottom baryons and pseudoscalar mesons, and find twelve nonzero coupling constants:

$$(a1)\ g_{\Sigma_b[\frac{1}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]\pi} = 1.40^{+0.98}_{-1.40}\ \text{GeV}^{-2},$$
$$(b2)\ g_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b[\frac{1}{2}^+]\pi} = 3.48^{+1.86}_{-1.47}\ \text{GeV}^{-2},$$
$$(b3)\ g_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]\pi} = 2.01^{+1.08}_{-0.85}\ \text{GeV}^{-2},$$
$$(d1)\ g_{\Xi_b'[\frac{1}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]\pi} = 1.44^{+0.66}_{-0.61}\ \text{GeV}^{-2},$$
$$(d2)\ g_{\Xi_b'[\frac{1}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]K} = 0.78\ \text{GeV}^{-2},$$
$$(e3)\ g_{\Xi_b'[\frac{3}{2}^-]\to\Xi_b'[\frac{1}{2}^+]\pi} = 2.22^{+0.96}_{-0.80}\text{GeV}^{-2},$$
$$(e4)\ g_{\Xi_b'[\frac{3}{2}^-]\to\Sigma_b[\frac{1}{2}^+]K} = 1.74\ \text{GeV}^{-2},$$
$$(e5)\ g_{\Xi_b'[\frac{3}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]\pi} = 1.28^{+0.58}_{-0.46}\ \text{GeV}^{-2},$$
$$(e6)\ g_{\Xi_b'[\frac{3}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]K} = 1.01\ \text{GeV}^{-2},$$
$$(g1)\ g_{\Omega_b[\frac{1}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]K} = 1.59\ \text{GeV}^{-2},$$
$$(h2)\ g_{\Omega_b[\frac{3}{2}^-]\to\Xi_b'[\frac{1}{2}^+]K} = 2.49\ \text{GeV}^{-2},$$
$$(h3)\ g_{\Omega_b[\frac{3}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]K} = 1.44\ \text{GeV}^{-2}. \qquad (61)$$

We show some of these coupling constants as functions of the Borel mass T in Fig. 3. Based on them, we further find six D-wave decay channels that are kinematically allowed:

TABLE IV. Decay properties of the P-wave bottom baryons belonging to the $[\mathbf{6}_F,1,1,\lambda]$ doublet. See the caption of Table II for detailed explanations.

Baryon (j^P)	Mass (GeV)	Difference (MeV)	Decay channels	S-wave width (MeV)	D-wave width (MeV)	Total width (MeV)	Candidate
$\Sigma_b(\frac{1}{2}^-)$	6.06 ± 0.13	6 ± 3	$\Sigma_b(\frac{1}{2}^-)\to\Sigma_b\pi$	$14.1^{+21.2}_{-10.9}$	⋯	$14.3^{+21.2}_{-10.9}$	⋯
			$\Sigma_b(\frac{1}{2}^-)\to\Sigma_b^*\pi$	⋯	$0.076^{+0.144}_{-0.076}$		
			$\Sigma_b(\frac{1}{2}^-)\to\Lambda_b\rho\to\Lambda_b\pi\pi$	$0.087^{+0.224}_{-0.085}$			
$\Sigma_b(\frac{3}{2}^-)$	6.07 ± 0.13		$\Sigma_b(\frac{3}{2}^-)\to\Sigma_b\pi$	⋯	$0.55^{+0.74}_{-0.36}$	$4.8^{+5.9}_{-2.9}$	⋯
			$\Sigma_b(\frac{3}{2}^-)\to\Sigma_b^*\pi$	$3.9^{+5.8}_{-2.9}$	$0.070^{+0.096}_{-0.047}$		
			$\Sigma_b(\frac{3}{2}^-)\to\Lambda_b\rho\to\Lambda_b\pi\pi$	$0.23^{+0.45}_{-0.20}$			
$\Xi_b'(\frac{1}{2}^-)$	6.21 ± 0.11	7 ± 2	$\Xi_b'(\frac{1}{2})\to\Xi_b'\pi$	$4.5^{+5.8}_{-3.3}$	⋯	$4.7^{+5.8}_{-3.3}$	⋯
			$\Xi_b'(\frac{1}{2}^-)\to\Xi_b^*\pi$	⋯	$0.16^{+0.18}_{-0.10}$		
			$\Xi_b'(\frac{1}{2}^-)\to\Xi_b\rho\to\Xi_b\pi\pi$	$0.043^{+0.079}_{-0.038}$			
$\Xi_b'(\frac{3}{2}^-)$	6.22 ± 0.11		$\Xi_b'(\frac{3}{2}^-)\to\Xi_b'\pi$	⋯	$0.34^{+0.35}_{-0.20}$	$1.8^{+1.07}_{-0.92}$	⋯
			$\Xi_b'(\frac{3}{2}^-)\to\Xi_b^*\pi$	$1.3^{+1.0}_{-0.9}$	$0.051^{+0.057}_{-0.030}$		
			$\Xi_b'(\frac{3}{2}^-)\to\Xi_b\rho\to\Xi_b\pi\pi$	$0.078^{+0.147}_{-0.068}$			
			$\Xi_b'(\frac{3}{2}^-)\to\Xi_b'\rho\to\Xi_b'\pi\pi$	$(5.5^{+6.4}_{-3.5})\times10^{-6}$			
$\Omega_b(\frac{1}{2}^-)$	6.34 ± 0.10	6 ± 2		⋯		~0	$\Omega_b(6330)^-$ [13]
$\Omega_b(\frac{3}{2}^-)$	6.34 ± 0.09			⋯		~0	$\Omega_b(6340)^-$ [13]

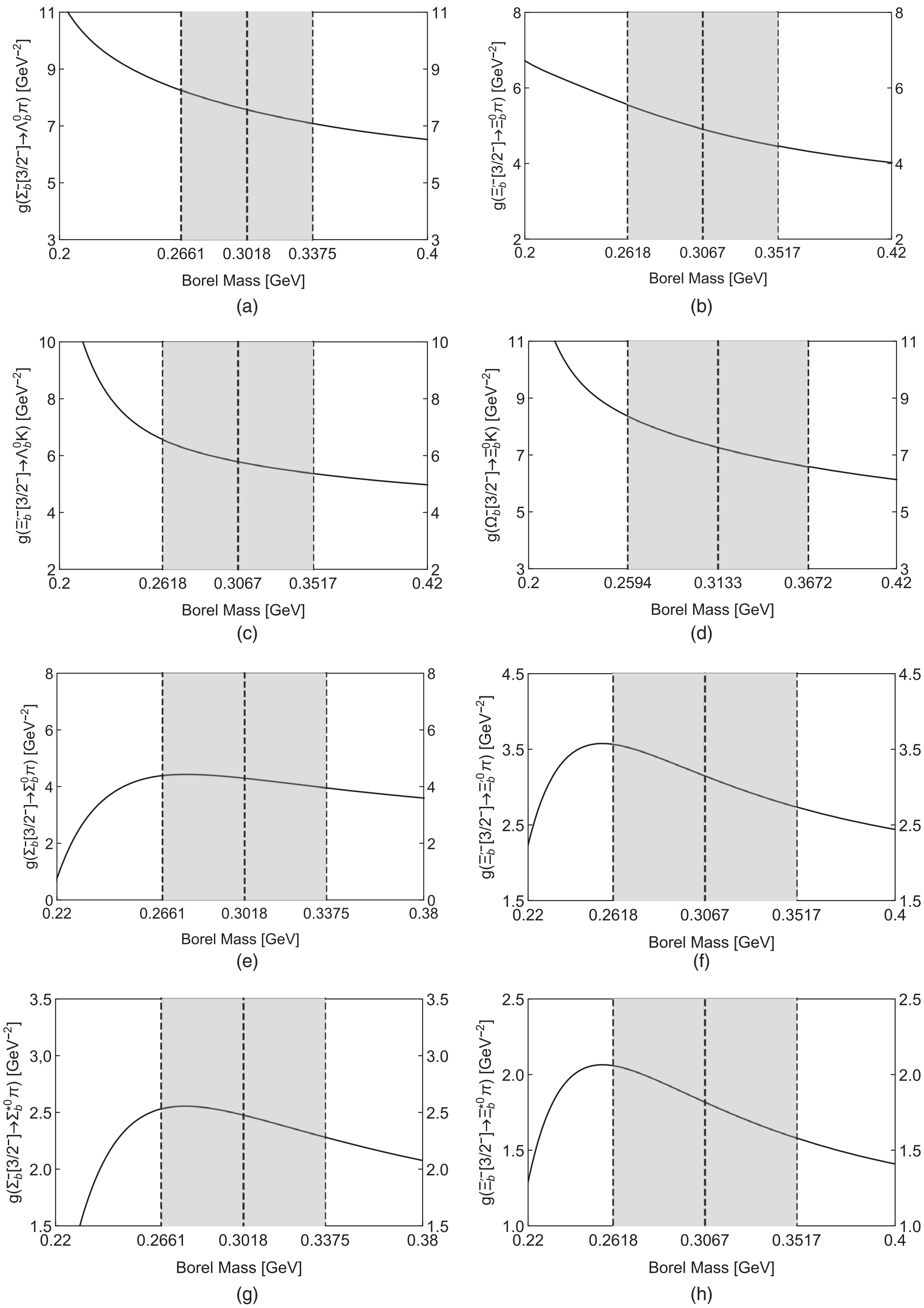


FIG. 4. The coupling constants as functions of the Borel mass T: (a) $g_{\Sigma_b^-[\frac{3}{2}^-]\to\Lambda_b^0\pi^-}$, (b) $g_{\Xi_b^{\prime-}[\frac{3}{2}^-]\to\Xi_b^0\pi^-}$, (c) $g_{\Xi_b^{\prime-}[\frac{3}{2}^-]\to\Lambda_b^0K^-}$, (d) $g_{\Omega_b^-[\frac{3}{2}^-]\to\Xi_b^0K^-}$, (e) $g_{\Sigma_b^-[\frac{3}{2}^-]\to\Sigma_b^0\pi^-}$, (f) $g_{\Xi_b^{\prime-}[\frac{3}{2}^-]\to\Xi_b^{\prime0}\pi^-}$, (g) $g_{\Sigma_b^-[\frac{3}{2}^-]\to\Sigma_b^{*0}\pi^-}$, and (h) $g_{\Xi_b^{\prime-}[\frac{3}{2}^-]\to\Xi_b^{*0}\pi^-}$. Here the bottom baryon doublet $[\mathbf{6}_F, 2, 1, \lambda]$ is investigated.

TABLE V. Decay properties of the P-wave bottom baryons belonging to the $[\mathbf{6}_F, 2, 1, \lambda]$ doublet. See the caption of Table II for detailed explanations.

Baryon (j^P)	Mass (GeV)	Difference (MeV)	Decay channels	S-wave width (MeV)	D-wave width (MeV)	Total width (MeV)	Candidate
$\Sigma_b(\frac{3}{2}^-)$	6.11 ± 0.16	12 ± 5	$\Sigma_b(\frac{3}{2}^-) \to \Lambda_b \pi$	$\cdots$	$49.6^{+76.4}_{-32.9}$	$51.4^{+76.5}_{-32.9}$	$\Sigma_b(6097)^{\pm}$ [12]
			$\Sigma_b(\frac{3}{2}^-) \to \Sigma_b \pi$	$\cdots$	$1.6^{+3.2}_{-1.1}$		
			$\Sigma_b(\frac{3}{2}^-) \to \Sigma_b^* \pi$	$0.019^{+0.065}_{-0.019}$	$0.23^{+0.36}_{-0.16}$		
			$\Sigma_b(\frac{3}{2}^-) \to \Sigma_b \rho \to \Sigma_b \pi\pi$	$(1.4^{+2.5}_{-1.1}) \times 10^{-4}$			
$\Sigma_b(\frac{5}{2}^-)$	6.12 ± 0.15			$\cdots$		~ 0	$\cdots$
$\Xi'_b(\frac{3}{2}^-)$	6.23 ± 0.15	11 ± 5	$\Xi'_b(\frac{3}{2}^-) \to \Xi_b \pi$	$\cdots$	$19.0^{+26.3}_{-13.3}$	$27.3^{+28.5}_{-14.2}$	$\Xi_b(6227)^-$ [11]
			$\Xi'_b(\frac{3}{2}^-) \to \Lambda_b K$	$\cdots$	$7.4^{+11.0}_{-4.8}$		
			$\Xi'_b(\frac{3}{2}^-) \to \Xi'_b \pi$	$\cdots$	$0.79^{+1.13}_{-0.79}$		
			$\Xi'_b(\frac{3}{2}^-) \to \Xi^*_b \pi$	$0.007^{+0.023}_{-0.007}$	$0.12^{+0.17}_{-0.08}$		
			$\Xi'_b(\frac{3}{2}^-) \to \Xi'_b \rho \to \Xi'_b \pi\pi$	$(5.6^{+9.1}_{-4.3}) \times 10^{-4}$			
$\Xi'_b(\frac{5}{2}^-)$	6.24 ± 0.14		$\Xi'_b(\frac{5}{2}^-) \to \Xi^*_b \rho \to \Xi^*_b \pi\pi$	$(2.8^{+4.6}_{-2.0}) \times 10^{-4}$		$(2.8^{+4.6}_{-2.0}) \times 10^{-4}$	$\cdots$
$\Omega_b(\frac{3}{2}^-)$	6.35 ± 0.13	10 ± 4	$\Omega_b(\frac{3}{2}^-) \to \Xi_b K$	$\cdots$	$4.6^{+3.3}_{-1.9}$	$4.6^{+3.3}_{-1.9}$	$\Omega_b(6350)^-$ [13]
$\Omega_b(\frac{5}{2}^-)$	6.36 ± 0.12			$\cdots$		~ 0	$\cdots$

$$
\begin{aligned}
&(a1)\ \Gamma_{\Sigma_b[\frac{1}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]\pi} = 0.076^{+0.144}_{-0.076}\ \text{MeV},\\
&(b2)\ \Gamma_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b[\frac{1}{2}^+]\pi} = 0.55^{+0.74}_{-0.36}\ \text{MeV},\\
&(b3)\ \Gamma_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]\pi} = 0.070^{+0.096}_{-0.047}\ \text{MeV},\\
&(d1)\Gamma_{\Xi'_b[\frac{1}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]\pi} = 0.16^{+0.18}_{-0.10}\ \text{MeV},\\
&(e3)\ \Gamma_{\Xi'_b[\frac{3}{2}^-]\to\Xi'_b[\frac{1}{2}^+]\pi} = 0.34^{+0.35}_{-0.20}\ \text{MeV},\\
&(e5)\ \Gamma_{\Xi'_b[\frac{3}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]\pi} = 0.051^{+0.057}_{-0.030}\ \text{MeV}.
\end{aligned} \qquad (62)
$$

We summarize these D-wave decay widths in Table IV, together with their mass spectrum, S-wave decay properties, and possible experimental candidates.

E. The $[\mathbf{6}_F, 2, 1, \lambda]$ doublet

There are six bottom baryons contained in the $[\mathbf{6}_F, 2, 1, \lambda]$ doublet, which are $\Sigma_b(\frac{3}{2}^-/\frac{5}{2}^-)$, $\Xi'_b(\frac{3}{2}^-/\frac{5}{2}^-)$, and $\Omega_b(\frac{3}{2}^-/\frac{5}{2}^-)$. We study their D-wave decays into ground-state bottom baryons and pseudoscalar mesons, and find twelve nonzero coupling constants:

$$
\begin{aligned}
&(b1)\ g_{\Sigma_b[\frac{3}{2}^-]\to\Lambda_b[\frac{1}{2}^+]\pi} = 7.57^{+4.50}_{-3.18}\ \text{GeV}^{-2},\\
&(b2)\ g_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b[\frac{1}{2}^+]\pi} = 4.29^{+3.13}_{-1.90}\ \text{GeV}^{-2},\\
&(b3)\ g_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]\pi} = 2.48^{+1.48}_{-1.09}\ \text{GeV}^{-2},\\
&(e1)\ g_{\Xi'_b[\frac{3}{2}^-]\to\Xi_b[\frac{1}{2}^+]\pi} = 4.92^{+2.68}_{-2.24}\ \text{GeV}^{-2},\\
&(e2)\ g_{\Xi'_b[\frac{3}{2}^-]\to\Lambda_b[\frac{1}{2}^+]K} = 5.79^{+3.36}_{-2.38}\ \text{GeV}^{-2},\\
&(e3)\ g_{\Xi'_b[\frac{3}{2}^-]\to\Xi'_b[\frac{1}{2}^+]\pi} = 3.15^{+1.76}_{-1.37}\ \text{GeV}^{-2},\\
&(e4)\ g_{\Xi'_b[\frac{3}{2}^-]\to\Sigma_b[\frac{1}{2}^+]K} = 2.43\ \text{GeV}^{-2},\\
&(e5)\ g_{\Xi'_b[\frac{3}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]\pi} = 1.82^{+0.99}_{-0.76}\ \text{GeV}^{-2},\\
&(e6)\ g_{\Xi'_b[\frac{3}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]K} = 1.40\ \text{GeV}^{-2},\\
&(h1)\ g_{\Omega_b[\frac{3}{2}^-]\to\Xi_b[\frac{1}{2}^+]K} = 7.27^{+3.92}_{-2.83}\ \text{GeV}^{-2},\\
&(h2)\ g_{\Omega_b[\frac{3}{2}^-]\to\Xi'_b[\frac{1}{2}^+]K} = 3.78\ \text{GeV}^{-2},\\
&(h3)\ g_{\Omega_b[\frac{3}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]K} = 2.18\ \text{GeV}^{-2}.
\end{aligned} \qquad (63)
$$

We show some of these coupling constants as functions of the Borel mass T in Fig. 4. Based on them, we further find eight D-wave decay channels that are kinematically allowed:

$$
\begin{aligned}
&(b1)\ \Gamma_{\Sigma_b[\frac{3}{2}^-]\to\Lambda_b[\frac{1}{2}^+]\pi} = 49.6^{+76.4}_{-32.9}\ \text{MeV},\\
&(b2)\ \Gamma_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b[\frac{1}{2}^+]\pi} = 1.6^{+3.2}_{-1.1}\ \text{MeV},\\
&(b3)\ \Gamma_{\Sigma_b[\frac{3}{2}^-]\to\Sigma_b^*[\frac{3}{2}^+]\pi} = 0.23^{+0.36}_{-0.16}\ \text{MeV},\\
&(e1)\ \Gamma_{\Xi'_b[\frac{3}{2}^-]\to\Xi_b[\frac{1}{2}^+]\pi} = 19.0^{+26.3}_{-13.3}\ \text{MeV},\\
&(e2)\ \Gamma_{\Xi'_b[\frac{3}{2}^-]\to\Lambda_b[\frac{1}{2}^+]K} = 7.4^{+11.0}_{-4.8}\ \text{MeV},\\
&(e3)\ \Gamma_{\Xi'_b[\frac{3}{2}^-]\to\Xi'_b[\frac{1}{2}^+]\pi} = 0.79^{+1.10}_{-0.79}\ \text{MeV},\\
&(e5)\ \Gamma_{\Xi'_b[\frac{3}{2}^-]\to\Xi_b^*[\frac{3}{2}^+]\pi} = 0.12^{+0.17}_{-0.08}\ \text{MeV},\\
&(h1)\ \Gamma_{\Omega_b[\frac{3}{2}^-]\to\Xi_b[\frac{1}{2}^+]K} = 4.6^{+3.3}_{-1.9}\ \text{MeV}.
\end{aligned} \qquad (64)
$$

We summarize these D-wave decay widths in Table V, together with their mass spectrum, S-wave decay properties, and possible experimental candidates.

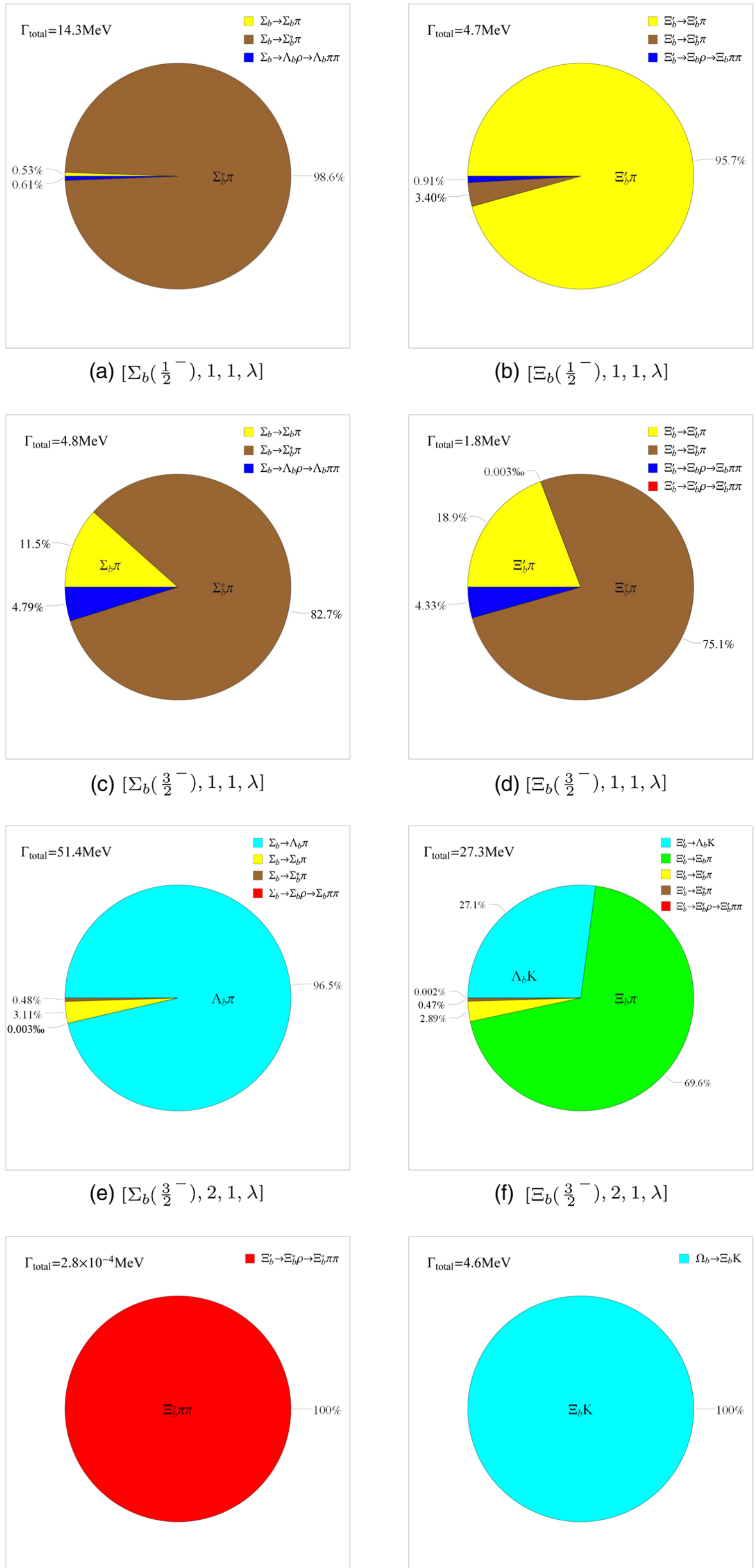
TABLE VI. Among all the possible P-wave bottom baryons of the flavor $\mathbf{6}_F$, we find altogether four Σ_b, four $\Xi_b^\prime$, and six Ω_b baryons, with limited widths (<100 MeV) and so capable of being observed. Their masses, mass splittings within the same multiplets, and decay properties are extracted for future experimental searches. We note that there are considerable uncertainties in our results for the absolute values of the bottom baryon masses due to their significant dependence on the bottom quark mass [68,69]; however, their mass splittings within the same doublets do not depend much on this, so they are produced quite well with much less uncertainties and give more useful information; moreover, we can extract even more useful information from S- and D-wave strong decay properties of P-wave bottom baryons.

B	Multiplet	Baryon (j^P)	Mass (GeV)	Difference (MeV)	Decay channel	Total width (MeV)	Candidate
Σ_b	$[\mathbf{6}_F, 1, 1, \lambda]$	$\Sigma_b(\frac{1}{2}^-)$	6.06 ± 0.13	6 ± 3	$\Gamma(\Sigma_b(\frac{1}{2}^-) \to \Sigma_b \pi) = 14.1^{+21.2}_{-10.9}$ MeV	$14.3^{+21.2}_{-10.9}$	⋯
					$\Gamma(\Sigma_b(\frac{1}{2}^-) \to \Sigma_b^* \pi) = 0.076^{+0.144}_{-0.076}$ MeV		
					$\Gamma(\Sigma_b(\frac{1}{2}^-) \to \Lambda_b \rho \to \Lambda_b \pi\pi) = 0.087$ MeV		
		$\Sigma_b(\frac{3}{2}^-)$	6.07 ± 0.13		$\Gamma(\Sigma_b(\frac{3}{2}^-) \to \Sigma_b \pi) = 0.55^{+0.74}_{-0.36}$ MeV	$4.8^{+5.9}_{-2.9}$	⋯
					$\Gamma(\Sigma_b(\frac{3}{2}^-) \to \Sigma_b^* \pi) = 4.0^{+5.8}_{-2.9}$ MeV		
					$\Gamma(\Sigma_b(\frac{3}{2}^-) \to \Lambda_b \rho \to \Lambda_b \pi\pi) = 0.23$ MeV		
	$[\mathbf{6}_F, 2, 1, \lambda]$	$\Sigma_b(\frac{3}{2}^-)$	6.11 ± 0.16	12 ± 5	$\Gamma(\Sigma_b(\frac{3}{2}^-) \to \Lambda_b \pi) = 49.6^{+76.4}_{-32.9}$ MeV	$51.4^{+76.5}_{-32.9}$	$\Sigma_b(6097)^\pm$
					$\Gamma(\Sigma_b(\frac{3}{2}^-) \to \Sigma_b \pi) = 1.6^{+3.2}_{-1.1}$ MeV		
					$\Gamma(\Sigma_b(\frac{3}{2}^-) \to \Sigma_b^* \pi) = 0.25^{+0.37}_{-0.16}$ MeV		
					$\Gamma(\Sigma_b(\frac{3}{2}^-) \to \Sigma_b \rho \to \Sigma_b \pi\pi) = 1.4 \times 10^{-4}$ MeV		
		$\Sigma_b(\frac{5}{2}^-)$	6.12 ± 0.15		⋯	~ 0	⋯
$\Xi_b^\prime$	$[\mathbf{6}_F, 1, 1, \lambda]$	$\Xi_b^\prime(\frac{1}{2}^-)$	6.21 ± 0.11	7 ± 2	$\Gamma(\Xi_b^\prime(\frac{1}{2}) \to \Xi_b^\prime \pi) = 4.5^{+5.8}_{-3.3}$ MeV	$4.7^{+5.8}_{-3.3}$	⋯
					$\Gamma(\Xi_b^\prime(\frac{1}{2}^-) \to \Xi_b^* \pi) = 0.16^{+0.18}_{-0.10}$ MeV		
					$\Gamma(\Xi_b^\prime(\frac{1}{2}^-) \to \Xi_b \rho \to \Xi_b \pi\pi) = 0.043$ MeV		
		$\Xi_b^\prime(\frac{3}{2}^-)$	6.22 ± 0.11		$\Gamma(\Xi_b^\prime(\frac{3}{2}^-) \to \Xi_b^\prime \pi) = 0.34^{+0.35}_{-0.20}$ MeV	$1.8^{+1.07}_{-0.96}$	⋯
					$\Gamma(\Xi_b^\prime(\frac{3}{2}^-) \to \Xi_b^* \pi) = 1.4^{+1.0}_{-0.9}$ MeV		
					$\Gamma(\Xi_b^\prime(\frac{3}{2}^-) \to \Xi_b \rho \to \Xi_b \pi\pi) = 0.078$ MeV		
					$\Gamma(\Xi_b^\prime(\frac{3}{2}^-) \to \Xi_b^\prime \rho \to \Xi_b^\prime \pi\pi) = 5.5 \times 10^{-6}$ MeV		
	$[\mathbf{6}_F, 2, 1, \lambda]$	$\Xi_b^\prime(\frac{3}{2}^-)$	6.23 ± 0.15	11 ± 5	$\Gamma(\Xi_b^\prime(\frac{3}{2}^-) \to \Xi_b \pi) = 19.0^{+26.3}_{-13.3}$ MeV	$27.3^{+28.5}_{-14.2}$	$\Xi_b(6227)^-$
					$\Gamma(\Xi_b^\prime(\frac{3}{2}^-) \to \Lambda_b K) = 7.4^{+11.0}_{-4.8}$ MeV		
					$\Gamma(\Xi_b^\prime(\frac{3}{2}^-) \to \Xi_b^\prime \pi) = 0.79^{+1.13}_{-0.79}$ MeV		
					$\Gamma(\Xi_b^\prime(\frac{3}{2}^-) \to \Xi_b^* \pi) = 0.13^{+0.17}_{-0.08}$ MeV		
					$\Gamma(\Xi_b^\prime(\frac{3}{2}^-) \to \Xi_b^\prime \rho \to \Xi_b^\prime \pi\pi) = 5.6 \times 10^{-4}$ MeV		
		$\Xi_b^\prime(\frac{5}{2}^-)$	6.24 ± 0.14		$\Gamma(\Xi_b^\prime(\frac{5}{2}^-) \to \Xi_b^* \rho \to \Xi_b^* \pi\pi) = 2.8 \times 10^{-4}$ MeV	~ 0	⋯
Ω_b	$[\mathbf{6}_F, 1, 0, \rho]$	$\Omega_b(\frac{1}{2}^-)$	6.32 ± 0.11	2 ± 1	⋯	~ 0	$\Omega_b(6316)^-$
		$\Omega_b(\frac{3}{2}^-)$	6.32 ± 0.11		⋯	~ 0	
	$[\mathbf{6}_F, 1, 1, \lambda]$	$\Omega_b(\frac{1}{2}^-)$	6.34 ± 0.10	6 ± 2	⋯	~ 0	$\Omega_b(6330)^-$
		$\Omega_b(\frac{3}{2}^-)$	6.34 ± 0.09		⋯	~ 0	$\Omega_b(6340)^-$
	$[\mathbf{6}_F, 2, 1, \lambda]$	$\Omega_b(\frac{3}{2}^-)$	6.35 ± 0.13	10 ± 4	$\Gamma(\Omega_b(\frac{3}{2}^-) \to \Xi_b K) = 4.6^{+3.3}_{-1.9}$ MeV	$4.6^{+3.3}_{-1.9}$	$\Omega_b(6350)^-$
		$\Omega_b(\frac{5}{2}^-)$	6.36 ± 0.12		⋯	~ 0	⋯

IV. SUMMARY AND DISCUSSIONS

To summarize this paper, we have investigated the P-wave bottom baryons belonging to the $SU(3)$ flavor $\mathbf{6}_F$ representation, and studied their D-wave decays into ground-state bottom baryons and pseudoscalar mesons. Together with Refs. [68–71], we have performed a rather complete study on both mass spectra and decay properties of P-wave bottom baryons using the method of QCD sum rules and light-cone sum rules within the framework of heavy quark effective theory.

Accordingly to the heavy quark effective theory, we categorize the P-wave bottom baryons of the $SU(3)$ flavor $\mathbf{6}_F$ into four multiplets: $[\mathbf{6}_F, 1, 0, \rho]$, $[\mathbf{6}_F, 0, 1, \lambda]$, $[\mathbf{6}_F, 1, 1, \lambda]$, and $[\mathbf{6}_F, 2, 1, \lambda]$. In this paper we have studied their D-wave decay properties, and the results are separately summarized in Tables II–V. Besides, in Refs. [68,69] we have studied the mass spectrum of P-wave bottom baryons, and the results are reanalyzed and summarized in these Tables; in Refs. [70,71] we have studied S-wave decay properties of P-wave bottom baryons



(a) $[\Sigma_b(\frac{1}{2}^-), 1, 1, \lambda]$ (b) $[\Xi_b(\frac{1}{2}^-), 1, 1, \lambda]$

(c) $[\Sigma_b(\frac{3}{2}^-), 1, 1, \lambda]$ (d) $[\Xi_b(\frac{3}{2}^-), 1, 1, \lambda]$

(e) $[\Sigma_b(\frac{3}{2}^-), 2, 1, \lambda]$ (f) $[\Xi_b(\frac{3}{2}^-), 2, 1, \lambda]$

(g) $[\Xi_b(\frac{5}{2}^-), 2, 1, \lambda]$ (h) $[\Omega_b(\frac{5}{2}^-), 2, 1, \lambda]$

FIG. 5. Branching ratios of eight P-wave bottom baryons of the flavor $\mathbf{6}_F$, with limited but nonzero widths ($0 < \Gamma < 100$ MeV). See the caption of Table VI for detailed explanations.

into ground-state bottom baryons together with pseudoscalar mesons or vector mesons, and the results are also reanalyzed and summarized in these Tables.

Before drawing our conclusions, we note that there are considerable (theoretical) uncertainties in our results for the absolute values of the bottom baryon masses due to their significant dependence on the bottom quark mass [68,69]; however, their mass splittings within the same doublets do not depend much on this, so they are produced quite well with much less (theoretical) uncertainties and give more useful information; moreover, we can extract even (much) more useful information from *S*- and *D*-wave strong decay properties of *P*-wave bottom baryons. Based on the results summarized in Tables II–V, we can well understand *P*-wave bottom baryons as a whole:

(i) The $[\mathbf{6}_F, 0, 1, \lambda]$ singlet contains three bottom baryons: $\Sigma_b(\frac{1}{2}^-)$, $\Xi'_b(\frac{1}{2}^-)$, and $\Omega_b(\frac{1}{2}^-)$. Their total widths are all calculated to be very large, preventing them from being observed in any experiment.

(ii) The $[\mathbf{6}_F, 1, 0, \rho]$ doublet contains six bottom baryons: $\Sigma_b(\frac{1}{2}^-/\frac{3}{2}^-)$, $\Xi'_b(\frac{1}{2}^-/\frac{3}{2}^-)$, and $\Omega_b(\frac{1}{2}^-/\frac{3}{2}^-)$. The total widths of $\Sigma_b(\frac{1}{2}^-/\frac{3}{2}^-)$ and $\Xi'_b(\frac{1}{2}^-/\frac{3}{2}^-)$ are all calculated to be very large, while the total widths of $\Omega_b(\frac{1}{2}^-/\frac{3}{2}^-)$ are both extracted to be zero.

(iii) The $[\mathbf{6}_F, 1, 1, \lambda]$ doublet contains six bottom baryons: $\Sigma_b(\frac{1}{2}^-/\frac{3}{2}^-)$, $\Xi'_b(\frac{1}{2}^-/\frac{3}{2}^-)$, and $\Omega_b(\frac{1}{2}^-/\frac{3}{2}^-)$. Their total widths are all calculated to be less than 100 MeV.

(iv) The $[\mathbf{6}_F, 2, 1, \lambda]$ doublet contains six bottom baryons: $\Sigma_b(\frac{3}{2}^-/\frac{5}{2}^-)$, $\Xi'_b(\frac{3}{2}^-/\frac{5}{2}^-)$, and $\Omega_b(\frac{3}{2}^-/\frac{5}{2}^-)$. Their total widths are all calculated to be less than 100 MeV.

Hence, among all the possible *P*-wave bottom baryons of the flavor $\mathbf{6}_F$, we find altogether four Σ_b, four Ξ'_b, and six Ω_b baryons, with limited widths (< 100 MeV) and so capable of being observed. Their masses, mass splittings within the same multiplets, and decay properties are summarized in Table VI. Their possible experimental candidates are also given in this Table for comparison. Among these fourteen bottom baryons, eight of them have nonzero decay widths, whose branching ratios are shown in Fig. 5 using pie charts. We suggest the LHCb and CMS collaborations to search for these excited bottom baryons, but note that it still depends on the production rates whether these baryons can be observed or not. Especially, it is interesting to further investigate the $\Lambda_b(6072)^0$, i.e., the broad excess of events in the $\Lambda_b^0\pi^+\pi^-$ mass distribution in the region of 6040–6100 MeV [15,16].

In the present study the ρ-mode doublet $[\mathbf{6}_F, 1, 0, \rho]$ is found to be lower than the two λ-mode doublets $[\mathbf{6}_F, 1, 1, \lambda]$ and $[\mathbf{6}_F, 2, 1, \lambda]$, a behavior which is consistent with our previous results for their corresponding doublets of the $SU(3)$ flavor $\bar{\mathbf{3}}_F$ [68,69], but in contrast to the quark model expectation [2,21]. However, this is possible simply because the mass differences between different multiplets have considerable uncertainties in our framework, similar to the absolute values of baryon masses, but unlike the mass differences within the same multiplet. We propose to verify whether the ρ-mode doublet $[\mathbf{6}_F, 1, 0, \rho]$ exists or not by investigating: (a) the spin-parity quantum number of the $\Omega_b(6316)^-$, (b) whether it can be separated into two states almost degenerate, and (c) whether its Σ_b and Ξ'_b partner states can be observed.

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APPENDIX: SUM RULE EQUATIONS

In this Appendix we give several examples of sum rule equations, which are used to study *D*-wave decays of *P*-wave bottom baryons into ground-state bottom baryons and pseudoscalar mesons.

The sum rule equation for the $\Xi'^-_b[\frac{1}{2}^-]$ belonging to $[\mathbf{6}_F, 1, 0, \rho]$ is

$$
\begin{aligned}
G_{\Xi'^-_b[\frac{1}{2}^-]\to\Xi^{*0}_b\pi^-}(\omega,\omega') &= g_{\Xi'^-_b[\frac{1}{2}^-]\to\Xi^{*0}_b\pi^-} \times \frac{f_{\Xi^-_b[\frac{1}{2}^-]} f_{\Xi^{*0}_b}}{(\bar{\Lambda}_{\Xi^-_b[\frac{1}{2}^-]} - \omega')(\bar{\Lambda}_{\Xi^{*0}_b} - \omega)} \\
&= \int_0^\infty dt \int_0^1 du e^{i(1-u)\omega' t} e^{iu\omega t} \times 4 \times \left(\frac{f_\pi m_s^2 m_\pi^2 u}{96(m_u+m_d)\pi^2} \phi^\sigma_{3;\pi}(u) + \frac{f_\pi m_s u}{128\pi^2} \phi_{4;\pi}(u) \right. \\
&\quad + \frac{f_\pi u}{24} \langle\bar{s}s\rangle \phi_{2;\pi}(u) + \frac{f_\pi m_s u}{8\pi^2 t^2} \phi_{2;\pi}(u) + \frac{f_\pi m_\pi^2 u}{24(m_u+m_d)\pi^2 t^2} \phi^\sigma_{3;\pi}(u) + \frac{f_\pi m_s m_\pi^2 u t^2}{576(m_u+m_d)} \langle\bar{s}s\rangle \phi^\sigma_{3;\pi}(u) \\
&\quad \left. + \frac{f_\pi u t^2}{384} \langle\bar{s}s\rangle \phi_{4;\pi}(u) + \frac{f_\pi u t^2}{384} \langle g_s\bar{s}\sigma G s\rangle \phi_{2;\pi}(u) + \frac{f_\pi u t^4}{6144} \langle g_s\bar{s}\sigma G s\rangle \phi_{4;\pi}(u) \right)
\end{aligned}
$$

$$
\begin{aligned}
&-\int_0^\infty dt\int_0^1 du\int \mathcal{D}\underline{\alpha} e^{i\omega' t(\alpha_2+u\alpha_3)}e^{i\omega t(1-\alpha_2-u\alpha_3)}\times\frac{1}{2}\times\left(\frac{if_{3\pi}u^2\alpha_3}{4\pi^2 t v\cdot q}\Phi_{3;\pi}(\underline{\alpha})+\frac{if_{3\pi}u\alpha_2 v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})\right.\\
&+\frac{if_{3\pi}u\alpha_3 v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})-\frac{if_{3\pi}uv\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})+\frac{if_{3\pi}\alpha_2 v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})-\frac{if_{3\pi}v\cdot q}{4\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})\\
&\left.-\frac{f_{3\pi}u}{4\pi^2 t^2}\Phi_{3;\pi}(\underline{\alpha})\right). \qquad \text{(A1)}
\end{aligned}
$$

The sum rule equation for the $\Omega_b^-[\frac{1}{2}^-]$ belonging to $[\mathbf{6}_F,0,1,\lambda]$ is

$$
G_{\Omega_b^-[\frac{1}{2}^-]\to\Xi_b^{*0}K^-}(\omega,\omega')=g_{\Omega_b^-[\frac{1}{2}^-]\to\Xi_b^{*0}K^-}\times\frac{f_{\Omega_b^-[\frac{1}{2}^-]}f_{\Xi_b^{*0}}}{(\bar\Lambda_{\Omega_b^-[\frac{1}{2}^-]}-\omega')(\bar\Lambda_{\Xi_b^{*0}}-\omega)}=0. \qquad \text{(A2)}
$$

The sum rule equation for the $\Sigma_b^-[\frac{3}{2}^-]$ belonging to $[\mathbf{6}_F,1,1,\lambda]$ is

$$
\begin{aligned}
G_{\Sigma_b^-[\frac{3}{2}^-]\to\Sigma_b^0\pi^-}(\omega,\omega')&=g_{\Sigma_b^-[\frac{3}{2}^-]\to\Sigma_b^0\pi^-}\times\frac{f_{\Sigma_b^-[\frac{3}{2}^-]}f_{\Sigma_b^0}}{(\bar\Lambda_{\Sigma_b^-[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Sigma_b^0}-\omega)}\\
&=\int_0^\infty dt\int_0^1 du e^{i(1-u)\omega' t}e^{iu\omega t}\times 8\times\left(\frac{if_\pi u}{4\pi^2 t^3}\phi_{2;\pi}(u)+\frac{if_\pi u}{64\pi^2 t}\phi_{4;\pi}(u)\right.\\
&\left.-\frac{if_\pi m_\pi^2 ut}{144(m_u+m_d)}\langle\bar qq\rangle\phi_{3;\pi}^\sigma(u)-\frac{if_\pi m_\pi^2 ut^3}{2304(m_u+m_d)}\langle g_s\bar q\sigma Gq\rangle\phi_{3;\pi}^\sigma(u)\right)\\
&-\int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha}e^{i\omega' t(\alpha_2+u\alpha_3)}e^{i\omega t(1-\alpha_2-u\alpha_3)}\times\left(\frac{if_\pi u^2\alpha_3}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha})+\frac{if_\pi u\alpha_2}{8\pi^2 t}\Phi_{4;\pi}(\underline{\alpha})\right.\\
&+\frac{if_\pi u\alpha_3}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha})+\frac{if_\pi u\alpha_3}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha})-\frac{if_\pi u}{8\pi t}\Phi_{4;\pi}(\underline{\alpha})+\frac{if_\pi\alpha_2}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha})+\frac{if_\pi\alpha_2}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha})\\
&-\frac{if_\pi}{16\pi^2 t}\Phi_{4;\pi}(\underline{\alpha})-\frac{if_\pi}{16\pi^2 t}\tilde\Phi_{4;\pi}(\underline{\alpha})+\frac{f_\pi u}{8\pi^2 t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha})+\frac{3f_\pi u}{8\pi^2 t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha})-\frac{f_\pi}{8\pi^2 t^2 v\cdot q}\Phi_{4;\pi}(\underline{\alpha})\\
&\left.-\frac{3f_\pi}{8\pi^2 t^2 v\cdot q}\tilde\Phi_{4;\pi}(\underline{\alpha})+\frac{f_\pi}{8\pi^2 t^2 v\cdot q}\Psi_{4;\pi}(\underline{\alpha})-\frac{f_\pi}{8\pi^2 t^2 v\cdot q}\tilde\Psi_{4;\pi}(\underline{\alpha})\right). \qquad \text{(A3)}
\end{aligned}
$$

The sum rule equation for the $\Sigma_b^-[\frac{3}{2}^-]$ belonging to $[\mathbf{6}_F,2,1,\lambda]$ is

$$
\begin{aligned}
G_{\Sigma_b^-[\frac{3}{2}^-]\to\Lambda_b^0\pi^-}(\omega,\omega')&=g_{\Sigma_b^-[\frac{3}{2}^-]\to\Lambda_b^0\pi^-}\times\frac{f_{\Sigma_b^-[\frac{3}{2}^-]}f_{\Lambda_b^0}}{(\bar\Lambda_{\Sigma_b^-[\frac{3}{2}^-]}-\omega')(\bar\Lambda_{\Lambda_b^0}-\omega)}\\
&=\int_0^\infty dt\int_0^1 du e^{i(1-u)\omega' t}e^{iu\omega t}\times 8\times\left(\frac{f_\pi m_\pi^2 u}{12(m_u+m_d)\pi^2 t^2}\phi_{3;\pi}^\sigma(u)+\frac{f_\pi ut^2}{12}\langle\bar qq\rangle\phi_{2;\pi}(u)\right.\\
&\left.+\frac{f_\pi ut^2}{192}\langle\bar qq\rangle\phi_{4;\pi}(u)+\frac{f_\pi u}{192}\langle g_s\bar q\sigma Gq\rangle\phi_{2;\pi}(u)+\frac{f_\pi ut^4}{3072}\langle g_s\bar q\sigma Gq\rangle\phi_{4;\pi}(u)\right)\\
&-\int_0^\infty dt\int_0^1 du\int\mathcal{D}\underline{\alpha}e^{i\omega' t(\alpha_2+u\alpha_3)}e^{i\omega t(1-\alpha_2-u\alpha_3)}\times\left(\frac{f_3\pi u}{2\pi^2 t^2}\Phi_{3;\pi}(\underline{\alpha})-\frac{f_{3\pi}}{2\pi^2 t^2}\Phi_{3;\pi}(\underline{\alpha})\right.\\
&\left.+\frac{if_{3\pi}u^2\alpha_3 v\cdot q}{2\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})+\frac{if_{3\pi}u\alpha_2 v\cdot q}{2\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})-\frac{if_{3\pi}uv\cdot q}{2\pi^2 t}\Phi_{3;\pi}(\underline{\alpha})\right). \qquad \text{(A4)}
\end{aligned}
$$

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