Comment on "Observing a wormhole"

S. Krasnikov \bullet ^{[*](#page-0-0)}

Central Astronomical Observatory at Pulkovo, St. Petersburg, 196140, Russia

 \bigcirc (Received 17 October 2019; accepted 4 November 2019; published 20 March 2020)

In their recent paper Dai and Stojkovic discuss an interesting possibility: a star near a wormhole mouth may gravitationally feel an object located near the other mouth. This means that a star's trajectory may tell an observer that the star orbits a wormhole mouth and not a black hole. I argue that within the approximation used in the paper the effect is, in fact, unobservable irrespective of how accurate the measurements are.

DOI: [10.1103/PhysRevD.101.068301](https://doi.org/10.1103/PhysRevD.101.068301)

There is almost consensus that the massive compact objects in the centers of galaxies are giant black holes. There is, however, an alternative point of view, advocated by Kardashev et al. [\[1\]](#page-1-0). According to it, these objects are mouths of wormholes. It would be important, therefore, to find effects distinguishing between these two possibilities; see [\[2,3\],](#page-1-1) in particular. One such effect has been discovered recently by Dai and Stojkovic (DS) [\[4\].](#page-1-2) They noticed that a star orbiting a wormhole mouth may be affected by perturbations in the gravitational field that are produced by an object orbiting the other mouth.

To estimate the effect, DS consider a wormhole obtained by gluing together the tubes $r_{1,2} = R$ in a pair of equal portions $r_{1,2} \ge R > r_g$ of Schwarzschild space. Here $r_{2,1}$ are the radial coordinates in, correspondingly, "our universe" (the half of the spacetime in which the test star orbits) and the "other universe" to which the wormhole leads. Now suppose there is an object of a small mass μ in the other universe at $r_1 = A > R$ (so, the object is approximated by a light sphere); the radius, A, can—quasistatically —vary. The metrics in our and in the other universes differ from that in the case $\mu = 0$ by perturbations h^{our} and h^{oth} which are assumed to obey the following conditions: (i) only the components h_{tt}^{our} , h_{rr}^{our} , h_{tt}^{ath} , and h_{rr}^{orth} are
nonzero: (2) the perturbations depend only on r_{tot} nonzero; (2) the perturbations depend only on $r_{1,2}$;
(3) $h^{our}(R) - h^{oth}(R)$ and $\frac{\partial}{\partial h^{our}}|_{h} = -\frac{\partial}{\partial h^{our}}|_{h}$ (3) $h_{\alpha\beta}^{our}(R) = h_{\alpha\beta}^{orth}(R)$ and $\partial_{r_2}h_{\alpha\beta}^{our}|_{r_2=R} = \partial_{r_1}h_{\alpha\beta}^{our}|_{r_1=R}$
(5) Applying these conditions to the expression for the [\[5\]](#page-1-3). Applying these conditions to the expression for the monopole perturbations borrowed from Ref. [35] of [\[4\],](#page-1-2) DS infer that

$$
a \approx -\mu \frac{R}{A} \frac{1}{r_2^2},\tag{1}
$$

where the "additional acceleration" $a(M, \tau)$ is the difference between the total acceleration $a_{\text{tot}}(\tau)$ of a (nonrelativistic) test star, and the acceleration $a_M(\tau)$

experienced by the same star [\[6\]](#page-1-4) in the Schwarzschild space with mass M

$$
a(M, \tau) \equiv a_{\text{tot}}(\tau) - a_M(\tau) \tag{2}
$$

 (τ) parametrizes the world line of the star). *a* serves as an indicator: the spacetime in question is a wormhole, not ^a Schwarzschild black hole, if $a(M) \neq 0$ for all M (and for some τ). Note that the indicator is imperfect: if $a(M) = 0$ for some M, both geometries are possible.

Acceleration is measured with very high accuracy. So, it might seem that Eq. [\(1\)](#page-0-1) solves the problem of the remote detection of supermassive wormholes. Unfortunately, it does not. Indeed, our universe is empty, by construction, and spherically symmetric, by Eqs. (30)–(35) in [\[4\]](#page-1-2). Therefore, by Birkhoff's theorem, the test star moves in a static region of the Schwarzschild space of some mass M_{\ast} . This is by no means anomatods and is range consistent M_{*} . This is by no means anomalous and is fully consistent mere black hole of mass M_* . Or, formally speaking,
 $a_{\mu\nu}(\tau) = a_{\mu\nu}(\tau)$ and hence by (2) $a_{\text{tot}}(\tau) = a_{M_*}(\tau)$ and hence, by [\(2\),](#page-0-2)

$$
a(M_*, \tau) = 0 \quad \forall \ \tau \tag{3}
$$

(from this equation it follows, in particular, that the right-hand sides of Eqs. (36)–(38) in [\[4\]](#page-1-2) are actually zeroes) which means, as mentioned above, that the space may or may not be a wormhole.

Remark.—To identify the error in DS's argument note that our reasoning fully applies to the region $R < r_1 < A_{\min}$ of the other universe. The region is spherically symmetric and empty; therefore it is static. Thus the perturbations h^{oth} are actually zero there. This is in perfect agreement with Ref. [35] in [\[4\]](#page-1-2) to which DS refer in justifying their Eqs. (28)–(35) [\[4\]](#page-1-2) and which, in fact, reads as follows, see item 10.1: "Inside the orbit, the perturbation vanishes." [*](#page-0-3)

krasnikov.xxi@gmail.com

ACKNOWLEDGMENTS

I am grateful to RFBR for financial support under Grant No. 18-02-00461"Rotating black holes as the sources of particles with high energy."

- [1] N. S. Kardashev, I. D. Novikov, and A. Shatskiy, [Int. J. Mod.](https://doi.org/10.1142/S0218271807010481) Phys. D 16[, 909 \(2007\)](https://doi.org/10.1142/S0218271807010481).
- [2] T. Damour and S. N. Solodukhin, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.76.024016) 76, 024016 [\(2007\)](https://doi.org/10.1103/PhysRevD.76.024016).
- [3] S. Krasnikov, Phys. Rev. D 99[, 069901 \(2019\).](https://doi.org/10.1103/PhysRevD.99.069901)
- [4] D. C. Dai and D. Stojkovic, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.100.083513) 100, 083513 [\(2019\)](https://doi.org/10.1103/PhysRevD.100.083513).
- [5] Note that no justification is given in [\[4\]](#page-1-2) for the last condition.
- [6] "the same star" \equiv "a star with the same radial coordinate and the same velocity."