

Effects of a parallel electromagnetic field in the three-flavor Nambu–Jona-Lasinio model

Gaoqing Cao *School of Physics and Astronomy, Sun Yat-sen University, Guangzhou 510275, China*

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In this work, we explore the ground state of the QCD system and the relevant collective modes in a parallel electromagnetic (EM) field within the effective three-flavor Nambu–Jona-Lasinio model. From the features of neutral chiral condensates, three critical EM fields are identified in our study: $e\bar{I}_2^{c_i}$ ($i = 1, 2, 3$). Moreover, competition between QCD and QED anomalies is found: the negativeness of π_d^* and π_s^* is a signal of QCD anomaly dominance for all of the flavors, and the positiveness of π_s^* beyond $e\bar{I}_2^{c_1}$ indicates QED anomaly dominance for the strange quark. For the lowest-lying neutral collective modes, the masses of Π and H modes reduce to zero around the critical EM fields and the scalar-pseudoscalar components are exchanged between Σ and H modes, as can be seen in the crossing structure.

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I. INTRODUCTION

In recent years, a lot of scientific interest has focused on the properties of QCD systems under extreme conditions, including those existing in certain types of astrophysical objects and those created in relativistic heavy-ion collisions. Large baryon densities (of the same order as the saturation density $\rho_0 = 0.16 \text{ fm}^{-3}$) are expected to occur in neutron stars, especially in the inner cores. Pasta structures [1,2], chiral symmetry restoration [3,4], partial-wave nucleon pairings [5,6], and color superconductivity [7,8] have been extensively studied in such circumstances. These explorations mainly focused on the equilibrium ground states of the QCD system, which are related to the equation of state (EoS). Actually, there is some ambiguity in the determination of the EoS, but some new insights have been gained from recent multimessenger observations of binary neutron star mergers, such as GW170817 [9]. As a matter of fact, the detection of the associated gravitational wave (GW) constrains the tidal deformabilities and radii of neutron stars to $\tilde{\Lambda} < 800$ and $R = 8.9\text{--}13.2 \text{ km}$ [10,11], and the combined optical and GW observations of GW170817 constrain the maximum mass of neutron stars to $2.16\text{--}2.28 M_\odot$ [12,13] and the EoS is found to be slightly soft at large baryon densities, $\rho > 5\rho_0$ [14,15].

In the large terrestrial facilities, very strong electromagnetic (EM) fields ($10^{18}\text{--}10^{20} \text{ G}$) [16–19] and fast rotation ($\sim 10^{22} \text{ s}^{-1}$) [20] can be produced in peripheral heavy-ion collisions (HICs), and the chiral magnetic effect (CME) [21–24] and particle polarization effect [25–35] are hot topics for both theoretical and experimental studies. Besides, the effects of an EM field and rotation on QCD ground states and the associated extended phase diagrams have been widely studied and several intriguing phenomena

were discovered, such as the inverse magnetic catalysis effect [36,37], anomalous neutral pion superfluidity [38–41], magnetic anisotropic confinement [42–44], and rotational magnetic inhibition [45]. Directly related to the ground states, neutral and charged meson masses have also been studied in different scenarios [46–54]. Take two important discoveries, for example: the charged rho meson mass never reduces to zero in a pure magnetic field according to the lattice QCD simulations [46–48], but the charged pion meson mass might vanish under the conditions of a parallel magnetic field and rotation [55–57].

Among all of the circumstances explored, the case with a parallel EM (PEM) field is special because neutral pseudoscalar condensations can be induced through a chiral anomaly [38–41]. On the experimental side, a PEM field was used to produce chirality imbalance in a Weyl semimetal system, and the well-known CME was discovered for the first time [58]. Besides, it was found that the observed charge-dependent elliptic flow of pions might be understood from the PEM field distribution in HICs [59]. The present work is just the three-flavor version of our previous researches [38–41]: the advantages are that this case is more realistic and the QCD $U_A(1)$ anomaly is automatically accounted for through the 't Hooft determinant [3,4]. With regards to the latter point, the relative significances and effects of QCD and QED anomalies can be well compared in the three-flavor case. As PEM fields are indeed relevant in HICs [59], we hope that in the future more attention will be paid to the possibility of anomalous pseudoscalar condensations in experiments in addition to the CME signals.

The paper is organized as follows. In Sec. II we present the whole formalism for the studies of chiral condensates and collective modes, with gap equations in Sec. II A and

polarization functions in Sec. II B. The numerical results are shown in Sec. III, and finally we conclude in Sec. IV.

II. THREE-FLAVOR FORMALISM

A. Gap equations

In order to improve upon previous studies [38–41] and study the QCD system more realistically, we adopt the three-flavor Nambu–Jona-Lasinio (NJL) model where more low-lying collective modes are involved and the QCD $U_A(1)$ anomaly has been taken into account automatically through the 't Hooft term. The corresponding Lagrangian density is [3,4]

$$\begin{aligned}\mathcal{L}_{\text{NJL}} &= \bar{\psi}(i\not{D} - m_0)\psi + G \sum_{a=0}^8 [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] + \mathcal{L}_{\text{tH}}, \\ \mathcal{L}_{\text{tH}} &= -K \sum_{s=\pm} \det \bar{\psi} \Gamma^s \psi,\end{aligned}\quad (1)$$

where $\psi = (u, d, s)^T$ represents the three-flavor quark field and \mathcal{L}_{tH} is the 't Hooft term with coupling K . In the kinetic term, $m_0 = \text{diag}(m_{0u}, m_{0d}, m_{0s})$ is the current quark mass matrix and

$$D_\mu = \partial_\mu + iQA_\mu \quad (2)$$

is the covariant derivative, with A_μ representing a PEM field and the charge matrix

$$Q = \text{diag}(q_u, q_d, q_s) = \text{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)e.$$

Without loss of generality, we choose the gauge in Euclidean space as $A_\mu = (iEz, 0, Bx, 0)$. In the four-quark interaction terms with coupling G , $\lambda^0 = \sqrt{2/3}\text{diag}(1, 1, 1)$ and $\lambda^i (i = 1, \dots, 8)$ are Gell-Mann matrices in flavor space. Finally, in the 't Hooft term, the determinant is also performed in flavor space and the interaction vertices are $\Gamma^\pm = 1 \pm \gamma_5$ for right- and left-handed channels, respectively.

In the setup $\mathbf{B} \cdot \mathbf{E} \neq 0$, we already found π^0 and η^0 condensations in two-flavor NJL models due to a chiral anomaly [38,39]. With the additional inclusion of strangeness, another neutral pseudoscalar field—the η^8 meson—should also be expected to condense in the same setup, though the effect might be small. Thus, for a consistent exploration, we should set $\pi_f^5 = \langle \bar{\psi}_f i\gamma^5 \psi_f \rangle / N_c$ to be non-zero besides the scalar condensates $\sigma_f = \langle \bar{\psi}_f \psi_f \rangle / N_c$, with $f = u, d, s$. To facilitate the study, we would first like to reduce \mathcal{L}_{tH} to an effective form of four-fermion interactions in the Hartree approximation. By following the derivations in Ref. [3] where a quark-antiquark pair is contracted in each six-fermion interaction term, we immediately find

$$\begin{aligned}\mathcal{L}_{\text{tH}}^4 &= -\frac{K}{2} \sum_{s=\pm} \epsilon_{ijk} \epsilon_{imn} \langle \bar{\psi}^i \Gamma^s \psi^i \rangle \langle \bar{\psi}^j \Gamma^s \psi^m \rangle \langle \bar{\psi}^k \Gamma^s \psi^n \rangle \\ &= -N_c K \epsilon_{ijk} \epsilon_{imn} \{ \sigma_i [(\bar{\psi}^j \psi^m)(\bar{\psi}^k \psi^n) - (\bar{\psi}^j i\gamma^5 \psi^m)(\bar{\psi}^k i\gamma^5 \psi^n)] - 2\pi_i^5 (\bar{\psi}^j i\gamma^5 \psi^m)(\bar{\psi}^k \psi^n) \} \\ &= -N_c \frac{K}{6} \left\{ 2 \sum_{i=1}^3 \sigma_i (\bar{\psi} \lambda^0 \psi)^2 - 3\sigma_s \sum_{i=1}^3 (\bar{\psi} \lambda^i \psi)^2 - 3\sigma_d \sum_{i=4}^5 (\bar{\psi} \lambda^i \psi)^2 - 3\sigma_u \sum_{i=6}^7 (\bar{\psi} \lambda^i \psi)^2 \right. \\ &\quad \left. - \sum_{i=1}^3 \sigma_i \left[\frac{(\bar{\psi} \lambda^8 \psi)^2}{\sqrt{3}/2\lambda_{ii}^8} + \sqrt{6}\lambda_{ii}^8 (\bar{\psi} \lambda^0 \psi)(\bar{\psi} \lambda^8 \psi) + \sqrt{6}\lambda_{ii}^3 (\bar{\psi} \lambda^3 \psi)(\bar{\psi} \lambda^0 \psi - \sqrt{2}\bar{\psi} \lambda^8 \psi) \right] - (\lambda^a \rightarrow i\lambda^a \gamma^5) \right\} \\ &\quad + N_c \frac{K}{3} \left\{ 2 \sum_{i=1}^3 \pi_i^5 (\bar{\psi} \lambda^0 \psi)(\bar{\psi} i\lambda^0 \gamma^5 \psi) - 3\pi_s^5 \sum_{i=1}^3 (\bar{\psi} \lambda^i \psi)(\bar{\psi} i\lambda^i \gamma^5 \psi) - 3\pi_d^5 \sum_{i=4}^5 (\bar{\psi} \lambda^i \psi)(\bar{\psi} i\lambda^i \gamma^5 \psi) \right. \\ &\quad \left. - 3\pi_u^5 \sum_{i=6}^7 (\bar{\psi} \lambda^i \psi)(\bar{\psi} i\lambda^i \gamma^5 \psi) - \sum_{i=1}^3 \pi_i^5 \left[\frac{(\bar{\psi} \lambda^8 \psi)(\bar{\psi} i\lambda^8 \gamma^5 \psi)}{\sqrt{3}/2\lambda_{ii}^8} + \frac{\sqrt{6}}{2} \lambda_{ii}^8 ((\bar{\psi} \lambda^0 \psi)(\bar{\psi} i\lambda^8 \gamma^5 \psi) + (\bar{\psi} \lambda^8 \psi)(\bar{\psi} i\lambda^0 \gamma^5 \psi)) \right. \right. \\ &\quad \left. \left. + \frac{\sqrt{6}}{2} \lambda_{ii}^3 ((\bar{\psi} \lambda^3 \psi)(\bar{\psi} i\lambda^0 \gamma^5 \psi - \sqrt{2}\bar{\psi} i\lambda^8 \gamma^5 \psi) + (\bar{\psi} i\lambda^3 \gamma^5 \psi)(\bar{\psi} \lambda^0 \psi - \sqrt{2}\bar{\psi} \lambda^8 \psi)) \right] \right\},\end{aligned}\quad (3)$$

where ϵ_{ijk} is the Levi-Civita symbol with $\epsilon_{123} = 1$ and we used the Einstein summation convention for the flavor indices i, j, k, m, n in the first two steps. The correspondences between 1,2,3 and u, d, s should be understood for the subscripts here and throughout the rest of the paper. Substituting \mathcal{L}_{tH} in Eq. (1) with $\mathcal{L}_{\text{tH}}^4$, the effective Lagrangian density with only four-fermion interactions is

$$\begin{aligned} \mathcal{L}_{\text{NJL}}^4 = & \bar{\psi}(i\not{D} - m_0)\psi + \sum_{a=0}^8 \{G_{aa}^- (\bar{\psi}\lambda^a\psi)^2 + G_{aa}^+ (\bar{\psi}i\gamma_5\lambda^a\psi)^2 + G_{aa}^5 [(\bar{\psi}\lambda^a\psi)(\bar{\psi}i\lambda^a\gamma_5\psi) + (\bar{\psi}i\lambda^a\gamma_5\psi)(\bar{\psi}\lambda^a\psi)]\} \\ & + \sum_{a,b=0,3,8}^{a\neq b} \{G_{ab}^- (\bar{\psi}\lambda^a\psi)(\bar{\psi}\lambda^b\psi) + G_{ab}^+ (\bar{\psi}i\lambda^a\gamma_5\psi)(\bar{\psi}i\lambda^b\gamma_5\psi) + G_{ab}^5 [(\bar{\psi}\lambda^a\psi)(\bar{\psi}i\lambda^b\gamma_5\psi) + (\bar{\psi}i\lambda^a\gamma_5\psi)(\bar{\psi}\lambda^b\psi)]\}, \quad (4) \end{aligned}$$

where the effective and symmetric couplings for the pure scalar-pseudoscalar channels G_{ab}^\mp and the mixing ones G_{ab}^5 are, respectively,

$$\begin{aligned} G_{00}^\mp &= G \mp N_c \frac{K}{3} \sum_{i=1}^3 \sigma_i, & G_{11}^\mp &= G_{22}^\mp = G_{33}^\mp = G \pm N_c \frac{K}{2} \sigma_s, & G_{44}^\mp &= G_{55}^\mp = G \pm N_c \frac{K}{2} \sigma_d, & G_{66}^\mp &= G_{77}^\mp = G \pm N_c \frac{K}{2} \sigma_u, \\ G_{88}^\mp &= G \mp N_c \frac{K}{6} (\sigma_s - 2\sigma_u - 2\sigma_d), & G_{08}^\mp &= \mp N_c \frac{\sqrt{2}K}{12} (2\sigma_s - \sigma_u - \sigma_d), & G_{38}^\mp &= -\sqrt{2}G_{03}^\mp = \mp N_c \frac{\sqrt{3}K}{6} (\sigma_u - \sigma_d), \\ G_{00}^5 &= N_c \frac{K}{3} \sum_{i=1}^3 \pi_i^5, & G_{11}^5 &= G_{22}^5 = G_{33}^5 = -N_c \frac{K}{2} \pi_s^5, & G_{44}^5 &= G_{55}^5 = -N_c \frac{K}{2} \pi_d^5, & G_{66}^5 &= G_{77}^5 = -N_c \frac{K}{2} \pi_u^5, \\ G_{88}^5 &= N_c \frac{K}{6} (\pi_s^5 - 2\pi_u^5 - 2\pi_d^5), & G_{08}^5 &= N_c \frac{\sqrt{2}K}{12} (2\pi_s^5 - \pi_u^5 - \pi_d^5), & G_{38}^5 &= -\sqrt{2}G_{03}^5 = N_c \frac{\sqrt{3}K}{6} (\pi_u^5 - \pi_d^5). \end{aligned} \quad (5)$$

Notice that pseudoscalar condensates further develop couplings between the scalar channels (mediated by σ, a_0, a_8 mesons) and pseudoscalar channels (mediated by π^0, η_0, η_8 mesons), and thus these isospin-parity eigenstates will mix with each other in the mass eigenstates.

Armed with the reduced Lagrangian density, all of the necessary analytic derivations are just parallel to those given in two-flavor NJL models [38,39]. By contracting a quark-antiquark pair further in each isospin-diagonal interaction term of Eq. (4), the effective mass and pion condensate of each quark flavor are, respectively,

$$\begin{aligned} m_i^* &= m_{0i} - 4N_c G \sigma_i + 2N_c^2 K (\sigma_j \sigma_k - \pi_j^5 \pi_k^5), \\ \pi_i^* &= -4N_c G \pi_i^5 - 2N_c^2 K (\sigma_j \pi_k^5 + \pi_j^5 \sigma_k), \end{aligned} \quad (6)$$

with $i \neq j \neq k$. The G - and K -dependent terms in Eq. (6) correspond to the $U_A(1)$ -symmetric and anomalous violating interactions, respectively. Then, the gap equations

are given through the following six self-consistent conditions [3]:

$$\sigma_i = \langle \bar{\psi}^i \psi^i \rangle / N_c = -\text{tr} S_i, \quad (7)$$

$$\pi_i^5 = \langle \bar{\psi}^i i\gamma_5 \psi^i \rangle / N_c = -\text{tri} \gamma_5 S_i, \quad (8)$$

where $S_i(x) = -[i\not{D}_i - m_i^* - i\gamma_5 \pi_i^*]^{-1}$ is the effective propagator of a given quark flavor in the PEM field and the trace should be taken over the spinor and coordinate spaces. With only neutral condensates involved, the propagators can be evaluated independently for quarks with different flavors or colors. In principle, we should work in the in-in formalism when an electric field is involved [40,60]; here, we simply adopt the simpler in-out formalism to avoid numerical difficulties.

In Euclidean space, the effective quark propagator has already been evaluated in momentum space as [61]

$$\begin{aligned} \hat{S}_f(p) = & i \int_0^\infty ds \exp \left\{ -iM_f^{*2} s - \frac{i \tanh(q_f \bar{I}_2 s)}{q_f \bar{I}_2} (p_4^2 + p_3^2) - \frac{i \tan(q_f \bar{I}_2 s)}{q_f \bar{I}_2} (p_1^2 + p_2^2) \right\} [m_f^* - i\gamma_5 \pi_f^* - \gamma^4 (p_4 - i \tanh(q_f \bar{I}_2 s) p_3) \\ & - \gamma^3 (p_3 + i \tanh(q_f \bar{I}_2 s) p_4) - \gamma^2 (p_2 + \tan(q_f \bar{I}_2 s) p_1) - \gamma^1 (p_1 - \tan(q_f \bar{I}_2 s) p_2)] \\ & \times [1 + i\gamma_5 \tan(q_f \bar{I}_2 s) \tanh(q_f \bar{I}_2 s) + \gamma^1 \gamma^2 \tan(q_f \bar{I}_2 s) + i\gamma^4 \gamma^3 \tanh(q_f \bar{I}_2 s)]. \end{aligned} \quad (9)$$

Here, we define the chiral mass $M_f^* = (m_f^{*2} + \pi_f^{*2})^{1/2}$ and the field strength is chosen to be $E = B = \bar{I}_2$ without loss of generality. Inserting it into Eqs. (7) and (8) and transforming the integral variable $s \rightarrow -is$, the explicit forms of the gap equations are

$$-\sigma_f = \frac{m_f^*}{4\pi^2} \int_0^\infty ds \frac{(q_f \bar{I}_2 s)^2 e^{-M_f^{*2} s}}{s^2 \tan(q_f \bar{I}_2 s) \tanh(q_f \bar{I}_2 s)} - \frac{q_f^2 \bar{I}_2}{4\pi^2} \frac{\pi_f^*}{M_f^{*2}}, \quad (10)$$

$$-\pi_f^5 = \frac{\pi_f^*}{4\pi^2} \int_0^\infty ds \frac{(q_f \bar{I}_2 s)^2 e^{-M_f^{*2} s}}{s^2 \tan(q_f \bar{I}_2 s) \tanh(q_f \bar{I}_2 s)} + \frac{q_f^2 \bar{I}_2}{4\pi^2} \frac{m_f^*}{M_f^{*2}}, \quad (11)$$

where $I_2 = \mathbf{E} \cdot \mathbf{B}$ is the second Lorentz invariant. These forms are divergent, so we would like to use the vacuum regularization scheme [62] to proceed to numerical evaluations; then, the gap equations become

$$-\sigma_f = m_f^* F_\Lambda(M_f^*) - \frac{q_f^2 I_2}{4\pi^2} \frac{\pi_f^*}{M_f^{*2}}, \quad (12)$$

$$-\pi_f^5 = \pi_f^* F_\Lambda(M_f^*) + \frac{q_f^2 I_2}{4\pi^2} \frac{m_f^*}{M_f^{*2}}, \quad (13)$$

with the auxiliary function defined as

$$F_\Lambda(M_f^*) = \frac{M_f^*}{2\pi^2} \left[\Lambda \left(1 + \frac{\Lambda^2}{M_f^{*2}} \right)^{\frac{1}{2}} - M_f^* \ln \left(\frac{\Lambda}{M_f^*} + \left(1 + \frac{\Lambda^2}{M_f^{*2}} \right)^{\frac{1}{2}} \right) \right] + \frac{1}{4\pi^2} \int_0^\infty \frac{ds}{s^2} e^{-M_f^{*2}s} \left[\frac{(q_f \bar{I}_2 s)^2}{\tan(q_f \bar{I}_2 s) \tanh(q_f \bar{I}_2 s)} - 1 \right].$$

Getting rid of the cutoff terms, model-independent results follow from Eqs. (12) and (13):

$$\sigma_f \pi_f^* - \pi_f^5 m_f^* = \frac{q_f^2 I_2}{4\pi^2}. \quad (14)$$

The two-flavor results in Ref. [39] can then be reproduced by setting $K = 0$.

Finally, the overall thermodynamic potential can be derived consistently as

$$\begin{aligned} \Omega = 2N_c G \sum_{f=u,d,s} (\sigma_f^2 + (\pi_f^5)^2) - 2N_c^2 K \left(2 \prod_{f=u,d,s} \sigma_f - \epsilon_{ijk}^2 \sigma_i \pi_j^5 \pi_k^5 \right) - \sum_{f=u,d,s} \left\{ \frac{M_f^{*3}}{8\pi^2} \left[\Lambda \left(1 + \frac{2\Lambda^2}{M_f^{*2}} \right) \left(1 + \frac{\Lambda^2}{M_f^{*2}} \right)^{\frac{1}{2}} \right. \right. \\ \left. \left. - M_f^* \ln \left(\frac{\Lambda}{M_f^*} + \left(1 + \frac{\Lambda^2}{M_f^{*2}} \right)^{\frac{1}{2}} \right) \right] + \frac{1}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-M_f^{*2}s} \left[\frac{(q_f \bar{I}_2 s)^2}{\tan(q_f \bar{I}_2 s) \tanh(q_f \bar{I}_2 s)} - 1 \right] - \frac{q_f^2 I_2}{4\pi^2} \tan^{-1} \left(\frac{\pi_f^*}{m_f^*} \right) \right\} \end{aligned} \quad (15)$$

by combining the integrations over m_f^* of Eq. (12) and the integrations over π_f^* of Eq. (13). The expression $\tan^{-1}(\frac{\pi_f^*}{m_f^*})$ in the last term of Eq. (15) represents chiral angles for different flavors [41] and would change randomly beyond the ends of corresponding chiral rotations [40]. Since the quark mass m_f^* is usually positive before the completion of chiral rotation, this term indicates that a pion condensate π_f^* always prefers the same sign as I_2 if the QED anomaly dominates over the QCD one, regardless of the charge. This has already been well checked in the two-flavor $U_A(1)$ -symmetric NJL model [39].

B. Polarization functions

Now we focus on the collective modes, especially the neutral scalars σ, a_0, a_8 and pseudoscalars π^0, η_0, η_8 which mix with each other. In the well-known random phase approximation [3], the kinetic terms of and mixings among these mesons are completely determined by the 6×6 polarization function matrix, which can be more conveniently derived from the reduced Lagrangian density (4). Actually, only 21 elements of the matrix are independent due to the transpose symmetry. To obtain these functions, we first evaluate the traces over spinor space for each flavor,

$$\begin{aligned} T_f^- &\equiv -\text{tr} \hat{S}_f(p+q/2) \hat{S}_f(p-q/2) \\ &= 4 \int_0^\infty ds \int_0^\infty ds' \exp \left\{ -iM_f^{*2}(s+s') - \frac{ith}{q_f \bar{I}_2} (p_4^{+2} + p_3^{+2}) - \frac{it}{q_f \bar{I}_2} (p_1^{+2} + p_2^{+2}) - \frac{ith'}{q_f \bar{I}_2} (p_4^{-2} + p_3^{-2}) - \frac{it'}{q_f \bar{I}_2} (p_1^{-2} + p_2^{-2}) \right\} \\ &\quad \times [(m_f^{*2} - \pi_f^{*2})(1-tt')(1+\text{th th}') + 2m_f^* \pi_f^*(t+t')(\text{th} + \text{th}') - (P_4^+ P_4^- + P_3^+ P_3^-)(1-tt')(1-\text{th th}') \\ &\quad + i(P_4^+ P_3^- - P_3^+ P_4^-)(1-tt')(\text{th} - \text{th}') - (P_2^+ P_2^- + P_1^+ P_1^-)(1+tt')(1+\text{th th}') \\ &\quad + (P_1^+ P_2^- - P_2^+ P_1^-)(t-t')(1+\text{th th}')], \end{aligned} \quad (16)$$

$$\begin{aligned}
T_f^+ &\equiv -\text{tr}[i\gamma^5 \hat{S}_f(p+q/2) i\gamma^5 \hat{S}_f(p-q/2)] \\
&= 4 \int_0^\infty ds \int_0^\infty ds' \exp \left\{ -iM_f^{*2}(s+s') - \frac{i\text{th}}{q_f \bar{I}_2} (p_4^{+2} + p_3^{+2}) - \frac{it}{q_f \bar{I}_2} (p_1^{+2} + p_2^{+2}) - \frac{i\text{th}'}{q_f \bar{I}_2} (p_4^{-2} + p_3^{-2}) - \frac{it'}{q_f \bar{I}_2} (p_1^{-2} + p_2^{-2}) \right\} \\
&\quad \times [-(m_f^{*2} - \pi_f^{*2})(1-tt')(1+\text{thth}') - 2m_f^* \pi_f^* (t+t')(\text{th}+\text{th}') - (P_4^+ P_4^- + P_3^+ P_3^-)(1-tt')(1-\text{thth}') \\
&\quad + i(P_4^+ P_3^- - P_3^+ P_4^-)(1-tt')(\text{th}-\text{th}') - (P_2^+ P_2^- + P_1^+ P_1^-)(1+tt')(1+\text{thth}') + (P_1^+ P_2^- - P_2^+ P_1^-)(t-t')(1+\text{thth}')], \quad (17)
\end{aligned}$$

$$\begin{aligned}
T_f^5 &\equiv -\text{tr}[\hat{S}_f(p+q/2) i\gamma^5 \hat{S}_f(p-q/2)] = -\text{tr}[i\gamma^5 \hat{S}_f(p-q/2) \hat{S}_f(p+q/2)] \\
&= 4 \int_0^\infty ds \int_0^\infty ds' \exp \left\{ -iM_f^{*2}(s+s') - \frac{i\text{th}}{q_f \bar{I}_2} (p_4^{+2} + p_3^{+2}) - \frac{it}{q_f \bar{I}_2} (p_1^{+2} + p_2^{+2}) - \frac{i\text{th}'}{q_f \bar{I}_2} (p_4^{-2} + p_3^{-2}) - \frac{it'}{q_f \bar{I}_2} (p_1^{-2} + p_2^{-2}) \right\} \\
&\quad \times [2m_f^* \pi_f^* (1-tt')(1+\text{thth}') - (m_f^{*2} - \pi_f^{*2})(t+t')(\text{th}+\text{th}') + (P_4^+ P_4^- + P_3^+ P_3^-)(t+t')(\text{th}-\text{th}') \\
&\quad - i(P_4^+ P_3^- - P_3^+ P_4^-)(t+t')(1-\text{thth}') + (P_2^+ P_2^- + P_1^+ P_1^-)(t-t')(\text{th}+\text{th}') + (P_1^+ P_2^- - P_2^+ P_1^-)(1+tt')(\text{th}+\text{th}')], \quad (18)
\end{aligned}$$

where the following denotations should be understood:

$$\begin{aligned}
t &= \tan(q_f \bar{I}_2 s), \quad \text{th} = \tanh(q_f \bar{I}_2 s), \quad t' = \tan(q_f \bar{I}_2 s'), \quad \text{th}' = \tanh(q_f \bar{I}_2 s'), \quad p_\mu^\pm = p_\mu \pm q_\mu/2, \\
P_4^+ &= p_4^+ - ip_3^+ \text{th}, \quad P_3^+ = p_3^+ + ip_4^+ \text{th}, \quad P_2^+ = p_2^+ + p_1^+ t, \quad P_1^+ = p_1^+ - p_2^+ t, \\
P_4^- &= p_4^- - ip_3^- \text{th}', \quad P_3^- = p_3^- + ip_4^- \text{th}', \quad P_2^- = p_2^- + p_1^- t', \quad P_1^- = p_1^- - p_2^- t'.
\end{aligned}$$

For the evaluations of pole masses, we set the three-momentum $\mathbf{q} = \mathbf{0}$; then, the four-momentum integrated forms of the trace functions $\Pi_f^n(q_4) \equiv -\int \frac{d^4 p}{(2\pi)^4} T_f^n(p, q_4) (n = \pm, 5)$ are, respectively,

$$\begin{aligned}
\Pi_f^\mp(q_4) &= \frac{1}{4\pi^2} \int_0^\infty ds \int_0^\infty ds' \frac{q_f \bar{I}_2}{\text{th} + \text{th}'} \frac{q_f \bar{I}_2}{t + t'} e^{-iM_f^{*2}(s+s') + \frac{i(\text{th}-\text{th}')^2 q_4^2}{q_f \bar{I}_2 (\text{th}+\text{th}')^4}} \\
&\quad \times \left[\pm (m_f^{*2} - \pi_f^{*2})(1-tt')(1+\text{thth}') \pm 2m_f^* \pi_f^* (t+t')(\text{th}+\text{th}') \right. \\
&\quad \left. - \left(\frac{q_f \bar{I}_2}{i(\text{th}+\text{th}')} - \frac{\text{thth}'}{(\text{th}+\text{th}')^2} q_4^2 \right) (1-tt')(1-\text{th}^2)(1-\text{th}'^2) - \frac{q_f \bar{I}_2}{i(t+t')} (1+\text{thth}')(1+t^2)(1+t'^2) \right], \quad (19)
\end{aligned}$$

$$\begin{aligned}
\Pi_f^5(q_4) &= \frac{1}{4\pi^2} \int_0^\infty ds \int_0^\infty ds' \frac{q_f \bar{I}_2}{\text{th} + \text{th}'} \frac{q_f \bar{I}_2}{t + t'} e^{-iM_f^{*2}(s+s') + \frac{i(\text{th}-\text{th}')^2 q_4^2}{q_f \bar{I}_2 (\text{th}+\text{th}')^4}} [2m_f^* \pi_f^* (1-tt')(1+\text{thth}') - (m_f^{*2} - \pi_f^{*2})(t+t')(\text{th}+\text{th}')]. \quad (20)
\end{aligned}$$

Transforming the integral variables as $s \rightarrow s(1+u)/2$, $s' \rightarrow s(1-u)/2$ and regularizing the functions Π_f^n with the help of the corresponding terms in the limit $\bar{I}_2 \rightarrow 0$, we find [40]

$$\begin{aligned}
\Pi_{\Lambda f}^\mp(q_4) &= \frac{q_f^2 \bar{I}_2}{8\pi^2} \int_0^\infty s ds \int_{-1}^1 du e^{-iM_f^{*2}s + \frac{i(\bar{\text{th}}_f - \bar{\text{th}}_f')^2 q_4^2}{q_f \bar{I}_2 (\bar{\text{th}}_f + \bar{\text{th}}_f')^4}} \left[\frac{\pm (m_f^{*2} - \pi_f^{*2})}{\tan(q_f \bar{I}_2 s) \tanh(q_f \bar{I}_2 s)} \pm 2m_f^* \pi_f^* - \frac{q_f \bar{I}_2 (1 - \frac{i\bar{\text{th}}_f \bar{\text{th}}_f'}{q_f \bar{I}_2 (\bar{\text{th}}_f + \bar{\text{th}}_f')}) q_4^2}{i \tan(q_f \bar{I}_2 s) \sinh^2(q_f \bar{I}_2 s)} \right. \\
&\quad \left. - \frac{q_f \bar{I}_2}{i \tanh(q_f \bar{I}_2 s) \sin^2(q_f \bar{I}_2 s)} \right] - (\bar{I}_2 \rightarrow 0) - 8 \int^\Lambda \frac{d^3 p}{(2\pi)^3} \frac{E_f^2(p) - \frac{1}{2} [M_f^{*2} \pm (m_f^{*2} - \pi_f^{*2})]}{E_f(p) (q_4^2 + 4E_f^2(p))}, \quad (21)
\end{aligned}$$

$$\begin{aligned}
\Pi_{\Lambda f}^5(q_4) &= \frac{q_f^2 \bar{I}_2}{8\pi^2} \int_0^\infty s ds \int_{-1}^1 du e^{-iM_f^{*2}s + \frac{i(\bar{\text{th}}_f - \bar{\text{th}}_f')^2 q_4^2}{q_f \bar{I}_2 (\bar{\text{th}}_f + \bar{\text{th}}_f')^4}} \left[\frac{2m_f^* \pi_f^*}{\tan(q_f \bar{I}_2 s) \tanh(q_f \bar{I}_2 s)} - (m_f^{*2} - \pi_f^{*2}) \right] \\
&\quad - (\bar{I}_2 \rightarrow 0) + 8 \int^\Lambda \frac{d^3 p}{(2\pi)^3} \frac{m_f^* \pi_f^*}{E_f(p) (q_4^2 + 4E_f^2(p))}, \quad (22)
\end{aligned}$$

where the dispersion relationship in the cutoff terms is $E_f(p) = (p^2 + M_f^{*2})^{1/2}$ and we define $\bar{t}_f = \tanh((1+u)q_f\bar{I}_2s/2)$ and $\bar{t}'_f = \tanh((1-u)q_f\bar{I}_2s/2)$ for brevity. Note that the integrations over the Euclidean energy p_4 have already been carried out in the vacuum terms.

For convenience, we define three diagonal matrices

$$\Pi^n(q_4) = \text{diag}(\Pi_{\Lambda u}^n(q_4), \Pi_{\Lambda d}^n(q_4), \Pi_{\Lambda s}^n(q_4)) \quad (n = \pm, 5).$$

Then, the polarization functions among the isospin eigenstates, that is,

$$\Pi_{ij}^- \equiv \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \hat{S}(p+q/2) \lambda^i \hat{S}(p-q/2) \lambda^j, \quad (23)$$

$$\Pi_{ij}^+ \equiv \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \hat{S}(p+q/2) i\gamma^5 \lambda^i \hat{S}(p-q/2) i\gamma^5 \lambda^j, \quad (24)$$

$$\Pi_{ij}^5 \equiv \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \hat{S}(p+q/2) \lambda^i \hat{S}(p-q/2) i\gamma^5 \lambda^j, \quad (25)$$

can be evaluated directly through traces over the flavor space:

$$\Pi_{ij}^n(q_4) = N_c \text{tr}_f [\lambda^i \Pi^n(q_4) \lambda^j]. \quad (26)$$

Since Π^n and λ^i ($i = 0, 3, 8$) are all diagonal, Π_{ij}^n are all symmetric under the exchange of the subscript indices i and j . Gathering all of the polarization functions into a matrix for the whole scalar-pseudoscalar sector with the generalized meson field $(\sigma, a_0, a_8, \eta_0, \pi^0, \eta_8)^T$, we have

$$\Pi_{\text{SP}}(q_4) = \begin{pmatrix} \Pi_{00}^- & \Pi_{03}^- & \Pi_{08}^- & \Pi_{00}^5 & \Pi_{03}^5 & \Pi_{08}^5 \\ \Pi_{03}^- & \Pi_{33}^- & \Pi_{38}^- & \Pi_{03}^5 & \Pi_{33}^5 & \Pi_{38}^5 \\ \Pi_{08}^- & \Pi_{38}^- & \Pi_{88}^- & \Pi_{08}^5 & \Pi_{38}^5 & \Pi_{88}^5 \\ \Pi_{00}^5 & \Pi_{03}^5 & \Pi_{08}^5 & \Pi_{00}^+ & \Pi_{03}^+ & \Pi_{08}^+ \\ \Pi_{03}^5 & \Pi_{33}^5 & \Pi_{38}^5 & \Pi_{03}^+ & \Pi_{33}^+ & \Pi_{38}^+ \\ \Pi_{08}^5 & \Pi_{38}^5 & \Pi_{88}^5 & \Pi_{08}^+ & \Pi_{38}^+ & \Pi_{88}^+ \end{pmatrix}. \quad (27)$$

Then, the corresponding propagator matrix is given by [63]

$$P_{\text{SP}}(q_4) = [1 + 2\mathcal{G}_{\text{SP}}\Pi_{\text{SP}}(q_4)]^{-1}\mathcal{G}_{\text{SP}}, \quad (28)$$

where \mathcal{G}_{SP} is the coupling matrix with the elements G_{ij}^n listed in Eq. (5) and arranged in the same order as Π_{ij}^n in $\Pi_{\text{SP}}(q_4)$. Eventually, the pole masses of the collective modes can be obtained by solving the equation

$$\det[1 + 2\mathcal{G}_{\text{SP}}\Pi_{\text{SP}}(iM_m)] = 0,$$

and there are six independent solutions in principle.

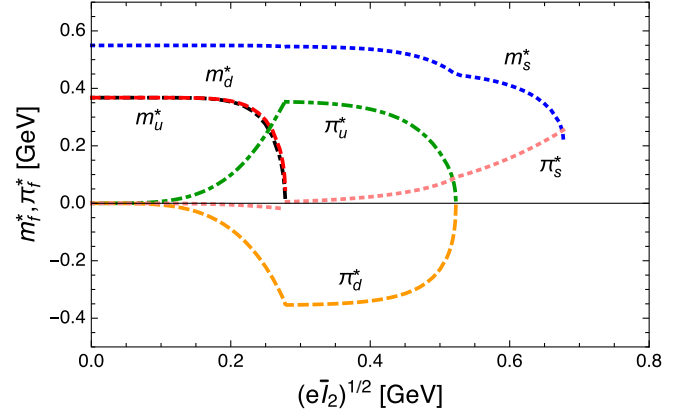


FIG. 1. The effective masses m_f^* and pion condensates π_f^* as functions of the electromagnetic field $e\bar{I}_2$ for u (dash-dotted), d (dashed), and s (dotted) quarks, respectively. For brevity, the restorations of the corresponding order parameters to their current values are not shown beyond the critical points.

III. NUMERICAL RESULTS

For the numerical calculations, we choose the following model parameters: $m_{0u} = m_{0d} = 5.5$ MeV, $m_{0s} = 140.7$ MeV, $\Lambda = 602.3$ MeV, $G\Lambda^2 = 1.835$, and $K\Lambda^5 = 12.36$ [64]. The effective masses and pion condensates of different flavors are solved self-consistently from Eqs. (12) and (13) and shown together in Fig. 1. The features of the more interesting pion condensates can be explained by adopting the cutoff-independent results (14) and the fact that, in Eq. (6), the $U_A(1)$ -symmetric term of π_f^* dominates over the anomalous violating term as $G \gg |K\sigma_f|$. Substituting Eq. (6) into Eq. (14), we get

$$-4N_c G \pi_i^5 = 4N_c G \frac{\frac{q_i^2 l_2}{4\pi^2} + 2N_c^2 K(\sigma_j \pi_k^5 + \pi_j^5 \sigma_k) \sigma_i}{m_{0i} + 2N_c^2 K(\sigma_j \sigma_k - \pi_j^5 \pi_k^5)}. \quad (29)$$

As the denominator on the right-hand side of Eq. (29) is always positive, the sign of π_f^* just follows the numerator. For small EM field, the QCD anomaly dominates over the QED one, that is, the numerator is mainly determined by the second term. According to the numerical results, σ_i are always negative and the QCD anomaly terms in the numerators are dominated by

$$\sigma_s \pi_d^5, \sigma_s \pi_u^5, (\sigma_u \pi_d^5 + \sigma_d \pi_u^5) \quad (30)$$

for u , d , s quarks, respectively. Then, it is easy to find that the signs of π_u^* and π_d^* are opposite to each other, and $\pi_s^* < 0$ as the numerical calculations show that $\sigma_u \sim \sigma_d$ and $0 < \pi_d^5 < -\pi_u^5$. These are just the characteristics up to the end of the first chiral rotation with the critical strength $e\bar{I}_2^c = (0.278 \text{ GeV})^2$. However, beyond this point with both σ_u and σ_d vanishing, π_s^* changes sign because the QCD anomaly loses significance for the strange quark.

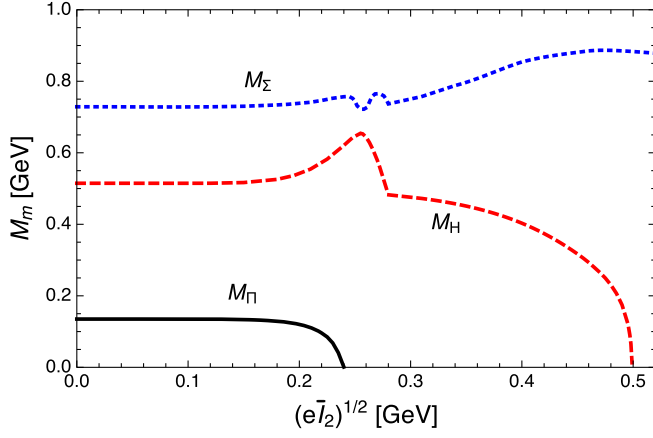


FIG. 2. The meson masses as functions of the electromagnetic field $e\bar{I}_2$ for the lowest-lying neutral collective modes Σ , Π , and H in the scalar-pseudoscalar sector.

In the one-flavor case, we have explained that the π_u condensate only stands for the magnitude of the chiral condensate and the chiral angle can be randomly chosen [40]. But, for the three-flavor case, the constancy of the s -quark dynamics requires $\sigma_u\sigma_d - \pi_u^5\pi_d^5$ to be almost a constant to keep the change of m_s^* small. In other words, the QCD anomaly correlates the chiral angles of u and d quarks in the three-flavor NJL model. Eventually, the chiral symmetry for u and d quarks is restored at the second critical EM field $e\bar{I}_2^{c_2} = (0.523 \text{ GeV})^2$ and that for s quarks is restored at the third critical field $e\bar{I}_2^{c_3} = (0.677 \text{ GeV})^2$. Note that the transitions at $e\bar{I}_2^{c_1}$, $e\bar{I}_2^{c_2}$, and $e\bar{I}_2^{c_3}$ are weak first-order, weak second-order, and strong first-order transitions, respectively. Due to the large current mass of s quarks, the corresponding end of chiral rotation and the chiral restoration point merge into one, that is, $e\bar{I}_2^{c_3}$.

There is one thing that needs to be clarified in an external EM field: the flavor separations of the collective modes, which were usually assumed in lattice QCD simulations at large magnetic field [46–48]. We rearrange the interaction terms of Eq. (4) in flavor eigenchannels and find the couplings of mixing terms to be

$$G_{ij}^\mp = \mp KN_c\sigma_k, \quad G_{ij}^5 = KN_c\pi_k,$$

with $i \neq j \neq k$. So, u and d quarks never separate from each other up to $e\bar{I}_2^{c_3}$, beyond which all of the condensates almost vanish, and s quarks never separate from them up to $e\bar{I}_2^{c_2}$ where chiral symmetry for u and d quarks is eventually restored. In real QCD, the effective couplings would decrease with the EM field due to asymptotic freedom and the flavor separations might occur earlier.

The masses of the lowest-lying collective modes Σ , Π , and H are shown in Fig. 2. Note that these modes separately

correspond to σ , π^0 , and η mesons in the vanishing-EM-field limit. As we can see, the masses of Π and H modes reduce to zero around $e\bar{I}_2^{c_1}$ and $e\bar{I}_2^{c_2}$, which actually signal the instabilities induced by QCD and QED anomalies for u and d quarks, respectively. There is an extra feature: around $e\bar{I}_2^{c_1}$, the scalar-pseudoscalar components are exchanged between the mass eigenstates Σ and H , which demonstrates itself through the peak-to-dip crossing structure. Concretely, the component $\bar{u}i\gamma^5 u + \bar{d}i\gamma^5 d$ dominates in the Σ mode, whereas $\bar{u}u + \bar{d}d$ dominates in the H mode beyond $e\bar{I}_2^{c_1}$. Furthermore, the peak right after the dips in M_Σ is just the hierarchy of that found in the one-flavor case around $e\bar{I}_2^{c_1}$ [40], and there is a second, soft one around $e\bar{I}_2^{c_2}$.

IV. CONCLUSIONS

The neutral chiral condensates and lowest-lying collective modes were explored in the three-flavor NJL model, which is characterized by the 't Hooft determinant for the QCD $U_A(1)$ anomaly. Three critical EM fields were found for the chiral condensates: $e\bar{I}_2^{c_1}$ is the end of chiral rotation of u and d quarks, $e\bar{I}_2^{c_2}$ is the chiral restoration point of u and d quarks, and $e\bar{I}_2^{c_3}$ is the chiral restoration point of s quarks. We performed a detailed mathematical analysis and found that the signs of pion condensates π_f^* reflect the relative significances of the QCD and QED anomalies: the opposite signs of π_u^* and π_d^* and the negativeness of π_s^* both indicate the dominance of the QCD anomaly in the small-EM-field region, and the positiveness of π_s^* implies the dominance of the QED anomaly for s quarks. Along with the changes of chiral condensates, the masses of the two lowest-lying modes Π and H reduce to zero at $e\bar{I}_2^{c_1}$ and $e\bar{I}_2^{c_2}$, which actually signal the instabilities induced by the QCD and QED anomalies, respectively. Besides, there is a component-exchange structure for the two modes Σ and H with their masses close to each other. Since we only studied the QED anomaly in our previous work [40], the different features of the lightest meson near $e\bar{I}_2^{c_1}$ here can be considered as the signal of QCD anomaly dominance, that is, the absence of a peak structure in M_Π .

Finally, it should be pointed out that, as we have demonstrated that the in-in and in-out results are almost the same when the EM field is relatively small [40], the most important discoveries of this work are reliable around the first critical point $e\bar{I}_2^{c_1}$.

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