Strong cosmic censorship conjecture in higher curvature gravity

Akash K. Mishra[1,*] and Sumanta Chakraborty[2,3,†]

[1]*Indian Institute of Technology, Gandhinagar-382355, Gujarat, India*
[2]*School of Physical Sciences, Indian Association for the Cultivation of Science, Kolkata-700032, India*
[3]*School of Mathematical and Computational Sciences, Indian Association for the Cultivation of Science, Kolkata-700032, India*

The deterministic nature of general relativity is ensured by the strong cosmic censorship conjecture, which asserts that spacetime cannot be extended beyond the Cauchy horizon with the square integrable connection. Although this conjecture holds true for asymptotically flat black hole spacetimes in general relativity, a potential violation of this conjecture occurs in charged asymptotically de Sitter spacetimes. Since it is expected that the Einstein-Hilbert action will involve higher curvature corrections, in this article we have studied whether one can restore faith in the strong cosmic censorship when higher curvature corrections to general relativity are considered. Contrary to our expectations, we have explicitly demonstrated that not only a violation to the conjecture occurs near extremality, but the violation appears to become stronger as the strength of the higher curvature term increases.

I. INTRODUCTION AND MOTIVATION

The Strong Cosmic Censorship Conjecture, broadly speaking, asserts that all the physically reasonable solutions of Einstein's equations with regular initial data are globally hyperbolic, which in turn implies that general relativity is deterministic in nature [1,2]. However, the existence of a Cauchy horizon in several realistic solutions of Einstein's equations, may indicate a possible violation of strong cosmic censorship conjecture, since a Cauchy horizon is regarded as the boundary of maximum Cauchy development of an initial data given on a Cauchy hypersurface. Therefore the breakdown of strong cosmic censorship conjecture or, equivalently, understanding the deterministic nature of the theory boils down to the question of whether the spacetime can be extended beyond the Cauchy horizon. If the metric is regular at the Cauchy horizon, it is possible to construct a geodesic that can be extended beyond the Cauchy horizon into regions where any further evolution of the geodesic cannot be uniquely obtained from the initial data [3]. This scenario can be considered as a potential violation of the strong cosmic censorship conjecture. One possible resolution to this problem, as proposed by Penrose, has to do with the unstable nature of the Cauchy horizon with respect to any small perturbation [4]. More precisely, if the perturbations at the Cauchy horizon grow unboundedly, then ultimately they will turn into a curvature singularity and, hence, the problem of

crossing the Cauchy horizon can be avoided. This process of turning a Cauchy horizon to a curvature singularity is known as the mass inflation in the literature [5–8]. This can also be understood from the fact that all the incoming waves will be blue shifted by an infinite amount as they approach the Cauchy horizon, leading to the existence of a singularity. This feature will survive for asymptotically flat spacetimes, where the perturbations have a power-law decay at late times, and, hence, the exponential growth $\Phi \sim e^{\kappa_- u}$, with κ_- being the surface gravity at the Cauchy horizon, always dominates, leading to singular behavior.

On the other hand, for asymptotically de Sitter spacetimes, the perturbations at late times also decay exponentially, which has the possibility of being canceled by the exponential growth at the Cauchy horizon, leading to extension of the spacetime beyond the Cauchy horizon [9,10]. More precisely, for the case of asymptotically de Sitter spacetimes, e.g., the Reissner-Norsdröm-de Sitter black hole, the perturbation attains an exponentially decaying late-time tail $\Phi \sim e^{-\alpha u}$, where $\alpha = -\text{Im}(\omega)$ is the spectral gap related to the lowest-lying quasinormal frequency. Therefore it is indeed possible, at least for a certain range of parameters, where the exponential decay of perturbation is balanced by the exponential growth of perturbation at the Cauchy horizon, thus avoiding any mass inflation singularity. Hence the quantity of interest in such an analysis is the relative strength between the decay and growth of the perturbation at the Cauchy horizon, which is determined by the quantity $\beta \equiv (\alpha/\kappa_-)$. It so happens that for $\beta > 1/2$, the late-time decay of the perturbation becomes strong enough to overcome the

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[*]akash.mishra@iitgn.ac.in
[†]sumantac.physics@gmail.com

growth at Cauchy horizon, thereby leading to a violation of strong cosmic censorship conjecture [11]. In the absence of a general proof for the strong cosmic censorship conjecture, such an analysis plays a very crucial role in order to test the conjecture, i.e., by looking for possible counterexamples. This approach has been used recently by several authors in order to test the validity of the strong cosmic censorship conjecture for general relativity on various asymptotically de Sitter black hole spacetimes in four and higher dimensions with different test fields [12–26]. The central result arising out of these analyses is the realization that the strong cosmic censorship conjecture is violated in the near extremal regime for nonrotating black holes, while for rotating black holes, the violation can be avoided. For black holes in the Born-Infeld–de Sitter and the Horndeski theories, the strong cosmic censorship conjecture has also been recently studied in [27,28].

Surprisingly, no such analysis for the validity of the strong cosmic censorship conjecture has ever been extended to black holes in higher curvature gravity theories. Even though general relativity describes the gravitational interaction around us very nicely, it also has several shortcomings. The most notable among various ones are the singularity problem and late time acceleration of the Universe. Besides there are also numerous other motivations for looking for gravity theories beyond general relativity, including nonrenormalizability of the gravitational action [29–32]. Thus it is reasonable to believe that general relativity is only an effective field theory, which must be supplemented by higher curvature corrections at the strong gravity regime. The most natural generalization of the Einstein-Hilbert action involving higher curvature corrections is the Lanczos-Lovelock gravity, containing at most the second derivative of the metric [33–38]. This motivates us to study whether the strong cosmic censorship conjecture holds in the presence of higher curvature terms. There is a hope that even though the strong cosmic censorship conjecture is violated for certain solutions in general relativity, when the higher curvature corrections are taken into account it may be respected.

Following which, in this work, we have studied the strong cosmic censorship conjecture in the context of two higher curvature theories. As our first example, we consider the case of Einstein-Gauss-Bonnet gravity in five and higher spacetime dimensions, which is the second-order term of the Lanczos-Lovelock Lagrangian. The Einstein-Gauss-Bonnet theory admits spherically symmetric charged black hole solutions with a cosmological constant [39–44], which involves a Cauchy horizon. Thus one can ask whether the solution can be extended beyond the Cauchy horizon. Our second example involves the study of pure Lovelock black hole solutions [45–50] in dimensions $d \geq (3k + 1)$, with "k" being the Lovelock order, i.e., $k = 1$ is the pure Einstein Gravity, while $k = 2$ is the pure Gauss-Bonnet Gravity and so on. We would like to emphasize that, although the pure Lovelock solutions may not represent a physical black hole, it does provide a natural platform to study the effect of higher curvature terms to the strong cosmic censorship conjecture, which is the ultimate aim of our work.

The article is arranged as follows: In Sec. II we start by reviewing the relationship between the quasinormal frequency of the photon sphere modes and the Lyapunov exponent associated with the photon sphere. This is a general result for any spherically symmetric spacetime irrespective of the underlying theory of gravity. Subsequently in Sec. III we present a detailed analysis for obtaining the quasinormal frequency for Einstein-Gauss-Bonnet black holes numerically and demonstrate the violation of the strong cosmic censorship conjecture. The above procedure has been repeated in order to obtain the quasinormal frequencies for pure Lovelock black holes in an appropriate spacetime dimension and show the violation of the strong cosmic censorship conjecture in Sec. IV. We have compared this violation with the corresponding scenario in Einstein gravity to illustrate the effect of higher curvature terms. From both of our examples, we conclude that the violation of the strong cosmic censorship conjecture becomes even stronger when higher curvature terms are added. We end with a brief discussion and possible future outlooks in Sec. V.

Notations and Conventions: We have set the fundamental constants $c = 1 = G$. The Roman indices $(a, b, c, \cdots)$ are used to denote spacetime indices. The Greek indices $(\mu, \nu, \alpha, \cdots)$, on the other hand, are used to denote spatial indices on a spacelike hypersurface.

II. STRONG COSMIC CENSORSHIP CONJECTURE AND QUASINORMAL MODES: A BRIEF OVERVIEW

The stability of black holes under small perturbation, one of the most important areas of research in black hole physics, requires the computation of the quasinormal modes. These modes are the eigenfunctions of the perturbation equation with respect to some special set of boundary conditions, i.e., only ingoing modes at the event horizon and outgoing modes at infinity. The real part of the associated eigenvalues, known as quasinormal mode frequencies, determines the time period of oscillation, while the imaginary part dictates the decay rate of the perturbation. It is the decay rate of the perturbation, which is central to the stability of a black hole. Thus the question of stability of a black hole spacetime is linked with the sign of the imaginary part of the quasinormal mode frequency, ω. For most black hole spacetimes, because of the complex structure of the perturbation equation, it is a daunting task to obtain an analytical expression for the quasinormal frequencies by solving the perturbation equation. A relatively more straightforward task is to obtain the quasinormal mode frequencies by solving the perturbation equation numerically, and various numerical techniques have been developed over the last few decades to compute the

quasinormal mode frequencies accurately. However, in certain limiting cases, it is indeed possible to obtain an analytical expression of the quasinormal mode frequency. One such limiting case is the ray optics approximation or the eikonal limit, where both the real and imaginary parts of the quasinormal mode frequency is related to various geometric constructs associated with the photon sphere. This stems from the fact that the effective potential experienced by a photon in a black hole spacetime is identical to the potential experienced by a test field in this black hole spacetime, in the large angular momentum limit. Thus the quasinormal mode frequencies, which are directly connected with the potential in the perturbation equation in the eikonal limit (this also corresponds to the large angular momentum limit), get related to the potential a photon experiences. In particular, the imaginary part of the quasinormal mode frequency is related to the Lyapunov exponent associated with the instability of the photon sphere and the real part to the angular velocity of the photon sphere [51–56], such that

$$\omega_n = \Omega_{\rm ph}\ell - i\left(n + \frac{1}{2}\right)\lambda_{\rm ph}, \tag{1}$$

where ℓ is the angular momentum and $n = 0, 1, 2...$ represents the overtone number. We also refer the reader to [57,58], where the above correspondence is discussed in the context of higher curvature gravity.

The Lyapunov exponent $\lambda_{\rm ph}$, associated with the photon sphere, determines the rate at which a geodesic located at the photon sphere diverges or converges with respect to a nearby geodesic. Further, $\Omega_{\rm ph}$ is the angular velocity of a photon located at the maxima of the photon sphere. In a d-dimensional static and spherically symmetric spacetime one can explicitly write down the expressions for $\lambda_{\rm ph}$ and $\Omega_{\rm ph}$ in terms of the metric coefficients, which read

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{d-2}^2. \tag{2}$$

Exploiting the fact that the spacetime possesses spherical symmetry, it is convenient to restrict our attention only to the equatorial plane, which is identified by setting the azimuthal angles to $\pi/2$, and, hence, the Lagrangian associated with the geodesic motion takes the form,

$$L = \frac{1}{2}(-f(r)\dot{t}^2 + f(r)^{-1}\dot{r}^2 + r^2\dot{\phi}^2), \tag{3}$$

where "dot" represents the derivative with respect to the affine parameter. Here t and ϕ are cyclic coordinates and correspondingly the energy and angular momentum, $p_t = -E$ and $p_\phi = L$ are the constants of motion. The unstable circular null trajectory or, more commonly, the photon sphere, is determined by the equation $V'_{\rm eff}(r_{\rm ph}) = V''_{\rm eff}(r_{\rm ph}) = 0$, where $V_{\rm eff}$ is the effective potential a photon experiences. These equations further reduce to,

$$\frac{E^2}{L^2} = \frac{f(r)}{r^2}$$
$$2f(r) = rf'(r). \tag{4}$$

The Lyapunov exponent is determined by taking the variation of the effective potential $V_{\rm eff}$ as $r \rightarrow r_{\rm ph} + \delta r$, and, hence, one can show the following time evolution, $\delta r \sim \exp(\pm\lambda_{\rm ph}t)$, where the Lyapunov exponent $\lambda_{\rm ph}$ has the following expression in terms of the effective potential [51]:

$$\lambda_{\rm ph} = \sqrt{\frac{V''_{\rm eff}}{2\dot{t}^2}}\bigg|_{r=r_{\rm ph}} = \sqrt{\frac{f(r_{\rm ph})}{2}\left(\frac{2f(r_{\rm ph})}{r_{\rm ph}^2} - f''(r_{\rm ph})\right)}. \tag{5}$$

As per our convention the time dependence of the perturbation goes as $\exp(-i\omega_n t)$, and, hence, the imaginary part of the quasinormal mode frequency must be negative ensuring stability. Since the longest lived quasinormal mode frequency corresponds to the $n = 0$ mode in (1), the quantity of interest for the strong cosmic censorship conjecture, i.e., $\beta \equiv \{-\min(\mathrm{Im}\,\omega_n)/\kappa_{\rm ch}\}$, is given by,

$$\beta_{\rm ph} = \frac{\lambda_{\rm ph}}{2\kappa_{\rm ch}} = \frac{1}{2\kappa_{\rm ch}}\left\{\sqrt{\frac{f(r_{\rm ph})}{2}\left(\frac{2f(r_{\rm ph})}{r_{\rm ph}^2} - f''(r_{\rm ph})\right)}\right\}. \tag{6}$$

This finishes one part of the story, since (1) yielding an analytical expression for quasinormal mode frequencies holds true only in the large ℓ limit. Since the quasinormal mode frequencies in this context solely depend on the photon sphere these are generally referred to as the photon sphere modes. However, in the presence of the electromagnetic charge and cosmological constant, the quasinormal spectrum of a black hole spacetime possesses two other characteristic quasinormal modes, namely, the de Sitter modes and the near extremal modes. The de Sitter mode becomes relevant when the cosmological horizon lies far away from the event horizon, i.e., in the limit when the cosmological constant goes to zero. On the other hand, the near extremal modes dominate the spectrum in the extremal limit, i.e., when the Cauchy horizon approaches the event horizon. The quasinormal mode frequency associated with a de Sitter mode can be solely determined from the asymptotic structure of spacetime, which has the following form [59–62]:

$$\omega_{n,\rm dS} = -i(\ell + 2n)\kappa_c, \tag{7}$$

where κ_c is the surface gravity associated with the cosmological horizon and the quasinormal mode frequencies are purely imaginary, as is evident from (8). The minimum value for the imaginary part of the quasinormal mode frequency corresponds to $\omega_{n=0,\rm dS} = -i\ell\kappa_c$. Note that the analytical expressions presented in this section for the photon sphere and de Sitter modes are not exact and have been obtained with some approximation. Hence, in the

subsequent sections we compute the quasinormal frequencies numerically which can be further compared with their corresponding analytical expressions given in this section. The final set of modes which are of importance in this context are the near extremal modes. These appear when the Cauchy horizon and the event horizon approach each other. In the context of black holes in general relativity it was possible to provide an analytical estimation for these modes; however, in the present context it turns out to be difficult to write down the analytical form for the near extremal modes due to the complicated nature of the equation determining the event horizon. Hence, we will not attempt to write down any analytical expression for the near extremal modes; rather, we will compute it numerically.

Violation of the strong cosmic censorship conjecture requires the existence of weak solutions across the Cauchy horizon. These weak solutions are said to exist when the integral of the nonlinear field equations multiplied with a smooth function over a region around the Cauchy horizon is finite. For the case of mth order Lovelock theory, the gravitational field equations will involve m curvatures terms, and, hence, the existence of weak solutions will demand,

$$\int_{\mathcal{V}} d^d x \sqrt{-g} \psi (\partial \Gamma + \Gamma^2)^m = \int_{\mathcal{V}} d^d x \sqrt{-g} \psi (\partial \phi)^2, \quad (8)$$

where $\mathcal{V}$ is a volume around the Cauchy horizon and ψ is a smooth function. Thus for the above integrals to exist, ϕ should be square integrable and, hence, must belong to H^1_{loc}, while Γ should be a function in L^{2m}_{loc} and, thus, $g_{\mu\nu}$ should be H^m_{loc}. Thus, for gravitational perturbation to extend weakly across the Cauchy horizon one requires a stronger condition than that for the scalar field. Since extension of any perturbation, e.g., scalar, electromagnetic or gravitational, across the Cauchy horizon would constitute a violation of the strong cosmic censorship conjecture, it follows that if $\beta > (1/2)$ for the scalar perturbation, then the strong cosmic censorship conjecture will be violated. However, for gravitational perturbation, one must be careful before analyzing the limit on β, which we leave for the future.

To see how the choice for β can be related to the validity of the strong cosmic censorship conjecture, consider the field equation $\Box \Phi = 0$ that a test scalar field obeys in the d dimensional static and spherically symmetric spacetime. Due to the existence of timelike and angular Killing vectors in the spacetime, the scalar field can be expressed as $\Phi(t, r, \Omega) = e^{-i\omega t} R(r) h(\Omega)$, where $h(\Omega)$ corresponds to spherical harmonics associated with the $(d-2)$ dimensional unit sphere. The function $R(r)$ satisfies a second order differential equation, whose two independent solutions, regular at the Cauchy horizon, read

$$\Phi^{(1)}(t, r, \Omega) = e^{-i\omega u} R^{(1)}(r) h(\Omega);$$
$$\Phi^{(2)}(r) = e^{-i\omega u} R^{(2)}(r)(r - r_{\mathrm{ch}})^{i\omega_n/\kappa_{\mathrm{ch}}} h(\Omega). \quad (9)$$

Here ω_n is the quasinormal mode frequency and κ_{ch} is the surface gravity associated with the Cauchy horizon. As a consequence, the integral of the kinetic term of the scalar field corresponds to the integral of $(r - r_{\mathrm{ch}})^{2(i\omega_n/\kappa_{\mathrm{ch}} - 1)}$, which in turn corresponds to $(r - r_{\mathrm{ch}})^{2(\beta - 1)}$. Here β has already been defined above in terms of $\mathrm{Im} \, \omega_n$ and surface gravity κ_{ch} as $\beta \equiv \{-\min(\mathrm{Im} \, \omega_n)/\kappa_{\mathrm{ch}}\}$. For $\beta > (1/2)$, the scalar field Φ is regular at the Cauchy horizon and can be extended beyond the horizon. Hence, the condition $\beta > (1/2)$ signifies whether the strong cosmic censorship conjecture is respected in the spacetime or not. As emphasized above, the addition of higher curvature terms to the field equation leads to a higher regularity requirement for the metric, in order to have the weak solution near the Cauchy horizon [28]. Therefore, extendibility of the metric perturbation beyond the Cauchy horizon would yield a different bound on β. However, since in our analysis we consider only the case of scalar perturbation, the above bound on β still ensures the violation of the strong cosmic censorship conjecture.

Another technical point must be emphasized here, for the $\beta > (1/2)$ condition to have any relevance with the violation of the strong cosmic censorship conjecture, it is necessary that the late time decay of the perturbations is exponential. This is certainly true for asymptotically de Sitter black holes in general relativity [63], but whether such an exponential decay holds for asymptotically de Sitter black holes in higher curvature theory as well must be properly addressed. This is important, since the late time exponential decay of the perturbation is one of the essential ingredients in the analysis of the strong cosmic censorship conjecture, and, thus, for our results to make sense, we must establish such a late time exponential tail for the asymptotically de Sitter black hole under consideration. Interestingly, such an exponential tail has already been reported for asymptotically de Sitter black holes in Einstein-Gauss-Bonnet gravity in [62]. Since the black hole solutions we will study in this work are also asymptotically de Sitter black holes in the Einstein-Gauss-Bonnet gravity, thus following [62], we can safely argue that $\beta > (1/2)$ will also characterize the violation of the strong cosmic censorship conjecture in the present situation. Since the properties of Gauss-Bonnet gravity, which is the second order term in the Lovelock polynomial, closely match with the higher order terms in the full Lovelock polynomial, it is reasonable to expect that the same exponential tail would appear even for the asymptotically de Sitter black holes in pure Lovelock theories, which are also considered in this work. It is certainly possible to support the argument by further numerical analysis, e.g., time evolution of the perturbation. However, given the complicated nature of the Lovelock polynomial and the associated field equations, such an analysis is beyond the scope of this work.

In the subsequent sections we have carried out the analysis presented above in the context of a scalar field

living on the charged Einstein-Gauss-Bonnet-de Sitter black hole background and subsequently for a pure Lovelock black hole background. The strategy we follow here is identical to [11], i.e., we start by computing the quasinormal mode frequencies associated with the photon sphere modes, de Sitter modes, and near extremal modes numerically. Having determined each of these modes individually, we look for any possible region of parameter space for which violation of strong cosmic censorship conjecture occurs, i.e., the parameter β becomes greater than $(1/2)$. Since the quasinormal mode spectra for Einstein-Gauss-Bonnet as well as pure Lovelock black holes are different from those in Einstein's gravity and strongly depends on the Gauss-Bonnet coupling constant [62,64–70], it is reasonable to expect that the fate of the strong cosmic censorship conjecture in such theories would be different and, hence, a detailed analysis in this context is very important. For numerical computation, we follow the Mathematica package developed in [71]. Since it is expected that the Einstein-Hilbert action must be supplemented by higher curvature terms, it is reasonable for one to expect that problems like violation of the strong cosmic censorship conjecture should be settled in such higher curvature theories. This is what we explore next.

III. STRONG COSMIC CENSORSHIP CONJECTURE IN EINSTEIN-GAUSS-BONNET GRAVITY

The statement of the strong cosmic censorship conjecture, i.e., the assertion that solutions of Einstein's equations are nonextendible beyond the Cauchy horizon, has been tested for numerous black hole solutions, but mostly within the realm of general relativity. Even though certain nontrivial matter couplings are taken into account, the influence of higher curvature terms on the strong cosmic censorship conjecture has not been studied earlier. Since general relativity is not a complete theory of gravity, it is crucial to understand the effects of these higher curvature modifications to general relativity and, hence, on the strong cosmic censorship conjecture. In this work we will be interested in the higher curvature corrections within the domain of the Lanczos-Lovelock Lagrangian, since they represent the most general extension to general relativity in dimensions higher than four with field equations containing up to the second derivatives of the metric. The Lanczos-Lovelock Lagrangian is a homogeneous polynomial in the Riemann tensor and is given by [33–37,72],

$$\mathcal{L} = \sqrt{-g} \sum_{k=0}^{k_{\max}} c_k L_k, \qquad (10)$$

where,

$$L_k = \frac{1}{2^k} \delta^{a_1 b_1 \cdots a_k b_k}_{c_1 d_1 \cdots c_k d_k} R^{c_1 d_1}_{a_1 b_1} \cdots R^{c_k d_k}_{a_k b_k}. \qquad (11)$$

Here R^{cd}_{ab} represents the Riemann tensor in d spacetime dimensions and $\delta^{a_1 b_1 \cdots a_k b_k}_{c_1 d_1 \cdots c_k d_k}$ denotes the totally antisymmetric Kronecker delta. The zeroth order $(k = 0)$ term of the Lanczos-Lovelock polynomial is the cosmological constant, and the first order term $(k = 1)$ represents the Einstein-Hilbert Lagrangian, and the second order term $(k = 2)$ is the Gauss-Bonnet Lagrangian. Further, $k_{\max}$ appearing in the Lanczos-Lovelock Lagrangian is related to the spacetime dimensions as $2k_{\max} \leq d$. The action for such a theory involving the first three nontrivial contributions to the Lanczos-Lovelock Lagrangian is of the following form:

$$\mathcal{A} = \frac{1}{16\pi} \int d^d x \sqrt{-g} [R + \alpha(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}) - 2\Lambda - 4\pi F_{pq}F^{pq}], \qquad (12)$$

where we have included a matter Lagrangian of the form $-(1/4)F_{ab}F^{ab}$ and α is the Gauss-Bonnet coupling parameter. There exists a spherically symmetric and static black hole solution in d spacetime dimensions arising out of the above action, with the line element in the form presented in (2), where the function $f(r)$ becomes [40,73–75],

$$f(r) = 1 + \frac{r^2}{2\tilde{\alpha}} \left[1 - \sqrt{1 + \frac{64\pi\tilde{\alpha}M}{(d-2)\Sigma_{d-2}r^{d-1}} - \frac{2\tilde{\alpha}Q^2}{(d-2)(d-3)r^{2d-4}} + \frac{8\tilde{\alpha}\Lambda}{(d-1)(d-2)}} \right]. \qquad (13)$$

Here "Q" is the electromagnetic charge corresponding to the field tensor $F_{\mu\nu}$, and $\tilde{\alpha} = (d-3)(d-4)\alpha$ is the rescaled Gauss-Bonnet coupling constant, and M is the mass of the black hole. Further, Σ_{d-2} is the volume of a $(d-2)$ dimensional unit sphere. The location of the horizons is given by the equation $f(r) = 0$, which further reduces to

$$\frac{4\Lambda}{(d-1)(d-2)} r^{(2d-4)} - 2r^{(2d-6)} - 2\alpha r^{(2d-8)}$$

$$+ \frac{32\pi M}{(d-2)\Sigma_{d-2}} r^{(d-3)} - \frac{Q^2}{(d-2)(d-3)} = 0. \qquad (14)$$

Since for our analysis we require the black hole under consideration to have three horizons, namely the event

horizon, cosmological horizon, and Cauchy horizon, (14) must give rise to three real positive roots. This is guaranteed from the Descartes rule of sign.

Given this black hole spacetime, which is an exact solution of the higher curvature gravitational field equations, we are interested in studying if there is any violation of the strong cosmic censorship conjecture in this spacetime. It should be emphasized that when $\alpha = 0$, i.e., in the absence of higher curvature terms, the spacetime reduces to a Reissner-Nordström -de Sitter configuration in $d -$ dimensions and admits violation of the strong cosmic censorship conjecture [20]. It is therefore interesting to see whether the addition of higher curvature terms may cure the violation of the strong cosmic censorship conjecture when $\alpha \neq 0$. Given the choice for $f(r)$, one can explicitly determine the quantity β for the photon sphere modes by computing the Lyapunov exponent and surface gravity at the Cauchy horizon following (6). However, as emphasized earlier in this section, it is better to determine the quasinormal mode frequencies numerically and then further obtain β to demonstrate the violation of the strong cosmic censorship conjecture.

Let us start by describing the dynamics of a massless scalar field Φ on a d dimensional spherically symmetric black hole background as given in (2). The evolution of the perturbation is governed by the Klein-Gordon equation $\Box\Phi = 0$. For the spherically symmetric background, one can always expand the field in terms of a natural basis on the $(d - 2)$ sphere, namely the spherical harmonics $\mathcal{Y}_{lm}(\theta, \phi)$ as follows:

$$\Phi(t, r, \Omega) = \sum_{\ell,m} e^{-i\omega t} \frac{\phi(r)}{r^{(d-2)/2}} \mathcal{Y}_{\ell m}(\Omega), \qquad (15)$$

which leads to the following master equation:

$$\left(\frac{\partial^2}{\partial r_*^2} + \omega^2 - V_{\text{eff}}(r)\right)\phi(r) = 0;$$

$$V_{\text{eff}}(r) = f(r)\left\{\frac{\ell(\ell + d - 3)}{r^2} + \frac{(d-2)(d-4)}{4r^2}f(r)\right.$$
$$\left. + \frac{(d-2)}{2r}f'(r)\right\}, \qquad (16)$$

where $dr_* = \{dr/f(r)\}$ is the tortoise coordinate and $V_{\text{eff}}(r)$ is the effective potential expressed above in terms of the metric function $f(r)$. The quasinormal mode frequency ω_n is defined as the eigenvalue of (16) that corresponds to ingoing modes at the event horizon, r_{h}, and outgoing modes at the cosmological horizon, r_{c}, i.e.,

$$\phi(r \to r_h) \sim e^{-i\omega r_*} \quad \text{and} \quad \phi(r \to r_c) \sim e^{i\omega r_*}. \qquad (17)$$

As mentioned earlier, for computing the quasinormal modes numerically we follow the procedure and use the Mathematica package developed in [71]. This requires one to work with the redefined radial coordinate $u = 1/r$ and impose the quasinormal mode boundary conditions appropriately at the event horizon and at the cosmological horizon. Thus, we obtain the complex frequencies of the quasinormal modes, which are the first ingredients that go into the definition of β. The computation of κ_{ch} can also be performed in a similar manner, and, hence, the numerical estimation for $\{-(\text{Im}\,\omega_{n,\ell})/\kappa_{\text{ch}}\}$ can be obtained, whose minimum value would yield an estimation for β. This has been shown explicitly in Table I for the three modes of interest, namely

TABLE I.   Numerical values of $\{-(\text{Im}\,\omega_{n,\ell})/\kappa_{\text{ch}}\}$ have been presented for the lowest lying quasinormal modes for different choices of ℓ. We have also presented them for various choices of the rescaled Gauss-Bonnet coupling constant $\tilde{\alpha}$, the cosmological constant Λ, and the rescaled electric charge (Q/Q_{max}), for $M = 1$ and $d = 5$. The numerical estimation of β, for a given $\tilde{\alpha}$, Λ, and (Q/Q_{max}) would correspond to the lowest entry in that respective row. The values presented in the first column, with $\ell = 0$, correspond to the near extremal modes, while the second column, with $\ell = 1$, depicts the de Sitter modes. Finally, the numerical estimation for $\{-(\text{Im}\,\omega_{n,\ell})/\kappa_{\text{ch}}\}$ associated with the photon sphere modes has been presented for $\ell = 10$. To see the direct correspondence with the analytical results presented in Sec. II, in the last column we provide the analytical estimate of the same as well. As evident the numerical and analytical estimations of $\{-(\text{Im}\,\omega_{n,\ell})/\kappa_{\text{ch}}\}$ are in close agreement, thereby justifying the use of analytical techniques for black holes in higher curvature theories of gravity.

α	Λ	Q/Q_{max}	$\ell = 0$	$\ell = 1$	$\ell = 10$	$\ell = 10$ (Analytical)
0.1	0.06	0.99	0.849266	0.467428	0.678101	0.6770764
		0.995	0.8860955	0.7059601	1.02414	1.018981
	0.1	0.99	0.850344	0.6296578	0.6674398	0.6661463
		0.995	0.8841346	0.9521732	1.00561	1.00365144
0.2	0.06	0.99	0.861229	0.510842	0.734559	0.7334486
		0.995	0.8952727	0.7683285	1.10481	1.09940317
	0.1	0.99	0.8608527	0.685808	0.7215474	0.7201545
		0.995	0.893013	1.0327756	1.0865954	1.0806729
0.3	0.06	0.99	0.8714403	0.555417	0.796272	0.79027861
		0.995	0.9035516	0.83237279	1.18615	1.1804926
	0.1	0.99	0.8703119	0.743391	0.776177	0.774676
		0.995	0.9010936	1.1544405	1.160704	1.1584423

the near extremal modes ($\ell = 0$), the de Sitter modes ($\ell = 1$), and the photon sphere modes ($\ell = 10$). Analytical estimations for $\{-(\mathrm{Im}\,\omega_{n,\ell})/\kappa_{\mathrm{ch}}\}$ have also been presented in Table I and, as evident from the numerical results, the analytical and numerical values match quite well, within an error of 6%. Further, from Table I we see that β crosses the value $(1/2)$ for the near extremal values of the charge parameter Q and, hence, the violation of the strong cosmic censorship conjecture does occur in the context of asymptotically de Sitter black holes in Einstein-Gauss-Bonnet gravity inheriting the Cauchy Horizon.

Let us now verify the violation of the strong cosmic censorship conjecture in an explicit manner. This can be achieved by plotting $\{-(\mathrm{Im}\,\omega_{n,\ell})/\kappa_{\mathrm{ch}}\}$ (which for brevity have been labeled as β) with respect to (Q/Q_{max}), where Q_{max} is the extremal limit of the electric charge Q for a given cosmological constant and Gauss-Bonnet parameter

α, in Fig. 1. The left column of the figure depicts the photon sphere modes, the plots in the middle column depict the de Sitter modes, and finally the plots on the right-most column illustrate the near extremal modes. It is again obvious that all of these modes cross the $\beta = (1/2)$ line and, hence, the strong cosmic censorship conjecture is violated for charged, asymptotically de Sitter black holes in Einstein-Gauss-Bonnet gravity. Thus presence of higher curvature terms does *not* help to restore the strong cosmic censorship conjecture. Furthermore, the violation gets severe as the Gauss-Bonnet coupling parameter α is increased, since the curves for β cross the line $\beta = (1/2)$ earlier, thus, allowing for a larger parameter space where the violation of the strong cosmic censorship conjecture can be perceived. Further, for photon sphere modes the violation becomes stronger as the spacetime dimension is increased from $d = 5$ to $d = 6$ (see the last
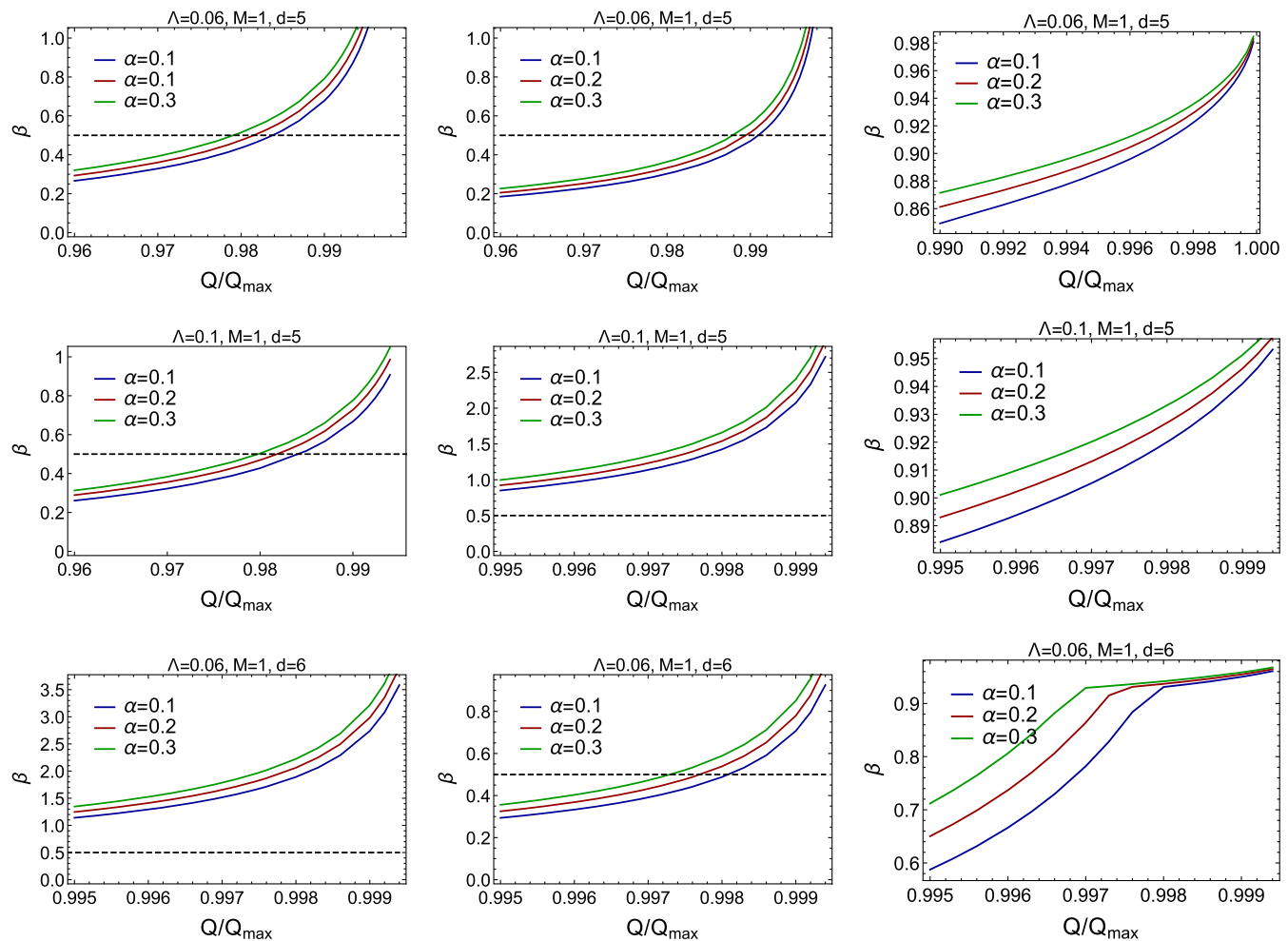

FIG. 1. We have plotted the quantity $\{-(\mathrm{Im}\,\omega_n)/\kappa_{\mathrm{ch}}\}$, whose minima provides an estimation for β, with the ratio (Q/Q_{max}) for all three quasinormal modes of different origins. The plots on the leftmost column depict the variation of the ratio $\{-(\mathrm{Im}\,\omega_n)/\kappa_{\mathrm{ch}}\}$ for the photon sphere modes, the plots in the middle column are for the variation of the same quantity with the de Sitter modes, and, finally, the plots on the right column are showing the variations with the near extremal modes. All the plots in a certain row are for a fixed value of the cosmological constant Λ and all three curves in a given plot are for the three choices of the rescaled Gauss-Bonnet parameter. See the text for discussion.

row of Fig. 1), which is reminiscent of the result presented in [15].

To see these results from a different perspective, we have again plotted the ratio of the imaginary part of the quasi-normal and the surface gravity at the Cauchy horizon against (Q/Q_{max}), but this time with all the three modes depicted in the same plot for various choices of the cosmological constant Λ and the Gauss-Bonnet Parameter α in Fig. 2 for $d = 5$. As evident from Fig. 2, for smaller values of Λ, the de Sitter mode dominates over and above the other two for smaller (Q/Q_{max}). Subsequently, the de Sitter mode crosses the $\beta = (1/2)$ line, thus violating the strong cosmic censorship conjecture, and finally the near extremal modes take over. On the other hand, for larger values of the cosmological constant, the photon sphere mode dominates, which crosses the $\beta = (1/2)$ line and finally gets subdominant to the near extremal modes. The effect of higher curvature terms on the

violation of the strong cosmic censorship conjecture can be easily realized from Fig. 2 as well. Since it clearly illustrates that as the Gauss-Bonnet coupling parameter α increases, the violation of the strong cosmic censorship conjecture happens at smaller and smaller values of (Q/Q_{max}). This implies that the parameter space available for violating the strong cosmic censorship conjecture is larger in higher curvature theories in comparison to general relativity. For this purpose, we have plotted β against (Q/Q_{max}) for three choices of the Gauss-Bonnet coupling parameter α, including $\alpha = 0$, which represents the general relativistic scenario.

IV. STRONG COSMIC CENSORSHIP CONJECTURE IN PURE LOVELOCK GRAVITY

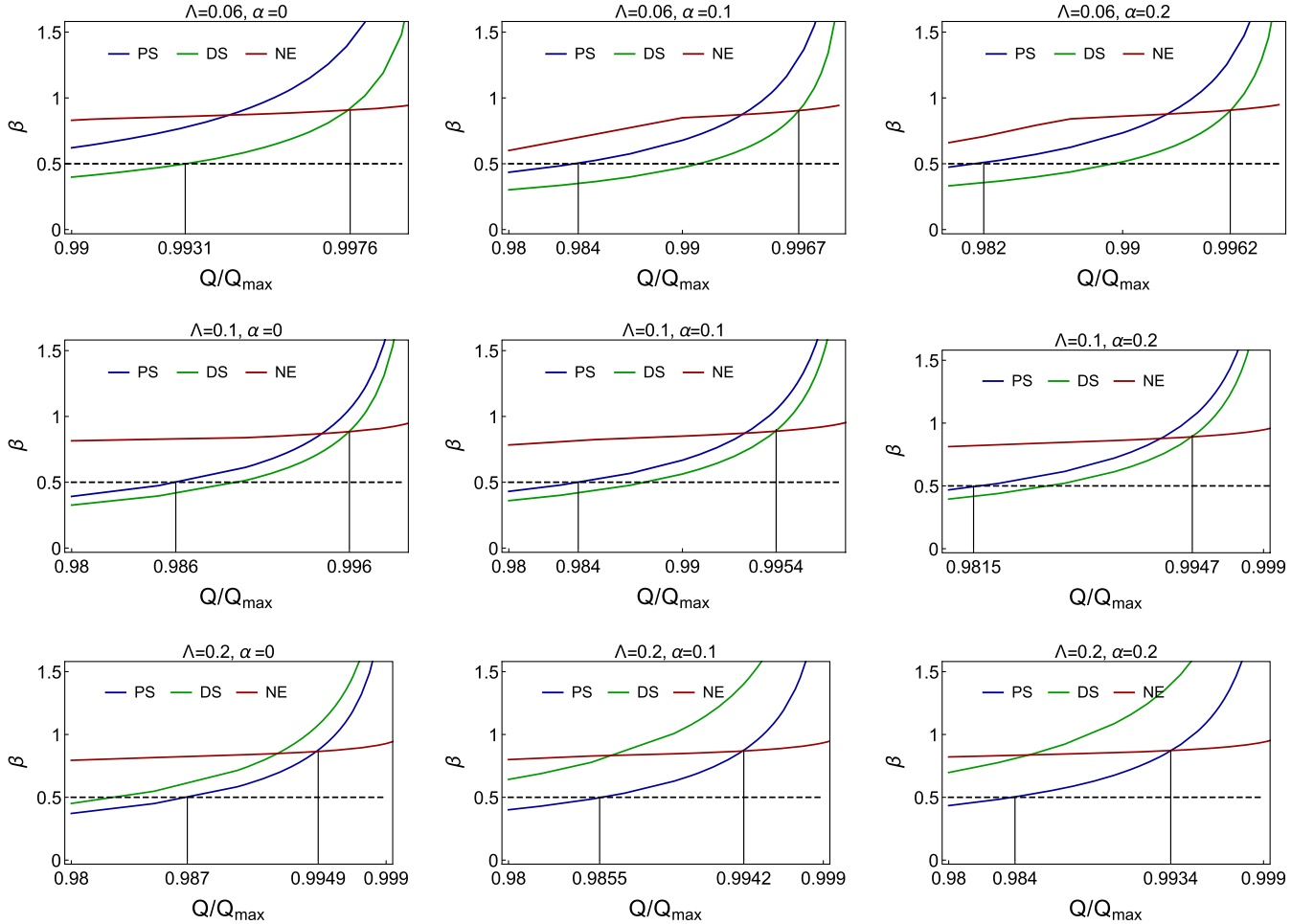So far, our discussion has been on the violation of the strong cosmic censorship conjecture in the context of five

FIG. 2. In this figure we have demonstrated the variation of $\{-(\text{Im}\,\omega_n)/\kappa_{ch}\}$, whose minima correspond to the parameter β, with respect to (Q/Q_{max}) for all three quasinormal modes of interest in each single plot. The photon sphere modes are denoted by blue curves, the de Sitter modes are represented by green curves, and the near extremal modes are depicted by brown curves. Each of these plots is for various choices of the cosmological constant Λ and the Gauss-Bonnet parameter α. The first vertical line in each of these plots corresponds to the value of (Q/Q_{max}), where β becomes greater than $(1/2)$ for the first time and, hence, the strong cosmic censorship conjecture is violated. While the second vertical line corresponds to the value of (Q/Q_{max}), where the near extremal modes starts to dominate. In this plot, the mass is taken to be unity and the spacetime dimension to be $d = 5$.

and higher dimensional Einstein-Gauss-Bonnet gravity. As illustrated in the previous section, the Einstein-Gauss-Bonnet gravity leads to even stronger violation of the strong cosmic censorship conjecture than that of a black hole solution in general relativity. This still kept room for the question, what happens for the other higher curvature terms in the Lanczos-Lovelock Lagrangian? In this section, we wish to study the effect of the other higher curvature terms in the Lanczos-Lovelock Lagrangian on the violation of the strong cosmic censorship conjecture, and for this purpose we wish to consider the case of pure Lovelock gravity [45,46], which refers to a single term in the full Lovelock polynomial. More precisely, the kth order pure Lovelock term corresponds to the Lagrangian $\mathcal{L} = \sqrt{-g}L_k$, without the sum. For example, the second-order pure Lovelock theory has the Lagrangian of the form, $\mathcal{L}_2 = \sqrt{-g}\{R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}\}$. Thus the action for such a theory involving a pure Lovelock Lagrangian with a positive cosmological constant term is given by,

$$\mathcal{A} = \frac{1}{16\pi}\int d^d x \sqrt{-g}[-2\Lambda + L_k - 4\pi F_{ab}F^{ab}]. \quad (18)$$

Such a theory admits the spherically symmetric black hole solution in d spacetime dimensions with the line element expressed in the form of (2), where the function $f(r)$ is given by [76],

$$f(r) = 1 - \left(\tilde{\Lambda}r^{2k} + \frac{2M^k}{r^{d-2k-1}} - \frac{\tilde{Q}^2}{r^{2d-2k-4}}\right)^{\frac{1}{k}}, \quad (19)$$

where $\tilde{\Lambda}$ and $\tilde{Q}$ are some rescaled version of the cosmological constant and $U(1)$ the electromagnetic charge associated with the Maxwell field coupled with gravity. It is well known that such a black hole solution admits instabilities with respect to small perturbations in dimensions $d < (3k + 1)$ [45]. Hence, in our analysis, we restrict our attention only to black holes in $d \geq (3k + 1)$. In particular, we demonstrate our result for the pure Gauss-Bonnet solution ($k = 2$) in seven spacetime dimensions. To illustrate the effect of pure Gauss-Bonnet term on the strong cosmic censorship conjecture, we compare it with that of the corresponding black hole solution of Einstein gravity in seven dimensions, i.e., a Reissner-Nordström de Sitter solution. In $d = 7$, the metric component $f(r)$ for the charged pure Gauss-Bonnet-de-Sitter and charged Einstein-de-Sitter black hole solution takes the form,

$$f_{\text{GB}}(r) = 1 - \left(\tilde{\Lambda}r^4 + \frac{2M^2}{r^2} - \frac{\tilde{Q}^2}{r^6}\right)^{\frac{1}{2}};$$

$$f_{\text{EH}}(r) = 1 - \left(\tilde{\Lambda}r^2 + \frac{2M}{r^4} - \frac{\tilde{Q}^2}{r^8}\right). \quad (20)$$

The horizons of both of these black hole solutions correspond to the solutions of the equations $f_{\text{GB}}(r) = 0$ and $f_{\text{EH}}(r) = 0$.
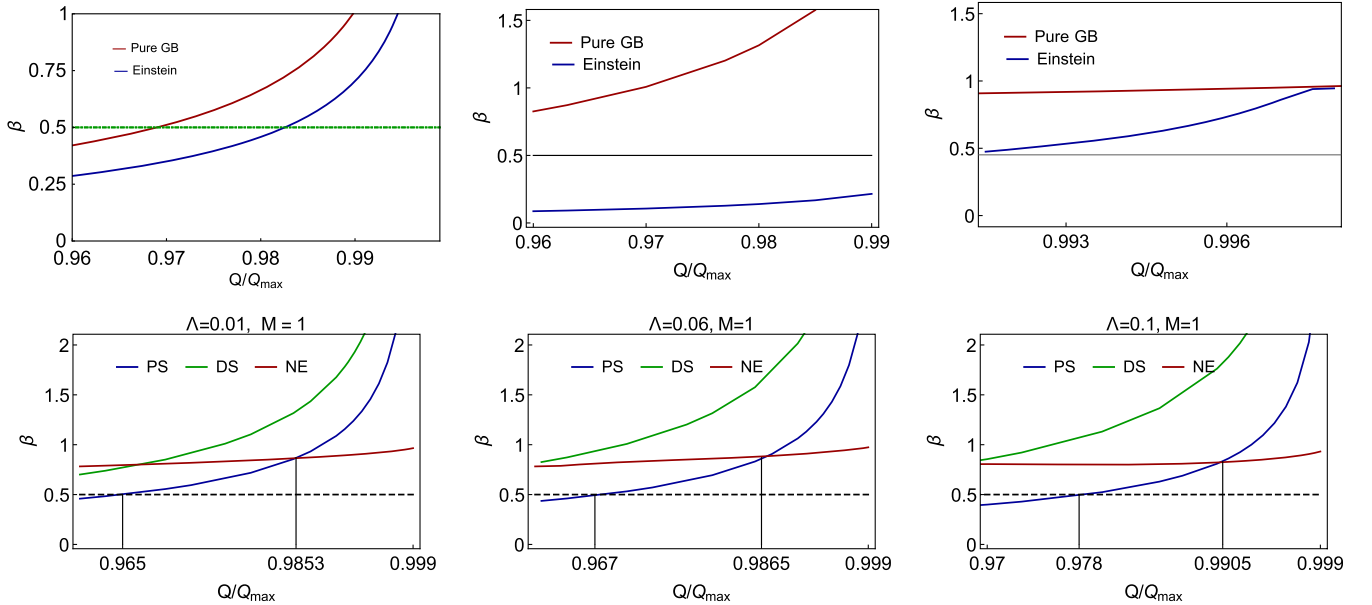

FIG. 3. We have plotted $\{-(\text{Im}\omega_n)/\kappa_{\text{ch}}\}$ (for notational convenience, we have labeled the axis as β) in the context of pure Gauss-Bonnet gravity as well as Einstein gravity, with the ratio (Q/Q_{max}), for the photon sphere (top left), de Sitter (top middle), and near extremal (top right) modes in the upper row. As evident from the plots of all the three modes, the strong cosmic censorship conjecture is violated in pure Gauss-Bonnet gravity. Moreover, all these plots demonstrate that the violation of the strong cosmic censorship conjecture is more strong for pure Gauss-Bonnet gravity than the Einstein gravity in $d = 7$ dimensions. Here we have taken the cosmological constant to be $\Lambda = 0.06$ and the mass being $M = 1$. The bottom panel demonstrates the variation of $\{-(\text{Im}\omega_n)/\kappa_{\text{ch}}\}$ for seven dimensional pure Gauss-Bonnet gravity with various choices of the cosmological constant. As the figures clearly demonstrate, the photon sphere mode always dominates till the near extremal modes take over. Note that the dominant quasinormal modes, which are the near extremal mode in all three plots always stays smaller than unity.

The existence of three positive real roots of these equations can be easily realized from the Descartes rule of the sign applied to the solutions of the above equations.

To check the validity of the strong cosmic censorship conjecture we follow the identical procedure adopted for the case of Einstein-Gauss-Bonnet gravity in the previous section. We start by computing the quasinormal mode frequencies and then the relative ratio between the late-time decay rate governed by the imaginary part of the lowest lying quasinormal modes and growth at the Cauchy horizon corresponding to a massless scalar perturbation. Following which we have demonstrated the variation of $\{-(\operatorname{Im} \omega_n)/\kappa_{\mathrm{ch}}\}$, whose minima provides an estimation for β, with the electric charge $(Q/Q_{\max})$, where $Q_{\max}$ corresponds to the extremal value of the electric charge in Fig. 3. We have depicted the variation of $\{-(\operatorname{Im} \omega_n)/\kappa_{\mathrm{ch}}\}$ for a given Λ, for the photon sphere, de Sitter, and near extremal modes. As the figures in the upper panel of Fig. 3 explicitly demonstrate, the strong cosmic censorship conjecture is indeed violated in the pure Lovelock spacetimes as well. Further, as evident from Fig. 3, the violation of the strong cosmic censorship conjecture in a pure Lanczos-Lovelock theory occurs at smaller values of $Q/Q_{\max}$ in comparison with the similar solution for Einstein gravity in the same spacetime dimensions. Thus we can safely conclude that the violation of the strong cosmic censorship conjecture is more severe in theories involving higher order Lovelock terms than in general relativity. For completeness, we have also plotted the variation of $\{-(\operatorname{Im} \omega_n)/\kappa_{\mathrm{ch}}\}$ with $(Q/Q_{\max})$ for the three modes, with different choices of the cosmological constant, in the context of the pure Gauss-Bonnet black hole in seven spacetime dimensions. As these figures in the bottom panel of Fig. 3 explicitly demonstrate, the strong cosmic censorship conjecture is indeed violated. In the initial phase, β is dominated by the photon sphere modes, while later on the near extremal modes take over. However, the β value for the dominating mode never crosses the value unity, which suggests that the field can be extended beyond the Cauchy horizon, with regularity as the H^1_{loc} function, but not smoother than that. This is consistent with the expectation from the general relativistic scenario as well. Thus in this respect the higher curvature gravity follows the footsteps of general relativity, even though the parameter space available for the violation of the strong cosmic censorship conjecture is larger in higher curvature theories of gravity.

V. CONCLUSION

Given an initial field configuration, predicting what happens to the field in the future through the field equation is one of the essential features that any well-behaved theory of nature must possess. For the case of general relativity, this is ensured by the strong cosmic censorship conjecture, which states that the extension of the spacetime metric across the Cauchy horizon keeping the Christoffel symbols

as square integrable functions is impossible. However, recently it has appeared that this version of the strong cosmic censorship conjecture is seemingly violated for charged black hole spacetimes with a positive cosmological constant. This suggests that the classical fate of an observer is not completely determined from the initial data in general relativity. This is a cause for alarm, since it depicts that the deterministic nature of general relativity may break down under certain situations. At the same time one must take cognizance of the fact that general relativity is only an effective theory and it must be supplemented by higher curvature corrections. Thus, there is a tantalizing possibility that the strong cosmic censorship conjecture may be respected when these higher curvature terms are taken into account.

Following this possibility, we have considered two well known higher curvature theories of gravity and have explored whether the strong cosmic censorship conjecture is respected for charged asymptotically de Sitter black hole solutions in them. In particular, we have studied the fate of the strong cosmic censorship conjecture in the context of a spherically symmetric black hole spacetime in Einstein-Gauss-Bonnet gravity as well as in pure Gauss-Bonnet theory. We have started by computing the quasinormal frequencies in these black hole spacetimes both analytically as well as numerically, which shows good agreement between them (see Table I). Following which we have determined the minimum of the imaginary part of the quasinormal mode frequency as well as the surface gravity at the Cauchy horizon, which has helped to study the variation of β with the electromagnetic charge $(Q/Q_{\max})$, where $Q_{\max}$ corresponds to the extremal value of the charge. Surprisingly, we find that the strong cosmic censorship conjecture is violated even in these contexts. Moreover, as the Gauss-Bonnet coupling constant, which characterizes the strength of the higher curvature terms, increases the violation of the strong cosmic censorship conjecture becomes stronger. This conclusion has been achieved by considering all three families of modes, namely the photon sphere modes, the de Sitter modes, and the near extremal modes. Thus, our analysis concludes that the addition of higher curvature terms does *not* cure the problem with the strong cosmic censorship conjecture, rather it leads to an even stronger violation of the strong cosmic censorship conjecture.

To further strengthen our result, we have considered one more type of higher curvature theory, namely, the pure Lanczos-Lovelock theory of gravity. In particular, we have studied the case of a static, spherically symmetric black hole solution with a positive cosmological constant in seven-dimensional pure Gauss-Bonnet gravity. Following an identical approach to the Einstein-Gauss-Bonnet case, i.e., by first computing the quasinormal modes and then computing β for the photon sphere modes and the near extremal modes, we see that the violation of the strong

cosmic censorship conjecture is present even in this context. Furthermore, by comparing this result to that of the pure Einstein gravity in seven dimensions we could conclude that the violation of the strong cosmic censorship conjecture is stronger in the case of pure Gauss-Bonnet gravity. Thus, our result suggests that the strong cosmic censorship conjecture is violated even when higher curvature terms are included and the violation is stronger than general relativity. Moreover, finally the near extremal modes dominate and they reach the value unity asymptotically as Q approaches Q_{max}. This suggests that the fields are extendible across the Cauchy horizon as functions while belonging to H^1_{loc}, but not smoother than that. In this respect the results are at per with the case of general relativity. As a future outlook, one may consider a similar scenario for rotating black hole solutions in the presence of higher curvature and see whether the strong cosmic censorship conjecture is still violated. This will help us to understand the origin of this violation in a better manner. Further, the above result was derived for the scalar field and we hope that similar analysis will go through for the Dirac and electromagnetic fields, but for gravitational perturbations one needs to be careful due to the presence of higher curvature terms. These we leave for the future.

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