Study of the charged super-Chandrasekhar limiting mass white dwarfs in the $f(R,\mathcal{T})$ gravity

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(Received 24 November 2019; accepted 8 April 2020; published 5 May 2020)

The equilibrium configuration of white dwarfs composed of a charged perfect fluid is investigated in the context of the $f(R,\mathcal{T})$ gravity, for which R and $\mathcal{T}$ stand for the Ricci scalar and the trace of the energy-momentum tensor, respectively. By considering the functional form $f(R,\mathcal{T}) = R + 2\chi\mathcal{T}$, where χ is the matter-geometry coupling constant, and for a Gaussian ansatz for the electric distribution, some physical properties of charged white dwarfs were derived, namely, mass, radius, charge, electric field, effective pressure, and energy density; their dependence on the parameter χ was also derived. In particular, the χ value important for the equilibrium configurations of charged white dwarfs has the same scale of 10^{-4} of that for noncharged stars and the order of the charge was 10^{20} C, which scales with the value of one solar mass, i.e., $\sqrt{G}M_{\odot} \sim 10^{20}$ C. We have also shown that charged white dwarf stars in the context of the $f(R,\mathcal{T})$ have surface electric fields below the Schwinger limit of 1.3×10^{18} V/m. In particular, a striking feature of the coupling between the effects of charge and $f(R,\mathcal{T})$ gravity theory is that the modifications in the background gravity increase the stellar radius, which in turn diminishes the surface electric field, thus enhancing stellar stability of charged stars in comparison with general relativity (GR) theory. Most importantly, our study reveals that the present $f(R,\mathcal{T})$ gravity model can suitably explain the super-Chandrasekhar limiting mass white dwarfs, which are supposed to be the reason behind the overluminous SNeIa and remain mostly unexplained in the background of GR.

DOI: 10.1103/PhysRevD.101.104008

I. INTRODUCTION

With the recent pioneering observations, such as supernovae of type Ia [1,2], baryon acoustic oscillations [3], Planck data [4], cosmic microwave background radiation [5,6], and redshift supernovae [7], it is evident that presently our Universe is going through the accelerated expanding phase which hardly can be explained through the most successful general relativity (GR) theory. The most standard way out to explain the present observed cosmological dynamics appeared as the inclusion of the cosmological constant (Λ) into the Einstein gravitational field equation which also provided fine agreement with the observed data by considering the presence of a hypothetical component known as the *dark matter* [8,9]. It is also largely accepted that the sole reason behind the present accelerated expansion phase of the Universe is actually another mysterious component widely known as dark energy [10–14] and appeared as the most successful avenue in explaining the present cosmic dynamical phase until it faced the major setback due to a huge mismatch of the values of 120 orders of magnitude between the observationally achieved and theoretically predicted values of Λ [15,16].

To overcome this situation, different researchers came up with more sophisticated gravity theories by modifying the Einstein-Hilbert action which gave rise to a new avenue known as modified/extended gravity theories. Extended theories of gravity have aroused as an opportunity to solve problems which are still without convincing explanation within the GR framework. The most famous modified theory of gravity is the $f(R)$ theory, which consists of choosing a more general action to replace the Einstein-Hilbert one, this is made by assuming that the gravitational action is given by an arbitrary function of the Ricci scalar R which can be found in literature Refs. [17–20]. Besides $f(R)$ gravity theory in recent times, the extended gravity models that attracted attention of the researchers are $f(R,G)$ gravity [21,22], Brans-Dicke gravity [23,24], $f(\mathbb{T})$ gravity [25–27], etc., where G and $f(\mathbb{T})$ are Gauss-Bonnet and torsion scalar, respectively.

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Recently, Harko *et al.* [28] developed a further generalization of the $f(R)$ theory of gravity by choosing a gravitational action as an arbitrary function of the Ricci scalar and also the trace of the energy-momentum tensor $\mathcal{T}$, which is called $f(R,\mathcal{T})$ theory of gravity. Within this theory, Solar System tests have been already performed [29,30]. Studies on compact astrophysical objects have also been considered in the literature [31–34]. In particular, modified theories of gravity have been shown to significantly elevate the maximum mass of compact objects [34–38], which means that $f(R,\mathcal{T})$ is of particular interest for the hydrostatic equilibrium configuration of compact stars.

In what concerns to white dwarfs they are the final evolution state of main sequence stars with initial masses up to 8.5–10.6 $M_\odot$. However, if the white dwarf (WD) mass grows over 1.44 $M_\odot$—known as Chandrasekhar mass limit [39]—as in binary systems, where the main star is receiving mass from a nearby star, a type Ia supernova explosion may occur. However, with the recently observed peculiar highly overluminous SNeIa, such as, SN 2003fg, SN 2006gz, SN 2007if, SN 2009dc [40,41], it is possible to confirm the existence of a huge Ni-mass which leads to the possibility of massive super-Chandrasekhar white dwarfs with mass 2.1–2.8 $M_\odot$ as their most feasible progenitors.

To provide some physical mechanism where a super-Chandrasekhar white dwarf could support the gravitational collapse, a lot of works have bubbled in the literature with different proposals. To cite some of them, we have general relativistic [42,43], strong magnetic field [44–50], modified theories of gravity [32,38,51–54], background gravity corrections [55], rotation [56,57], noncommutativity [58], and charge effects [59,60].

In addition, several authors have studied charged stars. Within them, there are investigations about the influence of the electrical charge distribution at the stellar structure of polytropic stars [61–63], anisotropic stars [64], strange stars [62,65] and white dwarfs [59,60]. In what concerns to charged WDs, Liu and collaborators [59] found that the charge contained in WDs can affect their structure; they have larger masses and radii than the uncharged ones. Moreover, Carvalho *et al.* have shown in their previous work [60] that the increment of the total charge from 0 to $\approx 2\times10^{20}$ C allows to increase the total mass in approximately 55.58%, and for the large total charge, more massive stellar objects are found.

Some works have also approached the coupling between charge and $f(R,\mathcal{T})$ gravity effects for stellar equilibrium [33,66–68]. Those works showed in particular, that charged objects have more stable configurations than noncharged ones. They also showed that the energy conditions are respected inside the compact objects.

Here in this work, we are particularly interested to study the charge effects within the framework of the $f(R,\mathcal{T})$ gravity, for the hydrostatic equilibrium configurations of white dwarfs. A few works [38,51,52,54,69,70] have achieved stable stellar models to explain super-Chandrasekhar white dwarfs in the background of the different modified theories of gravity. Although few researchers [53,71–73] have studied WD properties via scalar-tensor or Horndeski theories, they have only derived constraints on the parameters of the theories by comparing their results with WD observational data and not discussed the issue of super-Chandrasekhar white dwarfs that lie in the range 2.1–2.8 $M_\odot$. $f(R,\mathcal{T})$ gravity has remarkably explained both the late-time accelerated expanding phase of the Universe in the large scale and also passed the solar system test. Thus, it is also very important to study compact stellar objects as the WDs in the framework of $f(R,\mathcal{T})$ gravity theory that has been done recently [32]. However, the effects of $f(R,\mathcal{T})$ gravity theory on the charged WDs have never been done, and it is our primary motivation in the present article. We shall find that our investigation reveals that the present $f(R,\mathcal{T})$ gravity model can suitably explain the highly super-Chandrasekhar mass white dwarfs. It will also be interesting to explore the effects of other modified gravity theories, viz., scalar-tensor, Horndeski theories, etc., on the WDs in future projects and compare with the results of the present study.

The formalism of the $f(R,\mathcal{T})$ gravity is revisited in Sec. II, showing the basic equations and deriving the hydrostatic equilibrium configurations for the charged case. In Sec. III, we describe stellar properties that we assume, namely, equation of state and electric charge distribution. In Sec. IV, we outline our results and in Sec. V we present our conclusions.

II. BASIC FORMALISM

A. $f(R,\mathcal{T})$ gravity

The modified form of the Einstein-Hilbert action in the Einstein-Maxwell space-time is as follows [28]:

$$S=\frac{1}{16\pi}\int d^4x f(R,\mathcal{T})\sqrt{-g}+\int d^4x\mathcal{L}_m\sqrt{-g}+\int d^4x\mathcal{L}_e\sqrt{-g}, \tag{1}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the matter distribution, $\mathcal{L}_m$ represents the Lagrangian for the matter distribution, and $\mathcal{L}_e$ denotes the Lagrangian for the electromagnetic field.

Now, varying the action (1) with respect to the metric tensor component $g_{\mu\nu}$, we obtain the field equations of the model in $f(R,\mathcal{T})$ gravity theory as follows [28]:

$$G_{\mu\nu} = \frac{1}{f_R(R,\mathcal{T})}\left[8\pi T_{\mu\nu} + \frac{1}{2}f(R,\mathcal{T})g_{\mu\nu} - \frac{1}{2}Rf_R(R,\mathcal{T})g_{\mu\nu} - (T_{\mu\nu}+\Theta_{\mu\nu})f_{\mathcal{T}}(R,\mathcal{T}) + 8\pi E_{\mu\nu}\right], \tag{2}$$

where we define $f_R(R,\mathcal{T}) = \frac{\partial f(R,\mathcal{T})}{\partial R}$, $\Theta_{\mu\nu} = \frac{g^{\alpha\beta}\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}$, and $f_{\mathcal{T}}(R,\mathcal{T}) = \frac{\partial f(R,\mathcal{T})}{\partial \mathcal{T}}$. Here $\Box \equiv \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)/\sqrt{-g}$ is the D'Alambert operator, $R_{\mu\nu}$ is the Ricci tensor, ∇_μ represents the covariant derivative associated with the Levi-Civita connection of $g_{\mu\nu}$, $G_{\mu\nu}$ is the Einstein tensor, and $E_{\mu\nu}$ is the electromagnetic energy-momentum tensor.

We define $T_{\mu\nu}$ and $E_{\mu\nu}$ as follows:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \tag{3}$$

$$E_{\mu\nu} = \frac{1}{4\pi}\left(F^{\gamma}_{\mu}F_{\nu\gamma} - \frac{1}{4}g_{\mu\nu}F_{\gamma\beta}F^{\gamma\beta}\right), \tag{4}$$

where u_μ is the four velocity which satisfies the conditions $u_\mu u^\mu = 1$ and $u^\mu \nabla_\nu u_\mu = 0$, respectively, ρ and p represent matter density and pressure, respectively. In the present work, we consider $\mathcal{L}_m = -p$ and we obtain $\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu}$.

Now, the covariant divergence of Eq. (2) reads

$$\nabla^\mu T_{\mu\nu} = \frac{f_{\mathcal{T}}(R,\mathcal{T})}{8\pi - f_{\mathcal{T}}(R,\mathcal{T})}\left[(T_{\mu\nu}+\Theta_{\mu\nu})\nabla^\mu \ln f_{\mathcal{T}}(R,\mathcal{T}) + \nabla^\mu\Theta_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\nabla^\mu T - \frac{8\pi}{f_{\mathcal{T}}(R,\mathcal{T})}\nabla^\mu E_{\mu\nu}\right]. \tag{5}$$

Now, if we consider the simplest linear form of the function $f(R,\mathcal{T})$ as $f(R,\mathcal{T}) = R + 2\chi\mathcal{T}$, where χ is the matter-geometry coupling constant, and the field equation for $f(R,\mathcal{T})$ gravity theory reads

$$G_{\mu\nu} = (8\pi + 2\chi)T_{\mu\nu} + 2\chi pg_{\mu\nu} + \chi\mathcal{T}g_{\mu\nu} + 8\pi E_{\mu\nu} = 8\pi(T^{\rm eff}_{\mu\nu} + E_{\mu\nu}) = 8\pi T_{ab}, \tag{6}$$

where $T_{ab} = T^{\rm eff}_{\mu\nu} + E_{\mu\nu}$ represents the energy-momentum tensor of the charged effective matter distribution and $T^{\rm eff}_{\mu\nu}$ represents energy-momentum tensor of the effective fluid, i.e., "normal" matter and the new kind of fluid which originates due to the matter geometry coupling, given as

$$T^{\rm eff}_{\mu\nu} = T_{\mu\nu}\left(1 + \frac{\chi}{4\pi}\right) + \frac{\chi}{8\pi}(\mathcal{T} + 2p)g_{\mu\nu}. \tag{7}$$

Substituting $f(R,\mathcal{T}) = R + 2\chi\mathcal{T}$ in Eq. (5), we obtain

$$(4\pi + \chi)\nabla^\mu T_{\mu\nu} = -\frac{1}{2}\chi\left[g_{\mu\nu}\nabla^\mu\mathcal{T} + 2\nabla^\mu(pg_{\mu\nu}) + \frac{8\pi}{\chi}E_{\mu\nu}\right]. \tag{8}$$

B. Stellar equilibrium equations

Let consider the interior space-time is described by the metric as follows [74]:

$$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{9}$$

where the metric potentials ν and λ are the function of the radial coordinate r only.

Now substituting Eqs. (3) and (4) into Eq. (6), we find the explicit form of the Einstein field equation for the interior metric (9) as follows [31,32]:

$$e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} = (8\pi + 3\chi)\rho - \chi p + \frac{q^2}{r^4} = 8\pi\rho^{\rm eff} + \frac{q^2}{r^4}, \tag{10}$$

$$e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} = (8\pi + 3\chi)p - \chi\rho - \frac{q^2}{r^4} = 8\pi p^{\rm eff} - \frac{q^2}{r^4}, \tag{11}$$

where "$\prime$" denotes differentiation with respect to the radial coordinate r. Here $\rho^{\rm eff}$ and $p^{\rm eff}$ represent effective density and pressure of the effective matter distribution, respectively, and are given by

$$\rho^{\rm eff} = \rho + \frac{\chi}{8\pi}(3\rho - p), \tag{12}$$

$$p^{\rm eff} = p - \frac{\chi}{8\pi}(\rho - 3p). \tag{13}$$

The further essential stellar structure equations required to describe static and charged spherically symmetric sphere in $f(R,\mathcal{T})$ gravity theory are given as [33,75,76]

$$\frac{dm}{dr} = 4\pi\rho r^2 + \frac{q}{r}\frac{dq}{dr} + \frac{\chi}{2}(3\rho - p)r^2, \tag{14}$$

$$\frac{dq}{dr} = 4\pi\rho_e r^2 e^{\lambda/2}, \tag{15}$$

$$\frac{dp}{dr} = \frac{1}{[1+\frac{\chi}{8\pi+2\chi}(1-\frac{d\rho}{dp})]}\left\{-(\rho+p)\left[\left\{4\pi\rho r + \frac{m}{r^2} - \frac{q^2}{r^3} - \frac{\chi}{2}(\rho-3p)r\right\}\Big/\left(1-\frac{2m}{r}+\frac{q^2}{r^2}\right)\right] + \frac{8\pi}{8\pi+2\chi}\frac{q}{4\pi r^4}\frac{dq}{dr}\right\}, \tag{16}$$

where the metric potential e^{λ} have the usual Reisner-Nordström form

$$e^{-\lambda} = 1 - \frac{2m}{r} + \frac{q^2}{r^2}. \tag{17}$$

We describe the exterior space-time by the exterior Reissner-Nordström metric which is given as follows [74]:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 - \frac{1}{(1-\frac{2M}{r}+\frac{Q^2}{r^2})}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{18}$$

In the present case, the modified Tolman-Oppenheimer-Volkof equation [33,75,76] reads

$$-\frac{\mathrm{d}p}{\mathrm{d}r} - \frac{1}{2}\nu'(\rho+p) + \frac{\chi}{8\pi+2\chi}(\rho' - p') + \frac{8\pi}{8\pi+2\chi}\frac{q}{4\pi r^4}\frac{\mathrm{d}q}{\mathrm{d}r} = 0. \tag{19}$$

III. STELLAR PROPERTIES

A. Equation of state

It is considered that the pressure and the energy density of the fluid contained in the spherical object are as follows [39,77]:

$$p(k_F) = \frac{1}{3\pi^2\hbar^3}\int_0^{k_F}\frac{k^4}{\sqrt{k^2+m_e^2}}dk, \tag{20}$$

$$\rho(k_F) = \frac{1}{\pi^2\hbar^3}\int_{0s}^{k_F}\sqrt{k^2+m_e^2}k^2dk + \frac{m_N\mu_e}{3\pi^2\hbar^3}k_F^3, \tag{21}$$

where m_e represents the electron mass, m_N the nucleon mass, $\hbar$ is the reduced Planck constant, μ_e is the ratio between the nucleon number and atomic number for ions, and k_F represents the Fermi momentum of the electron. Equation (20) establishes the electric degeneracy pressure and (21) give the total energy density, as the sum of the relativistic electron energy density (first term of the right-hand side) and the energy density related to the rest mass of nucleons (second term of the right-hand side).

For numerical purposes, we rewrite Eqs. (20) and (21) as [60,78]

$$p(x) = \epsilon_0 f(x), \tag{22}$$

$$\rho(x) = \epsilon_0 g(x), \tag{23}$$

where

$$f(x) = \frac{1}{24}\left[(2x^3 - 3x)\sqrt{x^2+1} + 3\,\mathrm{asinh}\,x\right], \tag{24}$$

$$g(x) = \frac{1}{8}\left[(2x^3 + x)\sqrt{x^2+1} - \mathrm{asinh}\,x\right] + 1215.26x^3, \tag{25}$$

with $\epsilon_0 = m_e/\pi^2\lambda_e^3$ and $x = k_F/m_e$ is the dimensionless Fermi momentum, λ_e represents the electron Compton wavelength. In the above equation, we take $\mu_e = 2$.

B. Electric charge profile

We assume as in previous works that the star is mainly composed of degenerate material, so any charge present in the white dwarf would be concentrated close to the star's surface. Thus, following [60,79], we model the electric charge distribution in terms of a Gaussian distribution,

$$\rho_e = k \exp\left[-\frac{(r-R)^2}{b^2}\right], \tag{26}$$

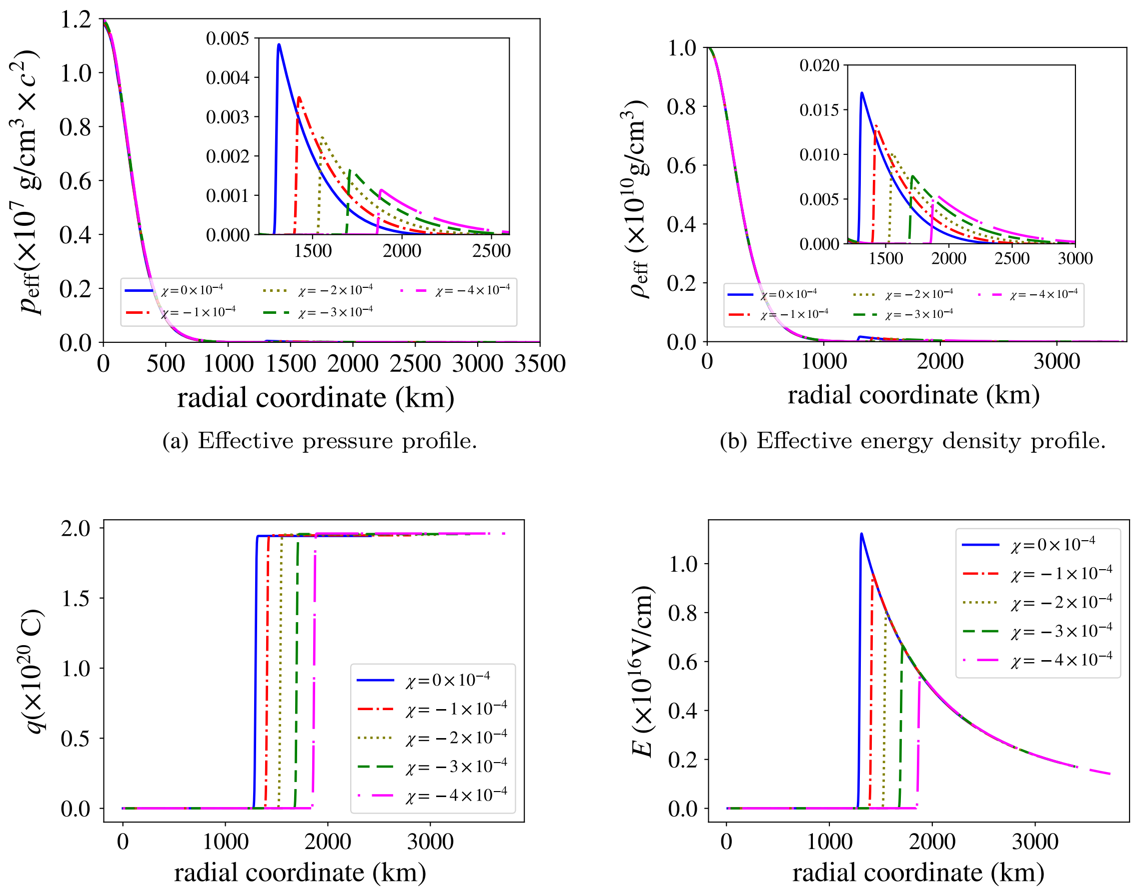
where R is the radius of the star in the uncharged case, and b is the width of the electric charge distribution. The parameter is considered to be $b = 10$ km since this is the order of magnitude of a WD's atmosphere and represents less than 1% of the star's radius. We are considering in the WD a very small charge fluctuation from the neutral case with a very tiny excess of electrons. Since electrons are lighter than ions, they move near the star surface producing the small charge layer. For comparable widths b of this layer, the WD structure does not change significantly, as we test it for values between 5 and 50 km and within this range mass and radius results have changed only ~0.01%. The chosen charge profile mostly does not change the magnitude of the total charge of the stars, as we will see later in Fig. 5, i.e., employing a different charge profile it yields the same order of the total charge for the charged stellar system ($Q \sim 10^{20}$, see Refs. [33,59,60,62,64,79]). As one can check by comparing our previous work [60] with the work of Liu *et al.* [59] that how the charge is distributed inside the star has no significant effect on its macroscopic features.

In our paper, we have defined a quantity given by σ as follows:

$$\sigma = \int_0^\infty 4\pi r^2 \rho_e dr, \tag{27}$$

where σ would be the total charge of the star if we were working on a flat background space-time. So, in the framework of GR within the finite limit of the stellar radius, we can write $\frac{dQ}{dr} = e^{\frac{\lambda}{2}} \frac{d\sigma}{dr}$. Since curvature effects are negligible in white dwarfs, σ is perfectly associated with the total charge (Q) of the star. So, σ represents the total charge and it is calculated from Eq. (27) we can certainly state that the chosen charge distribution leads to finite values of total charge and this choice of charge distribution has no infinite charge. We can estimate the proportionality constant k. Considering σ as a comparison parameter, we can estimate k as

$$8\pi k = \sigma\left(\frac{\sqrt{\pi} b R^2}{2} + \frac{\sqrt{\pi} b^3}{4}\right)^{-1}. \tag{28}$$



(a) Effective pressure profile.

(b) Effective energy density profile.

(c) Charge profile.

(d) Electric field profile.

FIG. 1. Profiles for several values of χ, $\sigma = 2 \times 10^{20}$ C and central density of $\rho_C = 10^{10}$ g/cm^3.

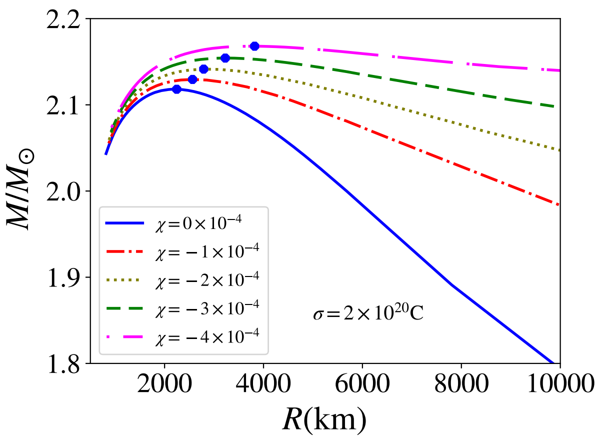
IV. RESULTS

The variation of the total mass of the charged WDs as a function of their total radii is shown in Fig. 2 for the parametric values of χ and σ. It is worth to cite that $\chi = 0$ recovers GR results for charged and noncharged stars.

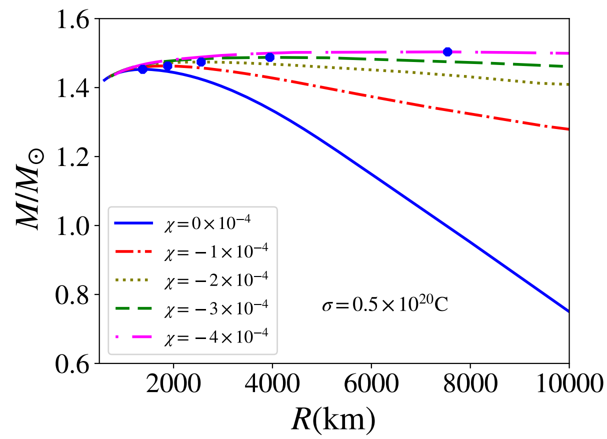
In order to observe the electric charge distribution in the star, the effective pressure inside the WD as a function of the radial coordinate is showed in Fig. 1(a), where few values of χ and $\rho_C = 10^{10}$ g/cm^3 are considered. In the figure, we can note that the pressure decays monotonically toward the baryonic surface, when it is attained, the pressure grows abruptly due to the beginning of the electrostatic layer. After this point, the pressure decreases with the radial coordinate until it attains the surface of the stars, which results in an electric charge distribution as a spherical shell close to the surface of the WD. For Fig. 1, we took into account $\rho_C = 10^{10}$ g/cm^3 and different values of χ. Figure 1(b) shows the effective energy density is as a function of the radial coordinate.

In Fig. 1(d), the behavior of the electric field in the star is presented. We can note in the figure that the electric field exhibit a very abrupt increase from zero to $10^{16\text{–}17}$ V/m; this indicates that the baryonic surface ends and starts the electrostatic layer. The same behavior can be observed in Fig. 1(c)—the interface between baryonic and electrostatic layers—where we present the charge profile.

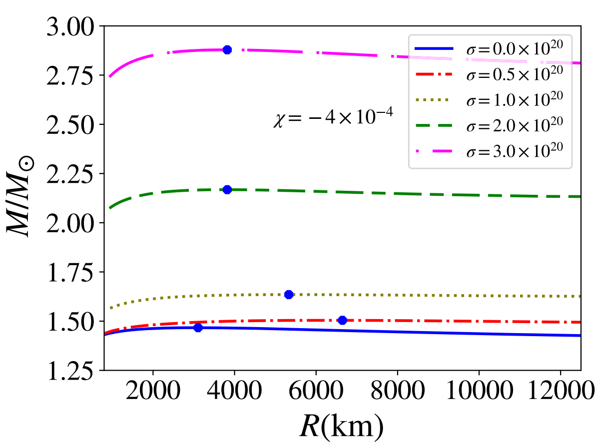
As we can see in Fig. 2 the mass of the stars grows as the total radius decreases until it attains a maximum mass point. It is important to remark that the maximum mass grows with the decrement of χ. The total radius increases when fixed star masses are considered, which implies that the effects of the $f(R, \mathcal{T})$ gravity are very important in the determination of the stellar radius. In addition, curves in Fig. 2 present a similar behavior in comparison with the mass-radius relations of the white dwarfs as reported by Carvalho and collaborators in Ref. [32]. Here in this work, we consider values of $\sigma = 2 \times 10^{20}$ C and $\sigma = 0.5 \times 10^{20}$ C. The value of total charge 10^{20} C has shown to saturate the



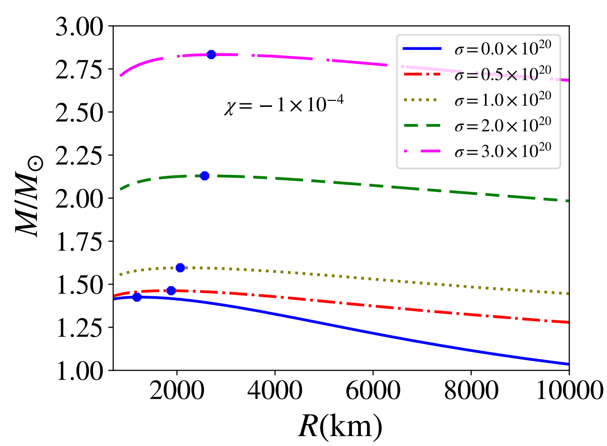
(a) Mass-radius relation of white dwarfs with varying χ and charge $\sigma = 2 \times 10^{20}$C fixed.



(b) Mass-radius relation of white dwarfs with varying χ and charge $\sigma = 0.5 \times 10^{20}$C fixed.



(c) Mass-radius relation of white dwarfs with varying σ and $\chi = -4 \times 10^{-4}$.



(d) Mass-radius relation of white dwarfs with varying σ and $\chi = -1 \times 10^{-4}$.

FIG. 2. Mass-radius relation of white dwarfs for the parametric chosen values of χ and σ.

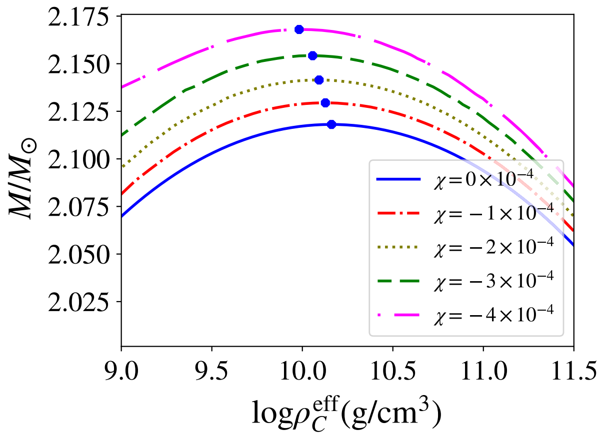


FIG. 3. Mass-central density relation of white dwarfs for several values of χ and $\sigma = 2\times10^{20}$ C.

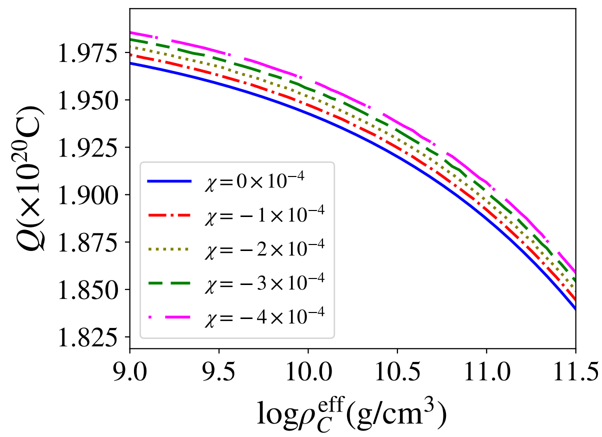


FIG. 5. Total charge versus central energy density of white dwarfs for several values of χ and $\sigma = 2\times10^{20}$ C.

electric field limit at the surface of the star, i.e., the Schwinger limit ($\sim 1.3\times10^{18}$ V/m) for a mass of 2.199 $M_\odot$ [60]. We also can see in Fig. 2 that the mass-radius curves tend to a plateau when χ is $\approx -4\times10^{-4}$. This result is corroborated by the one obtained in Ref. [32].

In Fig. 3, we present the mass-central density relation of static, charged, and noncharged WDs for five different values of χ and $\sigma = 2\times10^{20}$ C. As in previous works [33,59,60,62,64,79], we can see that the charge produces a force, repulsive in nature, which helps the one generated by the radial pressure to support more mass against the gravitational collapse, so the masses in the charged case can be larger than in the noncharged one. We present also the radius-central density relation in Fig. 4. To construct Figs. 3 and 4, we used effective central energy density, defined as in Eq. (12).

In Fig. 5, it is shown the total charge of the star as a function of the central effective energy density for the chosen parametric values of χ and $\sigma = 2\times10^{20}$ C. One can see that the total charge slightly varies with the increasing effective central density. The values of total charge may seem to be huge as it is 39–40 orders of magnitude larger than the elementary one. However, if we calculate the total number of electrons inside the neutral core of the WDs, we obtain $N\sim10^{56}$ electrons, and considering the total charge of the stars to be 10^{20} C, the exceeding number of electrons are of order $N\sim10^{39}$, which means deviations from charge neutrality are actually negligible and the apparent high total charge is feasible. Instead of such high surface charge of the order $\sim10^{20}$ C, the charged stellar system should be more stable due to the balance of the forces, viz., the inward and attractive gravitational force would be counterbalanced by the combined effect of the exterior and repulsive hydrodynamic force, electric force, and the force originates due to coupling between the matter and geometric terms. Hence, the present system is stable and capable of sustaining the apparently large amount of charge. Importantly, the study of similar kind of charged astrophysical systems is also found in several recent articles, such as [33,59,60,62,64,79]. On the other hand, to explain the super-Chandrasekhar white dwarf in this work, we have considered strongly charged WD model in the background of $f(R,\mathcal{T})$ gravity theory. Although, till this date, no charged WD has been observed, still, the present study is important in the theoretical aspect in explaining the super-Chandrasekhar white dwarfs which are hardly explained in the framework of GR.

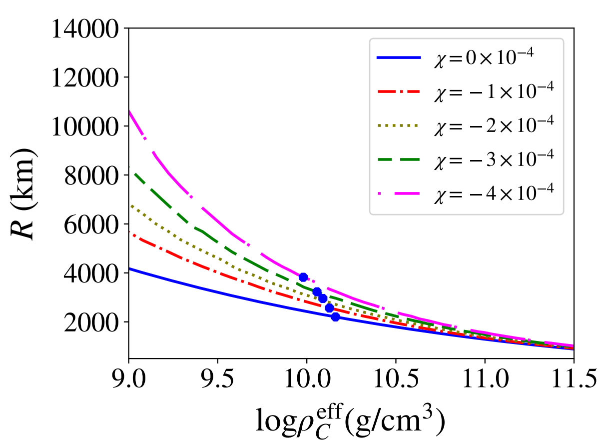


FIG. 4. Central energy density versus total radius of white dwarfs for several values of χ and $\sigma = 2\times10^{20}$ C.

In Table I, we present the maximum masses ($M_{\rm max}$) for the charged WD in $f(R,\mathcal{T})$ gravity with their total radii (R) and effective central energy densities ($\rho_C^{\rm eff}$) for each value of χ used in this work. It is possible to note that more massive and large charged WDs are found with the decrement of χ. We also note an important effect caused by the $f(R,\mathcal{T})$ gravity theory that is the increase of the radii, which contributes to the stability of the star, since it reduces the surface electric field. From Table I, one can realize that as the values of χ decrease the stellar system becomes more massive and larger in size turning itself into a less dense

TABLE I. The values for the constant χ and the maximum masses of the charged white dwarfs in $f(R,\mathcal{T})$ gravity with their respective radii, effective central densities, charges, and electric fields at the surface of the stars for the value of $\sigma = 2 \times 10^{20}$ C.

χ	$M_{\max}/M_{\odot}$	R (km)	$\rho_C^{\rm eff}$(g/cm^3)	Q(C)	E(V/m)
-0×10^{-4}	2.11	2201	1.45×10^{10}	1.94×10^{20}	3.59×10^{17}
-1×10^{-4}	2.13	2565	1.34×10^{10}	1.94×10^{20}	2.65×10^{17}
-2×10^{-4}	2.14	2954	1.23×10^{10}	1.95×10^{20}	2.01×10^{17}
-3×10^{-4}	2.15	3227	1.14×10^{10}	1.95×10^{20}	1.69×10^{17}
-4×10^{-4}	2.17	3820	9.60×10^{9}	1.96×10^{20}	1.21×10^{17}

TABLE II. The values for the constant σ and the maximum masses of the charged white dwarfs in $f(R,\mathcal{T})$ gravity with their respective radii, effective central densities, charges, and electric fields at the surface of the stars for $\chi = -4 \times 10^{-4}$.

σ (C)	$M_{\max}/M_{\odot}$	R (km)	$\rho_C^{\rm eff}$(g/cm^3)	Q(C)	E(V/m)
0.0×10^{20}	1.47	2940	3.37×10^{9}	...	...
0.5×10^{20}	1.50	6647	2.60×10^{9}	4.94×10^{19}	1.00×10^{16}
1.0×10^{20}	1.63	5330	4.29×10^{9}	9.86×10^{19}	3.12×10^{16}
2.0×10^{20}	2.17	3820	9.60×10^{9}	1.96×10^{20}	1.21×10^{17}
3.0×10^{20}	2.88	3770	9.94×10^{9}	2.94×10^{20}	1.86×10^{17}

compact stellar object as predicted by Carvalho *et al.* in their study [60]. We have also predicted in Table II different physical parameters of the compact stellar system due to the variation of σ for a chosen parametric value of $\chi = -4 \times 10^{-4}$. Table II features that with the increasing values of σ as usually the mass of the white dwarfs increase along with their surface charge and electric field, whereas the stellar system becomes gradually denser as its central density increase gradually with the increasing values of σ. Note that $f(R,\mathcal{T})$ should affect all the stars from low mass WDs to the super-Chandrasekhar limit of the mass for WDs and our study can suitably explain all the WDs. In Table II, we have predicted the maximum mass points for $\chi = -4 \times 10^{-4}$ and parametric chosen values of σ which also can be seen from Fig. 2(c). Readers should carefully notice that for $\chi = -4 \times 10^{-4}$ and $\sigma = 0$ the M-R curve is not predicting the WDs have far low mass compared to the maximum mass point 1.47 $M_{\odot}$. However, this happened only because with the appropriate choice of χ and σ as we wanted to show WDs in the super-Chandrasekhar mass interval. We find for σ in the range 2×10^{20}–3×10^{20} C the present $f(R,\mathcal{T})$ model is suitable to predict different physical parameters of the highly super-Chandrasekhar white dwarfs having mass 2.17–2.88 $M_{\odot}$. It is worth mentioning that the maximum electric field obtained in this work does not surpass the Schwinger limit of 1.3×10^{18} V/m for charge screening by pair production [60,80] (see Tables I and II), which means that the $f(R,\mathcal{T})$ gravity enhances the stability of the charged stars. However, it is possible to show WDs even in the low mass limit with the appropriate choice of χ and σ as shown in Figs. 2(b) and 2(d).

V. CONCLUSIONS

In this paper, we investigate the effects of a specific modified theory of gravity, namely, the $f(R,\mathcal{T})$ gravity, in the structure of charged white dwarfs. The procedure started from the derivation of the hydrostatic equilibrium equation for such a theory, with the addition of the charged effects. We suppose a Gaussian ansatz for the net charge distribution.

The main goal was to check the imprints of the extra material terms that come from the $\mathcal{T}$ dependence of the theory on charged WD properties.

The equilibrium configurations of charged white dwarfs were analyzed for $f(R,\mathcal{T}) = R + 2\chi\mathcal{T}$ with different values of χ and central densities. We observed that the charged white dwarfs can be affected by the extended theory of gravity in the maximum mass and radius depending on the value of χ.

We found that for $\chi = -4 \times 10^{-4}$ and $\sigma = 3 \times 10^{20}$ C, the maximum mass of the charged WD is 2.88 $M_{\odot}$, and the radii have considerable increasing. This larger radius yields a smaller surface electric field, thus enhancing the stellar stability of charged stars.

The possibility of explaining the highly super-Chandrasekhar limiting mass white dwarfs as a progenitor of the peculiar overluminous super-SNeIa in the framework of $f(R,\mathcal{T})$ gravity theory was first raised by Deb and collaborators in their work [33]. In the present work, we have successfully explained the highly super-Chandrasekhar limiting mass white dwarfs having mass 2.17–2.88 $M_{\odot}$ which remained hardly explained in the framework of GR. However, when explaining super-Chandrasekhar white dwarfs with modified theories of gravity one may wonder superluminous supernovae would be more common since modified gravity would affect all the stars. However, our present study reveals that one can easily explain WDs having the sub and super-Chandrasekhar masses by employing suitable choices of parametric values for σ and χ. Since gravitational fields are smaller for WDs than for neutron stars (NS) or strange stars (SS), the scale parameter χ used for WDs is small when compared to the values used for NSs and WDs, and also the values of χ used for NSs are smaller than the ones used for SSs. Solar system constraints also indicate χ must be of order $\sim 10^{-13}$ [81]. This indicates that more compact the system more deviations from GR theory are needed and the parameter χ may mimic a kind of chameleon mechanism, where the parameter scale depends on the density (or compactness/field regime) of the system [60,82,83]. As a final comment, the present work not only pushes the maximum mass limit for white dwarfs beyond the standard value of the Chandrasekhar mass limit but also plausibly explaining the requirement of the application of $f(R,\mathcal{T})$ gravity theory in studying astrophysical observations.

ACKNOWLEDGMENTS

F. R. would like to thank CAPES (Coordenação de Aperfeiçoamento Pessoal de Nível Superior) for financial support. G. A. C. thanks CAPES Grants No. PDSE/88881.188302/2018-01 and No. PNPD/88887.368365/2019-00. Research of D. D. is funded by the C. V. Raman Postdoctoral Fellowship (Reg. No. R(IA)CVR-PDF/2020/222) from Department of Physics, Indian Institute of Science. A part of this work was completed by D. D. when he was visiting the IUCAA, and D. D. gratefully acknowledges warm hospitality there. One of the authors (D. D.) is thankful to Banibrata Mukhopadhyay for his pertinent suggestions which helped to upgrade the paper substantially. M. M. acknowledge CAPES, CNPq, and FAPESP thematic Project No. 13/26258-4. All computations were performed in open source software, and the authors are sincerely thankful to open source community [84–86]. We are also thankful to the anonymous referee for several pertinent suggestions which helped us to upgrade this paper.

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