


Nonsingular bouncing cosmology from general relativity: Scalar metric perturbations

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We derive the equations of motion for scalar metric perturbations in a particular nonsingular bouncing cosmology, where the big bang singularity is replaced by a spacetime defect with a degenerate metric. The adiabatic perturbation solution is obtained for nonrelativistic hydrodynamic matter. We get the same result by working with conformal coordinates. This last method is also valid for vector and tensor metric perturbations, and selected results are presented. We, finally, discuss several new effects from the linear perturbations of this nonsingular bouncing cosmology, such as across-bounce information transfer and the possible imprint on cosmological perturbations from a new phase responsible for the effective spacetime defect.

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I. INTRODUCTION

The expanding universe appears to be described reasonably well by the Friedmann solution [1,2]. This solution has, however, a big bang singularity with diverging curvature and energy density. Recently, it has been shown [3] that the Friedmann big bang singularity can be replaced by a spacetime defect, where the spacetime metric is degenerate but the curvature finite, as is the energy density (further details are given in Ref. [4]).

This “regularized” big bang singularity corresponds, in fact, to a nonsingular bounce [5,6]. There is then a prebounce phase where the positive cosmic scale factor decreases, the bounce moment at which the positive cosmic scale factor is stationary, and a postbounce phase where the positive cosmic scale factor increases again. This particular nonsingular bouncing cosmology is obtained from an extended version of general relativity, which allows for degenerate metrics (further remarks and references will be given in Sec. II).

The urgent question, now, is if the bounce is stable under small perturbations of the metric and the matter. Perturbations are, therefore, the main topic of this article. The focus will be on scalar metric perturbations, while vector and tensor metric perturbations are briefly mentioned in one of the Appendixes. Having obtained the behavior of the metric perturbations, it is also possible to address the issue of information transfer across the bounce.

At this moment, there is an important point that we would like to make. The degenerate-metric *Ansatz* [3] gives modified Friedmann equations which correspond to *singular* differential equations. Even though these modified

Friedmann equations have a nonsingular solution for the background spacetime, there is still the potential danger of singularities appearing in perturbations of the metric. In this article, we will find that also the metric perturbations have nonsingular solutions, which is a nontrivial result (different and potentially catastrophic behavior has been found in a dynamic-vacuum-energy model without big bang curvature singularity [7,8]).

The outline of our article is now as follows. In Sec. II, we review the degenerate-metric *Ansatz* (together with the heuristics of the resulting bounce solution) and discuss general metric perturbations. In Sec. III, we derive the equations of motion for scalar metric perturbations and get the adiabatic perturbation solution for the case of nonrelativistic hydrodynamic matter. In Secs. IV and V, we briefly discuss the issues of bounce stability and across-bounce information transfer. In Sec. VI, we summarize our results and discuss how they may be relevant to the generation of a scale-invariant power spectrum of cosmological perturbations. In Appendix A, we provide certain details for the calculation of Sec. III. In Appendix B, we rederive our results for scalar metric perturbations by use of conformal coordinates, and also give some results for vector and tensor metric perturbations. In Appendix C, we present a scenario of how a possible new phase (“quantum spacetime” or something entirely different) may leave an imprint on the spectrum of cosmological perturbations.

II. BACKGROUND METRIC AND PERTURBATIONS

The modified spatially flat Robertson–Walker (RW) metric is given by [3]

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$$ds^2|_{\text{mod. RW}} \equiv g_{\mu\nu}(x)dx^\mu dx^\nu|_{\text{mod. RW}} \\ = -\frac{t^2}{b^2 + t^2} dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \quad (2.1a)$$

$$b^2 > 0, \quad (2.1b)$$

$$a(t) \in \mathbb{R}, \quad (2.1c)$$

$$t \in (-\infty, \infty), \quad (2.1d)$$

$$x^i \in (-\infty, \infty), \quad (2.1e)$$

where we set $c = 1$ and let the spatial indices i, j run over $\{1, 2, 3\}$. The cosmic time coordinate was denoted “ T ” in Refs. [3,4], but, here, we simply write “ t ,” while emphasizing that the coordinate range is given by (2.1d).

The metric from (2.1) is degenerate (with a vanishing determinant at $t = 0$) and describes a spacetime defect with characteristic length scale $b > 0$; see Ref. [9] for a general review of this type of spacetime defect and Ref. [10] for a detailed discussion of related mathematical aspects (other mathematical aspects of degenerate metrics have been discussed in Ref. [11]). The defect length scale b in the metric (2.1a) will, for the moment, be considered as an external parameter (see Sec. VI for further discussion).

At this moment, it may be helpful to recall some details of the nonsingular bouncing cosmology obtained in Refs. [3–5]. Inserting the metric Ansatz (2.1a) into the Einstein gravitational field equation [2] and taking the energy-momentum tensor of a homogeneous perfect fluid [with energy density $\rho(t)$, pressure $P(t)$, and a constant equation-of-state parameter w], the following modified Friedmann equations are obtained:

$$\left(1 + \frac{b^2}{t^2}\right) \left(\frac{1}{a(t)} \frac{da(t)}{dt}\right)^2 = \frac{8\pi G_N}{3} \rho(t), \quad (2.2a)$$

$$\left(1 + \frac{b^2}{t^2}\right) \left[\frac{1}{a(t)} \frac{d^2 a(t)}{dt^2} + \frac{1}{2} \left(\frac{1}{a(t)} \frac{da(t)}{dt}\right)^2\right] \\ - \frac{b^2}{t^3} \frac{1}{a(t)} \frac{da(t)}{dt} = -4\pi G_N P(t), \quad (2.2b)$$

$$\frac{d}{da} [a^3 \rho(a)] + 3a^2 P(a) = 0, \quad (2.2c)$$

$$\frac{P(t)}{\rho(t)} = w = \text{const}, \quad (2.2d)$$

where G_N is Newton’s gravitational coupling constant and where the last equation will later be specialized the case of nonrelativistic matter with $w = 0$.

The Eqs. (2.2) have a bounce solution which is perfectly smooth at $t = 0$ as long as $b \neq 0$. For the nonrelativistic-matter case ($w = 0$), this solution is given by

$$a(t)|_{\text{bounce}}^{(w=0)} = \sqrt[3]{\frac{b^2 + t^2}{b^2 + t_0^2}}, \quad (2.3a)$$

$$\rho(t)|_{\text{bounce}}^{(w=0)} = \rho_0 \frac{b^2 + t_0^2}{b^2 + t^2}, \quad (2.3b)$$

with normalization $a(t_0) = 1$ at $t_0 > 0$ and boundary condition $\rho(t_0) = \rho_0 > 0$. The bouncing behavior of the positive scale factor $a(t)$ from (2.3a) is manifest: $a(t)$ decreases for negative t approaching $t = 0^-$, the bounce occurs at $t = 0$ with a vanishing time derivative of $a(t)$ at $t = 0$, and $a(t)$ increases for positive t moving away from $t = 0^+$. Similar nonsingular bounce solutions for other equation-of-state parameters have been discussed in Refs. [5,6] and the detailed dynamics of the bounce has been studied analytically in Ref. [4].

The heuristics of this particular type of bounce solution is as follows. As noted in Sec. II of Ref. [4], the modified Friedmann equations (2.2a) and (2.2b) can be rewritten as the standard Friedmann equations with an additional effective energy density ρ_{defect} and an additional effective pressure P_{defect} . Specifically, the resulting equations read

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} [\rho + \rho_{\text{defect}}], \quad (2.4a)$$

$$\left[\frac{\ddot{a}}{a} + \frac{1}{2} \frac{\dot{a}^2}{a^2}\right] = -4\pi G_N [P + P_{\text{defect}}], \quad (2.4b)$$

$$\rho_{\text{defect}} \equiv -\frac{b^2}{b^2 + t^2} \rho, \quad (2.4c)$$

$$P_{\text{defect}} \equiv -\frac{b^2}{b^2 + t^2} \left(P + \frac{1}{4\pi G_N} \frac{1}{t} \frac{\dot{a}}{a}\right), \quad (2.4d)$$

where the overdot stands for the derivative with respect to t . We observe that ρ_{defect} and P_{defect} have “matter-induced” terms [proportional to the matter quantities ρ and P , respectively] and that P_{defect} also has a “vacuum” term [proportional to $(4\pi G_N)^{-1} t^{-1} \dot{a}/a$]. For the sum of the total effective energy density $\rho_{\text{total}} \equiv \rho + \rho_{\text{defect}}$ and the total effective pressure $P_{\text{total}} \equiv P + P_{\text{defect}}$, we have

$$\rho_{\text{total}} + P_{\text{total}} = \frac{t^2}{b^2 + t^2} (\rho + P) - \frac{1}{4\pi G_N} \frac{b^2}{b^2 + t^2} \frac{1}{t} \frac{\dot{a}}{a}. \quad (2.5)$$

With finite values of ρ and P at the moment of the bounce ($t = 0$) and the near-bounce behavior $a(t) \sim a_B + c_2 t^2$, for $a_B > 0$ and $c_2 > 0$ (or $a_B < 0$ and $c_2 < 0$), the following inequality holds:

$$[\rho_{\text{total}} + P_{\text{total}}]_{t=0} < 0, \quad (2.6)$$

which can be extended to a finite interval around $t = 0$. The inequality (2.6) corresponds to an effective violation of the null energy condition ($\rho_{\text{total}} + P_{\text{total}} \geq 0$), in agreement with the general discussion on bounces and energy conditions in Ref. [12].

We now return to the general background metric from (2.1), which will be called the unperturbed metric. Henceforth, a bar over a quantity denotes its unperturbed value. The perturbed metric can then be written as

$$g_{\mu\nu}(x)|_{\text{mod. RW}}^{(\text{perturbed})} = \bar{g}_{\mu\nu}(t) + h_{\mu\nu}(x), \quad (2.7)$$

where $h_{\mu\nu} = h_{\nu\mu}$ is a small perturbation compared to the unperturbed metric $\bar{g}_{\mu\nu}$ from (2.1a). The spatially isotropic and homogeneous background allows us to decompose the metric perturbations into scalars, divergenceless vectors, and divergenceless traceless symmetric tensors [13,14]. The main focus of this article will be on scalar metric perturbations.

III. SCALAR METRIC PERTURBATIONS

A. Metric Ansatz

The *Ansatz* for the metric with scalar perturbations is taken as follows:

$$ds^2|_{\text{mod. RW}}^{(\text{scalar pert})} = -(1 + E) \frac{t^2}{b^2 + t^2} dt^2 + 2\bar{a} \frac{\partial F}{\partial x^i} dx^i dt + \bar{a}^2 \left[(1 + A) \delta_{ij} + \frac{\partial B^2}{\partial x^i \partial x^j} \right] dx^i dx^j, \quad (3.1)$$

where the perturbations E , F , A and B are functions of all spacetime coordinates $\{t, x^1, x^2, x^3\}$ and the background scale factor \bar{a} is a function of only t .

B. Newtonian gauge

Consider the following transformation of the spacetime coordinates:

$$x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu, \quad (3.2)$$

where the parameters $\xi^\mu \equiv \xi^\mu(x)$ are infinitesimal functions of the spacetime coordinates. By decomposing the spatial part of ξ^μ into the gradient of a spatial scalar and a divergenceless vector [13,14],

$$\xi^i = \partial^i \xi_S + \xi_V^i, \quad (3.3a)$$

$$\partial_i \xi_V^i = 0, \quad (3.3b)$$

we have the following transformations of the metric functions from (3.1) under the change of coordinates (3.2):

$$\tilde{E} = E - \frac{2b^2}{t} \xi^0 - \frac{\partial \xi^0}{\partial t}, \quad (3.4a)$$

$$\tilde{F} = F - \frac{\bar{a}}{2} \frac{\partial \xi_S}{\partial t} + \frac{t^2}{b^2 + t^2} \frac{\xi^0}{2\bar{a}}, \quad (3.4b)$$

$$\tilde{A} = A - \frac{2\dot{\bar{a}}}{\bar{a}} \xi^0, \quad (3.4c)$$

$$\tilde{B} = B - 2\xi_S, \quad (3.4d)$$

where the overdot stands for the partial derivative with respect to t . Note that only ξ^0 and ξ_S contribute to the transformations of scalar metric perturbations.

Following Sec. 7.1.2 of Ref. [13], we construct the following gauge-invariant quantities:

$$2\Phi \equiv E - \frac{\partial}{\partial t} \left[2\bar{a} \frac{b^2 + t^2}{t^2} \left(F - \frac{\bar{a}}{4} \dot{B} \right) \right] - \frac{2b^2}{t} \left[2\bar{a} \frac{b^2 + t^2}{t^2} \left(F - \frac{\bar{a}}{4} \dot{B} \right) \right], \quad (3.5a)$$

$$2\Psi \equiv A - 4\dot{\bar{a}} \frac{b^2 + t^2}{t^2} \left(F - \frac{\bar{a}}{4} \dot{B} \right). \quad (3.5b)$$

In this article, we will use the Newtonian gauge (the origin of the name will become clear later on),

$$F = B = 0, \quad (3.6)$$

which can be reached by, first, choosing an appropriate ξ_S in (3.4d) and, then, an appropriate ξ^0 in (3.4b). In this gauge, the line element (3.1) reduces to

$$ds^2|_{\text{mod. RW}}^{(\text{scalar pert Newtonian-gauge})} = -(1 + 2\Phi) \frac{t^2}{b^2 + t^2} dt^2 + \bar{a}^2 (1 + 2\Psi) \delta_{ij} dx^i dx^j. \quad (3.7)$$

Three remarks are in order. First, note that, for later use, the perturbations Φ and Ψ are assumed to be spatially localized. Second, remark that, after choosing the Newtonian gauge, there is no further freedom to make coordinate transformations, while remaining within the *Ansatz* (3.7). Third, observe that the perturbed metric from (3.7) still has, for finite Φ and Ψ , a vanishing determinant at $t = 0$.

C. Hydrodynamic matter perturbations

1. General results

Now, consider a perfect fluid with energy-momentum tensor

$$T_{\mu\nu} = Pg_{\mu\nu} + (\rho + P)U_\mu U_\nu, \quad (3.8)$$

where P is the pressure, ρ the energy density, and U^μ the four-velocity. With the perturbed metric (3.7), the first-order perturbations of the 00 and ij components of the energy-momentum tensor are

$$\delta T_{00} = 2\frac{t^2}{b^2 + t^2}\bar{\rho}\Phi + \frac{t^2}{b^2 + t^2}\delta\rho, \quad (3.9a)$$

$$\delta T_{ij} = 2\bar{a}^2\bar{P}\Psi\delta_{ij} + \bar{a}^2\delta P\delta_{ij}, \quad (3.9b)$$

which gives

$$\delta T^0_0 = -\delta\rho, \quad (3.10a)$$

$$\delta T^i_j = \delta P\delta^i_j. \quad (3.10b)$$

The unperturbed energy density $\bar{\rho}(t)$ and pressure $\bar{P}(t)$ are determined by the following modified Friedmann equations [3,4]:

$$\left(1 + \frac{b^2}{t^2}\right)\left(\frac{\dot{\bar{a}}}{\bar{a}}\right)^2 = \frac{8\pi G_N}{3}\bar{\rho}, \quad (3.11a)$$

$$\left(1 + \frac{b^2}{t^2}\right)\left[\frac{2\ddot{\bar{a}}}{\bar{a}} + \frac{\dot{\bar{a}}^2}{\bar{a}^2}\right] - \frac{2b^2\dot{\bar{a}}}{t^3\bar{a}} = -8\pi G_N\bar{P}, \quad (3.11b)$$

which were already given as (2.2a) and (2.2b) in Sec. II. From a direct calculation of the perturbed Einstein tensor for the perturbed metric (3.7), together with (3.10) and (3.11), we get the following equations of motion for scalar metric perturbations:

$$4\pi G_N\delta\rho = \frac{\Delta\Phi}{\bar{a}^2} - 3\frac{\dot{\bar{a}}^2 b^2 + t^2}{\bar{a}^2 t^2}\Phi - 3\frac{\dot{\bar{a}} b^2 + t^2}{\bar{a} t^2}\dot{\Phi}, \quad (3.12a)$$

$$4\pi G_N\delta P = \frac{b^2 + t^2}{t^2}\ddot{\Phi} + \frac{b^2 + t^2}{t^2}\left(\frac{\dot{\bar{a}}^2}{\bar{a}^2} + \frac{2\ddot{\bar{a}}}{\bar{a}}\right)\Phi + 4\frac{\dot{\bar{a}} b^2 + t^2}{\bar{a} t^2}\dot{\Phi} - 2\frac{b^2\dot{\bar{a}}}{t^3\bar{a}}\Phi - \frac{b^2}{t^3}\dot{\Phi}, \quad (3.12b)$$

where Δ is the Laplace operator in three-dimensional Euclidean space and where we have used

$$\Psi = -\Phi, \quad (3.13)$$

which follows from the perturbed off-diagonal spatial Einstein equation. Details of this calculation are relegated to Appendix A. Note that (3.12a) for constant $\bar{a}(t)$ reproduces the Poisson equation of Newtonian gravity, which explains the name of the gauge [13].

Considering adiabatic perturbations, we have

$$\delta P = c_s^2\delta\rho, \quad (3.14)$$

where c_s^2 is the square of the speed of sound [13]. Combining (3.12) and (3.14), we get the following equation of motion for $\Phi(t, \mathbf{x})$:

$$\frac{b^2 + t^2}{t^2}\ddot{\Phi} - c_s^2\frac{\Delta\Phi}{\bar{a}^2} + \frac{b^2 + t^2}{t^2}\left[\frac{\dot{\bar{a}}^2}{\bar{a}^2}(1 + 3c_s^2) + \frac{2\ddot{\bar{a}}}{\bar{a}}\right]\Phi + \frac{\dot{\bar{a}} b^2 + t^2}{\bar{a} t^2}(4 + 3c_s^2)\dot{\Phi} - 2\frac{b^2\dot{\bar{a}}}{t^3\bar{a}}\Phi - \frac{b^2}{t^3}\dot{\Phi} = 0, \quad (3.15)$$

which is the basic equation for adiabatic perturbations.

Equations (3.12) and (3.15) are *singular* differential equations (the singularity appears at $t = 0$), but they have *nonsingular* solutions that will be presented shortly. As mentioned in Sec. I, the same behavior has been observed for the differential equations and solutions of the background spacetime [3,4].

2. Nonrelativistic matter

For nonrelativistic matter, we have the following background solution from (2.3):

$$P = c_s^2 = 0, \quad (3.16a)$$

$$\bar{\rho}(t) \propto [\bar{a}(t)]^{-3}, \quad (3.16b)$$

$$\bar{a}(t) = \sqrt[3]{\frac{b^2 + t^2}{b^2 + t_0^2}}, \quad (3.16c)$$

where $\bar{a}(t)$ has been normalized to unity at $t = t_0 > 0$. In this case, (3.15) has the solution

$$\Phi(t, \mathbf{x}) = C_1(\mathbf{x}) + \frac{b^{5/3}C_2(\mathbf{x})}{(b^2 + t^2)^{5/6}}, \quad (3.17)$$

where $C_1(\mathbf{x})$ and $C_2(\mathbf{x})$ are arbitrary dimensionless functions of the spatial coordinates \mathbf{x} . Notice that both modes in (3.17) are nonsingular at $t = 0$, which will be discussed further in Sec. IV.

As a special case of (3.17), consider a plane-wave perturbation with a single comoving wave vector \mathbf{k} ,

$$C_{1,2}(\mathbf{x}) = \hat{C}_{\mathbf{k},1,2} \exp(i\mathbf{k} \cdot \mathbf{x}), \quad (3.18)$$

where $\hat{C}_{\mathbf{k},1}$ and $\hat{C}_{\mathbf{k},2}$ are the dimensionless amplitudes. The amplitude of such a plane-wave scalar metric perturbation is given by

$$\Phi_{\mathbf{k}}(t) = \hat{C}_{\mathbf{k},1} + \frac{b^{5/3}\hat{C}_{\mathbf{k},2}}{(b^2 + t^2)^{5/6}}. \quad (3.19)$$

From (3.12a), the corresponding energy density perturbation has the following amplitude:

$$\begin{aligned} \frac{\delta\rho_{\mathbf{k}}(t)}{\bar{\rho}(t)} = & - \left[2 + \frac{3}{2}k^2(b^2 + t_0^2)^{2/3}(b^2 + t^2)^{1/3} \right] \hat{C}_{\mathbf{k},1} \\ & + \left[3 - \frac{3}{2}k^2(b^2 + t_0^2)^{2/3}(b^2 + t^2)^{1/3} \right] \frac{b^{5/3}\hat{C}_{\mathbf{k},2}}{(b^2 + t^2)^{5/6}}, \end{aligned} \quad (3.20)$$

with $k \equiv |\mathbf{k}|$. The perturbation results for different wave vectors \mathbf{k} can be superposed, in order to obtain localized wave packets.

For $t \neq 0$ and a physical wavelength much larger than the Hubble horizon ($1/H \equiv \bar{a}/\dot{\bar{a}}$),

$$\frac{\bar{a}^2}{k^2} \gg \frac{1}{H^2} > \frac{t^2/(b^2 + t^2)}{H^2}, \quad (3.21)$$

we have from (3.20)

$$\left. \frac{\delta\rho_{\mathbf{k}}(t)}{\bar{\rho}(t)} \right|_{(\text{long-wavelength})} \sim -2\hat{C}_{\mathbf{k},1} + \frac{3\hat{C}_{\mathbf{k},2}}{(1 + t^2/b^2)^{5/6}}. \quad (3.22a)$$

For a short physical wavelength, we get

$$\begin{aligned} \left. \frac{\delta\rho_{\mathbf{k}}(t)}{\bar{\rho}(t)} \right|_{(\text{short-wavelength})} \sim & -\frac{3}{2}k^2(t_0 + b^2)^{2/3} \\ & \times \left[\hat{C}_{\mathbf{k},1} \sqrt[3]{b^2 + t^2} + \frac{b^{2/3}\hat{C}_{\mathbf{k},2}}{\sqrt{1 + t^2/b^2}} \right]. \end{aligned} \quad (3.22b)$$

Remark that the growing mode in (3.22b) is proportional to the cosmic scale factor $\bar{a}(t)$ from (3.16c), just as happens for the standard matter-dominated Friedmann universe; cf. Eq. (7.56) of Ref. [13].

The results (3.22a) and (3.22b) will be rederived with an auxiliary cosmic time coordinate in Appendix B, which also contains results on vector and tensor metric perturbations.

IV. BOUNCE STABILITY

The results from Sec. III C 2 show that plane-wave scalar metric perturbations and the corresponding adiabatic density perturbations are *finite* at the moment of the bounce, $t = 0$. Specifically, these results are given by (3.19) and (3.20), for an arbitrary comoving wave vector \mathbf{k} .

But the *magnitude* of the metric perturbations at $t = 0$ must also be small enough, so as to keep the background metric essentially unchanged. For scalar metric perturbations in the Newtonian gauge, the perturbed metric is given by (3.7). In order to keep the bounce essentially unchanged,

the absolute value of both perturbations in (3.7) must be significantly less than unity at $t = 0$,

$$|2\Phi(0, \mathbf{x})| \ll 1, \quad (4.1a)$$

$$|2\Psi(0, \mathbf{x})| \ll 1. \quad (4.1b)$$

With the solutions (3.13) and (3.19), there are then the following bounds on the amplitudes of the plane-wave scalar metric perturbations:

$$|2\hat{C}_{\mathbf{k},1}| \ll 1, \quad (4.2a)$$

$$|2\hat{C}_{\mathbf{k},2}| \ll 1. \quad (4.2b)$$

In other words, having small enough amplitudes $|\hat{C}_{\mathbf{k},1}|$ and $|\hat{C}_{\mathbf{k},2}|$ does not disturb the bounce. As mentioned in Sec. I, a similar conclusion does not hold for a dynamic-vacuum-energy model [7], which is found to have a violent instability [8] at the moment of the big bang, where the cosmic scale factor vanishes.

Returning to our degenerate-metric bounce, there is, however, a puzzle. Consider, at $t = t_{\text{start}} < 0$ in the pre-bounce phase, the hypothetical generation of plane-wave scalar metric perturbations with an initial amplitude $\hat{\Phi}_{\mathbf{k},\text{start}}$. Generically, both modes in (3.19) are excited, but let us focus on the nonconstant mode, so that we have

$$\Phi_{\mathbf{k}}(t) \sim \hat{\Phi}_{\mathbf{k},\text{start}} \frac{(b^2 + t_{\text{start}}^2)^{5/6}}{(b^2 + t^2)^{5/6}}. \quad (4.3)$$

Perturbations generated at $t_{\text{start}} \ll -b < 0$ will grow with cosmic time t and give at the moment of the bounce, $t = 0$,

$$\Phi_{\mathbf{k}}(0) \sim \hat{\Phi}_{\mathbf{k},\text{start}} \frac{(b^2 + t_{\text{start}}^2)^{5/6}}{b^{5/3}} \sim \hat{\Phi}_{\mathbf{k},\text{start}} \frac{|t_{\text{start}}|^{5/3}}{b^{5/3}}. \quad (4.4)$$

The stability condition (4.1) then requires an extremely small start amplitude,

$$|\hat{\Phi}_{\mathbf{k},\text{start}}| \ll \frac{b^{5/3}}{|t_{\text{start}}|^{5/3}}. \quad (4.5)$$

The puzzle, now, is how to guarantee a sufficiently small amplitude $|\hat{\Phi}_{\mathbf{k},\text{start}}|$ if the scalar metric perturbations are generated in the prebounce phase at early times, $t_{\text{start}} \ll -b < 0$.

The puzzle outlined in the previous paragraph is, of course, well known to practitioners of nonsingular-bouncing-cosmology scenarios (cf. Refs. [15–19] and references therein). In fact, this is the motivation for postulating an exotic component in the prebounce phase, with an equation-of-state parameter $w_{\text{ex}} \equiv P_{\text{ex}}/\rho_{\text{ex}} \geq 1$. Approaching the bounce from the prebounce side, the energy density

of an exotic component with $w_{\text{ex}} > 1$ “grows to dominate all other forms of energy, including inhomogeneities, anisotropy and spatial curvature” (quote from Ref. [17]). For this reason, we have also considered [5,6] asymmetric versions of the degenerate-metric bounce with $w_{\text{prebounce}} = 1$ and $w_{\text{postbounce}} \sim 1/3$.

An alternative physical interpretation of the degenerate-metric bounce is as follows. The thermodynamic arrow of time may be considered to run in the direction of growing density perturbations (cf. the discussion in Appendix B of Ref. [7], which contains further references). From our result (3.22b) on the short-wavelength growing mode, we can then define the thermodynamic time $\mathcal{T}(t)$ as $\mathcal{T}(t) \equiv t$ for $t > 0$ and $\mathcal{T}(t) \equiv -t$ for $t \leq 0$. In short, we have $\mathcal{T}(t) \equiv |t|$. Now assume that small metric perturbations can *only* be generated near the bounce at $\mathcal{T} = 0$. Then, the corresponding matter perturbations will simply grow with $\mathcal{T} \equiv |t|$. In other words, they grow equally on both “sides” of the bounce, not endangering the bounce any further. The physical motivation of this alternative scenario will be discussed in the last paragraph of Sec. VI.

V. ACROSS-BOUNCE INFORMATION TRANSFER

Having established the propagation of scalar metric perturbations in Sec. III C 2 and of tensor metric perturbations in Appendix B 2, we are able to address the question of information transfer *across* the bounce.

Consider a plane-wave scalar metric perturbation triggered at

$$t = t_{\text{start}} < 0, \quad (5.1)$$

with amplitude

$$\Phi_{\mathbf{k}}(t_{\text{start}}) = \hat{\Phi}_{\mathbf{k},\text{start}} \quad (5.2)$$

and an appropriate nonvanishing value of $\dot{\Phi}_{\mathbf{k}}(t_{\text{start}})$ [from (4.3), we have $\dot{\Phi}_{\mathbf{k}}(t_{\text{start}})/\hat{\Phi}_{\mathbf{k},\text{start}} \sim -(5/3)t_{\text{start}}/(b^2 + t_{\text{start}}^2)$]. According to the solution (4.3), this perturbation grows until the bounce at $t = 0$ is reached and then decreases as t increases further.

Taking the observation time symmetrically for illustrative purposes,

$$t_{\text{obs}} = -t_{\text{start}} > 0, \quad (5.3)$$

we find, from (4.3), the following observed perturbation amplitude:

$$\Phi_{\mathbf{k}}(t_{\text{obs}}) = \hat{\Phi}_{\mathbf{k},\text{start}}. \quad (5.4)$$

In this way, we are, in principle, able to transfer information across the bounce. With a sequence of scalar-metric-perturbation pulses, for example, it is possible to compose

a message in Morse code that starts in the prebounce phase, passes across the bounce, and ends up in the postbounce phase.

More realistic would be to use short-wavelength gravitational waves, and the same result on across-bounce information transfer is obtained from (B7). In fact, we have already discussed in Ref. [5] the across-bounce effects of “gravitational standard candles,” so that the results of this article fill in one of the missing details (the other missing detail [*sic*] is the actual existence of these standard candles).

VI. DISCUSSION

In the present article, we have obtained first results for the metric perturbations of a particular nonsingular bouncing cosmology. This type of bounce [3] relies on an extension of general relativity, namely by allowing for degenerate metrics (see, e.g., Ref. [11] for an earlier discussion of general relativity with degenerate metrics). In fact, our degenerate-metric *Ansatz* describes what may be called a “spacetime defect” (see Ref. [9] for a review). The matter content of the corresponding nonsingular bouncing cosmology is entirely standard, without any problem whatsoever as regards unitarity and microcausality (see, e.g., Sec. III of Ref. [18] for an overview of other bounce realizations, some with potentially problematic matter content). In our case, singularity theorems are circumvented by having a degenerate metric with a vanishing determinant on a three-dimensional submanifold (further discussion and references can be found in Ref. [3]).

The main open question for our degenerate-metric bounce is the physical origin of the corresponding spacetime defect, notably its length scale b in the metric *Ansatz* (2.1). It could very well be that the spacetime defect at $t = 0$ (in the notation of our *Ansatz*) traces back to a new phase. But nothing is known for sure about such a phase. For the moment, we make no assumption about the physical origin of the defect length scale b and only require that the numerical value of b is large enough, so that Einstein’s classical gravity holds.

As it stands, we have with the degenerate-metric *Ansatz* (2.1) from Ref. [3] an economic way to describe a nonsingular bounce, assuming such a bounce to be relevant to our Universe. In that case, it is worthwhile to study the perturbations of the metric, and we have initiated that analysis in the present article.

For any cosmological model aiming to describe the evolution of the very early universe, it is crucial to be able to produce a scale-invariant power spectrum of cosmological perturbations. The postbigbang exponential expansion of the inflationary scenario does the job [13,14]. But prebounce or ekpyrotic scenarios can also get a scale-invariant power spectrum of fluctuations, using either axions or an additional scalar field (see, e.g., Sec. II of Ref. [18]). Recently, there have been effective-field-theory

calculations of linear perturbations in null-energy-condition-violating bounce models; see Refs. [20–22] and references therein.

Let us return to the degenerate-metric bounce [3], where the matter obeys the standard energy conditions. Even though the matter-dominated prebounce contraction phase considered in this article is far from perfect (as discussed in Sec. IV), vacuum fluctuations in the prebounce phase appear to be converted into a scale-invariant power spectrum of density perturbations in the postbounce phase (cf. Sec. II B of Ref. [18]). The actual mechanism which generates the required scale-invariant power spectrum of cosmological perturbations may depend on certain details of the prebounce phase (cf. Refs. [17,19]). But, regardless of the prebounce generation mechanism of the cosmological perturbations, our explicit degenerate-metric model of the bounce, with the time-symmetric results (3.19), (3.20), and (B7), has shown that these particular perturbations are unaffected by the dynamics of the bounce itself. In our degenerate-metric bounce scenario (with a large enough value of the defect length scale b , so as to remain in the classical-gravity phase), the cosmological perturbations from the prebounce phase can safely cross the bounce, and the present article has shown this by concrete examples.

If, however, the defect length scale b of our metric (2.1) is relatively small and related to an entirely new phase (“quantum spacetime” or something different), then another scenario may be envisioned, which was already mentioned in the last paragraph of Sec. IV. Taking non-relativistic matter as a toy model, we obtain from the density perturbation result (3.20) at $t = 0$ that the perturbations have a critical comoving wavelength $\lambda_{\text{crit}} \sim (ct_0)^{2/3}(b)^{1/3}$ (the corresponding physical wavelength is $\tilde{\lambda}_{\text{crit}} \sim b$). If the perturbations (all or only part of them) are generated by the new phase and emerge at the classical time $t = 0$, then we may expect to observe a different behavior for wavelengths below or above this critical wavelength (see Appendix C for a possible scenario). This would, in principle, provide a way to determine the numerical value of b [23], in addition to the *Gedankenexperiment* presented in Ref. [5]. As mentioned before, the crucial open question is the physical origin of the defect length scale b , related to a possible new phase or not. But, even if the defect length scale b is related to such a new phase, this does not necessarily imply that b is of the order of the Planck length [24].

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APPENDIX A: CALCULATIONAL DETAILS

In this Appendix, we provide some details for the calculation of (3.12) and (3.13) in Sec. III C 1.

From (3.7) and (3.10), we get the following perturbed Einstein equations (up to first-order perturbations):

$$8\pi G_N(\bar{\rho} + 2\Phi\bar{\rho} + \delta\rho) = 3\frac{b^2 + t^2\dot{\bar{a}}^2}{t^2\bar{a}^2} + 6\frac{b^2 + t^2\dot{\bar{a}}}{t^2\bar{a}}\dot{\Psi} - \frac{2\Delta\Psi}{\bar{a}^2}, \quad (\text{A1a})$$

$$8\pi G_N(\bar{P} + 2\Psi\bar{P} + \delta P)\delta_{ij} = \left[\frac{2b^2}{t^3}\dot{\Psi} - 2\frac{b^2 + t^2}{t^2}\ddot{\Psi} - 6\frac{b^2 + t^2\dot{\bar{a}}}{t^2\bar{a}}\dot{\Psi} + 2\frac{b^2 + t^2\dot{\bar{a}}}{t^2\bar{a}}\dot{\Phi} + 2\left(\frac{b^2\dot{\bar{a}}}{t^3\bar{a}} - \frac{b^2 + t^2\ddot{\bar{a}}}{t^2\bar{a}}\right)(1 + 2\Psi - 2\Phi) + \frac{\Delta(\Phi + \Psi)}{\bar{a}^2} - \frac{b^2 + t^2\dot{\bar{a}}^2}{t^2\bar{a}^2}(1 + 2\Psi - 2\Phi) \right] \delta_{ij} - \frac{1}{\bar{a}^2}\frac{\partial^2}{\partial x^i\partial x^j}(\Phi + \Psi), \quad (\text{A1b})$$

where Δ is the Laplace operator in three-dimensional Euclidean space.

With the assumption that Φ and Ψ vanish at spatial infinity, we obtain from (A1b) for $i \neq j$ that

$$\Psi = -\Phi, \quad (\text{A2})$$

as stated in (3.13) of the main text.

Without perturbations, the leading order terms in (A1) are precisely the background equations (3.11). From the first-order perturbations in (A1), together with the background equations (3.11) and the result (A2), we get (3.12) of the main text.

APPENDIX B: PERTURBATIONS WITH CONFORMAL COORDINATES

1. Scalar metric perturbations

The modified spatially flat Robertson–Walker metric (2.1a) can be written as

$$ds^2|_{\text{mod. RW}} = \Omega^2(\eta)(-d\eta^2 + \delta_{ij}dx^i dx^j), \quad (\text{B1a})$$

where η is the conformal time defined by

$$\Omega(\eta)d\eta = \sqrt{\frac{t^2}{b^2 + t^2}}dt, \quad (\text{B1b})$$

for $t \in \mathbb{R}$ and defect length scale $b > 0$ (see below for further details on η). Perturbations of the conformally flat metric (B1a) have been widely studied in the literature; see, in particular, Ref. [13].

For our nonsingular degenerate-metric bouncing cosmology, the metric perturbation solutions take the same form as in the standard hot-big-bang model but now with η given by (B1b). For example, the plane-wave adiabatic

scalar perturbations for nonrelativistic hydrodynamical matter have the following solutions given by Eqs. (7.55) and (7.56) in Ref. [13]:

$$\frac{\delta\rho_{\mathbf{k}}(\eta)}{\bar{\rho}(\eta)} \Big|_{(\text{long-wavelength})} \sim -2\hat{C}_{\mathbf{k},1} + 3\hat{C}_{\mathbf{k},2}\text{sgn}(\eta)\eta^{-5}, \quad (\text{B2a})$$

$$\frac{\delta\rho_{\mathbf{k}}(\eta)}{\bar{\rho}(\eta)} \Big|_{(\text{short-wavelength})} \sim -\frac{k^2}{6}(\hat{C}_{\mathbf{k},1}\eta^2 + \hat{C}_{\mathbf{k},2}\text{sgn}(\eta)\eta^{-3}), \quad (\text{B2b})$$

with $k \equiv |\mathbf{k}|$ and constants $\hat{C}_{\mathbf{k},1,2}$ (the extra sign factor multiplying $\hat{C}_{\mathbf{k},2}$ is needed to get the correct boundary conditions at the spacetime defect, as will be explained below). For nonrelativistic matter, (3.16c) and (B1b) give

$$\Omega(\eta) = \frac{1}{9} \frac{\eta^2}{b^2 + t_0^2}, \quad (\text{B3a})$$

$$\eta = \begin{cases} +3\sqrt[3]{b^2 + t_0^2}\sqrt[6]{b^2 + t^2}, & \text{for } t \geq 0, \\ -3\sqrt[3]{b^2 + t_0^2}\sqrt[6]{b^2 + t^2}, & \text{for } t \leq 0, \end{cases} \quad (\text{B3b})$$

$$\eta \in (-\infty, \eta_-] \cup [\eta_+, \infty), \quad (\text{B3c})$$

$$\eta_{\pm} \equiv \pm 3\sqrt[3]{b(b^2 + t_0^2)}, \quad (\text{B3d})$$

where the points $\eta = \eta_-$ and $\eta = \eta_+$ are identified (in this way, the topology becomes \mathbb{R}).

The coordinate transformation (B3b) is not a diffeomorphism, so that the differential structure from (2.1a) differs from that of (B1a). In fact, t from (2.1) is a good coordinate, but η from (B3b) is *not* (there are different values η_{\pm} for the single point $t = 0$). Still, η appears to be a useful auxiliary coordinate away from the spacetime defect at $\eta = \eta_{\pm}$. The implication is that the η domains $(-\infty, \eta_-)$ and (η_+, ∞) are disconnected, so that the boundary conditions at $\eta = \eta_{\pm}$ require special care. See Refs. [3,4,10] for an extensive discussion of these issues.

At this moment, we remark that the extra minus signs for the $\hat{C}_{\mathbf{k},2}$ terms in (B2) make for proper boundary conditions at $\eta = \eta_{\pm}$. Indeed, inserting the η expression from (B3b) into the perturbations (B2a) and (B2b), we observe the resulting expressions to be even with respect to t . Introducing dimensionless constants $\hat{c}_{\mathbf{k},1,2}$ by the definitions $\{\hat{C}_{\mathbf{k},1}, \hat{C}_{\mathbf{k},2}\} \equiv \{\hat{c}_{\mathbf{k},1}, b^5\hat{c}_{\mathbf{k},2}\}$, the final expressions are

$$\frac{\delta\rho_{\mathbf{k}}(t)}{\bar{\rho}(t)} \Big|_{(\text{long-wavelength})} \sim -2\hat{c}_{\mathbf{k},1} + 3^{-4}b^5\hat{c}_{\mathbf{k},2}(b^2 + t_0^2)^{-5/3}(b^2 + t^2)^{-5/6}, \quad (\text{B4a})$$

$$\begin{aligned} & \frac{\delta\rho_{\mathbf{k}}(t)}{\bar{\rho}(t)} \Big|_{(\text{short-wavelength})} \\ & \sim -\frac{3}{2}k^2(b^2 + t_0^2)^{2/3}[\hat{c}_{\mathbf{k},1}\sqrt[3]{b^2 + t^2} \\ & \quad + 3^{-5}b^5\hat{c}_{\mathbf{k},2}(b^2 + t_0^2)^{-5/3}(b^2 + t^2)^{-1/2}], \end{aligned} \quad (\text{B4b})$$

in agreement with our previous results (3.22a) and (3.22b).

2. Vector and tensor metric perturbations

Results on vector and tensor metric perturbations can be directly taken over from Sec. 7.3.2 in Ref. [13], where, for the nonrelativistic-matter case, the conformal factor $a(\eta)$ is replaced by our factor $\Omega(\eta)$ from (B3a) and η is given by (B3b).

Here, we give some explicit results for the radiation-dominated case. With the energy-momentum-tensor perturbation $\delta T_i^0 = \Omega^{-1}(\bar{\rho} + \bar{P})\delta u_{\perp i}$ and the definition $\delta v^i \equiv \Omega^{-1}\delta u_{\perp i}$, the result from Eq (7.94) in Ref. [13] is that plane-wave vector metric perturbations for the radiation-dominated case are constant with respect to the conformal time η ,

$$\delta v_{\mathbf{k}}^i = \frac{\hat{C}_{\mathbf{k},3}^i}{\Omega^4(\bar{\rho} + \bar{P})} = \hat{c}_{\mathbf{k},3}^i, \quad (\text{B5})$$

where the last equality uses the radiative behavior $\bar{P} = \bar{\rho}/3 \propto \Omega^{-4}$ and where the $\hat{c}_{\mathbf{k},3}^i$ are appropriate dimensionless constants.

Turning to plane-wave tensor metric perturbations for the radiation-dominated case, the result from Eq. (7.98) in Ref. [13] is as follows:

$$h_{\mathbf{k}}^{ij} = \frac{1}{\eta} [\hat{C}_{\mathbf{k},4} \sin(k\eta) + \hat{C}_{\mathbf{k},5} \text{sgn}(\eta) \cos(k\eta)] e_{\mathbf{k}}^{ij}, \quad (\text{B6a})$$

$$\eta = \begin{cases} +2\sqrt[4]{b^2 + t_0^2}\sqrt[4]{b^2 + t^2}, & \text{for } t \geq 0, \\ -2\sqrt[4]{b^2 + t_0^2}\sqrt[4]{b^2 + t^2}, & \text{for } t \leq 0, \end{cases} \quad (\text{B6b})$$

$$\eta \in (-\infty, \tilde{\eta}_-] \cup [\tilde{\eta}_+, \infty), \quad (\text{B6c})$$

$$\tilde{\eta}_{\pm} \equiv \pm 2\sqrt[4]{b^2(b^2 + t_0^2)}, \quad (\text{B6d})$$

with $k \equiv |\mathbf{k}|$ and constant polarization tensor $e_{\mathbf{k}}^{ij}$ (the polarization may be different for different wave vectors \mathbf{k}). Remark that, just as in Appendix B 1, we have added a sign factor to the coefficient $\hat{C}_{\mathbf{k},5}$ in (B6a), in order to get the proper boundary conditions at $\eta = \tilde{\eta}_{\pm}$.

From (B6a), we see that short-wavelength gravitational waves ($k|\eta| \gg 1$) have an averaged amplitude that goes as $1/|\eta|$, so that

$$h_{\mathbf{k}}^{ij} \Big|_{(\text{short-wavelength})} \sim \frac{\hat{c}_{\mathbf{k},6}^{ij}}{\sqrt[4]{1 + t^2/b^2}} e_{\mathbf{k}}^{ij}, \quad (\text{B7})$$

for a dimensionless constant $\hat{c}_{k,6}$. With the cosmic scale factor $\bar{a}(t) \propto \sqrt[4]{b^2 + t^2}$, we observe that the amplitudes of short-wavelength gravitational waves from (B7) go as $1/\bar{a}(t)$, which matches the behavior of the standard radiation-dominated Friedmann universe; see the second and third lines below Eq. (7.100) in Ref. [13].

APPENDIX C: IMPRINT FROM A NEW PHASE?

The present appendix expands on the discussion of the last paragraph in Sec. VI by sketching one possible scenario, purely as illustration.

Suppose that a new phase is responsible for the effective spacetime defect and that scalar metric perturbations (in the Newtonian gauge) emerge in the corresponding classical spacetime (3.7) at cosmic time coordinate $t = 0$. Matter perturbations in both universes (described by $t > 0$ and $t < 0$) may grow with thermodynamic time $\mathcal{T}(t) \equiv |t|$, as discussed in the last paragraph of Sec. IV.

Suppose also that this new phase sets the value of the metric perturbation $\Phi_{\mathbf{k}}(t)$ at $t = 0$ differently for comoving wave number $k \equiv |\mathbf{k}|$ above or below the critical value $k_{\text{crit}} \equiv 2\pi/\lambda_{\text{crit}}$ (the critical physical wave number is given by $\bar{k}_{\text{crit}} \sim 2\pi/b$),

$$\Phi_{\mathbf{k}}(0) = \begin{cases} 0, & \text{for } k > k_{\text{crit}}, \\ 1, & \text{for } k \leq k_{\text{crit}}. \end{cases} \quad (\text{C1a})$$

The new phase is, moreover, supposed to fix the value of the following quantity involving the time derivative of the metric perturbation $\Phi_{\mathbf{k}}(t)$:

$$\lim_{t \rightarrow 0} \left| \frac{b^2}{t} \dot{\Phi}_{\mathbf{k}}(t) \right| \ll 1, \quad (\text{C1b})$$

which holds equally for all values of \mathbf{k} and is a stronger condition than $b\dot{\Phi}_{\mathbf{k}}(0) = 0$.

From (C1a) and (C1b), the subsequent metric perturbations (3.19) for $|t| > 0$ have the following amplitudes:

$$\hat{C}_{\mathbf{k},1} = \begin{cases} -\hat{C}_{\mathbf{k},2}, & \text{for } k > k_{\text{crit}}, \\ 1 - \hat{C}_{\mathbf{k},2}, & \text{for } k \leq k_{\text{crit}}, \end{cases} \quad (\text{C2a})$$

$$|\hat{C}_{\mathbf{k},2}| \ll 1. \quad (\text{C2b})$$

The implication is that $\hat{C}_{\mathbf{k},1}$ is close to 0 for relatively small wavelengths and close to 1 for relatively large wavelengths, as long as the assumptions hold true. From (C2), the spectrum of adiabatic density perturbations (3.20) then shifts as k changes from above k_{crit} to below k_{crit} , with a jump of the coefficient multiplying the growing mode proportional to $(b^2 + t^2)^{1/3}$.

It is an interesting problem to see if it is possible to match the perturbations of the degenerate metric to the results from loop quantum gravity [25], string theory [26], or emergent gravity [27]. For homogeneous models, Appendix B of Ref. [4] has already given a first comparison between extended general relativity with an appropriate degenerate metric and the effective theories from loop quantum cosmology and string cosmology.

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