Tensor supercurrent in QCD

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An external Abelian magnetic field excites in the QCD vacuum a tensor supercurrent that represents the tensor polarization of the chiral condensate. This tensor supercurrent can be deduced from the chiral Lagrangian in the presence of anomalies; a similar tensor supercurrent emerges in rotating systems at finite chemical potential. We discuss the microscopic origin of this supercurrent and argue that it screens the instanton–anti-instanton molecules $I\bar{I}$ in the QCD vacuum, similarly to the vector supercurrent screening Abrikosov vortices in a superconductor. A number of possible experimental manifestations of the tensor supercurrent are discussed: (i) spin alignment of axial-vector and vector mesons in heavy ion collisions; (ii) tensor charge of the nucleon; (iii) transversity of quark distributions in polarized nucleons.

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I. INTRODUCTION

Since the early work of 't Hooft [1] and Mandelstam [2], it is widely believed that the QCD vacuum in the confined phase can be viewed as a dual superconductor. Due to the dual Meissner effect, the chromoelectric field gets repelled by the condensate of chromomagnetic degrees of freedom, and the emerging chromoelectric strings confine the quarks and bind them into mesons and baryons. While this qualitative picture is simple and attractive, it is not yet clear how to realize it microscopically in QCD. In particular, the nature of the chromomagnetic objects that condense in the vacuum is still not entirely understood, in spite of significant advances made over the last decades (see e.g., [3,4] and references therein). For example, apart from magnetic monopoles proposed originally, the confinement can also arise from the condensation of closed chromomagnetic strings [5].

In Londons' theory of conventional superconductivity, the flux of magnetic field inside an Abrikosov vortex is surrounded by the electric supercurrent proportional to the condensate that screens the magnetic field in the bulk of a superconductor. In the dual superconductor model of confinement, it is thus natural to expect that the confining chromoelectric flux is surrounded by the supercurrent of chromomagnetic charges that shields the vacuum from the chromoelectric field. Does this supercurrent exist in QCD? If so, what are the consequences for the structure of hadrons? The answers to these questions are still lacking.

Quarks interact with the chromomagnetic degrees of freedom, and this interaction should affect both the chiral condensate and the hadrons. Since quarks, in addition to color charges, possess also the electric charge, they respond to an external magnetic field that can thus be used as a probe of nonperturbative QCD dynamics.

Indeed, an external Abelian magnetic field has emerged as a powerful probe of the QCD vacuum [6–8]. In chiral theory, one expects that the chiral condensate increases [9] in an external magnetic field (in accord with the "magnetic catalysis" scenario [8,10]), and a constant density of magnetic moment gets generated [11]. At finite chemical potentials for the chiral and vector charges, the chiral magnetic effect [12] and chiral separation effect [13] are induced, see [6,14] for reviews. These effects have been discussed not only in the deconfined phase, but also in confined, chirally broken phases, see e.g., [15–17]. It was argued that in the confined phase the pionic effective strings could play an important role providing their core for the propagation of dissipationless chiral currents [18].

In this paper, we focus on a vacuum tensor current that emerges at zero temperature and zero chemical potential in an external abelian magnetic field. The nonvanishing v.e.v. of this tensor current gives rise to both the dipole magnetic moment of the vacuum and the current circulating in the plane perpendicular to the external magnetic field. The existence of this tensor supercurrent can be implied from the relations between the magnetic [19] and vortical [20] susceptibilities of the quark condensate and the quantum anomalies. Once the term describing the anomalous response of the quark condensate is added to the effective chiral Lagrangian [19], it leads to the tensor current

$$J_{\mu\nu} = \chi \langle \bar{\Psi}\Psi \rangle F_{\mu\nu}, \tag{1}$$

where an external magnetic field is described by the field strength tensor $F_{\mu\nu}$, and $\langle \bar{\Psi}\Psi \rangle$ is the quark condensate. The value of magnetic susceptibility of the condensate was discussed within several approaches; the Vainshtein relation [21]

$$\chi = -\frac{N_c}{4\pi^2 f_\pi^2}, \tag{2}$$

was derived from the VVA anomalous triangle diagram with the use of pion dominance in the axial channel.

In this paper we will address the following two questions:

(1) What are the microscopic carriers of the tensor current in confined QCD? Since the current can be excited by an arbitrarily weak magnetic field, it has to be carried by very light degrees of freedom.

(2) The tensor supercurrent is proportional to the chiral condensate which breaks the global chiral symmetry. Can we interpret this current in analogy with superfluidity or superconductivity? Does this current emerge at the boundary of a domain where the chiral symmetry is partially or completely restored, similar to supercurrents around strings and vortices in superfluidity and superconductivity?

These questions can be addressed both in Euclidean and Minkowski spaces. The Euclidean QCD vacuum with a quark condensate, according to the Casher-Banks relation [22], is characterized by a finite density of quasizero Dirac operator eigenmodes which are delocalized in 4d Euclidean space-time. It was shown in [23] that magnetic susceptibility of the quark condensate is saturated by the zero modes of 4D Dirac operator; therefore the key contribution to the tensor current should involve the defects supporting such zero modes. The simplest Euclidean defect supporting fermionic zero mode is the instanton. The behavior of the fermion in the background of a single instanton in the external magnetic field has been considered in [24]. It was found that a single instanton gives rise to a dipole electric moment of the quark quasizero modes, in agreement with the lattice QCD study [25]; see [26] for a related observation for a polarized nucleon. Since an anti-instanton develops an electric dipole moment of an opposite orientation, it can be expected that a pair of an instanton and an anti-instanton develops a tensor electric moment, in accord with the emergence of the tensor current from the effective theory. While the microscopic picture can become quite complicated due to the instanton–anti-instanton interactions, the effective chiral Lagrangian allows us to fix the magnitude of the tensor current in terms of the quark condensate.

We advocate here the following interpretation: in an external magnetic field, the tensor supercurrent in 4D Euclidean space-time screens the $I\bar{I}$ molecules in the vacuum ensemble. Indeed, the $I\bar{I}$ molecule supports the fermion zero modes [27] that in an external magnetic field develop the tensor electric moment [24]. Since this tensor moment should be absent in the empty vacuum surrounding the molecule, it is screened by the tensor supercurrent. In other words, one can say that magnetic field probes the instanton molecule component of the QCD vacuum and induces the tensor supercurrent surrounding the individual molecules.

The paper is organized as follows. First we describe how the 2-form supercurrent emerges in confined phase of QCD within the low-energy effective theory in Sec. II. In Sec. III we consider the microscopic aspects of tensor supercurrent and argue its relevance for the screening of the $I\bar{I}$ molecules. In Sec. IV we describe the possible experimental manifestations of the tensor supercurrent. The comparison with the screening currents familiar in superconductivity and superfluidity, as well as interpretation of the tensor supercurrent as a conserved 2-form current of broken 1-form global symmetry in hydrodynamics is presented in Discussion. Some open questions are formulated in the Conclusion.

II. TENSOR SUPERCURRENT IN THE CONFINED PHASE OF QCD

A. Tensor currents and dipole moments

In this section we explain how the 2-form currents emerge from the polarization of the chiral condensate [19,20]. In hadronic phase of QCD, the chiral condensate $\langle \bar{\Psi}\Psi \rangle$ breaks the $SU(N_f)_L \times SU(N_f)_R$ symmetry of the Lagrangian to the diagonal $SU(N_f)$ subgroup. Let us consider the response of this chiral condensate to an external electromagnetic field:

$$\langle 0|\bar{\Psi}_f \sigma_{\mu\nu} \Psi_f|0\rangle = \chi e_f \langle \bar{\Psi}\Psi \rangle F_{\mu\nu}, \tag{3}$$

and

$$\langle 0|\bar{\Psi}_f \sigma_{\mu\nu}\gamma_5 \Psi_f|0\rangle = \tilde{\chi} e_f \langle \bar{\Psi}\Psi \rangle \tilde{F}_{\mu\nu}, \tag{4}$$

where e_f is the electric charge of the quark with flavor f, and $\sigma_{\mu\nu} = \frac{1}{2i}[\gamma_\mu, \gamma_\nu]$ is the relativistic spin operator. Since in four dimensions

$$\sigma_{\mu\nu} = i\epsilon_{\mu\nu\alpha\beta}\sigma_{\alpha\beta}\gamma_5, \tag{5}$$

the electric and magnetic susceptibilities of the condensate χ and $\tilde{\chi}$ are related. The value of magnetic susceptibility introduced in [11] has been derived from the anomalous $\langle VVA \rangle$ triangle [28] [as given by (2)], in holography via 5d Chern-Simons term [29], in an extended holographic model [20,30] and in lattice QCD [23].

The physical interpretation of the vacuum tensor currents (3) and (4) is as follows: the $\bar{\Psi}_f \sigma_{0i} \Psi_f$ component of (3) that is a tensor charge corresponds to the electric dipole moment in the external electric field $E_i = F_{0i}$. Similarly, the $\bar{\Psi}_f \sigma_{0i} \gamma_5 \Psi_f$ component of (4) corresponds to the magnetic dipole moment in the external magnetic field $B_i = \tilde{F_{oi}}$. Due to the kinematic relation (5), the nonvanishing dipole magnetic moment implies the "electric" spatial tensor current $\sim \bar{\Psi}_f \sigma_{ji} \Psi_f$ in the plane transverse to the applied magnetic field B_i while similarly the electric dipole moment implies the "magnetic" spatial tensor current $\sim \bar{\Psi}_f \sigma_{ji} \gamma_5 \Psi_f$ in the plane transverse to the applied electric field E_i.

Since there is no CP violation in QCD, there are no CP-odd terms in (3) and (4)—for instance, there is no induced electric dipole moment in a magnetic field. However the electric dipole moment squared does not vanish due to fluctuations, as was demonstrated in the lattice study [25]. The fluctuations of the electric dipole moment emerge for example from an ensemble of instantons and anti-instantons that have opposite electric dipole moments in an external magnetic field [24]. While the total electric dipole moment vanishes upon the averaging over the instanton–anti-instanton ensemble, the correlator of the electric dipole moments does not vanish due to the nonvanishing correlator of topological charges. The emergence of the spatial electric tensor current in the plane transverse to an external magnetic field in the presence of an instanton-anti-instanton pair is illustrated in Fig. 1.

The duality between electric and magnetic dipole moments is similar to the familiar duality between the vector and axial currents in the Schwinger model, the $(1+1)$-dimensional QED. In this model there exist the vector and axial vacuum currents in the external gauge field that at one-loop level are given by

$$\langle 0 | \bar{\Psi}_f \gamma_\nu \Psi_f | 0 \rangle \sim A_\nu, \tag{6}$$

$$\langle 0 | \bar{\Psi}_f \gamma_\nu \gamma_5 \Psi_f | 0 \rangle \sim \epsilon_{\mu\nu} A_\mu, \tag{7}$$

which are related kinematically due to the identity

$$\gamma_\nu = i \epsilon_{\mu\nu} \gamma_\mu \gamma_5. \tag{8}$$

This means that a non-vanishing vector charge in the external field implies the nonvanishing axial current and vice versa. This is similar to our four-dimensional case, where the nonvanishing dipole magnetic moment in an external magnetic field implies the spatial electric tensor current.

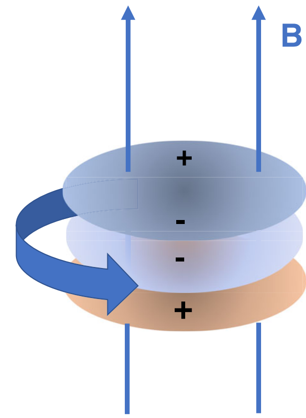

FIG. 1. Tensor supercurrent in the QCD vacuum. The external magnetic field induces electric dipole moments of opposite orientation in the instanton and anti-instanton, creating a tensor polarization of the quark zero modes. This tensor polarization is absent in the "empty" vacuum surrounding the instanton-anti-instanton pair, and is thus screened by the tensor supercurrent proportional to the quark condensate.

B. Tensor current from magnetic susceptibility

It is useful to introduce the antisymmetric rank two external field tensor $\mathcal{B}_{\mu\nu}$ as a source for the microscopic quark tensor current (3)

$$\langle 0 | \bar{\Psi} \sigma_{\mu\nu} \Psi | 0 \rangle = \frac{\delta L_{\text{anom}}}{\delta \mathcal{B}_{\mu\nu}} \tag{9}$$

and similarly external field pseudotensor $\tilde{\mathcal{B}}_{\mu\nu}$ as the source for the pseudotensor quark current (4). Implementing (9) amounts to the additional terms in the effective chiral Lagrangian which can be derived at the quark level from the triangle diagram involving the vector, tensor and scalar currents. Taking into account nonvanishing magnetic susceptibility, the resulting anomalous term in the chiral Lagrangian in the external $\mathcal{B}_{\mu\nu}$ field is [19]

$$\begin{aligned} L_{\text{anom}} = {} & \chi \langle \bar{\Psi}\Psi \rangle F_{\mu\nu} \mathcal{B}_{\mu\nu} \operatorname{Tr} \mathbf{BQ}(U + U^{-1}) \\ & + \tilde{\chi} \langle \bar{\Psi}\Psi \rangle \tilde{F}_{\mu\nu} \tilde{\mathcal{B}}_{\mu\nu} \operatorname{Tr} \mathbf{BQ}(U + U^{-1}); \end{aligned} \tag{10}$$

here $\mathbf{B}$ is the flavor matrix and $\mathbf{Q}$ is the charge matrix; $U = \exp(\frac{i\pi^a t^a}{f_\pi})$.

The usual treatment of currents in standard chiral perturbation theory includes the external scalar S, pseudoscalar P, vector V, and axial-vector A sources, without coupling to an external tensor source that is present in (10). Nevertheless, to describe within the chiral perturbation theory the relations (3) and (4) that follow from the $\langle$VVA$\rangle$

triangle anomaly we have to add to the usual chiral Lagrangian the term (10) describing the coupling to an external tensor $\mathcal{B}_{\mu\nu}$ source. The anomalous term (10) yields in particular an effective mass of the pion in external fields in the chiral limit

$$m^2_{\pi,\mathrm{eff}} = \chi \langle \bar{\Psi}\Psi \rangle F_{\mu\nu} \mathcal{B}_{\mu\nu} f_\pi^{-2} \tag{11}$$

that arises from the tensor polarization of the quark condensate.

The chiral Lagrangian can be derived in the holographic framework from the 5d gauge theory with $SU(N_f) \times SU(N_f)$ gauge group. The stringy currents in the holographic picture appear if we take into account the specific mixed CS-like term in the 5d bulk Lagrangian for the extended hard-wall model [19,30,31]

$$\delta S = \int d^5x \sqrt{-g} \mathrm{Tr}(X^+ F_L \mathcal{B} + \mathcal{B} F_R X) \tag{12}$$

where X is scalar in bifundamental representation, $\mathcal{B}$ is self-dual antisymmetric rank 2 field in the bifundamental representation and $F_{L,R}$ are the field strengths for the left and right gauge groups. At the boundary of the holographic 5d space it yields the corresponding term (10) in the chiral Lagrangian.

The tensor $\mathcal{B}$ field is sourced by stringy degrees of freedom due to the term $\int \mathcal{B}_{\mu\nu} d\Sigma_{\mu\nu}$ in the string worldsheet action. Therefore we could introduce the stringy current as

$$J_{\mu\nu} = \frac{\partial L_{\mathrm{anom}}}{\partial \mathcal{B}_{\mu\nu}}. \tag{13}$$

From the anomalous term (10) in the chiral Lagrangian we then immediately get the conserved stringy supercurrent in an external magnetic field:

$$J_{\mu\nu} = \chi \langle \bar{\Psi}\Psi \rangle F_{\mu\nu}. \tag{14}$$

The mixed anomalous term also yields the vector current proportional to the chiral condensate if the external rank-two pseudotensor field has nonvanishing curvature:

$$J_\nu = \chi \langle \bar{\Psi}\Psi \rangle \epsilon_{\nu\mu\alpha\beta} \partial_\mu \tilde{\mathcal{B}}_{\alpha\beta}. \tag{15}$$

Since this current is proportional to the condensate, it can be considered as analog of vector supercurrent. Similarly there is an axial current proportional to the curvature of the tensor field and the chiral condensate.

III. TOWARD A MICROSCOPIC PICTURE

A. Dual Lagrangians and examples of tensor currents

Let us present two examples in which the tensor stringy currents in an external magnetic field can be constructed explicitly. The stringy currents are usually hidden in the original formulation of the theory but emerge clearly in the dual formulation. The first example concerns the Polyakov's $(2+1)$-dimensional compact QED [32]. In that theory there is a natural 2-form current

$$J_{\mu\nu} = \epsilon_{\mu\nu\alpha} \partial_\alpha \phi, \tag{16}$$

where ϕ is the pseudoscalar that is dual to the photon:

$$\epsilon_{\mu\nu\alpha} \partial_\alpha \phi = F_{\mu\nu}. \tag{17}$$

This current is conserved perturbatively, apart from the points where the vortices-monopoles are localized, and it counts the number of strings. The natural microscopic carrier of this 2-form current is the string of finite length with monopole and antimonopole at its ends. It is the string that provides confinement in the theory, and we thus see clearly the two roles played by the tensor current. The monopole-antimonopole pair at the ends of the string amounts to the magnetic dipole structure described by the tensor current. On the other hand, this tensor current is carried by the confining string—this suggests a stringy interpretation for it.

The second example concerns the Abelian Higgs model in $(3+1)$-dimensional space-time. The Lagrangian reads

$$L = -\frac{1}{4} F^2 - |\partial_\mu - igA_\mu \phi|^2 - (g|\phi|^2 - v^2)^2, \tag{18}$$

where the potential supports the v.e.v. of the scalar field. There are effective strings in this theory, and the v.e.v. of the scalar field vanishes at their cores. It is useful to introduce the following parametrization for the complex scalar

$$\phi = \rho e^{i\theta}; \tag{19}$$

the scalar field ϕ can then be dualized into the 2-form gauge field $\mathcal{B}_{\mu\nu}$, and the dual Lagrangian reads as

$$\begin{aligned} L_{\mathrm{dual}} = & \frac{1}{2\rho^2} (d\mathcal{B})^2 - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} - g \epsilon_{\mu\nu\alpha\beta} \mathcal{B}_{\mu\nu} F_{\alpha\beta} \\ & - \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \mathcal{B}_{\mu\nu} \partial_\alpha \partial_\beta \theta, \end{aligned} \tag{20}$$

where the last term corresponds to the interaction of the 2-form field with the effective strings; $d\mathcal{B} = \frac{1}{2} \sum_{\alpha\beta\gamma} \partial_\alpha \mathcal{B}_{\beta\gamma} dx_\alpha \wedge dx_\beta \wedge dx_\gamma$ is the usual exterior derivative.

Now we can define the stringy current in the standard manner in terms of this Lagrangian. If we consider the vacuum currents we can put $\rho = v$ (v is the v.e.v. of the ρ field), and, to keep the analogy with QCD more close, rescale the 2-form field as

$$\mathcal{B}'_{\mu\nu} = v\mathcal{B}_{\mu\nu} \tag{21}$$

The stringy dimension-3 pseudotensor current in the external magnetic field now reads as

$$J_{\mu\nu} = gv\tilde{F}_{\mu\nu}, \tag{22}$$

which can be seen as the analog of the tensor current (1) in low-energy QCD that we discussed above.

The microscopic interpretation of this current is subtle. However one can view the scalar condensate in the vacuum in the Abelian Higgs model as a condensate of small closed loops of magnetic flux. In such a vacuum it is natural to assume that it is these small closed strings that get excited by magnetic field and provide a 2-form current.

B. Euclidean picture. Stringy current and the Dirac operator spectrum

In Euclidean space, it is useful to interpret magnetic susceptibility in terms of the spectrum of 4D Euclidean Dirac operator [23]:

$$\hat{D}(A)\psi_n = i\lambda_n\psi_n \tag{23}$$

which coincides with Dirac equation with an imaginary fermion mass $m = i\lambda$. The spectral density is defined as

$$\rho(\lambda) = \left\langle \sum_n \delta(\lambda - \lambda_n) \right\rangle_{\mathrm{QCD}}; \tag{24}$$

in the confined phase, according to the Casher-Banks relation, it is related to the quark condensate [22]:

$$\langle\bar{\Psi}\Psi\rangle = \Sigma = \frac{\pi\rho(0)}{V}. \tag{25}$$

It was found in [23] that magnetic susceptibility of the condensate has the following representation in terms of the Dirac operator spectrum

$$\left\langle 0|\bar{\Psi}_f\sigma_{\mu\nu}\Psi_f|0\rangle = \mathrm{lim}_{\lambda\to 0}\langle\rho(\lambda)\int d^4x\bar{\Psi}_\lambda\sigma_{\mu\nu}\Psi_\lambda\right\rangle_{\mathrm{QCD}}, \tag{26}$$

which involves the tensor current of zero modes of the Dirac operator. Assuming factorization (which has been checked numerically in [23]) on the rhs of (26), we arrive at the conclusion that the tensor current is saturated by the zero modes of the Dirac operator in the external magnetic field in the background of $I\bar{I}$ molecule. We have already advocated for this interpretation above, see also Fig. 1.

C. Minkowski space. Light degrees of freedom?

We have discussed above the nature of the stringy tensor current in Euclidean space-time. However it is important to identify the carriers of tensor current in the QCD ground state in Minkowski space. Because the tensor current appears as a response to even very weak magnetic field $eB \ll \Lambda^2_{\mathrm{QCD}}$, the carriers of the current have to be very light. On the other hand, the Euclidean analysis suggests the dominant role of some kind of topological objects, since the spectral Dirac operator representation of the tensor current is saturated by the quark zero modes.

Two possibilities that come to mind are

(i) The loop corrections to the ground state energy get deformed by the magnetic field and provide the magnetic moment and the tensor current;

(ii) There are specific stable light extended semiclassical configurations which can be excited in a weak magnetic field and serve as the carriers or the stringy tensor current. The example of rearrangement of the ground state at very weak magnetic field has been discussed in [33] where the stabilization of the pionic domain walls has been demonstrated.

The scenario based on the virtual correction generalizes the evaluation of the condensate in a magnetic field [9]. In [9], the loop of charged pions was found to generate the leading correction to the condensate in magnetic field. In our case, we have to analyze the dependence of the vacuum energy on the external $\mathcal{B}_{\mu\nu}$ field through the one-loop correction to the chiral lagrangian. The anomalous term in the Lagrangian yields the effective pion mass (11); therefore, one contribution of the desired type can be found by substituting this mass into the Heisenberg-Euler lagrangian in the external magnetic field.

The contribution of the pion loop to the vacuum energy density in an external magnetic field is given by the Heisenberg-Euler theory as

$$\epsilon_{\mathrm{vac}} = -\frac{1}{16\pi^2}\int_0^\infty \frac{ds}{s^3}\exp[-m^2_{\pi,\mathrm{eff}}s]\left[\frac{eBs}{\sinh(eBs)} - 1\right], \tag{27}$$

where the integral is over the Schwinger's proper time s. Substituting into this expression the effective mass of the pion in an external magnetic field (11) and taking derivative with respect to the external $\mathcal{B}_{\mu\nu}$ field (not to be confused with magnetic field B), we get the tensor current up to the second power of the magnetic field:

$$J_{\mu\nu} = \chi\langle\bar{\Psi}\Psi\rangle F_{\mu\nu}\left(1 + \frac{eB\ln 2}{16\pi^2 f^2_\pi}\right). \tag{28}$$

The physical interpretation of this formula is the following. The first term, which is already familiar to us, describes the virtual charged pions that rotate in magnetic field on Landau levels and give rise to the magnetic moment of the vacuum. The second term describes the correction that arises from the polarization of the pion, which is a composite particle of size $R_\pi \simeq (4\pi f_\pi)^{-1} \simeq 0.2$ fm, by the magnetic field. It is clear from (28) that this expression represents an expansion in powers of the ratio R_π/R_B of the pion size to the magnetic length $R_B = (eB)^{-1/2}$.

The quarks inside the pion are confined, so the second term in (28) can also be seen as originating from a tensor current carried by a string with quark and antiquark at its ends. It has been argued [34] that the chiral condensate gets suppressed by the confining string—therefore it is not surprising to find a correction to the condensate response that originates from the composite nature of the pion.

In addition to the polarization of the charged pion, there also exists a contribution to the tensor current that is induced by the π^0-photon mixing in the magnetic field. Iterating this vertex, we will get the loop involving the photon and pion propagators. This opens an interesting possibility to describe the tensor current in terms of neutral pions. Indeed, it was shown in [33] that low-energy QCD in an external magnetic field is unstable with respect to formation of the π^0 domain walls. In the limit of the massless quarks, the ground state is always unstable, the domain walls carry the baryon charge and are more energetically favorable than the ordinary nuclear matter. Moreover, the domain wall with the disc topology is enclosed with the π-mesonic string which completely screens the induced baryonic charge of the wall [18].

Very recently such pancake configurations were suggested as the candidates for high spin baryons [35]. The stabilization of the disc size occurs due to the edge chiral currents at its boundary. We speculate that the polarization of the edge chiral currents at the boundary by magnetic field can also give rise to the tensor current, similarly to the case of charged pions considered above. It will be interesting to investigate this scenario further.

IV. EXPERIMENTAL MANIFESTATIONS

A. Vector meson spin alignment in heavy ion collisions

Heavy ion collisions produce QCD matter subjected to strong magnetic field and large vorticity. In both cases, we expect the emergence of the tensor supercurrent. One manifestation of this tensor current is the polarization of Λ hyperons that was estimated in [20] for the case of finite vorticity. It was found that the resulting polarization is consistent with the experimental results of STAR Collaboration at RHIC.

There exists another, potentially even more prominent, signature of the tensor supercurrent in heavy ion collisions. Indeed, consider a state $|\Omega\rangle$ of QCD matter with a nonzero tensor charge (4), i.e., the state with a finite magnetic dipole moment $\langle\Omega|\bar{\Psi}_f\sigma_{0i}\gamma_5\Psi_f|\Omega\rangle$. The quantum numbers of this state suggest that it should hadronize with a copious emission of spin-polarized axial vector $J^{PC} = 1^{+-}$ mesons V, such as $a_1(1260)$, $h_1(1170)$, $b_1(1235)$ and so on; the corresponding meson emission amplitude is

$$\langle\Omega|\bar{\Psi}_f\sigma_{\mu\nu}\gamma_5\Psi_f|A\rangle = if_A(\epsilon_\mu k_\nu - \epsilon_\nu k_\mu), \tag{29}$$

where f_A is the meson decay constant, ϵ_μ is the meson polarization vector, and k_ν is the meson's four-momentum.

Let us consider this amplitude in the rest frame of matter characterized by a dipole magnetic moment along the direction of the magnetic field (axis i). Assuming that the produced meson in this reference frame is slow, with three-momentum much smaller than the meson mass M_A, $k_i \ll M_A$, we find

$$\langle\Omega|\bar{\Psi}_f\sigma_{0i}\gamma_5\Psi_f|A\rangle \simeq -if_A M_A \epsilon_i, \tag{30}$$

i.e., the axial-vector mesons are produced with polarization along the direction of the magnetic field. It may be hard to measure the polarization of the axial-vector mesons directly in heavy ion experiments, where only the polarization of vector mesons has been measured so far. Fortunately, for the mesons listed above the dominant decay mode is $A \to V + \pi$, i.e., a vector meson and a pion (or kaon, for strange axial-vector mesons). Since this is an s-wave decay, and pion has a zero spin, the produced vector mesons should inherit the polarization of the parent axial-vector mesons, and should thus be characterized by a spin alignment along the magnetic field. A remarkably strong spin alignment of vector mesons has been observed recently by STAR Collaboration at RHIC. It significantly exceeds the predictions based on both the statistical hadron model and the recombination of polarized quarks.

To get a rough estimate of the resulting from (30) spin alignment of the produced axial vector mesons, let us estimate the spin polarization of the quark condensate in an external magnetic field $B_z = \tilde{F}_{0z}$ from the definition of magnetic susceptibility (4) and its value predicted by (2):

$$\langle\sigma_z\rangle = \frac{\langle 0|\bar{\Psi}\sigma_{oz}\gamma_5\Psi|0\rangle}{\langle\bar{\Psi}\Psi\rangle} = -\frac{N_c}{4\pi^2 f_\pi^2} eB_z. \tag{31}$$

The predicted value of magnetic field at freeze-out in Au-Au collisions at RHIC energy of $\sqrt{s} = 200$ GeV per nucleon pair within the relativistic magnetohydrodynamics approach [36] is about $eB \simeq 2f_\pi^2$. Using this value in (31) yields the spin alignment of about $\sim 7\%$. Assuming that all of this alignment gets transferred to the axial-vector, and then, via $A \to V + \pi$ decays, to vector mesons, we expect that tensor supercurrent should be a significant source of the observed vector meson alignment. The prediction of our approach is a spin alignment of axial vector mesons, such as a_1, that should be even larger than that observed for the vector mesons.

It is hard to estimate the axial-vector and vector meson polarization resulting from the tensor supercurrent in a model-independent way, but it should greatly exceed the predictions of a statistical model where the polarization originates from the coupling of vector meson's spin to vorticity. Indeed, in the statistical model the vector mesons are excited states of the system, whereas the tensor current and the resulting polarization characterize its ground state, even at zero temperature.

B. Tensor charge of the nucleon and transversity

The forward matrix element of the tensor current operator $\hat{J}_{\mu\nu} = \bar{\Psi}_f \sigma_{\mu\nu}\gamma_5 \Psi_f$ over the transversely polarized nucleon defines the tensor charge of the nucleon:

$$\langle S^T|\bar{\Psi}_f \sigma_{\mu\nu}\gamma_5 \Psi_f|S^T\rangle = 2\delta q_f (P_\mu S_\nu^T - P_\nu S_\mu^T), \quad (32)$$

where P_μ is the four-momentum of the nucleon, S^T describes the nucleon's transverse polarization, and δq_f is quark transversity given by the integral over the difference of quark $\delta q_f(x)$ and antiquark $\delta \bar{q}_f(x)$ transversity distribution functions:

$$\delta q_f = \int_0^1 dx(\delta q_f(x) - \delta \bar{q}_f(x)), \quad (33)$$

where x is Bjorken x.

In this case the role of magnetic field or vorticity is played by the spin of the nucleon. Nevertheless, just as the magnetic dipole moment of the vacuum is greatly enhanced by nonperturbative effects (tensor polarization of the quark condensate), we expect that a similar enhancement should occur for the tensor charge of the nucleon. Because this enhancement results microscopically from extended topological objects, transversity has to be a higher twist effect, in accord with perturbative analysis. Nevertheless, at moderate momentum transfers $Q^2 \sim 4\pi f_\pi^2 \simeq 1$ GeV, basing on the analysis in Sec. II B, we expect that transversity should be very large—in accord with experimental observations.

Moreover, in analogy with our discussion of the axial vector meson production in the decay of QCD matter, one can expect a strong coupling of axial vector mesons to the nucleons. The value of the corresponding coupling constant has been extracted from the data for the a_1 meson and is indeed large [37]:

$$g_{a_1 NN} = 9.3 \pm 1. \quad (34)$$

We can compare this to the vector ρ meson coupling to the nucleon, which is known to be significantly smaller [38]:

$$g_{\rho NN} = 2.52 \pm 0.06. \quad (35)$$

To summarize this section, it appears that transversity and the tensor charge of the nucleon are enhanced by the quantum chiral anomaly.

V. DISCUSSION AND FUTURE DIRECTIONS

A. Role of instanton molecules

In a conventional superconductor, the persistent electric current can be expressed in terms of the phase of charged condensate

$$J_\mu = n_s(\partial_\mu \Phi - A_\mu) \quad (36)$$

where n_s is proportional to the condensate of the Cooper pairs $\langle\Psi\Psi\rangle$ and Φ is the phase of the order parameter. The charge current turns to be proportional to the gauge field in the London limit, the photon gets massive and magnetic field cannot penetrate the superconductor due to the screening by electric supercurrent. The supercurrent also screens the magnetic field inside the Abrikosov vortex where the condensate vanishes in its core. The supercurrent arises due to the Abelian gauge symmetry in this case. The example of the supercurrent arising from the global symmetry is provided by the superfluid where the superfluid velocity is proportional to the neutral condensate. The superfluid supercurrent is proportional to the superfluid density and the gradient of the neutral condensate phase ϕ:

$$J_\nu \propto \rho_s \partial_\nu \phi. \quad (37)$$

This supercurrent screens the angular velocity (which can be treated as an external gravimagnetic field) resulting in the emergence of vortices. Note that the superfluid density ρ_s is the analog of the topological susceptibility of the QCD vacuum.

The chiral symmetry is broken locally inside the instanton molecule [27]; therefore necessarily there are gradients of the pion field which is the phase of the chiral condensate at the boundary of the 4D region occupied by the molecule. The pions are pseudoscalar Goldstone bosons related to the spontaneous breaking of the global chiral symmetry. The tensor supercurrent we are interested in would arise if there were vector (photonlike) Goldstone bosons associated with a broken 1-form symmetry. The possible mechanism for the emergence of these vector excitations is provided by the π^0-photon mixing in the external magnetic field due to the chiral anomaly. Hence the pseudoscalar and vector Goldstone modes are coupled in a magnetic field. Within this interpretation our tensor current acquires the "standard" interpretation (37), becoming a 2-form current of the vector Goldstone mode.

The tensor supercurrent (which screens the instanton molecules in the Euclidean version) amounts to the alignment of the instanton molecules in the external magnetic field. A potentially related phenomenon in the language of the chiral effective theory is the emergence of a pancakelike configuration of pionic domain walls and antiwalls in Minkowski space in weak magnetic field [33]. Another related phenomenon discussed previously within the chiral effective theory is the anomaly-induced quadrupole moment of the neutron in an external magnetic field [39].

B. Tensor supercurrent and magnetohydrodynamics

Recently hydrodynamics involving electromagnetism has been reformulated in an interesting way [40–42] which is based on the 1-form global symmetry [43] and the corresponding conserved 2-form stringy current. The 1-form symmetry is responsible for the conservation of the number of strings and the Bianchi identity yields the conservation of 2-form current (in the absence of magnetic monopoles). Two-form charge density is identified as the density of magnetic dipole charge which counts the number of magnetic field lines through an arbitrary surface.

The fate of the global 1-form symmetry determines three different regimes of the theory:

(1) The 1-form symmetry is unbroken;
(2) The 1-form symmetry is partially broken;
(3) The 1-form symmetry is completely broken.

In these different symmetry breaking patterns one can obtain, respectively, magneto-hydrodynamics (MHD), stringy fluid, and bound-charge plasma (see [42] for the classification of regimes and the connections between them). The vector Goldstone for the 1-form symmetry was identified with the photon.

Which situation gets realized in QCD? The quark tensor current in general is not conserved hence in the quark sector the symmetry is broken. However it is conserved in the vacuum sector in the external magnetic field, since we do not assume the presence of magnetic monopoles—indeed, the current (1) is conserved due to the Bianchi identity. Hence we have the main ingredient of the 1-form hydrodynamics in the vacuum sector and may hypothesize that the QCD vacuum in magnetic field can be described as a kind of stringy superfluid. Within this regime of 1-form hydrodynamics the magnetic susceptibility of the condensate defines the density of the instanton molecules. We postpone discussion of the 1-form symmetry in the hydrodynamic approach to low-energy QCD in magnetic field for a separate study.

VI. CONCLUSION

In this paper we have addressed the microscopic origin of the tensor supercurrent that arises in low-energy QCD in an external magnetic field or in a rotating frame and that is proportional to the chiral condensate. We have suggested the following microscopic picture. There is a finite density of instanton–anti-instanton $I\bar{I}$ molecules in the QCD vacuum. As it was argued long time ago [27], the instanton and anti-instanton host the pair of fermionic zero-modes. When the external magnetic field is switched on, the fermion zero modes become polarized and develop a quadrupole moment that gets screened by the tensor supercurrent. All of the vacuum $I\bar{I}$ molecules become aligned in the external magnetic field. Hence the external magnetic field probes the molecule component of the QCD vacuum via the tensor supercurrent.

We have also argued that the tensor supercurrent manifests itself in experiment through the polarization of axial-vector and vector mesons in heavy ion collisions, and through the tensor charge of the nucleon and transversity in deep-inelastic scattering. All of these phenomena appear strongly affected by the chiral anomaly. The tensor supercurrent may play an important role in the stringy interpretation of magnetic hydrodynamics; we will discuss this interesting issue elsewhere.

The localization of zero modes of Dirac operator on extended defects has been investigated in lattice QCD. There were evidences that the Euclidean 4-dimensional zero modes "live" on 2d surfaces and 3d volumes [44,45] corresponding to the worldsheets of effective strings and effective domain walls, correspondingly. In this study we considered the tensor supercurrents screening the instanton molecules but similar tensor current could exist on a pair of extended objects in the Euclidean space with the opposite topological charges.

The stringy current considered here has a counterpart in condensed matter physics. The chiral condensate in QCD is an analog of the neutral exciton condensate and the current we have considered corresponds to the polarization of the exciton condensate in magnetic field. Such currents have been indeed discussed [46,47], so it would be interesting to pursue this analogy further. One more question concerns the possible role of the charged tensor current discussed in the holographic framework in [48].

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