Nature of the vector resonance $Y(2175)$

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Spectroscopic parameters and decay channels of the vector resonance $Y(2175)$ are studied by considering it as a diquark-antidiquark state with the quark content $su\bar{s}\bar{u}$. The mass and coupling of the tetraquark $Y(2175)$ are calculated using the QCD two-point sum rules by taking into account various quark, gluon and mixed condensates up to dimension 15. Partial widths of its strong decays to $\phi f_0(980)$, $\phi\eta$, and $\phi\eta'$ are computed as well. To this end, we explore the vertices $Y\phi f_0(980)$, $Y\phi\eta$, and $Y\phi\eta'$, and calculate the corresponding strong couplings by means of the QCD light-cone sum rule method. The coupling $G_{Y\phi f}$ of the vertex $Y\phi f_0(980)$ is found using the full version of this method, and by treating the scalar meson $f_0(980)$ as a diquark-antidiquark tetraquark state. The couplings $g_{Y\phi\eta}$ and $g_{Y\phi\eta'}$, however, are calculated by applying the soft-meson approximation to the light-cone sum rule method. Prediction for the mass of the resonance $m_Y = (2173 \pm 85)$ MeV is in excellent agreement with the data of the *BABAR* Collaboration [Phys. Rev. D **74**, 091103 (2006)], and within errors of calculations is compatible with the result reported by BESIII [Phys. Rev. D **91**, 052017 (2015)]. The full width $\Gamma_{\rm full} = (91.1 \pm 20.5)$ MeV of the $Y(2175)$ saturated by its three strong decay channels is in a reasonable agreement with existing experimental data.

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I. INTRODUCTION

The resonances $\{Y\}$ with the quantum numbers $J^{\rm PC} = 1^{--}$ constitute two families of particles, interpretation of which is one of interesting and yet unsettled problems of the high energy physics. Members of the first family populate the mass region $m = 4.2$–4.7 GeV, and were observed by different collaborations. These resonances reside very close to each other, and are more numerous than vector charmonia $\bar{c}c$ from this mass range. Hence, at least some of these resonances have different quark-gluon structure, and are presumably states built of four valence quarks. Besides a suggestion about the tetraquark nature of heavy $\{Y\}$ states, there are various alternative models to account for their parameters and decay channels.

Another family of the $\{Y\}$ resonances occupies the light segment of meson spectroscopy and incorporates the famous "old" state $Y(2175)$, and new ones $X(2239)$ and $X(2100)$ seen recently. The structure $Y(2175)$ was discovered by the *BABAR* collaboration in the initial-state radiation process $e^+e^- \to \gamma_{\rm ISR}\phi f_0(980)$ as a resonance in the $\phi f_0(980)$ invariant mass spectrum [1]. The mass and width of this resonance measured by *BABAR* amount to $m = (2175 \pm 10 \pm 15)$ and $\Gamma = (58 \pm 16 \pm 20)$ MeV, respectively. The same structure was seen also by the BESIII collaboration in the exclusive decay $J/\psi \to \eta\phi\pi^+\pi^-$ [2]. The spectroscopic parameters of the $Y(2175)$ extracted in this experiment differ from original results and are $m = (2200 \pm 6 \pm 5)$ and $\Gamma = (104 \pm 15 \pm 15)$ MeV. Recently, anomalously high cross section at $\sqrt{s} = 2232$ MeV was observed by the BESIII collaboration in the channel $e^+e^- \to \phi K^+K^-$, which may be explained by interference of different resonances [3]: more data are necessary to decide whether $Y(2175)$ contributes to enhancement of this cross section or not. Because the $Y(2175)$ was seen by *BABAR* and confirmed by the BESII, BESIII, and Belle collaborations [2,4,5], its existence is not in doubt, but an uncertain situation with the mass and full width of this resonance requires further experimental and theoretical studies.

Other resonances that may be considered as candidates to light exotic vector mesons were discovered by the BESIII collaboration. The first of them, i.e., $X(2239)$, was fixed in the process $e^+e^- \to K^+K^-$ as a resonant structure in the cross section shape line [6]. The second resonance $X(2100)$ was seen in the $\phi\eta'$ mass spectrum in the process $J/\psi \to \phi\eta\eta'$ [7]. The quantum numbers of $X(2239)$ were determined unambiguously, whereas a situation with $X(2100)$ remains unclear. Indeed, because of a scarcity of experimental information the collaboration could not clearly distinguish two 1^+ and 1^- assumptions for the spin-parity J^P of the resonance $X(2100)$. Hence, BESIII extracted its mass and full width using both of these options. Obtained results differ from each other and depend on assumption about the parity of the state $X(2100)$.

Theoretical interpretations of the light vector resonances comprise all available models and approaches of the high energy physics. Thus, the $Y(2175)$ was considered as 2^3D_1 excitation of the ordinary $\bar{s}s$ meson [8,9]. It was explained also as a dynamically generated state in the $\phi K\bar{K}$ system [10], or as a resonance appeared due to self-interaction between ϕ and $f_0(980)$ mesons [11]. A hybrid meson with structure $\bar{s}sg$ [12] and a baryon-antibaryon $qqs\bar{q}\bar{q}\bar{s}$ state which couples strongly to the $\Lambda\bar{\Lambda}$ channel are among alternative models of the $Y(2175)$ resonance. There were attempts to interpret $Y(2175)$ as a vector tetraquark with $s\bar{s}s\bar{s}$ or $ss\bar{s}\bar{s}$ contents [13–15] (see Ref. [6] for other models). The resonance $X(2100)$ was examined in the framework of the QCD sum rule method in Refs. [16,17].

Recently, we explored the light resonances $X(2100)$ and $X(2239)$ as the axial-vector and vector $ss\bar{s}\bar{s}$ tetraquarks [18], respectively. Besides spectroscopic parameters we also investigated the strong decays $X(2100) \to \phi\eta'$ and $X(2100) \to \phi\eta$, and calculated their partial widths. Predictions obtained for the mass and width of the axial-vector state allowed us to identify it with the resonance $X(2100)$, because our theoretical predictions are very close to its parameters measured by the BESIII collaboration. We classified $X(2239)$ as the vector tetraquark $ss\bar{s}\bar{s}$ and found a reasonable agreement between theoretical and experimental results.

In the present work, we continue our investigations of the light vector resonances and concentrate on features of the state $Y(2175)$ (hereafter, Y). Our treatment of this state differs from existing analyses. Thus, we consider it as a vector tetraquark with content $su\bar{s}\,\bar{u}$ rather than as a state $ss\bar{s}\bar{s}$. The traditional assumption about the quark content of the Y is inspired by the fact that it was discovered in $\phi f_0(980)$ invariant mass distribution. Because in the standard model of mesons one treats the ϕ and $f_0(980)$ as vector and scalar particles with the same $\bar{s}s$ structure, then it is natural to assume that Y is built of four valence s quarks.

But the conventional quark-antiquark model of mesons in the case of light scalar nonets meets with evident difficulties. In fact, the nonet of scalar mesons in the $\bar{q}q$ model may be realized as 1^3P_0 states. In accordance with various computations, masses of the scalars 1^3P_0 are higher than 1 GeV. They were identified with the isoscalar mesons $f_0(1370)$ and $f_0(1710)$, the isovector $a_0(1450)$ or isospinor $K_0^*(1430)$ states, i.e., with scalars from the second light nonet. But masses of the mesons from the first nonet are lower than 1 GeV, and they cannot be included into this scheme. Therefore, to explain experimental information on their masses, and an unusual mass hierarchy inside of the nonet Jaffe made a suggestion on a four-quark nature of these particles [19].

An updated model of the light scalar nonets is based on assumption about a diquark-antidiquark structure of these particles, which appear as mixtures of spin-0 diquarks from $(\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_f)$ representation with spin-1 diquarks from $(\mathbf{6}_c, \bar{\mathbf{3}}_f)$ representation of the color-flavor group [20]. In Refs. [21,22] we investigated the scalar mesons $f_0(500)$ and $f_0(980)$ as admixtures of the $SU_f(3)$ basic light $\mathbf{L} = [ud][\bar{u}\bar{d}]$ and heavy $\mathbf{H} = ([su][\bar{s}\bar{u}] + [ds][\bar{d}\bar{s}])/\sqrt{2}$ tetraquark states, and calculated their spectroscopic parameters and full widths. Obtained predictions agree with existing experimental data, therefore we consider the $f_0(980)$ as the exotic four-quark meson. Once we accept this model, a treatment of the Y as a vector tetraquark $Y = [su][\bar{s}\,\bar{u}]$ becomes quite reasonable.

We calculate the spectroscopic parameters of the vector tetraquark $Y = [su][\bar{s}\bar{u}]$ and explore some of its decay channels. The mass and coupling of the Y are evaluated using the QCD two-point sum rule method [23,24]. We investigate the strong decays $Y \to \phi f_0(980)$,$Y \to \phi\eta$, and $Y \to \phi\eta'$, and find their partial widths. To this end, we use the QCD light-cone sum rule (LCSR) method [25], and calculate the couplings $G_{Y\phi f}$, $g_{Y\phi\eta}$, and $g_{Y\phi\eta'}$ corresponding to the strong vertices $Y\phi f_0(980)$, $Y\phi\eta$, and $Y\phi\eta'$, respectively. The coupling $G_{Y\phi f}$ is computed by employing the full version of the LCSR method, whereas in the case of $g_{Y\phi\eta}$, and $g_{Y\phi\eta'}$ this method is supplemented by a technique of the soft-meson approximation [26–28]. Because the light component of $f_0(980)$ is irrelevant for analysis of the decay $Y \to \phi f_0(980)$, we treat $f_0(980)$ as a pure $\mathbf{H}$ state.

This article is organized as the following way: In Sec. II we calculate the mass and coupling of the tetraquark Y. The strong decays of this state are considered in Secs. III and IV. In Sec. III we analyze the process $Y \to \phi f_0(980)$ using the LCSR method and find the partial decay width of this channel. The partial widths of the decay modes $Y \to \phi\eta$, and $Y \to \phi\eta'$ are calculated in Sec. IV. In Sec. V we analyze the obtained results, and give our conclusions.

II. SPECTROSCOPIC PARAMETERS OF THE TETRAQUARK Y: THE MASS m_Y AND CURRENT COUPLING f_Y

To evaluate the mass m_Y and coupling f_Y of the vector tetraquark Y, we use the QCD two-point sum rule method and start our calculations from analysis of the correlation function,

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \langle 0|\mathcal{T}\{J_\mu^Y(x) J_\nu^{Y\dagger}(0)\}|0\rangle, \quad (1)$$

where $J_\mu^Y(x)$ is the interpolating current for the Y state.

The current for a tetraquark with $J^{\mathrm{P}}=1^{-}$ can be built of a scalar diquark and vector antidiquark or/and a vector diquark and scalar antidiquark. There are several options to construct alternative currents with required spin-parities, but because a scalar diquark (antidiquark) is a most stable two-quark state [29], for J_{μ}^{Y} we use the structure

$$C\gamma_5\otimes\gamma_\mu\gamma_5 C-C\gamma_\mu\gamma_5\otimes\gamma_5 C. \tag{2}$$

This current consists of two components, and each of them describes a vector tetraquark. The whole structure corresponds to a vector tetraquark with definite charge-conjugation parity $J^{\mathrm{PC}}=1^{--}$. Indeed, the charge-conjugation transforms diquarks to antidiquarks and vice versa, therefore the minus sign between two components in Eq. (2) generates the current with $C=-1$.

The last question to be solved is a color structure of constituent diquarks and antidiquarks. Thus, to get the color-singlet current J_{μ}^{Y} they should have the same color structures and be either in color triplet $[\bar{\mathbf{3}}_c]\otimes[\mathbf{3}_c]$ or sextet $[\mathbf{6}_c]\otimes[\bar{\mathbf{6}}_c]$ configurations. The current of the type (2) and built of color-sextet diquark-antidiquark has the following form [30]:

$$\begin{aligned}J_{1\mu}&=u_a^T C\gamma_5 s_b[\bar{u}_a\gamma_\mu\gamma_5 C\bar{s}_b^T+\bar{u}_b\gamma_\mu\gamma_5 C\bar{s}_a^T]\\&\quad-u_a^T C\gamma_\mu\gamma_5 s_b[\bar{u}_a\gamma_5 C\bar{s}_b^T+\bar{u}_b\gamma_5 C\bar{s}_a^T].\end{aligned} \tag{3}$$

The triplet current (2) is given by the expression

$$\begin{aligned}J_{3\mu}&=u_a^T C\gamma_5 s_b[\bar{u}_a\gamma_\mu\gamma_5 C\bar{s}_b^T-\bar{u}_b\gamma_\mu\gamma_5 C\bar{s}_a^T]\\&\quad-u_a^T C\gamma_\mu\gamma_5 s_b[\bar{u}_a\gamma_5 C\bar{s}_b^T-\bar{u}_b\gamma_5 C\bar{s}_a^T].\end{aligned} \tag{4}$$

In Eqs. (3) and (4) a and b are color indices, and C is the charge-conjugation matrix.

The $J_{1\mu}$ and $J_{3\mu}$ are color-singlet currents composed of color-sextet and -triplet diquark-antidiquark pairs, respectively. To see this, let us consider in a detailed form $J_{1\mu}$. The color-sextet, i.e., color-symmetric $a\leftrightarrow b$ nature of the antidiquark fields in Eq. (3) is evident. The first component of $J_{1\mu}$, for example, in the explicit color-singlet form is

$$(u_a^T C\gamma_5 s_b+u_b^T C\gamma_5 s_a)[\bar{u}_a\gamma_\mu\gamma_5 C\bar{s}_b^T+\bar{u}_b\gamma_\mu\gamma_5 C\bar{s}_a^T], \tag{5}$$

where both the diquark and antidiquark are symmetric in color indices. It is not difficult to see that diquarks $u_a^T C\gamma_5 s_b$ and $u_b^T C\gamma_5 s_a$ lead to identical results, hence it is enough in $J_{1\mu}$ to keep one of them. The similar analysis is valid for the second component of $J_{1\mu}$ as well. In the case of the current $J_{3\mu}$, we see that the antidiquark fields in Eq. (4) are color-triplet or color-antisymmetric constructions. The color-triplet diquark field, for example, in the first component of $J_{3\mu}$ is $(u_a^T C\gamma_5 s_b-u_b^T C\gamma_5 s_a)$, and both $u_a^T C\gamma_5 s_b$ and $-u_b^T C\gamma_5 s_a$ give again the same results. Therefore, we use one of them in the current $J_{3\mu}$ and get (4).

An appropriate form of the current J_{μ}^{Y} that ensures stability and convergence of the sum rules, which are actual in the case of light tetraquarks [31], is superposition of $J_{1\mu}$ and $J_{3\mu}$. In the present work we use $J_{\mu}^{Y}=(J_{1\mu}+J_{3\mu})/2$, and get

$$\begin{aligned}J_{\mu}^{Y}(x)&=[u_a^T(x)C\gamma_5 s_b(x)][\bar{u}_a(x)\gamma_\mu\gamma_5 C\bar{s}_b^T(x)]\\&\quad-[u_a^T(x)C\gamma_\mu\gamma_5 s_b(x)][\bar{u}_a(x)\gamma_5 C\bar{s}_b^T(x)].\end{aligned} \tag{6}$$

The $J_{\mu}^{Y}(x)$ is a sum of two colorless terms, but belongs neither to sextet nor to triplet representations of the color group being the admixture of such states $J_{1\mu}$ and $J_{3\mu}$.

To obtain sum rules for the mass and coupling of Y, we should express the correlation function in terms of these spectral parameters, and also calculate $\Pi_{\mu\nu}(p)$ using quark-gluon degrees of freedom. The first expression forms the physical side of the sum rules $\Pi_{\mu\nu}^{\mathrm{Phys}}(p)$, whereas the second one constitutes their QCD side $\Pi_{\mu\nu}^{\mathrm{OPE}}(p)$. In terms of the tetraquark's parameters the correlation function has the following form:

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(p)=\frac{\langle 0|J_{\mu}^{Y}|Y(p)\rangle\langle Y(p)|J_{\nu}^{Y\dagger}|0\rangle}{m_Y^2-p^2}+\cdots. \tag{7}$$

Equation (7) is derived by saturating the correlation function with a complete set of $J^{\mathrm{PC}}=1^{--}$ states and carrying out integration in Eq. (1) over x. As usual, contributions arising from higher resonances and continuum states are denoted above by dots.

The correlator $\Pi_{\mu\nu}^{\mathrm{Phys}}(p)$ can be further simplified if one introduces the matrix element,

$$\langle 0|J_{\mu}^{Y}|Y(p)\rangle=f_Y m_Y\epsilon_\mu, \tag{8}$$

where ϵ_μ is the polarization vector of the Y state. Then the correlation function $\Pi_{\mu\nu}^{\mathrm{Phys}}(p)$ takes the simple form

$$\Pi_{\mu\nu}^{\mathrm{Phys}}(p)=\frac{m_Y^2 f_Y^2}{m_Y^2-p^2}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{m_Y^2}\right)+\cdots, \tag{9}$$

and contains the Lorentz structure corresponding to the vector state. Because a part of this structure proportional to $g_{\mu\nu}$ receives contribution only from the vector states, we work with this term and corresponding invariant amplitude $\Pi^{\mathrm{Phys}}(p^2)$.

The QCD side of the sum rules is given by the same correlation function $\Pi_{\mu\nu}(p)$ but expressed in terms of the quark propagators. Substituting the interpolating current into Eq. (1), and contracting the quark fields, we get

$$\Pi_{\mu\nu}^{\rm OPE}(p) = i\int d^4x e^{ipx}\{\mathrm{Tr}[\gamma_5\tilde{S}_s^{b'b}(-x)\gamma_5\gamma_\nu S_u^{a'a}(-x)]\mathrm{Tr}[S_u^{aa'}(x)\gamma_5\tilde{S}_s^{bb'}(x)\gamma_5\gamma_\mu] + \mathrm{Tr}[\gamma_\mu\gamma_5\tilde{S}_s^{b'b}(-x)\gamma_5 S_u^{a'a}(-x)]\mathrm{Tr}[S_u^{aa'}(x)\gamma_\nu\gamma_5\tilde{S}_s^{bb'}(x)\gamma_5] + \mathrm{Tr}[S_u^{aa'}(x)\gamma_5\tilde{S}_s^{bb'}(x)\gamma_5]\mathrm{Tr}[\gamma_\mu\gamma_5\tilde{S}_s^{b'b}(-x)\gamma_5\gamma_\nu S_u^{a'a}(-x)] + \mathrm{Tr}[\gamma_5\tilde{S}_s^{b'b}(-x)\gamma_5 S_u^{a'a}(-x)]\mathrm{Tr}[S_u^{aa'}(x)\gamma_\nu\gamma_5\tilde{S}_s^{bb'}(x)\gamma_5\gamma_\mu]\}, \quad (10)$$

where

$$\tilde{S}_q(x) = CS_q^T(x)C. \quad (11)$$

In the formula above $S_q(x)$ is the light quark propagator, for which we employ the expression

$$S_q^{ab}(x) = i\frac{\not{x}}{2\pi^2x^4}\delta_{ab} - \frac{m_q}{4\pi^2x^2}\delta_{ab} - \frac{\langle\bar{q}q\rangle}{12}\left(1 - i\frac{m_q}{4}\not{x}\right)\delta_{ab} - \frac{x^2}{192}\langle\bar{q}g_s\sigma Gq\rangle\left(1 - i\frac{m_q}{6}\not{x}\right)\delta_{ab} - \frac{\not{x}x^2g_s^2}{7776}\langle\bar{q}q\rangle^2\delta_{ab} - \frac{ig_sG_{ab}^{\mu\nu}}{32\pi^2x^2}[\not{x}\sigma_{\mu\nu} + \sigma_{\mu\nu}\not{x}] - \frac{x^4\langle\bar{q}q\rangle\langle g_s^2G^2\rangle}{27648}\delta_{ab} + \frac{m_qg_s}{32\pi^2}G_{ab}^{\mu\nu}\sigma_{\mu\nu}\left[\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right] + \cdots, \quad (12)$$

where $\gamma_E \simeq 0.577$ is the Euler constant and Λ is the QCD scale parameter. In Eq. (12) $G_{ab}^{\alpha\beta} = G_A^{\alpha\beta}t_{ab}^A$, where $t^A = \lambda^A/2$ with λ^A being the Gell-Mann matrices and $A, B, C = 1, 2, \ldots 8$. Let us note that the gluon field strength tensor is fixed at $x = 0$, i.e., $G_{\alpha\beta}^A \equiv G_{\alpha\beta}^A(0)$.

To find the required sum rules, we extract the invariant amplitude $\Pi^{\rm OPE}(p^2)$ corresponding to the structure $g_{\mu\nu}$, and equate it to $\Pi^{\rm Phys}(p^2)$. We apply the Borel transformation to both sides of the obtained equality, which is necessary to suppress contributions of the higher resonances and continuum states. At the next stage, using an assumption on quark-hadron duality, we carry out the continuum subtraction. After these standard manipulations the sum rule depends on new parameters M^2 and s_0: the first of them M^2 is the Borel parameter generated by the Borel transformation, whereas s_0 is the continuum threshold parameter that dissects contributions of the ground state and higher resonances from each another. Remaining operations to find the sum rules for m_Y and f_Y are similar to ones presented numerously in the literature, and therefore, we skip further details. It is worth noting that calculation of $\Pi^{\rm OPE}(p^2)$ in the present article is performed by taking into account nonperturbative terms up to dimension 15.

The obtained sum rules contain various vacuum condensates, and depend on the s quark's mass and on two auxiliary parameters M^2 and s_0. Values of the vacuum condensates and the mass of s quark used in numerical computations are collected in Table I. Here, we also write down the parameters of the ϕ, $f_0(980)$, η, and η' mesons which are necessary to calculate partial widths of the decay processes.

The condensates characterize nonperturbative features of the vacuum and do not depend on a problem under consideration. On the contrary, the Borel and continuum threshold parameters M^2 and s_0 should be chosen for each sum rule computation individually and must meet restrictions imposed on them by the QCD sum rule method. The main constraints on M^2 and s_0 are connected with convergence of the operator product expansion (OPE) which we fix by means of the ratio

$$R(M^2) = \frac{\Pi^{\rm DimN}(M^2, s_0)}{\Pi(M^2, s_0)}, \quad (13)$$

and with the restriction on the pole contribution (PC)

$$\mathrm{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}. \quad (14)$$

In Eqs. (13) and (14) $\Pi(M^2, s_0)$ is the invariant amplitude $\Pi^{\rm OPE}(p^2)$ obtained after the Borel transformation and

TABLE I. Vacuum condensates and spectroscopic parameters of the mesons used in numerical computations.

Quantity	Value
$\langle\bar{q}q\rangle$	$-(0.24 \pm 0.01)^3$ GeV^3
$\langle\bar{s}s\rangle$	$0.8\langle\bar{q}q\rangle$
m_0^2	(0.8 ± 0.1) GeV^2
$\langle\bar{q}g_s\sigma Gq\rangle$	$m_0^2\langle\bar{q}q\rangle$
$\langle\bar{s}g_s\sigma Gs\rangle$	$m_0^2\langle\bar{s}s\rangle$
$\langle\frac{\alpha_sG^2}{\pi}\rangle$	(0.012 ± 0.004) GeV^4
m_s	93_{-5}^{+11} MeV
m_ϕ	(1019.461 ± 0.019) MeV
m_f	(990 ± 20) MeV
m_η	(547.862 ± 0.018) MeV
$m_{\eta'}$	(957.78 ± 0.06) MeV
f_ϕ	(215 ± 5) MeV

subtraction procedures, and $\Pi^{\mathrm{DimN}}(M^2, s_0)$ denotes a last term (or a sum of last few terms) in OPE. At the minimum of the working window for the Borel parameter, we require $R(M^2) \simeq 0.01$ and use a sum of three terms $\mathrm{DimN} = \mathrm{Dim}(13+14+15)$ to estimate $R(M^2_{\mathrm{min}})$. At maximum allowed value of M^2, we demand fulfillment of the condition $\mathrm{PC} > 0.2$.

In general, m_Y and f_Y extracted from the sum rules should not depend on the Borel parameter M^2. But in actual computations the best thing one can do is find a plateau where dependence of physical quantities on M^2 is minimal. The continuum threshold parameter s_0 separates a ground-state contribution from the ones due to higher resonances and continuum states. In other words, s_0 should be below the first excited state of the particle under discussion Y. In the case of conventional hadrons, masses of excited states are known either from experimental measurements or from alternative theoretical studies. For exotic particles the situation is more complicated: there is not information on their radial and/or orbital excitations. It is worth noting that for tetraquarks this problem was addressed only in few publications [32–34]. Therefore, one chooses s_0 by demanding maximum for PC and, at the same time, a stability of an extracting physical quantity. In such analysis very important is control over self-consistency of the prediction for m_Y and s_0 used for these purposes: $\sqrt{s_0}$ may exceed m_Y approximately $[0.3, 0.6]$ MeV to be below a first excited state of Y. Uncertainties in the choice of the M^2 and s_0 are the main sources of theoretical errors in the sum rule calculations, which however can be systematically kept under control.

Numerical analysis allows us to fix the regions

$$M^2 \in [1.2, 1.7]\ \mathrm{GeV}^2, \qquad s_0 \in [6, 6.5]\ \mathrm{GeV}^2 \tag{15}$$

as ones which obey the constraints imposed on M^2 and s_0. Thus, at $M^2 = 1.2\ \mathrm{GeV}^2$ the convergence of the OPE is fulfilled, because a contribution of the last three terms to the Borel transformed and subtracted invariant amplitude $\Pi(M^2, s_0)$ does not exceed 0.3% of its value. At $M^2 = 1.2\ \mathrm{GeV}^2$ the pole contribution forms 60% of $\Pi(M^2, s_0)$, whereas at $M^2 = 1.7\ \mathrm{GeV}^2$ it amounts to approximately 30% of the whole result.

The mass m_Y and coupling f_Y are plotted in Figs. 1 and 2 as functions of M^2 and s_0: one can inspect their dependence on the Borel and continuum threshold parameters which is considerable for f_Y.

Our results for the spectroscopic parameters of the tetraquark Y are

$$m_Y = (2173 \pm 85)\ \mathrm{MeV},$$
$$f_Y = (2.8 \pm 0.5) \times 10^{-3}\ \mathrm{GeV}^4. \tag{16}$$

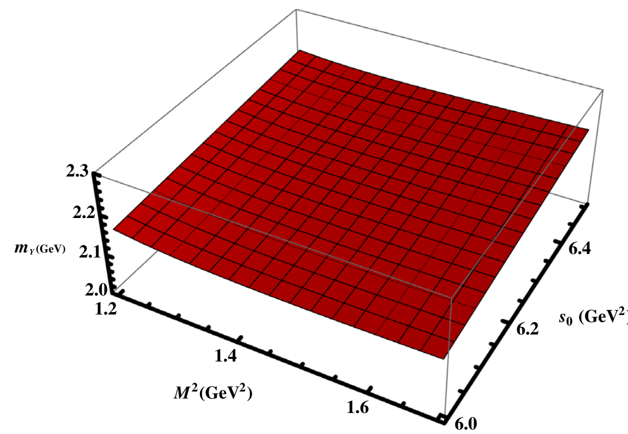

FIG. 1. The mass m_Y of the tetraquark Y as a function of the Borel and continuum threshold parameters.

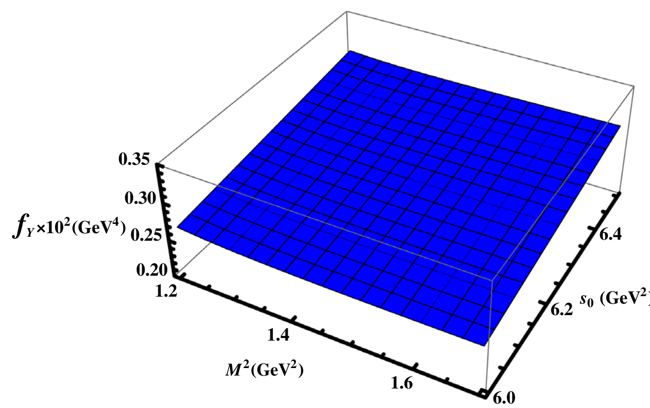

FIG. 2. Dependence of the coupling f_Y on M^2 and s_0.

Comparing m_Y with $\sqrt{s_0}$ we see that $\sqrt{s_0} - m_Y = [0.28, 0.38]$ MeV is a reasonable mass gap to separate Y from its excitations.

As is seen, the m_Y is in excellent agreement with the datum of the *BABAR* collaboration $(2175 \pm 10 \pm 15)$ MeV. It is lower than the new result $(2200 \pm 6 \pm 5)$ MeV reported by BESIII, but within errors of calculations is compatible with this measurement as well. In this situation decays of the tetraquark Y become of special interest, because predictions for partial widths of the different channels and for the full width of the Y are important to verify our assumption on its structure.

III. THE DECAY $Y \to \phi f_0(980)$

The process $Y \to \phi f_0(980)$ is one of dominant strong decays of the tetraquark Y. To calculate partial width of this channel, we extract the strong coupling $G_{Y\phi f}$ of the vertex $Y\phi f_0(980)$ in the context of the LCSR method and express it in terms of various vacuum condensates and distribution amplitudes (DAs) of the ϕ meson.

To derive the light-cone sum rule for the coupling $G_{Y\phi f}$, we start from analysis of the correlation function,

$$\Pi_\mu(p,q) = i\int d^4x e^{ipx}\langle\phi(q)|\mathcal{T}\{J^f(x)J_\mu^{Y\dagger}(0)\}|0\rangle, \qquad (17)$$

where $J_\mu^Y(x)$ is the interpolating current of Y introduced in Eq. (6).

As it has been emphasized above, we consider the scalar meson $f_0(980)$ [in formulas we use $f = f_0(980)$] as a pure **H** state. Interpolating current for such state is given by expression

$$J^f(x) = \frac{\epsilon\tilde{\epsilon}}{\sqrt{2}}\{[u_a^T(x)C\gamma_5 s_b(x)][\bar{u}_c(x)\gamma_5 C\bar{s}_e^T(x)] + [d_a^T(x)C\gamma_5 s_b(x)][\bar{d}_c(x)\gamma_5 C\bar{s}_e^T(x)]\}, \qquad (18)$$

where $\epsilon\tilde{\epsilon} = \epsilon^{dab}\epsilon^{dce}$.

Then, the phenomenological side of the sum rule is determined by the formula

$$\Pi_\mu^{\rm Phys}(p,q) = \frac{\langle 0|J^f|f(p)\rangle}{p^2 - m_f^2}\langle f(p)\phi(q)|Y(p')\rangle \times \frac{\langle Y(p')|J_\mu^{Y\dagger}|0\rangle}{p'^2 - m_Y^2} + \cdots, \qquad (19)$$

where p', and p, q are 4-momenta of the initial and final particles, respectively. To simplify $\Pi_\mu^{\rm Phys}(p,q)$ we express the matrix elements in terms of physical parameters of the particles involved into the decay process. The matrix element $\langle Y(p')|J_\mu^\dagger|0\rangle$ is given by Eq. (8), whereas for $\langle 0|J^f|f(p)\rangle$ we use

$$\langle 0|J^f|f(p)\rangle = F^f m_f. \qquad (20)$$

We parametrize the vertex $\langle f(p)\phi(q)|Y(p')\rangle$ by means of the expression

$$\langle f(p)\phi(q)|Y(p')\rangle = G_{Y\phi f}[(p'\cdot q)(\varepsilon^*\cdot\varepsilon') - (q\cdot\varepsilon')(p'\cdot\varepsilon^*)], \qquad (21)$$

where $G_{Y\phi f}$ is the strong coupling which should be determined using the sum rule, and ε_μ^* is the polarization vector of the ϕ meson. This information on the matrix elements is enough to get the phenomenological side of the sum rule which reads

$$\Pi_\mu^{\rm Phys}(p,q) = G_{Y\phi f}\frac{m_Y f_Y m_f F_f}{2(p'^2 - m_Y^2)(p^2 - m_f^2)} \times \left[(m_f^2 - m_Y^2 - m_\phi^2)\varepsilon_\mu^* + \frac{m_Y^2 + m_f^2 - m_\phi^2}{m_Y^2} p\cdot\varepsilon^* q_\mu\right]. \qquad (22)$$

It is seen that the function $\Pi_\mu^{\rm Phys}(p,q)$ contains two Lorentz structures which can be employed to derive the required sum rule. In the present study we choose the structure proportional to the polarization vector ε_μ^*.

The second component of the sum rule $\Pi_\mu^{\rm OPE}(p,q)$ is obtained by substituting the interpolating currents into the correlation function (17), contracting the relevant quark fields, and expressing a final expression in terms of quarks' light-cone propagators $\mathcal{S}_q(x)$, and distribution amplitudes of the ϕ meson.

After contracting the quark fields the matrix element in Eq. (17) contains numerous terms of the forms

$$[A(x)]_{\alpha\beta}^{ab}\langle\phi(q)|\bar{s}_\alpha^a(x)s_\beta^b(0)|0\rangle, \quad [B(x)]_{\alpha\beta}^{ab}\langle\phi(q)|\bar{s}_\alpha^a(0)s_\beta^b(x)|0\rangle, \qquad (23)$$

where α and β are the spinor indices. Here $A(x)$ and $B(x)$ are some combinations of the propagators $\mathcal{S}_q(\pm x)$, $\tilde{\mathcal{S}}_q(\pm x) = C\mathcal{S}_q^T(\pm x)C$, and $\gamma_{5(\sigma)}$ matrices. In calculations we use the light-cone propagator of the u, d, and s quarks, which is determined by the formula

$$\begin{aligned}\mathcal{S}_q^{ab}(x) = {}& \frac{i\not{x}}{2\pi^2x^4}\delta_{ab} - \frac{m_q}{4\pi^2x^2}\delta_{ab} - \frac{\langle\bar{q}q\rangle}{12}\left(1 - i\frac{m_q}{4}\not{x}\right)\delta_{ab} \\ & - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle\left(1 - i\frac{m_q}{6}\not{x}\right)\delta_{ab} \\ & - ig_s\int_0^1 du\left\{\frac{\not{x}G_{ab}^{\mu\nu}(ux)\sigma_{\mu\nu}}{16\pi^2x^2} - \frac{iux_\mu}{4\pi^2x^2}G_{ab}^{\mu\nu}(ux)\gamma_\nu \right. \\ & \left. - \frac{im_q}{32\pi^2}G_{ab}^{\mu\nu}(ux)\sigma_{\mu\nu}\left[\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right]\right\}. \end{aligned} \qquad (24)$$

The first two terms in (24) are the perturbative components of the propagator, whereas others are nonperturbative contributions. The terms $\sim G^{\mu\nu}$ appear due to expansion of $\mathcal{S}_q(x)$ on the light cone and describe interactions with the gluon field. In our analysis, we neglect terms proportional to m_q, but, at the same time, take into account the ones $\sim m_s$.

Apart from propagators the function $\Pi_\mu^{\rm OPE}(p,q)$ depends also on nonlocal matrix elements of the quark operator $\bar{s}s$ sandwiched between the vacuum and ϕ state. To express these matrix elements using the ϕ meson's distribution amplitudes, we expand $\bar{s}(x)s(0)$ [this analysis is valid for $\bar{s}(0)s(x)$ as well] over the full set of Dirac matrices Γ^J and project them onto the color-singlet states,

$$\bar{s}_\alpha^a(x)s_\beta^b(0) \to \frac{1}{12}\delta^{ab}\Gamma_{\beta\alpha}^J[\bar{s}(x)\Gamma^J s(0)], \qquad (25)$$

where Γ^J

$$\Gamma^J = \mathbf{1}, \gamma_5, \gamma_\mu, i\gamma_5\gamma_\mu, \sigma_{\mu\nu}/\sqrt{2}. \qquad (26)$$

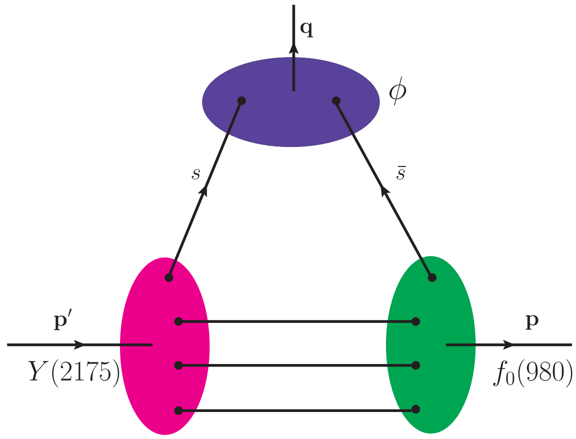

FIG. 3. The leading order diagram contributing to $\Pi_\mu^{\mathrm{OPE}}(p,q)$.

The matrix element of the operators $\bar{s}(x)\Gamma^J s(0)$ can be expanded over x^2 and written down in terms of the ϕ meson's two- and three-particle DAs of different twist. In the case $\Gamma^J = \mathbf{1}$ and $i\gamma_\mu\gamma_5$ we use the definitions

$$\langle 0|\bar{s}(x)s(0)|\phi(q)\rangle = -if_\phi^\perp \varepsilon\cdot x m_\phi^2 \int_0^1 du e^{i\bar{u}qx}\psi_3^\parallel(u), \tag{27}$$

and

$$\langle 0|\bar{s}(x)\gamma_\mu\gamma_5 s(0)|\phi(q)\rangle = \frac{1}{2} f_\phi^\parallel m_\phi \epsilon_{\mu\nu\alpha\beta}\varepsilon^\nu q^\alpha x^\beta \times \int_0^1 du e^{i\bar{u}qx}\psi_3^\perp(u). \tag{28}$$

For the structures $\Gamma^J = \gamma_\mu$ and $\sigma_{\mu\nu}$ we have

$$\begin{aligned}\langle 0|\bar{s}(x)\gamma_\mu s(0)|\phi(q)\rangle &= f_\phi^\parallel m_\phi \left\{ \frac{\varepsilon\cdot x}{q\cdot x} q_\mu \int_0^1 du e^{i\bar{u}qx}\left[\phi_2^\parallel(u) + \frac{m_\phi^2 x^2}{4}\phi_4^\parallel(u)\right] \right. \\ &+ \left(\varepsilon_\mu - q_\mu \frac{\varepsilon\cdot x}{q\cdot x}\right)\int_0^1 du e^{i\bar{u}qx}\phi_3^\perp(u) \\ &\left. - \frac{1}{2} x_\mu \frac{\varepsilon\cdot x}{(q\cdot x)^2} m_\phi^2 \int_0^1 du e^{i\bar{u}qx} C(u) + \cdots \right\}, \end{aligned} \tag{29}$$

and

$$\begin{aligned}\langle 0|\bar{s}(x)\sigma_{\mu\nu} s(0)|\phi(q)\rangle &= if_\phi^\perp \left\{ (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int_0^1 du e^{i\bar{u}qx}\left[\phi_2^\perp(u) + \frac{m_\phi^2 x^2}{4}\phi_4^\perp(u)\right] \right. \\ &+ \frac{1}{2}(\varepsilon_\mu x_\nu - \varepsilon_\nu x_\mu)\frac{m_\phi^2}{q\cdot x}\int_0^1 du e^{i\bar{u}qx}[\psi_4^\perp(u) - \phi_2^\perp(u)] \\ &\left. + (q_\mu x_\nu - q_\nu x_\mu)\frac{\varepsilon\cdot x}{(q\cdot x)^2} m_\phi^2 \int_0^1 du e^{i\bar{u}qx} D(u) + \cdots \right\}, \end{aligned} \tag{30}$$

respectively. Here $\bar{u} = 1 - u$, and m_ϕ and ε are the mass and polarization vector of the ϕ meson, respectively. In the equations above the functions $C(u)$ and $D(u)$ denote the combinations of the two-particle DAs

$$\begin{aligned} C(u) &= \psi_4^\parallel(u) + \phi_2^\parallel(u) - 2\phi_3^\perp(u), \\ D(u) &= \phi_3^\parallel(u) - \frac{1}{2}\phi_2^\perp(u) - \frac{1}{2}\psi_4^\perp(u). \end{aligned} \tag{31}$$

The twists of the distribution amplitudes are shown as subscripts in the relevant functions. It is seen that the $C(u)$ and $D(u)$ include the two-particle leading twist DAs $\phi_2^{\parallel(\perp)}(u)$, the twist-3 distribution amplitudes $\phi_3^{\parallel(\perp)}(u)$ and $\psi_3^{\parallel(\perp)}(u)$, as well as twist-4 distributions $\phi_4^{\parallel(\perp)}(u)$ and $\psi_4^{\parallel(\perp)}(u)$. Expressions of the matrix elements $\langle 0|\bar{s}(x)\Gamma^J G_{\mu\nu}(vx)s(0)|\phi(q)\rangle$ in terms of the higher twist DAs of the ϕ meson, as well as detailed information on their properties, were reported in Refs. [35–39].

The main contribution to $\Pi_\mu^{\mathrm{OPE}}(p,q)$ comes from the terms (23), where all of the propagators are replaced by their perturbative components (see Fig. 3). Contribution of this diagram can be computed using the ϕ meson two-particle distribution amplitudes. The one gluon-exchange diagrams shown in Fig. 4 are corrections, which can be expressed and calculated by utilizing three-particle DAs of the ϕ meson. An analytic expression of the $\Pi_\mu^{\mathrm{OPE}}(p,q)$ in

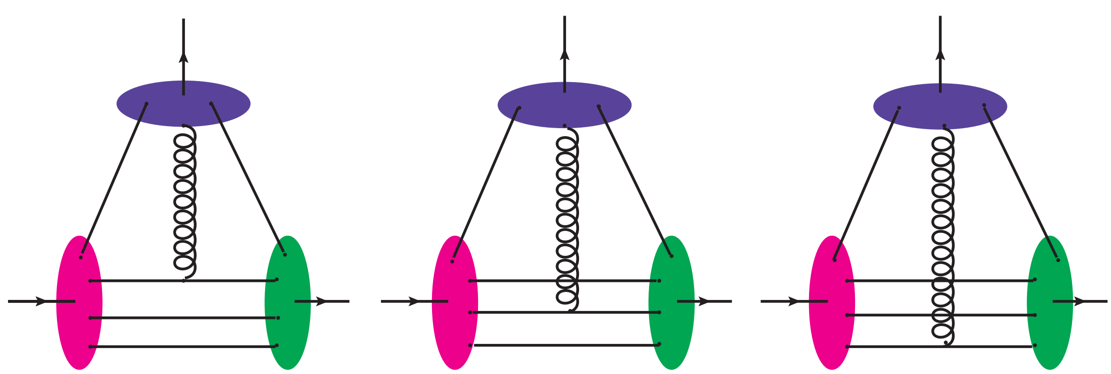

FIG. 4. The one-gluon exchange diagrams, which can be expressed in terms of the ϕ meson's three-particle DAs.

terms of the ϕ meson's DAs is rather cumbersome, therefore we do not provide it here.

In our analysis we employ the invariant amplitude $\Pi^{\rm OPE}(p'^2, p^2)$ proportional to ε_μ^* and match it to the corresponding function from $\Pi_\mu^{\rm Phys}(p,q)$. These amplitudes depend on p'^2 and p^2, therefore one should perform the double Borel transformation over the variables p'^2 and p^2:

$$\Pi^{\rm OPE}(M_1^2, M_2^2) = \mathcal{B}_{p'^2}^{M_1^2}\mathcal{B}_{p^2}^{M_2^2}\Pi^{\rm OPE}(p'^2, p^2). \tag{32}$$

The Borel transformed amplitude $\Pi^{\rm OPE}(M_1^2, M_2^2)$ can be calculated in accordance with a scheme explained in Ref. [40], and expressed as a double dispersion integral. But to simplify manipulations following after the Borel transformation, we can relate the parameters M_1^2 and M_2^2 to each other using $\frac{M_1^2}{M_2^2} = \frac{m_Y^2}{m_f^2}$ and introduce a common parameter M^2 through the relation

$$\frac{1}{M^2} = \frac{1}{M_1^2} + \frac{1}{M_2^2}. \tag{33}$$

This implies replacements

$$M_1^2 = \frac{m_f^2 + m_Y^2}{m_f^2}M^2,$$
$$M_2^2 = \frac{m_f^2 + m_Y^2}{m_Y^2}M^2, \tag{34}$$

in the sum rules, and allows us to perform integration over one of variables in the double dispersion integral. The obtained expressions in this step depend also on the parameter u_0 with

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2} = \frac{m_Y^2}{m_f^2 + m_Y^2}. \tag{35}$$

As a result of the above procedure we get a single integral representation for $\Pi^{\rm OPE}(M^2)$ which considerably simplifies the continuum subtraction: formulas necessary to carry out this procedure can be found in Appendix B of Ref. [40].

The DAs of the ϕ meson depend on numerous parameters. For example, the leading twist DAs of the longitudinally and transversely polarized ϕ meson are given by the expression

$$\phi_2^{\parallel(\perp)}(u) = 6u\bar{u}\left[1 + \sum_{n=2}^{\infty} a_n^{\parallel(\perp)} C_n^{3/2}(2u-1)\right], \tag{36}$$

where $C_n^m(2u-1)$ are the Gegenbauer polynomials. Equation (36) is the general expression for $\phi_2^{\parallel(\perp)}(u)$. In our calculations we employ twist-2 DAs with a nonasymptotic term $a_2^{\parallel(\perp)} \neq 0$. The models for the higher twist DAs and values of the relevant parameters at the normalization scale $\mu_0 = 1$ GeV are taken from Refs. [38,39] (see Tables 1 and 2 in Ref. [39]).

The sum rule for the coupling $G_{Y\phi f}$ contains the quark, gluon and mixed condensates and the s-quark mass which are moved to Table I. The spectroscopic parameters of the particles involved into the decay $Y \to \phi f_0(980)$ are also input information of computations. The mass and coupling of the tetraquark Y have been evaluated in the present work. For the mass of the ϕ and $f_0(980)$ mesons we use their experimental values (see Table I). The coupling F_f of the meson $f_0(980)$ is borrowed from Ref. [21], where it was treated as the four-quark system,

$$F_f \equiv F_{\mathbf{H}} = (1.35 \pm 0.34) \times 10^{-3}\ \text{GeV}^4. \tag{37}$$

Finally, the sum rule depends on the Borel and continuum threshold parameters: M^2 and s_0 are auxiliary parameters of computations, and the result should be insensitive to their choices. But in real analysis we can only minimize these effects and fix convenient working windows for the M^2 and s_0:

$$M^2 \in [2.4, 3.4]\ \text{GeV}^2, \qquad s_0 \in [6, 6.5]\ \text{GeV}^2. \tag{38}$$

In accordance with our studies the strong coupling $G_{Y\phi f}$ is equal to

$$G_{Y\phi f} = (1.62 \pm 0.41)\ \text{GeV}^{-1}. \tag{39}$$

The width of the decay $Y \to \phi f_0(980)$ is determined by the expression

$$\Gamma(Y \to \phi f) = \frac{G_{Y\phi f}^2 m_\phi^2}{24\pi}\lambda\left(3 + \frac{2\lambda^2}{m_\phi^2}\right), \tag{40}$$

where

$$\lambda \equiv \lambda(m_Y, m_\phi, m_f) = \frac{1}{2m_Y}[m_Y^4 + m_\phi^4 + m_f^4$$
$$- 2(m_Y^2 m_\phi^2 + m_Y^2 m_f^2 + m_\phi^2 m_f^2)]^{1/2}. \tag{41}$$

Then computations yield

$$\Gamma(Y \to \phi f) = (49.2 \pm 17.6)\ \text{MeV}. \tag{42}$$

The prediction for $\Gamma(Y \to \phi f)$ is the main result of this section which will be used to estimate the full width of the tetraquark Y.

IV. THE DECAYS $Y \to \phi\eta$ AND $Y \to \phi\eta'$

The next two strong decays of the tetraquark Y are the channels $Y \to \phi\eta$ and $Y \to \phi\eta'$. Here, we consider in a detailed form the dominant process $Y \to \phi\eta$, and write down final results for the second mode $Y \to \phi\eta'$.

In the framework of the LCSR method the correlation function necessary to study the vertex $Y\phi\eta$ is given by the expression

$$\Pi_{\mu\nu}(p,q) = i\int d^4x e^{ipx}\langle\eta(q)|\mathcal{T}\{J_\mu^{\phi}(x)J_\nu^{Y\dagger}(0)\}|0\rangle, \quad (43)$$

where $J_\mu^{\phi}(x)$ is the interpolating current for the vector ϕ meson

$$J_\mu^{\phi}(x) = \bar{s}(x)\gamma_\mu s(x). \quad (44)$$

The phenomenological side of the sum rule can be written down in the form

$$\Pi_{\mu\nu}^{\rm Phys}(p,q) = \frac{\langle 0|J_\mu^{\phi}|\phi(p)\rangle}{p^2-m_\phi^2}\langle\phi(p)\eta(q)|Y(p')\rangle \times\frac{\langle Y(p')|J_\nu^{Y\dagger}|0\rangle}{p'^2-m_Y^2}+\cdots, \quad (45)$$

and simplified further using the matrix elements:

$$\langle 0|J_\mu^{\phi}|\phi(p)\rangle = f_\phi m_\phi \epsilon_\mu, \quad (46)$$

and

$$\langle\phi(p)\eta(q)|Y(p')\rangle = g_{Y\phi\eta}\varepsilon_{\mu\nu\alpha\beta}p^\mu q^\nu \epsilon^{*\alpha}\epsilon'^{\beta}, \quad (47)$$

where ϵ'^{β} is the polarization vector of the tetraquark Y, and $g_{Y\phi\eta}$ is the strong coupling corresponding to the vertex $Y\phi\eta$.

Simple manipulations allow us to recast $\Pi_{\mu\nu}^{\rm Phys}(p,q)$ into the form

$$\Pi_{\mu\nu}^{\rm Phys}(p,q) = g_{Y\phi\eta}\frac{f_\phi m_\phi f_Y m_Y}{(p^2-m_\phi^2)(p'^2-m_Y^2)} \times\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta+\cdots, \quad (48)$$

where the only term is the contribution arising from the ground-state particles: effects of the higher resonances and continuum states are denoted by dots. The correlation function $\Pi_{\mu\nu}^{\rm Phys}(p,q)$ has a simple Lorentz structure. The invariant amplitude $\Pi^{\rm Phys}(p'^2,p^2)$, which will be used to derive the sum rule for the coupling $g_{Y\phi\eta}$, can be obtained from Eq. (48) by factoring out the structure $\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta$.

We extract the second component of the sum rule, i.e., the invariant amplitude $\Pi^{\rm OPE}(p'^2,p^2)$ from the correlation function $\Pi_{\mu\nu}^{\rm OPE}(p,q)$. In the case under analysis it is given by the following expression:

$$\Pi_{\mu\nu}^{\rm OPE}(p,q) = -i\int d^4x e^{ipx}[\gamma_5\tilde{S}_s^{ib}(x)\gamma_\mu\tilde{S}_s^{bi}(-x)\gamma_5\gamma_\nu +\gamma_\nu\gamma_5\tilde{S}_s^{ib}(x)\gamma_\mu\tilde{S}_s^{bi}(-x)\gamma_5]_{\alpha\beta} \times\langle\eta(q)|\bar{u}_\alpha^a(0)u_\beta^a(0)|0\rangle. \quad (49)$$

As is seen, the correlation function is written down in terms of the s quark propagators and local matrix elements of the η meson. Dependence of $\Pi_{\mu\nu}^{\rm OPE}(p,q)$ on the local matrix elements of a final meson is the distinctive feature of the LCSR method when applied to tetraquark-meson-meson vertices. Treatment of such vertices requires some additional manipulations, which we are going to explain below. But before that we have to find $\Pi_{\mu\nu}^{\rm OPE}(p,q)$ by rewriting the matrix elements $\langle\eta(q)|\bar{u}_\alpha^a(0)u_\beta^a(0)|0\rangle$ in terms of the η meson's parameters. To this end, we expand $\bar{u}_\alpha^a(0)u_\beta^a(0)$ and determine the standard matrix elements of the η meson that contribute to the correlation function. These operations have been discussed in the previous section, therefore here we omit further details.

The performed analysis shows that the matrix element $\langle\eta(q)|\bar{u}\gamma_\mu\gamma_5 u|0\rangle$ contributes to the correlation function $\Pi_{\mu\nu}^{\rm OPE}(p,q)$. It is defined by the formula

$$\langle\eta(q)|\bar{u}\gamma_\mu\gamma_5 u|0\rangle = -i\frac{f_\eta^q}{\sqrt{2}}q_\mu, \quad (50)$$

where f_η^q is the decay constant of the η meson's q component. The matrix element (50) differs from similar expressions of other pseudoscalar mesons. This is connected with the mixing in the $\eta-\eta'$ system which can be described using either the octet-singlet or quark-flavor basis of the flavor $SU_f(3)$ group. The latter is more convenient and simple for applications, and was used in Refs. [41–43] to explore different exclusive processes. This scheme is utilized in the present work as well.

In the quark-flavor basis the decay constants of the mesons η and η' can be extracted from the equality

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = U(\varphi)\begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix}, \quad (51)$$

where $U(\varphi)$ is the mixing matrix

$$U(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}, \quad (52)$$

with $\varphi = 39^\circ.3\pm 1^\circ.0$ being the mixing angle in the quark-flavor basis. The constants f_q and f_s in Eq. (51) are given by the formulas

$$f_q = (1.07\pm 0.02)f_\pi, \qquad f_s = (1.34\pm 0.06)f_\pi, \quad (53)$$

where $f_\pi = 131$ MeV is the pion decay constant.

Using Eqs. (49) and (50), we can obtain the invariant amplitude $\Pi^{\rm OPE}(p'^2, p^2)$ which should be equated to $\Pi^{\rm Phys}(p'^2, p^2)$ in order to derive the sum rule for the strong coupling $g_{Y\phi\eta}$. But, as we have been emphasized above, a treatment of tetraquark-meson-meson vertices in the context of the LCSR method differs from standard analysis of the previous section [28]. In fact, the LCSR for vertices of conventional mesons depends on distribution amplitudes of one of final mesons, which contain all information about nonperturbative dynamical features of the meson. The same arguments are valid for the tetraquark-tetraquark-meson vertices as well [40]. But in the case of the tetraquark-meson-meson vertices, after contracting relevant quark fields, due to a four-quark structure of the tetraquark, the correlation function contains only local matrix elements of one of final mesons. Then the momentum of this meson should be set $q = 0$ which is necessary to satisfy the four-momentum conservation at the vertex. This leads to essential modifications in the calculational scheme, because now we have to complete the LCSR method with technical methods of the soft-meson approximation [26,28].

In the soft limit $q \to 0$, we get $p' = p$, as a result we have to perform one-variable Borel transformation of the invariant amplitudes [28]. For the physical (phenomenological) side this leads to the formula

$$\mathcal{B}\Pi^{\rm Phys}(p^2) = g_{Y\phi\eta} f_\phi m_\phi f_Y m_Y \frac{e^{-m^2/M^2}}{M^2} + \cdots, \tag{54}$$

where $m^2 = (m_\phi^2 + m_Y^2)/2$.

In the soft-meson approximation the phenomenological side of the sum rule has a more complicated organization than in the case of the full LCSR method. The reason is that in the soft limit contributions connected with higher resonances and continuum states demonstrate complicated behavior. Indeed, some of these terms even after the Borel transformation remain unsuppressed and appear as contaminations in the phenomenological side [26]. Therefore, before carrying out the continuum subtraction they should be excluded from $\mathcal{B}\Pi^{\rm Phys}(p^2)$ by means of some manipulations. This problem is solved by acting on the phenomenological side of the sum rule by the operator [26,27]

$$\mathcal{P}(M^2, m^2) = \left(1 - M^2 \frac{d}{dM^2}\right) M^2 e^{m^2/M^2}, \tag{55}$$

which eliminates contaminating terms. Then contributions of higher resonances with regular behavior can be subtracted from the QCD side by benefiting from the quark-hadron duality assumption.

The operator $\mathcal{P}(M^2, m^2)$ should also be applied to the QCD side of the sum rule. Then the strong coupling $g_{Y\phi\eta}$ can be determined from the sum rule

$$g_{Y\phi\eta} = \frac{1}{f_\phi m_\phi f_Y m_Y} \mathcal{P}(M^2, m^2) \Pi^{\rm OPE}(M^2, s_0), \tag{56}$$

where $\Pi^{\rm OPE}(M^2, s_0)$ is the invariant amplitude $\Pi^{\rm OPE}(p^2)$ after the Borel transformation and continuum subtraction procedures. Our calculations carried out by taking into account nonperturbative terms up to dimension 5 yield

$$\Pi^{\rm OPE}(M^2, s_0) = \frac{f_\eta^q m_s}{8\sqrt{2}\pi^2} \int_{4m_s^2}^{s_0} ds e^{-s/M^2} + \frac{f_\eta^q m_s^2}{6\sqrt{2}M^2} \langle \bar{s}s \rangle + \frac{f_\eta^q}{12\sqrt{2}M^2} \langle \bar{s} g_s \sigma G s \rangle. \tag{57}$$

The width of the decay $Y \to \phi\eta$ is given by the following expression:

$$\Gamma(Y \to \phi\eta) = \frac{g_{Y\phi\eta}^2 \lambda^3(m_Y, m_\phi, m_\eta)}{12\pi}. \tag{58}$$

Numerical analysis leads to the results

$$g_{Y\phi\eta} = (1.85 \pm 0.38)\ {\rm GeV}^{-1},$$
$$\Gamma(Y \to \phi\eta) = (35.8 \pm 10.4)\ {\rm MeV}. \tag{59}$$

It is worth noting that in computations of $g_{Y\phi\eta}$, we have used the following working regions for M^2 and s_0:

$$M^2 \in [1.3, 1.8]\ {\rm GeV}^2, \qquad s_0 \in [6, 6.5]\ {\rm GeV}^2. \tag{60}$$

The partial width of the second process $Y \to \phi\eta'$ can be computed by utilizing the expressions obtained for the first decay. The corrections are connected with mass of the η' meson and coupling $f_{\eta'}^q$, and required replacements,

$$f_{\eta'}^q = f_q \sin\varphi, \qquad \lambda \to \lambda(m_Y, m_\phi, m_{\eta'}), \tag{61}$$

can be easily implemented into analysis. For the parameters of the second process we obtain

$$g_{Y\phi\eta'} = (1.59 \pm 0.31)\ {\rm GeV}^{-1},$$
$$\Gamma(Y \to \phi\eta') = (6.1 \pm 1.7)\ {\rm MeV}. \tag{62}$$

Saturating the full width of the Y resonance by three decay channels considered in the present work, we get

$$\Gamma_{\rm full} = (91.1 \pm 20.5)\ {\rm MeV}. \tag{63}$$

This estimate coincides neither with the *BABAR* data nor with measurements of the BESIII collaboration, but is close to the latter.

V. ANALYSIS AND CONCLUDING NOTES

We have explored the resonance Y by modeling it as a light vector tetraquark with the content $[su][\bar{s}\bar{u}]$. In the tetraquark model it was considered until now as a vector $[ss][\bar{s}\bar{s}]$ or $(s\bar{s})(s\bar{s})$ particles. Our treatment is motivated by the dominant decay channel $Y \to \phi f_0(980)$ of the Y, where it was observed as a resonant structure in the $\phi f_0(980)$ invariant mass distribution. A suggestion on the quark content of the Y depends on the structures of the final-state particles: one can consider the $f_0(980)$ either as a scalar meson $\bar{s}s$ or as a particle composed of the four valence quarks. In the second picture the vector compound $Y = [su][\bar{s}\bar{u}]$ emerges as a quite natural assignment for this resonance. Calculations carried out in the present work lead to the following predictions for m_Y and Γ_{full} of such a state:

$$m_Y = (2173 \pm 85)\ \text{MeV}, \qquad \Gamma_{\text{full}} = (91.1 \pm 20.5)\ \text{MeV}. \tag{64}$$

The result for the mass m_Y is in accord with the *BABAR* data, but is compatible with BESIII measurements as well. The full width Γ_{full} has the small overlapping region with $\Gamma = (58 \pm 16 \pm 20)$ MeV extracted in Ref. [1], but agreement with data of the BESIII collaboration is considerably better. In calculations of the Γ_{full}, we have taken into account only three strong decays of the resonance Y. But decay modes $Y \to \phi\pi\pi, K^+K^-\pi^+\pi^-, K^*(892)^0\bar{K}^*(892)^0$ of Y (seen experimentally and/or theoretically possible) and other channels have not been included into analysis. Partial width of these decays may significantly improve the present prediction for Γ_{full}.

Encouraging is our estimate for the ratio

$$\frac{\Gamma(Y \to \phi\eta)}{\Gamma(Y \to \phi f)} \approx 0.73, \tag{65}$$

which almost coincides with its experimental value ≈ 0.74. The latter has been extracted from available information on the ratios [44] [Y is denoted there $\phi(2170)$]

$$\frac{\Gamma(Y \to \phi\eta) \times \Gamma(Y \to e^+e^-)}{\Gamma_{\text{total}}} = 1.7 \pm 0.7 \pm 1.3, \tag{66}$$

and

$$\frac{\Gamma(Y \to \phi f) \times \Gamma(Y \to e^+e^-)}{\Gamma_{\text{total}}} = 2.3 \pm 0.3 \pm 0.3. \tag{67}$$

Unfortunately, precision of the experimental data and uncertainties of the theoretical results do not allow us to make more strong statements about decay modes of the tetraquark Y.

As is seen, our suggestion on a nature of the resonance $Y(2175)$ as the vector tetraquark with the content $[su][\bar{s}\bar{u}]$ has led to reasonable agreements with existing experimental data. Theoretical analyses of decay channels left beyond the scope of the present work, as well as their detailed experimental studies, will be of great help to answer open questions about the structure of the resonance $Y(2175)$.

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