Implications of a proton blazar inspired model on correlated observations of neutrinos with gamma-ray flaring blazars

Prabir Banik,[1,*] Arunava Bhadra,[2,†] Madhurima Pandey,[3,‡] and Debasish Majumdar[3,§]

[1]*Department of Physics, Surendra Institute of Engineering & Management, Dhukuria, Siliguri, West Bengal 734009, India*

[2]*High Energy & Cosmic Ray Research Centre, University of North Bengal, Siliguri, West Bengal 734013, India*

[3]*Astroparticle Physics and Cosmology Division, Saha Institute of Nuclear Physics, HBNI, 1/AF Bidhannagar, Kolkata, West Bengal 700064, India*

(Received 4 September 2019; revised manuscript received 3 February 2020; accepted 2 March 2020; published 24 March 2020)

The recent detections of the neutrino event IceCube-170922A, 13 muon-neutrino events observed in 2014–2015 and the IceCube-141209A by IceCube observatory from the Blazars TXS $0506+056$, PKS $0502+049$/TXS $0506+056$, and GB6 J1040+0617 respectively, all of which were in the state of enhanced gamma-ray emission, indicate that cosmic rays are accelerated in the blazar jets. The photomeson ($p\gamma$) interaction cannot explain the IceCube observations of 13 neutrino events. However, the nondetection of broadline emission in the optical spectra of the IceCube blazars questions the hadronuclear (pp) interaction interpretation, in which a relativistic jet interacts with a high density cloud. In this work, we investigate the proton blazar model in which the nonrelativistic protons that come into existence under the charge neutrality condition of the blazar jet can offer sufficient target matter for pp interactions with shock-accelerated protons, to describe the observed high-energy gamma rays and neutrino signals from said blazars. Our findings suggest that the model can consistently explain the observed electromagnetic spectrum in combination with the appropriate number of neutrino events from the corresponding blazars.

DOI: 10.1103/PhysRevD.101.063024

I. INTRODUCTION

Recently the IceCube Neutrino Observatory at the South Pole reported the detection of a few reconstructed high-energy (TeV energy and above) neutrino events in spatial coincidence with a couple of known gamma-ray blazars, which provides the first direct identification of sources of high-energy cosmic rays. Gamma-ray blazars, a class of active galactic nuclei (AGN) with powerful relativistic jets oriented close to the line of sight of the observer, are considered as one of the promising contributors to the diffuse flux of high-energy neutrinos detected by IceCube [1]. Blazars are usually subclassified into BL Lac objects and flat spectrum radio quasars (FSRQs), depending on the emission line properties [2]. A common feature of the nonthermal electromagnetic (EM) spectral energy distribution (SED) of blazars is the double-hump structure: one hump in the IR/optical/UV or x-ray and the other in the high-energy gamma-ray bands. The lower-energy bump is usually believed to be produced from synchrotron radiation of primary electrons, while the second one can be explained by several different mechanisms. The most popular explanation of the higher-energy hump is inverse Compton (IC) scattering of synchrotron or external photons [3–5]. The leptonic scenarios, however, cannot explain some observed characteristics such as the very fast variability in almost all observed bands [6,7].

The first detection of cosmic-ray sources occurred on 22 September, 2017 when the IceCube Collaboration observed a high-energy muon-neutrino event—IceCube-170922A—of energy $\sim$290 TeV [8,9] coming from the direction of the sky location of the known blazar TXS $0506+056$, a BL Lac object [8,10]. A follow-up observation by the *Fermi* Large Area Telescope (LAT) Collaboration [11] revealed that the gamma-ray source TXS $0506+056$ was in a state of enhanced emission at GeV energies with day-scale variability [12] on September 28, 2017. A significant very high-energy (VHE) γ-ray signal was observed by the Major Atmospheric Gamma Imaging Cherenkov Telescopes [13] with energies up to about 400 GeV on 28 September, 2017. As high-energy neutrinos are believed to be produced only in hadronic processes, the observed association of the neutrino event with the gamma-ray flaring blazar TXS $0506+056$ has opened a new window to study the origin of cosmic rays in blazars using multimessenger astronomy.

*pbanik74@yahoo.com
†aru_bhadra@yahoo.com
‡madhurima.pandey@saha.ac.in
§debasish.majumdar@saha.ac.in

Triggered by the discovery of the 2017 flare from TXS 0506 + 056, the IceCube Collaboration reinvestigated the their 9.5 years of data at the position of TXS 0506 + 056 and reported significant evidence for a flare of 13 muon-neutrino events between September 2014 and March 2015 [8]. Assuming a power-law distribution of the signal between 32 TeV and 3.6 PeV, a statistically significant 3.5σ excess over the atmospheric neutrino background was found during a 158-day box-shaped time window from MJD 56937.81 to MJD 57096.21 [8]. Surprisingly, at the arrival time window of such a neutrino flare, the blazar TXS 0506 + 056 was found to be in a quiescent state of both radio and GeV emission [14]. A nearby FSRQ blazar, PKS 0502 + 049, which is only $\sim 1.2^0$ away from TXS 0506 + 056, was in a state of enhanced gamma-ray emission just before and after the period of the neutrino excess in 2014–2015 [14,15]. Thus, in principle, the production of neutrinos in the jet of PKS 0502 + 049 could contribute to such a neutrino flare in 2014–2015 as the position of PKS 0502 + 049 was found to be spatially consistent with the directional reconstruction uncertainties of such observed muon neutrinos.

Very recently, IceCube reported the detection of a reconstructed high-energy neutrino event—designated as IceCube-141209A [16]—in spatial coincidence with another known gamma-ray blazar, GB6 J1040 + 0617, with a coincidence detection probability of just 30%. This observed association suggests that the blazar GB6 J1040 + 0617 is another plausible neutrino source candidate.

All of these correlated observations of high-energy neutrinos with blazars during a gamma-ray flaring stage revealed that blazars may indeed be one of the most probable extragalactic sources of very high-energy cosmic rays. A relevant question is what is the production scenario of these detected neutrinos and gamma rays from the blazars. The high-energy neutrinos can be produced in either lepto-hadronic ($p\gamma$) or pure hadronic (pp) interactions. The high-energy neutrinos and TeV gamma rays are (totally or partially) produced in the former scenario through the interaction of blazar-accelerated cosmic rays with surrounding EM radiation, whereas in the latter scenario they are produced in the interaction of the blazar-accelerated cosmic rays with the ambient matter. Another issue is the maximum energy of the accelerated particles in the detected sources that led to the creation of such high-energy neutrinos together with observed EM radiation from the sources.

In previous work, it was demonstrated that a proton blazar model can consistently describe the observed high-energy gamma rays and neutrino signal from the blazar TXS 0506 + 056 [17]. In the present work, we would like to demonstrate that the proton blazar model can consistently explain the spectral behavior of the observed higher-energy bump of the EM SED along with the observed association of neutrinos from all three IceCube blazars assuming that the association of the observed neutrino events with the corresponding blazars at the flaring stage is genuine. Such a scenario appears to be more realistic than the scenario of the cloud-in-jet model, as we discuss later.

The plan of the paper is as follows. In the next section, we briefly review the models proposed in the literature to explain the observed high-energy gamma rays and neutrino signal from the IceCube blazars. The proton blazar model is described in Sec. III. In the same section, we describe the methodology for evaluating the gamma-ray and neutrino fluxes produced in the interaction of cosmic rays with the ambient matter in the AGN jet under the framework of the proton blazar model. The numerical estimated fluxes of hadronically produced gamma rays and neutrinos from the IceCube blazars over the GeV-to-TeV energy range are shown in Sec. IV, along with the observations. The findings of the present work are discussed in Sec. V, and we conclude in Sec. VI.

II. THE MODELS OF GAMMA RAY AND NEUTRINO PRODUCTION IN ICECUBE BLAZARS

Several efforts have been made so far to interpret the production of the detected neutrino events together with the EM observations from TXS 0506 + 056. A common feature of all of the proposed models is that protons are accelerated along with electrons to relativistic energies in the acceleration sites. The observations of the high-energy component have been explained as due to the interaction of protons with either low-energy photons in the blazar's environment [lepto-hadronic ($p\gamma$) interaction] [10,12,18,19] or ambient matter [hadronic (pp)] [20,21].

Ansoldi *et al.* [10] described the detected neutrino event along with the EM observations from TXS 0506 + 056 by assuming a dense field of external low-energy photons originating in a possible structured layer surrounding the jet as targets for photohadronic interactions. Keivani *et al.* [12] assumed a hybrid leptonic scenario for TXS 0506 + 056 where the production of high-energy gamma rays are described by external inverse-Compton processes and high-energy neutrinos are accounted for via a radiatively subdominant hadronic component. The observation was interpreted recently by Gao *et al.* [18] by considering a compact radiation core for high photohadronic interaction rates.

In the hadronic (pp) interaction scenario, a high thermal plasma density is required for efficient high-energy γ-ray production in an AGN jet. Recently, Liu *et al.* [20] described the observed EM and neutrino fluxes from TXS 0506 + 056 by assuming the presence of clouds in the vicinity of the supermassive black hole (SMBH) that provides targets for inelastic pp collisions once they enter the jet. However, the nondetection of broadline region (BLR) emission from TXS 0506 + 056 and other BL Lac objects [12] casts doubt on the presence of BLR clouds in the vicinity of the SMBH of TXS $0506+056$ [12]. Murase *et al.* [22] considered the cosmic-ray-induced neutral beam model in which beamed neutrons, which are produced via the photodisintegration of nuclei in the blazar zone, interact with an external radiation field/cloud after escaping from the blazar zone and thereby produce neutrinos. Their model can naively explain both the 2017 and 2014–2015 neutrino flares from TXS $0506+056$ when the effective optical depth to the photodisintegration process is taken as 0.1 and ≥ 1, respectively.

The main reason behind the difficulties in understanding the interaction mechanism for gamma-ray and neutrino production is the composition of the bulk of the jet medium, which is not clearly known. For highly luminous blazars the proton component of plasma is necessary (see, e.g., Refs. [23,24]) in order to maintain the radiated power, which would not exceed that carried by jet. Under such a scenario, recently two of us (Banik and Bhadra) have demonstrated that the detected neutrino event together with the EM observations from TXS $0506+056$ can be consistently described by assuming a proton blazar model where nonrelativistic protons that come into existence under the charge neutrality condition of the blazar jet can offer sufficient target matter for pp interactions with shock-accelerated protons [17].

So far no proper explanation of the flare of 13 muon-neutrino events observed by IceCube is available in the literature. Rodrigues *et al.* [25] concluded from their analysis that the high event numbers of neutrinos quoted by IceCube can not be explained by any other model of the source. Considering an EM spectral hardening of the source TXS $0506+056$ above 2 GeV during the neutrino flare, as predicted by Padovani *et al.* [14] based on *Fermi* data, Rodrigues *et al.* [25] recently demonstrated that roughly two to five neutrino events during the flare can be described with different lepto-hadronic models: a one-zone model, a compact-core model, and an external radiation field model. Garrappa *et al.* [16] pointed out that the feature of spectral hardening in the SED of the source may in fact not be significant. On the other hand, Liang *et al.* [26] and He *et al.* [15] interpreted the 2014 neutrino flare and the gamma-ray flare using a jet-cloud interaction model assuming that the 2014 detection was actually from the nearby source PKS $0502+049$.

No detailed production model for neutrinos from GB6 J$1040+0617$ is available in the literature yet.

III. THE PROTON BLAZAR MODEL AND COMPUTATION TECHNIQUE FOR FLUX ESTIMATION

The overall composition of the AGN jet is currently unknown. In almost all hadronic models of AGN jets, it is generally assumed that the relativistic jet material is composed of relativistic protons (p) and electrons (e^-). In principle, cold (nonrelativistic) protons that arose from the charge neutrality condition also exist as described in the adopted proton blazar inspired model [17], which is developed from the proton blazar model [27]. The existence of cold protons is supported by the fact that only a small fraction of the protons in the system (roughly 4%) are accelerated to nonthermal energies at diffusive shocks, as demonstrated in hybrid simulation studies [28]. The adjustable parameters of the model are the ratio of the number of relativistic protons to electrons, the maximum energies attained by protons/electrons in the acceleration process, the slope of their energy spectrum, and the luminosities of electrons and protons. We consider that the region of a blazar jet that is responsible for nonthermal emission is a spherical blob of size R'_b (primed variables are for jet frame) and it contains a tangled magnetic field of strength B'. The magnetic field energy density can be written as $u'_B = B'^2/8\pi = 3p'_B$, where p'_B is the corresponding pressure. If θ is the angle between the line of sight and the jet axis, then the Doppler factor of the moving blob can be written as $\delta = \Gamma_j^{-1}(1-\beta_j\cos\theta)^{-1}$, where $\Gamma_j = 1/\sqrt{1-\beta_j^2}$ is the bulk Lorentz factor [29].

In the proton blazar framework, we have assumed a broken power-law energy distribution of accelerated relativistic electrons in the blazar jet to explain the low-energy bump of the SED by synchrotron radiation, which can be written as [17,30]

$$\begin{aligned} N'_e(\gamma'_e) &= K_e\gamma_e'^{-\alpha_1} \quad \text{if } \gamma'_{e,\min} \leq \gamma'_e \leq \gamma'_b, \\ &= K_e\gamma_b'^{\alpha_2-\alpha_1}\gamma_e'^{-\alpha_2} \quad \text{if } \gamma'_b < \gamma'_e \leq \gamma'_{e,\max}, \end{aligned} \tag{1}$$

where α_1 and α_2 are the spectral indices before and after the spectral break at Lorentz factor γ'_b, and $\gamma'_e = E'_e/m_ec^2$ is the Lorentz factor of electrons of energy E'_e. The normalization constant K_e can be found using the relation [17,31]

$$L'_e = \pi R_b'^2\beta_j c\int_{\gamma'_{e,\min}}^{\gamma'_{e,\max}} m_ec^2\gamma'_e N'_e(\gamma'_e)d\gamma'_e, \tag{2}$$

where L'_e represents the kinetic power of accelerated electrons in the comoving blazar jet frame. The energy density and number density of relativistic ("hot") electrons are $u'_e = \int m_ec^2\gamma'_eN'_e(\gamma'_e)d\gamma'_e$ and $n'_{e,h} = \int N'_e(\gamma'_e)d\gamma'_e$, respectively.

It is often considered that all of the electrons in a system undergo Fermi acceleration. However, as pointed out by Eichler and Waxman [32] in the context of gamma-ray bursts, the exact fraction of electrons (χ_e) that participate in diffusive shock (Fermi) acceleration cannot be evaluated by current observations; the observationally admissible range is $m_e/m_p \leq \chi_e \leq 1$. When thermal ions/electrons encounter a shock barrier, only about 25% of them are reflected. When thermal ions/electrons impinge upon a shock barrier that is too weak to reflect them, they move downstream and thus do not participate in the acceleration. A portion of the impinged ions/electrons are reflected by shocks and energized up to a certain level via diffusive shock acceleration (DSA), and finally ejected downstream. Only a small fraction of injected ions/electrons achieve sufficient energy via DSA and escape upstream [28]. In the present work we take $\chi_e \approx 10^{-3}$, which is within the allowed range and consistent with the hybrid simulation results of DSA by a parallel collisionless shock [33,34]. Such a low value is also supported by the fact that the total energy of electrons is 2 orders of magnitude smaller than the total energy of protons [33,35], and as already mentioned only 4% of protons undergo Fermi acceleration. Note that it is not necessary to strictly consider such a low $\chi_e \sim 10^{-3}$; we may also choose up to $\chi_e \sim 10^{-2}$. In the latter case, we have to increase the cosmic-ray flux appropriately, i.e., we need higher jet power. The total number electrons including "hot" and nonrelativistic ("cold") electrons is $n'_e = n'_{e,h}/\chi_e$.

The detailed computational technique for electromagnetic and neutrino spectra at the Earth from a blazar in the framework of a proton blazar inspired model was given in our earlier work [17]. Basically, the parameters are chosen in such a way that synchrotron emission of the relativistic electrons gives the low-energy component of the EM SED of the blazar, which is computed here following the formulation given in Ref. [31].

The inverse Compton (IC) scattering of primary accelerated electrons with the seed photons comoving with the AGN jet is employed to describe the lower part of the high-energy component of the EM SED of the blazar. The emissivity $Q_c(\epsilon'_c)$ of produced gamma-ray photons of energy $E'_c = m_e c^2 \epsilon'_c$ due to IC scattering of primary accelerated electrons with the seed photons was given in Refs. [36,37]. The seed photon density and spectra are estimated directly from the observed photon flux from the blazar [17,38].

In the proton blazar framework, we have assumed a power-law behavior of the cosmic-ray protons which are supposed to be accelerated to very high energies $E'_p = m_p c^2 \gamma'_p$ in the blob of a blazar jet [39,40],

$$N'_p(\gamma'_p) = K_p {\gamma'_p}^{-\alpha_p}, \tag{3}$$

where γ'_p is the Lorentz factor of the accelerated protons, α_p represents the spectral index, and K_p is a proportionality constant which can be obtained from Eq. (2) (as the expression also holds for protons) using the corresponding jet power L'_p for relativistic protons. The energy density of relativistic protons is $u'_p = \int m_p c^2 \gamma'_p N'_p(\gamma'_p) d\gamma'_p$, and $n'_p = \int N'_p(\gamma'_p) d\gamma'_p$ is the corresponding number density of relativistic protons.

Secondary particles (mainly pions) are produced when the shock-accelerated cosmic rays interact with the cold matter (protons) of density $n_H = (n'_e - n'_p)$ in the blob of the AGN jet. The emissivity of secondary particles is calculated in this work following Refs. [20,41–43].

The π^0 mesons subsequently decays to gamma rays. The produced TeV–PeV gamma rays are likely to be absorbed due to internal photon-photon ($\gamma\gamma$) interactions [45] while propagating through an isotropic source of low-frequency radiation which is generally assumed to be the observed synchrotron radiation photons produced by the relativistic electron population in the comoving jet. The gamma-ray emissivity as a function of gamma-ray energy $E'_\gamma (= m_e c^2 \epsilon'_\gamma)$ is computed following Ref. [44] and the emissivity of escaped gamma rays after internal $\gamma\gamma$ absorption within the source region is estimated following Ref. [31]:

$$Q'_{\gamma,\mathrm{esc}}(\epsilon'_\gamma) = Q'_\gamma(\epsilon'_\gamma).\left(\frac{1 - e^{-\tau_{\gamma\gamma}}}{\tau_{\gamma\gamma}}\right), \tag{4}$$

where $\tau_{\gamma\gamma}(\epsilon'_\gamma)$ is the optical depth for the interaction [17,45].

The total number of high-energy injected electrons/positrons (Q'_e) in the emission region of the AGN jet is the sum of those created in $\gamma\gamma$ pair production and those produced directly due to the decay of $\pi^\pm$ mesons created in pp interactions. These injected electrons/positrons will initiate EM cascades in the AGN blob via synchrotron radiation and IC scattering. The secondary-pair cascade processes are determined following the self-consistent formalism of Böttcher *et al.* [31] after the inclusion of the IC mechanism.

The observable differential flux of gamma rays reaching the Earth from a blazar can be written as

$$E_\gamma^2 \frac{d\Phi_\gamma}{dE_\gamma} = \frac{V' \delta^2 \Gamma_j^2}{4\pi d_L^2} \frac{{E'_\gamma}^2}{m_e c^2} Q'_{\gamma,\mathrm{esc}}(\epsilon'_\gamma).e^{-\tau_{\gamma\gamma}^{\mathrm{EBL}}} \tag{5}$$

where $Q'_{\gamma,\mathrm{esc}}(\epsilon'_\gamma)$ is the total gamma-ray emissivity from the blob of the AGN jet with photon energy $E'_\gamma = m_e c^2 \epsilon'_\gamma$ in the comoving jet frame including all processes stated above, i.e., the synchrotron and IC radiation of accelerated electrons, the gamma rays produced in pp interactions, and the synchrotron photons of EM cascade electrons. $V' = \frac{4}{3}\pi R_b'^3$ is the volume of the emission region, $E_\gamma = \delta E'_\gamma/(1+z)$ [46] describes photon energies in the observer frame, d_L is the luminosity distance between the AGN and the Earth and z is the redshift parameter of the comoving jet frame respectively. Here we introduced the effect of the absorption of gamma-ray photons by the extragalactic background light (EBL), and $\tau_{\gamma\gamma}^{\mathrm{EBL}}(\epsilon_\gamma, z)$ is the corresponding optical depth which can be obtained using the Franceschini-Rodighiero-Vaccari model [47,48].

We have used the recent results on neutrino mixing angles to compute the flavor ratio of various neutrino flavors after oscillations. The oscillation probability for a neutrino $|\nu_\alpha\rangle$ of flavor α to a neutrino $|\nu_\beta\rangle$ of flavor β after traversing a baseline distance d_L is given by [49,50]

$$P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2\left(\frac{\pi d_L}{\lambda_{ij}}\right), \quad (6)$$

where $U_{\alpha i}$, $U_{\beta i}$, etc., in the above are the elements of the neutrino mass-flavor mixing matrix (Pontecorvo-Maki-Nakagawa-Sakata matrix) [51] with i, j, etc., being the neutrino mass indices. A neutrino $|\nu_\alpha\rangle$ of flavor α is related to its mass eigenstates $|\nu_i\rangle$ ($i = 1, 2, 3$, for the three-flavor case) by

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle. \quad (7)$$

If the neutrinos originate at a distant blazar with the flavor ratio

$$\varphi_{\nu_e} : \varphi_{\nu_\mu} : \varphi_{\nu_\tau} = 1:2:0,$$

the flux $\Phi^3_{\nu_\alpha}$ at the Earth can be expressed as

$$\begin{aligned}
\Phi^3_{\nu_e} = {} & [|U_{e1}|^2(1 + |U_{\mu 1}|^2 - |U_{\tau 1}|^2) \\
& + |U_{e2}|^2(1 + |U_{\mu 2}|^2 - |U_{\tau 2}|^2) \\
& + |U_{e3}|^2(1 + |U_{\mu 3}|^2 - |U_{\tau 3}|^2)]\varphi_{\nu_e}, \\
\Phi^3_{\nu_\mu} = {} & [|U_{\mu 1}|^2(1 + |U_{\mu 1}|^2 - |U_{\tau 1}|^2) \\
& + |U_{\mu 2}|^2(1 + |U_{\mu 2}|^2 - |U_{\tau 2}|^2) \\
& + |U_{\mu 3}|^2(1 + |U_{\mu 3}|^2 - |U_{\tau 3}|^2)]\varphi_{\nu_e}, \\
\Phi^3_{\nu_\tau} = {} & [|U_{\tau 1}|^2(1 + |U_{\mu 1}|^2 - |U_{\tau 1}|^2) \\
& + |U_{\tau 2}|^2(1 + |U_{\mu 2}|^2 - |U_{\tau 2}|^2) \\
& + |U_{\tau 3}|^2(1 + |U_{\mu 3}|^2 - |U_{\tau 3}|^2)]\varphi_{\nu_e}. \quad (8)
\end{aligned}$$

The elements $U_{\alpha i}$ in Eq. (8) can be computed using

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \quad (9)$$

where $c_{ij(i,j=1,2,3)} = \cos\theta_{ij}$ and $s_{ij(i,j=1,2,3)} = \sin\theta_{ij}$, where θ_{ij} is the mixing angle between the ith and jth neutrinos.

In this work we adopt the following values for the mixing angles: $\theta_{12} = 32.96°$, $\theta_{23} = 40.7°$, and $\theta_{13} = 8.43°$ [52]. Hence, the flavor ratio of the neutrino flux reaching the Earth from a distant blazar after neutrino oscillation is evaluated to be $\Phi^3_{\nu_e} : \Phi^3_{\nu_\mu} : \Phi^3_{\nu_\tau} = 1.052:0.992:0.955$.

The corresponding muon neutrino flux reaching the Earth can be expressed as

$$E^2_\nu \frac{d\Phi_{\nu_\mu}}{dE_\nu} = \xi . \frac{V'\delta^2\Gamma^2_j}{4\pi d^2_L} \frac{E'^{\,2}_\nu}{m_e c^2} Q'_{\nu,pp}(\epsilon'_\nu), \quad (10)$$

where the fraction $\xi = 0.992/3$ is included due to neutrino oscillation and $E_\nu = \delta E'_\nu/(1+z)$ [46] relates neutrino energies in the observer and comoving jet frames. If the differential flux of muon neutrinos is known, then the number of expected muon neutrino events at IceCube over a time interval τ can be obtained from the relation

$$N_{\nu_\mu} = \tau \int_{\epsilon_{\nu,\min}}^{\epsilon_{\nu,\max}} A_{\rm eff}(\epsilon_\nu) . \frac{d\Phi_{\nu_\mu}}{d\epsilon_\nu} d\epsilon_\nu, \quad (11)$$

where $A_{\rm eff}$ is the IceCube detector's effective area at the declination of the blazars [9,14,53].

IV. GAMMA-RAYS AND NEUTRINOS FROM THE ICECUBE DETECTED BLAZARS

The gamma-ray variability time scale of all three blazars can generally be assumed to be $t_{\rm ver} \leq 10^5$ s, as was found for TXS 0506 + 056 by analyzing the x-ray and gamma-ray light curves [12]. The best-fit spectral slope of the astrophysical neutrinos between 194 TeV and 7.8 PeV observed by IceCube [54,55] suggests that the spectral index of the energy spectrum of AGN-accelerated cosmic rays can be taken as $\alpha_p \sim -2.1$ for all blazars. As the declinations of all of the blazars are nearly same, we use the same $A_{\rm eff}$ as provided by the IceCube Collaboration at the declination of TXS 0506 + 056 for all three blazars [9,53].

A. GB6 J1040 + 0617

An energy of 97.4 ± 9.6 TeV was deposited in the IceCube detector by the neutrino event IceCube-141209A [16]. Although two neighboring FSRQs of the object GB6 J1040 + 0617 − 4C + 06.41 and SDSS J104039.54 + 061521.5—were found to be located within the 90% uncertainty band of the well-reconstructed neutrino event IceCube-141209A, they are less favored as the likely neutrino counterpart [16] because no significant high-energy gamma-ray emission was observed at the arrival time of IceCube-141209A. On the other hand, being a BL Lac object, GB6 J1040 + 0617 displayed a bright optical flare detected by the All Sky Automated Survey for SuperNovae associated with modest gamma-ray activity at the arrival time. When IceCube-141209A was detected the blazar showed increased gamma-ray activity that started a few days before the neutrino event and lasted for 93 days, i.e., from MJD 56997 to 57090 with respect to the 9.6-year average flux [16]. Moreover, the blazar is located near the equatorial plane at a similar declination as TXS 0506 + 056, which is the sky region that IceCube is most sensitive to. If IceCube-141209A is astrophysical in origin, the low-synchrotron peaked gamma-ray blazar GB6

J1040 + 0617 appears to be a plausible neutrino source candidate based on its energetics and multiwavelength characteristics [16]. The redshift of the blazar was recently estimated to be $z = 0.73$ [56,57] and the luminosity distance of the blazar was evaluated to be $d_L \sim 4612.1$ Mpc with a consensus cosmology.

To explain the EM SED of GB6 J1040 + 0617 over the optical to gamma-ray energy range, we consider an emission region size of $R'_b = 5.2 \times 10^{16}$ cm with a bulk Lorentz factor for the AGN jet of $\Gamma_j = 21$ and Doppler boosting factor $\delta = 30$, which are strongly consistent with the size suggested from the variability, namely, $R'_b \lesssim \delta c t_{\rm ver}/(1+z) \simeq 5.2 \times 10^{16}$ cm assuming $t_{\rm ver} \simeq 10^5$ s (similar to that of the blazar TXS 0506 + 05).

The lower-energy bump of the experimental EM SED data can be well explained if we model the distribution of the primary accelerated electrons by a broken power law as given by Eq. (1), with spectral indices $\alpha_1 = 1.45$ and $\alpha_2 = 6.2$ before and after the spectral break, respectively, with a Lorentz factor $\gamma'_b = 1.35 \times 10^4$. The magnetic field and kinematic power of relativistic electrons in the blazar jet required to fit the observed data are found to be $B' = 0.01$ G and $L'_e = 1.5 \times 10^{43}$ erg/s, respectively. Also, IC scattering of primary relativistic electrons with synchrotron photons comoving with the AGN jet is found to produce the lower part of the high-energy bump of the EM spectrum.

The higher-energy bump of the EM SED data can be well reproduced by the model given in Eq. (5), and the required accelerated primary proton injection luminosity is estimated to be $L'_p = 2.7 \times 10^{45}$ erg/s along with a best-fit spectral slope of $\alpha_p = -2.1$. The cold proton number density in the jet turns out to be 4.2×10^5 particles/cm^3 under the charge neutrality condition, which provides a sufficient number of targets for hadronuclear interactions with accelerated protons under the assumption of a low acceleration efficiency of electrons in the AGN jet of $\chi_e \approx 10^{-3}$. The contributions of synchrotron and IC emission of the stationary electron/positron pairs produced in the EM cascade induced by protons are estimated following Ref. [31], as discussed in the previous section. The radiative cooling time due to inverse Compton losses varies inversely with the energy density of photons in the comoving jet frame in equilibrium. To evaluate the IC spectrum from cascade electrons we consider seed photons from the whole observed electromagnetic energy spectrum rather than the just synchrotron seed photons, i.e., we consider a higher energy density of seed photons. This leads to a smaller IC cooling time and yields higher IC and lower synchrotron fluxes from cascade electrons compared to prevailing studies. We incorporate IC process with the full Klein-Nishina formulation while implementing the self-consistent formalism of Böttcher *et al.* [31].

The total jet power, given by the kinetic energy of relativistic electrons and protons and the energy of the magnetic field, is evaluated as $L^k_{\rm jet} = \Gamma_j^2 \beta_j c \pi R'^2_b [u'_e + u'_p + u'_B]$ [10] and is found to be 1.2×10^{48} erg/s. This is consistent with the Eddington luminosity of the blazar if we assume that the system hosts a supermassive black hole of mass $M_{\rm BH} \gtrsim 9.5 \times 10^9 \, M_\odot$, like AGN NGC 1281. During outbursts or for a collimated outflow in a jet, and if the jet does not interfere with the accretion flow, the jet power may moderately exceed the Eddington luminosity, within a factor of 10 [18,58].

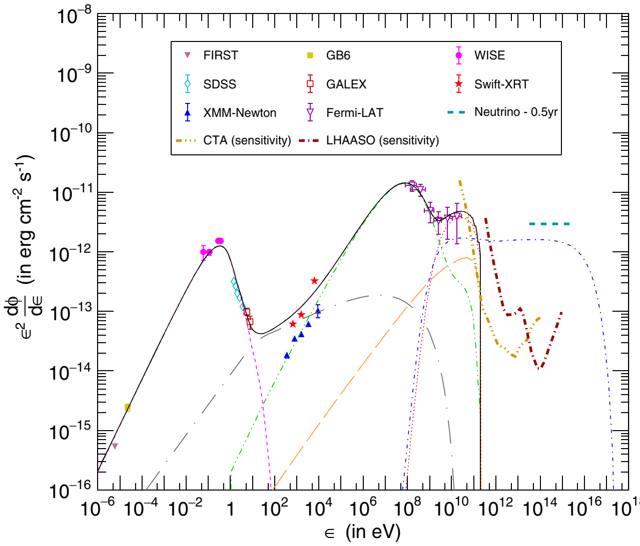

FIG. 1. The estimated differential energy spectrum of gamma rays and neutrinos reaching the Earth from the blazar GB6 J1040 + 0617. The pink small-dashed and green long-dash-double-dotted lines indicate the EM spectrum due to synchrotron emission and inverse Compton emission of relativistic electrons, respectively. The red dotted line shows the gamma-ray flux produced from pp interactions after $\gamma\gamma$ absorption. The gray large-dash-dotted and orange large-dashed lines denote the flux for synchrotron and inverse Compton emission by electrons/positrons in the EM cascade after $\gamma\gamma$ absorption, respectively. The black continuous line represents the estimated overall differential EM SED. The blue small-dash-single-dotted line indicates the differential muon neutrino flux at the Earth. The yellow dash-triple-dotted and brown long-dash-single-dotted lines denote the detection sensitivity of the CTA detector for 1000 hours and the LHAASO detector for one year, respectively. The cyan long-dashed line indicates the expected level and energy range of the neutrino flux reaching the Earth to produce one muon neutrino at IceCube over 0.5 years, as observed.

The calculated differential gamma-ray and neutrino spectrum reaching Earth from this blazar along with the different space- and ground-based observations is displayed in Fig. 1. The x-ray data points shown in the figure are not contemporaneous and the displayed *XMM-Newton* and the *Swift-XRT* data were collected in May 2003 and during 2007–2011, respectively [16]. Note that *XMM-Newton* and *Swift-XRT* data are not consistent, which may be due to the dynamical behavior of the source. Using Eq. (11), the expected number of muon neutrino events at IceCube from the blazar is evaluated to be about $N_{\nu_\mu} = 0.52$ in the 32 TeV–7.5 PeV energy range over 0.5 years for the flaring VHE emission state with $E'_{p,\max} = 20$ PeV.

B. TXS 0506 + 056

The high-energy neutrino-induced muon track IceCube-170922A detected by IceCube was found to be coincident with the known flaring γ-ray blazar TXS 0506 + 056 with chance coincidence being rejected at the 3σ confidence level [8]. No additional excess of neutrinos was found from the direction of TXS 0506 + 056 near the time of the alert. Considering a spectral index of −2.13 (−2.0) for the spectrum of diffuse astrophysical muon neutrinos, the most probable energy of the neutrino event was estimated to be 290 TeV (311 TeV), with the 90% C.L. lower and upper limits being 183 TeV (200 TeV) and 4.3 PeV (7.5 PeV), respectively [8,10]. The *Fermi*-LAT observations suggest that the integrated gamma-ray flux above 0.1 GeV from TXS 0506 + 056 increased to $(5.3 \pm 0.6) \times 10^{-7}$ cm^{-2} s^{-1} in the week from 4–11 July, 2017 from its average integrated flux of $(7.6 \pm 0.2) \times 10^{-8}$ cm^{-2} s^{-1} above 0.1 GeV during 2008–2017. The Astro-Rivelatore Gamma a Immagini Leggero gamma-ray telescope observed a flux of $(5.3 \pm 2.1) \times 10^{-7}$ cm^{-2} s^{-1} during 10–23 September, 2017. Thus, all of the extensive follow-up observations revealed that TXS 0506 + 056 was active in all EM bands during the period from July to September 2017, whereas the source was found to be in a quiescent stage most of the time based on an archival study of the time-dependent γ-ray data over the last 10 years or so. The redshift of the blazar was recently estimated to be $z = 0.3365$ [59] and the luminosity distance of the blazar was evaluated to be $d_L \sim 1750$ Mpc [12] with a consensus cosmology.

Here we have chosen the size of the emission region to be $R'_b = 5.5 \times 10^{16}$ cm, with a bulk Lorentz factor of the AGN jet $\Gamma_j = 30$ and Doppler boosting factor $\delta = 30$, which are strongly consistent with the size inferred from the variability, namely, $R'_b \lesssim \delta c t_{\rm ver}/(1+z) \simeq 6.7 \times 10^{16} (\delta/30)(t_{\rm ver}/10^5\ {\rm s})$ cm [12] to describe the EM SED of TXS 0506 + 056 over the optical to gamma-ray energy range.

We found that we can well reproduce the lower-energy bump of the experimental EM SED data by modeling the distribution of primary accelerated electrons by a broken power law. The IC scattering of primary relativistic electrons with synchrotron photons comoving with the AGN jet is found to produce the lower part of the high-energy bump of the EM spectrum. Our proton blazar model in which gamma rays are produced in interactions of relativistic protons with the ambient cold protons in the blob can consistently explain the observed higher-energy bump of the observed EM SED data, as estimated following Eq. (5). The required accelerated primary proton injection luminosity is estimated to be $L'_p = 1.65 \times 10^{45}$ erg/s, with a best-fit spectral slope $\alpha_p = -2.1$ and total kinetic jet power of 1.5×10^{48} erg/s. The synchrotron and IC emissions of the stationary-state electron/positron pairs produced in the EM cascade induced by protons are found to well explain the x-ray data, as shown in Fig. 2. The results for this blazar were also discussed in detail in Ref. [17], where we did not consider the full Klein-Nishina scattering cross section when computing the energy loss rate of electrons in the EM cascade initiated by high-energy protons. Our estimated flux from the electromagnetic cascade for TXS 0506 + 056 is comparable with that of Liu *et al.*, in which electromagnetic radiation absorption was also considered. The softer proton spectrum considered in the present work seems to balance the effect of electromagnetic radiation absorption considered by Liu *et al.* Our estimated cascade synchrotron emission is almost the same as that given by Sahakyan [21]. The effect of the softer primary proton spectrum considered by Sahakyan seems to be compensated by the absence of the IC process in his work. The model fitting parameters are shown in Table I.

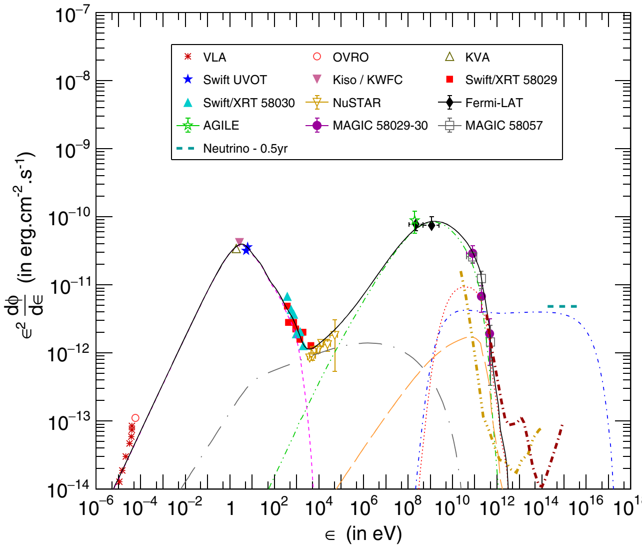

FIG. 2. Same as Fig. 1, but for the blazar TXS 0506 + 056 during the active phase. The cyan long-dashed line represents the expected level and energy range of the neutrino flux at the Earth to produce one muon neutrino event at IceCube over 0.5 years, as observed.

The calculated differential gamma-ray and neutrino spectrum reaching Earth from this blazar along with the different space and ground based observations are given in Fig. 2. Using Eq. (11), the expected number of muon neutrino events at IceCube from the blazar is found to be about $N_{\nu_\mu} = 0.74$ in the 200 TeV–7.5 PeV energy range over 0.5 years for the flaring VHE emission state with $E'_{p,\max} = 20$ PeV.

TABLE I. Model fitting parameters for TXS 0506 + 056, PKS 0502 + 049, and GB6 J1040 + 0617 according to the proton blazar model.

	TXS 0506 + 056		PKS 0502 + 049	GB6 J1040 + 0617
Parameters	Active	Quiescent	Active	Active
δ	30	30	40	30
Γ_j	30	30	30	21
θ	1.9^0	1.9^0	1.35^0	1.7^0
z	0.3365	0.3365	0.954	0.73
R'_b (in cm)	5.5×10^{16}	5.5×10^{16}	6×10^{16}	5.2×10^{16}
B (in G)	0.034	0.068	0.023	0.01
α_1	−1.6	−1.7	−1.7	−1.45
α_2	−4.1	−4.5	−3.8	−6.2
γ'_b	1.8×10^4	10^4	1.3×10^4	1.35×10^4
$\gamma'_{e,\min}$	1	1	1	1
$\gamma'_{e,\max}$	3×10^5	10^5	1.4×10^5	10^5
L'_e (in erg/s)	2.3×10^{42}	9.1×10^{41}	1.7×10^{43}	1.5×10^{43}
n_H (in cm^{-3})	1.1×10^5	10^5	1.4×10^6	4.2×10^5
α_p	−2.1	−2.2	−2.1	−2.1
$E'_{p,\max}$ (in eV)	2×10^{16}	2×10^{16}	2×10^{16}	2×10^{16}
L'_p (in erg/s)	1.65×10^{45}	6.8×10^{44}	9.2×10^{45}	2.7×10^{45}
L^k_{jet} (in erg/s)	1.5×10^{48}	6.1×10^{47}	8.3×10^{48}	1.2×10^{48}
N_{ν_μ}	0.74	0.19	10.85	0.52

C. TXS 0506 + 056/PKS 0502 + 049

Reanalyzing the historical 9.5 years of data at the position of TXS 0506 + 056, the IceCube Collaboration reported significant evidence for a flare of 13 ± 5 muon-neutrino events between September 2014 and March 2015 [8]. Moreover, it was also reported that the observed neutrino flare has a 3.5σ excess over atmospheric neutrinos in the energy range 32 TeV–3.6 PeV (68% C.L.). However, TXS 0506 + 056 appeared to be in a quiescent state in both the radio and GeV emission bands during the arrival-time window of this neutrino flare [8]. The energy spectrum of detected neutrinos exhibits a power-law behavior, with a spectral slope of around −2.1 to −2.2 depending on the choice of fitting window [9]. The spectral index of the accelerated proton spectrum has to be chosen accordingly.

The calculated differential gamma-ray and neutrino spectrum reaching at the Earth from TXS 0506 + 056 during MJD 56949-57059 along with the different space- and ground-based observations are given in Fig. 3. It is found that the overall differential multiwavelength EM SED of the blazar in a quiescent state can be fitted using our model, where the parameters Γ, δ, θ, and R'_b are kept fixed to that of an active state of the blazar. The magnetic field has to increase slightly to $B' = 0.068$ G, and the other parameters are adjusted as $\alpha_1 = 1.7$, $\alpha_2 = 4.5$, $\gamma'_b = 10^4$, $L'_e = 9.1 \times 10^{41}$ erg/s, $L'_p = 6.8 \times 10^{44}$ erg/s, and $L^k_{\mathrm{jet}} = 6.1 \times 10^{47}$ erg/s. The archival observed x-ray data restrict the spectral slope to $\alpha_p = -2.2$ and the expected number of muon-neutrino events at IceCube from the blazar is found to be about $N_{\nu_\mu} = 0.19$ in the 32 TeV–3.6 PeV energy range over 158 days.

Thus, it is unlikely that the blazar TXS 0506 + 056 (which was in a quiescent state during MJD 56949-57059)

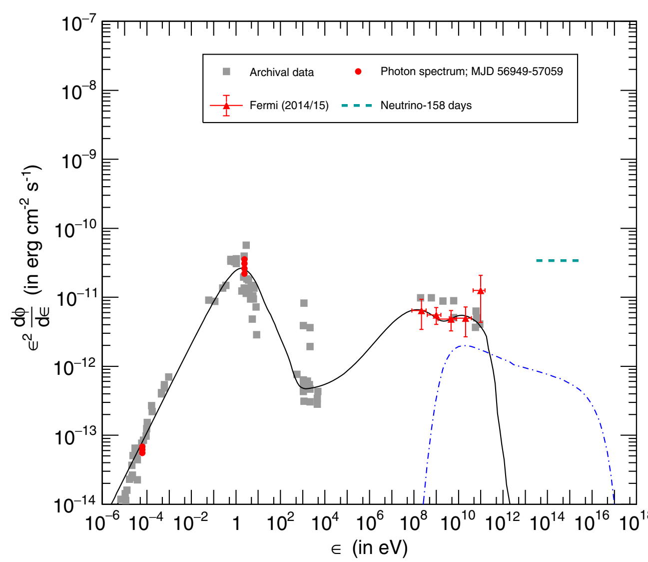

FIG. 3. Evaluated differential energy spectrum of gamma rays and neutrinos reaching the Earth from the blazar TXS 0506 + 056 in a quiescent state (during MJD 56949–57059). The black continuous and blue dash-dotted lines indicate the estimated overall differential EM SED and neutrino spectrum, respectively, following our model. The cyan long-dashed line represents the expected level and energy range of the neutrino flux reaching Earth to produce 13 muon-neutrino events at IceCube over 158 days, as observed.

contributed to the detected neutrino flare during the same period. It was found that a bright source PKS 0502 + 049 blazar located $\sim 1.2^0$ away from TXS 0506 + 056 had strong GeV flares around the neutrino flare phase in 2014–2015. The light curve of the blazar displays two major active phases—one during the period MJD 56860–56960, and another in the period MJD 57010–57120—which partly overlap with the 158-day box-shaped time window of the IceCube neutrino flare from MJD 56937–57096 [60]. The redshift of the blazar has been recently evaluated to be $z = 0.954$ and the luminosity distance of the blazar is estimated to be $d_L \sim 6.4$ Gpc with a consensus cosmology [61].

To describe the EM SED of PKS 0502 + 049 over the optical to gamma-ray energy range during MJD 56860–56960, we have considered the size of emission region of $R'_b = 6 \times 10^{16}$ cm with bulk Lorentz factor of AGN jet $\Gamma_j = 30$ and Doppler boosting factor $\delta = 40$. The size of emission region R'_b is strongly consistent with the size inferred from the variability, namely $R'_b \lesssim \delta c t_{\rm ver}/(1+z) \simeq 6.14 \times 10^{16}$ cm assuming $t_{\rm ver} \simeq 10^5$ s (similar to the blazar TXS 0506 + 05).

During the first active phase, the distribution of primary accelerated electrons is found to well reproduce the lower-energy bump of the experimental EM SED data of PKS 0502 + 049. The magnetic field and kinematic power of relativistic electrons in the blazar jet required to reproduce the EM spectrum [as given by Eq. (2)] are $B' = 0.023$ G and $L'_e = 1.7 \times 10^{43}$ erg/s, respectively. The IC scattering of primary relativistic electrons with synchrotron photons comoving with the AGN jet are found to produce the lower part of the high-energy bump of the EM spectrum.

Following the model given in Eq. (5), the observed higher-energy bump of the EM SED data during the first active phase can be reproduced well and the required accelerated primary proton injection luminosity is estimated to be $L'_p = 9.2 \times 10^{45}$ erg/s, with a spectral slope of $\alpha_p = -2.1$ to -2.2. Here we have taken the acceleration efficiency of the electrons in the AGN jet to be $\chi_e \approx 10^{-3}$ and the cold proton number density in the jet turns out to be 1.4×10^6 particles/cm^3 under the charge neutrality condition, which provides a sufficient number of targets for hadronuclear interactions with accelerated protons. The expected number of muon-neutrino events at IceCube from the blazar is found to be about $N_{\nu_\mu} = 10.85$ and 5.2 using $\alpha_p = -2.1$ and -2.2, respectively, in the 32 TeV–3.6 PeV energy range over 158 days for the flaring VHE emission state with $E'_{p,\rm max} = 20$ PeV, as estimated using Eq. (11).

The total jet power, given by the kinetic energy of relativistic electrons and protons and the energy of the magnetic field, is $L^k_{\rm jet} = 8.3 \times 10^{48}$ erg/s, which is about 84 times higher then the Eddington luminosity of the blazar PKS 0502 + 049 ($L_{\rm Edd} \simeq 9.8 \times 10^{46}$ erg/s considering a black hole mass of $M_{\rm BH} \simeq 7.53 \times 10^8 \, M_\odot$ [62]). The synchrotron and cascade emissions of the electrons/positrons pairs produced in the EM cascade initiated by primary accelerated protons can explain the observed x-ray data.

On the other hand, the quiescent state of the blazar during MJD 56949–57059 was found to be of leptonic origin, where the observed low- and high-energy bumps of the EM SED data can be well explained by the synchrotron emission and IC scattering of primary accelerated electrons with an injection luminosity $L'_e = 1.3 \times 10^{43}$ erg/s, with $B' = 0.017$ G. Because of the harder spectral slope at GeV energies of the observed EM SED, the second active phase during MJD 57010–57120 may also have a leptonic origin, as suggested by Sahakyan [60]. The gamma-ray and neutrino production via pp interactions may be subdominant, and hence the emission of neutrinos is not significant during this phase. The expected number of muon-neutrino events at IceCube from the blazar is found to be about $N_{\nu_\mu} = 0.13$, with $L'_p = 8 \times 10^{43}$ erg/s and $\alpha_p = -2.1$ in the 32 TeV–3.6 PeV energy range over 158 days with $E'_{p,\rm max} = 20$ PeV. The calculated differential gamma-ray

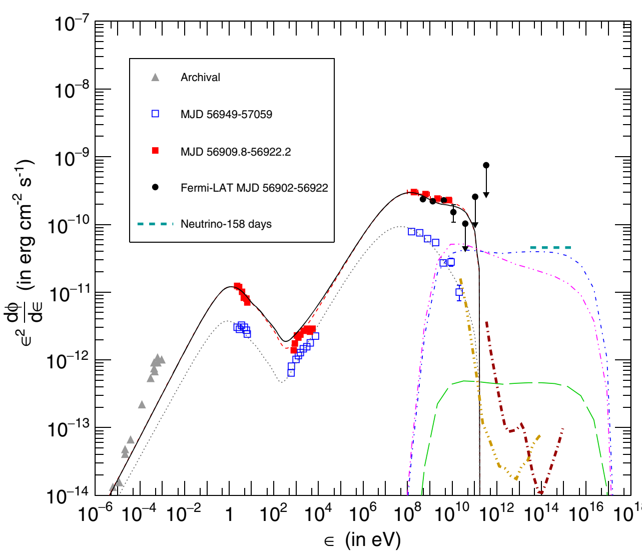

FIG. 4. The estimated differential energy spectrum of gamma rays and neutrinos reaching the Earth from the blazar PKS 0502 + 049. The black continuous line (red small-dashed line) and blue small-dash-dotted line (pink small dash-double-dotted line) indicate the estimated overall differential EM SED and the differential muon neutrino flux at the Earth following our model using $\alpha_p = -2.1$ (-2.2) during the first active phase of MJD 56909.8–56922.2. The gray dotted and green very long-dashed lines represent the same with $\alpha_p = -2.1$ but for the quiescent state of the blazar during MJD 56949–57059. The yellow dash-triple-dotted and brown long-dash-single-dotted lines indicate the detection sensitivity of the CTA detector for 1000 hours and the LHAASO detector for one year, respectively. The cyan long-dashed line represents the expected level and energy range of the neutrino flux reaching the Earth to produce 13 muon-neutrino events at IceCube over 158 days, as observed.

and neutrino spectrum at the Earth from this blazar along with the different space- and ground-based observations are shown in Fig. 4.

V. DISCUSSION

The observation of high-energy neutrinos along with gamma rays from flaring blazars suggests a hadronic mechanism as their origin in blazar jets. Although the AGN jet composition is still unknown, the general convention is that there is a dense radiation target within the jet, unless it is of an external origin [60]. The photomeson reaction ($p\gamma$) is more widely used to explain the emission from blazars. To describe the SED and neutrino events, the conventional one-zone models offer too low neutrino rates in combination with excessively high neutrino energies or sustained super-Eddington injection luminosities. The current theoretical study generally suggests that the geometry of the radiation zone must be more complex, involving an external radiation field boosted into the jet frame, either thermal [12] or nonthermal with structured jet modeling [10] or a compact radiation core with high photohadronic interaction rates [18]. By adopting such a complex geometry for the radiation zone one may interpret the neutrino events from TXS $0506+056$ (during 2017) and GB6 J1040 + 0617, but the observation of flaring events during the period 2014–2015 from the direction of TXS 0506 + 056/PKS 0502 + 049 cannot be explained using leptohadronic models [25,60].

The inelastic hadronic (pp) interaction scenario requires a high thermal plasma density, but offers high neutrino rates in combination with a neutrino energy range similar to that detected by IceCube. The can be understood as a cloud-jet interaction scenario, where the presence of clouds in the vicinity of the SMBH provides targets for inelastic pp collisions once they enter the jet. However, the presence of BLR clouds in the vicinity of the SMBH of TXS 0506 + 056 is doubtful due to the nondetection of the BLR emission from TXS 0506 + 056, GB6 J1040 + 0617, and other BL Lac objects. Consequently, the hadronuclear (pp) interaction interpretation (similar to relativistic jets meeting a high density cloud) is unlikely to be a common scenario for neutrino production in all IceCube blazars.

In the original proton blazar model [27] high-energy gamma rays are produced via synchrotron radiation by high-energy protons in a strong magnetic field environment. However, the gamma-ray spectrum of the IceCube blazars cannot be modeled using proton synchrotron radiation due to the low magnetic field strength of the source as obtained from the fitting of the low-energy hump of the SED. In the present cases, the photomeson ($p\gamma$) interaction is also found to be inefficient due to the low amplitude of the target synchrotron photon field. Instead, the proton blazar inspired model (as elaborately discussed in Ref. [17]) appears to be more appropriate as the high-energy gamma rays are produced along with a relatively higher neutrino event rate in interactions of relativistic protons with the ambient cold protons in the blob of the AGN jet. In the framework of the proton blazar model, our findings suggest that the relative contributions to the total jet power of cold protons, accelerated protons, the magnetic field, and accelerated electrons obtained based on charge neutrality can consistently describe both the low- and high-energy bumps of the multiwavelength EM SED, as well as the neutrino events IceCube-170922A and IceCube-141209A from the flaring blazars TXS 0506 + 056 and GB6 J1040 + 0617, respectively. On the other hand, the emission of 13 muon-neutrino events observed in 2014–2015 from the direction of the blazar TXS 0506 + 056 cannot be explained by any existing models assuming a correlation with the neutrino flare from TXS 0506 + 056, which was found to be in a quiescent state during this time. The estimated number of neutrino events is conservative because of possible contributions from interactions of tau neutrinos, which create muons with a branching ratio of 17.7% [10]. With this consideration, the total number of muon-like neutrino events can be estimated as $N_\mu^{\rm like} = N_\mu + 17.7\% \times \frac{0.955}{0.992} N_\mu$, and we find values of 0.61, 0.86 and 12.7 from GB6 J1040 + 0617, TXS 0506 + 056 in the 2017 flare, and PKS 0502 + 049 in the 2014–2015 flare, respectively, for a cosmic-ray spectral index $\alpha_p = -2.1$. We found that the nearby flaring blazar PKS 0502 + 049 can effectively contribute 13 neutrino events (depending on the cosmic-ray spectral slope) to the neutrino flare reported by IceCube over 158 days during its first active phase (MJD 56860–56960). The second active phase of PKS 0502 + 049 during MJD 57010–57120 is assumed to be leptonic in origin, as suggested by Sahakyan [60], and consequently no neutrino emission is expected during this stage. We find that the maximum energy $[E_p = \delta E'_p/(1+z)]$ of cosmic-ray particles achievable in the blazars TXS 0506 + 056, PKS 0502 + 049, and GB6 J1040+0617 is 4.5×10^{17}, 4.1×10^{17}, and 3.5×10^{17} eV, respectively, in the observer frame, and these value are required to consistently explain the observed gamma-ray and neutrino signals from these sources. The model fitting parameters that match the EM SED, as well as the muon-neutrino events from each of the blazars considered here, are shown in Table I.

VI. CONCLUSION

The coincident detections of the neutrino event, IceCube-170922A, 13 muon-neutrino events observed in 2014–2015, and IceCube-141209A by the IceCube Observatory with the gamma-ray flaring blazars TXS 0506 + 056, PKS 0502 + 049 and GB6 J1040 + 0617, respectively, provide support to the interpretation that the cosmic rays in the blazar jet underwent a diffusive shock acceleration process [8]. The photomeson reaction ($p\gamma$) is often used to explain the emissions blazars; however, low neutrino rates or a complex geometry of the blazar jet are often required. More importantly, the lepto-hadronic model cannot reproduce the neutrino flaring events from the direction of TXS 0506 + 056/PKS 0502 + 049 during the period 2014–2015. On the other hand, the cloud-jet interaction scenario seems unlikely to be a common scenario for neutrino production in all IceCube blazars due to the absence of the broadline emission in the optical spectra of the sources.

In the framework of the proton blazar model, our findings suggest that both the low- and high-energy bumps of the multiwavelength EM SED as well as the observed neutrino events from the corresponding blazars can be consistently explained with the relative contributions to the total jet power of cold protons, accelerated protons, accelerated electrons, and the magnetic field obtained based on charge neutrality. The (present) model-predicted flux is in minor tension with the observed x-ray data, which was found in earlier $p\gamma$ and hadronuclear models as well. Such a minor discrepancy may be due to the absorption of x-rays (mainly by the hydrogen column) while traveling to the Solar System from the source. The neutrino flare observed during 2014–2015 from the direction of TXS 0506 + 056 can not be described by any model considering TXS 0506 + 056 as the source. It has been found that PKS 0502 + 049 which is nearby to TXS 0506 + 056 may be responsible for the stated flare [25].

Our findings suggest that the maximum achievable energy of the cosmic-ray particles in the blazars is nearly 1 order smaller than the ankle energy of the cosmic-ray energy spectrum in the observer frame and is required to consistently explain the observed gamma-ray and neutrino signals from the IceCube sources. Upcoming gamma-ray experiments like CTA [63] and LHAASO [64], which are very sensitive up to 100 TeV energies, may provide clearer evidence of the physical origin of gamma rays and the maximum achievable energy of cosmic rays in AGN jets if more such events are detected from other blazars.

ACKNOWLEDGMENTS

The authors would like to thank an anonymous reviewer for insightful comments and very useful suggestions that helped us to improve and correct the manuscript. M. P. thanks the DST-INSPIRE fellowship grant (DST/INSPIRE/FELLOWSHIP/IF160004) from the DST, Govt. of India. A. B. acknowledges financial support from SERB (DST), Govt. of India vide approval number CRG/2019/004944.

[1] P. L. Biermann and P. A. Strittmatter, Astrophys. J. **322**, 643 (1987).
[2] C. M. Urry and P. Padovani, Publ. Astron. Soc. Pac. **107**, 803 (1995).
[3] G. Ghisellini, L. Maraschi, and A. Treves, Astron. Astrophys. **146**, 204 (1985), http://articles.adsabs.harvard.edu/full/1985A%26A...146..204G.
[4] G. Ghisellini and F. Tavecchio, Mon. Not. R. Astron. Soc. **397**, 985 (2009).
[5] M. Sikora, M. C. Begelman, and M. J. Rees, Astrophys. J. **421**, 153 (1994).
[6] J. Albert, E. Aliu, H. Anderhub *et al.*, Astrophys. J. **669**, 862 (2007).
[7] F. Aharonian, A. G. Akhperjanian, A. R. Bazer-Bachi *et al.*, Astrophys. J. **664**, L71 (2007).
[8] The IceCube Collaboration, Science **361**, eaat1378 (2018).
[9] IceCube Collaboration, Science **361**, 147 (2018).
[10] S. Ansoldi *et al.*, Astrophys. J. **863**, L10 (2018).
[11] Y. T. Tanaka, S. Buson, and D. Kocevski, The Astronomers Telegram **10791**, 1 (2017), https://ui.adsabs.harvard.edu/abs/2017ATel10791....1T/abstract
[12] A. Keivani *et al.*, Astrophys. J. **864**, 84 (2018).
[13] R. Mirzoyan, Astronomers Telegram **10817**, 1 (2017), https://ui.adsabs.harvard.edu/abs/2017ATel10817....1M/abstract.
[14] P. Padovani, P. Giommi, E. Resconi, T. Glauch, B. Arsioli, N. Sahakyan, and M. Huber, Mon. Not. R. Astron. Soc. **480**, 192 (2018).
[15] H. He, Y. Inoue, S. Inoue, and Y. Liang, arXiv:1808.04330v2.
[16] S. Garrappa *et al.*, Astrophys. J. **880**, 103 (2019).
[17] P. Banik and A. Bhadra, Phy. Rev. D **99**, 103006 (2019).
[18] S. Gao, A. Fedynitch, W. Winter, and M. Pohl, Nat. Astron. **3**, 88 (2019).
[19] M. Cerruti, A. Zech, C. Boisson, G. Emery, S. Inoue, and J.-P. Lenain, Mon. Not. R. Astron. Soc. **483**, L12 (2019).
[20] R. Liu, K. Wang, R. Xue, A. M. Taylor, X.-Y. Wang, Z. Li, and H. Yan, Phys. Rev. D **99**, 063008 (2019).
[21] N. Sahakyan, Astrophys. J. **866**, 109 (2018).
[22] K. Murase, F. Oikonomou, and M. Petropoulou, Astrophys. J. **865**, 124 (2018).
[23] A. Celotti and A. C. Fabian, Mon. Not. R. Astron. Soc. **264**, 228 (1993).
[24] G. Ghisellini, F. Tavecchio, L. Foschini, G. Ghirlanda, L. Maraschi, and A. Celotti, Mon. Not. R. Astron. Soc. **402**, 497 (2010).
[25] X. Rodrigues, S. Gao, A. Fedynitch, A. Palladino, and W. Winter, Astrophys. J. Lett. **874**, L29 (2019).
[26] Y. Liang, H. He, N. Liao, Y. Xin, Q. Yuan, and Y. Fan, arXiv:1807.05057v2.
[27] A. Mücke and R. J. Protheroe, Astropart. Phys. **15**, 121 (2001).
[28] D. Caprioli, A.-R. Pop, and A. Spitkovsky, Astrophys. J. **798**, L28 (2015).

[29] M. Petropoulou and A. Mastichiadis, Mon. Not. R. Astron. Soc. **447**, 36 (2015).
[30] K. Katarzyński, H.Sol, and A. Kus, Astron. Astrophys. **367**, 809 (2001).
[31] M. Böttcher, A. Reimer, K. Sweeney, and A. Prakash, Astrophys. J. **768**, 54 (2013).
[32] David Eichler and Eli Waxman, Astrophys. J. **627**, 861 (2005).
[33] A. M. Bykov and P. Mészáros, Astrophys. J. **461**, L37 (1996).
[34] J. Giacalone, D. Burgess, S. J. Schwartz, and D. C. Ellison, Geophys. Res. Lett. **19**, 433 (1992).
[35] F. Vazza, D. Eckert, and M. Brüggen, and B. Huber, Mon. Not. R. Astron. Soc. **451**, 2198 (2015).
[36] G. R. Blumenthal and R. J. Gould, Rev. Mod. Phys. **42**, 237 (1970).
[37] S. Inoue and F. Takahara, Astrophys. J. **463**, 555 (1996).
[38] C. D. Dermer and R. Schlickeiser, Astrophys. J. **575**, 667 (2002).
[39] M. A. Malkov and L. O. Drury, Rep. Prog. Phys. **64**, 429 (2001).
[40] M. Cerruti, A. Zech, C. Boisson, and S. Inoue, Mon. Not. R. Astron. Soc. **448**, 910 (2015).
[41] L. A. Anchordoqui, J.F. Beacom, H. Goldberg, S. Palomares-Ruiz, and T. J. Weiler, Phys. Rev. D **75**, 063001 (2007).
[42] P. Banik, B. Bijay, S. K. Sarkar, and A. Bhadra, Phys. Rev. D **95**, 063014 (2017).
[43] S. R. Kelner, F. A. Aharonian, and V. V. Bugayov, Phys. Rev. D **74**, 034018 (2006).
[44] P. Banik and A. Bhadra, Phys. Rev. D **95**, 123014 (2017).
[45] F. A. Aharonian, D. Khangulyan, and L. Costamante, Mon. Not. R. Astron. Soc. **387**, 1206 (2008).
[46] A. M. Atoyan and C. D. Dermer, Astrophys. J. **586**, 79 (2003).
[47] A. Franceschini, G. Rodighiero, and M. Vaccari, Astron. Astrophys. **487**, 837 (2008).
[48] http://www.astro.unipd.it/background/.
[49] D. Majumdar and A. Ghosal, Phys. Rev. D **75**, 113004 (2007).
[50] M. Pandey, D. Majumdar, and A. D. Banik, Phys. Rev. D **97**, 103015 (2018).
[51] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
[52] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
[53] A. Albert *et al.*, Mon. Not. R. Astron. Soc. **482**, 184 (2019).
[54] F. Halzen, Nat. Phys. **13**, 232 (2017).
[55] M. G. Aartsen, Astrophys. J. **833**, 3 (2016).
[56] A. Maselli, F. Massaro, R. D'Abrusco, G. Cusumano, V. La Parola, A. Segreto, and G. Tosti, Astrophys. Space Sci. **357**, 141 (2015).
[57] C. P. Ahn, R. Alexandroff, C. Allende Prieto *et al.*, Astrophys. J. Suppl. **203**, 21 (2012).
[58] A. Sadowski and R. Narayan, Mon. Not. R. Astron. Soc. **453**, 3213 (2015).
[59] S. Paiano, R. Falomo, A. Treves, and R. Scarpa, Astrophys. J. Lett. **854**, L32 (2018).
[60] N. Sahakyan, Astron. Astrophys. **622**, A144 (2019).
[61] M. J. Drinkwater, R. L. Webster, P. J. Francis, J. J. Condon, S. L. Ellison, D. L. Jauncey, J. Lovell, B. A. Peterson, and A. Savage, Mon. Not. R. Astron. Soc. **284**, 85 (1997).
[62] A. Y. K. N. Oshlack, R. L. Webster, and M. T. Whiting, Astrophys. J. **576**, 81 (2002).
[63] R. A. Ong, Proc. Sci., ICRC2017 (**2017**) 1071.
[64] C. Liu (LHAASO Collaboration), Proc. Sci., ICRC2017 (**2017**) 424.