What can we learn from the global spin alignment of ϕ mesons
in heavy-ion collisions? in heavy-ion collisions?

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We propose that a significant positive deviation from $1/3$ for the spin density matrix element ρ_{00} of the ϕ meson may indicate the existence of a mean field of the ϕ meson generated in heavy-ion collisions. This explains why STAR preliminary data for the ϕ meson's ρ_{00} are much larger than 1/3, while the data of Λ and Λ polarization seem not to allow such a significant and positive deviation. The contribution is from the polarization of the strange quark and antiquark through the spin-orbit interaction in the ϕ field, a similar interaction that is responsible for nuclear shell structure at the nucleon level. We show that ρ_{00} for the ϕ meson is a good analyzer for fields even if they may strongly fluctuate in space-time.

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The rotation and spin polarization are inherently correlated and can be converted from one to another in materials as manifested in the Barnett effect [\[1\]](#page-4-0) and the Einsteinde Haas effect [\[2\]](#page-4-1). One of the most recent examples is that an electric voltage from the spin current is observed to be generated from the vortical motion in a liquid metal [\[3\].](#page-4-2) In ultrarelativistic heavy-ion collisions (HIC), a huge orbital angular momentum (OAM) can also be generated mainly along the direction perpendicular to the reaction plane $[4–9]$ $[4–9]$ (see, e.g., $[10]$ for a recent review). Such a huge OAM is distributed into the hot and dense quark matter and converted to global polarization of hadrons through the spin-orbit coupling [\[4,9,11\]](#page-4-3) in a microscopic approach or spin-vorticity coupling in a macroscopic approach [\[12](#page-4-5)–17]. The STAR Collaboration has recently measured a nonvanishing global polarization of Λ hyperons in Au + Au collisions at $\sqrt{s_{NN}} = 7.7{\text -}200 \text{ GeV}$ [\[18,19\].](#page-4-6)

Accompanying a huge OAM in HIC, a strong magnetic field is also formed, pointing to the same direction [\[20](#page-4-7)–27]. The OAM and magnetic field lead to chiral effects of massless fermions: the chiral magnetic effect (CME) which probes the topological fluctuation of quantum chromodynamics vacuum [\[28](#page-4-8)–30] (see, e.g., [\[31\]](#page-5-0) for a recent review) and the chiral vortical effect [\[32](#page-5-1)–39] which probes the vorticity field of the fluid. One of the most active researches in HIC experiments is to search for the CME [\[40](#page-5-2)–48]. However, the CME has not been observed due to dominant backgrounds. Furthermore, no direct and definite effects from electric and magnetic fields have been found so far. The challenge comes from the fact that the lifetime of the electric and magnetic field is so short (≤ 1 fm/c) that they can be regarded as a pulse.

While the polarization of Λ can be measured by its weak decay, the polarization of vector mesons cannot be measured since they mainly decay through strong interaction. However, the spin alignment of a vector meson can only be measured through ρ_{00} , the 00 element of its spin density matrix, encoded in the angular distribution of its decay daughters [\[5,49\]](#page-4-9). If $\rho_{00} \neq 1/3$, the distribution is anisotropic and the spin of the vector meson is aligned to the spin quantization direction. In 2008, the STAR Collaboration measured ρ_{00} for the vector meson $\phi(1020)$ in Au + Au collisions at 200 GeV, which is consistent to $1/3$ indicating no spin alignment within errors [\[50\]](#page-5-3). Recent STAR's preliminary data for the ϕ meson's ρ_{00} or ρ_{00}^{ϕ} at lower energies show a significant POSITIVE deviation from $1/3$, which is far beyond our current understanding of the polarization [\[51\].](#page-5-4) In this note, we will show that such a large POSITIVE deviation of ρ_{00}^{ϕ} from 1/3 may imply the existence of a mean field for the ϕ meson in heavy-ion collisions.

SPIN ALIGNMENT OF ϕ MESON

The 00 element of the spin density matrix ρ_{00} for the vector meson enters the angular distribution of its decay daughter as

$$
\frac{dN}{d\cos\theta} = \frac{3}{4}[(1-\rho_{00}) + (3\rho_{00} - 1)\cos^2\theta],\qquad(1)
$$

where θ is the angle between the daughter's momentum and the spin quantization direction [\[5,49\].](#page-4-9) The STAR preliminary data imply that $\rho_{00}^{\phi} > 1/3$ and significantly deviate from $1/3$. In the coalescence or combination model, the s and \bar{s} quark form a ϕ meson, and ρ_{00}^{ϕ} is related to the polarization P_s and $P_{\bar{s}}$ for s and \bar{s} , respectively,

$$
\rho_{00}^{\phi} \approx \frac{1}{3} - \frac{4}{9} P_s P_{\bar{s}}, \qquad (2)
$$

if P_s and $P_{\bar{s}}$ are both small. In a simple model, the spin polarization of Λ and Λ is carried by s and \bar{s} , respectively, so we have $P_{\Lambda} = P_s$ and $P_{\bar{\Lambda}} = P_{\bar{s}}$. Therefore, ρ_{00}^{ϕ} in [\(2\)](#page-1-0) is approximately

$$
\rho_{00}^{\phi} \approx \frac{1}{3} - \frac{4}{9} P_{\Lambda} P_{\bar{\Lambda}} \lesssim \frac{1}{3},\tag{3}
$$

where P_{Λ} and $P_{\overline{\Lambda}}$ can be estimated by using the STAR data $P_\Lambda \approx (1.08 \pm 0.15 \pm 0.11)\%$ and $P_{\bar{\Lambda}} \approx (1.38 \pm 0.30 \pm 0.30)\$ 0.13)% [\[18,19\]:](#page-4-6) $(4/9)P_{\Lambda}P_{\bar{\Lambda}} \approx 6.6 \times 10^{-5}$. So, the STAR data for P_{Λ} and $P_{\bar{\Lambda}}$ seem to imply that ρ_{00}^{ϕ} cannot be significantly larger than $1/3$, which contradicts the STAR preliminary data on ρ_{00}^{ϕ} . We will show that the key to reconcile such a conflict is that P_s and $P_{\bar{s}}$ will have additional contributions which have never been considered before.

III. SPIN POLARIZATION IN VORTICITY AND ELECTROMAGNETIC FIELD

We take xz plane as the reaction plane with one nucleus moving along $+z$ direction at $x = -b/2$ while the other nucleus moving along $-z$ direction at $x = b/2$. The OAM is along $+y$ direction.

From Eq. (64) in Ref. [\[49\]](#page-5-5), the spin polarization vector (normalized to 1) for massive fermions (upper sign) and antifermions (lower sign) in the vorticity and electromagnetic field is

$$
P_{\pm}^{\mu}(x, p) = \frac{1}{2m} \left(\tilde{\omega}_{\text{th}}^{\mu\nu} \pm \frac{1}{E_p T} Q \tilde{F}^{\mu\nu} \right) p_{\nu} [1 - f_{FD}(E_p \mp \mu)], \tag{4}
$$

where Q is the electric charge of the fermion, $p^{\mu} =$ $(E_p, \pm \mathbf{p})$ for fermion/antifermion with $E_p \equiv \sqrt{\mathbf{p}^2 + m^2}$ being the energy of the fermion or antifermion, $\tilde{\omega}_{\text{th}}^{\mu\nu}$ = being the energy of the fermion or antifermion, $\omega_{\text{th}}^2 = \frac{1}{2} \epsilon^{\mu\nu\sigma\rho} \omega_{\sigma\rho}^{\text{th}}$ is the dual thermal vorticity tensor with the thermal vorticity tensor given by $\omega_{\sigma\rho}^{\text{th}} = \frac{1}{2} [\partial_{\sigma} (\beta u_{\rho}) \partial_{\rho}(\beta u_{\sigma})]$ with $\beta \equiv 1/T$, $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\sigma\rho} F_{\sigma\rho}$ is the dual electromagnetic field strength tensor, and f_{FD} is the Fermi-Dirac distribution. The electric and magnetic field as three-vectors are defined as $E^i = E_i = F^{i0}$ and $B^i = B_i = -\frac{1}{2} \epsilon_{ijk} F^{jk}$

with $i, j, k = x, y, z$. In a similar way, one can define the thermal vorticity three-vector $\omega^i = \omega_i = \tilde{\omega}_{\text{th}}^{i0}$, the "magnetic" part of the thermal vorticity tensor, and the "electric" part of the thermal vorticity tensor $\varepsilon^i = \varepsilon_i = \omega_{\text{th}}^{i0}$, which are $\boldsymbol{\omega} = \frac{1}{2} \nabla \times (\beta \mathbf{u})$ and $\boldsymbol{\varepsilon} = -(1/2)[\partial_t(\beta \mathbf{u}) + \nabla(\beta u^0)]$ in three-vector forms.

Applying Eq. [\(4\)](#page-1-1) to the strange and antistrange quark s and \bar{s} , we obtain the polarization along the y direction,

$$
P_{s/\bar{s}}^{y}(t, \mathbf{x}, \mathbf{p}_{s/\bar{s}}) = \frac{1}{2}\omega_{y} \pm \frac{1}{2m_{s}}\hat{\mathbf{y}} \cdot (\boldsymbol{\varepsilon} \times \mathbf{p}_{s/\bar{s}})
$$

$$
\pm \frac{Q_{s}}{2m_{s}T}B_{y} + \frac{Q_{s}}{2m_{s}^{2}T}\hat{\mathbf{y}} \cdot (\mathbf{E} \times \mathbf{p}_{s/\bar{s}}), \quad (5)
$$

where $Q_s = -e/3$ is the electric charge of the s quark $(e > 0)$, and we have taken the nonrelativistic limit $E_p \simeq m_s$ and the Boltzmann limit $1 - f_{FD}(E_p \mp \mu) \simeq 1$. The last term of Eq. [\(5\)](#page-1-2) is the spin-orbit term for quarks in electric fields, the similar term is the key to the nuclear shell structure if applying to nucleons in meson fields [\[52,53\].](#page-5-6)

In the coalescence model, the polarization of Λ or $\bar{\Lambda}$ in its rest frame is given by [\[49\]](#page-5-5)

$$
P_{\Lambda/\bar{\Lambda}}^{y}(t, \mathbf{x}) = \frac{1}{3} \int \frac{d^{3} \mathbf{r}}{(2\pi)^{3}} \frac{d^{3} \mathbf{q}}{(2\pi)^{3}} |\psi_{\Lambda/\bar{\Lambda}}(\mathbf{q}, \mathbf{r})|^{2} \times [P_{s/\bar{s}}^{y}(t, \mathbf{x}, \mathbf{p}_{1}) + P_{s/\bar{s}}^{y}(t, \mathbf{x}, \mathbf{p}_{2}) + P_{s/\bar{s}}^{y}(t, \mathbf{x}, \mathbf{p}_{3})] = \frac{1}{2} \omega_{y} \pm \frac{Q_{s}}{2m_{s}T} B_{y},
$$
\n(6)

where $\psi_{\Lambda/\bar{\Lambda}}(\mathbf{q},\mathbf{r})$ are wave functions of $\Lambda/\bar{\Lambda}$ in momentum space with the normalization condition $\int d^3 \mathbf{r} d^3 \mathbf{q} |\psi_{\Lambda/\bar{\Lambda}}(\mathbf{q}, \mathbf{r})|^2 = (2\pi)^6$, and internal momenta of three quarks are denoted as $\mathbf{p}_1 = \mathbf{r}/2 + \mathbf{q}$, $\mathbf{p}_2 = \mathbf{r}/2 - \mathbf{q}$ and $\mathbf{p}_3 = -\mathbf{r}$ which satisfy $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$ in the rest frame of Λ/Λ . In the square bracket of Eq. [\(6\)](#page-1-3), $P_{s/\bar{s}}^{y}(t, \mathbf{x}, \mathbf{p}_1)$ means that \mathbf{p}_1 is the momentum of the s/\bar{s} quark in $\Lambda/\bar{\Lambda}$ (the momenta of two light quarks/antiquarks are then \mathbf{p}_2 and \mathbf{p}_3), and $P_{s/\overline{s}}^y(t, \mathbf{x}, \mathbf{p}_2)$ and $P_{s/\overline{s}}^y(t, \mathbf{x}, \mathbf{p}_3)$ have similar meanings. Comparing Eq. [\(6\)](#page-1-3) with Eq. [\(5\),](#page-1-2) we see that there are no contributions from ϵ and \mathbf{E} in $P_{\Lambda/\bar{\Lambda}}^y$. The reason is that both ϵ and **E** terms in $P_{s/\bar{s}}^{y}$ are linearly proportional to p, so these terms in the square bracket of Eq. [\(6\)](#page-1-3) are vanishing due to $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$ in the rest frame of Λ and $\overline{\Lambda}$.

The 00 element of the spin density matrix for the ϕ meson is calculated by [\[49\]](#page-5-5)

$$
\rho_{00}^{\phi}(t, \mathbf{x}) \approx \frac{1}{3} - \frac{4}{9} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} P_s^y(\mathbf{p}) P_{\bar{s}}^y(-\mathbf{p}) |\psi_{\phi}(\mathbf{p})|^2, \tag{7}
$$

where $\psi_{\phi}(\mathbf{p})$ is the wave function in momentum space for the ϕ meson with the normalization $\int d^3 \mathbf{p} |\psi_{\phi}(\mathbf{p})|^2 = (2\pi)^3$, and we have put $\mathbf{p}_s = \mathbf{p}$ and

 $\mathbf{p}_{\bar{s}} = -\mathbf{p}$ in the center of mass frame of ϕ . Note that it is the correlation between $P_s^y(\mathbf{p})$ and $P_{\bar{s}}^y(-\mathbf{p})$ [\[54\]](#page-5-7) that is essential to resolve the puzzle in ρ_{00}^{ϕ} . Inserting [\(5\)](#page-1-2) into [\(7\)](#page-1-4) and taking an average of $\rho_{00}^{\phi}(t, \mathbf{x})$ over the fireball volume V and the polarization time t with an effective temperature T_{eff} , we obtain

$$
\rho_{00}^{\phi} \approx \frac{1}{3} - \frac{4}{9} \langle P_{\bar{\Lambda}}^{\nu} P_{\Lambda}^{\nu} \rangle - \frac{1}{27 m_s^2} \langle \mathbf{p}^2 \rangle_{\phi} \langle \varepsilon_z^2 + \varepsilon_x^2 \rangle + \frac{e^2}{243 m_s^4 T_{\text{eff}}^2} \langle \mathbf{p}^2 \rangle_{\phi} \langle E_z^2 + E_x^2 \rangle, \tag{8}
$$

where we have used $\langle \mathbf{p} \rangle_{\phi} = 0$, $\langle p_{z,x}^2 \rangle_{\phi} = (1/3) \langle \mathbf{p}^2 \rangle_{\phi}$, $\langle p_z p_x \rangle_{\phi} = 0$, with $\langle a(\mathbf{p}) \rangle_{\phi} \equiv (2\pi)^{-3} \int d^3 \mathbf{p} |\psi_{\phi}(\mathbf{p})|^2 a(\mathbf{p})$ being the mean value of a momentum function $a(\mathbf{p})$ in the ϕ meson wave function in momentum space, and replaced T by the effective temperature T_{eff} of the fireball. From the ϕ meson wave function in the quark potential model [\[55,56\]](#page-5-8), we have $\langle \mathbf{p}^2 \rangle_{\phi} \approx 0.18 \text{ GeV}^2 \approx 9.18 m_{\pi}^2$. Using Eq. [\(6\)](#page-1-3), the second term in the right-hand side of Eq. [\(8\)](#page-2-0) is denoted as $c_{\Lambda} \equiv -(4/9) \langle P_{\bar{\Lambda}}^{\gamma} P_{\Lambda}^{\gamma} \rangle$,

$$
c_{\Lambda} = -\frac{1}{9} \langle \omega_{y}^{2} \rangle + \frac{Q_{s}^{2}}{9m_{s}^{2}T_{\text{eff}}^{2}} \langle B_{y}^{2} \rangle. \tag{9}
$$

We see that the contribution to ρ_{00}^{ϕ} from the vorticity is always negative while that from the magnetic field is always positive. We also see that the magnitudes of $\langle \omega_y^2 \rangle$ and $\langle B_y^2 \rangle$ are constrained by the data of P_Λ and $P_{\bar{\Lambda}}$, but this is not the case for $\langle \varepsilon_z^2 + \varepsilon_x^2 \rangle$ and $\langle E_z^2 + E_x^2 \rangle$ in Eq. [\(8\).](#page-2-0)

We denote the third and fourth terms in the right-hand side of Eq. [\(8\)](#page-2-0) as c_e and c_E , respectively. Note that all these terms are either positive or negative definite, which is a good feature of ρ_{00}^{ϕ} . The c_{Λ} term provides a negative contribution to ρ_{00}^{ϕ} relative to 1/3. This can be estimated by using the STAR data [\[18,19\]](#page-4-6) as is done after Eq. [\(3\)](#page-1-5): $c_A \sim -6.6 \times 10^{-5}$, which is very small compared to 1/3. This also means that the contribution from the vorticity is larger than that from the magnetic field. The c_{ε} term also provides a negative contribution to ρ_{00}^{ϕ} but not constrained by the data of Λ polarization. This term comes from the fluid vorticity and can be estimated by the hydrodynamic simulation. We use CLVisc [\[57,58\],](#page-5-9) a $(3 + 1)D$ viscous hydrodynamic model, to calculate $\langle \varepsilon_z^2 + \varepsilon_x^2 \rangle$ at the freezeout. The numerical results show $\langle \varepsilon_z^2 + \varepsilon_x^2 \rangle \sim 10^{-4}$. Using the constituent quark mass for m_s of about 450 MeV, c_e is even more suppressed. The c_E term is from the electric field which is also absent in the Λ polarization [\(6\)](#page-1-3) and therefore not constrained by the data of Λ polarization. The peak value for $eE \equiv e \sqrt{\langle E_z^2 + E_x^2 \rangle}$ is about m_π^2 according to the simulation based on the Parton-Hadron-String Dynamics (PHSD) transport model [\[59\]](#page-5-10) which includes a dynamical generation of retarded electromagnetic fields [\[21,22\]](#page-4-10), where we set $m_s \approx 450$ MeV and $T_{\text{eff}} \approx 100-300$ MeV for Au $+$ Au collisions in the collision energy range 20– 200 GeV. Then we obtain $c_E \sim 10^{-5}$, which cannot give a large deviation of ρ_{00}^{ϕ} from 1/3.

MESON FIELD OF ϕ

Like the electromagnetic field, a mean field of the ϕ meson, if exists, can also polarize s and \bar{s} and contribute to ρ_{00}^{ϕ} . The role of the mean field of vector mesons in the polarization of the Lambda hyperon was proposed in Ref. [\[60\].](#page-5-11) The electric and magnetic part of the ϕ meson field \mathbf{E}_{ϕ} and \mathbf{B}_{ϕ} can be obtained by the field potential ϕ^{μ} in the same way as for the electromagnetic field $F^{\mu\nu}_{\phi} =$ $\frac{\partial^{\mu} \phi^{\nu}}{\partial \phi^{\mu}}$. This is in analogy with the vector dominance model [\[61,62\].](#page-5-12) Similar to the meson field out of the baryon current in Ref. [\[60\]](#page-5-11), ϕ^{μ} can be approximately proportional to the current density of the strangeness quantum number, $\phi^{\mu} \approx -\left(g_{\phi}/m_{\phi}^{2}\right)J_{s}^{\mu}$, known as the current-field identity [\[63,64\]](#page-5-13) in the vector dominance model [\[61,62\].](#page-5-12) Here m_{ϕ} is the ϕ meson mass, and g_{ϕ} is the coupling constant of the s quark to the ϕ meson in the quark-meson model [\[65,66\]](#page-5-14).

Note that the contribution from s and \bar{s} to J_s^{μ} is negative and positive, respectively. The strangeness current density in the central rapidity region is assumed to be a function of time and space,

$$
J_s^{\mu}(t, \mathbf{x}) = (\rho_s, \mathbf{J}_s) = (\rho_s, j_s^x, j_s^y, j_s^z). \tag{10}
$$

It must satisfy strangeness conservation $\partial_{\mu}J_{s}^{\mu} = 0$ with the condition $\int d^3\mathbf{x}\rho_s(t,\mathbf{x})=0$. The electric and magnetic part of the ϕ field that contributes to the spin alignment along $+y$ direction is given by

$$
\mathbf{E}_{\phi} = \hat{\mathbf{z}} \frac{g_{\phi}}{m_{\phi}^2} \tilde{E}_{\phi}^z + \hat{\mathbf{x}} \frac{g_{\phi}}{m_{\phi}^2} \tilde{E}_{\phi}^x,
$$

$$
\mathbf{B}_{\phi} = \hat{\mathbf{y}} \frac{g_{\phi}}{m_{\phi}^2} \left(\frac{\partial j_s^z}{\partial x} - \frac{\partial j_s^x}{\partial z} \right),
$$
 (11)

where $\tilde{E}^i_{\phi} = \tilde{E}_{\phi,i} \equiv \nabla_i \rho_s + \partial_j j_s^i / \partial t$ with $i = x, y, z$. The z component of J_s in [\(10\)](#page-2-1) is the result of the difference in the parton distribution function for s and \bar{s} in nucleons: $s(x_B) \neq \bar{s}(x_B)$ in different regions of x_B , where x_B is the momentum fraction (Bjorken variable) carried by s and \bar{s} in the proton. Although the uncertainty in extracting $s(x_B)$ and $\bar{s}(x_B)$ in the nucleon sea from experimental data [\[67](#page-5-15)–69] is large, there are strong evidences [\[69,70\]](#page-5-16) for $s(x_B) \neq \bar{s}(x_B)$. Extensive theoretical studies have been done on the asymmetry of $s(x_B)$ and $\bar{s}(x_B)$ in the past 30 years [71–[79\].](#page-5-17) In nucleus-nucleus collisions, this leads to a nonzero strangeness current j_s^z which may depend on

time. We have also generalized this feature by introducing ρ_s , j_s^x , and j_s^y in Eq. [\(10\).](#page-2-1)

Then the contribution from the ϕ meson field can be obtained from Eqs. [\(5\),](#page-1-2) [\(6\)](#page-1-3), [\(8\),](#page-2-0) [\(9\)](#page-2-2) by replacements: $\mathbf{B} \to \mathbf{B}_{\phi}$, $\mathbf{E} \to \mathbf{E}_{\phi}$, and $Q_s = -\frac{1}{3}e \to g_{\phi}$. Now $P_{s/\overline{s}}^{y}$ in Eq. [\(5\)](#page-1-2) has two additional terms: $\pm g_{\phi} B_{\phi}^{y}/(2m_{s}T)^{'}$ and $g_{\phi}\hat{\mathbf{y}} \cdot (\mathbf{E}_{\phi} \times \mathbf{p}_{s/\bar{s}})/(2m_s^2 T)$. Correspondingly, $P_{\Lambda/\bar{\Lambda}}^{y}(t, \mathbf{x})$ in [\(6\)](#page-1-3) has an additional term $\pm g_{\phi} B_{\phi}^{y}/(2m_s T)$, and c_{Λ} in Eq. [\(9\)](#page-2-2) contains an additional term $g_{\phi}^2 \langle B_{\phi,y}^2 \rangle / (9 m_s^2 T_{\text{eff}}^2)$. We see that it is \mathbf{B}_{ϕ} instead of \mathbf{E}_{ϕ} that contributes to $P_{\Lambda/\bar{\Lambda}}^{y}(t, \mathbf{x})$. Equation [\(8\)](#page-2-0) becomes

$$
\rho_{00}^{\phi} \approx \frac{1}{3} + c_{\Lambda} + c_{\varepsilon} + c_{E} + c_{\phi},\tag{12}
$$

where c_{ϕ} is from the electric part of the mean ϕ field,

$$
c_{\phi} \equiv \frac{g_{\phi}^{4}}{27m_{\phi}^{4}m_{\phi}^{4}T_{\rm eff}^{2}} \langle \mathbf{p}^{2} \rangle_{\phi} \langle \tilde{E}_{\phi,z}^{2} + \tilde{E}_{\phi,x}^{2} \rangle. \tag{13}
$$

Note that the average is taken over the space-time volume. In deriving [\(12\),](#page-3-0) we have assumed that there are no correlations among different fields (fluid field, electromagnetic field, ϕ field), e.g., between fluid and electromagnetic field, between **B** and \mathbf{B}_{ϕ} , and between **E** and \mathbf{E}_{ϕ} , etc. We have also assumed that there is no correlation between the electric and magnetic part of the same field. The most important feature is that c_{ϕ} in [\(13\)](#page-3-1) is always positive and is related to \mathbf{E}_{ϕ} which is absent in $P_{\Lambda/\bar{\Lambda}}^{y}(t, \mathbf{x})$. We note that Eq. [\(12\)](#page-3-0) is for ρ_{00}^{ϕ} in the y direction; one can obtain ρ_{00}^{ϕ} in the x or z direction as well. For ρ_{00}^{ϕ} in the x direction, one can just replace ω_y , B_y , and B_{ϕ}^y in c_A by ω_x , B_x , and B_{ϕ}^x , respectively, and replace ε_x , E_x , and E_{ϕ}^x in c_{ε} , c_E , and c_{ϕ} by ε_y , E_y , and E_{ϕ}^y , respectively.

As we have shown in Sec. [III](#page-1-6) that c_A , c_{ε} , and c_E in Eq. (12) are negligibly small compared with $1/3$ for $Au + Au$ collisions in the collision energy range 20–200 GeV. If the data show that ρ_{00}^{ϕ} is larger than 1/3 by at least a few percent, according to our model, the deviation must be solely from c_{ϕ} involving the electric part of the mean ϕ field. A good feature of ρ_{00}^{ϕ} is that each contribution is in square up to a sign, so it is either positive or negative definite. This property does not depend on the procedure of taking an average or on choices of parameters. It exists even for fluctuating fields (the vorticity, electromagnetic, and ϕ field). Therefore, ρ_{00}^{ϕ} is a good analyzer for fields even if they may fluctuate strongly in space-time.

We can estimate in a simple model the dominant contribution to ρ_{00}^{ϕ} from the last term of Eq. [\(12\)](#page-3-0). We choose the effective temperature as $T_{\text{eff}} \propto \tau_0^{-1/3} (dn_{\text{ch}}/d\eta)_{\eta=0}^{1/3}$, where $\tau_0 \sim s_{NN}^{-1/2}$ and $(dn_{\text{ch}}/d\eta)_{\eta=0} \propto -0.4 + 0.39 \ln s_{NN}$ is the pseudorapidity density of charged particles at the central pseudorapidity $\eta = 0$ and the collision energy $s_{NN}^{1/2}$ should take the dimensionless number when expressed in the unit GeV [\[80\].](#page-5-18) We set $T_{\text{eff}} = 300 \text{ MeV}$ at $s_{NN}^{1/2} = 200 \text{ GeV}$ for calibration. In this way, the collision energy behavior of ρ_{00}^{ϕ} is solely from T_{eff} which is a strong assumption in this order of magnitude estimate. As an approximation, we assume that $\partial j_s^{z,x}/\partial t$ do not depend on the collision energy. We set the values of the following parameters: $m_s = 450$ MeV and $C_s^{(y)} = 400,600,1000$ fm⁻⁸ where $C_s^{(y)} \equiv g_{\phi}^4 \langle \tilde{E}_{\phi,z}^2 + \tilde{E}_{\phi,x}^2 \rangle$. Note that the value of g_{ϕ} can be taken from the constraint by the compact star properties in the quark-meson model [\[65,66\].](#page-5-14) With these values of parameters, the dominant contribution to ρ_{00}^{ϕ} , c_{ϕ} in Eq. [\(12\)](#page-3-0), as a function of collision energy in $Au + Au$ collisions is shown in Fig. [1.](#page-3-2) We see in Fig. [1](#page-3-2) that ρ_{00}^{ϕ} decreases with the collision energy.

A natural question arises: are the theory and conclusion in this paper valid for another vector meson $K^{*0}(892)$? The answer would be no. There are a few reasons for it. First, due to unequal masses of \bar{s} and d, one cannot derive similar formula to Eq. [\(8\)](#page-2-0) in which terms of vorticity and those of electric and magnetic field are decoupled. Therefore, one cannot build up a simple relationship between ρ_{00} for K^{*0} , $\rho_{00}^{K^{*0}}$, and the hyperon polarization. In $\rho_{00}^{K^{*0}}$, each contribution can be either positive or negative, so it is not easy to single out a specific contribution from ρ_{00} which belongs to the vorticity, electromagnetic field, or mesonic filed without ambiguity. Second, the interaction of K^{*0} with the surrounding matter is much stronger than the ϕ meson. In this sense, the ϕ meson is a cleaner probe than K^{*0} to the state of the fireball. Actually, preliminary data from the ALICE experiment show that ρ_{00} for the K^{*0} meson is less than $1/3$ at LHC energies [\[81,82\]](#page-5-19), which is very different from the ϕ meson. Another question is: what happens for

FIG. 1. The spin matrix element ρ_{00} for the ϕ meson in heavyion collisions from Eq. [\(12\).](#page-3-0) The thin horizontal solid line shows the no-alignment value $\rho_{00} = 1/3$. Three values of $C_s^{(y)}$ are chosen.

 ρ_{00} at LHC energies? From the energy behavior in Fig. [1](#page-3-2), we expect that negative c_A and c_{ε} would be comparable to positive c_{ϕ} at LHC energies. In this case, whether ρ_{00} is larger or smaller than $1/3$ depends on a fine-tuning of each terms.

V. SUMMARY

Due to the difference in the parton distribution function of s and \bar{s} in high energy proton-proton collisions, the longitudinal momenta carried by s and \bar{s} are not equal. This leads to a nonvanishing collective strangeness current in the beam direction in high energy heavy-ion collisions. We generalize this feature to transverse directions. Such a strangeness current gives rise to a nonvanishing electric and magnetic part of the mean ϕ field, \mathbf{E}_{ϕ} and \mathbf{B}_{ϕ} , respectively. Like the magnetic field, \mathbf{B}_{ϕ} can also polarize s and \bar{s} through their magnetic moments which contributes to the polarization of Λ and Λ , while the contribution from E_{ϕ} is absent and therefore is not constrained by the polarization of Λ and $\bar{\Lambda}$. However, the contribution from \mathbf{E}_{ϕ} to ρ_{00}^{ϕ} is always positive through the correlation between the spin-orbit term involving \mathbf{E}_{ϕ} that polarizes s and \bar{s} . The spin-orbit force at the nucleon level is responsible for the nuclear shell structure. We then propose that a significant and positive deviation of ρ_{00}^{ϕ} from 1/3 could indicate presence of a mean ϕ field in heavy-ion collisions which polarizes s and \bar{s} through the spin-orbit interaction. The contribution is positive definite even for a fluctuating field. In this sense, ρ_{00}^{ϕ} is a good analyzer for fluctuating fields.

ACCEPT CHEMICS

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