

## Multimessenger tests of gravity with weakly lensed gravitational waves

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General relativity (GR) predicts concordant trajectories for photons and gravitational waves (GWs). We propose a new multimessenger avenue (GW-CMB-CMB) to prove this aspect of fundamental physics by cross-correlating the GW signal of astrophysical origin with the lensing field derived from the cosmic microwave background (CMB). This new window will allow robust measurement of the prediction from GR with high signal-to-noise ratio and will be able to unveil the true nature of gravity using the GW sources detected by missions such as the Laser Interferometer Space Antenna, Einstein Telescope, and Cosmic Explorer.

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### I. INTRODUCTION

Our endeavor to understand the Universe through electromagnetic waves (EWs) over a wide frequency band ranging from radio to gamma rays has enabled us to construct the standard model of cosmology over a large redshift range and unveiled various cosmic secrets. Several probes such as supernovae, the cosmic microwave background (CMB), galaxy surveys, and quasars are the founding pillars of the Lambda cold dark matter (ΛCDM) standard model of the Universe. This model matches the observational evidence of enigmatic late-time acceleration and invisible, dark matter.

Gravitational waves (GWs) are a new avenue capable of probing the Universe through white dwarfs, neutron stars, and binary black holes (BBHs). In the framework of general relativity (GR), GWs (like EWs) propagate along the geodesics defined by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. However, due to the presence of the matter distribution in the Universe, GWs interact gravitationally with the matter distribution and hence propagate through the perturbed FLRW metric, written as

$$ds^2 = (1 - 2\Phi)dt^2 - a(t)^2(1 - 2\Psi)(dx^2 + dy^2 + dz^2), \quad (1)$$

where  $a(t)$  is the scale factor, and  $\Phi$  and  $\Psi$  are the scalar perturbations due to the matter distribution in the Universe.

We propose a new method for probing the propagation of GWs in a perturbed metric. General relativity predicts that weak lensing due to the intervening matter distribution affects the geodesics of EWs and GWs in the same manner [1] (see the schematic diagram in Fig. 1). To test this fundamental prediction, the cross-correlation of GW and EW signals we propose is the *only known avenue* capable to detect the *gravitational lensing of GW* unambiguously.

Furthermore, this new avenue will be capable to explore multiple theoretical aspects such as (i) alternative theories to GR [2,3] (by probing the running of the Planck mass, anisotropic stress, graviton mass), (ii) dimensions of the space-time [4], and (iii) the difference between the two scalar potentials  $\Phi$  and  $\Psi$  [5–9]. The above mentioned theoretical aspects lead to several observable effects. Alternative theories of gravity lead to a different propagation equation of GWs from GR which can be written in terms of a few additional parameters  $\alpha_M, c_T, \mu$ , and  $\Gamma_{ij}$  [3,10–14] as

$$h''_{ij} + (2 + \alpha_M)\mathcal{H}h'_{ij} + (c_T^2 k^2 + a^2 \mu^2)h_{ij} = a^2 \Gamma_{ij}. \quad (2)$$

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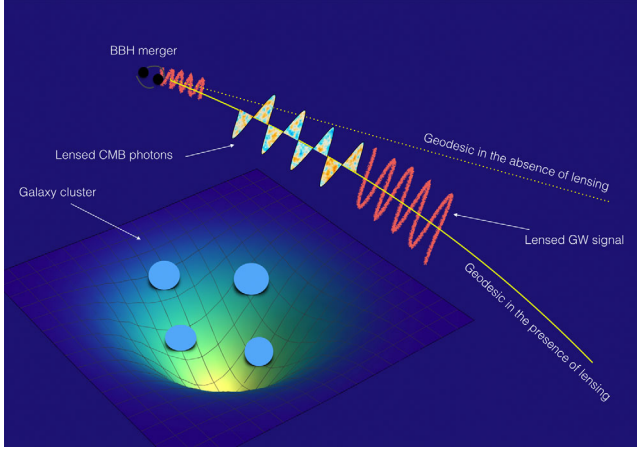


FIG. 1. GR predicts identical geodesics of the lensed GWs as for the lensed photons due to cosmic perturbations and hence should be correlated. This schematic diagram depicts the physical mechanism behind the correlation between the lensed CMB photons and the lensed GW signal from astrophysical sources.

The single combined EW-GW observation of a binary neutron star ruled out vast swathes of alternative gravity models via the time delay between EW and GW signals [13] and constraining the term  $c_T$  and  $\mu$  which are related to the speed of GWs and the graviton mass. However, the other two parameters such as  $\alpha_M$  and  $\Gamma_{ij}$  remain unconstrained. Along with the effects on GW propagation, alternate theories of gravity also affect the Poisson equation

$$\begin{aligned}\nabla^2(\Phi + \Psi) &= 8\pi G_{\text{light}} a^2 \rho \delta, \\ \nabla^2\Phi &= 4\pi G_{\text{matter}} a^2 \rho \delta,\end{aligned}\quad (3)$$

where  $G_{\text{light}}$  and  $G_{\text{matter}}$  are equal in GR, but differ in alternative theories of gravity,  $\rho$  is the matter density, and  $\delta$  denotes the density contrast. The gravitational lensing of GWs is affected by both  $\Phi$  and  $\Psi$ . As a consequence, in order to understand the theory of gravity it is essential to measure all the effects on the GW signal as it propagates through the space-time. For a given theory of gravity, all the parameters  $\alpha_M$ ,  $\Gamma_{ij}$ ,  $G_{\text{matter}}$ , and  $G_{\text{light}}$  lead to an observable signature and are not necessary to be independent from each other. The method proposed by us in this paper enables one to probe both Eqs. (2) and (3). As a result, the modification in the luminosity distance due to the modified theories of gravity and the effects on the GW strain [which can be related to the luminosity distance, see the discussion around Eq. (17)] can be jointly estimated in this method. It will also be a direct test of the equivalence principle by comparing the concurrent trajectory of EWs and GWs up to a high cosmological redshift. This method opens a new scientific window to study fundamental physics with GW binaries by exploring the two-point correlation between GW strain and CMB lensing (or equivalently the three-point correlation between GW strain and CMB fields such

as temperature and polarization anisotropies) and goes beyond the luminosity distance-redshift test which probes only the background cosmology. It also enables a correct estimate of the signatures of modified gravity theories from the strain of the GW signal by eliminating the degeneracy with the weak lensing field. Finally, our proposed scheme makes it possible to remove the effect of weak lensing (or *delense*) from the GW signal and hence reduce the additional uncertainty in the GW source parameters.

## II. WEAK LENSING OF CMB

Cosmological observables like the CMB and the galaxy field carry the imprint of lensing and were measured recently [15]. The weak lensing of the CMB temperature ( $T$ ) and the polarization field ( $E$ ,  $B$ ) can be written as

$$\tilde{X}(\hat{n}) = X(\hat{n} + \vec{\nabla}\phi(\hat{n})), \quad (4)$$

where  $X \in T, E, B$  and  $\vec{\nabla}\phi(\hat{n})$  is the deflection angle and  $\phi(\hat{n})$  is the lensing potential. Different lensing estimators [16–18] are developed to reconstruct this signal by using the off-diagonal correlations between  $T$ ,  $E$ ,  $B$  ( $EB, TT, EE, TE, BB, TB$ ). The commonly used quadratic minimum variance estimator [16] reconstructs the lensing potential with the corresponding lensing reconstruction noise  $N_l^{\kappa\kappa}$  [16] which is obtained from all the cross- and autocorrelations between  $T$ ,  $E$ , and  $B$  mentioned above.

The deflection angle  $\vec{\nabla}\phi$  is related to the lensing convergence field  $\kappa$  by the relation  $\kappa = -\nabla^2\phi/2$  [or in the spherical harmonic space<sup>1</sup> by the relation  $\kappa_{lm} = l(l+1)\phi_{lm}/2$ ]. This in turn is related to the intervening matter distribution between us and the CMB source plane by the relation

$$\kappa_{\text{CMB}}(\hat{n}) = \int_0^{z_s} dz W_{\text{CMB}}(\chi(z)) \delta(\chi(z)\hat{n}, z), \quad (5)$$

where  $z_s = 1089$  is the source redshift of CMB, and  $W_{\text{CMB}}(\chi(z))$  is defined as

$$W_{\text{CMB}}(\chi(z)) = \frac{3\Omega_m H_0^2 (1+z)\chi(z)}{2cH(z)} \left[ \frac{\chi(z_s) - \chi(z)}{\chi(z_s)} \right], \quad (6)$$

where  $H_0$  is the Hubble constant,  $\Omega_m$  is the matter density,  $\chi(z)$  is the comoving distance to redshift  $z$ , and  $\delta$  is the perturbation in the matter distribution. Here we have used the Poisson equation  $\nabla^2\Phi = 3\Omega_{m0}H_0^2\delta/2a$  to connect the potential  $\Phi$  with the matter density perturbations  $\delta$ .

<sup>1</sup>Any spin-0 field  $P(\hat{n})$  in the sky can be written in the spherical harmonics basis as  $P(\hat{n}) = \sum_{lm} P_{lm} Y_{lm}(\hat{n})$ .

### III. EFFECT OF COSMOLOGICAL PERTURBATIONS ON GWs

GW strain from the inspiraling binaries can be written in Newtonian order<sup>2</sup> in frequency domain ( $\nu$ ) as [1,19–21]

$$h(\nu_z) = \mathcal{Q}(\text{angles}) \sqrt{\frac{5}{24}} \frac{G^{5/6} M_z^2 (\nu_z M_z)^{-7/6}}{c^{3/2} \pi^{2/3} d_L} e^{i\phi_z}, \quad (7)$$

where  $\nu_z$  and  $M_z$  are redshifted frequency and redshifted chirp mass, respectively, which are related to the source chirp mass  $M_c$  and emitted frequency  $\nu_e$  by  $M_z = (1+z)M_c$  and  $\nu_z = \nu_e/(1+z)$ .  $\phi_z = 2\pi\nu_z t_0 + \phi_r(t_0)$  is the phase of the GWs with  $t_0$  as the stationary point of the phase.  $\mathcal{Q}(\text{angles})$  is the factor which depends upon the source orientation. The above equation is valid only in the inspiraling phase of the binaries and not during its merger. The information regarding the background cosmology can be inferred from the luminosity distance independent of the chirp mass by using the relationship [22]

$$d_L \propto \frac{1}{\bar{h}(t)\tau\nu^2}, \quad \text{where } \tau \equiv \left(\frac{d\nu/dt}{\nu}\right)^{-1} \propto \frac{\pi M_z^2}{(\pi M_z)^{11/3} \nu^{8/3}},$$

$$\text{and } \bar{h}(t) \propto \frac{M_z (\pi \nu_z M_z)^{2/3}}{d_L}. \quad (8)$$

Here  $\bar{h}$  is the GW strain averaged over detectors and source orientations and  $\tau$  is the timescale related to the change of the frequency. The above quantity is independent of the chirp mass and is a useful estimator of the luminosity distance [22]. A more general luminosity distance estimator using the two polarization states of the GW ( $h_+$ ,  $h_\times$ ) and the source orientation has also been studied [23].

The observed GW strain can be modeled as  $h^{\text{obs}}(t) = h(t) + n(t)$  where  $h(t)$  is the signal strain and  $n(t)$  is the noise strain. The signal-to-noise ratio ( $\rho$ ) for the GW signal can be written in the frequency domain as [21,24]<sup>3</sup>

$$\rho^2 = 4 \int_0^\infty \frac{\nu^2 |h(\nu)|^2}{|h_n(\nu)|^2} d \ln \nu, \quad (9)$$

where  $h_n$  is the dimensionless noise strain which depends upon the experimental noise power spectrum ( $S_n$ ) as  $|h_n(\nu)|^2 = \nu S_n$ .  $|h(\nu)|^2$  is the power spectrum of the signal strain and depends upon the GW source properties and luminosity distance as mentioned in Eq. (7).

<sup>2</sup>For this paper, we will use the Newtonian waveform [19], which can be generalized to the waveform including post-Newtonian (PN) corrections and also with the waveform generated using numerical relativity. Inclusion of the PN corrections will not change the primary concept of this paper.

<sup>3</sup>The factor 4 appears due to the definition of the SNR with only one-sided noise density. Usually the characteristic strain  $h_c(\nu) \equiv 2\nu h(\nu)$  is used in the literature.

GWs propagate through the geodesics defined by the perturbed FLRW metric defined in Eq. (1). The presence of cosmological perturbations in this metric leads to change in the emitted GW frequency  $\tilde{\nu}$  which can be written as [25]

$$\tilde{\nu} = \nu \left( 1 - \left( \Phi|_e^r - (\vec{n} \cdot \vec{v})|_e^r - \int_{\lambda_e}^{\lambda_r} \partial_\eta (\Psi + \Phi) d\lambda' \right) \right), \quad (10)$$

where the first term is the Sachs-Wolfe (SW) effect, the second term is the Doppler effect due to the difference in the velocity of the source and the observer, and the third term is the integrated-SW effect due to the presence of dark energy. The rms fluctuations in  $\nu$  is of the order  $10^{-5}$  [25] and can be considered to be a negligible effect for the current discussion.

The GW strain also gets modified by the matter perturbations [25–28], with the dominant contributions arising from lensing for the GW sources at high redshift

$$\tilde{h}(\hat{n}, \nu_z) = h(\nu_z) [1 + \kappa_{\text{gw}}(\hat{n})], \quad (11)$$

where  $\kappa_{\text{gw}}(\hat{n})$  is the convergence field due to weak lensing, which can be written in terms of the intervening matter density field  $\delta$  by the relation

$$\kappa_{\text{gw}}(\hat{n}) = \int_0^{z_s} dz W_{\text{gw}}(\chi(z)) \delta(\chi(z)\hat{n}, z), \quad (12)$$

where  $W_{\text{gw}}(\chi(z))$  is the lensing kernel defined as

$$W_{\text{gw}}(\chi(z)) = \frac{3 \Omega_m H_0^2 (1+z) \chi(z)}{2 c H(z)} \times \int_z^\infty dz' \frac{dn_{\text{gw}}(z')}{dz'} \frac{(\chi(z') - \chi(z))}{(\chi(z'))}. \quad (13)$$

Here  $\frac{dn_{\text{gw}}(z)}{dz}$  is the normalized ( $\int dz \frac{dn_{\text{gw}}(z)}{dz} = 1$ ) redshift distribution of the GW sources,  $\chi(z)$  is the comoving distance to redshift  $z$ ,  $\Omega_m \equiv \rho_m/\rho_c$  is the matter density in terms of critical density  $\rho_c$ , and  $H_0$  is the current value of the Hubble parameter.

The all-sky lensing convergence map can be expressed as  $\kappa(\hat{n}) = \sum_{lm} \kappa_{lm} Y_{lm}(\hat{n})$ , where  $Y_{lm}(\hat{n})$  are the spherical harmonics. The correlation between the convergence lensing field of the CMB and GWs due to the perturbed geodesics can be written as

$$C_l^{\kappa_{\text{gw}} \kappa_{\text{CMB}}} \equiv \langle (\kappa_{\text{gw}})_{lm} (\kappa_{\text{CMB}}^*)_{l'm'} \rangle \delta_{ll'} \delta_{mm'}, \quad (14)$$

where the angular bracket denotes ensemble average and  $C_l^{\kappa_{\text{gw}} \kappa_{\text{CMB}}}$  is the power spectrum of the cross-correlation field between CMB and GWs which can be written as

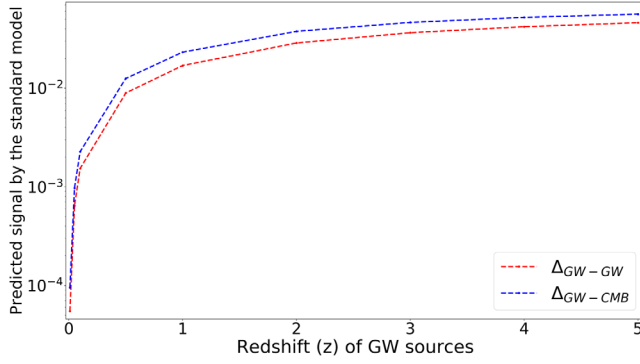


FIG. 2. The theoretical rms signal of the CMB lensing-GW correlation and GW-GW correlation due to the convergence field is shown in blue and red, respectively, as a function of redshift for best-fit cosmological parameters using the nonlinear matter power spectrum.

$$C_l^{\kappa_{\text{gw}}\kappa_{\text{CMB}}} = \int \frac{dz}{\chi(z)^2} \frac{H(z)}{c} [W_{\text{gw}}(\chi(z))W_{\text{CMB}}(\chi(z)) \times P_{\delta}((l+1/2)/\chi(z))]. \quad (15)$$

Here  $P_{\delta}((l+1/2)/\chi)$  is the nonlinear matter power spectrum of the cosmic density field evaluated at  $k = (l+1/2)/\chi(z)$  in the Limber approximation, obtained using the numerical code CLASS [29–31].<sup>4</sup> This correlation is observable as a GW-CMB-CMB three-point correlation since the estimate of the CMB lensing potential is quadratic in the CMB anisotropies. The predicted rms signal strength  $\Delta_{\text{GW-CMB}} \equiv [\sum_l (2l+1)C_l^{\kappa_{\text{gw}}\kappa_{\text{CMB}}}/4\pi]^{1/2}$  for the LCDM model in the framework of general relativity is shown as a function of redshift  $z$  of the source in Fig. 2. We have taken the GW source redshift distribution as  $\frac{dn}{dz} = \delta(z-z')$  for the plot in Fig. 2. The theoretical signal strength of CMB lensing-GW correlation for the LCDM model is greater than  $10^{-2}$  at redshift above 0.5. Since the CMB source redshift is at  $z_s = 1089$ , the GW sources present at high redshift have more overlap with the CMB lensing kernel and hence exhibit a stronger signal. The autocorrelation between the GW signal is also depicted in Fig. 2, in accordance with [25]. Along with CMB lensing-GW correlation, galaxy-GW cross-correlation is another avenue to study the lensing of GW strain [37].

#### IV. ESTIMATOR OF THE CONVERGENCE FIELD FROM GW STRAIN

In order to make an estimate of the convergence field from the luminosity distance, we need to make an estimate of the true luminosity distance  $d_L^{\text{es}}$  to the GW source by

<sup>4</sup>We have used the following packages in this analysis: CLASS [29–31], IPython [32], Mathematica [33], Matplotlib [34], NumPy [35], and SciPy [36].

using the redshift  $z$  of the source and the best-fit cosmological parameters. The massive BBHs which can be detected by Laser Interferometer Space Antenna (*LISA*) are expected to have electromagnetic signatures [38–43] and we can identify the host galaxy [44] and its redshift ( $z$ ) using upcoming missions [45–48] or other dedicated spectroscopic surveys. For stellar origin BBHs which can be probed from Cosmic Explorer, we may not have an electromagnetic counterpart and, as a result, the redshift error will be large.

So, by using the redshift from the electromagnetic follow-up and the value of the background cosmological parameters from large scale structure and CMB upcoming missions [45,49–51],<sup>5</sup> we can estimate the luminosity distance  $d_L^{\text{es}}$  as

$$d_L^{\text{es}} = \frac{c}{H_0} (1+z) \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_{de}}}. \quad (16)$$

By using Eqs. (8) and (11), we can write the estimator of the convergence field for a distribution of the GW sources as

$$\hat{D}_L(\hat{n}) = \kappa_{\text{gw}}(\hat{n}) + \epsilon(\hat{n}) - \epsilon(\hat{n})\kappa_{\text{gw}}(\hat{n}), \quad (17)$$

where  $\hat{D}_L(\hat{n}) \equiv 1 - \frac{D_L(\hat{n})}{d_L^{\text{es}}(\hat{n})}$  is a probe to the convergence field along with an additional term which is related to the error  $\epsilon = 1 - d_L/d_L^{\text{es}}$  in the estimate of the true luminosity distance. We expect the error  $\epsilon$  to be small ( $\epsilon \ll 1$ ) if the source redshift and cosmological parameters are measured accurately. More details about the estimator can be found in [37]. Here we have assumed that the waveform of the GW signal can be modeled according to GR. Alternate theories of gravity can have imprints on the GW signal in the strong gravity regime. But it is often assumed that such effects can be avoided by a screening mechanism [52]. The test proposed in this paper can be also done by including the effects in the waveform along with the effects from GW propagation and lensing. This we will address in future work.

Studies from simulations have shown that BBH sources detected by *LISA* can have an electromagnetic counterpart [38–43] which will allow us to obtain the redshift of these objects.<sup>6</sup> As a result, the second term in Eq. (17) can be negligible for  $10^4$ – $10^7 M_{\odot}$ . The error in the redshift measurement from a follow-up photometric survey is considered as  $\sigma_z/(1+z) = 0.03$  in this analysis. However, the redshift error can be negligible ( $\sigma_z \approx 0$ ) for a

<sup>5</sup>These cosmological probes will reach a much better accuracy in the cosmological parameters than the current estimates.

<sup>6</sup>However there can be scenarios where the electromagnetic counterpart is not present for the *LISA* source. In this case, we need to consider large redshift errors as considered for the forecast for Cosmic Explorer.

spectroscopic follow-up mission, resulting in an improvement in the SNR. For stellar origin BBHs which can be probed from Cosmic Explorer, we may not have any electromagnetic counterpart and, as a result, the  $d_l^{\text{es}}$  will be noise dominant leading to a large value of  $\epsilon$ . In our analysis, we have taken this into account by considering 100% error in the estimate of source redshift for the forecast of Cosmic Explorer.

The cross-correlation  $\hat{\mathcal{E}}^{\kappa_{\text{gw}}\kappa_{\text{CMB}}}$  between the convergence field from CMB  $\hat{\kappa}_{\text{CMB}}(\hat{n})$  and the  $\hat{D}_L(\hat{n})$  signal can be written as

$$\hat{\mathcal{E}}^{\kappa_{\text{gw}}\kappa_{\text{CMB}}} = \int \frac{d^2\hat{n}}{4\pi} (\epsilon(\hat{n}) + \hat{\kappa}_{\text{gw}}(\hat{n}))\hat{\kappa}_{\text{CMB}}(\hat{n}'). \quad (18)$$

The correlation of the GW strain with the convergence map is a three-point correlation function (also called the bispectrum [53,54]) between two CMB fields ( $T, E, B$ ) and the GW strain. This equation implies that the sources of GW compact objects detected at a direction  $\hat{n}$  with a strain  $\tilde{h}(\nu, \hat{n})$  will show a correlated signal with the convergence map obtained from the CMB. As the convergence field is uncorrelated with the error  $\epsilon(\hat{n})$ , the first term on the right-hand side goes to zero. The second term is a cosmological signal which captures the rms fluctuations due to the convergence field of the CMB and GWs.

The corresponding covariance matrix with a diagonal approximation can be calculated using

$$(\sigma_l^{\text{gw-CMB}})^2 = \frac{1}{f_{\text{sky}}(2l+1)} ((C_l^{\kappa_{\text{gw}}\kappa_{\text{gw}}} + N_l^{\text{DD}})(C_l^{\kappa_{\text{CMB}}} + N_l^{\text{KK}}) + (C_l^{\kappa_{\text{gw}}\kappa_{\text{CMB}}})^2), \quad (19)$$

where  $C_l^{\kappa_{\text{gw}}\kappa_{\text{gw}}} = \langle (\kappa_{\text{gw}})_{lm}(\kappa_{\text{gw}})_{l'm'} \rangle \delta_{ll'}\delta_{mm'}$  is the convergence power spectrum from the autocorrelation of GW-GW,  $N_l^{\text{DD}}$  is the measurement error associated with the GW luminosity distance determination (we explain this quantity in detail later),  $N_l^{\text{KK}}$  is the reconstruction noise due to lensing estimation [16] (as we mentioned previously), and  $f_{\text{sky}}$  is the sky fraction available common between the GW sources and CMB. For a number of GW sources  $N_{\text{gw}}$  with the same value of  $\sigma_{d_l}$ , the spatial GW noise mentioned in Eq. (19) can be written as

$$N_l^{\text{DD}} = \frac{4\pi}{N_{\text{gw}}} \left( \frac{\sigma_{d_l}^2}{d_l^2} + \frac{\sigma_b^2}{d_l^2} \right) e^{l^2\theta_{\text{min}}^2/8 \ln 2}, \quad (20)$$

where  $\sigma_{d_l}^2$  can be obtained using Eq. (9) and  $\sigma_b^2$  is the error due to the estimation of the background cosmological parameters, lensing, and redshift of the GW source. The presence of the sky-localization error for the GW sources will lead to a poor angular resolution of the GW sources, and as a result the spatial correlations cannot be probed for scales smaller than the angular scale of the sky-localization

error. This translates to a maximum value of  $l_{\text{max}} \approx 180^\circ/\theta_{\text{min}}$ , beyond which there is no signal and the CMB lensing-GW correlation is noise dominated.

The SNR of the CMB lensing-GW correlation can be written in terms of  $C_l^{\kappa_{\text{gw}}\kappa_{\text{CMB}}}$  and  $\sigma_l^{\text{gw-CMB}}$  (and are defined in Eqs. (15) and (19)), respectively, as  $(\text{SNR})^2 = \sum_l^{l_{\text{max}}} \left( \frac{C_l^{\kappa_{\text{gw}}\kappa_{\text{CMB}}}}{\sigma_l^{\text{gw-CMB}}} \right)^2$ . Using the *LISA* noise and the Newtonian GW waveform of the form mentioned in Eq. (7), we make a Fisher estimate of the luminosity distance error for different masses of BBHs up to a maximum frequency equal to the merger frequency defined as  $\nu_{\text{merge}} = 205(20 M_\odot/M)$  Hz [24]. We do not consider the merger and ring-down phase of the BBHs in the estimate of the SNR for this paper. Inclusion of the merger and ring-down phase of BBHs will improve the SNR [55].

## V. FORECAST FOR *LISA* AND COSMIC EXPLORER

The CMB lensing-GW correlation is strong for GW sources at high redshift (as depicted in Fig. 2) and hence we

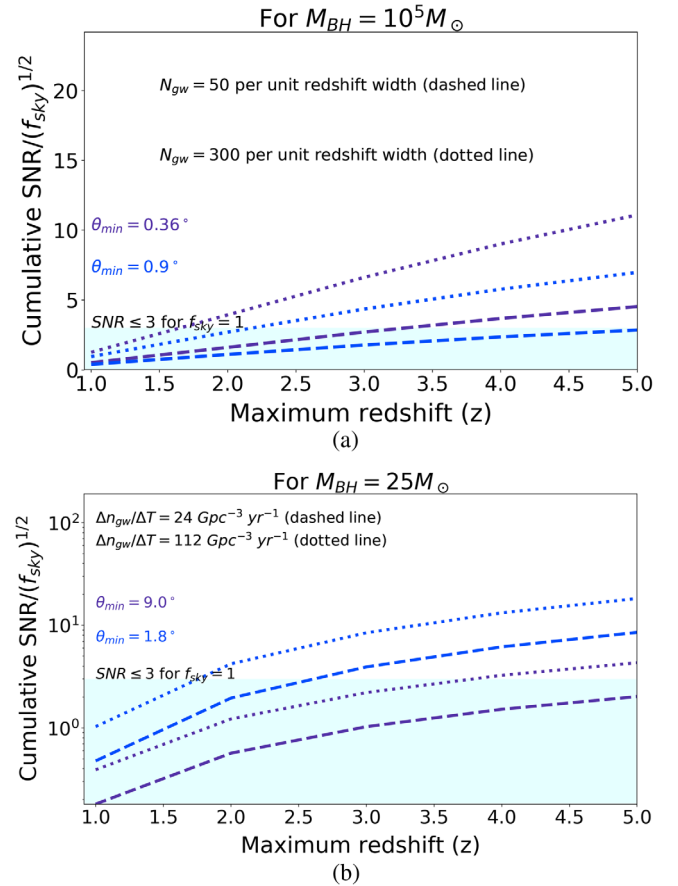


FIG. 3. We show the cumulative SNR as a function of the maximum cosmological redshift ( $z$ ). We have considered only the inspiral phase of the unit mass-ratio BBH with total masses (a)  $2 \times 10^5 M_\odot$  and (b)  $50 M_\odot$ . The area within the shaded region has SNR less than three for  $f_{\text{sky}} = 1$ .

expect that the signal will be more easily accessible from *LISA* [56] and Cosmic Explorer [57] than from Laser Interferometer Gravitational Wave Observatory (*LIGO*) [58] (for the currently predicted merger rates [59,60]). However, if the number of GW sources are more by about an order of magnitude than the current estimate of the event rate [59] then this signal is also accessible from Advanced *LIGO*. In this analysis, we treat the number of GW sources  $N_{\text{gw}}$  and the smallest angular scale  $\theta_{\text{min}}$  as free parameters. The numbers of GW sources per unit redshift for four years of *LISA* operational time are motivated from a few recent theoretical studies [61,62]. For the Cosmic Explorer-like survey, we have taken GW detection rate  $\Delta n_{\text{gw}}/\Delta T = 24\text{--}112 \text{ Gpc}^{-3} \text{ yr}^{-1}$  (comoving volume) [60]. For *LISA* and Cosmic Explorer we plot the cumulative  $\text{SNR}/\sqrt{f_{\text{sky}}}$  in Fig. 3 for a total mass of  $2 \times 10^5 M_{\odot}$  and  $50 M_{\odot}$  ( $M_{\text{total}} \equiv M_1 + M_2$ ) of the BBHs (with  $M_{\text{ratio}} \equiv M_1/M_2 = 1$ ) as a function of the maximum redshift (considered in the cumulative SNR),  $N_{\text{gw}}$  and  $\theta_{\text{min}}$ . The region in cyan indicate  $\text{SNR} \leq 3$  for  $f_{\text{sky}} = 1$ . This plot indicates that there is a large measurable window for different possibilities of  $N_{\text{gw}}$ ,  $M_{\text{BH}}$ , and  $\theta_{\text{min}}$  which can probe the CMB lensing-GW correlation signal with high statistical significance. The detection of the GW sources with electromagnetic counterparts is going to improve the sky localization of the source [63]. This will result in further improvement of the lensing signal from gravitational waves.

## VI. CONCLUSION

The cross-correlation of CMB photons and GW signal can be a path-breaking probe of fundamental physics. First, to observe the expected correlation proves that the GWs and EWs propagate on identical space-time geodesics. The existence of a nonzero correlation between these two signals will manifestly verify a fundamental prediction

of general relativity. Second, the strength of the correlated signal will also probe alternative theories to general relativity. Under the framework of general relativity and the  $\Lambda$ CDM model of cosmology, the predicted signal as a function of GW source redshift is shown in Fig. 1. Any variation from this predicted signal for the known BBHs distributions will be a signature of alternative theories of general relativity [2–4]. Third, this probes the gravitational influence of matter on GWs or graviton-graviton interactions in the perturbation regime [64]. Our approach also makes it possible to measure any deviation in the scalar potentials  $\Phi$  and  $\Psi$  from the prediction of general relativity. This method can also be more generally applied to study the cross-correlation of the GW signal from neutron star binaries, black hole-neutron star binaries, and with other probes of cosmic density field such as galaxy surveys [37].

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