Cosmological unification, dark energy, and the origin of neutrino mass

J. G. Salazar-Arias [*] and A. Pérez-Lorenzana [†]

Centro de Investigación y de Estudios Avanzados del I.P.N. Apartado Postal 14-740, 07000 Mexico City, Mexico

(Received 9 January 2020; revised manuscript received 10 March 2020; accepted 26 March 2020; published 16 April 2020)

We suggest that quintessential vacuum energy could be the source of right-handed neutrino masses that feed the seesaw mechanism, which may provide the observed small masses of light standard neutrinos. This idea is naturally implemented in the cosmological unification model based on the global $SO(1,1)$ symmetry, where the early inflation and late-time accelerated expansion of the Universe are driven by the degrees of freedom of a doublet scalar field. In this model, the $SO(1,1)$ custodial symmetry naturally provides the coupling between the Standard Model singlet fermion and quintessence, which sources neutrino masses. We also show that the model predicts a highly suppressed contribution to the relativistic degrees of freedom from quintessential quanta during any late Universe epoch, ensuring the consistency of the model.

DOI: 10.1103/PhysRevD.101.083526

I. INTRODUCTION

Contemporary cosmological surveys [1–6] have shown that the energy density of our Universe, in the framework of general relativity, consists mainly of an unknown substance having the exotic property of overcoming the pull of gravity, compelling our Universe to a stage of accelerated expansion. Whatever this component is made of, it is known as dark energy (DE). The simplest candidate for DE is the cosmological constant (Λ) [7,8]; other more elaborate proposals invoke the existence of scalar fields [9–11] which near their vacuum state behave like Λ and additionally have the advantage of allowing a dynamics that could alleviate the problems of smallness and fine-tuning that Λ has to deal with [12–15].

Since it was proposed, the feasibility of dynamic DE has been studied extensively (see, for instance, Ref. [16] or the more recent Ref. [17]). It is also expected to be analyzed in the near future through scheduled high-precision probes like DESI [18].

Several scalar fields have been proposed as DE—such as k-essence [19,20], Chaplygin gas [21,22], phantom dark energy [23,24], and h-essence [25,26]—but among them the most well known and studied is *quintessence* (Q) [27–29], which is thought of as a canonical scalar field minimally coupled to gravity with a potential that is flat enough to guarantee the slow-rolling evolution of the field, which in turn is necessary to violate the strong energy condition and realize the accelerated cosmic expansion.

The cosmological evolution of Q has been widely studied regardless of its origin or the phenomenology of the high-energy theory it may come from [30–33]. It is possible to do this because identifying Q as DE only requires the existence of a vacuum state that can be used as a classical source in Einstein's equations.

On the other hand, an underlying theory has to be considered when interactions between DE and other fields are taken into account; see, for instance, Refs. [34,35] for DE and dark matter (DM) interactions (for a review about DE and DM, see Ref. [36]). Another example is the effect of coupling Q with ordinary matter, which was studied in Ref. [37]. The first mention of and a posterior study on the possible connection between active neutrinos and Q, grounding the mass-varying neutrinos models, can be found in Refs. [38,39]. A series of related studies can be found, for instance, in Ref. [40] and Refs. [41,42]. A study of Yukawa couplings between DE and fermionic DM and the effects of radiative corrections on the mass of Q, as well as the proposal of multiaxion DE/DM models and their cosmological evolution, were addressed in Ref. [43]. Early ideas regarding a possible connection between sterile Majorana neutrino masses and ultralight bosons that could be Q were presented in Ref. [44], although no reference to any governing principle was given there. Studies of Q as an axionic particle and its connection with higher-energy theories like string, superstring, or M-theory can be found, for instance, in Refs. [45–47].

The dynamics of Q resembles that of the inflaton, which is the scalar field hypothesized in order to solve (among other things) the horizon and flatness problems that affect

[*]jsalazar@fis.cinvestav.mx
[†]aplorenz@fis.cinvestav.mx

noninflationary (Friedmann) cosmologies [48–52]. Inflation assumes that the early Universe underwent an exponential expansion phase driven by a state of almost pure vacuum energy—which behaves like a cosmological constant—generated through the slow-rolling evolution of the inflaton, which (unlike Q) occurred at a higher energy scale and totally dominated the content of the Universe. Despite these facts, both dynamics are evidently similar, and it seems reasonable to assume that Q and the inflaton may be deeply interrelated.

Such is the line of thought of the cosmological unification idea presented in Ref. [53]. According to this idea, by using symmetries one can unify (in the field theory sense) both stages of accelerated expansion by relating inflation and quintessence fields with the degrees of freedom of a unique scalar field representation. In such an approach, DE would be just the remnant of the very early stages of cosmological evolution (see also Ref. [54]). Although the original model based on the $SO(1,1)$ global symmetry (as discussed in Ref. [53]) was intended for phantom energy instead of Q as DE, on the basis of the same symmetry the unification of inflation and Q is very well possible (as we will show below) by describing both fields as associated with the components of a doublet scalar representation.

The interesting aftermath of this symmetry-guided cosmological unification model is that, solely due to the symmetry, all possible interactions become very well defined at the Lagrangian level when written in terms of a few field-invariant couplings. This is the case for both scalar self-interactions and scalar-to-fermionic matter couplings. Hence, all possible physics derived from the model can mostly be determined with just a few fundamental parameters. Exploring these parameters and determining to what extent the $SO(1,1)$ cosmological unification model can provide acceptable physical consequences is the main goal of the present paper.

Interestingly enough, as we will discuss later on, $SO(1,1)$ symmetry does provide a set of bilinear field invariants that accommodate inflation and quintessence dynamics from the most generic quadratic scalar potential. As we will explain below, more general potentials can be built by choosing higher-order invariants; nonetheless, we study the simplest one as a first approximation to the phenomenology of our model, despite the fact that the quadratic potential in the inflation sector is disfavored by *Planck* data [55].

To allow for a fermion-to-scalar coupling the symmetry enforces the introduction of a fermion doublet and a singlet. We assume that these fermions are right-handed and singlets under the Standard Model (SM) symmetries, and we naturally identify them as neutrinos. As expected, such Yukawa couplings would provide an inflaton decay channel for reheating after inflation. However, as we shall discuss, due to the symmetry the same set of couplings would keep right-handed neutrinos coupled to the quintessence field. The latter would remain trapped in a false vacuum configuration throughout the evolution of the observable Universe. According to the quintessence model, such a false vacuum is the actual source of the observed DE; however, even more interesting is the observation that in the context of our model this explanation of DE would also introduce a natural way to generate large masses for right-handed neutrinos, which would become connected to the cosmological accelerated expansion.

Right-handed neutrino masses are the main known ingredient of the seesaw mechanism [56–62], which provides a natural explanation of the tiny standard neutrino masses observed in neutrino oscillation experiments (for a detailed discussion, see Ref. [63]), which are so far bounded to be on the sub-eV scale. (See also Refs. [64,65] for very strong constraints on the sum of neutrino masses from cosmological data in the context of both constant and dynamical DE.)

In its simplest one-family formulation, a right-handed singlet neutrino N is added to the SM particle content, and the most general Lagrangian terms that contribute to neutrino masses are then written as $y^\nu \bar{L}_\alpha \tilde{H} N + (\mathrm{H.c.}) + M_R \overline{N^c} N$, where L is the SM lepton doublet, H is the Higgs boson, and y^ν are the Yukawa couplings. By introducing the Higgs vacuum $\langle H\rangle$, the first term becomes a Dirac mass term for the neutrino, $m_D \bar{\nu} N$, where $m_D = y^\nu \langle H\rangle/\sqrt{2}$, which together with the Majorana mass term provides a small effective mass for the standard neutrino, $m_\nu = (m_D)^2/M_R$. Assuming an order-one Yukawa, the only way to understand a sub-eV m_ν is to have M_R as large as 10^{13} GeV or so. Smaller values are possible if smaller Yukawa couplings are considered. Nevertheless, notice that a Majorana mass enters as a free parameter in the theory, with no connection to the Higgs mechanism whatsoever. Therefore, understanding neutrino masses with the seesaw mechanism becomes equivalent to searching for the origin of M_R. This is where the outcome of the $SO(1,1)$ model becomes relevant, as it suggests that such a mass could actually have a cosmological origin associated with the source of DE. This is a striking observation that deserves to be closely analyzed in order to establish its consistency in the cosmological setup, and that is the main goal of this paper.

This paper is organized as follows. In the next section we introduce the cosmological unification model based on the $SO(1,1)$ symmetry. There we present the Lagrangian of the model, which is based on the most general bilinear invariants built upon a dimension-two fundamental representation to which cosmological scalar fields are assigned. We then discuss how inflation and quintessence emerge in the model. Right-handed neutrinos are introduced into the model in Sec. III. Yukawa couplings to the cosmological scalars are explored and the conditions under which these get masses from the cosmic vacuum energy is discussed. As this mechanism also implies that quintessence quanta $\mathcal{X}$ can be excited in the primordial plasma from out-of-equilibrium right-handed neutrino interactions due to the same couplings that provide neutrino masses, in Sec. IV we explore the consequences of this by studying the production of relativistic $\mathcal{X}$ fields through Boltzmann equations, which shows the consistency of the scenario with big bang nucleosynthesis requirements. Furthermore, since the see-saw mechanism also implies small $\mathcal{X}$ to active neutrino couplings due to heavy-to-light neutrino mixing, in Sec. V we quantify the thermal corrections to the quintessence mass due to the cosmological neutrino background, and check that, despite this, the slow-roll condition for $\mathcal{X}$ is always fulfilled, and hence the field keeps its DE behavior. Section VI contains a short discussion and some final remarks about our proposal. Finally, the two appendices contain some technical details and relevant calculations.

II. THE $SO(1,1)$ COSMOLOGICAL MODEL

Following the motivations of the $SO(1,1)$ model as presented in Ref. [53], we consider the scalar doublet

$$\Phi = \begin{pmatrix} \phi \\ \varphi \end{pmatrix}, \tag{1}$$

where ϕ and φ are complex scalar fields, which for convenience can be written in terms of four real fields as

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \qquad \varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2).$$

This representation transforms under the global $SO(1,1)$ group as $\Phi \to g_\alpha \Phi$, where g_α stands for an arbitrary element in the corresponding $SO(1,1)$ matrix representation, whose exponential mapping is in general given by

$$g_\alpha = e^{i\alpha\sigma_1}, \qquad \alpha \in \mathbb{R}, \tag{2}$$

where σ_1 is the first Pauli matrix.

There are four bilinear invariants formed with this doublet [53]:

$$\Phi^\dagger\Phi = |\phi|^2 + |\varphi|^2, \qquad \Phi^\dagger\sigma_1\Phi = \phi^*\varphi + \varphi^*\phi,$$
$$\Phi^T i\sigma_2\Phi = \phi\varphi - \varphi\phi, \qquad \Phi^T\sigma_3\Phi = \phi^2 - \varphi^2, \tag{3}$$

where σ_2 and σ_3 are the other two Pauli matrices. Clearly, the kinetic term $\partial_\mu\Phi^\dagger\partial^\mu\Phi$ belongs to the first class of invariants in the above equation. The potential of the model, on the other hand, is restricted to be built out of these invariants in order to keep the symmetry.

It is worth noticing that these terms still allow for some diversity in the possible cosmological potentials one may consider. In the case of real field representations, for instance, first and third invariants can be added together to provide a whole class of systems where the fields have an independent evolution, simply because one can write $\phi^2 = \Phi^\dagger\Phi + \Phi^T\sigma_3\Phi$ and $\varphi^2 = \Phi^\dagger\Phi - \Phi^T\sigma_3\Phi$. In such a case, the potentials $U(\phi^2)$ and $V(\varphi^2)$ written in terms of such combinations will always have a quadratic dependence on the fields. Of course, such a scenario implies the removal of the $\Phi^\dagger\sigma_1\Phi$ term from the theory, but (as stated in Ref. [53]) this can be done by noticing that such a term is actually a pseudoscalar bilinear under the parity transformation defined as $\Phi \to \sigma_3\Phi$, which can easily be added to the model. Such a construction, however, ignores the most general complex nature of the cosmological field Φ and we will avoid it.

Next, for our model we consider the most general theory we can build with the invariant terms in Eq. (3), but considering for simplicity only mass-like terms in the potential. As should be clear, more general potentials based on these same bilinears are also possible, but considering this simplest form (although it is disfavored by *Planck* data) will suffice for our propose. Therefore, the Lagrangian we consider is

$$\mathcal{L}_\Phi = \partial^\mu\Phi^\dagger\partial_\mu\Phi - V(\Phi), \tag{4}$$

where the potential is formed from the most general linear combination of the nontrivial invariants,

$$V(\Phi) = \Phi^\dagger(\alpha_0\mathbb{I} + \alpha_1\sigma_1)\Phi + \alpha_3\Phi^T\sigma_3\Phi + \text{H.c.} \tag{5}$$

Here $\alpha_{i=0,1,3}$ are mass-dimension-two quantities which in general can be complex. As the model intends to incorporate inflation, we should assume that the involved scales are naturally large, perhaps a few orders below the Planck scale, $m_{\rm pl}$. Also, a contraction with the background Friedmann-Robertson-Walker metric should be understood in the kinetic terms. In order to identify the dynamics of the so-constructed cosmological model, we need to explore the potential in detail and identify the proper set of initial conditions that give rise to inflation and DE.

As explained in detail in Appendix A, the above generic potential can be diagonalized using an orthogonal rotation, $\mathbb{S}'$, on the four-dimensional field space of initial real field components, such that we can use the new fields defined as $(\mathrm{Q}_1, \mathrm{Q}_2, \xi_1, \xi_2)^T = \mathbb{S}'(\phi_1, \phi_2, \varphi_1, \varphi_2)^T$ to build the mass eigenstate complex scalars

$$\mathrm{Q} = \frac{1}{\sqrt{2}}(\mathrm{Q}_1 + i\mathrm{Q}_2) \quad \text{and} \quad \xi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2), \tag{6}$$

from which the Lagrangian simply becomes [see Eq. (A6)]

$$\mathcal{L}_\Phi = \partial^\mu\boldsymbol{\varphi}^\dagger\partial_\mu\boldsymbol{\varphi} - \boldsymbol{\varphi}^\dagger\mathbb{M}\boldsymbol{\varphi} \tag{7}$$

$$= \partial^\mu\mathrm{Q}^\dagger\partial_\mu\mathrm{Q} + \partial^\mu\xi^\dagger\partial_\mu\xi - m^2|\mathrm{Q}|^2 - M^2|\xi|^2, \tag{8}$$

where

$$\boldsymbol{\varphi} = \begin{pmatrix} \mathrm{Q} \\ \xi \end{pmatrix} \quad \text{and} \quad \mathbb{M} = \begin{pmatrix} m^2 & 0 \\ 0 & M^2 \end{pmatrix}. \tag{9}$$

As stated in Appendix A, the above masses can be written in terms of α_i as

$$M^2 = \mu_0^2 + \mu^2, \quad \text{whereas} \quad m^2 = \mu_0^2 - \mu^2, \tag{10}$$

where $\mu_0^2 = 2\mathrm{Re}\alpha_0$ and $\mu^2 = 2\sqrt{(\mathrm{Re}\alpha_1)^2 + |\alpha_3|^2}$.

Notice that even though in Eq. (4) we started with a coupled system of complex fields, after the field rotation we ended up with a new description where the Q and ξ degrees of freedom are decoupled. However, we should also notice that, even though this is a more suitable way of writing the potential, it is at the cost of hiding the $SO(1,1)$ symmetry, which now is not explicit in the Lagrangian.

Furthermore, the potential in Eq. (7) shows no explicit dependence on the phase fields, which suggests that they do not play any fundamental role in the slow-roll evolution phase of the background cosmological system. In accordance with this, for simplicity, we shall proceed with the analysis of the cosmological model by only considering the modular field components as a good first approximation, fixing the phases to zero. However, as one may still wish to know the role played by these field phases in other effects of cosmological interest, such as in the DE sector where this phase could play a regulatory role (as it is done, for instance, in spintessence models [66,67]), we address the issue in some detail in Appendix B, where it is shown that the system dynamics of the background universe indicates that it is indeed consistent to set the initial phase value to zero, such that the phase does not evolve.

The supplementary condition $\mu_0^2 \approx \mu^2$, which is consistent with the assumption that all involved scales are naturally of about the same order, allows to incorporate a fine-tuning in the masses to have $M^2 \gg m^2$, which permits to identify ξ as the inflation field and Q as the quintessence source of DE. As a matter of fact, in such a case the cosmological system involves the independent evolution of two fields that fall on a paraboloidal potential from some given initial condition towards the absolute minimum located at $\xi = \mathrm{Q} = 0$. Clearly, for $M^2 \gg m^2$ the potential is steeper along the ξ direction, with Q behaving almost like a flat direction. Assuming that the initial condition is such that $\langle\xi\rangle \sim \langle \mathrm{Q}\rangle \sim m_{\mathrm{pl}}$, in the slow-roll regime the source of inflation in the model would then be proportional to the squared modulus of the inflaton, as it is done in chaotic inflation. Similarly, the source of DE is proportional to the squared modulus of $\langle \mathrm{Q}\rangle$. According to the standard dynamics, ξ should slow roll down the potential towards the local minimum at $\xi = 0$ but where Q is frozen at its initial value $\langle \mathrm{Q}\rangle$ due to its small mass since $m \ll \mathsf{H} \approx M$, where H is the Hubble parameter, and thus $\dot{\mathrm{Q}}/\mathrm{Q} \approx m^2/\mathsf{H} \approx 0$, until H catches up with the scale m. Effectively, most of the time Q behaves as a perfect fluid with an equation of state $p = \omega\rho$, with $\omega = -1$. Eventually, ξ exits inflation and suddenly evaporates and reheats the Universe. As is usual for chaotic inflation, the observed amount of density perturbations in the cosmic microwave background would require $M \approx 10^{-5} m_{\mathrm{pl}}$. Q, on the other hand, should stay fixed at its initial value during most eras of evolution, until the matter density ρ_m catches up with the quintessence false vacuum energy density,

$$\rho_{\langle \mathrm{Q}\rangle} = \frac{1}{2} m^2 \langle \mathrm{Q}\rangle^2, \tag{11}$$

close the coincidence era. After that, Q is released and starts slow rolling down towards its true minimum at zero. Most of the Q models use this expression to rewrite the observed DE density [63], $\rho_{\mathrm{DE}} = M_{\mathrm{pl}}^2 \Lambda \approx 2.53 \times 10^{-47}\ \mathrm{GeV}^4$, where $M_{\mathrm{pl}} = m_{\mathrm{pl}}/\sqrt{8\pi}$ is the reduced Planck mass, such that

$$m^2 = 2\frac{M_{\mathrm{pl}}^2}{\langle \mathrm{Q}\rangle^2}\Lambda.$$

Therefore, with $\langle \mathrm{Q}\rangle \sim m_{\mathrm{pl}}$, the mass of Q should be

$$m \approx 5.8 \times 10^{-34}\ \mathrm{eV} \tag{12}$$

to provide a successful scenario. The smallness of this parameter indicates the need for a fine-tuning as large as in the cosmological constant problem.

We would like to finish this section by making some comments about the smallness of this scale. It has been noticed that the tiny mass of Q is unstable under radiative corrections due to quadratic divergences, in such a way that the required flatness of the potential—and thus the slow-roll condition—would disappear [68]. This is a generic illness of any interacting scalar field theory, commonly known as the hierarchy problem. In order to keep the physical mass around the required order, it is necessary to introduce further large fine-tunings for all loop order corrections.

This problem is commonly overcome in the context of supersymmetry, where quadratic divergences are exactly canceled by superpartner contributions, and hence the mass is kept under control. Such is the case for, e.g., the Higgs mass in the context of the supersymmetric Standard Model extension. Nevertheless, in the case of Q, it has been observed that simply adding supersymmetry might not solve the whole problem, since remaining corrections due to supersymmetry breaking might still be large (see Refs. [68,69]). Additionally, the Q mass could be stabilized by invoking Goldstone symmetries, where Q is assumed to be a pseudo-Goldstone boson belonging to a higher-dimensional space in which the supersymmetric breaking scales are suppressed; thus, this boson appears as an effective boson that remains stable in four dimensions (see, for instance, Refs. [29,69,70]).

Whether our model could be extended to consider supersymmetry or be embedded in a more fundamental theory (perhaps supergravity or even string theory)—where the ξ and Q fields arise as effective degrees of freedom such that the large hierarchy problem is resolved—is a question we still have to explore, and so it remains a potentially troublesome issue for the model, as is the case for most quintessence models.

III. ADDING FERMIONS: REHEATING AND NEUTRINO MASS

Reheating after inflation in the usual approach uses the sudden decay of the inflaton into other particles in order to inject matter into an otherwise empty universe. In accordance with the global $SO(1,1)$ symmetry we adopted for our cosmological model, the minimal fermionic matter content is accounted for by introducing a total of three spinorial fields $N^{\dot{a}}_{i=0,1,2}$, two of which are arranged into a doublet

$$\Psi = \begin{pmatrix} N_1^{\dot{a}} \\ N_2^{\dot{a}} \end{pmatrix}, \tag{13}$$

and the remaining one is treated as a singlet. We choose fermions to be (two-component) right-handed Weyl fields, such that they can be identified with those usually introduced in extensions of the Standard Model of particle physics in order to have massive neutrinos through the seesaw mechanism. Thus, the two-component spinorial index $\dot{a} = 1, 2$ and Dirac matrices are written as

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{a\dot{c}} \\ \bar{\sigma}^{\mu\dot{a}c} & 0 \end{pmatrix}, \tag{14}$$

with

$$\sigma^\mu{}_{a\dot{a}} = (\mathbb{I}, \boldsymbol{\sigma}), \qquad \bar{\sigma}^{\mu\dot{a}a} = (\mathbb{I}, -\boldsymbol{\sigma}), \qquad \bar{\sigma}^{\mu\dot{a}a} = \epsilon^{\dot{a}\dot{b}}\epsilon^{ab}\sigma^\mu{}_{b\dot{b}},$$

and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$. In this notation, the charge-conjugation matrix and β matrix (which is numerically equal to γ^0 but carries a different index structure) are, respectively, given by

$$C = \begin{pmatrix} \epsilon_{ac} & 0 \\ 0 & \epsilon^{\dot{a}\dot{c}} \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & \delta^{\dot{a}}_{\dot{c}} \\ \delta^c_a & 0 \end{pmatrix}. \tag{15}$$

From each Weyl field, a four-component $(a, \dot{a} = 1, 2)$ sterile Majorana neutrino is built by writing

$$\psi_i = \begin{pmatrix} N^\dagger_{ia} \\ N^{\dot{a}}_i \end{pmatrix}, \tag{16}$$

where $N^\dagger_{ia}$ is the charge conjugate of the right-handed Weyl field, given by $N^\dagger_{ia} = (N^{\dot{a}}_i)^{\mathcal{C}}$. The previous can be seen from Eq. (16) and $\psi^{\mathcal{C}} = C\bar{\psi}^T$ with the application of Eq. (15). The doublet in Eq. (13) transforms under $g_\alpha \in SO(1,1)$ as

$$\begin{pmatrix} N_1^{\dot{a}} \\ N_2^{\dot{a}} \end{pmatrix} \xrightarrow{g_\alpha} e^{i\alpha\sigma_1} \begin{pmatrix} N_1^{\dot{a}} \\ N_2^{\dot{a}} \end{pmatrix} = \begin{pmatrix} N_1'^{\dot{a}} \\ N_2'^{\dot{a}} \end{pmatrix}, \tag{17}$$

with the new Weyl fields arising from combinations and global phase changes of the previous ones. It is important to note that since the Weyl fields admit global phase transformations, it will always be possible to build a new four-component sterile Majorana neutrino

$$\psi'_i = \begin{pmatrix} N'^\dagger_{ia} \\ N'^{\dot{a}}_i \end{pmatrix}, \quad \text{such that} \quad \psi_i = \psi_i^{\mathcal{C}} \xrightarrow{g_\alpha} \psi'_i = \psi_i'^{\mathcal{C}},$$

and therefore the transformation of the field ψ_i induced by the $SO(1,1)$ rotation in Eq. (17) does not violate the Majorana condition.

With these conventions, the general fermion kinetic terms for the Majorana fields become $\frac{1}{2}\bar{\psi}_i i\gamma^\mu\partial_\mu\psi_i = N_i^{\dagger a} i\sigma^\mu_{a\dot{c}}\partial_\mu N_i^{\dot{c}}$, where (as before) a background metric contraction should be understood. Next, it is easy to see that one can write the kinetic terms in a clearly $SO(1,1)$- and Lorentz-invariant form as

$$\mathcal{L}_\Psi = N_0^{\dagger a} i\sigma^\mu_{a\dot{c}}\partial_\mu N_0^{\dot{c}} + \Psi^\dagger i\sigma^\mu\partial_\mu\Psi. \tag{18}$$

On the other hand, by taking the Hermitian conjugate of $N_0^{\dot{a}}$ and the fermion and scalar doublets, the most general Yukawa interaction terms from the linear combination of the invariants one can build are

$$-\mathcal{L}_I = N_{0\dot{a}}\{a_0\Phi^\dagger\Psi + a_1\Phi^\dagger\sigma_1\Psi + a_2\Phi^T i\sigma_2\Psi + a_3\Phi^T\sigma_3\Psi\} + \text{H.c.}, \tag{19}$$

where $a_{i=0,\dots,3}$ are complex dimensionless couplings.

Notice that, analogous to the invariant terms which appear in Eq. (3), there exist bilinear $SO(1,1)$ invariants that are formed from the fermion doublet taken with itself, $\Psi^\dagger\Psi$, $\Psi^\dagger\sigma_1\Psi$, $\Psi^T i\sigma_2\Psi$, and $\Psi^T\sigma_3\Psi$, which, however, are not Lorentz-invariant objects and therefore we remove them from the Lagrangian.

It is also worth asking if there are allowed mass terms for the fermions. We note that such terms can be built by defining an additional doublet formed from the charge-conjugate fields of $N^{\dot{a}}_{i=1,2}$ as

$$\Psi^{\mathcal{C}} = \begin{pmatrix} N^\dagger_{1a} \\ N^\dagger_{2a} \end{pmatrix}. \tag{20}$$

The Lorentz-invariant scalar

$$\Psi^{\mathcal{C}\dagger}\Psi + \text{H.c.} = N_{1\dot{a}}N_1^{\dot{a}} + N_{2\dot{a}}N_2^{\dot{a}} + \text{H.c.} \qquad (21)$$

clearly produces Majorana mass terms ($\psi_i^T C^\dagger \psi_i$) for the fields $\psi_{i=1,2}$; however, in order to get a consistent transformation of $\Psi^{\mathcal{C}}$ under the symmetry, it is necessary to impose the condition that

$$N'^{\dagger}_{ia} = (N'^{\dot{a}}_i)^{\mathcal{C}},$$

which means that the components of the charge-conjugate rotated doublet $\Psi'^{\mathcal{C}}$ must to be equal to the charge-conjugate components of the rotated doublet Ψ'. In order to achieve this, the doublet in Eq. (20) has to transform with the Hermitian-conjugate matrix $g_\alpha^\dagger$, as can be checked by means of the two-dimensional matrix representations. Consequently, the term in Eq. (21) is not invariant under $SO(1,1)$ rotations and we must remove it from the Lagrangian. The same occurs for all of the terms formed from Eqs. (20) and (13). On the other hand, a mass term for $N_0^{\dot{a}}$ is allowed by the $SO(1,1)$ symmetry because it transforms as a singlet; however, we note that the interaction sector in Eq. (19) is invariant under the $U(1)$ transformation

$$\Psi \to e^{iq}\Psi, \qquad N_0^{\dot{a}} \to e^{iq_0}N_0^{\dot{a}} \qquad (22)$$

as long as $q = -q_0$. So the fields $N_{i=1,2}^{\dot{a}}$ transform with the same charge and $N_0^{\dot{a}}$ transforms with the opposite charge. Therefore, by imposing invariance under $U(1)$ in the fermion sector, which implies lepton number conservation, we remove the singlet's mass term. We note that the same argument can be invoked in order to forbid mass terms for the fermions $\psi_{i=1,2}$, but this only confirms what the $SO(1,1)$ symmetry suggests.

Finally, the complete Lagrangian we are left with is

$$\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_\Psi + \mathcal{L}_I, \qquad (23)$$

where the three sectors are given by Eqs. (4), (18), and (19), respectively. The above Lagrangian is the most general one that can be written with $SO(1,1)$ bilinear invariant terms, and it is Lorentz, P (as long as both scalar fields transform with the same parity phase), and CP invariant. As mentioned above, the fermionic sector is $U(1)$ invariant; similarly, there is $U(1)$ invariance in the scalar sector, as long as both ϕ and φ transform with the same charge.

A. Reheating

By performing the rotation in field space that diagonalizes the scalar sector and allows to identify the inflaton and quintessence fields, one also has to redefine the general Yukawa couplings introduced in Eq. (19). As explained in detail in Appendix A, after some algebra the scalar-to-fermion couplings [see Eq. (A16)] can be put into the following simple form:

$$-\mathcal{L}_I = N_{0\dot{a}}\{\boldsymbol{\varphi}^\dagger \mathbb{G}_1 \mathbf{F} + \boldsymbol{\varphi}^T \mathbb{G}_2 \mathbf{F}\} + \text{H.c.}, \qquad (24)$$

where the new coupling constants, which are just simple linear combinations of the original a_i constants written in Eq. (19), are contained in the matrices [see Eq. (A17)]

$$\mathbb{G}_1 = \begin{pmatrix} 0 & g_2 \\ h_1 & 0 \end{pmatrix}, \qquad \mathbb{G}_2 = \begin{pmatrix} g_1 & 0 \\ 0 & -h_2 \end{pmatrix}.$$

Here, the Weyl fields $F_{i=1,2}^{\dot{a}}$ are the components of the doublet

$$\mathbf{F} = \begin{pmatrix} F_1^{\dot{a}} \\ F_2^{\dot{a}} \end{pmatrix}, \qquad (25)$$

which arises from Eq. (13) after performing a $SO(2)$ rotation, $e^{-i\sigma_2\pi/4}\Psi = \mathbf{F}$, as can be seen in Eq. (A15). Notice that this rotation also transforms the spinor kinetic terms, which remain diagonal [see Eq. (A18)]. Clearly, as for the scalar sector, after the transformations the $SO(1,1)$ symmetry is no longer explicit in the Yukawa Lagrangian. On the other hand, the assumed $U(1)$ symmetry imposed in the fermion sector remains explicit.

The former couplings can be written in a more useful way as

$$-\mathcal{L}_I = N_{0\dot{a}}\{g_1 \mathrm{Q} F_1^{\dot{a}} + g_2 \mathrm{Q}^* F_2^{\dot{a}} + h_1 \xi^* F_1^{\dot{a}} - h_2 \xi F_2^{\dot{a}}\} + \text{H.c.} \qquad (26)$$

The last two terms of Eq. (26) provide the inflaton decay channels, $\xi \to N_0 F_i$, which are required for reheating after inflation. The sudden evaporation of inflaton energy would inject entropy into the emptied universe by inflation. Assuming that such a process is efficient enough, the reheating temperature should be $T_r \sim 6\times 10^{-3} \max\{|h_1|,|h_2|\}M_{\rm pl}$. Since the fermions in final states are assumed to be right-handed neutrinos they should provide the portal through the standard couplings $\bar{L}\tilde{H}N_0$ and $\bar{L}\tilde{H}F_i$ to produce all types of SM fields, which in turn should thermalize, producing the primordial plasma.

B. $SO(1,1)$ as a flavor symmetry?

Notice that the $\bar{L}\tilde{H}F_i$ couplings explicitly violate the $SO(1,1)$ symmetry unless the SM matter fields have nontrivial transformations. Of course, we can proceed with our study by assuming this, in which case $SO(1,1)$ would be a symmetry of the cosmological and sterile sectors only, which is broken by the Yukawa couplings to the standard fields. Such a scenario mimics the construction of models used to explain supersymmetry breaking, where two sectors exist in the theory: the visible sector (where the symmetry is respected) and the hidden sector (where it is violated). A messenger that couples the sectors communicates the breaking of the symmetry in the second sector to the first sector, producing a rich phenomenology and possibly even dark matter candidates [71]. Here, on the contrary, the hidden sector (cosmological plus right-handed neutrinos) has the symmetry, whereas the SM sector does not transform under $SO(1,1)$. Thus, the coupling among them would be the source of the breaking and the right-handed neutrinos themselves would be the messengers that carry this explicit breaking to the cosmological sector. The consequences of this mechanism would appear through loop quantum corrections, and so we do not expect them to be relevant enough to affect the classical configuration of the quintessential field, nor the physics that we discuss below. Yet, this is an issue that may deserve further analysis.

The alternative, on the other hand, is quite interesting. Global flavor symmetries have long been considered as a way to understand the fermion mass spectrum as well as flavor mixings appearing in charged weak interactions. In the lepton sector, such mixings are responsible for neutrino oscillation phenomena. Extending the cosmological symmetry to also involve SM fields falls into this class of symmetries. Indeed, it turns out that it is possible to imagine a simple model extension where $SO(1,1)$ is promoted to a global flavor symmetry for the leptonic SM sector, at least.

To be specific, let us consider the following lepton matter content assignments under $SO(1,1)$. We assume that both left- and right-handed μ- and τ-type flavors belong to doublets,

$$2_L = \begin{pmatrix} L_\mu \\ L_\tau \end{pmatrix} \quad \text{and} \quad 2_R = \begin{pmatrix} \mu_R \\ \tau_R \end{pmatrix}.$$

Electron-type fermions L_e and e_R belong to singlets. Right-handed neutrinos, of course, are given by the representations introduced in our cosmological model above. Furthermore, we take an extended Higgs sector consisting of the standard $SO(1,1)$ singlet Higgs H and a doublet of Higgses, written as

$$2_H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}.$$

With this matter content, we write the most general Yukawa couplings as

$$\bar{2}_L \cdot 2_H e_R + \bar{2}_L \cdot 2_R H + \bar{L}_e 2_H \cdot 2_R + \bar{L}_e H e_R + \text{H.c.} \tag{27}$$

and

$$\bar{2}_L \cdot \tilde{2}_H N_0 + \bar{2}_L \cdot \Psi \tilde{H} + \bar{L}_e \tilde{2}_H \cdot \Psi + \bar{L}_e \tilde{H} N_0 + \text{H.c.}, \tag{28}$$

where, in each term, the contribution of all relevant $SO(1,1)$ invariants should be understood. For instance, for the two fermion doublet couplings we have

$$\begin{aligned} \bar{2}_L \cdot 2_R &= 2_L^\dagger \gamma^0 (f_1 + f_2\sigma_1) 2_R \\ &= \begin{pmatrix} \bar{L}_\mu & \bar{L}_\tau \end{pmatrix} (f_1 + f_2\sigma_1) \begin{pmatrix} \mu_R \\ \tau_R \end{pmatrix}, \end{aligned}$$

where $f_{1,2}$ are Yukawa couplings. Clearly, this invariant contains bilinears of the first and second types shown in Eq. (3). Similar expressions can be written for $\bar{2}_L \cdot \Psi$, $\bar{2}_L \cdot 2_H$, and $\tilde{2}_H \cdot \Psi$. For the other bilinears, like those of the third and fourth types shown in Eq. (3), we get invariants such as

$$\begin{aligned} 2_H \cdot 2_R &= 2_H^T (ik_1\sigma_2 + k_2\sigma_3) 2_R, \\ &= \begin{pmatrix} H_1 & H_2 \end{pmatrix} (ik_1\sigma_2 + k_2\sigma_3) \begin{pmatrix} \mu_R \\ \tau_R \end{pmatrix}, \end{aligned}$$

where $k_{1,2}$ are the Yukawa couplings. The same type of couplings should be understood from $\bar{2}_L \cdot \tilde{2}_H$. Note that all terms in Eq. (28) are exactly of the $\bar{L}\,\tilde{H}\,F_i$ type, as required for a successful reheating process in our cosmological model through the Higgs portal.

After spontaneous symmetry breaking, the above terms provide the nine mass terms of the charged lepton mass matrix M_ℓ, as well as the nine Dirac neutrino mass terms that contribute to the seesaw mechanism. Without further assumptions, all such terms are expected to be nonzero. Additional flavor symmetries might be introduced in order to generate specific textures. After diagonalizing both the sectors, flavor mixings appear in the charged weak interactions given by the mixing matrix $U_{\text{mix}} = V_\ell^\dagger U_\nu$, where U_ν (V_ℓ) denotes the rotation matrix of the left-handed neutrino (charged lepton) sector used in the diagonalization process, as is well known.

As the interest in the present work is to analyze the cosmological setup, we will not discuss further the details and the phenomenology that should arise from this flavor model. An extended analysis of this will be presented elsewhere. Nevertheless, what we have discussed above does serve to show that $SO(1,1)$ symmetry may also work as a SM flavor symmetry, allowing enough freedom to accommodate lepton masses and mixings, at the cost, of course, of extending the Higgs sector. The extra Higgses would have little impact on what follows, and thus we will proceed with our discussion without considering them explicitly.

C. Sourcing neutrino mass with DE

At the end of inflation the ξ field evaporates completely, such that its energy density becomes null, leaving the inflaton field value at zero which makes its couplings of no further relevance for thermal history. On the other hand, as we have already discussed in the previous section, the Q field remains trapped in its initial homogeneous configuration throughout the Universe's evolution, perhaps changing quite slowly until recent times when it is still slow-rolling down its almost flat potential and causing the Universe's accelerated expansion.

By inserting the Q false vacuum (conveniently defined as $\langle Q\rangle/\sqrt{2}$) back into Eq. (26), one immediately realizes that due to the couplings provided by the $SO(1,1)$ model DE naturally generates masses for the right-handed neutrinos, given as

$$\mathcal{L}_m = m_1 N_{0\dot{a}} F_1^{\dot{a}} + m_2 N_{0\dot{a}} F_2^{\dot{a}} + \text{H.c.}, \tag{29}$$

where $m_i = g_i \langle Q\rangle/\sqrt{2}$. As discussed in detail in Appendix B, these mass terms give rise to two degenerate massive Majorana neutrinos $\nu_{1,2}$, for which one can write

$$-\mathcal{L}_m = \frac{1}{2} m_k (\bar{\nu}_1 \nu_1 + \bar{\nu}_2 \nu_2). \tag{30}$$

This is a striking result that connects the seesaw mechanism (and thus the origin of standard neutrino mass) to the origin of DE.

Here we have implicitly written the Majorana condition, namely, $\bar{\nu} = \nu^T C$, where C is the charge-conjugation matrix [see Eq. (15)]. Likewise, the mass m_k appearing in Eq. (30) [as defined in Eq. (B23)] is given by

$$m_k = \frac{a_c}{\sqrt{2}} \langle Q\rangle, \tag{31}$$

where the effective coupling $a_c = \sqrt{|g_1|^2 + |g_2|^2}$. We note that by choosing a_c in the interval $10^{-5} \lesssim a_c \lesssim 10^{-3}$, which seems reasonable, we can get right-handed neutrino masses in the range of 10^{14} GeV $\lesssim m_k \lesssim 10^{16}$ GeV, which are values around those needed to implement the standard seesaw mechanism.

Another immediate outcome of the present model is the alignment of couplings among quintessence quantum excitations $\mathcal{X}$ and neutrinos with the mass terms. By redefining $Q = (\langle Q\rangle + \mathcal{X})/\sqrt{2}$ we set the $\mathcal{X}$ excitations around the quintessence false vacuum, so it is clear that after diagonalization fermion masses, one gets

$$-\mathcal{L}_{I\mathcal{X}} = \frac{a_c}{2\sqrt{2}} \mathcal{X} (\bar{\nu}_1 \nu_1 + \bar{\nu}_2 \nu_2). \tag{32}$$

This coupling has relevance for thermal history. Equations (30) and (32) show that only two neutrinos are massive and interact with the DE field. The third neutrino remains massless and decoupled. Heavy neutrinos will eventually become nonrelativistic in the very early stages of the Universe and decay. The main decay process is into SM particles, $\nu_i \to LH$, injecting entropy into the primordial plasma. However, there could be an increase in the relativistic energy density due to out-of-equilibrium processes allowed by Eq. (32), since quintessence is a rather ultralight field, and the coannihilation process $\nu\nu \to \mathcal{X}\mathcal{X}$ will populate this degree of freedom, as we will examine in the next section.

Notice that, after SM symmetry breaking, the seesaw mechanism will produce a mixing among the heavy and standard neutrino states of order m_D/m_k. Such a mixing would in turn introduce an effective coupling among $\mathcal{X}$ and the light active neutrinos, which, together with the photons of the CMB, permeate the Universe in the form of radiation. As a consequence, a thermal mass correction to the $\mathcal{X}$ potential due to the cosmological neutrino background has to be taken into account since it could eventually overcome the Hubble parameter, breaking the slow-roll condition. We will also address this issue later on (see Sec. V).

It is worth mentioning that, although the above analysis neglected phases for the fields, their inclusion has little impact on our main conclusions. To state our point, in Appendix B we include a detailed discussion of the changes and effects that are involved when the phases of the scalar fields are considered. In particular, we notice that the phase of the scalar DE field does not take part in the interaction sector beyond the term that involves the inflaton [see Eq. (B31)], where the value of the phase ϑ can change the rate of decay of the complex inflaton into neutrinos. Both mass and $\mathcal{X}$ interaction terms [as expressed by Eqs. (30) and (32)] remain unchanged [see Eqs. (B29) and (B30)]. On the other hand, this phase could impact the evolution of the homogeneous background universe, since it appears as part of the total DE density, as shown in Eq. (B38). However, as can be seen from the first slow-roll condition, which in the polar base, where we define

$$Q = \frac{(\langle Q\rangle + \mathcal{X})}{\sqrt{2}} e^{i\vartheta/\langle Q\rangle}, \tag{33}$$

takes the form [see Eq. (B42)],

$$\frac{1}{2}\dot{\mathcal{X}}^2 + \frac{1}{2}\left(1 + \frac{\mathcal{X}}{\langle Q\rangle}\right)^2 \dot{\vartheta}^2 \ll \frac{1}{2} m^2 (\langle Q\rangle + \mathcal{X})^2, \tag{34}$$

the phase does not directly contribute to the DE density but rather controls it indirectly, because the fulfillment of the condition depends on the initial values of the phase and its velocity. The condition (34) is fulfilled during the DE-dominated age for most of the initial values of the phase and its velocity, as can be checked by the evolution of the dynamic system (B43).

In particular, for the simplest $\vartheta_{\text{ini}} = \dot{\vartheta}_{\text{ini}} = 0$, the phase ϑ remains null throughout the history of the Universe; therefore, for these values, the condition (B42) is simplified to the expected one for the usual case of a real scalar field.

Since in the rest of the present work we will only focus on Eq. (32), the value of the phase will not play a crucial role, and thus we can choose the simplest initial condition without losing generality. Nonetheless, our model is completely compatible with different values (as shown in Appendices A and B), and although it is not developed here we believe that a deeper analysis of the initial conditions could be related to studies of the problem of coincidence, as well as effects beyond the homogeneous limit.

IV. QUINTESSENCE QUANTA AND SM PARTICLE PRODUCTION

Because $\mathcal{X}$ particles have the same mass associated with Q they are ultrarelativistic, and thus right-handed pair annihilation constitutes a source that can inject an extra degree of freedom during the radiation-dominated age. Hence, it is necessary to check whether the presence of such radiation is compatible with the predictions of big bang nucleosynthesis (BBN).

In order to do that, we consider standard BBN (SBBN) [72–74] (for a recent review, see Ref. [75]), in which all of the input parameters—namely, the number of relativistic degrees of freedom at equilibrium (g_*), the neutron lifetime, the cross sections of the involved nuclear processes, the mass difference between neutrons and protons, and the strength of both the weak force and gravity—are in accordance with the Standard Model of particle physics and Einstein gravity. In SBBN all of these parameters are well determined. The unique input free parameter is the baryon-to-photon ratio, which determines the primordial abundances of the four light nuclei, namely, ^{4}He, ^{3}He, H or D, and ^{7}Li. None of these are modified directly in our model, apart from, perhaps, g_*.

Since SBBN assumes a Friedmann-Lemaître-Robertson-Walker (FLRW) universe and it occurs during the radiation-dominated age, any increment of g_* increases the value of the Hubble parameter H; consequently, the value of the freeze-out temperature of the neutron-to-proton ratio also increases, which in turn implies an increment of the final primordial helium abundance. The same is accomplished if there is some net increase in the total radiation energy density due to any process beyond thermal equilibrium, e.g., neutrino pair annihilation.

Once the system formed by the neutrinos and $\mathcal{X}$ particles goes out of equilibrium, the energy density of the latter becomes relevant; otherwise, the pair annihilation can be reversed, yielding a net increment of zero for the total radiation energy density. Therefore, to evaluate the total impact on the Hubble parameter, it is necessary to determine the out-of-equilibrium radiation production along with that in equilibrium by evolving the Boltzmann equation for the *radiation number density* $n_{\mathcal{X}}$ as a function of the temperature in an FLRW universe. As the whole process is controlled solely by the coupling a_c, and thus by the scale of right-handed neutrino masses, the analysis of such a process should constrain this parameter in order to avoid perturbing the predictions of SBBN through an excess of injected $\mathcal{X}$. Nevertheless, as we show below, the process is already so inefficient that no additional constraints on a_c are needed, such that our model appears to be consistent with SBBN. Now we proceed with the detailed analysis.

In order to write the Boltzmann equation, we have to explicitly calculate the collision term, which in turn involves the thermally averaged cross section for the pair annihilation. (For the last calculation we follow Refs. [76,77]). We start by calculating the total cross section for the part of the Lagrangian (32) that corresponds to only one of the neutrinos, namely,

$$-\mathcal{L}_{I\mathcal{X}_i} = \frac{a_c}{2\sqrt{2}}\mathcal{X}\bar{\nu}_i\nu_i.$$

For this Lagrangian, the total cross section of neutrino pairs annihilating into a pair of $\mathcal{X}$ particles, calculated in the center-of-mass (CM) frame, is

$$\sigma_{\mathcal{X}} \equiv \sigma_{\bar{\nu}_i\nu_i\to\mathcal{X}\mathcal{X}} = \frac{1}{2048\pi s}\frac{a_c^4}{v_r(s)\sqrt{\lambda(s,m_k^2)}}F(s), \tag{35}$$

where $v_r(s)$ is the relative velocity between the neutrinos and

$$F(s) = \left[s + 16m_k^2\left(1 - \frac{2m_k^2}{s}\right)\right]\log\left[\frac{s+\sqrt{\lambda(s,m_k^2)}}{s-\sqrt{\lambda(s,m_k^2)}}\right] - 2\left(1+\frac{8m_k^2}{s}\right)\sqrt{\lambda(s,m_k^2)}, \tag{36}$$

where the ultrarelativistic mass of $\mathcal{X}$, m, has been neglected with respect to the nonrelativistic neutrino mass m_k. In the previous equations s was a Mandelstam variable, which in the CM frame corresponds to $s = 4E^2$, where E is the energy of each incoming neutrino and $\lambda(s, m_k^2)$ is the Mandelstam triangular function, which is given by

$$\lambda(s,m_k^2) = s(s - 4m_k^2).$$

Next, the thermally averaged cross section becomes

$$\langle\sigma_{\mathcal{X}}v_r\rangle = \frac{a_c^4}{4096\pi m_k T K_2^2(m_k/T)}\mathcal{I}_{\mathcal{X}}(m_k;T), \tag{37}$$

where K_2 is the modified Bessel function of the second kind of order 2, and where we have defined the integral

$$\mathcal{I}_{\mathcal{X}}(m_k;T) \equiv \int_0^1 dx\frac{g_{\mathcal{X}}(x)}{x\sqrt{x}}K_1\left(\frac{2m_k}{T\sqrt{x}}\right), \tag{38}$$

where K_1 is the modified Bessel function of the second kind of order 1, and $g_{\mathcal{X}}(x)$ is a function coming from Eq. (36) after the change of integration variable

$$s \to 4m_k^2/x, \qquad F(s) \to 4m_k^2 g_{\mathcal{X}}(x). \tag{39}$$

On the other hand, the out-of-equilibrium number density for the $\mathcal{X}$ particles $n_{\mathcal{X}}$, by means of the Boltzmann equation in an FLRW universe, is given as

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_{\mathcal{X}}) = \langle \sigma_{\mathcal{X}} v_r \rangle \left[n_\nu^2 - (n_\nu)_{\rm eq}^2 \frac{n_{\mathcal{X}}^2}{(n_{\mathcal{X}})_{\rm eq}^2} \right], \tag{40}$$

where $a = a(t)$ is the universal scale factor and the source n_ν is the neutrino number density, which in turn must be calculated through its own Boltzmann equation.

In order to write this last equation we have to calculate the corresponding collision term by considering all of the involved processes, namely, annihilation and decay of neutrinos into SM particles, together with those of the $\mathcal{X}$ channel. For this we consider the most general Yukawa couplings of our heavy neutrinos, $(y^\nu)^{ni}\bar{L}_n \tilde{H} \nu_{iR} + \text{H.c.}$, for $i = 1, 2$ where L_n stands for the three standard left-handed lepton doublets, with $n = 1, 2, 3$, and H stands for the Higgs doublet, whose components are denoted as

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}, \quad \text{with} \quad \tilde{H} = i\sigma_2 H^\dagger, \quad \text{and} \quad L_n = \begin{pmatrix} \nu_{nL} \\ \ell_{nL} \end{pmatrix}.$$

Thus, there are actually two channels for the decay of heavy neutrinos into SM particles,

$$\nu_{iR} \to \nu_{nL} h^0 \quad \text{and} \quad \nu_{iR} \to \ell_{nL} h^+.$$

On the other hand, the annihilation processes can be written as

$$\nu_i \nu_j \to h^{0\dagger} h^0 \quad \text{and} \quad \nu_i \nu_j \to h^+ h^-.$$

By assuming that all of the Yukawa couplings are about the same order, namely, $(y^\nu)^{ni} \sim y^\nu$, and because there are six similar processes of disintegration of a heavy neutrino, the total decay width is

$$\Gamma_d = \frac{3}{32\pi}(y^\nu)^2 m_k. \tag{41}$$

Next, for the annihilation process $\bar{\nu}_i \nu_j \to h^{0\dagger} h^0$ the total cross section in the CM frame is given as

$$\sigma_H \equiv \sigma_{\nu_i \nu_j \to h^{0\dagger} h^0} = \frac{3}{32\pi} \frac{1}{s v_r(s)} \frac{(y^\nu)^4}{\sqrt{\lambda(s, m_k^2)}} T(s),$$

where

$$T(s) = \frac{m_k^2 - 2s}{m_k^2} \sqrt{\lambda(s, m_k^2)} + 3m_k^2 \log \frac{t_-}{t_+},$$

with

$$\frac{t_-}{t_+} = \frac{(s - 2m_k^2) + \sqrt{\lambda(s, m_k^2)}}{(s - 2m_k^2) - \sqrt{\lambda(s, m_k^2)}}.$$

The thermally averaged cross section involving the three contributions to the same channel is then given by

$$\langle \sigma_H v_r \rangle = \frac{9}{4\pi} \frac{(y^\nu)^4}{m_k T K_2^2(m_k/T)} \mathcal{I}_H(m_k; T), \tag{42}$$

where, as previously, we have defined the integral

$$\mathcal{I}_H(m_k; T) = \int_0^1 dx \frac{g_H(x)}{x\sqrt{x}} K_1\left(\frac{2m_k}{T\sqrt{x}}\right),$$

where

$$g_H(x) = \frac{2 - x}{x}\sqrt{1 - x} + \frac{3}{4}\log \frac{1 - \frac{x}{2} + \sqrt{1 - x}}{1 - \frac{x}{2} - \sqrt{1 - x}}$$

is the function coming from $T(s)$ after changing the variable as it was defined in Eq. (39).

It turns out that for the other coannihilation process $\nu_i \nu_j \to h^+ h^-$, we have

$$\sigma_{\nu_i \nu_j \to h^+ h^-} = \sigma_{\nu_i \nu_j \to h^0 h^{0\dagger}} = \sigma_H \tag{43}$$

due to the $SU(2)$ standard symmetry. With these, the Boltzmann equation involving both the quintessence and Higgs channel becomes

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_\nu) = C[T], \tag{44}$$

where the collision term is given by

$$C[T] = \frac{1}{4}[\langle \sigma_{\rm TOT} v_r \rangle ((n_\nu)_{\rm eq}^2 - n_\nu^2) - \Gamma_d n_\nu],$$

with the total thermally averaged cross section given in terms of Eqs. (37) and (42) as

$$\langle \sigma_{\rm TOT} v_r \rangle = 2\langle \sigma_{\mathcal{X}} v_r \rangle + 2\langle \sigma_H v_r \rangle,$$

wherein the factor of 2 in the $\mathcal{X}$ term accounts for the two involved Majorana neutrinos [see Eq. (32)], and the second factor of 2 is there because of Eq. (43). The last term to be defined in the collision term is the neutrino number density at equilibrium, which is given by

$$(n_\nu)_{\rm eq} = 4\pi m_k^2 T K_2(m_k/T). \tag{45}$$

After changing the time evolution in favor of the temperature in Eq. (44), which is possible to do during the radiation-dominated age, one gets

$$\frac{d}{dT}(a^3 n_\nu) = -\frac{M_{\rm pl}}{\pi}\left(\frac{90}{g_*(T)}\right)^{1/2}\frac{a^3}{T}C[T], \qquad (46)$$

where $g_*(T)$ is the number of relativistic degrees of freedom of the energy density at equilibrium.

It turns out that y^ν is always greater than the coupling a_c, as can be checked by considering that $\langle Q\rangle \sim m_{\rm pl}$ and the Higgs vacuum expectation value $\langle H\rangle = 246$ GeV in the seesaw formula, according to which the light neutrino mass is given as $m_\nu \sim m_D^2/m_k$. For a m_ν well within the observed bounds [63], which for the heaviest of the light states is given as 5×10^{-2} eV $\lesssim m_\nu \lesssim 10^{-1}$ eV, we arrive at

$$\frac{(y^\nu)^2}{a_c} \sim (1.4 - 2.8)\times 10^4, \qquad (47)$$

with the largest value corresponding to the cosmological neutrino mass bound, and the lowest to the atmospheric neutrino oscillation scale. This means that both the decomposition and coannihilation channels dominate over that of $\mathcal{X}$-quanta production. By taking, for instance, $y^\nu \sim 1$, we get $a_c \sim 10^{-4}$ in accordance with our assumptions.

Next, in order to calculate the evolution of the number density n_ν, we shall set $y^\nu = 1$ from now on. Thus, by numerically evolving Eq. (46) we find that the number density n_ν never overrides that at equilibrium $(n_\nu)_{\rm eq}$, as is shown in Fig. 1, and thus the system evolves in thermal equilibrium at early times and then leaves equilibrium to get highly suppressed due to the decay channel characterized by Eq. (41).

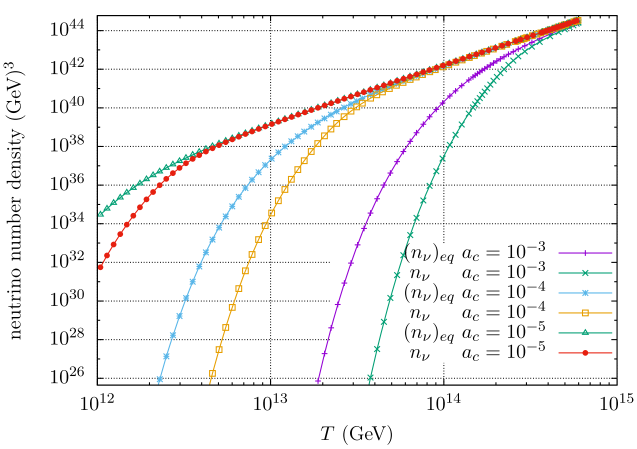

FIG. 1. The neutrino number density n_ν vs temperature driven by Eq. (46) and the equilibrium number density given in Eq. (45). Notice that the system goes out of equilibrium at early times but the number density gets strongly suppressed due to the decay term proportional to Eq. (41).

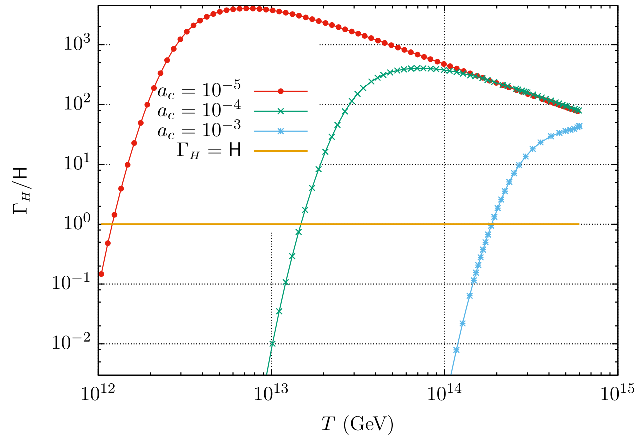

FIG. 2. The out-of-equilibrium condition given in Eq. (48) for some values of the parameter a_c. As stated in the text, the greater the mass m_k, the earlier the beginning of the out-of-equilibrium epoch, due to the decay processes into SM particles.

In Fig. 2 we plot, for a few values of the coupling a_c, the out-of-equilibrium condition

$$\Gamma_H \equiv 2\times\langle\sigma_H v_r\rangle \times n_\nu \lesssim \mathsf{H}, \qquad (48)$$

where Γ_H is the neutrino interaction rate for the Higgs channel and H is the Hubble parameter. As is shown there, because of the decay of neutrinos into Higgs bosons and leptons, the greater the coupling a_c (and so the mass m_k), the earlier the out-of-equilibrium epoch. This also shows that, as expected, coannihilation of neutrinos into SM fields is efficient enough at higher temperatures to thermalize the heavy neutrinos.

In Fig. 3 we illustrate the behavior of the system by plotting the behavior of the parameter a_c vs temperature. As said before, the inflaton decays into neutrinos ν and reheats the Universe at temperature $T_r \sim 10^{15}$ GeV; below this temperature and above the upper line that stands for the value of the mass m_k, the population of neutrinos behaves like pure radiation in thermal equilibrium and stays that way until the temperature drops into the region below the line m_k and above the line $\Gamma_H \approx \mathsf{H}$, where the system becomes nonrelativistic but still stays in thermal equilibrium. Below the bottom line the system goes out of equilibrium, and the population of neutrinos decreases due to the coannihilation into quintessence and Higgs pairs, as well as the decay into Higgs bosons and leptons. As a final note, we mention that adding extra Higgses to the model will add to the number of coannihilation channels, introducing an overall factor to the corresponding rates which will not affect our conclusions above.

Turning back to the Boltzmann equation (40), we notice that, because we are interested in maximizing the production of $\mathcal{X}$ quanta, which, if increases dangerously would state the worst possible scenario for the model, and because $(n_\nu)_{\rm eq} \gtrsim n_\nu$ [see Fig. 1], we can choose $(n_\nu)_{\rm eq}$ instead of n_ν

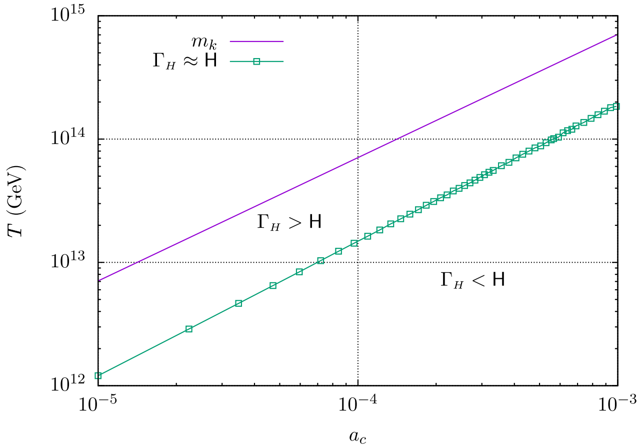

FIG. 3. The behavior of the population of Majorana sterile neutrinos as a function of the parameter a_c vs temperature. As stated in the text, between $T_r \sim 10^{15}$ GeV and $T \approx m_k$ the number density of neutrinos corresponds to that of radiation in thermal equilibrium; between $T \approx m_k$ and the temperature of equality $\Gamma_H \approx \mathsf{H}$ the system becomes nonrelativistic but still stays in thermal equilibrium. Below this line the system goes out of equilibrium and the population of neutrinos gets suppressed due to the processes of annihilation and decay populating the universe with SM particles.

in the collision term, and we can neglect the ratio $n_{\mathcal{X}}^2/(n_{\mathcal{X}})_{\rm eq}^2$ which accounts for a tiny fraction of $n_{\mathcal{X}}$ produced due to the inverse process $\mathcal{X}\mathcal{X} \to \bar{\nu}\nu$. In this way we overestimate the production of $\mathcal{X}$ quanta. As we will show next, this approximation will be enough to establish the cosmological consistency of the model, because the result does not conflict with the requirements of big bang nucleosynthesis. Then Eq. (40) becomes

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_{\mathcal{X}}) = \langle \sigma_{\mathcal{X}} v_r \rangle (n_\nu)_{\rm eq}^2. \tag{49}$$

By using Eqs. (37) and (45), the Boltzmann equation (49) becomes

$$\frac{1}{a^3}\frac{d}{dt}(a^3 n_{\mathcal{X}}) = \frac{\pi}{256} m_k^3 a_c^4 T \mathcal{I}_{\mathcal{X}}(m_k;T). \tag{50}$$

As before, after changing the time evolution in favor of the temperature, the Boltzmann equation (50) becomes

$$\frac{d}{dT}(a^3 n_{\mathcal{X}}) = -\frac{M_{\rm pl}}{256}\left(\frac{90}{g_*(T)}\right)^{1/2} m_k^3 a_c^4 \frac{a^3}{T^2}\mathcal{I}_{\mathcal{X}}(m_k;T). \tag{51}$$

Since the Universe is cooling, we perform the integration on both sides backward in T, from $T_{\rm out}$ to a certain temperature $T' < T_{\rm out}$, so we have

$$\int_{(a^3 n_{\mathcal{X}})(T_{\rm out})}^{(a^3 n_{\mathcal{X}})(T')} d(a^3 n_{\mathcal{X}}) = -\frac{M_{\rm pl}}{256}\sqrt{90} m_k^3 a_c^4 \int_{T_{\rm out}}^{T'} dT \frac{a^3(T)}{\sqrt{g_*(T)}T^2}\mathcal{I}_{\mathcal{X}}(m_k;T), \tag{52}$$

where on the rhs we have explicitly written the universal scale factor's dependence on T. Such a dependence, during the radiation-dominated age, is given by

$$a(T) = \frac{b_0}{g_{*s}^{1/3}(T)T}, \tag{53}$$

where b_0 is a constant and $g_{*s}(T)$ is the number of relativistic degrees of freedom of the entropy density at equilibrium.

When the cooling Universe reaches the temperature $T_{\rm out}$ the density $n_{\mathcal{X}}$ starts to increase, i.e., the system $\bar{\nu}\nu \leftrightarrow \mathcal{X}\mathcal{X}$ goes out of equilibrium, which is true whenever

$$\Gamma_{\mathcal{X}} \equiv 2 \times \langle \sigma_{\mathcal{X}} v_r \rangle \times n_\nu \lesssim \mathsf{H}, \tag{54}$$

where $\Gamma_{\mathcal{X}}$ is the neutrino interaction rate of the channel, which can be calculated using Eq. (37) and the numerical output of Eq. (46). It turns out that for any value of $T \lesssim T_r$, both the integral (38) and the rate $\Gamma_{\mathcal{X}}$ are very suppressed, as is shown in the Fig. 4. Then the inequality (54) is always fulfilled and we can use the temperature $T_{\rm out} \sim T_r$ as the lower limit to obtain a good estimate of the integral that appears on the rhs of Eq. (52).

Furthermore, as the initial state of the $\mathcal{X}$ field is pure vacuum and it is not coupled to the inflaton, there are no initial quanta; consequently, we can impose the condition

$$(a^3 n_{\mathcal{X}})(T_{\rm out}) = 0,$$

which together with Eq. (53) allows us to express the integral in Eq. (52) as

$$n_{\mathcal{X}}(T') = N a_c^7 g_{*s}(T') T'^3 \int_{T'}^{T_r} \frac{dT}{T^5}\frac{\mathcal{I}_{\mathcal{X}}(m_k;T)}{g_{*s}(T)\sqrt{g_*(T)}}, \tag{55}$$

where N is a constant factor given by

$$N = 2 \times \frac{M_{\rm pl}}{512}\sqrt{45}\langle \mathrm{Q}\rangle^3, \tag{56}$$

and where we have multiplied it by 2 because there are two Majorana neutrinos involved [see Eq. (32)].

By considering $g_{*s} \sim g_{*n}$, where g_{*n} is the relativistic degrees of freedom of the number density at equilibrium, the integral (55) can be written as

$$n_{\mathcal{X}}(T') = n_r(T') \times f(T'), \tag{57}$$

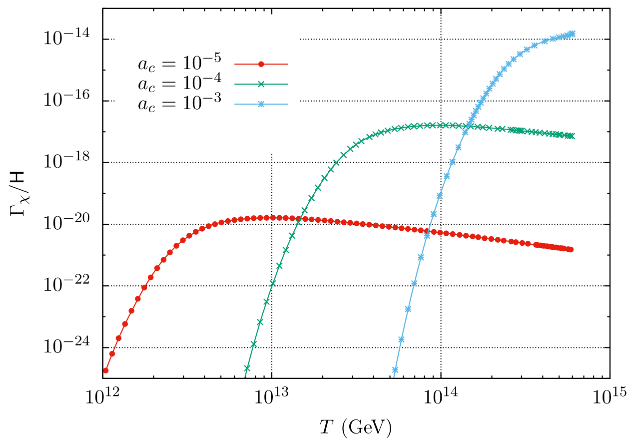

FIG. 4. The out-of-equilibrium condition given in Eq. (54) for some values of the parameter a_c. As stated in the text, the integral (38) is very suppressed, and hence the system $\bar{\nu}\nu \leftrightarrow \chi\chi$ is always out of equilibrium, even for temperatures as high as the reheating temperature.

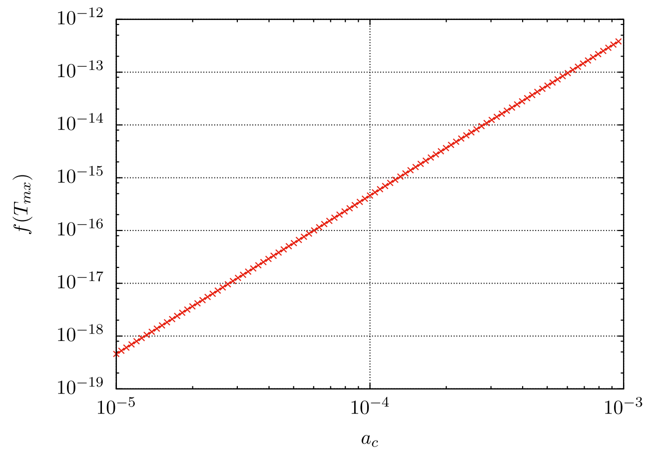

FIG. 5. The maxima of the function $f(T)$ given in Eq. (59) for different values of the parameter a_c. Notice that the integral is always less than 1 and so the increase in $n_{\rm TOT}$ given in Eq. (60) is negligible.

where $n_r(T')$ is the relativistic number density at equilibrium, given by

$$n_r(T') = \frac{\zeta(3)}{\pi^2} g_{*n}(T')T'^3, \tag{58}$$

where $\zeta(3)$ is Apéry's constant, and

$$f(T') = N a_c^7 \frac{\pi^2}{\zeta(3)} \int_{T'}^{T_r} \frac{dT}{T^5} \frac{\mathcal{I}_{\mathcal{X}}(m_k;T)}{g_{*s}(T)\sqrt{g_*(T)}}. \tag{59}$$

By means of Eq. (57) we write the total relativistic number density of our model $n_{\rm TOT}$ in terms of Eq. (59) as

$$n_{\rm TOT}(T') = n_r(T')(1 + f(T')). \tag{60}$$

The integral (59) can be calculated numerically for different values of the a_c parameter, with the result that for each value of the latter the integral depends smoothly on the temperature and it is easy to maximize.

Since $n_r(T)$ is a growing monotonic function, it is enough to know whether, for a certain a_c, the value of $f(T_{\rm mx})$ exceeds that of $n_r(T_{\rm mx})$, where $T_{\rm mx}$ is the temperature that maximizes the integral (59). What we find is that $f(T_{\rm mx})$ is always several orders of magnitude below one for any value of a_c within the range we are interested in, as shown in Fig. 5. Thus, the increase in the total relativistic number density of $\mathcal{X}$ particles due to the coannihilation of right-handed neutrinos is of no cosmological consequence. Clearly, once neutrino decay into SM fields is switched on, the actual $\mathcal{X}$ would be much smaller than the value we have just calculated. To this extent, the model appears to be consistent with the cosmological constraints.

V. THERMAL CORRECTIONS TO QUINTESSENCE MASS

As stated earlier, the Q field actually couples to light active Majorana neutrinos ν_l that emerge from the seesaw mechanism. By setting the corresponding seesaw mixing as in Eq. (32), we get the effective coupling $-\mathcal{L}_{I\mathcal{X}} = \lambda\mathcal{X}\bar{\nu}_l\nu_l + \cdots$, where

$$\lambda = \frac{a_c}{2\sqrt{2}}\left(\frac{m_D}{m_k}\right)^2.$$

Then, the thermal contribution to the quintessence potential due to the cosmological neutrino background (see, for instance, Refs. [78,79]) is given by

$$V^T(\mathrm{Q},T) = \frac{g}{48}\lambda^2 T^2 \langle \mathrm{Q}\rangle^2,$$

where g are the degrees of freedom of the Majorana neutrinos coupled to the scalar field and T corresponds to the neutrino bath temperature T_ν, which scales as

$$T_\nu = T_{\nu,0}\left(\frac{a_0}{a}\right) = (1+z)T_{\nu,0},$$

where a_0 is the current value of the scale factor a, z is the cosmological redshift, and $T_{\nu,0}$ is the current neutrino temperature, which is related to the current CMB temperature as $T_{\nu,0} = (4/11)^{1/3}T_{\gamma,0}$, and so (see Ref. [63])

$$T_{\nu,0} = 1.676 \times 10^{-4}\ \mathrm{eV}.$$

Thus, the effective potential, defined with the addition of the above thermal correction, becomes

$$V^{\rm eff}(\mathrm{Q},T) = \frac{1}{2}M_{\rm eff}^2\langle \mathrm{Q}\rangle^2,$$

where the effective mass is defined as

$$M_{\rm eff}^2 = m^2 + m_{\rm Th}^2,$$

with the *thermal mass* given by

$$m_{\rm Th}(T_\nu) = \sqrt{\frac{g}{24}}\lambda T_\nu.$$

Clearly, $m_{\rm Th}$ should scale down as $m_{\rm Th} = (1+z)m_{\rm Th,0}$ where, obviously, its current value $m_{\rm Th,0} = m_{\rm Th}(T_{\nu,0})$.

By using the corresponding mass expressions, it is straightforward to write the effective coupling in terms of Yukawa couplings and vacuum scalar values as

$$\lambda = \frac{1}{2\sqrt{2}}\frac{(y^\nu)^2}{a_c}\left(\frac{\langle H\rangle}{\langle \mathrm{Q}\rangle}\right)^2.$$

Then, by using that $\langle H\rangle/\langle \mathrm{Q}\rangle \approx 2.05\times 10^{-17}$ and the range limits imposed from neutrino masses given in Eq. (47), we obtain that the effective coupling should be in the narrow range

$$\lambda \approx (2.05\text{–}4.1)\times 10^{-30}.$$

Considering the contribution of the three active neutrinos to the internal degrees of freedom, we get $g = 6$, and thus the current thermal mass is

$$m_{\rm Th,0} \approx (1.7\text{–}3.4)\times 10^{-34}, \tag{61}$$

which is the same range needed to source the current cosmological constant [see Eq. (12)] by $\rho_{\rm DE} = \frac{1}{2}M_{\rm eff,0}^2\langle \mathrm{Q}\rangle^2$. As a further note, since λ depends linearly on the light neutrino mass $m_{\rm Th,0}$ gets closer to the required $M_{\rm eff,0}$ for larger masses. In particular, for $m_\nu \sim 0.17$ eV one already gets $m_{\rm Th,0} \sim 5.8\times 10^{-34}$ eV. This would then be the largest allowed value for the neutrino mass in our model.

A. Slow-roll condition

Next, we consider the redshift of the thermal mass to explore the evolution of the slow-roll condition to ensure that the effective mass remains smaller than the Hubble parameter throughout the Universe's history, which in turn ensures that the Q field behaves as a true DE. As it should be clear, the main concern is just the a^{-1} scaling of the thermal mass value throughout the age of the Universe.

First, during the radiation-dominated age the Hubble parameter evolves as

$$\mathsf{H}^2 = \mathsf{H}_0^2\Omega_{R,0}\left(\frac{a}{a_0}\right)^{-4}.$$

Then, during that age, the quotient between the thermal mass and the Hubble parameter becomes

$$\frac{m_{\rm Th}}{\mathsf{H}} = \frac{m_{\rm Th,0}}{\mathsf{H}_0\sqrt{\Omega_{R,0}}}\left(\frac{a}{a_0}\right).$$

By using $\mathsf{H}_0 = 1.44\times 10^{-33}$ eV and $\Omega_{R,0} = 1.4\times 10^{-3}$ together with Eq. (61), we arrive at

$$\left.\frac{m_{\rm Th}}{\mathsf{H}}\right|_{\rm RAD} \approx (3.2\text{–}6.4)\left(\frac{a}{a_0}\right), \qquad a \le a_{\rm eq},$$

where $a_{\rm eq}$ is the scale factor at radiation-matter equality. When $a = a_{\rm eq}$, we have that $(a_{\rm eq}/a_0) = 4.45\times 10^{-3}$. With this value, at the end of the radiation-dominated age

$$\left.\frac{m_{\rm Th}}{\mathsf{H}}\right|_{a=a_{\rm eq}} \approx (1.42\text{–}2.84)\times 10^{-2},$$

and thus we see that the slow-roll condition is fulfilled during throughout the radiation-dominated age.

During the matter-dominated era, the Hubble parameter evolves as

$$\mathsf{H}^2 = \mathsf{H}_0^2\Omega_{M,0}\left(\frac{a}{a_0}\right)^{-3}.$$

Then, the ratio of interest scales as

$$\frac{m_{\rm Th}}{\mathsf{H}} = \frac{m_{\rm Th,0}}{\mathsf{H}_0\sqrt{\Omega_{M,0}}}\left(\frac{a}{a_0}\right)^{1/2}.$$

By taking $\Omega_{M,0} = 0.3142$, one gets

$$\left.\frac{m_{\rm Th}}{\mathsf{H}}\right|_{\rm MAT} \approx (0.21\text{–}0.43)\left(\frac{a}{a_0}\right)^{1/2}.$$

Considering that in our model [see Eq. (63)] at the epoch of the transition to DE domination $(a_{\rm DE}/a_0) \sim 0.45$, we roughly estimate that

$$\left.\frac{m_{\rm Th}}{\mathsf{H}}\right|_{a=a_{\rm DE}} \sim (0.14\text{–}0.28).$$

This means that even during the era of the matter domination, the condition of slow-roll is still fulfilled, and this would remain so until today since thermal mass keeps scaling down as the Universe expands. As a matter of fact, Eq. (61) implies that nowadays

$$\frac{m_{\rm Th,0}}{\mathsf{H}_0} = (0.12\text{–}0.24).$$

Last but not least, by considering our above-mentioned upper bound on the neutrino mass ($m_\nu = 1.7\times 10^{-1}$ eV),

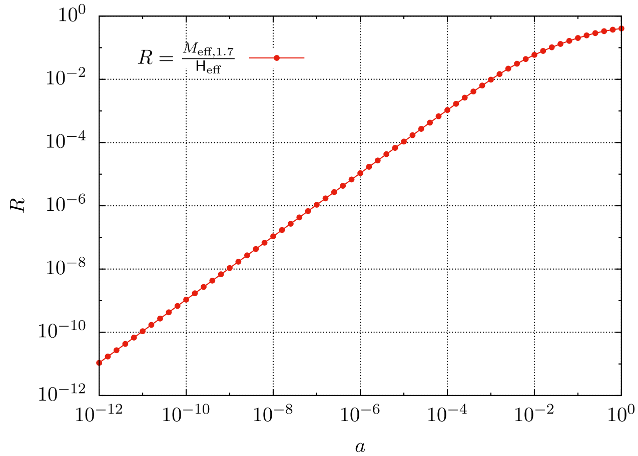

FIG. 6. The ratio R as defined in Eq. (62). The slow-roll condition is satisfied throughout the evolution of the Universe. Lower values are expected when $m_\nu \to 0.5\times 10^{-2}$.

which saturates the DE density such that $M_{\rm eff,1.7} \approx m_{\rm Th,0}$, we note that such a case provides a natural upper bound on the slow-roll condition. For this given value, we can make a more careful estimate of the evolution of the mass-to-Hubble-constant ratio

$$R = \frac{M_{\rm eff,1.7}}{\mathsf{H}_{\rm eff}} \tag{62}$$

by observing that the DE density then scales as

$$\rho_{\rm DE,1.7} = \frac{1}{2} M^2_{\rm eff,0} \left(\frac{a_0}{a}\right)^2 \langle \mathrm{Q} \rangle^2,$$

corresponding to a DE relative density parameter

$$\Omega_{\rm DE,1.7} = \frac{\rho_{\rm DE,1.7}}{\rho_{\rm crit}} = \Omega_{\rm DE,0}\left(\frac{a_0}{a}\right)^2,$$

where, by using that $\rho_{\rm crit} = 3.69\times 10^{-47}$ GeV^4, it is obtained that $\Omega^{\rm eff}_{\rm DE,0} \approx 0.685$, as expected in our model.

On the other hand, using the actual observed values of $\Omega_{R,0}$ and $\Omega_{M,0}$ mentioned above, the effective Hubble parameter to calculate R at any given a can be written as

$$\mathsf{H}^2_{\rm eff} = \mathsf{H}^2_0 \left[1.4\times 10^{-3}\left(\frac{a_0}{a}\right)^4 + 0.314\left(\frac{a_0}{a}\right)^3 + 0.685\left(\frac{a_0}{a}\right)^2\right]. \tag{63}$$

The numerical evolution of the ratio is plotted in Fig. 6, from which we can see that the slow-roll condition is satisfied throughout the evolution of the Universe.

VI. SUMMARY AND CONCLUDING REMARKS

We have presented a cosmological model that unifies early inflation and late accelerated expansion, driven by a quintessence field, where both cosmological scalar fields belong to the degrees of freedom of the same fundamental field representation Φ of the $SO(1,1)$ symmetry. This symmetry, as is usual in particle physics model building and in particular in the construction of the Standard Model, is the guiding principle that dictates and governs the dynamics of the system. It is really interesting that such a simple principle allows one to reproduce chaotic-type potentials for both inflation and DE, which are derived from considering all possible bilinear field operators based on Φ that are invariant under the symmetry. As a matter of fact, the field system of the model can be rewritten in terms of two scalar fields with independent evolutions, which in the cosmological setup will fall down a simple mass-type potential. Upon fine-tuning, one can easily understand the reason why one of these fields breaks the slow-roll condition at large scales and thus ends inflation, whereas the other stays trapped in a false vacuum configuration that we see as a cosmological constant nowadays.

The need for reheating after inflation, which requires the coupling of the inflaton to matter fields, is fulfilled by introducing a set of fermions which, in order to be consistent with the symmetry, belong to a doublet and singlet of $SO(1,1)$. Enforcing the symmetry to build the Yukawa couplings as invariant terms has two outstanding implications. First, since the cosmological field does not belong to the Standard Model particle sector, neither will the new fermions, and thus they are naturally identified as right-handed neutrinos. Second, the invariant couplings among Φ and the fermions do provide the appropriate inflaton couplings to allow inflaton decay and reheating, but they also mean that right-handed neutrinos couple to the cosmological DE field. Without any further assumptions, beyond the use of symmetries, our model introduces a way to naturally understand the existence of large sterile Majorana neutrino masses as sourced by DE which, on the other hand, is needed for the standard seesaw mechanism to work. This is the simplest known mechanism that provides very small masses to the standard neutrinos, which are required to explain neutrino oscillation phenomena.

Here we have studied in some detail the mechanism contained in the $SO(1,1)$ cosmological unification model that underlies the generation of neutrino masses. Our analysis shows that the origin of the mass is independent of the field phases and their dynamics. However, it may not be the only possible mechanism in nature, as the $SO(1,1)$ symmetry does not prohibit writing an independent mass associated with any singlet fermion. Such a mass seems unnatural since there is no *a priori* mass scale associated with it, an issue already present in the seesaw mechanism. Nevertheless, as we have argued, such a mass can easily be removed if additional global symmetries are involved in the fermion sector. In such a scenario, DE arises as the natural source of such a neutrino mass, through its false vacuum energy that supports the current accelerated expansion of the Universe. As the scale of this expansion is quite large, the right-handed neutrinos would have masses of order $\sim 10^{14}$ GeV.

Our study has also looked at the possible impact that the model—and in particular quintessence quanta $\mathcal{X}$—may have on the thermal history of the Universe. The inflaton in the model does not couple to the quintessence field, and thus it does not inject entropy through that channel upon decay. As a matter of fact, the only allowed decay channel for the inflaton is into heavy right-handed neutrinos, which would eventually create the primordial plasma through Higgs and Standard Model lepton couplings of the form $\bar{L}\tilde{H}N$. After this, we expect the standard thermal history to proceed as usual, except for the possible entropy contributions from the right-handed neutrinos in the form of quintessence quanta through out-of-equilibrium coannihilation processes ($\bar{\nu}\nu \to \mathcal{X}\mathcal{X}$). To further estimate this effect, we have calculated the thermally averaged cross section for the process, which depends on the same Yukawa coupling that provides neutrino masses, a_c. As discussed in this paper, the numerical integration of the Boltzmann equations with a_c varying over a wide range of values shows that the process is so suppressed that, the total contribution to the total primordial plasma number density (due to injection of quintessence quanta) is negligible. This clearly indicates that the model, without any further constraints or assumptions, remains consistent with the conditions required for successful big bang nucleosynthesis.

The present model uses complex scalars to realize the symmetry and thus it involves dynamical phases, but we have not explored their possible role in the Universe's evolution. Our analysis does show that they are not potentially relevant for the after-inflation evolution, provided the initial conditions fix them to zero, at least for the mechanism that generates neutrino masses and the production of quintessence quanta. However, other roles may be possible that would be interesting to look at.

Our analysis also explored the thermal corrections of the mass of Q coming from interactions with the thermal bath of neutrinos permeating the Universe. By calculating the slow-roll condition with these corrections, we found that the Q field preserves its DE behavior. At the present epoch, the thermal mass coincides with that commonly required in Q models.

As a final comment, we note that one of the issues that remains to be explored in detail to make the model more realistic is the phenomenology concerning the $SO(1,1)$ flavor sector, introduced above as the connection of our heavy neutrinos with the Standard Model particles. We have shown that heavy neutrinos and Standard Model particles should be assigned into a set of doublets and singlets under $SO(1,1)$; thus, it would be interesting to explore if such symmetry may account for the masses and mixings of the light neutrinos as well.

ACKNOWLEDGMENTS

This work was done in part with financial support from Consejo Nacional de Ciencia y Tecnología, Mexico, under Grant No. 237004.

APPENDIX A: DIAGONALIZATION OF THE LAGRANGIAN

In this Appendix, we present in detail the diagonalization analysis of our model Lagrangian, whose results are used in the main text. First, we consider the scalar sector, whose Lagrangian (4) in terms of the doublet complex field components becomes

$$\mathcal{L}_\Phi = \partial^\mu \phi^* \partial_\mu \phi + \partial^\mu \varphi^* \partial_\mu \varphi - V(\phi, \varphi),$$

with the potential

$$V(\phi, \varphi) = \alpha_0(|\phi|^2 + |\varphi|^2) + \alpha_1(\phi^* \varphi + \varphi^* \phi) + \alpha_3(\phi^2 - \varphi^2) + \text{c.c.} \tag{A1}$$

Next, we rewrite the Lagrangian in terms of the Hermitian base

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \qquad \varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2),$$

where $\phi_i, \varphi_i, i = 1, 2$ are real scalar fields. This lets us put the potential in a matrix form which we will diagonalize in order to identify physical fields having separated dynamics. The potential (A1) becomes

$$V = \frac{1}{2}\Phi_R^T A \Phi_R,$$

with Φ_R being the vector formed from the above real scalar field components of ϕ and φ, given by $\Phi_R^T = (\phi_1, \phi_2, \varphi_1, \varphi_2)$, and A is the 4×4 mass coupling matrix

$$A = \begin{pmatrix} m_1^2 & \lambda^2 & \mu_1^2 & 0 \\ \lambda^2 & m_2^2 & 0 & \mu_1^2 \\ \mu_1^2 & 0 & m_2^2 & -\lambda^2 \\ 0 & \mu_1^2 & -\lambda^2 & m_1^2 \end{pmatrix},$$

where we have defined

$$m_1^2 = \mu_0^2 + \mu_3^2, \qquad m_2^2 = \mu_0^2 - \mu_3^2, \qquad \lambda^2 = 2\text{Re}(i\alpha_3)$$

and

$$\mu_0^2 \equiv 2\mathrm{Re}(\alpha_0), \qquad \mu_1^2 \equiv 2\mathrm{Re}(\alpha_1), \qquad \mu_3^2 = 2\mathrm{Re}(\alpha_3).$$

Notice that by definition all of the involved mass terms (m_1^2, m_2^2, λ^2, μ_0^2, μ_3^2 and μ_1^2) are real, and by construction we have made them positive.

Since the matrix A is real and symmetric, by means of a proper orthogonal rotation of the field base $\mathbb{S}$, through which we redefine

$$\Phi_D = \mathbb{S}\Phi_R, \qquad A_D = \mathbb{S}A\mathbb{S}^T,$$

we should get a diagonal mass sector. It is not difficult to check that such a matrix can be expressed as

$$\mathbb{S} = (\mathbb{I}_{2\times 2} \otimes \mathbb{B} - i\sigma_2 \otimes \mathbb{H})\cos(\omega),$$

where

$$\mathbb{B} = \begin{pmatrix} \cos(\rho) & 0 \\ 0 & \cos(\rho) \end{pmatrix}, \qquad \mathbb{H} = \begin{pmatrix} \tan(\omega) & \sin(\rho) \\ \sin(\rho) & -\tan(\omega) \end{pmatrix}.$$

In the above, we have made use of the shorthand notation

$$\cos(\rho) = \frac{\mu_1^2}{\sqrt{\mu_1^4 + \lambda^4}}, \qquad \sin(\rho) = \frac{\lambda^2}{\sqrt{\mu_1^4 + \lambda^4}},$$
$$\cos(\omega) = \frac{\alpha^2}{\sqrt{2h^2(h^2 + \Delta^2)}}, \qquad \sin(\omega) = \frac{\alpha^2}{\sqrt{2h^2(h^2 - \Delta^2)}},$$

and

$$\alpha^4 = 4(\mu_1^4 + \lambda^4), \qquad \Delta^2 = m_1^2 - m_2^2, \qquad h^4 = \Delta^4 + \alpha^4.$$

After performing the $\mathbb{S}$ rotation, the potential becomes

$$V = \frac{1}{2}\Phi_D^T A_D \Phi_D,$$

with $\Phi_D^T = (\mathrm{Q}_1, \xi_1, \xi_2, \mathrm{Q}_2)^T$ and

$$A_D = \mathrm{diag}\Big(m^2, \quad M^2, \quad M^2, \quad m^2 \Big),$$

where the eigenvalues m^2 and M^2 are given by

$$m^2 = \mu_0^2 - \mu^2 \quad \text{and} \quad M^2 = \mu_0^2 + \mu^2, \qquad \text{(A2)}$$

where $\mu^2 = \sqrt{\mu_3^4 + \mu_1^4 + \lambda^4}$. In terms of the α couplings, we get $\mu_0^2 = 2\mathrm{Re}\alpha_0$ and $\mu^2 = 2\sqrt{(\mathrm{Re}\alpha_1)^2 + |\alpha_3|^2}$.

The requirement that $M^2, m^2 > 0$, which guarantees that the potential is bounded from below, is fulfilled if $\mu_0^2 > \mu^2 > 0$. If both parameters were of the same order, $\mu_0^2 \approx \mu^2 > 0$, we would naturally get $M^2 \gg m^2 \approx 0$. In such a scenario it becomes natural to identify ξ with the inflaton and Q with the DE field, provided M is as large as the inflation scale.

Notice that the mass eigenstates in Φ_D can be ordered in a more natural way with the permutation matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

such that $(\mathrm{Q}_1, \mathrm{Q}_2, \xi_1, \xi_2)^T = \mathbb{S}'\Phi_R$, with $\mathbb{S}' = P\mathbb{S}$.

In terms of the diagonal base and given that there are two degenerate scalar degrees of freedom for each mass, the potential can finally be expressed as

$$V = m^2|\mathrm{Q}|^2 + M^2|\xi|^2, \qquad \text{(A3)}$$

where we have introduced the new complex scalar fields

$$\mathrm{Q} = \frac{1}{\sqrt{2}}(\mathrm{Q}_1 + i\mathrm{Q}_2) \quad \text{and} \quad \xi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2). \qquad \text{(A4)}$$

Analogously, after the $\mathbb{S}'$ rotation of Φ_R the scalar kinetic term can be easily put in terms of the new fields to get the diagonal terms $\partial^\mu \mathrm{Q}^* \partial_\mu \mathrm{Q} + \partial^\mu \xi^* \partial_\mu \xi$.

Finally, by introducing the doublet

$$\boldsymbol{\varphi} = \begin{pmatrix} \mathrm{Q} \\ \xi \end{pmatrix}, \qquad \text{(A5)}$$

the whole Lagrangian of the scalar sector becomes

$$\mathcal{L}_{\boldsymbol{\varphi}} = \partial^\mu \boldsymbol{\varphi}^\dagger \partial_\mu \boldsymbol{\varphi} - \boldsymbol{\varphi}^\dagger \mathbb{M} \boldsymbol{\varphi}, \qquad \text{(A6)}$$

where $\mathbb{M}$ is the diagonal mass matrix

$$\mathbb{M} = \begin{pmatrix} m^2 & 0 \\ 0 & M^2 \end{pmatrix}. \qquad \text{(A7)}$$

We should emphasize that this new doublet notation is not a faithful representation of $SO(1,1)$, since the $SO(4)$ rotation $\mathbb{S}'$ and the $SO(1,1)$ transformations do not commute. Therefore, the diagonal Lagrangian (A7), which provides the decoupled field system that evolves and thus explains inflation and the late-time accelerated expansion of the Universe, is not explicitly invariant under $SO(1,1)$, even though the original model is.

Let us now move on to analyzing the fermion sector of the theory, for which the corresponding kinetic terms, as given in Eq. (18), are

$$\mathcal{L}_{N_i} = \sum_{i=0}^{2} N_i^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu N_i^{\dot{c}}, \qquad \text{(A8)}$$

and the interaction terms (19) take the form

$$-\mathcal{L}_I = N_{0\dot{a}}\{a_0(\phi^* N_1^{\dot{a}} + \varphi^* N_2^{\dot{a}}) + a_1(\phi^* N_2^{\dot{a}} + \varphi^* N_1^{\dot{a}}) + a_2(\phi N_2^{\dot{a}} - \varphi N_1^{\dot{a}}) + a_3(\phi N_1^{\dot{a}} - \varphi N_2^{\dot{a}})\} + \text{H.c.} \tag{A9}$$

Last, written in terms of the real field components in Φ_R, this leads to

$$-\mathcal{L}_I = \frac{1}{\sqrt{2}} N_{0\dot{a}} \Phi_R^T \{\mathbb{V} N_1^{\dot{a}} + \mathbb{\Gamma}\mathbb{V} N_2^{\dot{a}}\} + \text{H.c.}, \tag{A10}$$

where $\mathbb{V}$ is the vector formed from the complex couplings a_i,

$$\mathbb{V} = \begin{pmatrix} a_3 + a_0 \\ i(a_3 - a_0) \\ a_1 - a_2 \\ -i(a_1 + a_2) \end{pmatrix},$$

and $\mathbb{\Gamma}$ is a 4×4 matrix given by $\mathbb{\Gamma} = -\sigma_1 \otimes \sigma_2$. After performing the $\mathbb{S}$ rotation in the scalar sector, and noticing that $\mathbb{\Gamma}$ is actually an invariant matrix (since $\mathbb{\Gamma} = \mathbb{S}\mathbb{\Gamma}\mathbb{S}^T$), the interaction Lagrangian becomes

$$-\mathcal{L}_I = \frac{1}{\sqrt{2}} N_{0\dot{a}} \Phi_D^T \{\mathbb{V}' N_1^{\dot{a}} + \mathbb{\Gamma}\mathbb{V}' N_2^{\dot{a}}\} + \text{H.c.}, \tag{A11}$$

where $\mathbb{V}' = \mathbb{S}\mathbb{V}$.

It is important to note that $\mathbb{V}'$ just corresponds to a redefinition of the Yukawa couplings, for which one can always assume a convenient parametrization, implicitly defined in terms of the initial $a_{i=0,\ldots,3}$ couplings. Hence, using this freedom we choose the following combinations to define the couplings in the rotated scalar base:

$$\mathbb{V}' = \frac{1}{\sqrt{2}} \begin{pmatrix} g_1 + g_2 \\ h_1 - h_2 \\ -i(h_1 + h_2) \\ i(g_1 - g_2) \end{pmatrix}, \tag{A12}$$

where $g_{i=1,2}$ and $h_{i=1,2}$ are complex numbers. Substituting the last expression and the redefinition of the scalar fields given in Eq. (A4) into Eq. (A11), and after some simple algebra, we finally rewrite the interaction terms as

$$-\mathcal{L}_I = N_{0\dot{a}}\{g_1 \mathrm{Q} F_1^{\dot{a}} + g_2 \mathrm{Q}^* F_2^{\dot{a}} + h_1 \xi^* F_1^{\dot{a}} - h_2 \xi F_2^{\dot{a}}\} + \text{H.c.}, \tag{A13}$$

where the new Weyl fields $F_{i=1,2}^{\dot{a}}$ are the components of the doublet

$$\mathbf{F} = \begin{pmatrix} F_1^{\dot{a}} \\ F_2^{\dot{a}} \end{pmatrix}, \tag{A14}$$

which in turn comes from the transformation

$$e^{-i\sigma_2 \pi/4} \mathbf{\Psi} = \mathbf{F}, \tag{A15}$$

i.e., the diagonalization of the scalar potential through $\mathbb{S}$ imposes an $SO(2)$ rotation on the doublet Eq. (13) by an angle $\pi/4$. Note that we can still define the $U(1)$ global transformation used in Eq. (22) with the same charge for the new Weyl fields as $\mathbf{F} \to e^{iq}\mathbf{F}$, and so this convenient transformation does not alter the argument used to remove the mass of N_0 in the main text. Nevertheless, as for the scalar sector, the transformations used to rewrite the interactions hide the $SO(1,1)$ symmetry of the theory; however, they allow us to write Eq. (A13) in a simple and compact way,

$$-\mathcal{L}_I = N_{0\dot{a}}\{\boldsymbol{\varphi}^\dagger \mathbb{G}_1 \mathbf{F} + \boldsymbol{\varphi}^T \mathbb{G}_2 \mathbf{F}\} + \text{H.c.}, \tag{A16}$$

where we have defined the coupling matrices as

$$\mathbb{G}_1 = \begin{pmatrix} 0 & g_2 \\ h_1 & 0 \end{pmatrix}, \qquad \mathbb{G}_2 = \begin{pmatrix} g_1 & 0 \\ 0 & -h_2 \end{pmatrix}. \tag{A17}$$

Finally, notice that the transformation given in Eq. (A15) keeps the diagonal form of fermion kinetic terms, as expected, which can now be expressed as

$$\mathcal{L}_F = N_0^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu N_0^{\dot{c}} + \mathbf{F}^\dagger i\sigma^\mu \partial_\mu \mathbf{F}. \tag{A18}$$

APPENDIX B: INCLUDING PHASE FIELDS IN THE $SO(1,1)$ MODEL

Here we explore some of the possible effects that using dynamical phase fields for the cosmological scalars may have on the outcomes of the model discussed in the main text, as well as other interesting aspects that we believe might be of further interest for field dynamics. For this, we assume that after reheating the Q field remains dynamically trapped in a homogeneous and isotropic false vacuum configuration, which sources DE and breaks the $U(1)$ global symmetry in the neutrino sector, whereas the inflaton field ξ has already settled to its null value, and thus quantum perturbations for our cosmological scalar fields can be conveniently introduced in a polar base as

$$\mathrm{Q} = \frac{(\langle \mathrm{Q} \rangle + \mathcal{X})}{\sqrt{2}} e^{i\vartheta/\langle \mathrm{Q} \rangle}, \qquad \xi = \frac{1}{\sqrt{2}} |\xi| e^{i\vartheta/\langle \mathrm{Q} \rangle}, \tag{B1}$$

where the degrees of freedom of the complex scalar field Q are now given by the real scalar field $\mathcal{X}$ and the dynamical phase ϑ. Similarly, the degrees of freedom of ξ are given by its modulus and dynamical phase θ.

Next, we proceed to rewrite the Lagrangian of our model in terms of the above parametrization. To do this we first notice that the doublet (A5) can be written as

$$\boldsymbol{\varphi} = \mathbb{P}\boldsymbol{\varphi}_R, \tag{B2}$$

where we have defined the radial field part as

$$\boldsymbol{\varphi}_R = \frac{1}{\sqrt{2}}\begin{pmatrix} \langle \mathrm{Q}\rangle + \mathcal{X} \\ |\xi| \end{pmatrix} \tag{B3}$$

and the field phase matrix is given by

$$\mathbb{P} = \begin{pmatrix} e^{i\vartheta/\langle \mathrm{Q}\rangle} & 0 \\ 0 & e^{i\theta/\langle \mathrm{Q}\rangle} \end{pmatrix}. \tag{B4}$$

By substituting Eq. (B2) into the scalar sector of the theory, it is straightforward to see that the Lagrangian (A6) simply becomes

$$\mathcal{L}_{\boldsymbol{\varphi}} = \partial^\mu \boldsymbol{\varphi}_R^T \partial_\mu \boldsymbol{\varphi}_R + \boldsymbol{\varphi}_R^T \mathbb{M} \boldsymbol{\varphi}_R + \mathcal{T}(\boldsymbol{\varphi}_R, \mathbb{P}), \tag{B5}$$

where $\mathcal{T}(\boldsymbol{\varphi}_R, \mathbb{P}) = \boldsymbol{\varphi}_R^T(\partial^\mu \mathbb{P}^\dagger)(\partial_\mu \mathbb{P})\boldsymbol{\varphi}_R$ is a dimension-six and highly suppressed operator. Thus, we do not expect it to be relevant for the later dynamics of DE.

Explicitly, in terms of the inflaton and DE fields, the above Lagrangian reads

$$\mathcal{L}_{\boldsymbol{\varphi}} = \frac{1}{2}\partial^\mu|\xi|\partial_\mu|\xi| + \frac{1}{2}\partial^\mu \mathcal{X}\partial_\mu \mathcal{X} + \frac{1}{2}m^2(\langle \mathrm{Q}\rangle + \mathcal{X})^2 + \frac{1}{2}M^2|\xi|^2 + \mathcal{T}_{(\xi,\mathcal{X},\vartheta,\theta)}, \tag{B6}$$

where the last term on the rhs is given by

$$\mathcal{T}_{(\xi,\mathcal{X},\vartheta,\theta)} = \frac{|\xi|^2}{2\langle \mathrm{Q}\rangle^2}\partial^\mu\theta\partial_\mu\theta + \frac{1}{2}\left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q}\rangle}\right)^2 \partial^\mu\vartheta\partial_\mu\vartheta. \tag{B7}$$

The interaction with fermions given by Eq. (A16) is now written as

$$-\mathcal{L}_I = N_{0\dot{a}}\boldsymbol{\varphi}_R^T\{\mathbb{P}^\dagger \mathbb{G}_1 + \mathbb{P}^T\mathbb{G}_2\}\mathbf{F} + \mathrm{H.c.} \tag{B8}$$

$$= N_{0\dot{a}}\boldsymbol{\varphi}_R^T \mathbb{G}\mathbf{F}' + \mathrm{H.c.}, \tag{B9}$$

where the new coupling matrix is given by

$$\mathbb{G} = \begin{pmatrix} g_1 & g_2 \\ h_1 e^{-i(\theta+\vartheta)/\langle \mathrm{Q}\rangle} & -h_2 e^{i(\theta+\vartheta)/\langle \mathrm{Q}\rangle} \end{pmatrix}, \tag{B10}$$

and where we have performed a local phase transformation over the fermions in the doublet to introduce

$$\mathbf{F}' = \begin{pmatrix} F_1'^{\dot{a}} \\ F_2'^{\dot{a}} \end{pmatrix}, \tag{B11}$$

with $F_1'^{\dot{a}} = e^{i\vartheta/\langle \mathrm{Q}\rangle}F_1^{\dot{a}}$ and $F_2'^{\dot{a}} = e^{-i\vartheta/\langle \mathrm{Q}\rangle}F_2^{\dot{a}}$. This redefinition of the fermion fields removes the dynamical phases in the $\mathcal{X}$ sector, as can be seen from Eq. (B10). Nonetheless, they will reappear as currents coming from the transformation of the kinetic terms (A18), which now read as

$$\mathcal{L}_F = N_0^{\dagger a} i\sigma^\mu_{a\dot{c}}\partial_\mu N_0^{\dot{c}} + \mathbf{F}'^\dagger i\sigma^\mu\partial_\mu \mathbf{F}' + \frac{\partial_\mu\vartheta}{\langle \mathrm{Q}\rangle}\mathbf{F}'^\dagger\sigma^\mu\sigma_3\mathbf{F}',$$

where in the last term the effect of σ_3 is to switch the sign of the lower entry of the doublet. Notice that once again the phase field enters in a suppressed way. Apart from these new terms where the phase fields are explicit, the part of the Lagrangian that matters for the model remains the same.

1. Revisiting massive neutrino base

Let us now perform a new transformation with the aim of removing the constant phases of the couplings g_1 and g_2 appearing in Eq. (B10), by means of a $SU(2)$ rotation of the doublet fermion sector

$$\boldsymbol{\eta} = \mathbb{R}\mathbf{F}' = \begin{pmatrix} \eta_1^{\dot{a}} \\ \eta_2^{\dot{a}} \end{pmatrix}, \tag{B12}$$

with

$$\mathbb{R} = \frac{1}{a_c}\begin{pmatrix} g_1 & g_2 \\ -g_2* & g_1* \end{pmatrix}, \tag{B13}$$

where $a_c = \sqrt{|g_1|^2 + |g_2|^2}$. After this rotation, the interaction term (B9) becomes

$$-\mathcal{L}_I = N_{0\dot{a}}\boldsymbol{\varphi}_R^T \mathbb{G}'\boldsymbol{\eta} + \mathrm{H.c.}, \tag{B14}$$

where now the coupling matrix is

$$\mathbb{G}' = \mathbb{G}\mathbb{R}^\dagger = \begin{pmatrix} a_c & 0 \\ C_1(\theta,\vartheta) & C_2(\theta,\vartheta) \end{pmatrix}. \tag{B15}$$

In the above we have used the shorthand notation

$$C_1(\theta,\vartheta) = (g_{11}e^{-i(\theta+\vartheta)/\langle \mathrm{Q}\rangle} - g_{22}e^{i(\theta+\vartheta)/\langle \mathrm{Q}\rangle})/a_c,$$
$$C_2(\theta,\vartheta) = -(g_{12}e^{i(\theta+\vartheta)/\langle \mathrm{Q}\rangle} + g_{21}e^{-i(\theta+\vartheta)/\langle \mathrm{Q}\rangle})/a_c,$$

where $g_{11} = g_1^* h_1$, $g_{22} = g_2^* h_2$, $g_{12} = g_1 h_2$, and $g_{21} = g_2 h_1$. On the other hand, upon the same rotation, the fermion kinetic terms are now written as

$$\mathcal{L}_F = N_0^{\dagger a} i\sigma^\mu_{a\dot{c}}\partial_\mu N_0^{\dot{c}} + \boldsymbol{\eta}^\dagger i\sigma^\mu\partial_\mu\boldsymbol{\eta} + \frac{\partial_\mu\vartheta}{\langle \mathrm{Q}\rangle}\boldsymbol{\eta}^\dagger\sigma^\mu\mathbb{Y}\boldsymbol{\eta}, \tag{B16}$$

where $\mathbb{Y}$ is a coupling matrix that comes from the transformation of σ_3 under Eq. (B13), given by

$$\mathbb{Y} = \begin{pmatrix} y_1 & -y_2 \\ -y_2^* & -y_1 \end{pmatrix},$$

where $y_1 = (|g_1|^2 - |g_2|^2)/a_c^2$ and $y_2 = 2g_1 g_2/a_c^2$, i.e., $y_1 \in \mathbb{R}$ and $y_2 \in \mathbb{C}$. (Notice that $y_1^2 + |y_2|^2 = 1$.)

Let us now concentrate our analysis on the interaction among neutrinos and the DE field, which after the above mathematical manipulations has the simple expression

$$-\mathcal{L}_{\nu\mathcal{X}} = \frac{a_c}{\sqrt{2}}(\langle \mathrm{Q} \rangle + \mathcal{X})\{N_{0\dot{a}}\eta_1^{\dot{a}} + \mathrm{H.c.}\}. \tag{B17}$$

The part between braces can also be expressed as

$$\begin{aligned} N_{0\dot{a}}\eta_1^{\dot{a}} + \mathrm{H.c.} &= N_{0\dot{a}}\eta_1^{\dot{a}} + \eta_1^{\dagger a} N_{0a}^{\dagger} \\ &= \frac{1}{2}\{N_{0\dot{a}}\eta_1^{\dot{a}} + N_{0\dot{a}}\eta_1^{\dot{a}} + \eta_1^{\dagger a} N_{0a}^{\dagger} + \eta_1^{\dagger a} N_{0a}^{\dagger}\} \\ &= \frac{1}{2}\{N_{0\dot{a}}\eta_1^{\dot{a}} + \eta_{1\dot{a}} N_0^{\dot{a}} + \eta_1^{\dagger a} N_{0a}^{\dagger} + N_0^{\dagger a}\eta_{1a}^{\dagger}\}, \end{aligned} \tag{B18}$$

where for both the second and fourth terms in the last line we have used the anticommutation properties plus an extra minus sign coming from the change from ${}^{\dot{a}}{}_{\dot{a}}$ to ${}_{\dot{a}}{}^{\dot{a}}$ (and similarly for the undotted indices). Now we define two four-component Dirac neutrinos as

$$u_1 = \begin{pmatrix} N_{0a}^{\dagger} \\ \eta_1^{\dot{a}} \end{pmatrix}, \qquad u_2 = \begin{pmatrix} \eta_{1a}^{\dagger} \\ N_0^{\dot{a}} \end{pmatrix}, \tag{B19}$$

in terms of which the last line in Eq. (B18) can be written as

$$N_{0\dot{a}}\eta_1^{\dot{a}} + \mathrm{H.c.} = \frac{1}{2}\{\bar{u}_1 u_1 + \bar{u}_2 u_2\}. \tag{B20}$$

As it can be seen from Eq. (B19), the neutrinos u_1 and u_2 are charge conjugates, which allows us to express them in terms of two Majorana neutrinos ν_1 and ν_2 via another rotation, which is given by

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}. \tag{B21}$$

Therefore, Eq. (B20) directly becomes

$$N_{0\dot{a}}\eta_1^{\dot{a}} + \mathrm{H.c.} = \frac{1}{2}\{\bar{\nu}_1\nu_1 + \bar{\nu}_2\nu_2\}, \tag{B22}$$

which explicitly provides the neutrino mass eigenstates, with the mass given by

$$m_k = \frac{a_c \langle \mathrm{Q} \rangle}{\sqrt{2}}. \tag{B23}$$

Notice that this same rearrangement of the neutrinos provides the interaction Lagrangian with $\mathcal{X}$ fields,

$$-\mathcal{L}_{I\mathcal{X}} = \frac{a_c}{2\sqrt{2}}\mathcal{X}(\bar{\nu}_1\nu_1 + \bar{\nu}_2\nu_2), \tag{B24}$$

which we use in our discussions throughout the paper. We stress that these results are independent of the phase fields and link the origin of the heavy right-handed neutrino masses with DE, as argued in the main text.

As a final note on this subject, notice that the Majorana neutrinos, in four-component notation, can be expressed as

$$\nu_i = \begin{pmatrix} \mathrm{K}_{ia}^{\dagger} \\ \mathrm{K}_i^{\dot{a}} \end{pmatrix}, \quad i = 1, 2. \tag{B25}$$

In the last equation, we have introduced the new right-handed Weyl field in two-component notation: $\mathrm{K}_{i=1,2}^{\dot{a}}$. Note that the transformation (B21) together with Eq. (B19) is equivalent to the transformations

$$\begin{pmatrix} \mathrm{K}_1^{\dot{a}} \\ \mathrm{K}_2^{\dot{a}} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\begin{pmatrix} N_0^{\dot{a}} \\ \eta_1^{\dot{a}} \end{pmatrix} \tag{B26}$$

and

$$\begin{pmatrix} \mathrm{K}_{1a}^{\dagger} \\ \mathrm{K}_{2a}^{\dagger} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\begin{pmatrix} \eta_{1a}^{\dagger} \\ N_{0a}^{\dagger} \end{pmatrix}. \tag{B27}$$

It is important to remark that these transformations do not respect the $U(1)$ invariance of the fermionic sector since they mix fields with different global charges.

In summary, we can either substitute Eq. (B25) into Eq. (B22) or directly operate over Eq. (B18) with Eqs. (B26) and (B27) to get

$$N_{0\dot{a}}\eta_1^{\dot{a}} + \mathrm{H.c.} = \frac{1}{2}\{\mathrm{K}_{1\dot{a}}\mathrm{K}_1^{\dot{a}} + \mathrm{K}_{2\dot{a}}\mathrm{K}_2^{\dot{a}}\} + \mathrm{H.c.} \tag{B28}$$

By substituting Eq. (B28) into Eq. (B17), one gets the mass terms

$$-\mathcal{L}_m = \frac{1}{2}m_k(\mathrm{K}_{1\dot{a}}\mathrm{K}_1^{\dot{a}} + \mathrm{K}_{2\dot{a}}\mathrm{K}_2^{\dot{a}}) + \mathrm{H.c.}, \tag{B29}$$

with the mass given as before and the interaction term

$$-\mathcal{L}_{I\mathcal{X}} = \frac{a_c}{2\sqrt{2}}\mathcal{X}(\mathrm{K}_{1\dot{a}}\mathrm{K}_1^{\dot{a}} + \mathrm{K}_{2\dot{a}}\mathrm{K}_2^{\dot{a}}) + \mathrm{H.c.} \tag{B30}$$

On the same footing and for future use, we use the inflaton-neutrino interactions [as derived from Eq. (B14)] with the redefinition $\mathrm{K}_3^{\dot{a}} \equiv \eta_2^{\dot{a}}$ to write

$$\begin{aligned} -\mathcal{L}_g = {} & \frac{1}{4}C_1(\theta,\vartheta)|\xi|(\mathrm{K}_{1\dot{a}}\mathrm{K}_1^{\dot{a}} + \mathrm{K}_{2\dot{a}}\mathrm{K}_2^{\dot{a}}) \\ & + \frac{1}{2\sqrt{2}}C_2(\theta,\vartheta)|\xi|(\mathrm{K}_{1\dot{a}} - i\mathrm{K}_{2\dot{a}})\mathrm{K}_3^{\dot{a}} + \mathrm{H.c.} \end{aligned} \tag{B31}$$

Similarly, by expanding Eq. (B16) and using the transformation (B26), whereas the kinetic terms for $\mathrm{K}^{\dot{a}}_{i=1,2,3}$ remain as usual,

$$\mathcal{L}_K = \sum_{i=1}^{3} \mathrm{K}_i^{\dagger a} i\sigma^\mu_{a\dot{c}} \partial_\mu \mathrm{K}_i^{\dot{c}}, \tag{B32}$$

the current couplings among the phase scalar $\partial_\mu \vartheta$ and the neutrinos are

$$\mathcal{L}_c = \mathcal{L}_{c_1} + \mathcal{L}_{c_2}, \tag{B33}$$

where

$$\mathcal{L}_{c_1} = y_1 \frac{\partial_\mu \vartheta}{\langle \mathrm{Q} \rangle} \left\{ \frac{1}{2} (\mathrm{K}_1^{\dagger a} \sigma^\mu_{a\dot{c}} \mathrm{K}_1^{\dot{c}} + \mathrm{K}_2^{\dagger a} \sigma^\mu_{a\dot{c}} \mathrm{K}_2^{\dot{c}}) + \frac{i}{2} (K_1^{\dagger a} \sigma^\mu_{a\dot{c}} \mathrm{K}_2^{\dot{c}} - \mathrm{K}_2^{\dagger a} \sigma^\mu_{a\dot{c}} \mathrm{K}_1^{\dot{c}}) - \mathrm{K}_3^{\dagger a} \sigma^\mu_{a\dot{c}} \mathrm{K}_3^{\dot{c}} \right\}, \tag{B34}$$

and

$$\mathcal{L}_{c_2} = -\frac{\partial_\mu \vartheta}{\langle \mathrm{Q} \rangle} \left\{ \frac{y_2}{\sqrt{2}} (\mathrm{K}_1^{\dagger a} - i\mathrm{K}_2^{\dagger a}) \sigma^\mu_{a\dot{c}} \mathrm{K}_3^{\dot{c}} + \mathrm{H.c.} \right\}. \tag{B35}$$

2. Energy density and equations of motion for the DE sector

We close this Appendix by presenting the results of the calculation of the equation-of-state parameter for DE in the present model. For this purpose, we make explicit use of the model Lagrangian defined in Eq. (B6), with the DE part written as

$$\mathcal{L}_{\mathcal{X},\vartheta} = \frac{1}{2} \partial^\mu \mathcal{X} \partial_\mu \mathcal{X} + \frac{1}{2} \left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 \partial^\mu \vartheta \partial_\mu \vartheta + V(\mathcal{X}), \tag{B36}$$

where the potential is defined as

$$V(\mathcal{X}) = \frac{1}{2} m^2 (\langle \mathrm{Q} \rangle + \mathcal{X})^2. \tag{B37}$$

From Eq. (B36) and by calculating the energy-momentum tensor in a FLRW universe, we obtain both the energy density and the pressure in terms of $\mathcal{X}$ and the phase ϑ. These are given by

$$\rho_{\mathrm{DE}} = \frac{1}{2} \dot{\mathcal{X}}^2 + \frac{1}{2} \left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 \dot{\vartheta}^2 + \frac{1}{2a^2} (\nabla \mathcal{X})^2 + V(\mathcal{X}) + \frac{1}{2a^2} \left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 (\nabla \vartheta)^2 \tag{B38}$$

and

$$P_{\mathrm{DE}} = \frac{1}{2} \dot{\mathcal{X}}^2 + \frac{1}{2} \left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 \dot{\vartheta}^2 - \frac{1}{6a^2} (\nabla \mathcal{X})^2 - V(\mathcal{X}) - \frac{1}{6a^2} \left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 (\nabla \vartheta)^2. \tag{B39}$$

In the homogeneous case the previous equations are reduced to

$$\rho_{\mathrm{DE}} = \frac{1}{2} \dot{\mathcal{X}}^2 + \frac{1}{2} \left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 \dot{\vartheta}^2 + V(\mathcal{X}) \tag{B40}$$

and

$$P_{\mathrm{DE}} = \frac{1}{2} \dot{\mathcal{X}}^2 + \frac{1}{2} \left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 \dot{\vartheta}^2 - V(\mathcal{X}). \tag{B41}$$

In order to realize the accelerated expansion, the DE field has to satisfy an equation of state such that

$$\omega \equiv \frac{P_{\mathrm{DE}}}{\rho_{\mathrm{DE}}} \approx -1,$$

which means that, according to Eqs. (B40) and (B41), the first slow-roll condition is of the form

$$\frac{1}{2} \dot{\mathcal{X}}^2 + \frac{1}{2} \left(1 + \frac{\mathcal{X}}{\langle \mathrm{Q} \rangle}\right)^2 \dot{\vartheta}^2 \ll \frac{1}{2} m^2 (\langle \mathrm{Q} \rangle + \mathcal{X})^2. \tag{B42}$$

The dynamics of the homogeneous background involving both $\mathcal{X}$ and ϑ is given by substituting Eq. (B40) into the first Friedman equation, after applying the first slow-roll condition, together with the terms coming from applying the Euler-Lagrange equations to Eq. (B36). For completeness, we also include DM, baryons (b), photons (γ), and active neutrinos (n). Taking the first slow-roll condition into account, the whole system is

$$\begin{aligned} &\mathsf{H}^2 = \frac{1}{3M_{\mathrm{pl}}^2} V(\mathcal{X}), \\ &\ddot{\mathcal{X}} + 3\mathsf{H}\dot{\mathcal{X}} + V(\mathcal{X})_{,\mathcal{X}} = 0, \\ &\ddot{\vartheta} + 3\mathsf{H}\dot{\vartheta} = 0, \\ &\dot{\mathsf{H}} = \frac{-1}{2M_{\mathrm{pl}}^2} \left(\rho_{\mathrm{DM}} + \rho_b + \frac{4}{3}\rho_\gamma + \frac{4}{3}\rho_\nu\right), \\ &\dot{\rho}_{\mathrm{DM},b} + 3\mathsf{H}\rho_{\mathrm{DM},b} = 0, \\ &\dot{\rho}_{\gamma,n} + 4\mathsf{H}\rho_{\gamma,n} = 0. \end{aligned} \tag{B43}$$

[1] A. G. Riess *et al.* (Supernova Search Team), Astron. J. **116**, 1009 (1998).
[2] S. Perlmutter *et al.* (Supernova Cosmology Project Collaboration), Astrophys. J. **517**, 565 (1999).
[3] D. N. Spergel *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **148**, 175 (2003).
[4] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **192**, 18 (2011).
[5] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **594**, A13 (2016).
[6] D. H. Weinberg, M. J. Mortonson, D. J. Eisenstein, C. Hirata, A. G. Riess, and E. Rozo, Phys. Rep. **530**, 87 (2013).
[7] S. Weinberg, in *Critical Dialogues in Cosmology*, edited by N. Turok (World Scientific, Singapore, 1996), pp. 195–203.
[8] S. M. Carroll, Living Rev. Relativity **4**, 1 (2001).
[9] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988).
[10] C. Wetterich, Nucl. Phys. **B302**, 668 (1988).
[11] J. A. Frieman, C. T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. **75** (1995) 2077.
[12] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[13] S. Weinberg, arXiv:astro-ph/0005265.
[14] D. Bernard and A. LeClair, Phys. Rev. D **87**, 063010 (2013).
[15] Q. Wang, Z. Zhu, and W. G. Unruh, Phys. Rev. D **95**, 103504 (2017).
[16] K. Coble, S. Dodelson, and J. A. Frieman, Phys. Rev. D **55**, 1851 (1997).
[17] S. Tsujikawa, A. De Felice, and J. Alcaniz, J. Cosmol. Astropart. Phys. 01 (2013) 030.
[18] A. Aghamousa *et al.* (DESI Collaboration), arXiv: 1611.00036.
[19] C. Armendariz-Picon, V. F. Mukhanov, and P. J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000).
[20] C. Armendariz-Picon, V. F. Mukhanov, and P. J. Steinhardt, Phys. Rev. D **63**, 103510 (2001).
[21] A. Y. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. B **511**, 265 (2001).
[22] M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D **66**, 043507 (2002).
[23] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002).
[24] S. M. Carroll, M. Hoffman, and M. Trodden, Phys. Rev. D **68**, 023509 (2003).
[25] H. Wei and R. G. Cai, Phys. Rev. D **72**, 123507 (2005).
[26] H. Wei, R. G. Cai, and D. F. Zeng, Classical Quantum Gravity **22**, 3189 (2005).
[27] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).
[28] E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).
[29] S. Tsujikawa, Classical Quantum Gravity **30**, 214003 (2013).
[30] T. Harko, F. S. N. Lobo, and M. K. Mak, Eur. Phys. J. C **74**, 2784 (2014).
[31] T. Chiba, Phys. Rev. D **79**, 083517 (2009); **80**, 109902(E) (2009).
[32] T. Chiba, Phys. Rev. D **81**, 023515 (2010).
[33] R. R. Caldwell and E. V. Linder, Phys. Rev. Lett. **95**, 141301 (2005).
[34] G. W. Anderson and S. M. Carroll, *Dark Matter with Time Dependent Mass* (World Scientific, Singapore, 1998), pp. 227–229.
[35] R. Bean, E. E. Flanagan, and M. Trodden, Phys. Rev. D **78**, 023009 (2008).
[36] V. Sahni, Lect. Notes Phys. **653**, 141 (2004).
[37] S. M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998).
[38] R. Fardon, A. E. Nelson, and N. Weiner, J. Cosmol. Astropart. Phys. 10 (2004) 005.
[39] R. D. Peccei, Phys. Rev. D **71**, 023527 (2005).
[40] N. Afshordi, M. Zaldarriaga, and K. Kohri, Phys. Rev. D **72**, 065024 (2005).
[41] H. Mohseni Sadjadi and V. Anari, Phys. Rev. D **95**, 123521 (2017).
[42] H. M. Sadjadi and V. Anari, J. Cosmol. Astropart. Phys. 10 (2018) 036.
[43] G. D'Amico, T. Hamill, and N. Kaloper, Phys. Rev. D **94**, 103526 (2016).
[44] G. D'Amico, T. Hamill, and N. Kaloper, Phys. Lett. B **797**, 134846 (2019).
[45] S. Kumar, S. Panda, and A. A. Sen, Classical Quantum Gravity **30**, 155011 (2013).
[46] A. Albrecht and C. Skordis, Phys. Rev. Lett. **84**, 2076 (2000).
[47] K. Choi, Phys. Rev. D **62**, 043509 (2000).
[48] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[49] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[50] A. D. Linde, Phys. Lett. **129B**, 177 (1983).
[51] A. D. Linde, Phys. Lett. **162B**, 281 (1985).
[52] A. D. Linde, Phys. Lett. B **175**, 395 (1986).
[53] A. Perez-Lorenzana, M. Montesinos, and T. Matos, Phys. Rev. D **77**, 063507 (2008).
[54] R. Rosenfeld and J. A. Frieman, J. Cosmol. Astropart. Phys. 09 (2005) 003.
[55] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A22 (2014).
[56] P. Minkowski, Phys. Lett. **67B**, 421 (1977).
[57] M. Gell-Mann, P. Ramond, and R. Slansky, Conf. Proc. C **790927**, 315 (1979).
[58] T. Yanagida, Conf. Proc. C **7902131**, 95 (1979).
[59] S. L. Glashow, NATO Sci. Ser. B **61**, 687 (1980).
[60] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).
[61] J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 774 (1982).
[62] R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).
[63] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
[64] S. Vagnozzi, E. Giusarma, O. Mena, K. Freese, M. Gerbino, S. Ho, and M. Lattanzi, Phys. Rev. D **96**, 123503 (2017).
[65] S. Vagnozzi, S. Dhawan, M. Gerbino, K. Freese, A. Goobar, and O. Mena, Phys. Rev. D **98**, 083501 (2018).
[66] R. R. Caldwell and M. Kamionkowski, Annu. Rev. Nucl. Part. Sci. **59**, 397 (2009).
[67] L. A. Boyle, R. R. Caldwell, and M. Kamionkowski, Phys. Lett. B **545**, 17 (2002).
[68] C. F. Kolda and D. H. Lyth, Phys. Lett. B **458**, 197 (1999).
[69] L. J. Hall, Y. Nomura, and S. J. Oliver, Phys. Rev. Lett. **95**, 141302 (2005).
[70] C. P. Burgess, P. Grenier, and D. Hoover, J. Cosmol. Astropart. Phys. 03 (2004) 008.
[71] For a review of interest in cosmology see for instance M. Battaglieri *et al.*, arXiv:1707.04591.

[72] R. V. Wagoner, W. A. Fowler, and F. Hoyle, Astrophys. J. **148**, 3 (1967).
[73] T. P. Walker, G. Steigman, D. N. Schramm, K. A. Olive, and H. S. Kang, Astrophys. J. **376**, 51 (1991).
[74] C. J. Copi, D. N. Schramm, and M. S. Turner, Science **267**, 192 (1995).
[75] R. H. Cyburt, B. D. Fields, K. A. Olive, and T. H. Yeh, Rev. Mod. Phys. **88**, 015004 (2016).
[76] P. Gondolo and G. Gelmini, Nucl. Phys. **B360**, 145 (1991).
[77] M. Srednicki, R. Watkins, and K. A. Olive, Nucl. Phys. **B310**, 693 (1988).
[78] I. Baldes, T. Konstandin, and G. Servant, Phys. Lett. B **786**, 373 (2018).
[79] M. H. Thoma, Z. Phys. C **66**, 491 (1995).