Rapid Communications

Manifestation of kaonium in the $e^+e^- \to K^+K^-$ process

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We analyze the precise data obtained by the CMD-3 experiment on the e^+e^- annihilation into two charged kaons in the vicinity of the ϕ peak. A perfect fit is obtained only if a pole on the real axis below the reaction threshold is assumed. This can be interpreted as proof of the existence of the 2p state of kaonium, a compound of K^+ and K^-. The *BABAR* Collaboration data on the same process supports this conclusion and, in addition, points to the strong interaction as a dominant source of the binding energy. The possibility of discovering 2p kaonium in the $e^+e^- \to \pi^+\pi^-$ process is discussed. The 2p state of K^0-onium is indicated on the basis of the CMD-3 $e^+e^- \to K_S^0 K_L^0$ data.

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I. INTRODUCTION

Kaonium is still a hypothetical compound system consisting of a positively charged and a negatively charged kaon. It belongs to a wide class of onia, systems made of a particle and an antiparticle. In the lepton sector, they are the well-known positronium, true muonium, and true tauonium (the last two have yet to be observed). Quarkonia, the bound states of a quark and its antiquark, are observed as truly neutral (all flavor quantum numbers vanishing) mesons. They are numerous and include, e.g., ϕ ($s\bar{s}$), η_c and J/ψ ($c\bar{c}$), Υ ($b\bar{b}$). Many theoretical studies, starting with the Fermi-Yang and Sakata models [1], have considered the possibilities of baryon-antibaryon bound states.

In the meson sector, pionium ($\pi^+\pi^-$) was discovered in 1993 at the 70 GeV proton synchrotron at Serpukhov, Russia [2], and intensively studied in the Dimeson Relativistic Atomic Complex (DIRAC) experiment [3] at the CERN Proton Synchrotron. Assuming pure Coulombic interaction, the binding energy of pionium can be calculated from the hydrogen-atom formula

$$b_n = \frac{m_r \alpha^2}{2n^2}, \tag{1}$$

where m_r is the reduced mass in energy units (used throughout this paper), $\alpha \approx 1/137$ is the fine-structure constant, and n is the principal quantum number. Putting $n = 1$ for the ground state and $m_r = m_{\pi^+}/2$, we get $b = 1.86$ keV. The decay to two neutral pions is dominant and the measured lifetime is $3.15^{+0.28}_{-0.26} \times 10^{-15}$ s [3]. The NA48/2 Collaboration at the CERN Super Proton Synchrotron [4] studied decays $K^\pm \to \pi^\pm\pi^0\pi^0$ and found an anomaly in the $\pi^0\pi^0$ invariant mass distributions that can be interpreted as the production of pionia in the kaon decays and their subsequent two-π^0 decay.

The DIRAC experiment also observed and studied π^-K^+ and π^+K^- atoms [5,6].

To date, the experiments concerning dimeson production have been performed at proton accelerators [2–7]. Electron-positron colliders, the machines that are famous for participating in the discovery of many new particles (notably quarkonia), have not yet contributed much to mesonia physics. The reason is that ground-state mesonia (1s in atomic notation) are objects with $J^{PC} = 0^{++}$ quantum numbers, and as such they cannot couple to the photon. In the e^+e^- annihilation processes, they must be accompanied by an additional particle or particles, or at least a photon [8]. However, the DIRAC Collaboration recently discovered [7] so-called long-lived $\pi^+\pi^-$ atoms, which are the 2p atomic states with quantum numbers $J^{PC} = 1^{--}$. Therefore, they can be produced in the e^+e^- collisions. The Coulombic binding energy of the 2p pionium is 0.464 keV; its lifetime was determined in Ref. [7] to be $\tau_{2p} = 0.45^{+1.08}_{-0.30} \times 10^{-11}$ s. Such a long lifetime is caused by the fact that the decay modes to the positive C-parity states $\pi^0\pi^0$ and $\gamma\gamma$ are now forbidden and the $2p \to 1s$ transition dominates [9].

No experimental evidence of kaonium has yet been found. In the simplest way, kaonium can be considered a hydrogenlike atom (a system held together due to Coulombic attraction between opposite electric charges).

Equation (1) gives the binding energy of the ground state $b \approx 6.57$ keV. Unlike the hydrogen atom and leptonic onia, the constituents of pionium and kaonium also interact via strong force. Krewald *et al.* [10] found "the ground state energy for the kaonium atom that is shifted above the Coulomb value by a few hundred eV." As a rule, an increase in bound-state energy means a drop in binding energy. On the contrary, Zhang *et al.* [11] found that kaonium binds more strongly ($b = 7.05$ keV) than it corresponds to Coulomb interaction.

Kaonium is not stable. Ground-state (1s) kaonium partly decays electromagnetically into two photons. In addition, the exchange of K^* between kaons generates the $\pi^+\pi^-$, $\pi^0\pi^0$, and $\eta\pi^0$ decay modes. Klevansky and Lemmer [12] used meson-meson interaction amplitudes taken from leading order chiral perturbation theory and found the resulting lifetime of $(2.2 \pm 0.9) \times 10^{-18}$ s. This is in conformity with the order of magnitude estimate 10^{-3} fs obtained by Deloff [13].

The Coulombic binding energy of the first excited state ($n = 2$) of kaonium is 1.64 keV. We will concentrate on the 2p state, which can be produced in the e^+e^- experiments. Its quantum numbers $J^{PC} = 1^{--}$ forbid decays to the $C = 1$ $\pi^0\pi^0$ and $\eta\pi^0$ states. The 2p kaonium width is thus determined by the decay rate into $\pi^+\pi^-$ only [14]. Its lifetime should be at least 2 times higher than that of the ground-state kaonium. The corresponding decay width is around 0.1 keV, about 4 orders of magnitude smaller than the decay width of the $\phi(1020)$, which is the dominant object in the process we are going to investigate. For our purposes, we can thus neglect the 2p kaonium decay width and consider it a stable particle.

II. 2p KAONIUM AS A POLE IN THE $e^+e^- \to K^+K^-$ AMPLITUDE

In this paper, we analyze the existing precise data on the $e^+e^- \to K^+K^-$ process with the aim of finding a pole in the amplitude corresponding to a bound state.

The reaction amplitude is a function of s, the invariant energy squared. Under very general conditions, the amplitude can be continued into the complex s-plane. The resonances are represented by the poles at imaginary s. The (quasi)stable particle, or bound state, appears as a pole at real s below the reaction threshold.

The formula for the cross section of the e^+e^- annihilation into a K^+K^- pair based on the vector-meson-dominance model with two resonances is

$$\sigma(s) = \frac{\pi\alpha^2}{3s}\left(1 - \frac{4m_K^2}{s}\right)^{3/2} \times \left|\frac{R_1 e^{i\delta}}{s - M_1^2 - iM_1\Gamma_1} + \frac{R_2}{s - M_2^2 - iM_2\Gamma_2}\right|^2, \quad (2)$$

where M_i and Γ_i determine the position and width of the ith resonance, respectively. The residuum R_i includes the product of two constants. One characterizes the coupling of the ith resonance to the photon (up to the elementary charge e, which is taken off to form, after squaring, an α in the prefactor) and the other is the coupling of the resonance to the K^+K^- pair. The phase δ regulates the interference between the resonances.

We use the $e^+e^- \to K^+K^-$ cross section data [15] obtained by the CMD-3 (Cryogenic Magnetic Detector) experiment at the VEPP-2000 e^+e^- collider in Novosibirsk, Russia. The advantage of the energy scan method used in this experiment is in getting high statistics data at precisely known energies, to which the collider is tuned up step by step. The number of data points is 24, and they are concentrated in a narrow region around the $\phi(1020)$ resonance. The data are shown in Fig. 1, together with two curves depicting our fits [16].

We first tried to fit the data assuming just one resonance, i.e., using three free parameters [17]. The result was disastrous: the usual χ^2 was equal to 341.8, which together with the number of degrees of freedom NDF $= 24 - 3 = 21$ implied a confidence level (C.L.) of zero. This fit is depicted by a dashed curve in Fig. 1. The parameters of the fit are listed in the middle column of Table I. It must be said that Kozyrev *et al.* [15] achieved a better result. They used a more sophisticated parametrization of the $\phi(1020)$-resonance shape based on an energy dependent width $\Gamma(s)$ and attained $\chi^2/\text{NDF} = 25/20$, which gave C.L. $= 20\%$.

We then tried to improve our fit by considering two resonances. To our surprise, the width of the second resonance came out close to zero (0.6 ± 1.7 MeV [18])

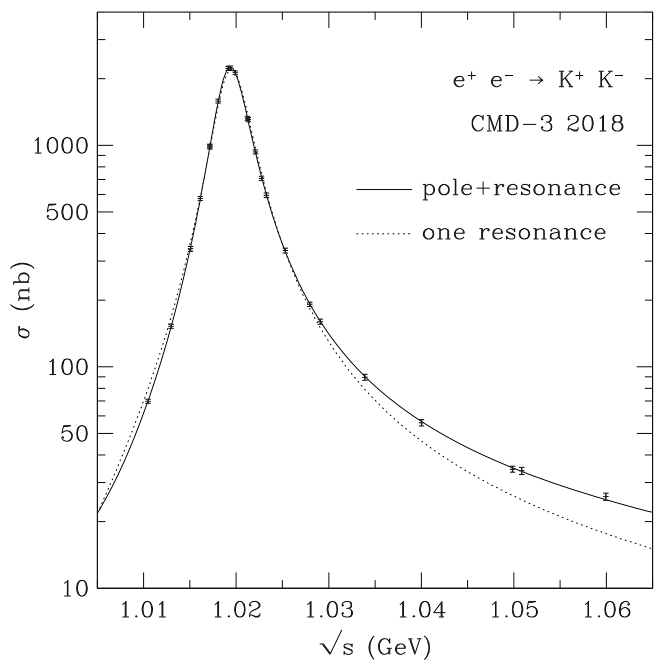

FIG. 1. Cross section for the e^+e^- annihilation into two charged kaons measured in the CMD-3 experiment [15] and two fits to it using Eq. (2). Their parameters are given in Table I.

TABLE I. Parameters of the two fits to the CMD-3 K^+K^- data [15] depicted in Fig. 1.

	One resonance	Resonance and pole
R_1 (GeV^2)	0.3679(15)	0.3759(15)
M_1 (GeV)	1.019393(14)	1.019247(18)
Γ_1 (MeV)	4.359(32)	4.172(38)
δ	0 (fixed)	1.334(34)
R_2 (GeV^2)	0 (fixed)	0.298(23)
b (keV)		1.64[a] (fixed)
Γ_2		0 (fixed)
χ^2	341.8	4.6
NDF	21	19
Confidence level	0	100%

[a]$M_2 = 2m_{K^+} - b$.

and the resonance position was below the threshold, which signalized the bound state. Inspired by that, we replaced the second resonance with a bound-state pole by putting $\Gamma_2 \equiv 0$ and $M_2 = 2m_{K^+} - b$, where b is the 2p kaonium binding energy. We took $b = 1.64$ keV as a first try. We attained perfect agreement with the data ($\chi^2/\text{NDF} = 4.6/19$, C.L. = 100%), depicted by the full curve in Fig. 1. All parameters of the fit are listed in the rightmost column of Table I. Minimizing the χ^2 with respect to the binding energy gives an estimate of $b = (10.8^{+14.8}_{-9.1})$ MeV [19]. Let us mention that in 1961 Uretsky and Palfrey [20] assumed a mesonium binding energy of 10 MeV.

A binding energy much larger that its Coulombic value of 1.64 keV would mean that the strong force between K^+ and K^- is attractive and responsible for keeping kaonium together. But our result is still not conclusive. The parameters of the fit with $b = 10.8$ MeV ($\chi^2/\text{NDF} = 2.4/19$, C.L. = 100%) are only marginally better than those with the Coulombic binding energy shown in Table I. To get more insight into this problem, we explored two other datasets.

The $e^+e^- \to K^+K^-$ data obtained by the CMD-2 experiment at the VEPP-2000 e^+e^- collider in Novosibirsk, Russia, were published in 2008 [21]. They contain 21 points in an energy range narrower than that of the later CMD-3 data [15] explored above. We have performed the fits to data [21] under three different assumptions; the results are shown in the middle panel of Table II.

TABLE II. Evidence for 2p kaonium from two other experiments. The last two rows show results of the fits (χ^2/NDF, C.L.) with kaonium considered in addition to the resonance.

	CMD-2 [21]	*BABAR* [22]
Energy range (GeV)	1.01136–1.03406	0.985–1.065
No. of points	21	48
One-resonance fit	7.5/18 98.5%	244.4/45 0%
Kaonium $b = 1.64$ keV	2.2/16 100%	184.5/43 0%
Kaonium $b = 10$ MeV	2.3/16 100%	48.1/43 27.4%

TABLE III. Parameters of the fits to the $\sqrt{s} < 1.065$ GeV subset of *BABAR* K^+K^- data [22] assuming no pole below the threshold (second column) and the 2p kaonium with the Coulombic binding energy b (third column) or with $b = 10$ MeV (fourth column).

	No kaonium	$b = 1.64$ keV	$b = 10$ MeV
R_1 (GeV^2)	0.3577(17)	0.3575(17)	0.3572(18)
M_1 (GeV)	1.019144(17)	1.019120(17)	1.019034(19)
Γ_1 (MeV)	4.461(37)	4.455(37)	4.300(40)
R_2 (GeV^2)		0.0140(13)	0.184(18)
M_2		$2m_{K^+} - b$	$2m_{K^+} - b$
Γ_2		0 (fixed)	0 (fixed)
δ_2		−0.07(18)	−1.070(56)

The one-resonance fit is already perfect. Adding kaonium with either Coulombic binding energy or the Uretsky-Palfrey value of 10 MeV further decreases χ^2. Owing to perfect fit without kaonium, this cannot be considered an unambiguous proof of kaonium's existence.

The *BABAR* data [22] from 2013 extend up to 5 GeV. We have used only 48 energies up to 1.065 GeV, which is the highest energy in the CMD-3 [15] data. The *BABAR* data also cover the low-energy region and provide 13 data points between the K^+K^- threshold and the lowest CMD-3 [15] energy. They are therefore more sensitive to the subthreshold behavior [given mainly by the position(s) and residuum (residua) of the pole(s)] of the reaction amplitude. Thanks to that, we have gotten the following results (see the rightmost panel of Table II): the "no kaonium" and "Coulombic kaonium" hypotheses are rejected, and the kaonium with a binding energy of 10 MeV gives an acceptable fit. The details of the fits are given in Table III.

III. 2p KAONIUM IN THE $e^+e^- \to \pi^+\pi^-$ DATA

There is also another possible way of seeing the 2p kaonium. Thanks to its $\pi^+\pi^-$ decay mode, the 2p kaonium may, in principle, reveal itself as an irregularity in the $e^+e^- \to \pi^+\pi^-$ excitation function slightly below $\sqrt{s} = 2m_{K^+}$. The contemporary experimental data do not show any anomaly that could be interpreted as a manifestation of 2p kaonium. The invariant energy region in question has been ignored by most experiments, which have concentrated on the measurement in the ρ/ω region or at energies higher than 1 GeV. The important exception is the *BABAR* experiment [23], which covered energies ranging from 0.3 to 3 GeV by using the initial state radiation method. Their data in the presumed signal region are shown in Fig. 2, together with our "conservative" fit that does not include kaonium. There is an insignificant excess at $\sqrt{s} \approx 973$ MeV, but no firm conclusion can be drawn yet. More precise and denser data would be necessary to confirm or reject the presence of 2p kaonium in the $e^+e^- \to \pi^+\pi^-$ data. According to Ref. [24], we may expect new data from the CMD-3 experiment soon. Figure 4 in Ref. [24] hints at their covering energies to almost 1 GeV.

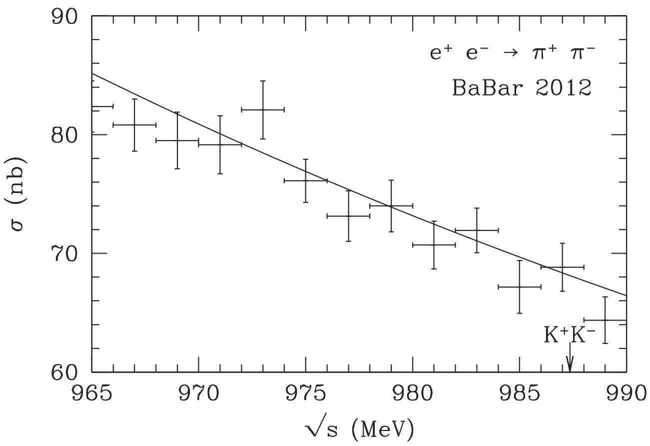

FIG. 2. Cross section for the e^+e^- annihilation into two charged pions measured in the *BABAR* experiment [23]. The curve is our fit to data from 0.3 to 1.6 GeV ($\chi^2/\mathrm{NDF} = 257.6/303$, C.L. $= 97.2\%$). The K^+K^- threshold is marked with an arrow.

IV. 2p K^0-ONIUM AS A POLE IN THE $e^+e^- \to K_S^0 K_L^0$ AMPLITUDE

The result we obtained when exploring the K^+K^- *BABAR* data [22] indicates that the strong interaction between a kaon and its antiparticle is attractive. One may therefore speculate about the existence of the $K^0\bar{K}^0$ bound state. This would be the first (and probably only) onium composed of neutral particles. To pursue this idea, we use the high-precision data on the $e^+e^- \to K_S^0 K_L^0$ process [25], again coming from the CMD-3 experiment at Budker Institute of Nuclear Physics in Novosibirsk, Russia, and comprising 25 data points in a narrow interval around the $\phi(1020)$ resonance.

Our simple one-resonance fit (the middle column in Table IV) provides $\chi^2/\mathrm{NDF} = 36.7/22$, C.L. $= 2.5\%$ [26]. The sophisticated fit by experimentalists [25] is much better ($\chi^2/\mathrm{NDF} = 15/21$, C.L. $= 82\%$). To explore the one-resonance–one-bound-state scenario, we choose again the Uretsky-Palfrey value of 10 MeV for the binding energy b. The result of the fit is $\chi^2/\mathrm{NDF} = 7.5/20$, C.L. $= 99.5\%$, and all parameters are listed in the rightmost column in Table IV. Keeping in mind a good fit by the experimenters themselves, we can say that the K^0-onium is not required by the CMD-3 [25] data. When looking for

TABLE IV. Parameters of the two fits to the CMD-3 $K_S^0 K_L^0$ data [25].

	One resonance	Resonance and pole
R_1 (GeV^2)	0.3488(14)	0.3500(39)
M_1 (GeV)	1.019221(14)	1.019263(16)
Γ_1 (MeV)	4.144(28)	4.168(29)
δ	0 (fixed)	3.2 ± 2.0
R_2 (GeV^2)	0 (fixed)	0.0289(53)
b (MeV)		10.0[a] (fixed)
Γ_2		0 (fixed)
χ^2	36.7	7.5
NDF	22	20
Confidence level	2.5%	99.5%

[a] $M_2 = 2m_{K^0} - b$.

K^0-onium in the $\pi^+\pi^-$ data, the signal region should be shifted upward by about 7.9 MeV.

V. CONCLUSIONS

To conclude, we have found an indication of the existence of kaonium made of charged kaons in the 2p state by analyzing the data on the $e^+e^- \to K^+K^-$ process. A similar analysis of the $e^+e^- \to K_S K_L$ data has provided somewhat weaker evidence for the 2p K^0-onium. To this point, we have ignored the fact that both 2p kaonia have the same quantum numbers. As a consequence, the K^+K^- kaonium should be present in the $e^+e^- \to K_S K_L$ amplitude as a bound-state pole, and the kaonium composed of neutral kaons should reveal itself in the $e^+e^- \to K^+K^-$ amplitude as a bound-state pole or a resonance, depending on its bounding energy. Both 2p kaonia decay into two charged pions. To get a clear and consistent picture of kaonia, the information should be combined from the e^+e^- annihilation data to the K^+K^-, $K_S K_L$, and $\pi^+\pi^-$ final states. Work in this direction is in progress.

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[8] This also applies to scalar mesons $f_0(980)$ and $a_0(980)$, the structure of which was studied in such processes. See, e.g., Ref. [27].
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[16] We combined in quadrature the statistical errors from Ref. [15]: Table 1 with total systematic uncertainties from Table 2 therein.
[17] As always, we found the program MINUIT by F. James and M. Roos [Comput. Phys. Commun. **10**, 343 (1975)] to be extremely useful.
[18] When determining the error of Γ_2, we kept the other parameters at their optimal values. This is a parabolic error, characterizing the behavior of the χ^2 in the proximity to minimum, not the allowed interval.
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