Nucleon decay and $n-\bar{n}$ oscillations in a left-right symmetric model with large extra dimensions

Sudhakantha Girmohanta and Robert Shrock
C. N. Yang Institute for Theoretical Physics and Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA

(Received 27 March 2020; accepted 29 April 2020; published 13 May 2020)

We study baryon-number-violating processes, including proton and bound neutron decays and $n-\bar{n}$ oscillations, in a left-right-symmetric (LRS) model in which quarks and leptons have localized wave functions in extra dimensions. In this model we show that, while one can easily suppress baryon-number-violating nucleon decays well below experimental bounds, this does not suppress $n-\bar{n}$ transitions, which may occur at levels comparable to current limits. This is qualitatively similar to what was found in an extra-dimensional model with a Standard-Model low-energy effective field theory (SMEFT). We show that experimental data imply a lower limit on the mass scale $M_{n\bar{n}}$ characterizing the physics responsible for $n-\bar{n}$ oscillations in the LRS model that is significantly higher than in the extra-dimensional model using a SMEFT and explain the reason for this. Our results provide further motivation for new experiments to search for $n-\bar{n}$ oscillations.

DOI: 10.1103/PhysRevD.101.095012

I. INTRODUCTION

The Standard Model (SM) conserves baryon number, B [1,2], but baryon-number violation (BNV) is expected to occur in nature, since this is one of the requisite conditions for producing the observed baryon-number asymmetry in the universe [3]. Indeed, many ultraviolet extensions of the Standard Model, such as grand unified theories (GUTs), do feature baryon-number violation (as well as the violation of total lepton number, L). In addition to the $\Delta B = -1$ decays of protons and bound neutrons, another type of baryon-number violation is neutron-antineutron oscillations, with $|\Delta B| = 2$. These $n-\bar{n}$ oscillations could explain baryogenesis [4]. Some early studies of $n-\bar{n}$ oscillations include [5–11]. The same physics beyond the Standard Model that gives rise to $n-\bar{n}$ oscillations also leads to matter instability via $\Delta B = -2$ decays of nn and np dinucleon states in nuclei. Several generations of experiments have searched for baryon-number-violating decays of protons and bound neutrons (henceforth denoted simply as nucleon decays) and have set limits on such decays [12]. There have also been searches for $n-\bar{n}$ oscillations using neutron beams from reactors [13] and for matter instability and various dinucleon decay modes using large underground detectors [12]. The best current limit on matter instability is from the Super-Kamiokande (SK) experiment [14].

The operators in the low-energy effective Hamiltonian (in four spacetime dimensions) for proton decay are four-fermion operators with Maxwellian (i.e., free-field) mass dimension 6 and hence coefficients of mass dimension -2, whereas the operators in $\mathcal{H}_{\rm eff}^{(n\bar{n})}$ are six-quark operators, with coefficients of dimension -5. Hence, if there were only a single mass scale characterizing BNV physics, then nucleon decays would generically be much more important as a manifestation of baryon-number violation than $n-\bar{n}$ oscillations and the corresponding dinucleon decays. However, the opposite order of importance of BNV processes may actually describe nature. In Ref. [6], Mohapatra and Marshak presented a model using a left-right symmetric gauge group (in four spacetime dimensions) in which $n-\bar{n}$ oscillations occur, while proton decay does not. In Ref. [15], Nussinov and Shrock presented an extra-dimensional model in which proton decay is suppressed well beyond observable levels while $n-\bar{n}$ oscillations occur at levels comparable to experimental limits. In the model used in [15], quarks and leptons have strongly localized wave function profiles in the extra dimensions [16,17]. In the models of both Refs. [6,15], it is the $n-\bar{n}$ oscillations and the corresponding nn and np dinucleon decays to multimeson final states that are the main manifestations of baryon-number violation, rather than individual BNV nucleon decays. Further examples of models in four spacetime dimensions with baryon-number violation but no proton decay were later given in [18].

Recently, in [19] we studied a number of related BNV nucleon and dinucleon decays to various final states in the extra-dimensional model used in [15].

In this paper we investigate nucleon decays and $n-\bar{n}$ oscillations in an extra-dimensional model with the left-right symmetric (LRS) gauge group

$$G_{\rm LRS} = {\rm SU}(3)_c \otimes {\rm SU}(2)_L \otimes {\rm SU}(2)_R \otimes {\rm U}(1)_{B-L}. \quad (1.1)$$

Our present work complements the study in Ref. [6], which was set in four spacetime dimensions, and also the previous studies [15] and [19], which used a low-energy effective field theory with the SM gauge group, $G_{\rm SM} = {\rm SU}(3)_c \otimes {\rm SU}(2)_L \otimes {\rm U}(1)_Y$ rather than $G_{\rm LRS}$. Anticipating our results in advance, we show that in the extra-dimensional LRS model, it is easy to suppress nucleon decays well below observable levels, but this does not suppress $n-\bar{n}$ oscillations, which can occur at levels comparable with current experimental limits. This is qualitatively similar to the conclusions reached in [15]. Here we find an interesting feature of the extra-dimensional LRS model that makes $n-\bar{n}$ oscillations even less suppressed than in the model of [15] with its Standard-Model low-energy effective field theory (SMEFT). The reason for this is that the integration of six-quark operators over the extra dimensions always led to exponential suppression factors in the model of [15], whereas, in contrast, we find that in the LRS model, there are some operators for which this integration does not lead to exponential suppression factors.

Our work here also complements our recent studies in [20], where we derived improved upper bounds on the rates for several nucleon-to-trilepton decay modes and in [21], where we presented improved upper bounds on the rates for several dinucleon-to-dilepton decay channels (see also [22]). References [20,21] were model-independent phenomenological analyses, whereas our present paper is a study within the context of a specific type of extra-dimensional model. Recent reviews of $n-\bar{n}$ oscillations include [23,24].

This paper is organized as follows. In Sec. II we briefly review the properties of the left-right symmetric model that will be needed for our analysis. In Sec. III we discuss the extra-dimensional model and low-energy effective field theory approach that serve as the theoretical framework for our calculations. In Sec. IV we extract constraints on the fermion wave functions in the model from limits on BNV nucleon decay modes. Section V contains our analysis of $n-\bar{n}$ oscillations. Our conclusions are presented in Section VI.

II. LEFT-RIGHT SYMMETRIC MODEL

In this section we recall some basic properties of the left-right symmetric model [6,25–27] that will be relevant here, and define our notation for the fermion and Higgs fields in the theory. The Lagrangian is invariant under the gauge group $G_{\rm LRS}$ in Eq. (1.1), with corresponding ${\rm SU}(2)_L$, ${\rm SU}(2)_R$, and ${\rm U}(1)_{B-L}$ gauge fields $\vec{A}_{L,\mu}$, $\vec{A}_{R,\mu}$ and U_μ, and respective gauge couplings g_L, g_R, and g_U. The quarks and leptons of each generation transform as

$$Q_L\colon\ (3,2,1)_{1/3,L}, \qquad Q_R\colon\ (3,1,2)_{1/3,R} \quad (2.1)$$

and

$$L_{\ell,L}\colon\ (1,2,1)_{-1,L}, \qquad L_{\ell,R}\colon\ (1,1,2)_{-1,R}, \quad (2.2)$$

where the numbers in the parentheses are the dimensionalities of the representations under the three non-Abelian factor groups in $G_{\rm LRS}$ and the numbers in the subscripts are the values of $B-L$. (No confusion should result from the use of the symbol L for both "lepton" and "left"; the context will make clear which meaning is intended.) For our purposes, we shall only need the first-generation quark fields, which are, explicitly,

$$Q_L^\alpha = \begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}_L, \qquad Q_R^\alpha = \begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}_R, \quad (2.3)$$

where Greek indices α, β, etc. are ${\rm SU}(3)_c$ color indices. The explicit lepton fields are

$$L_{\ell,L} = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L, \qquad L_{\ell,R} = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_R, \quad (2.4)$$

where $\ell = e, \mu, \tau$. We denote ${\rm SU}(2)_L$ and ${\rm SU}(2)_R$ gauge indices as Roman indices $i, j\ldots$ and primed Roman indices $i', j'\ldots$, respectively, so, e.g., $Q_L^{i\alpha} = u_L^\alpha$ for $i=1$ and $Q_R^{i'\alpha} = d_R^\alpha$ for $i'=2$. The electric charge is given by the elegant expression $Q_{em} = T_{3L} + T_{3R} + (B-L)/2$, where $\vec{T}_L$ and $\vec{T}_R$ denote the ${\rm SU}(2)_L$ and ${\rm SU}(2)_R$ weak isospin generators.

The Higgs sector contains a Higgs field Φ transforming as $(1,2,2)_0$, which can be written as $\Phi^{ij'}$, or equivalently, in matrix form, as

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}. \quad (2.5)$$

The Higgs sector also contains two Higgs fields, commonly denoted Δ_L and Δ_R, which transform as $(1,3,1)_2$ and $(1,1,3)_2$, respectively. Since the adjoint representation of SU(2) is equivalent to the symmetric rank-2 tensor representation, these may be written as $(\Delta_L)^{ij} = (\Delta_L)^{ji}$ and $(\Delta_R)^{i'j'} = (\Delta_R)^{j'i'}$ or, alternatively, as (traceless) matrices:

$$\Delta_\chi = \begin{pmatrix} \Delta_\chi^+/\sqrt{2} & \Delta_\chi^{++} \\ \Delta_\chi^0 & -\Delta_\chi^+/\sqrt{2} \end{pmatrix}, \qquad \chi = L, R. \quad (2.6)$$

The minimization of the Higgs potential to produce vacuum expectation values (VEVs) has been analyzed in a number of studies [27–32]. With appropriate choices of parameters in the Higgs potential, this minimization yields the following vacuum expectation values of the Higgs fields:

$$\langle\Phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 e^{i\theta_\Phi} \end{pmatrix}, \tag{2.7}$$

$$\langle\Delta_L\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_\Delta} & 0 \end{pmatrix} \tag{2.8}$$

and

$$\langle\Delta_R\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \tag{2.9}$$

(Here, the choices of which VEVs are real are made with the requisite rephasings.) The spontaneous symmetry breaking of the $G_{\rm LRS}$ gauge symmetry occurs in several stages. At the highest-mass stage, Δ_R picks up a VEV, thereby breaking the $\mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$ subgroup of $G_{\rm LRS}$ to $\mathrm{U}(1)_Y$, where Y denotes the weak hypercharge, i.e.,

$$\mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L} \to \mathrm{U}(1)_Y. \tag{2.10}$$

This gives the W_R a large mass, which, to leading order, is $m_{W_R} = g_R v_R/\sqrt{2}$. The second stage of symmetry breaking,

$$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to \mathrm{U}(1)_{em}, \tag{2.11}$$

occurs at a lower scale and results from the VEVs of the Φ field. This gives a mass $m_{W_L} = g_L v_{\rm EW}/2$, where $v_{\rm EW} = \sqrt{\kappa_1^2 + \kappa_2^2} = 246$ GeV is the electroweak symmetry breaking (EWSB) scale. The neutral gauge fields A_{3L}, A_{3R}, and U mix to form the photon, the Z, and a much more massive Z'. Since the VEV v_L of the $\mathrm{SU}(2)_L$ Higgs triplet Δ_L would modify the successful tree-level relation $\rho = 1$, where $\rho = m_W^2/(m_Z^2 \cos^2\theta_W) = 1$, one takes $v_L \ll \kappa_{1,2}$. It is also possible to consider dynamical breaking of the LRS gauge symmetry (e.g., [33,34]), but the conventional scenario with Higgs fields will be assumed here.

This LRS model has several interesting features as a UV extension of the Standard Model. The relation for Q_{em} entails charge quantization. Furthermore, one may impose left-right symmetry at some high ultraviolet (UV) scale, so the running gauge couplings for $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ are equal, i.e., $g_L = g_R$ at this scale, thereby reducing the number of parameters in the model. The left-right symmetry in the Lagrangian is of conceptual interest since it means that parity violation is due to spontaneous symmetry breaking, rather than being intrinsic, as in the Standard Model. The nonobservation of any right-handed charged currents in weak decays and the lower limits (of order several TeV) from the Large Hadron Collider on a $W_R^\pm$ and Z' can be accommodated by making v_R sufficiently large. Since the Δ_R has $B-L$ charge of 2, its VEV, v_R, breaks $B-L$ by two units. The gauge group $G_{\rm LRS}$ has a natural UV extension to a theory with gauge group $G_{422} = \mathrm{SU}(4)_{PS} \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$, where the Pati-Salam (PS) gauge group $\mathrm{SU}(4)_{PS}$ [35] contains $\mathrm{SU}(3)_c \otimes U(1)_{B-L}$ as a maximal subgroup. In turn, G_{422} is a maximal subgroup of the SO(10) GUT group, since $\mathrm{SO}(10) \supseteq \mathrm{SO}(6) \otimes \mathrm{SO}(4) \approx \mathrm{SU}(4) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(2)$. There are also supersymmetric extensions of the LRS model (e.g., [36]). However, since the LHC has not yet observed evidence of supersymmetric partners, and since we use a low-energy effective field theory framework for our analysis, the nonsupersymmetric version of the LRS model will be sufficient for our study.

III. EXTRA-DIMENSIONAL FRAMEWORK

In this section we describe the extra-dimensional model that we use. Some aspects of this discussion are similar to those of Refs. [15,19], but to make our presentation self-contained, we reiterate these here. The particular type of extra-dimensional model that was used for the study of $n-\bar{n}$ oscillations in [15,19] has the appeal that it can naturally explain the large hierarchy in quark and lepton masses by requisite properties of fermion wave functions in the extra dimensions, without the need for a large range of dimensionless Yukawa couplings in the fundamental theory [16,17].

A remark is in order concerning a difference in our use of the extra-dimensional model here and the use in Refs. [15] and [19]. Because the scale of baryon-number violation responsible for $n-\bar{n}$ oscillations is larger than the electroweak scale, Refs. [15] and [19] used a low-energy effective field theory analysis with six-quark operators that are invariant under the Standard-Model gauge group, $G_{\rm SM}$, i.e., an extra-dimension SMEFT. As noted above, in the Standard Model, B is a global symmetry, and the baryon-number-violating physics that gives rise to $n-\bar{n}$ oscillations is encoded in the six-quark operators and their coefficients. In contrast, in the LRS model, B and L are both gauged, as the combination $B-L$ in the $\mathrm{U}(1)_{B-L}$ factor group of $G_{\rm LRS}$. This gauge symmetry is spontaneously broken by the VEV of the Δ_R field at the high scale v_R. As mentioned above, since Δ_R has charge 2 under $\mathrm{U}(1)_{B-L}$, this VEV v_R breaks $\mathrm{U}(1)_{B-L}$ by two units. For a process that has $\Delta L = 0$, this means that it breaks B as $|\Delta B| = 2$. It follows that the mass scale, $M_{n\bar{n}}$, characterizing the physics responsible for $n-\bar{n}$ oscillations is v_R:

$$M_{n\bar{n}} = v_R. \tag{3.1}$$

We shall analyze $n-\bar{n}$ oscillations in this theory by writing down the relevant $G_{\rm LRS}$-invariant operators, which are six-quark operators multiplied by $(\Delta_R)^\dagger$, and then focusing on the resultant six-quark operators resulting from the VEV of $(\Delta_R)^\dagger$.

Proceeding with the description of the extra-dimensional model, the usual spacetime coordinates are denoted as x_ν, with $\nu = 0, 1, 2, 3$, and the n extra coordinates as y_λ with $1 \le \lambda \le n$; for definiteness, the latter are assumed to be compact. The fermion and boson fields are taken to have a factorized form. For fermions, this form is

$$\Psi(x,y) = \psi(x)\chi(y), \tag{3.2}$$

where here $\Psi(x,y)$ is a generic symbol standing for $Q_L(x,y)$, $Q_R(x,y)$, $L_{\ell,L}(x,y)$ or $L_{\ell,R}(x,y)$. In the extra dimensions these fields are restricted to the interval $0 \le y_\lambda \le L$ for all λ. We define an energy corresponding to the inverse of the compactification scale as $\Lambda_L \equiv 1/L$.

Starting from an effective Lagrangian in the $d = (4+n)$-dimensional spacetime, one obtains the resultant low-energy effective Lagrangian in four dimensions by integrating over the extra n dimensions. We use a low-energy effective field theory (EFT) approach that entails an ultraviolet cutoff, which we denote as M_*. In accordance with this low-energy EFT approach, as in Ref. [17], we focus on the lowest Kaluza-Klein modes of the boson (gauge and Higgs) fields and take these to have flat profiles in the extra dimensions. Recall that the Maxwellian mass dimension of a boson field in a $d = 4+n$ dimensional spacetime is $d_b = (d-2)/2 = 1 + (n/2)$. Therefore, in order to maintain canonical normalization of boson fields in four spacetime dimensions, a Higgs field in $4+n$ dimensions with a flat profile in the extra dimensions, generically denoted ϕ_{4+n}, has the form

$$\phi_{4+n}(x,y) = (\Lambda_L)^{n/2}\phi(x) = L^{-n/2}\phi(x). \tag{3.3}$$

It is readily seen that the integration of the quadratic terms in the Higgs field over the n extra dimensions yields the correct normalization for the resultant quadratic terms in the Lagrangian in four spacetime dimensions:

$$\int_0^L d^n y \mathrm{Tr}[\phi^\dagger_{4+n}\phi_{4+n}] = L^n[L^{-n}\mathrm{Tr}(\phi^\dagger\phi)] = \mathrm{Tr}(\phi^\dagger\phi). \tag{3.4}$$

The coefficients of higher-power products of Higgs fields can be expressed using similar methods. For example, the coefficient $\lambda_{1,4+n}$ of the quartic term $[\mathrm{Tr}(\Phi(x,y)^\dagger\Phi(x,y))]^2$ has dimensions $d_{\lambda_{1,4+n}} = 4 - d = -n$, and hence we set $\lambda_{1,4+n} = \Lambda_L^{-n}\lambda_1 = L^n\lambda_1$ so that the integration over the extra dimensions yields the standard quartic term in the Lagrangian:

$$\begin{aligned}\lambda_{1,4+n}&\int_0^L d^n y[\mathrm{Tr}(\Phi(x,y)^\dagger\Phi(x,y))]^2 \\ &= (L^n\lambda_1)(L^n)(L^{-n/2})^4\mathrm{Tr}(\Phi(x)^\dagger\Phi(x))] \\ &= \lambda_1[\mathrm{Tr}(\Phi(x)^\dagger\Phi(x))],\end{aligned} \tag{3.5}$$

and similarly with other terms in the Higgs potential. Corresponding statements apply for the covariant derivative terms. The VEV of the higher-dimensional Higgs field $(\Delta_R)_{4+n}$ is thus

$$\langle(\Delta_R)_{4+n}\rangle_0 = (\Lambda_L)^{n/2}v_R = L^{-n/2}v_R. \tag{3.6}$$

Since the Higgs fields are taken to have flat profiles in the extra dimensions as in [17] and since we will only need to make use of their VEVs for our purposes, we may simply replace the various Higgs fields by these VEVs in the four-spacetime-dimensional Lagrangian and deal only with the dependence of the fermion fields on the y coordinates. This simplified procedure will be followed henceforth.

The localization of the wave function of a fermion f in the extra dimensions has the form [16,17]

$$\chi_f(y) = Ae^{-\mu^2\|y-y_f\|^2}, \tag{3.7}$$

where A is a normalization factor and $y_f \in \mathbb{R}^n$ denotes the position vector of this fermion in the extra dimensions, with components $y_f = (y_{f,1}, \ldots, y_{f,n})$ and with the standard Euclidean norm of a vector in $\mathbb{R}^n$, namely $\|y_f\| \equiv (\sum_{\lambda=1}^n y^2_{f,\lambda})^{1/2}$. For $n = 1$ or $n = 2$, this fermion localization can result from appropriate coupling to a scalar localizer field with a kink or vortex solution, respectively [37–43]. Corrections due to Coulombic gauge interactions between fermions have been studied in [44]. The normalization factor A is determined by the condition that, after integration over the n higher dimensions, the four-dimensional fermion kinetic term has its canonical normalization. This yields the result

$$A = \left(\frac{2}{\pi}\right)^{n/4}\mu^{n/2}. \tag{3.8}$$

We define a distance inverse to the localization measure μ as $L_\mu \equiv 1/\mu$. The fermion wave functions are assumed to be strongly localized, with half-width $L_\mu \ll L$ at various points in the higher-dimensional space. We define $\xi \equiv L/L_\mu = \mu/\Lambda_L$. As in the earlier works [15,19], the choice $\xi \sim 30$ is made for sufficient separation of the various fermion wave functions while still fitting well within the size L of the compactified extra dimensions. The UV cutoff M_* is taken to be much larger than any mass scale in the model, to ensure the self-consistency of the low-energy effective field theory analysis. The choice $\Lambda_L \gtrsim 100$ TeV is consistent with bounds on extra dimensions from precision electroweak constraints and collider searches [12] and produces adequate suppression of flavor-changing neutral-current processes [45] (see also [46,47]). With $\xi = 30$, this yields $\mu \sim 3\times 10^3$ TeV. (The models considered here with SM fields propagating in the large extra dimensions are to be contrasted with models in which only the gravitons propagate in these dimensions (e.g., [48–51]) and models with noncompact extra dimensions and a warped metric [52,53].)

For integrals of products of fermion fields, although the range of integration over each of the n coordinates of a vector y is from 0 to L, the strong localization of each fermion field in the Gaussian form (3.7) means that, to a very good approximation, the restriction of the fermion wave functions to the form (3.7), the range of integration can be extended to the interval $(-\infty,\infty)$: $\int_0^L d^n y \to \int_{-\infty}^{\infty} d^n y$. We define the (dimensionless) vector

$$\eta = \mu y. \tag{3.9}$$

We next discuss the Yukawa terms and resultant mass terms for quarks in this extra-dimensional LRS model. These are

$$\mathcal{L}_{\rm Yuk} = \sum_{a,b=1}^{3} [\bar{Q}_{a,L}(y_{ab}^{(q)}\Phi + h_{ab}^{(q)}\tilde{\Phi})Q_{b,R}] + {\rm H.c.}, \tag{3.10}$$

where a, b are generation indices and $\tilde{\Phi} = \tau_2\Phi^*\tau_2$, and here $y_{ab}^{(q)}$ and $h_{ab}^{(q)}$ are Yukawa couplings. Inserting the VEV of Φ from Eq. (2.7) and performing the integration, over the extra dimensions, of the quark bilinears gives the mass terms

$$\frac{1}{\sqrt{2}}\sum_{a,b=1}^{3}[\bar{u}_{a,L}(y_{ab}^{(q)}\kappa_1 + h_{ab}^{(q)}\kappa_2 e^{i\theta_\Phi})u_{b,R}]e^{-S_{yQ,ab}} + \frac{1}{\sqrt{2}}\sum_{a,b=1}^{3}[\bar{d}_{a,L}(y_{ab}^{(q)}\kappa_2 e^{-i\theta_\Phi} + h_{ab}^{(q)}\kappa_1)d_{b,R}]e^{-S_{yQ,ab}} + {\rm H.c.}, \tag{3.11}$$

where

$$S_{yQ,ab} = \frac{1}{2}\|\eta_{Q_{a,L}} - \eta_{Q_{b,R}}\|^2. \tag{3.12}$$

For our study of $n-\bar{n}$ oscillations in this model, we will only need to deal with the first-generation quark fields, $Q_{1,L}$ and $Q_{1,R}$. Consequently, we will omit the generation indices on these fields, with the understanding that they are first-generation quarks: $Q_L = \binom{u}{d}_L$ and $Q_R = \binom{u}{d}_R$. Neglecting small Cabibbo-Kobayashi-Maskawa mixings, the relevant quark mass terms are then

$$\frac{1}{\sqrt{2}}\left[[\bar{u}_L(y_{11}^{(q)}\kappa_1 + h_{11}^{(q)}\kappa_2 e^{i\theta_\Phi})u_R] + [\bar{d}_L(y_{11}^{(q)}\kappa_2 e^{-i\theta_\Phi} + h_{11}^{(q)}\kappa_1)d_R]\right]e^{-(1/2)\|\eta_{Q_L}-\eta_{Q_R}\|^2} + {\rm H.c.} \tag{3.13}$$

Note that although one may impose left-right symmetry in the deep UV, this symmetry is broken at the scale v_R, so at this EWSB scale, η_{Q_L} is expected to be different from η_{Q_R}. In accordance with the original motivation for this type of extra-dimensional model, namely that the generational hierarchy in the quark and charged lepton masses is not due primarily to a hierarchy in the dimensionless Yukawa couplings, but instead to the different positions of the wave function centers in the extra dimensions, one may take $y_{11}^{(q)} \sim O(1)$ and $h_{11}^{(q)} \sim O(1)$. Then

$$\|\eta_{Q_L} - \eta_{Q_R}\| = \left[2\ln\left(\frac{|y_{11}^{(q)}\kappa_1 + h_{11}^{(q)}\kappa_2 e^{i\theta_\Phi}|}{\sqrt{2}m_u}\right)\right]^{1/2} \tag{3.14}$$

and

$$\|\eta_{Q_L} - \eta_{Q_R}\| = \left[2\ln\left(\frac{|y_{11}^{(q)}\kappa_2 e^{-i\theta_\Phi} + h_{11}^{(q)}\kappa_1|}{\sqrt{2}m_d}\right)\right]^{1/2}. \tag{3.15}$$

For given κ_1 and κ_2, the two Yukawa couplings $y_{11}^{(q)}$ and $h_{11}^{(q)}$, and the phase factor $e^{i\theta_\Phi}$ can be chosen to satisfy these relations. Taking $y_{11}^{(q)} \sim O(1)$ and $h_{11}^{(q)} \sim O(1)$ as above, and using the values of the running quark masses m_u and m_d at the EWSB scale from Ref. [54], one can then compute a value of $\|\eta_{Q_L} - \eta_{Q_R}\|$ that satisfies Eqs. (3.14) and (3.15). For our purposes, we will take the value

$$\|\eta_{Q_L} - \eta_{Q_R}\| \simeq 4.7. \tag{3.16}$$

For our analysis of baryon-number-violating processes, let us consider a generic operator product of fermion fields in the four-dimensional Lagrangian consisting of k fermion fields multiplied by a coefficient $c_{r,k}$, which we denote as $\mathcal{O}_{r,k}$. We denote the corresponding operator in the $d = (4+n)$-dimensional space as $O_{r,k}(x,y)$. The coefficient of this operator, $\kappa_{r,k}$, can be written in a form that exhibits its mass dimension explicitly, namely

$$\kappa_{r,k} = \frac{\bar{\kappa}_{r,k}}{(M_{\rm BNV})^{k(3+n)/2-4-n}}, \tag{3.17}$$

where $\bar{\kappa}_{r,k}$ is dimensionless and $M_{\rm BNV}$ is a relevant mass scale for the BNV process (nucleon decay or $n-\bar{n}$

oscillations). We denote the integral over the extra dimensions of this fermion operator product as $I_{r,k}$. Using Eq. (A2) of Ref. [19], we have $I_{r,k} = b_k e^{-S_{r,k}}$, where

$$b_k = A^k \mu^{-n} \left(\frac{\pi}{k}\right)^{n/2} = [2^{k/4}\pi^{-(k-2)/4}k^{-1/2}\mu^{(k-2)/2}]^n. \tag{3.18}$$

Then, as in [19],

$$c_{r,k} = \kappa_{r,k} I_{r,k} = \frac{\bar{\kappa}_{r,k}}{(M_{\rm BNV})^{(3k-8)/2}} \left(\frac{\mu}{M_{\rm BNV}}\right)^{(k-2)n/2} \times \left(\frac{2^{k/4}}{\pi^{(k-2)/4}k^{1/2}}\right)^n e^{-S_{r,k}}. \tag{3.19}$$

For cases where the number k is obvious, we will sometimes suppress this subscript in the notation.

IV. CONSTRAINTS FROM LIMITS ON BARYON-NUMBER-VIOLATING NUCLEON DECAYS

In this section we analyze the constraints on fermion wave functions that can be derived from the experimental upper limits on the rates for baryon-number-violating nucleon decays. We denote the relevant BNV mass scale $M_{\rm BNV}$ as M_{Nd}, where Nd stands for "nucleon decay." We assume that M_{Nd} is large compared with the highest gauge-symmetry breaking scale in the LRS model, namely v_R, so that the effective Lagrangian is invariant under the LRS gauge group, $G_{\rm LRS}$.

For the effective Lagrangian that is relevant for nucleon decays, we write

$$\mathcal{L}_{\rm eff}^{(Nd)}(x) = \sum_r c_r^{(Nd)} \mathcal{O}_r^{(Nd)}(x) + \text{H.c.}, \tag{4.1}$$

where $c_r^{(Nd)}$ are coefficients, and $\mathcal{O}_r^{(Nd)}(x)$ are the various four-fermion operators. Correspondingly, in the $d = (4+n)$-dimensional space, the effective Lagrangian is

$$\mathcal{L}_{{\rm eff},4+n}^{(Nd)}(x,y) = \sum_r \kappa_r^{(Nd)} O_r^{(Nd)}(x,y) + \text{H.c.} \tag{4.2}$$

Four-fermion operators $\mathcal{O}_r^{(Nd)}$ in $\mathcal{L}_{\rm eff}^{(Nd)}$ that contribute to nucleon decays in this LRS model and are invariant under $G_{\rm LRS}$ are listed below [where the unprimed and primed Roman indices are ${\rm SU}(2)_L$ and ${\rm SU}(2)_R$ gauge indices, as defined above]:

$$\mathcal{O}_{LL}^{(Nd)} = \epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{km}[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\gamma T}CL_{\ell,L}^m] = 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T}Cd_L^\beta]([u_L^{\gamma T}C\ell_L] - [d_L^{\gamma T}C\nu_{\ell,L}]), \tag{4.3}$$

$$\mathcal{O}_{RR}^{(Nd)} = \epsilon_{\alpha\beta\gamma}\epsilon_{i'j'}\epsilon_{k'm'}[Q_R^{i'\alpha T}CQ_R^{j'\beta}][Q_R^{k'\gamma T}CL_{\ell,R}^{m'}] = 2\epsilon_{\alpha\beta\gamma}[u_R^{\alpha T}Cd_R^\beta]([u_R^{\gamma T}C\ell_R] - [d_R^{\gamma T}C\nu_{\ell,R}]), \tag{4.4}$$

$$\mathcal{O}_{LR}^{(Nd)} = \epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{i'j'}[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_R^{i'\gamma T}CL_{\ell,R}^{j'}] = 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T}Cd_L^\beta]([u_R^{\gamma T}C\ell_R] - [d_R^{\gamma T}C\nu_{\ell,R}]) \tag{4.5}$$

and

$$\mathcal{O}_{RL}^{(Nd)} = \epsilon_{\alpha\beta\gamma}\epsilon_{i'j'}\epsilon_{ij}[Q_R^{i'\alpha T}CQ_R^{j'\beta}][Q_L^{i\gamma T}CL_{\ell,L}^j] = 2\epsilon_{\alpha\beta\gamma}[u_R^{\alpha T}Cd_R^\beta]([u_L^{\gamma T}C\ell_L] - [d_L^{\gamma T}C\nu_{\ell,L}]), \tag{4.6}$$

where C is the Dirac charge conjugation matrix satisfying $C\gamma_\mu C^{-1} = -(\gamma_\mu)^T$, $C = -C^T$; and $\epsilon_{\alpha\beta\gamma}$, ϵ_{ij}, and $\epsilon_{i'j'}$ are totally antisymmetric ${\rm SU}(3)_c$, ${\rm SU}(2)_L$, and ${\rm SU}(2)_R$ tensors, respectively.

To each of these operators $\mathcal{O}_r^{(Nd)}$ there corresponds an operator $O_r^{(Nd)}$ in $\mathcal{L}_{{\rm eff},4+n}^{(Nd)}$. These are four-fermion operators, and, as the $k = 4$ special case of Eq. (3.17), we have

$$\kappa_r^{(Nd)} = \frac{\bar{\kappa}_r^{(Nd)}}{(M_{Nd})^{2+n}}. \tag{4.7}$$

The dependence of $\kappa_r^{(Nd)}$ on the generational index of the lepton field that occurs in $\mathcal{O}_r^{(Nd)}$ is left implicit. From the factorized form of fermion fields in Eq. (3.2), it follows that

$$O_r^{(Nd)}(x,y) = U_r^{(Nd)}(x)V_r^{(Nd)}(y), \tag{4.8}$$

where $r = LL, RR, LR, RL$. To perform the integrals over y, we use the general integration formula given as Eq. (A2) in Ref. [19]. Carrying out the integration over the y components and using Eq. (3.8) for the relevant case $k = 4$, we obtain the following results for the nonvanishing operators:

$$I_{LL}^{(Nd)} = b_4 \exp\left[-\frac{3}{4}\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2\right], \tag{4.9}$$

$$I_{RR}^{(Nd)} = b_4 \exp\left[-\frac{3}{4}\|\eta_{Q_R} - \eta_{L_{\ell,R}}\|^2\right], \tag{4.10}$$

$$I_{LR}^{(Nd)} = b_4 \exp\left[-\frac{1}{4}\{2\|\eta_{Q_L} - \eta_{Q_R}\|^2 + 2\|\eta_{Q_L} - \eta_{L_{\ell_R}}\|^2 + \|\eta_{Q_R} - \eta_{L_{\ell_R}}\|^2\}\right] \tag{4.11}$$

and

$$I_{RL}^{(Nd)} = b_4 \exp\left[-\frac{1}{4}\{2\|\eta_{Q_R} - \eta_{Q_L}\|^2 + 2\|\eta_{Q_R} - \eta_{L_{\ell_L}}\|^2 + \|\eta_{Q_L} - \eta_{L_{\ell_L}}\|^2\}\right] \tag{4.12}$$

where $b_4 = (\pi^{-1/2}\mu)^n$, from the $k = 4$ special case of Eq. (3.18). It is convenient to write the integral $I_r^{(Nd)}$ in the form

$$I_r^{(Nd)} \equiv b_4 e^{-S_r^{(Nd)}}, \qquad (4.13)$$

where $S_r^{(Nd)}$ denotes the sum of squares of fermion wave function separation distances (rescaled via multiplication by μ to be dimensionless) in the argument of the exponent in $I_r^{(Nd)}$. Thus, for example, in the case of $O_{LL}^{(Nd)}$, the sum in the exponent is $S_{LL}^{(Nd)} = (3/4)\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2$, and similarly for the other $S_r^{(Nd)}$. Then, as the special case of (3.19) with $k=4$,

$$c_r^{(Nd)} = \frac{\bar{\kappa}_r^{(Nd)}}{(M_{Nd})^2}\left(\frac{\mu}{\pi^{1/2}M_{Nd}}\right)^n e^{-S_r^{(Nd)}}. \qquad (4.14)$$

We use the experimental lower bound [12] on the partial lifetime $(\tau/B)_{N\to \mathrm{f.s.}} = \Gamma^{-1}_{N\to \mathrm{f.s.}}$ for a given nucleon decay mode $N \to$ f.s. with branching ratio B to a final state denoted f.s. to infer upper bounds on the magnitudes of the $c_r^{(Nd)}$ coefficients. The strongest lower bounds on these partial lifetimes that are relevant here include $(\tau/B)_{p\to e^+\pi^0} > 1.6 \times 10^{34}$ yr and $(\tau/B)_{p\to\mu^+\pi^0} > 0.77\times 10^{34}$ yr [55]. The limits for the analogous decays of neutrons are $(\tau/B)_{n\to e^+\pi^-} > 0.53\times 10^{34}$ yr and $(\tau/B)_{n\to\mu^+\pi^-} > 0.35\times 10^{34}$ yr [56]. (These and other experimental limits quoted in this paper are at the 90% confidence level.) Since we do not not assume any cancellation between different terms $c_r^{(Nd)}\mathcal{O}_r^{(Nd)}$ occurring in $\mathcal{L}_{\mathrm{eff}}^{(Nd)}$, we impose the bounds from a given decay individually on each term that contributes to it. For given values of μ, M_{Nd}, and the dimensionless coefficients $\bar{\kappa}_r^{(Nd)}$, these constraints are upper bounds on the integrals $I_r^{(Nd)}$ and hence lower bounds on the sums of squares of distances in $S_r^{(Nd)}$ for each operator $\mathcal{O}_r^{(Nd)}$. Our analysis of these lower bounds on fermion separation distances in Ref. [19] can be taken over, with appropriate changes, for our present study; we refer the reader to [19] for the details. We find, for each r, $S_r^{(Nd)} > (S_r^{(Nd)})_{\mathrm{min}}$, where

$$(S_r^{(Nd)})_{\mathrm{min}} = 39 - \frac{n}{2}\ln\pi - 2\ln\left(\frac{M_{Nd}}{10^4\ \mathrm{TeV}}\right) - n\ln\left(\frac{M_{Nd}}{\mu}\right). \qquad (4.15)$$

The most direct bounds on fermion separation distances arises from the contribution of the operators $O_{LL}^{(Nd)}$ and $O_{RR}^{(Nd)}$, since, for a given ℓ ($= e$ or μ), the integrals $I_{LL}^{(Nd)}$ and $I_{RR}^{(Nd)}$ each involve only one fermion separation distance, namely $\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|$ and $\|\eta_{Q_R} - \eta_{L_{\ell,R}}\|$, respectively, for a given lepton generation $\ell = e$ or $\ell = \mu$. In this case, for the illustrative case of $n=2$ extra dimensions, we obtain the lower bound

$$\|\eta_{Q_\chi} - \eta_{L_{\ell,\chi}}\|^2 > 50 - \frac{8}{3}\ln\left(\frac{M_{Nd}}{10^4\ \mathrm{TeV}}\right) - \frac{8}{3}\ln\left(\frac{M_{Nd}}{\mu}\right)$$
$$\text{for } \chi = L,R \quad \text{and for } \ell = e,\mu. \qquad (4.16)$$

With the illustrative value $M_{Nd} = 10^4$ TeV, these are the inequalities $\|\eta_{Q_\chi} - \eta_{L_{\ell\chi}}\| > 6.8$ for each of the four possibilities $\chi = L, R$ and $\ell = e, \mu$. A conservative solution to the coupled quadratic inequalities would require that each of the relevant distances $\|\eta_{f_i} - \eta_{f_j}\|$ in Eq. (4.16) for both $\ell = e$ and $\ell = \mu$ would be larger than the square root of the right-hand side of Eq. (4.15):

$$\{\|\eta_{Q_L} - \eta_{L_{\ell_L}}\|, \|\eta_{Q_R} - \eta_{L_{\ell_R}}\|, \|\eta_{Q_L} - \eta_{L_{\ell_R}}\|,$$
$$\|\eta_{Q_R} - \eta_{L_{\ell_L}}\|\} > [(S_r^{(Nd)})_{\mathrm{min}}]^{1/2}. \qquad (4.17)$$

That is, this set of inequalities is sufficient, but not necessary, to satisfy experimental constraints on the model from lower limits on partial lifetimes for nucleon decays.

V. $n-\bar{n}$ OSCILLATIONS AND DINUCLEON DECAYS

In this section we analyze $n-\bar{n}$ oscillations and the resultant $\Delta B = -2$ dinucleon decays in this extra-dimensional LRS model. We refer the reader to Refs. [15] and [19] for relevant background; here we will review this background briefly. We consider a general theory in which baryon-number-violating physics can produce $n-\bar{n}$ transitions. We denote the relevant low-energy effective Lagrangian in 4D as $\mathcal{L}_{\mathrm{eff}}^{(n\bar{n})}$, and the transition matrix element as $|\delta m| = |\langle\bar{n}|\mathcal{L}_{\mathrm{eff}}^{(n\bar{n})}|n\rangle|$. In (field-free) vacuum, an initial state which is $|n\rangle$ at time $t=0$ has a nonzero probability to be an $|\bar{n}\rangle$ state at a later time $t>0$. This probability is given by $P(n(t)=\bar{n}) = |\langle\bar{n}|n(t)\rangle|^2 = [\sin^2(t/\tau_{n\bar{n}})]e^{-t/\tau_n}$, where $\tau_{n\bar{n}} = 1/|\delta m|$ and τ_n is the mean life of the neutron. The current direct limit on $\tau_{n\bar{n}}$, from a reactor experiment at the Institut Laue-Langevin (ILL) in Grenoble is $\tau_{n\bar{n}} \geq 0.86\times 10^8$ sec, i.e., $|\delta m| < 0.77 \times 10^{-29}$ MeV [13]. Because of the nonvanishing $n-\bar{n}$ transition amplitude, the physical eigenstate for the neutron state in matter has a small component of $\bar{n}$, i.e., $|n\rangle_{\mathrm{phys.}} = \cos\theta_{n\bar{n}}|n\rangle + \sin\theta_{n\bar{n}}|\bar{n}\rangle$, with $|\theta_{n\bar{n}}| \ll 1$. In turn, this leads to annihilation with an adjacent neutron or proton, and hence to $\Delta B = -2$ decays to nonbaryonic final states, predominantly involving pions. Experiments have searched for the resultant matter instability due to these dinucleon decays and have set lower limits on the matter instability (m.i.) lifetime, $\tau_{\mathrm{m.i.}}$. This lifetime is related to $\tau_{n\bar{n}}$ by the formula $\tau_{\mathrm{m.i.}} = R\tau_{n\bar{n}}^2$, where $R \sim O(10^2)$ MeV, or equivalently, $R \simeq 10^{23}$ sec^{-1}, depending on the nucleus. The best current limit on matter instability is from the Super-Kamiokande water Cherenkov experiment [14], namely $\tau_{\mathrm{m.i.}} > 1.9\times 10^{32}$ yr. Using the value $R \simeq 0.52 \times 10^{23}$ sec^{-1} for the ^{16}O nuclei in the water, Ref. [14] obtained the lower bound $\tau_{n\bar{n}} > 2.7 \times 10^{8}$ sec, or equivalently,

$$|\delta m| < 2.4 \times 10^{-30} \text{ MeV}. \tag{5.1}$$

As mentioned above, we shall analyze $n - \bar{n}$ oscillations in this theory by writing down the relevant $G_{\rm LRS}$-invariant operators, which are six-quark operators multiplied by $(\Delta_R)^\dagger$, and then focusing on the resultant $|\Delta B| = 2$ six-quark operators resulting from the VEV of $(\Delta_R)^\dagger$. The effective Lagrangian (in four-dimensional spacetime) that mediates $n - \bar{n}$ oscillations is a sum of six-quark operators,

$$\mathcal{L}_{\rm eff}^{(n\bar{n})}(x) = \sum_r c_r^{(n\bar{n})} \mathcal{O}_r^{(n\bar{n})}(x) + \text{H.c.} \tag{5.2}$$

The corresponding Lagrangian in the $(4+n)$-dimensional space is

$$\mathcal{L}_{\text{eff},4+n}^{(n\bar{n})}(x,y) = \sum_r \kappa_r^{(n\bar{n})} O_r^{(n\bar{n})}(x,y) + \text{H.c.} \tag{5.3}$$

We find, for the set $\mathcal{O}_r^{(n\bar{n})}$, the operators

$$\mathcal{O}_1^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}(\epsilon_{i'k'}\epsilon_{j'm'} + \epsilon_{j'k'}\epsilon_{i'm'})(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})[Q_R^{i'\alpha T} C Q_R^{j'\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}](\Delta_R^\dagger)^{r's'}, \tag{5.4}$$

$$\mathcal{O}_2^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{i'j'}\epsilon_{k'm'}(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})[Q_R^{i'\alpha T} C Q_R^{j'\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}](\Delta_R^\dagger)^{r's'}, \tag{5.5}$$

$$\mathcal{O}_3^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{ij}\epsilon_{k'm'}(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_R^{k'\gamma T} C Q_R^{m'\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}](\Delta_R^\dagger)^{r's'}, \tag{5.6}$$

$$\mathcal{O}_4^{(n\bar{n})} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}\epsilon_{ij}\epsilon_{km}(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}](\Delta_R^\dagger)^{r's'}, \tag{5.7}$$

$$\mathcal{O}_5^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}(\epsilon_{ik}\epsilon_{jm} + \epsilon_{jk}\epsilon_{im})(\epsilon_{p'r'}\epsilon_{q's'} + \epsilon_{q'r'}\epsilon_{p's'})[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][Q_R^{p'\rho T} C Q_R^{q'\sigma}](\Delta_R^\dagger)^{r's'} \tag{5.8}$$

where the $SU(3)_c$ color tensors are

$$(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta} \tag{5.9}$$

and

$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}. \tag{5.10}$$

To obtain the six-quark operators that mediate $n - \bar{n}$ transitions, we replace the Δ_R field by its VEV, v_R. To each of these $n - \bar{n}$ transition operators $\mathcal{O}_r^{(n\bar{n})}$ there corresponds an operator $O_r^{(n\bar{n})}$ in $\mathcal{L}_{\text{eff},4+n}^{(Nd)}$. We have

$$\kappa_r^{(n\bar{n})} = \frac{\bar{\kappa}_r^{(n\bar{n})}}{(M_{n\bar{n}})^{6+2n}}. \tag{5.11}$$

To each of these operators there is a corresponding $V_r^{(n\bar{n})}$ function; for example,

$$V_1^{(n\bar{n})} = V_2^{(n\bar{n})} = A^6 \exp[-6\|\eta - \eta_{Q_R}\|^2], \tag{5.12}$$

and so forth for the others. The integrals of these functions over the extra n dimensions comprise two classes. The integration of the $V_r^{(n\bar{n})}$ functions for the operators $\mathcal{O}_r^{(n\bar{n})}$ with $r = 1, 2$ are the same, defining class $C_{1s}^{(n\bar{n})}$, where the subscript s is appended to distinguish this and the other classes from the classes calculated in terms of the $G_{\rm SM}$-based low-energy effective field theory in [15,19]:

$$I_{C_{1s}}^{(n\bar{n})} = b_6 \tag{5.13}$$

where $b_6 = (2 \cdot 3^{-1/2}\pi^{-1}\mu^2)^n$ from the $k = 6$ special case of Eq. (3.18) and $I_{C_k}^{(n\bar{n})} \equiv I_{C_k^{(n\bar{n})}}$. The integrals of the operators $\mathcal{O}_r^{(n\bar{n})}$ with $r = 3, 4, 5$ are equal and yield a second class,

$$I_{C_{2s}}^{(n\bar{n})} = b_6 \exp\left[-\frac{4}{3}\|\eta_{Q_L} - \eta_{Q_R}\|^2\right]. \tag{5.14}$$

From the special case of Eq. (3.19) with $k = 6$, together with Eq. (3.1), it follows that

$$c_r^{(n\bar{n})} = \frac{\bar{\kappa}_r^{(n\bar{n})}}{v_R^5}\left(\frac{2\mu^2}{3^{1/2}\pi v_R^2}\right)^n e^{-S_r^{(n\bar{n})}}, \tag{5.15}$$

where

$$S_r^{(n\bar{n})} = 0 \quad \text{for } r = 1, 2 \tag{5.16}$$

and

$$S_r^{(n\bar{n})} = \frac{4}{3}\|\eta_{Q_L} - \eta_{Q_R}\|^2 \quad \text{for } r = 3, 4, 5. \tag{5.17}$$

An important result from this calculation is that because $S_r^{(n\bar{n})} = 0$ for $r = 1,\ 2$, there is no exponential wave function suppression from the integration over the n extra dimensions for $O_r^{(n\bar{n})}$ with $r = 1,\ 2$.

Then

$$|\delta m| = \frac{1}{v_R^5}\left(\frac{\mu}{v_R}\right)^{2n}\left(\frac{2}{3^{1/2}\pi}\right)^n \times\left|\sum_r \bar{\kappa}_r^{(n\bar{n})} e^{-S_r^{(n\bar{n})}} \langle\bar{n}|\mathcal{O}_r^{(n\bar{n})}|n\rangle\right|. \tag{5.18}$$

The dominant contribution to $|\delta m|$ comes from the operators $\mathcal{O}_r^{(n\bar{n})}$ with $r = 1,\ 2$ (provided that the coefficients $\bar{\kappa}_r^{(n\bar{n})}$ with $r = 1, 2$ are not negligibly small), since $S_r^{(n\bar{n})} = 0$ for $r = 1, 2$, so these operators do not incur any exponential suppression factors from the integration over the extra dimensions. The matrix elements $\langle\bar{n}|\mathcal{O}_r^{(n\bar{n})}|n\rangle$ have dimensions of $(\text{mass})^6$, and since they are determined by hadronic physics, one expects on general grounds that they are $\sim\Lambda_{\text{QCD}}^6$, where, as above, $\Lambda_{\text{QCD}} \simeq 0.25$ GeV. This expectation is confirmed by quantitative calculations [10,11,57]. Taking $\bar{\kappa}_r^{(n\bar{n})} \sim O(1)$ for $r = 1,\ 2$ and the illustrative value $n = 2$ extra dimensions, and requiring that $|\delta m|$ must be less than the experimental upper bound (5.1), we then derive the following lower bound on $M_{n\bar{n}} = v_R$:

$$v_R > (1\times 10^3\ \text{TeV})\left(\frac{\tau_{n\bar{n}}}{2.7\times 10^8\ \text{sec}}\right)^{1/9} \times\left(\frac{\mu}{3\times 10^3\ \text{TeV}}\right)^{4/9}\left(\frac{|\langle\bar{n}|\mathcal{O}_{1,2}^{(n\bar{n})}|n\rangle|}{\Lambda_{\text{QCD}}^6}\right)^{1/9}. \tag{5.19}$$

Thus, our analysis shows that, while it is easy to suppress $\Delta B = -1$ nucleon decay far below observable levels in this model by making the fermion wave function separation distances in Eq. (4.17) sufficiently large, this does not suppress the $|\Delta B| = 2$ $n-\bar{n}$ oscillations, which can occur at a level comparable with current experimental limits. We have used this fact to deduce the lower bound (5.19) on v_R and hence the scale of $|\Delta B| = 2$ baryon-number violation in this model. A similar comment applies to $\Delta B = -2$ dinucleon decays (occurring primarily to multipion final states), since these are induced by the fundamental $n-\bar{n}$ oscillations.

It is of interest to compare our new results for the extra-dimensional LRS model with the results that were previously obtained in Ref. [15] and studied further in [19] for an extra-dimensional model that used a Standard-Model low-energy field theory. A striking feature that is common to both of these types of models is that although one can easily arrange the fermion wave function separation distances to suppress nucleon decays, this does not suppress $n-\bar{n}$ oscillations. A basic difference between the model used in Refs. [15,19] and the present LRS model is that in the SM effective field theory framework of [15,19], baryon number is a global symmetry, while in the LRS model, B and L are gauged via the $\text{U}(1)_{B-L}$ symmetry, and the VEV of the Δ_R field spontaneously breaks B by 2 units in processes for which $\Delta L = 0$. Hence, while the SM Higgs VEV preserves B (and L), here the scale of baryon-number violation is set by v_R, as given in Eq. (3.1). We recall the corresponding limit from Ref. [15] (updated in [19] with the newer limit on $\tau_{\text{m.i.}}$ from the Super-Kamiokande experiment [14]), namely

$$M_{n\bar{n}} > (44\ \text{TeV})\left(\frac{\tau_{n\bar{n}}}{2.7\times 10^8\ \text{sec}}\right)^{1/9} \times\left(\frac{\mu}{3\times 10^3\ \text{TeV}}\right)^{4/9}\left(\frac{|\langle\bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle|}{\Lambda_{\text{QCD}}^6}\right)^{1/9} \quad \text{for SMEFT}. \tag{5.20}$$

The main reason why the lower bound on $M_{n\bar{n}} = v_R$ in Eq. (5.19) is substantially higher than the lower bound on $M_{n\bar{n}}$ in Eq. (5.20) is that all of the integrals of six-quark operators in the extra dimensions in the model of Refs. [15,19] involved exponential suppression factors, whereas, in contrast, here, $S_r^{(n\bar{n})} = 0$ for $r = 1,\ 2$, so the integrals of these operators $O_r^{(n\bar{n})}$ over the extra dimensions do not produce any exponential suppression factors.

VI. CONCLUSIONS

In this paper we have studied $n-\bar{n}$ oscillations in a left-right-symmetric model in which Standard-Model fermions have localized wave functions in extra dimensions. We have shown that in this extra-dimensional LRS model, even with fermion wave function positions chosen so as to render the rates for baryon-violating nucleon decays much smaller than experimental limits, $n-\bar{n}$ oscillations can occur at rates comparable to current bounds. Thus, this feature is common to both the present LRS model and the model with a SM low-energy effective field theory studied in [15,19]. An interesting difference between these models that we find is that certain six-quark operators in the LRS model are not suppressed by exponential factors resulting from the integration over the extra dimensions, in contrast to the SMEFT model of Refs. [15,19], where this integration yields exponential suppression factors for all six-quark operators. These findings provide further motivation for new experimental searches for $n-\bar{n}$ oscillations. In the future, one may look forward to such experiments using a neutron beam at the European Spallation Source [24] and searching for resultant matter instability in the water Cherenkov detector in Hyper-Kamiokande [58] and the liquid argon detector in the Deep Underground Neutrino Experiment, DUNE [59,60].

ACKNOWLEDGMENTS

This research was supported in part by the NSF Grant No. NSF-PHY-1915093.

[1] As analyzed in [2], $SU(2)_L$ instantons in the SM break B and L (conserving $B-L$), but this has a negligibly small effect at temperatures much smaller than the electroweak symmetry breaking scale.
[2] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976).
[3] A. D. Sakharov, Zh. Eksp. Teor. Fiz. Pis'ma **5**, 32 (1967) [JETP Lett. B **91**, 24 (1967)].
[4] V. Kuzmin, Zh. Eksp. Theor. Fiz. Pis'ma **12**, 335 (1970) [JETP Lett. **12**, 228 (1970)].
[5] S. L. Glashow, Report No. HUTP-79/A059.
[6] R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44**, 1316 (1980).
[7] L.-N. Chang and N.-P. Chang, Phys. Lett. **92B**, 103 (1980).
[8] T. K. Kuo and S. Love, Phys. Rev. Lett. **45**, 93 (1980).
[9] R. Cowsik and S. Nussinov, Phys. Lett. **101B**, 237 (1981).
[10] S. Rao and R. E. Shrock, Phys. Lett. **116B**, 238 (1982).
[11] S. Rao and R. E. Shrock, Nucl. Phys. **B232**, 143 (1984).
[12] Particle Data Group, Review of Particle Properties, http://pdg.lbl.gov.
[13] M. Baldo-Ceolin *et al.*, Z. Phys. C **63**, 409 (1994).
[14] K. Abe *et al.* (SuperKamiokande Collaboration), Phys. Rev. D **91**, 072006 (2015).
[15] S. Nussinov and R. Shrock, Phys. Rev. Lett. **88**, 171601 (2002).
[16] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61**, 033005 (2000).
[17] E. A. Mirabelli and M. Schmaltz, Phys. Rev. D **61**, 113011 (2000).
[18] J. M. Arnold, B. Fornal, and M. B. Wise, Phys. Rev. D **87**, 075004 (2013).
[19] S. Girmohanta and R. Shrock, Phys. Rev. D **101**, 015017 (2020).
[20] S. Girmohanta and R. Shrock, Phys. Rev. D **100**, 115025 (2019).
[21] S. Girmohanta and R. Shrock, Phys. Lett. B **803**, 135296 (2020).
[22] S. Girmohanta, S. Nussinov, and R. Shrock (to be published).
[23] R. N. Mohapatra, J. Phys. G **36**, 104006 (2009).
[24] D. G. Phillips *et al.*, Phys. Rep. **612**, 1 (2016).
[25] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 2558 (1975).
[26] G. Senjanović and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975).
[27] R. N. Mohapatra and G. Senjanović, Phys. Rev. D **23**, 165 (1981).
[28] J. F. Gunion, J. Grifols, A. Mendez, B. Kayser, and F. I. Olness, Phys. Rev. D **40**, 1546 (1989).
[29] N. G. Deshpande, J. F. Gunion, B. Kayser, and F. I. Olness, Phys. Rev. D **44**, 837 (1991).
[30] G. Barenboim, M. Gorbahn, U. Nierste, and M. Raidal, Phys. Rev. D **65**, 095003 (2002).
[31] A. Maiezza, G. Senjanović, and J. C. Vasquez, Phys. Rev. D **95**, 095004 (2017).
[32] P. S. Bhupal Dev, R. N. Mohapatra, W. Rodejohann, and X.-J. Xu, J. High Energy Phys. 02 (2019) 154.
[33] T. Appelquist and R. Shrock, Phys. Rev. Lett. **90**, 201801 (2003).
[34] N. C. Christensen and R. Shrock, Phys. Rev. Lett. **94**, 241801 (2005).
[35] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).
[36] K. S. Babu and R. N. Mohapatra, Phys. Lett. B **668**, 404 (2008).
[37] V. Rubakov and M. Shaposhnikov, Phys. Lett. **125B**, 136 (1983).
[38] D. Kaplan and M. Schmaltz, Phys. Lett. B **368**, 44 (1996).
[39] G. Dvali and M. Shifman, Phys. Lett. B **475**, 295 (2000).
[40] V. A. Rubakov, Usp. Fiz. Nauk **171**, 913 (2001) [Phys. Usp. **44**, 871 (2001)].
[41] Y. Grossman and G. Perez, Phys. Rev. D **67**, 015011 (2003).
[42] Z. Surujon, Phys. Rev. D **73**, 016008 (2006).
[43] D. P. George and R. R. Volkas, Phys. Rev. D **75**, 105007 (2007).
[44] S. Nussinov and R. Shrock, Phys. Lett. B **526**, 137 (2002).
[45] A. Delgado, A. Pomarol, and M. Quiros, J. High Energy Phys. 01 (2000) 030.
[46] T. Appelquist, H.-C. Cheng, and B. Dobrescu, Phys. Rev. D **64**, 035002 (2001).
[47] T. Appelquist, B. Dobrescu, E. Ponton, and H.-U. Yee, Phys. Rev. Lett. **87**, 181802 (2001).
[48] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).
[49] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **436**, 257 (1998).
[50] K. Dienes, E. Dudas, and T. Gherghetta, Phys. Lett. B **436**, 55 (1998).
[51] S. Nussinov and R. Shrock, Phys. Rev. D **59**, 105002 (1999).
[52] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[53] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[54] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).
[55] K. Abe *et al.* (Super-Kamiokande Collaboration), Phys. Rev. D **95**, 012004 (2017).
[56] K. Abe *et al.* (Super-Kamiokande Collaboration), Phys. Rev. D **96**, 012003 (2017).
[57] E. Rinaldi *et al.*, Phys. Rev. D **99**, 074510 (2019).
[58] K. Abe *et al.*, arXiv:1109.3262; arXiv:1805.04163; http://hyperk.org.
[59] B. Abi *et al.*, arXiv:1807.10334; arXiv:2002.03005.
[60] J. L. Barrow, E. S. Golubeva, E. Paryev, and J.-M. Richard, Phys. Rev. D **101**, 036008 (2020).