

Collective oscillations of Majorana neutrinos in strong magnetic fields and self-induced flavor equilibrium

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We study collective oscillations of Majorana neutrinos in some of the most extreme astrophysical sites such as neutron star merger remnants and magneto-rotational core-collapse supernovae which include dense neutrino media in the presence of strong magnetic fields. We show that neutrinos can reach flavor equilibrium if neutrino transition magnetic moment μ_ν is strong enough, namely when $\mu_\nu/\mu_B \gtrsim 10^{-14}-10^{-15}$ with μ_B being the Bohr magneton. This sort of flavor equilibrium, which is not necessarily flavor equipartition, can occur on (short) scales determined by the strength of the magnetic term in the Hamiltonian. Our findings can have interesting implications for the physics of such violent astrophysical environments.

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I. INTRODUCTION

Neutrinos can play a vital role in the physics of the most violent astrophysical phenomena such as neutron star mergers (NSM) and core-collapse supernovae (CCSNe) [1–4]. Due to their weak interactions, they can act as the major channel of energy transport. Moreover, they can be crucial to heavy elements nucleosynthesis since they can modify the neutron to proton ratio through the weak reactions $\bar{\nu}_e + p \rightleftharpoons n + e^+$ and $\nu_e + n \rightleftharpoons p + e^-$ [5].

Neutrinos can experience flavor conversions which can change their energy spectra. This, in principle, can change their interaction rates and consequently influence their effects on the dynamics and nucleosynthesis in the extreme astrophysical environments. In addition, on the observational side, any flavor conversions can modify the neutrino signal which may be observed from these events on the earth.

Neutrinos can experience collective flavor oscillations in NSM remnants and CCSNe due to their coherent forward scatterings by the high density background neutrino gas. The presence of neutrino-neutrino interaction makes the problem of neutrino evolution in a dense neutrino medium very demanding and remarkably different from the one in vacuum and matter. It, indeed, makes this problem a nonlinear one with strong coupling among different neutrino momenta [6–10].

The first studies on this problem were carried out in maximally symmetric models. For example, to study collective neutrino oscillations in the supernova context, a stationary spherically symmetric SN model, i.e., the so-called neutrino bulb model was used [7]. The most important feature of the results obtained in the bulb model

is the presence of the spectral swapping phenomenon in which ν_e ($\bar{\nu}_e$) exchanges its spectra with ν_x ($\bar{\nu}_x$) for a certain range of neutrino energies [8,9,11–15]. This phenomenon is a direct consequence of collective neutrino oscillations.

However, regarding the evolution of neutrinos in NSM remnant accretion disks, the geometry is much more complicated and a self-consistent one-dimensional model is unavailable. The first studies were done in the so-called single-angle approximation scenario [16] in which it is assumed that all neutrinos emitted from the neutrino emitting accretion disk experience similar flavor evolution. The salient characteristic of the results obtained in these calculations is the occurrence of matter-neutrino resonance (MNR) [17–26]. This phenomenon results from the cancellation between the neutrino-neutrino interaction and matter potentials which can happen since in the NSM environment, $\bar{\nu}_e$ can be more abundant than ν_e , i.e., $n_{\bar{\nu}_e}/n_{\nu_e} > 1$. The MNR phenomenon is currently thought to be absent in the supernova environment where normally $n_{\bar{\nu}_e}/n_{\nu_e} < 1$ and as a result, the neutrino and matter potentials have similar signs.¹

Nevertheless, it was then realized that such oversimplified maximally symmetric models are not appropriate to study neutrino flavor evolution in dense neutrino media. On the one hand, the spatial and time symmetries in the neutrino gas can be broken spontaneously in the presence of collective neutrino oscillations [10,30–39]. This can

¹Note that there can exist SN zones inside the protoneutron star for which $n_{\bar{\nu}_e}/n_{\nu_e} > 1$ [27–29]. Despite this, the matter density is much larger than the neutrino number densities at these zones and the cancellation between the neutrino-neutrino interaction and matter potentials seems to be inaccessible.

allow for neutrino flavor conversions at very large matter/neutrino densities. On the other hand, it has been shown that neutrinos can experience the so-called fast flavor conversion modes in dense neutrino media probably provided that ν_e and $\bar{\nu}_e$ angular distributions cross each other [38,40–56]. Such fast conversion modes can occur on scales $\sim G_F^{-1} n_\nu^{-1}$ which can be as short as a few cm's in the aforementioned extreme astrophysical environments. This must be compared with slow modes expected to occur on scales $\sim \mathcal{O}(1)$ km (for a 10 MeV neutrino) determined by the neutrino vacuum frequency $\omega = \Delta m_{\text{atm}}^2/2E$.

In addition, neutrinos are expected to have tiny but nonzero magnetic moments (see, e.g., [57–59] for a review) which can influence their flavor evolution in the presence of magnetic fields. In particular, the presence of ultrastrong magnetic fields ($B \gtrsim 10^{15}$ Gauss [60]) in NSM and magnetorotational CCSNe (in which rapid rotation and large magnetic fields are thought to play an important role) makes them ideal settings for studying the impact of the coupling between neutrinos and magnetic field (photon) on collective neutrino oscillations. While such a coupling leads to *active-sterile* neutrino oscillations in the case of Dirac neutrinos, it results in *neutrino-antineutrino* oscillations for Majorana neutrinos.

In the minimally-extended Standard Model (MESM), the diagonal magnetic moment of Dirac neutrinos can be written as [61]

$$\mu_\nu^{ii,D} = \frac{3eG_F m_\nu}{8\sqrt{2}\pi^2} \simeq 3.2 \times 10^{-19} \left(\frac{m_\nu}{1 \text{ eV}} \right) \mu_B, \quad (1)$$

where m_ν is the neutrino mass and $\mu_B = 5.788 \times 10^{-9} \text{ eV Gauss}^{-1}$ is the Bohr magneton. The transition magnetic moment is smaller than the diagonal one by approximately four orders of magnitude. As for Majorana neutrinos, while the diagonal magnetic moment is dictated to be zero, the transition magnetic moment is similar to the transition magnetic moment of Dirac neutrinos.

Although MESM predicts $\mu_\nu \lesssim 10^{-19} \mu_B$, some of the theories beyond SM predict (or at least can explain) much larger values for μ_ν .² In fact, the current experiments can only provide an upper bound on μ_ν (see, e.g., Refs. [65,66]),

²There can indeed exist some difficulties here [62–64]. In particular, since the neutrino magnetic moment can depend linearly on the neutrino mass, any attempt to increase the neutrino magnetic moment leads to an increase in the neutrino mass as well. Thus, to be consistent with current constraints on the neutrino mass and at the same time having large values for μ_ν , one may need a sort of fine-tuning. Though for Dirac neutrinos this necessary fine-tuning leads to theoretical difficulties to produce $\mu_\nu \gtrsim 10^{-15} \mu_B$, it is almost harmless to the case of Majorana neutrinos since it only becomes problematic when $\mu_\nu \gtrsim 10^{-9} \mu_B$ which is already excluded by experiments.

$$\mu_\nu \lesssim 3 \times 10^{-11} \mu_B, \quad (2)$$

which is many orders of magnitude larger than the value suggested by MESM. This constraint is valid for both Dirac and Majorana neutrinos and also diagonal and transition magnetic moments.

The coupling between neutrinos and magnetic field can provide new channels for changing neutrino lepton number and can possibly lead to new physics, if it is strong enough. A number of papers have studied this phenomenon in astrophysical environment [67–82]. In particular, in Refs. [78,79] the authors reported that collective oscillations of Majorana neutrinos can be nontrivially affected by the magnetic term for level-of-SM or even smaller μ_ν 's.

In this paper, we study collective oscillations of Majorana neutrinos in the presence of strong magnetic fields with $B \gtrsim 10^{15}$ Gauss, thought to be present in NSM remnants and magneto-rotational CCSNe. To achieve this goal, we use a schematic multiangle one-dimensional model for the neutrino gas in the two-flavor (Sec. II A) and three-flavor (Sec. II B) scenarios. We show that if the neutrino magnetic moment is large enough, the neutrino gas can reach a sort of flavor equilibrium (which is not necessarily equipartition) on scales determined by the magnetic term.

II. COLLECTIVE OSCILLATIONS OF MAJORANA NEUTRINOS IN THE PRESENCE OF MAGNETIC FIELDS

To study the evolution of Majorana neutrinos in the presence of strong magnetic fields, we consider a single-energy, multiangle neutrino gas in both two and three-flavor scenarios in which neutrinos are emitted with emission angles in the range $[-\vartheta_{\text{max}}, \vartheta_{\text{max}}]$. This model is similar to the one used in Ref. [49].

At each space-time point (t, \mathbf{r}) , the flavor state of a neutrino traveling in direction ϑ can be specified by its density matrix $\rho_\vartheta(t, \mathbf{r})$. The evolution of $\rho_\vartheta(t, \mathbf{r})$ in the absence of collisions is governed by the Liouville-von Neumann equation of motion [83–87]

$$iD_t \rho_\vartheta = [H_\vartheta, \rho_\vartheta], \quad (3)$$

where $D_t = \partial_t + \mathbf{v} \cdot \nabla$ and $H_\vartheta = H_{\text{vac}} + H_{\text{mat}} + H_{\nu\nu, \vartheta}$ is the total Hamiltonian, with H_{vac} , H_{mat} and $H_{\nu\nu, \vartheta}$ being the contributions from vacuum, matter, and neutrino-neutrino interaction potentials, respectively. Here, the contribution from the coupling between neutrinos and magnetic field is included in the vacuum term.

In our study, the evolution of neutrinos is considered in two models, namely a stationary one-dimensional model and a time-dependent homogenous neutrino gas. In the one-dimensional model $D_t = \cos \vartheta d_t$ while one has $D_t = d_t$ in the time-dependent homogenous gas. As will be seen in what follows, the occurrence and nature of the

equilibrium does not depend on the employed model since the outcome is purely determined by the presence of the strong magnetic coupling term. Nevertheless, the amplitude of the oscillations around the equilibrium can be smaller in the stationary one-dimensional model. We also assume that the physical quantities such as the matter/neutrino densities and magnetic field are constant during the propagation of neutrinos. This is justified by noting that the scales associated with neutrino oscillations in this problem (induced by strong magnetic coupling) are much shorter than the relevant scales of the astrophysical problems of interest.

A. Two-flavor scenario

To demonstrate the idea and to show how the presence of strong coupling between neutrinos and magnetic field can influence their oscillations in a dense neutrino medium, we first start with the case of two-flavor scenario. We follow the formalism developed in Ref. [78,79] and we take ρ to be a 4×4 matrix which includes the flavor content of neutrinos and antineutrinos

$$\rho = \begin{bmatrix} \rho_{\nu_e\nu_e} & \rho_{\nu_e\nu_x} & \rho_{\nu_e\bar{\nu}_e} & \rho_{\nu_e\bar{\nu}_x} \\ \rho_{\nu_x\nu_e} & \rho_{\nu_x\nu_x} & \rho_{\nu_x\bar{\nu}_e} & \rho_{\nu_x\bar{\nu}_x} \\ \rho_{\bar{\nu}_e\nu_e} & \rho_{\bar{\nu}_e\nu_x} & \rho_{\bar{\nu}_e\bar{\nu}_e} & \rho_{\bar{\nu}_e\bar{\nu}_x} \\ \rho_{\bar{\nu}_x\nu_e} & \rho_{\bar{\nu}_x\nu_x} & \rho_{\bar{\nu}_x\bar{\nu}_e} & \rho_{\bar{\nu}_x\bar{\nu}_x} \end{bmatrix}, \quad (4)$$

where the diagonal terms are basically the occupation numbers of the corresponding neutrino flavors and off-diagonal terms carry information on neutrino flavor mixing. This matrix has clearly the form

$$\rho = \begin{bmatrix} \rho_\nu & X \\ X^\dagger & \rho_{\bar{\nu}} \end{bmatrix}, \quad (5)$$

with

$$X = \begin{bmatrix} \rho_{\nu_e\bar{\nu}_e} & \rho_{\nu_e\bar{\nu}_x} \\ \rho_{\nu_x\bar{\nu}_e} & \rho_{\nu_x\bar{\nu}_x} \end{bmatrix}, \quad (6)$$

and ρ_ν and $\rho_{\bar{\nu}}$ being the usual 2×2 flavor matrices having information on the flavor content of neutrinos and antineutrinos, respectively. It is very convenient to follow this formalism here since for nonzero Majorana neutrino magnetic moment, neutrinos and antineutrinos are coupled in the presence of magnetic field and there is a nonzero $\nu - \bar{\nu}$ transition amplitude.

Within this formalism, the vacuum and matter potentials can be written as

$$H_{\text{vac}} = \begin{bmatrix} -\omega \cos 2\theta_\nu & \omega \sin 2\theta_\nu & 0 & \Omega \\ \omega \sin 2\theta_\nu & \omega \cos 2\theta_\nu & -\Omega & 0 \\ 0 & -\Omega & -\omega \cos 2\theta_\nu & \omega \sin 2\theta_\nu \\ \Omega & 0 & \omega \sin 2\theta_\nu & \omega \cos 2\theta_\nu \end{bmatrix}, \quad (7)$$

$$H_{\text{mat}} = \begin{bmatrix} (\lambda_e - \lambda_n/2) & 0 & 0 & 0 \\ 0 & -\lambda_n/2 & 0 & 0 \\ 0 & 0 & -(\lambda_e - \lambda_n/2) & 0 \\ 0 & 0 & 0 & \lambda_n/2 \end{bmatrix}, \quad (8)$$

where $\lambda_{e(n)} = \sqrt{2}G_F n_{e(n)}$, with n_e (n_n) being the electron (neutron) number density and θ_ν and $\omega = \Delta m_{\text{atm}}^2/2E$ are the neutrino vacuum mixing angle and the vacuum frequency ($\Delta m_{\text{atm}}^2 > 0$ (< 0) for the normal (inverted) mass hierarchy) for a neutrino with energy E . In our calculations, we set $\theta_\nu = 0.1$ and $\omega = 1$ though the results do not qualitatively depend on the choice of these parameters. Note that the vacuum term has the new contribution $\Omega = \mu_\nu B_T$ from the coupling of Majorana neutrino with the component of magnetic field transverse to the neutrino momentum, B_T . Furthermore, unlike the case of collective neutrino oscillations in the absence of magnetic field, the neutral current contribution from neutrons to the matter potential can not be ignored since it has different signs for neutrinos and antineutrinos and can not be removed as a common phase when these two are coupled.

In addition, the neutrino-neutrino interaction potential, $H_{\nu\nu,\vartheta}$, is³

$$H_{\nu\nu,\vartheta} = \sqrt{2}G_F \int_{-\vartheta_{\text{max}}}^{\vartheta_{\text{max}}} d\vartheta' (1 - \cos(\vartheta - \vartheta')) \times \left[G^\dagger (\rho_{\vartheta'} - \rho_{\vartheta'}^{c*}) G + \frac{1}{2} G^\dagger \text{tr}((\rho_{\vartheta'} - \rho_{\vartheta'}^{c*}) G) \right], \quad (9)$$

where

$$G = \begin{bmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad (10)$$

and ρ^c is defined as

³Here and in Eq. (8), we assume having a large μ_ν does not modify neutrino weak interactions (similar to, e.g., Refs. [71,72]).

$$\rho^c = \begin{bmatrix} \rho_{\bar{\nu}} & X^* \\ X^T & \rho_{\nu} \end{bmatrix}. \quad (11)$$

Note that the definition of ρ^c is somewhat different from the one in Refs. [78,79] so that there is no contribution to $\nu - \bar{\nu}$ transition from the neutrino-neutrino interaction term [80,88] (see also [87,89,90]). The last term in Eq. (9) refers to a phase factor which has different signs for neutrinos and antineutrinos and therefore, cannot be removed here. One can then recover the usual equations of motion of traditional collective oscillations if $B = 0$.

1. Results

In our simulations, we took $\vartheta_{\max} = \pi/3$ and a fixed magnetic field with $B_T = 5 \times 10^{15}$ Gauss. Such strong magnetic fields may not exist on very large scales in the astrophysical problems of interest. However, the scales associated with neutrino oscillations for strong Ω 's are much shorter than other relevant scales in the problem and therefore, we here intend to consider the local effects of large Ω 's rather than the global ones. Thus, the physical quantities are assumed to be constant. Note also that since Hamiltonian is only sensitive to B via $\mu_{\nu} B_T$, for smaller/larger magnetic fields one can just rescale μ_{ν} . We also set $n_{\nu_{\mu}} = n_{\bar{\nu}_{\mu}} = n_{\nu_{\tau}} = n_{\bar{\nu}_{\tau}} = 0.4 n_{\nu_e}$ in our calculations.

The angle-averaged neutrino survival probabilities of neutrinos and antineutrinos are shown in Figs. 1 and 2.

We considered two cases with $n_{\bar{\nu}_e}/n_{\nu_e} = 0.7$ and 2, for a number of Ω 's and two neutrino number densities specified by

$$\zeta = \sqrt{2} G_F n_{\nu_e}. \quad (12)$$

For each panel, the corresponding neutrino magnetic moment is

$$\mu_{\nu} = 6.8 \times 10^{-15} \mu_B \left(\frac{\Omega}{\zeta} \right) \left(\frac{\zeta}{10^{-2} \text{ cm}^{-1}} \right). \quad (13)$$

We have confirmed that our results do not qualitatively depend on the choice of $n_{\bar{\nu}_e}/n_{\nu_e}$ and n_{ν_x}/n_{ν_e} , as well as electron and neutron densities (as long as Ω is the dominant term) and the mass term in the Hamiltonian.

As can be clearly seen in Figs. 1 and 2, the magnetic term does not noticeably modify neutrino flavor evolution for small Ω 's ($\Omega \lesssim 0.5\zeta$). But if Ω term is comparable to the rest of the Hamiltonian (here dominated by $H_{\nu\nu}$), the neutrino gas experiences an interesting sort of flavor equilibrium in which ν_e ($\bar{\nu}_e$) reaches an approximate equalization with $\bar{\nu}_x$ (ν_x) so that

$$\begin{aligned} \rho_{\nu_e \nu_e} &\simeq \rho_{\bar{\nu}_x \bar{\nu}_x} \simeq \frac{n_{\nu_e} + n_{\bar{\nu}_x}}{2} \\ \rho_{\bar{\nu}_e \bar{\nu}_e} &\simeq \rho_{\nu_x \nu_x} \simeq \frac{n_{\bar{\nu}_e} + n_{\nu_x}}{2} \end{aligned} \quad (14)$$

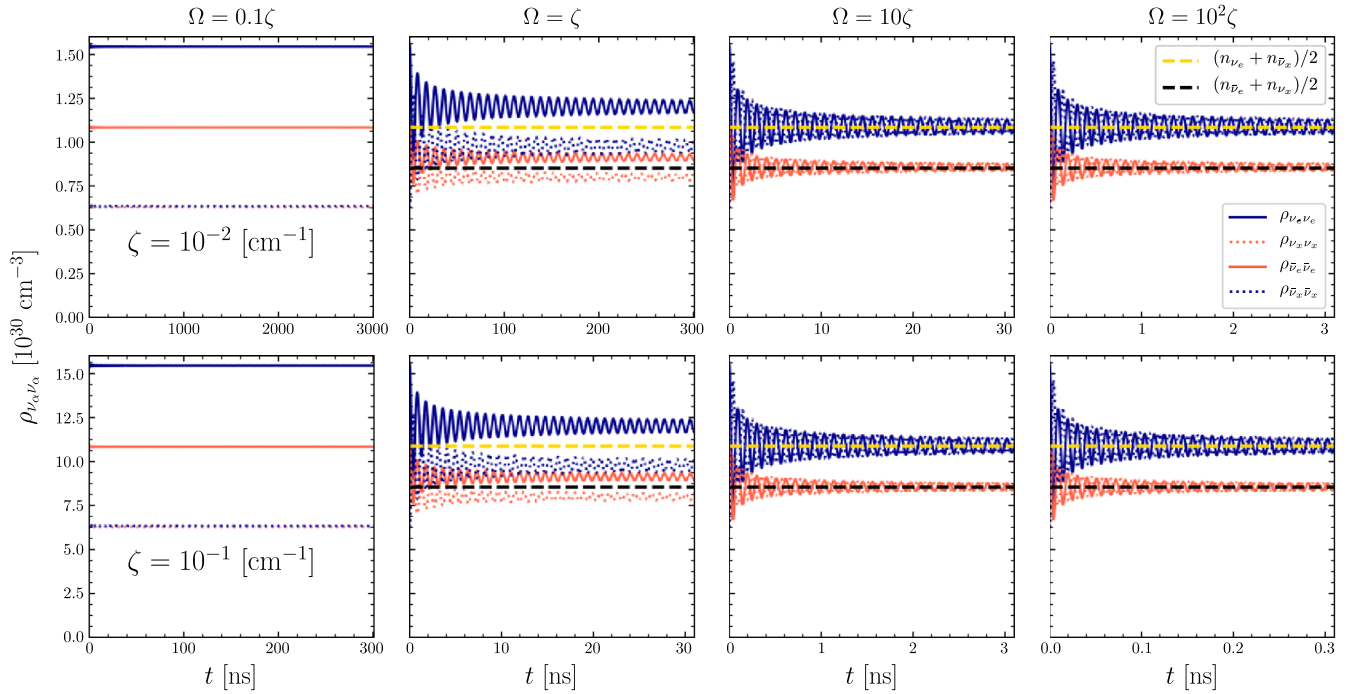
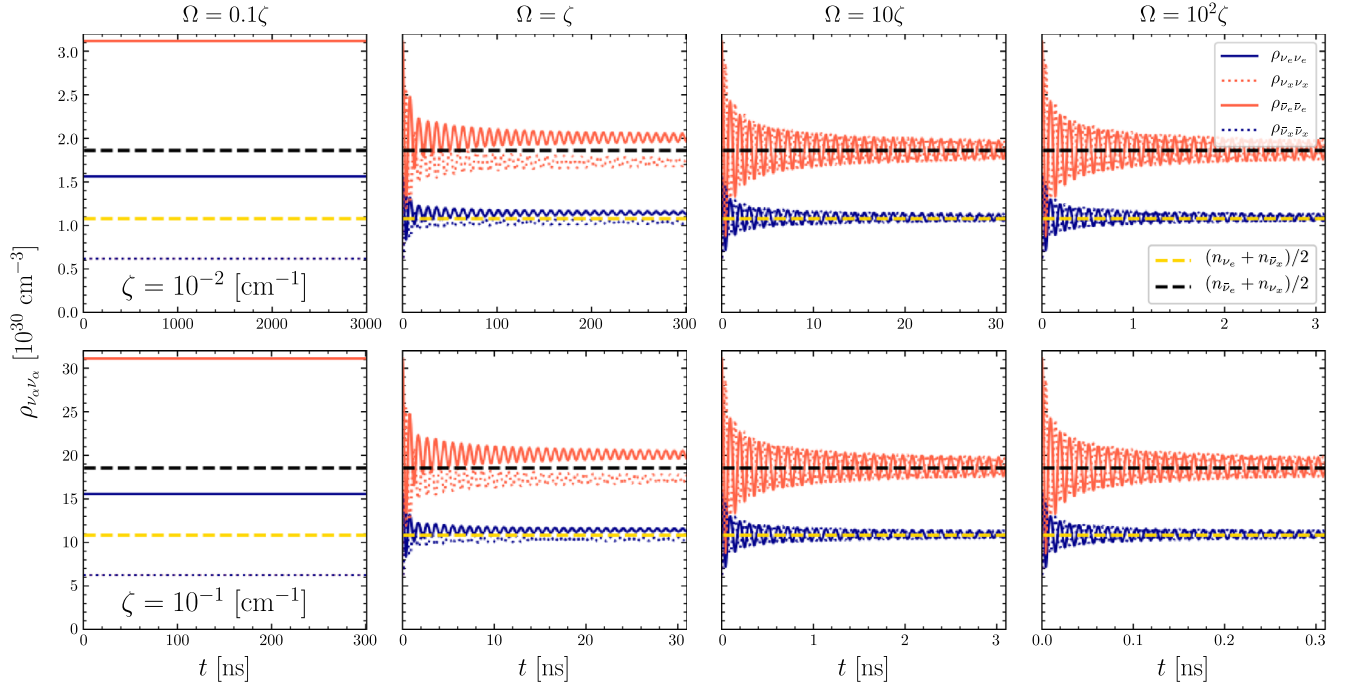


FIG. 1. Angle-averaged survival probabilities of neutrinos and antineutrinos for $n_{\bar{\nu}_e}/n_{\nu_e} = 0.7$. The evolution of neutrinos is studied in the homogenous time-dependent model and as mentioned in the text, μ_{ν} can be found from $\mu_{\nu} = 6.8 \times 10^{-15} \mu_B (\Omega/\zeta) (\zeta/10^{-2} \text{ cm}^{-1})$ in each panel. Note that the neutrino oscillations scale is $\sim 1/\Omega$ for strong Ω 's.

FIG. 2. The same information as in Fig. 1 for $n_{\bar{\nu}_e}/n_{\nu_e} = 2$.

with some small amplitude oscillations around the equilibrium value which can become smaller for $\Omega > \sqrt{2}G_{\text{F}}n_{\nu_e}$. The form of flavor equilibrium is indeed independent of the ratio $n_{\bar{\nu}_e}/n_{\nu_e}$.

The special form of the equilibrium arises from the specific structure of the vacuum Hamiltonian which couples $\nu_e \leftrightarrow \bar{\nu}_x$ and $\bar{\nu}_e \leftrightarrow \nu_x$. For strong Ω 's, the vacuum term dominates the evolution of neutrinos meaning that the neutrino oscillation is caused and triggered by the magnetic term which implies that the neutrino oscillation frequency is here determined by Ω . This, combined with the decoherence induced by the neutrino-neutrino interaction term can then lead to the flavor equilibrium. Note that here the flavor conversion does not arise from cancellation between the diagonal terms in the Hamiltonian as in Refs. [69,72], where resonant conversion is responsible for neutrino flavor oscillations.

B. Three-flavor scenario

Three-flavor oscillations of a dense neutrino gas in the presence of strong coupling between neutrinos and magnetic field can be studied as a straightforward generalization of the two-flavor case.

The 6×6 neutrino density matrix

$$\rho = \begin{bmatrix} \rho_{\nu} & X \\ X^{\dagger} & \rho_{\bar{\nu}} \end{bmatrix}, \quad (15)$$

includes the flavor contents of both neutrinos and antineutrinos of all three flavors with

$$X = \begin{bmatrix} \rho_{\nu_e \bar{\nu}_e} & \rho_{\nu_e \bar{\nu}_\mu} & \rho_{\nu_e \bar{\nu}_\tau} \\ \rho_{\nu_\mu \bar{\nu}_e} & \rho_{\nu_\mu \bar{\nu}_\mu} & \rho_{\nu_\mu \bar{\nu}_\tau} \\ \rho_{\nu_\tau \bar{\nu}_e} & \rho_{\nu_\tau \bar{\nu}_\mu} & \rho_{\nu_\tau \bar{\nu}_\tau} \end{bmatrix}, \quad (16)$$

and ρ_{ν} and $\rho_{\bar{\nu}}$ are the usual 3×3 flavor matrices of neutrinos and antineutrinos, respectively, defined as

$$\rho = \begin{bmatrix} \rho_{\nu_e \nu_e} & \rho_{\nu_e \nu_\mu} & \rho_{\nu_e \nu_\tau} \\ \rho_{\nu_\mu \nu_e} & \rho_{\nu_\mu \nu_\mu} & \rho_{\nu_\mu \nu_\tau} \\ \rho_{\nu_\tau \nu_e} & \rho_{\nu_\tau \nu_\mu} & \rho_{\nu_\tau \nu_\tau} \end{bmatrix}, \quad (17)$$

and similarly for antineutrinos.

In addition, the vacuum Hamiltonian can be written as

$$H_{\text{vac}} = \begin{bmatrix} \tilde{H}_{\text{vac}} & \tilde{H}_{\text{B}} \\ -\tilde{H}_{\text{B}} & \tilde{H}_{\text{vac}}^* \end{bmatrix}, \quad (18)$$

where \tilde{H}_{vac} is the usual 3×3 three-flavor vacuum Hamiltonian described by two mass-squared differences Δm_{12}^2 and Δm_{13}^2 , three mixing angles θ_{12} , θ_{13} and θ_{23} , and one CP -violating phase δ ,⁴ for which the values were taken from Particle Data Group [91]. Also,

⁴Although we set $\delta = 0$ in our calculations, we have confirmed that the results do not qualitatively depend on the choice of δ .

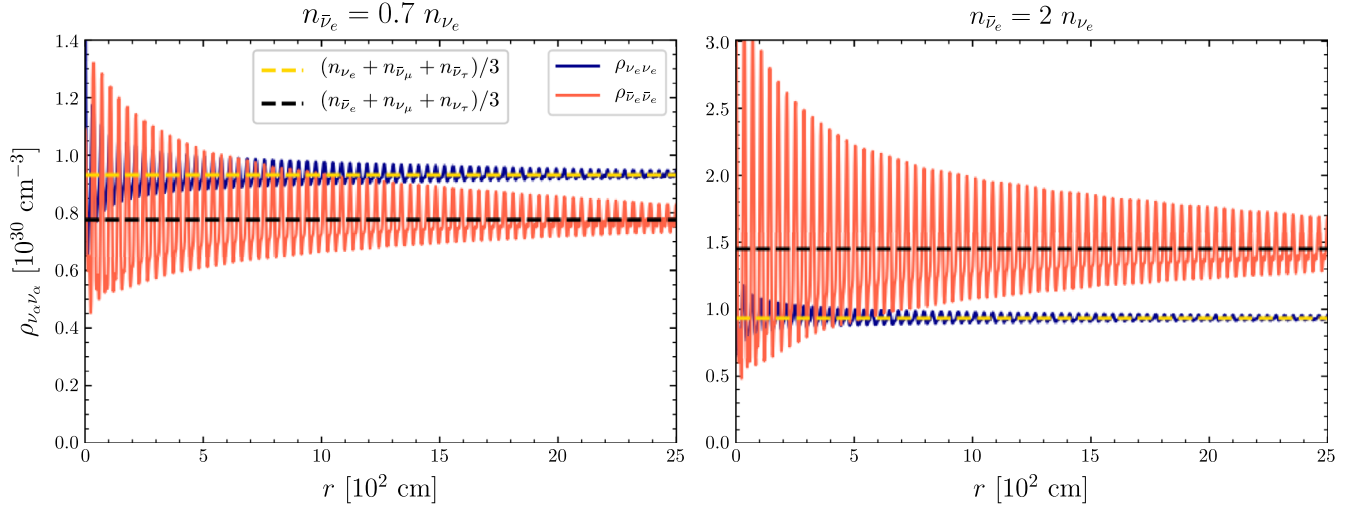


FIG. 3. Angle-averaged survival probabilities of neutrinos and antineutrinos in the three-flavor calculation in the stationary one-dimensional model with $\zeta = 10^3$, $\Omega = 10\zeta$ and two different values of $n_{\bar{\nu}_e}/n_{\nu_e}$. A similar flavor equilibrium can occur among the coupled neutrino/antineutrino species. $\Omega_{\alpha\beta}$'s are assumed to be equal here.

$$\tilde{H}_B = \begin{bmatrix} 0 & \Omega_{e\mu} & \Omega_{e\tau} \\ -\Omega_{e\mu} & 0 & \Omega_{\mu\tau} \\ -\Omega_{e\tau} & -\Omega_{\mu\tau} & 0 \end{bmatrix}, \quad (19)$$

describes the contribution from the magnetic term where $\Omega_{\alpha\beta} = \mu_{\alpha\beta} B_T$ are assumed to be real quantities. Moreover, in the neutrino-neutrino interaction term, Eq. (9), G and ρ^c are straightforward 6×6 generalizations of the corresponding 4×4 ones,

$$G = \begin{bmatrix} +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \quad (20)$$

and

$$\rho^c = \begin{bmatrix} \rho_{\bar{\nu}} & X^* \\ X^T & \rho_{\nu} \end{bmatrix}. \quad (21)$$

1. Results

Collective neutrino oscillations in strong magnetic field in the three-flavor scenario is very similar to the one in the two-flavor scenario. In particular, the angle-averaged survival probabilities can reach some sort of flavor equilibrium, as indicated in Fig. 3. However, the magnetic term is more complicated in the three-flavor scenario. Thus, different flavors can, in general, reach different equilibrium values. For example, in our calculations with

$n_{\nu_\mu} = n_{\bar{\nu}_\mu} = n_{\nu_\tau} = n_{\bar{\nu}_\tau}$, we observed that ν_e and $\bar{\nu}_e$ reach a flavor equilibrium in which

$$\begin{aligned} \rho_{\nu_e \nu_e} &\simeq \frac{n_{\nu_e} + n_{\bar{\nu}_\mu} + n_{\bar{\nu}_\tau}}{3} \\ \rho_{\bar{\nu}_e \bar{\nu}_e} &\simeq \frac{n_{\bar{\nu}_e} + n_{\nu_\mu} + n_{\nu_\tau}}{3}. \end{aligned} \quad (22)$$

This can be explained by noting that the magnetic term couples ν_e to $\bar{\nu}_\mu$, $\bar{\nu}_\tau$ and $\bar{\nu}_e$ to ν_μ , ν_τ . In general, the equilibrium values of different neutrino species are only functions of the neutrino number densities but independent of other quantities such as the mass term in the Hamiltonian, ϑ_{\max} , the matter density and so on.

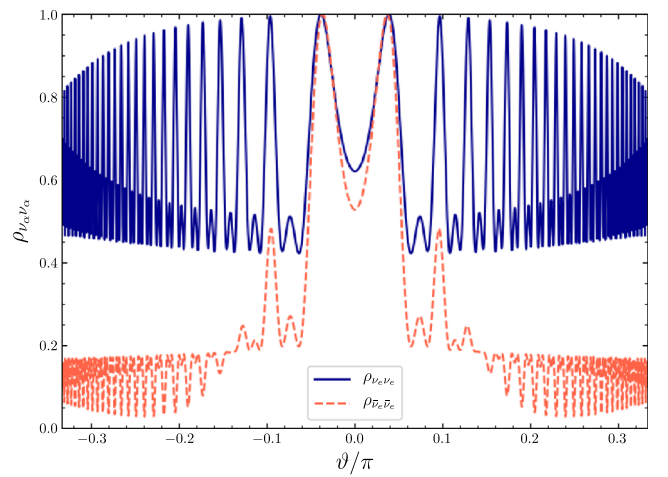


FIG. 4. Angular distributions of the survival probabilities of neutrinos and antineutrinos corresponding to the calculation in Fig. 3 with $n_{\bar{\nu}_e}/n_{\nu_e} = 0.7$, and at $r = 1000$ cm.

Although individual neutrino (angle) beams can experience large amplitude flavor oscillations, the rapid variations of the angular distributions of neutrino survival probabilities, as shown in Fig. 4, allow neutrinos to reach a flavor equilibrium with relatively small amplitude oscillations around the equilibrium value.

III. CONCLUSION

We have studied collective oscillations of Majorana neutrinos in a dense neutrino gas in the presence of strong magnetic fields. Such physical environment is thought to exist in NSM remnants and magnetorotational CCSNe.

Collective oscillations of Majorana neutrinos can lead to a sort of approximate flavor equilibrium in the presence of strong magnetic field provided that the neutrino transition magnetic moment is strong enough, i.e., when $\Omega = \mu_\nu B_T$ is comparable to the other terms in the Hamiltonian. The equilibrium state is determined by the number densities of the coupled (through the magnetic term) neutrino/antineutrino species.

In the presence of nonzero Majorana neutrino magnetic moment, although the total number of neutrinos plus antineutrinos is conserved, the number of neutrinos and antineutrinos are not individually conserved. This is different from the case of usual collective neutrino oscillations in the absence of magnetic fields (and other beyond SM terms). We do not consider the case of Dirac neutrinos in this study where the total number of active neutrinos can also be changed due to the possibility of transition between active and sterile neutrinos.

In our calculations, the magnetic coupling term can play a noticeable role only if Ω is not much smaller than the other terms in the Hamiltonian. This is different from the observation of Refs. [78,79] where the presence of the magnetic term can significantly modify collective neutrino

oscillations even if it is many orders of magnitude smaller than the other terms in the Hamiltonian. This means that one should observe a remarkable impact from the magnetic term even if the length scale associated with Ω is orders of magnitude larger than the size of the SN.⁵ Note, however, that any comparison between the results presented here and the ones in Refs. [78,79] must be made with great caution since the employed models are different in several respects.

Apart from providing an example of a physical situation in which collective neutrino oscillations can lead to generic flavor equilibrium, our findings provide useful insight on how the presence of strong lepton number violating channels can impact collective neutrino oscillations. Our results can have important implications for the physics of the most extreme astrophysical environments such as NSM remnants and magneto-rotational CCSNe.

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⁵In the case of usual collective neutrino oscillations (in the absence of magnetic fields), the presence of the off-diagonal term can have a switch-on effect on flavor conversion. Thus, even an infinitesimal nonzero off-diagonal term ($\theta_\nu \ll 1$) can significantly affect flavor evolution of neutrinos. This arises from the flavor instabilities induced by neutrino-neutrino interactions. However, one should not generally expect a similar effect from small $\nu - \bar{\nu}$ transition amplitude due to the presence of Ω unless neutrino-neutrino interaction has some contribution to $\nu - \bar{\nu}$ transition, as in Refs. [78,79].

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