Tetrahedral A_4 symmetry in anti-SU(5) GUT

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We construct a flavor model in an anti-SU(5) grand unified theory with a tetrahedral symmetry A_4. We choose a basis where $Q_{\rm em}=-\frac{1}{3}$ quarks and charged leptons are already mass eigenstates. This choice is possible from the A_4 symmetry. Then, matter representation $\overline{\mathbf{10}}_{-1}^{\rm matter}$ contains both a quark doublet and a heavy neutrino N, which enables us to use the A_4 symmetry to both $Q_{\rm em}=+\frac{2}{3}$ quark masses and neutrino masses (through the see saw via N). This is made possible because the anti-SU(5) breaking is achieved by the Higgs fields transforming as antisymmetric representations of SU(5), $\overline{\mathbf{10}}_{-1}^{H}\oplus\mathbf{10}_{+1}^{H}$, reducing the rank-5 anti-SU(5) group down to the rank-4 standard model group $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_W\times\mathrm{U}(1)_Y$. For possible mass matrices, the A_4 symmetry predictions on mass matrices at field theory level are derived. Finally, an illustration from string compactification is presented.

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I. INTRODUCTION

Recently, we pointed out analytically how the tetrahedral discrete symmetry A_4 results from the permutation symmetry S_4 [1]. The A_4 discrete symmetry [2–11] in connection with the tribimaximal form of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton mixing matrix [12–14] was observed a long time ago. The underlying permutation symmetry is useful in model building and furthermore it can be accommodated to string compactification. In string compactification, chiral fields can arise from fixed points also [15]. The multiplicity N in a fixed point should respect permutation symmetry S_N because the chiral fields at that fixed point are not distinguished. In this paper, we will use a specific grand unified theory (GUT) anti-SU(5) [16,17].

Georgi and Glashow's (GG) GUT SU(5) [18] is an important prototype in the consideration of GUTs. An initial success was attributed to the $b-\tau$ unification [19]. However, there may be two issues against the GG model when one tries to include it in an ultraviolet completed theory. The rank of the GG group is 4 which is identical to that of the Standard Model (SM) gauge group $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_W\times\mathrm{U}(1)_Y$. Therefore, string compactification, an ultraviolet completion of the GG SU(5), needs an adjoint representation for breaking the GG SU(5) down to the SM gauge group without changing the rank. First, in string compactification, it is not possible to obtain an adjoint representation at the level-1 construction [20]. Second, the $Q_{\rm em}=-\frac{1}{3}$ Georgi-Jarlskog quark mass relations [21] need another representation **45** beyond a quintet of Higgs fields. The need for this additional representation makes it difficult for it to be realized in the string compactification. Of course, one may argue that **45** may arise from nonrenormalizable interactions, which needs further fine-tuning.

Therefore, the anti-SU(5) or flipped SU(5) is preferred in string compactification. Barr commented that flipped-SU(5) is a subgroup of SO(10) [16], but here we consider it an independent GUT since string compactification may not go through an intermediate SO(10) which also needs an adjoint representation for spontaneous symmetry breaking to obtain Barr's flipped SU(5). On the other hand, for breaking anti-SU(5), we use a vectorlike representation $\mathbf{10}_{-1}$ and $\overline{\mathbf{10}}_{+1}$ (the subscripts are X charges) which are antisymmetric tensor representations of SU(5) and hence it is called "anti-SU(5)" in [17]. This generalization for spontaneous symmetry breaking by antisymmetric representations in string compactification stops at SU(7) [22].

Since the anti-SU(5) gauge group $\mathrm{SU}(5)\times\mathrm{U}(1)_X$ is rank 5, one can use antisymmetric representations to reduce rank 1 to arrive at the rank-4 SM gauge group via the Higgs fields,

$$\begin{aligned}\overline{\mathbf{10}}^H_{-1} &= \{(\mathbf{3},\mathbf{1})^c_L, (\mathbf{3},\mathbf{2})_L, B^{45}\}_{-1},\\ \mathbf{10}^H_{+1} &= \{(\mathbf{3},\mathbf{1})_L, (\mathbf{3},\mathbf{2})^c_L, B_{45}\}_{+1}.\end{aligned} \tag{1}$$

We use the X definition given in Ref. [15]. The vacuum expectation values (VEVs) of neutral singlets B^{45} and B_{45} (in $\overline{\mathbf{10}}^H_{-1}$ and $\mathbf{10}^H_{+1}$) break the anti-SU(5) down to the SM gauge group. But, there is no $b-\tau$ unification in this anti-SU(5).

One family in the anti-SU(5) in terms of left-handed (L-handed) fields is

$$\overline{\mathbf{10}}_{-1} = \{(d^\alpha)^c_L, Q^\alpha_L, N^c_L\}, \quad \mathbf{5}_{+3} = \{(u^\alpha)^c_L, \ell_L\}, \quad \bar{\mathbf{1}}_{-5} = e^c_L, \tag{2}$$

where

$$Q^\alpha_L = \begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}_L, \qquad \ell_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L. \tag{3}$$

Note that all SU(2) singlets are with superscript ^{c}. So, the singlet neutrino N^c_L is in $\bar{\overline{\mathbf{10}}}_{-1}$ and N_R has $X=+1$. To break the SM gauge group to $U(1)_{\rm em}$, we need a Higgs quintet(s) $\bar{\mathbf{5}}^H_{+2}$ and $\mathbf{5}^H_{-2}$.

The family problem or the flavor problem consists of two parts. First, why are there three families which have exactly the same gauge interactions. Second, why do these families have different Yukawa couplings? In GUTs, the first problem was formulated by Georgi [23] which was applied in extended GUTs [24,25]. In string theory, three family models have been searched in various compactification schemes [26–52]. The second problem is usually talked about in terms of flavor symmetry. The flavor symmetry is designed to calculate the Cabibbo-Kobayashi-Maskawa (CKM) and Pontecorvo-Maki-Nakagawa-Skata (PMNS) matrices. Permutation symmetry S_3 has been started to calculate the CKM matrix [53,54] but permutation symmetries blossomed recently in fitting the PMNS matrix [55].

In Sec. II, we summarize the results of Ref. [1]. In Sec. III, we discuss the A_4 symmetry at field theory level for three families in the anti-SU(5) GUT. We obtain possible forms of mass matrices of quarks and leptons, which are related by the anti-SU(5) representations. In Sec. IV, we present an example for possible quark and lepton mass matrices in a string derived spectra presented in Ref. [15]. Finally, a brief conclusion is given in Sec. V.

II. A_4 FROM S_4

The permutation symmetry S_3 was used in the leptonic sector for a bimaximal PMNS matrix in the late 1990s [57,58], and the A_4 symmetry was started in the early 2000s [3]. The flavor symmetry in the PMNS matrix of a tribimaximal form

$$V \sim \begin{pmatrix} \times & \times & \times \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix} \tag{4}$$

has led to an A_4 symmetry, as shown analytically in [1]. The key points of Ref. [1] are the following:

(i) We choose the bases such that $Q_{\rm em}=-\frac{1}{3}$ quarks and $Q_{\rm em}=-1$ leptons are mass eigenstates.
(ii) ℓ_L in $\mathbf{5}$ of Eq. (2) is a triplet under the tetrahedral group A_4.
(iii) All quark states in Eq. (2) are singlets under A_4.
(iv) The Higgs doublet(s) is a singlet under the permutation symmetry group A_4.

There are four representations in A_4: $\mathbf{3}$, $\mathbf{1}$, $\mathbf{1}'$, and $\mathbf{1}''$. Let us remark first that item i evades the problem encountered in the Georgi-Jarlskog relation. We choose the needed mass values in the definition of the $Q_{\rm em}=-\frac{1}{3}$ quark masses. Item iv requires that the Higgs quintet $\bar{\mathbf{5}}^H_{+2}$ is a tetrahedral group singlet. Then, Items ii and iii dictate the assignment u^c_L in the triplet representation $\mathbf{3}$ of A_4 since both ℓ_L and u^c_L belongs to the same representation $\mathbf{5}_{+3}$.

The tensor product of two $\mathbf{3}$'s of A_4 is [65]

$$\mathbf{3}\otimes\mathbf{3} = 2\cdot\mathbf{3}\oplus\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''. \tag{5}$$

We use the representations where three $Q_{\rm em}=-\frac{1}{3}$ quarks of each chirality form a representation $\mathbf{3}$ of A_4, so do charged leptons. Then, the tensor product Eq. (5) allows three parameters, viz. three singlets, for three $Q_{\rm em}=-\frac{1}{3}$ quark masses and choosing the diagonal basis for $Q_{\rm em}=-\frac{1}{3}$ quarks is guaranteed from A_4. The same applies to charged leptons also.

Note that the charged currents(CCs) in the SM are given by

$$\frac{g}{\sqrt{2}}(\bar{u}^{(0)}_L\gamma^\mu d^{(\rm mass)}_L + \bar{\nu}^{(0)}_L\gamma^\mu e^{(\rm mass)}_L)W^+_\mu + {\rm H.c.} \tag{6}$$

where

$$u^{(0)}_L = \begin{pmatrix} u^{(0)} \\ c^{(0)} \\ t^{(0)} \end{pmatrix}_L, \qquad \nu^{(0)}_L = \begin{pmatrix} \nu^{(0)}_e \\ \nu^{(0)}_\mu \\ \nu^{(0)}_\tau \end{pmatrix}_L. \tag{7}$$

With the anti-SU(5) representations of (2), these CC's are included in

$$g(\bar{\overline{\mathbf{10}}}_{-1}\gamma^\mu T^-_{\overline{10}}\overline{\mathbf{10}}_{-1} + \bar{\mathbf{5}}_{+3}\gamma^\mu T^-_5\mathbf{5}_{+3})W^+_\mu + {\rm H.c.} \tag{8}$$

where

$$T_5^- = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{9}$$

and $T_{\overline{10}}^-$ changes d^α to u^α. Three families are

$$\bar{\mathbf{T}} = (\overline{\mathbf{10}}_{-1}^d, \overline{\mathbf{10}}_{-1}^s, \overline{\mathbf{10}}_{-1}^b), \qquad \mathbf{F} = (\mathbf{5}_{+3}^e, \mathbf{5}_{+3}^\mu, \mathbf{5}_{+3}^\tau) \tag{10}$$

where d, s, b and e, μ, τ are family indices. In terms of mass eigenstates $Q_{\rm em} = +\frac{2}{3}$ quarks (u, c, t) and neutrinos (ν_1, ν_2, ν_3), the weak eigenstates of (7) are related by L-sector unitary matrices U and R-sector unitary matrices $\mathcal{U}$ by

$$\begin{pmatrix} u^{(0)} \\ c^{(0)} \\ t^{(0)} \end{pmatrix}_L = U^{(u)\dagger} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L, \qquad \begin{pmatrix} \nu_e^{(0)} \\ \nu_\mu^{(0)} \\ \nu_\tau^{(0)} \end{pmatrix}_L = U^{(\nu)\dagger} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L,$$
$$\begin{pmatrix} N_e^{(0)} \\ N_\mu^{(0)} \\ N_\tau^{(0)} \end{pmatrix}_R = \mathcal{U}^{(\nu)\dagger} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_R. \tag{11}$$

Now, Eq. (8) reads for three families as

$$g(\bar{\bar{\mathbf{T}}}\gamma^\mu T_{\overline{10}}^- \bar{\mathbf{T}} + \bar{\mathbf{F}}\gamma^\mu T_5^- \mathbf{F})W_\mu^+ + \text{H.c.} \tag{12}$$

The CKM and PMNS matrices are given by

$$V^{(\rm CKM)} = U^{(u)} U^{(d)\dagger} = U^{(u)},$$
$$V^{(\rm PMNS)} = U^{(\nu)} U^{(e)\dagger} = U^{(\nu)}. \tag{13}$$

The definitions of $U^{(u)}$ and $U^{(\nu)}$ in Eq. (13) have the required number of parameters. In $U^{(u)}$, there are just two phases of L-handed $u^{(0)}$ quarks for constraints because the baryon number phase cannot be used as a constraint. Also, three $u^{(0)}$ masses provide three constraints. Thus, out of 9 parameters in a 3×3 unitary matrix, the number of undetermined parameters are 4: 3 real angles and 1 phase. In $U^{(\nu)}$, we do not have any phase constraint because Majorana neutrinos are real. So, we have 9 parameters minus 3 mass parameters, leading to 3 real angles, 1 Dirac phase and 2 Majorana phases.

Let us consider the leptonic part first, which is included in the second term in Eq. (12). Since neutrinos belong to the triplet representation of A_4, $\mathbf{F}$ transforms as $\mathbf{3}$ under A_4. The $Q_{\rm em} = -1$ leptons being chosen as mass eigenstates, there remains to choose $\nu^{(0)}$. Thus, the A_4 symmetric property of $\bar{\mathbf{F}} \otimes \mathbf{F}$ is $\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus 2 \cdot \mathbf{3}$, from which we choose $\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$ for Eq. (12) to be A_4 symmetric. Thus, $\mathbf{F}$ can be chosen as

$$\mathbf{F}^{(0)} \ni a\nu_e^{(0)}, b\nu_\mu^{(0)}, c\nu_\tau^{(0)}, \tag{14}$$

which are matched with charged leptons e, μ, and τ.

In the quark sector, quarks are treated as singlets $\mathbf{1}, \mathbf{1}'$ and $\mathbf{1}''$. So, the first term of Eq. (12) is A_4 symmetric. With these CC couplings, the question to discuss next is how the quark and lepton Yukawa couplings are given.

III. YUKAWA COUPLINGS

To realize A_4 symmetry, we assign the Yukawa couplings such that the flavor indices of i respect the A_4 symmetry requirements. Since the A_4 symmetry was suggested from the PMNS matrix, let us first discuss the L violating neutrino masses. Since $\mathbf{F}$ is complex, it can have a global U(1) phase which is not violated by Eq. (12). The charged lepton in $\mathbf{F}$ obtains mass by the Yukawa coupling to $\bar{\mathbf{1}}_{-5} = e^c$ of Eq. (2), $\bar{\mathbf{1}}_{-5} C^{-1} \mathbf{F}_{+3} \bar{\mathbf{5}}_{+2}^H$. Since e^c, i.e., $\bar{\mathbf{1}}_{-5}$, carries lepton number $\mathrm{L} = -1$, $\mathbf{F}_{+3}$ carries $\mathrm{L} = +1$. But $\mathbf{F}_{+3}$ also contains u^c which is known to carry baryon number $\mathrm{B} = -1$. For consistency, we require no global anomaly. So, $\mathbf{F}_{+3}$ should carry a vanishing global charge which can be (B–L). $\mathbf{F}_{+3}$ couples to $\bar{\mathbf{T}}_{-1}$ by $\bar{\mathbf{T}}_{-1} C^{-1} \mathbf{F}_{+3} \mathbf{5}_{-2}^H$. Since $\mathbf{5}_{-2}^H$ is interpreted carrying no B and L charges, $\bar{\mathbf{T}}_{-1}$ carries $\mathrm{B} = +1$ or $\mathrm{L} = -1$. In particular N_L^c carries $\mathrm{L} = -1$. Namely, N_R carries $\mathrm{L} = +1$. The L violating source at the super-renormalizable level is given by $(m_N/2)N_R^2$. What is the A_4 representation of $\bar{\mathbf{T}}_{-1}$? To write $\frac{m_N}{2}(N_R)^2$, $\bar{\mathbf{T}}_{-1}$ transforms as a singlet(s) or $\mathbf{3}$ of A_4. These L violating heavy neutrino masses are contained in

$$(\bar{\mathbf{T}}_{-1})_i (H)_{ij} (\bar{\mathbf{T}}_{-1})_j, \tag{15}$$

where (H) is the heavy neutrino mass matrix. Since we do not introduce any triplet in the Higgs or fermion sectors, our neutrino mass matrix will be a type 1 see saw. The Dirac neutrino mass is given by

$$\mathbf{F}_{+3i} Y_{ij} \bar{\mathbf{T}}_{-1,j} \mathbf{5}_{-2}^H \tag{16}$$

where (Y) is the Yukawa coupling matrix. In Fig. 1, we show the tree diagram for the type 1 see-saw mechanism. In Fig. 1, the chiralities of ν and N are L and R, respectively. This diagram depends on the A_4 property of N. With these diagrams, we obtain the effective Weinberg operators,

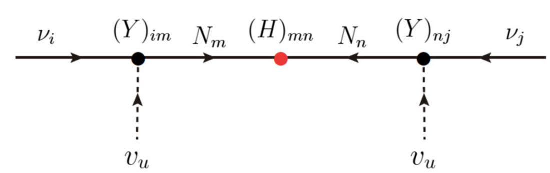

FIG. 1. The type 1 see-saw diagram.

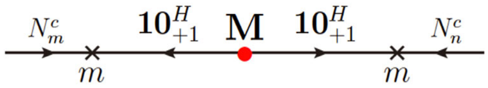

FIG. 2. The diagram for heavy neutrino masses. The fermionic partner of the GUT Higgs $\mathbf{10}_{+1}^H$, i.e., B_{45} of Eq. (1), is called "heavy heavy neutrino."

$$h_{ij}\frac{v_u^2}{m_N}\tilde{\nu}_{iL}^{(0)}C^{-1}\nu_{jL}^{(0)}. \tag{17}$$

We noted above that $\bar{\mathbf{T}}_{-1}$ of Eq. (2) transforms as $\mathbf{3}$ under A_4, so does N_L^c in $\bar{\mathbf{T}}_{-1}$. The Yukawa coupling in (H) of Fig. 1 is a constant because all three N_R's belong to $\mathbf{3}$ of A_4. But, we allow the difference among masses of three N. Thus, (H^{-1}) is just the inverse of the mass matrix of N. But, the mass term of N cannot arise at the renomalizable level. It occurs only through the dimension-5 term, from fields in Eqs. (1) and (2),

$$\sim\bar{\mathbf{T}}_{-1}(\text{fermion})\bar{\mathbf{T}}_{-1}(\text{fermion})\mathbf{10}_{+1}^H(\text{boson})\mathbf{10}_{+1}^H(\text{boson}) \tag{18}$$

where the VEV $\langle\mathbf{10}_{+1}^H(\text{boson})\rangle$is needed to break the anti-SU(5) to the SM gauge group. The gauge invariant super-renormalizable mass term breaking lepton number L is

$$\bar{\mathbf{T}}_{-1}(\text{fermion})\mathbf{m}\mathbf{10}_{+1}^H(\text{fermion}) \tag{19}$$

where $\mathbf{m}$ is a constant (or matrix). This dictates that $\mathbf{10}_{+1}^H$(fermion) of Eq. (19) transforms as $\mathbf{3}$ of A_4. In Fig. 2, we draw a schematic Feynman diagram generating the heavy neutrino masses from the anti-SU$(5)\times A_4$ symmetry.[1] The mass matrix $\mathbf{M}$ transforms, under A_4, as $\mathbf{3}\otimes\mathbf{3}$, $\mathbf{3}\otimes\mathbf{1}$, $\mathbf{1}\otimes\mathbf{3}$, or $\mathbf{1}$'s, where the left factor combines with N_m^c and the right factor combines with N_n^c. For each case, we study the L-violating neutrino masses.

Before discussing each neutrino mass matrix, we present the $Q_{\text{em}}=+\frac{2}{3}$ quark masses from the anti-SU(5) coupling which depends only on the coupling given in Eq. (16). The $Q_{\text{em}}=+\frac{2}{3}$ quark Yukawa couplings are determined from the A_4 tensor product $\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''\oplus 2\cdot\mathbf{3}$. There are three independent singlets, which are three independent Yukawa couplings. Y_{ij} in Eq. (16) are matrix elements. There is only one class for matrices which have Det $=1$ and Tr $=-1$ for entries with ±1,

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{20}$$

[1]Identifying $\mathbf{10}_{+1}^H$'s of Eqs. (18) and (19), we may be led to introduce supersymmetry.

For example, the matrix

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{21}$$

satisfies the required conditions but changing the indices $1\leftrightarrow 2$ gives the form Eq. (20). Similarly, all the other cases can be reduced to the form (20). For Eq. (20), the Yukawa couplings are defined as $Y_{11}=h_1$, $Y_{22}=h_2$, $Y_{33}=h_3$, and all the rest are zeros.

Now let us proceed to discuss each class of $\mathbf{M}$ on neutrino masses.

A. $\mathbf{M}\sim\mathbf{3}\otimes\mathbf{3}$

In this case, three values are the same for the left and right factors. In the matrix form,

$$\mathbf{M}=\begin{pmatrix} M & M & M \\ M & M & M \\ M & M & M \end{pmatrix} \tag{22}$$

which has eigenvalues of $3M$, 0, and 0. The above is a democratic form suggested in Refs. [59,60]. The heavy neutrino mass components are

$$\mathbf{M}_{mn}=\frac{m^2}{M}. \tag{23}$$

In this case Y_{ij} of Eq. (16) is $Y_{ij}=h\delta_{ij}$. Then, the SM neutrinos obtain masses through Fig. 1,

$$m_{ij}=h^2\frac{Mv_u^2}{m^2}. \tag{24}$$

The above universal mass matrix is diagonalized by

$$\begin{pmatrix} \frac{\sqrt{3}-1}{2\sqrt{3}} & \frac{-\sqrt{3}-1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{-\sqrt{3}-1}{2\sqrt{3}} & \frac{\sqrt{3}-1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \tag{25}$$

which is trimaximal.

B. $\mathbf{M}\sim\mathbf{3}\otimes\mathbf{1}$

The left factors give the same value and the right factors give three different values,

$$\mathbf{M}=\begin{pmatrix} M_1 & M_2 & M_3 \\ M_1 & M_2 & M_3 \\ M_1 & M_2 & M_3 \end{pmatrix} \tag{26}$$

which has eigenvalues of $M_1+M_2+M_3$, 0, and 0. All the heavy neutrinos have the same mass,

$$\mathbf{M}_{mn}=\frac{m^2}{M_n}. \tag{27}$$

In this case Y_{ij} of Eq. (16) is $Y_{ij}=h\delta_{ij}$ at the LHS vertex and h_j at the RHS vertex. Thus, the SM neutrinos obtain masses through Fig. 1 as

$$m_{ij}=hh_j\frac{M_j v_u^2}{m^2}, \tag{28}$$

which is proportional to

$$m\propto\begin{pmatrix} h_1 & h_2 & h_3\\ h_1 & h_2 & h_3\\ h_1 & h_2 & h_3\end{pmatrix} \tag{29}$$

whose eigenvalues are 0, 0, and $h_1+h_2+h_3$. Three column vectors of m^T with eigenvalues 0, 0, and $h_1+h_2+h_3$ are

$$\psi_1\sim\begin{pmatrix} h_2\\ -h_1\\ 0\end{pmatrix},\quad \psi_2\sim\begin{pmatrix} h_1\\ h_2\\ \frac{-h_1^2-h_2^2}{h_3}\end{pmatrix},\quad \psi_3\sim\begin{pmatrix} h_1\\ h_2\\ h_3\end{pmatrix}. \tag{30}$$

Note that Eq. (29) has a freedom to choose the scale. We fix such that the unitarity matrix results. The unitarity matrix diagonalizing m^T is

$$U=\frac{1}{\sqrt{h^2(1+h^2)}}\begin{pmatrix} h_2\sqrt{1+h^2} & h_1 & h_1 h\\ -h_1\sqrt{1+h^2} & h_2 & h_2 h\\ 0 & -h^2 & h\end{pmatrix}, \tag{31}$$

where we choose

$$h_3=1,\qquad h^2\equiv h_1^2+h_2^2. \tag{32}$$

Then, the diagonalized states and matrix are expressed in terms of the original ones as

$$\psi^{(\rm diag)}=U\psi_0,\qquad m^{(\rm diag)}=UmU^\dagger. \tag{33}$$

C. M ∼ 1 ⊗ 3

The left factors give three different values and the right factors give the same values,

$$\mathbf{M}=\begin{pmatrix} M_1 & M_1 & M_1\\ M_2 & M_2 & M_2\\ M_3 & M_3 & M_3\end{pmatrix} \tag{34}$$

which has eigenvalues of $3M$, 0, and 0. All the heavy neutrinos have the same mass,

$$\mathbf{M}_{mn}=\frac{m^2}{M_m}. \tag{35}$$

In this case Y_{ij} of Eq. (16) is $Y_{ij}=h_i$ at the LHS vertex and $h\delta_{ij}$ at the RHS vertex. Thus, the SM neutrinos obtain masses through Fig. 1 as

$$m_{ij}=hh_i\frac{M_i v_u^2}{m^2}. \tag{36}$$

As in case **B**, we obtain the following

$$U=\frac{1}{\sqrt{h^2(1+h^2)}}\begin{pmatrix} h_2\sqrt{1+h^2} & -h_1\sqrt{1+h^2} & 0\\ h_1 & h_2 & -h^2\\ h_1 h & h_2 h & h\end{pmatrix}, \tag{37}$$

where

$$\psi^{(\rm diag)}=U\psi_0,\qquad m^{(\rm diag)}=UmU^\dagger. \tag{38}$$

D. M ∼ 1's

In this case, both the left and right factors give three different values,

$$\mathbf{M}=\begin{pmatrix} M_{11} & M_{12} & M_{13}\\ M_{21} & M_{22} & M_{23}\\ M_{31} & M_{32} & M_{33}\end{pmatrix} \tag{39}$$

which in general gives three different nonzero eigenvalues. All the heavy neutrinos have the same mass,

$$\mathbf{M}_{ij}=\frac{m^2}{M_{ij}}. \tag{40}$$

In this case Y_{ij} of Eq. (16) is $Y_{ij}=h_i$ at the LHS vertex and h_j at the RHS vertex. Thus, the SM neutrinos obtain masses through Fig. 1 as

$$m_{ij}=h_i h_j\frac{M_{ij}v_u^2}{m^2}, \tag{41}$$

which is general enough to obtain any unitarity matrix U.

E. Allowed matrices for M from effective neutrino masses

In the above subsections, the heavy heavy neutrno mass matrix **M** of Fig. 2, leading to the heavy neutrino masses of N in Eq. (2), were given. On the other hand, the effective neutrino mass operator of Weinberg [61],

$$\sim \ell_i^T C^{-1} \ell_j, \tag{42}$$

is symmetric on the exchange $i \leftrightarrow j$. But, cases **B** and **C** allow asymmetric neutrino masses. Therefore, the heavy heavy neutrno mass matrix **M** can take only cases **A** and **D**. Since case **D** is not very much predictive at this stage, we present the A_4 from the anti-SU(5) prediction given in Eq. (25),

$$\begin{pmatrix} \frac{-\sqrt{3}-1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\sqrt{3}-1}{2\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\sqrt{3}-1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-\sqrt{3}-1}{2\sqrt{3}} \end{pmatrix} \tag{43}$$

where $|(U^\dagger)_{13}| = \frac{\sqrt{3}-1}{2\sqrt{3}} \simeq 0.211$. The best fit [55] gives 0.147 and the 3σ range is 0.138–0.156. Therefore, case **A** is ruled out. Only case **D**, which is general enough, is a viable mass pattern of the heavy neutrinos.

F. The CKM matrix

For case **D**, let us consider the CKM matrix. In Ref. [1], we argued that the CKM matrix is close to the identity because of the huge ratio of m_t/m_c. So, the mass matrix is of the form

$$\sim \begin{pmatrix} a\varepsilon^2 & b\varepsilon^{\frac{3}{2}} & c\varepsilon \\ d\varepsilon^{\frac{3}{2}} & e\varepsilon & f\varepsilon^{\frac{1}{2}} \\ g\varepsilon & h\varepsilon^{\frac{1}{2}} & 1 \end{pmatrix} \tag{44}$$

where ε is $\mathrm{O}(\frac{m_c}{m_t}) \approx 0.007$. The determinant of the above matrix is $D = (ae - bd + bfg + cdh - afh - ceg)\varepsilon^3$. We choose $e \simeq 1$ such that the trace is almost $m_t + m_c$. $D \simeq (a - bd - cg + bfg + cdh - afh)\varepsilon^3$, and hence $m_u \simeq m_c(m_c/m_t)^2(a - bd - cg + bfg + cdh - afh) \simeq 2.5$ MeV leading to $(a - bd - cg + bfg + cdh - afh) \simeq 43$. Since we follow case **D**, all these coefficients $a \sim g$ are arbitrary. Let us take a real symmetric matrix, choosing simple numbers just for an illustration,

$$a = -32.1615, \quad e = 1,$$
$$b = d = c = g = (43)^{1/3}, \quad f = h = 1.42, \tag{45}$$

where a is chosen to satisfy $(a - bd - cg + bfg + cdh - afh) \simeq 43$. In this case, the mass matrix is

$$M \sim \begin{pmatrix} -0.00157596, & 0.00205181, & 0.0245238 \\ 0.00205181, & 0.007, & 0.118806 \\ 0.0245238, & 0.118806, & 1 \end{pmatrix}. \tag{46}$$

Then, eigenvalues of M are

$$-0.00203125, \quad -0.00715645, \quad 1.01461, \tag{47}$$

where the first term can be corrected more by higher dimensional operators. Here, $m_c/m_t \simeq 0.007$, and the diagonalizing matrix, $UMU^\dagger = (\text{diagonal})$, is [62]

$$V^{(\mathrm{CKM})} = U^{(u)} = \begin{pmatrix} 0.985959 & -0.166933 & -0.00433802 \\ -0.165227 & -0.978988 & 0.119506 \\ 0.0241964 & 0.117111 & 0.992824 \end{pmatrix} \tag{48}$$

which gives the Cabibbo angle $|\theta_C| \simeq 9.61°$, roughly 3.4° smaller than the needed one. Note however that we neglected the CP phase δ and other higher dimensional contributions. Most importantly, it is for a specific set of parameters in Eq. (45). In general, the mass matrix is complex which can be diagonalized by bi-unitary matrices, by U and $\mathcal{U}$. In sum, we tried to show there can be a reasonable set of parameters fitting all the flavor data for case **D**.

IV. STRING COMPACTIFICATION

To discuss flavor symmetry from string compactification, one needs a compactification model where details of the SM field assignment are presented. In doing so, the key SM phenomenologies are automatically included, i.e., it is not ruled out from any well-established data. Here, we show a realization of A_4 symmetry based on an anti-SU(5) GUT [15] possessing the $\mathbf{Z}_{4R}$ discrete parity which is obtained from the $\mathbf{Z}_{12-I}$ compactification of the $\mathrm{E}_8 \times \mathrm{E}_8'$ heterotic string [63]. Anyway, for a detailed study of flavor physics, one has to specify every aspect of the flavors for which we do not find any reference except Ref. [15]. So, we show an example of A_4 symmetry based on an anti-SU(5) GUT of [15] based on the model [64]. Here, we just cite the needed information from Refs. [15,64]. In string compactification, the needed Yukawa couplings arise by satisfying all the selection criteria. The anti-SU(5) GUT of [15] does not allow any SM Yukawa couplings at the renormalizable level. But, at the level of dimension-5 there appear the SM Yukawa couplings which are proportional to the VEVs of $\langle \mathbf{10}_{+1}^H \rangle = \langle \overline{\mathbf{10}}_{-1}^H \rangle$. Since these VEVs are near the string scale, we obtain top quark mass at the order the electroweak scale. Since we are not attempting to discuss details of

TABLE I. Phases Θ_i of matter fields in the SM. U and T are twisted sectors. In T_4^0, there are two $\overline{\mathbf{10}}_{-1}$'s.

	State$(P+kV_0)$	Θ_i	$\mathbf{R}_X$(Sect.)
ξ_3	$(\underline{+++--};--+)(0^8)'$	0	$\overline{\mathbf{10}}_{-1}(U_3)$
$\bar{\eta}_3$	$(\underline{+----};+--)(0^8)'$	0	$\mathbf{5}_{+3}(U_3)$
τ^c	$(+++++;-+-)(0^8)'$	0	$\mathbf{1}_{-5}(U_3)$
ξ_2	$(\underline{+++--};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\overline{\mathbf{10}}_{-1}(T_4^0)$
$\bar{\eta}_2$	$(\underline{+----};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{5}_{+3}(T_4^0)$
μ^c	$(+++++;-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{1}_{-5}(T_4^0)$
ξ_1	$(\underline{+++--};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\overline{\mathbf{10}}_{-1}(T_4^0)$
$\bar{\eta}_1$	$(\underline{+----};-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{5}_{+3}(T_4^0)$
e^c	$(+++++;-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})(0^8)'$	$\frac{+1}{4}$	$\mathbf{1}_{-5}(T_4^0)$
H_{uL}	$(\underline{+1\,0\,0\,0\,0};0\,0\,0)(0^5;\frac{-1}{2}\frac{+1}{2}0)'$	$\frac{+1}{3}$	$2\cdot\mathbf{5}_{-2}(T_6)$
H_{dL}	$(\underline{-1\,0\,0\,0\,0};0\,0\,0)(0^5;\frac{+1}{2}\frac{-1}{2}0)'$	$\frac{+1}{3}$	$2\cdot\bar{\mathbf{5}}_{+2}(T_6)$

models in string compactification, we only pay attention to the multiplicities of the needed chiral fields.

First consider $\mathbf{10}^H_{+1}$ and $\overline{\mathbf{10}}^H_{-1}$ needed for breaking anti-SU(5). In the T_3 twisted sector, chiral fields are constructed in Eqs. (23) and (24) of Ref. [15],

$$\begin{array}{ccc} s & \text{Multiplicity} & P\cdot V \\ (\oplus\,|-+-): & 2, & \frac{+1}{4}\,(\Sigma_1^*) \\ (\ominus|---) & 1, & \frac{-1}{4}\,(\Sigma_2) \end{array} \tag{49}$$

$$\begin{array}{ccc} s & \text{Multiplicity} & P\cdot V \\ (\oplus\,|-+-): & 1, & \frac{+1}{4}\,(\Sigma_1^*) \\ (\ominus|---): & 2, & \frac{-1}{4}\,(\Sigma_2) \end{array} \tag{50}$$

where $\oplus$ and $\ominus$ denote L-handed and R-handed chiral fields respectively. So, here we consider only the number of chiral fields at the same fixed points. We cited only chirality and multiplicity. Θ_i in Table I and $P\cdot V$ in Eqs. (49) and (50) are used to calculate the multiplicity. From Eqs. (49) and (50), note that there appear three L-handed fields Σ_2, and three R-handed fields Σ_1^*. These chiral fields at the same fixed points are not distinguished. Thus, in the L-handed fields Σ_2 has the representation **3** of A_4, so do the R-handed fields have Σ_1^*. Σ_2 is the one for $\mathbf{10}^H_{+1}$ of Eq. (19). But $\bar{\mathbf{T}}_{-1}$ of Eq. (19) belongs to the matter fields in Table I of [15]. Two matter $\overline{\mathbf{10}}_{-1}$'s appear in T_4^0, viz. Table I. But it is better to check all $\overline{\mathbf{10}}_{-1}$'s before removing vectorlike representations, for which we go back to Ref. [64]. In fact, there was no vectorlike representations of $\overline{\mathbf{10}}_{-1}\oplus\mathbf{10}_{+1}$'s removed in Ref. [64]. So, from our string model, $\overline{\mathbf{10}}_{-1}$ is a doublet **2** of permutation symmetry S_3. We do not realize the coupling of Eq. (19).

TABLE II. Branching of S_4 representations $\mathbf{1},\mathbf{1}',\mathbf{2},\mathbf{3}$ and $\mathbf{3}'$ into the A_4 and S_3 representations [65].

S_4	A_4	S_3
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}'$
$\mathbf{2}$	$\mathbf{1}'\oplus\mathbf{1}''$	$\mathbf{2}$
$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}\oplus\mathbf{2}$
$\mathbf{3}'$	$\mathbf{3}$	$\mathbf{1}'\oplus\mathbf{2}$

From Table II, we note that the doublet representation **2** of the permutation group S_3 can be obtained from **3** of S_4. Also, **3** of A_4 is from **3** of S_4.

In Eq. (19), $\bar{\mathbf{T}}_{-1}$ transforms as **2** under the permutation group S_3 and $\mathbf{10}^H_{+1}$ transforms as **3** of A_4. Note that **2** and **3** of S_4 produce **2** of S_3. Out of two **1**'s of S_4, we restrict to only **1**. Let us consider the relevant tensor products of S_4,

$$\text{Tensor products in } S_4 \begin{cases} \mathbf{3}\times\mathbf{3}=\mathbf{1}\oplus\mathbf{2}\oplus\mathbf{3}\oplus\mathbf{3}' \\ \mathbf{3}\times\mathbf{3}'=\mathbf{1}'\oplus\mathbf{2}\oplus\mathbf{3}\oplus\mathbf{3}' \\ \mathbf{3}'\times\mathbf{3}'=\mathbf{1}\oplus\mathbf{2}\oplus\mathbf{3}\oplus\mathbf{3}' \\ \mathbf{2}\otimes\mathbf{3}=\mathbf{3}\oplus\mathbf{3}' \end{cases} \tag{51}$$

where the last line does not produce a singlet. The other three lines produce S_4 singlets and we consider the first line, $\mathbf{3}\otimes\mathbf{3}$. The other cases can be equivalent to this by redefining the origin of **3** of A_4. Then, $\bar{\mathbf{T}}_{-1}$ and $\mathbf{10}^H_{+1}$ can be traced back to **3** of S_4.

$$\begin{aligned} R(S_3)\otimes R(A_4)\colon\ & (\mathbf{1},\mathbf{2})\otimes\mathbf{3} \\ & \to(\mathbf{1}\otimes\mathbf{3})\oplus(\mathbf{2}\otimes\mathbf{3}) \\ & \to\mathbf{1}\otimes\mathbf{3}\oplus\mathbf{1}'\otimes\mathbf{3}\oplus\mathbf{1}''\otimes\mathbf{3} \end{aligned} \tag{52}$$

where the first line is the fourth line of Table II, written as S_3 and A_4 subgroups. In the second line, $\mathbf{1}\otimes\mathbf{3}$ is the S_4 product and can be interpreted as the A_4 triplet. In the second line, $\mathbf{2}\otimes\mathbf{3}$ is the S_4 product from the third and fourth lines of Table II. In terms of A_4, it produces $\mathbf{1}'\otimes\mathbf{3}\oplus\mathbf{1}''\otimes\mathbf{3}$, i.e., two independent **3**'s. In total, there are three independent **3**'s. Therefore, **M** of Fig. 2 is

$$\mathbf{M}=\begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{21} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} \tag{53}$$

which is case **D** of Sec. III, which is allowed from the neutrino mass data.

So far we paid attention to the heavy neutrino mass in $\bar{\mathbf{T}}_{-1}$. Now, let us check how this representation containing a quark doublet predicts on the Yukawa couplings through Eq. (16) with an R-handed $Q_{\rm em}=\frac{2}{3}$ quark in $\mathbf{F}_{+3}$. Both $\bar{\mathbf{T}}_{-1}$ and $\mathbf{F}_{+3}$ are doublets under S_3. It belongs to the third row of Table II. The S_4 tensor product is $\mathbf{2}\otimes\mathbf{2}=\mathbf{2}\oplus\mathbf{1}\oplus\mathbf{1}'$ which becomes $2 \cdot \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$ under A_4. Thus, there are three independent couplings[2] which can be of the form in the 2×2 subspace (due to doublets in the T_4^0 twisted sector),

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & a & b \\ 0 & c & a \end{pmatrix}, \tag{54}$$

which is general enough to allow the mixing between top and charm quarks. With higher dimensional operators [15], the 0 entries will be supplied with small numbers and may fulfill the needed 3×3 matrix for the $Q_{\rm em} = \frac{2}{3}$ quark matrix.

The above illustration from a compactification model was intended to show a possibility. To study the flavor problem from string compactification, one needs an explicit model locating all the SM fields in the sectors of the compactification as shown in this section.

V. CONCLUSION

We constructed quark and lepton mass matrices in an anti-SU(5) GUT with a tetrahedral symmetry A_4. In the previous paper [1], we showed the hint of the A_4 from the PMNS matrix form with one entry being zero. In this paper, for a convenience of presentation we chose a basis where $Q_{\rm em} = -\frac{1}{3}$ quarks and charged leptons are already diagonalized. Then, matter representation $\bar{\mathbf{T}}_{-1}$ contains both a quark doublet and a heavy neutrino N. For $Q_{\rm em} = +\frac{2}{3}$ quark masses $\bar{\mathbf{T}}_{-1}$ coupling to $\mathbf{F}_{+3}$ is used, and for neutrino masses the Weinberg operator of $(\mathbf{F}_{+3})^2$ is used through the see saw of $\bar{\mathbf{T}}_{-1}$. In this sense, the quark and neutrino masses are related by the symmetry A_4. One notable feature is the anti-SU(5) breaking achieved by the Higgs fields transforming as antisymmetric representations of SU(5), $\overline{\mathbf{10}}^H_{-1} \oplus \mathbf{10}^H_{+1}$. This set reduces the rank-5 anti-SU(5) group down to the rank-4 standard model group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$. Finally, a string compactification example is presented. As illustrated in this example, the definite assignments of the SM fields in the twisted sectors are needed to compare with the CKM and PMNS data.

[2]Two **1**'s are counted as the same entry.

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[1] P. H. Frampton and J. E. Kim, *Anticorrelation of Mass and Mixing Angle Hierarchies* (to be published); See also, J. E. Kim, Flavor physics and Jarlskog determinant, arXiv:2002.12519.
[2] E. Ma and G. Rajasekaran, Softly broken A_4 symmetry for nearly degenerate neutrino masses, Phys. Rev. D **64**, 113012 (2001).
[3] K. S. Babu, E. Ma, and J. W. F. Valle, Underlying A_4 symmetry for the neutrino mass matrix and the quark mixing matrix, Phys. Lett. B **552**, 207 (2003).
[4] E. Ma, A_4 symmetry and neutrinos with very different masses, Phys. Rev. D **70**, 031901 (2004).
[5] G. Altarelli and F. Feruglio, Tri-bimaximal neutrino mixing, A_4 and the modular symmetry, Nucl. Phys. **B741**, 215 (2006).
[6] C. S. Lam, Symmetry of lepton mixing, Phys. Lett. B **656**, 193 (2007).
[7] S. Morisi, M. Picariello, and E. Torrente-Lujan, Model for fermion masses and lepton mixing in $\mathrm{SO}(10) \times A_4$, Phys. Rev. D **75**, 075015 (2007).
[8] S. Morisi, M. Picariello, and E. Torrente-Lujan, Fermion masses and mixing in models with $\mathrm{SO}(10) \times A_4$ symmetry, Phys. Rev. D **78**, 116018 (2008).
[9] A. Blum, C. Hagedorn, and M. Lindner, Fermion masses and mixings from dihedral flavor symmetries with preserved subgroups, Phys. Rev. D **77**, 076004 (2008).
[10] G. Altarelli and D. Meloni, A simplest A_4 model for tri-bimaximal neutrino mixing, J. Phys. G **36**, 085005 (2009).
[11] Y. BenTov and A. Zee, Neutrino mixing and the double tetrahedral group, Int. J. Mod. Phys. A **28**, 1350157 (2013).
[12] B. Pontecorvo, Mesonium and anti-mesonium, Zh. Eksp. Teor. Fiz. **33**, 549 (1957) [Phys. JETP **6**, 429 (1957)].
[13] Z. Maki, M. Nakagawa, and S. Sakata, Remarks on the unified model of elementary particles, Prog. Theor. Phys. **28**, 870 (1962).
[14] B. Pontecorvo, Neutrino experiments and the problem of conservation of leptonic charge, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967) [Phys. JETP **26**, 984 (1968)].
[15] J. E. Kim, *R*-parity from string compactification, Phys. Rev. D **99**, 093004 (2019).
[16] S. M. Barr, A new symmetry breaking pattern for SO(10) and proton decay, Phys. Lett. B **112**, 219 (1982).
[17] J.-P. Derendinger, J. E. Kim, and D. V. Nanopoulos, anti-SU(5), Phys. Lett. B **139**, 170 (1984).
[18] H. Georgi and S. L. Glashow, Unity of All Elementary Particle Forces, Phys. Rev. Lett. **32**, 438 (1974).
[19] A. J. Buras, J. R. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Aspects of the grand unification of strong, weak and electromagnetic interactions, Nucl. Phys. **B135**, 66 (1978).
[20] K.-S. Choi and J. E. Kim, *Quarks and Leptons from Orbifolded Superstring*, Lecture Notes in Physics Vol. 696 (Springer-Verlag, Berlin, 2006).

[21] H. Georgi and C. Jarlskog, A new lepton-quark mass relation in a unified theory, Phys. Lett. B **86**, 297 (1979).
[22] J. E. Kim, Towards unity of families: Anti-SU(7) from $\mathbf{Z}_{12-I}$ orbifold compactification, J. High Energy Phys. 06 (2015) 114.
[23] H. Georgi, Towards a grand unified theory of flavor, Nucl. Phys. **B156**, 126 (1979).
[24] J. E. Kim, A Model of Flavor Unity, Phys. Rev. Lett. **45**, 1916 (1980).
[25] P. H. Frampton, Unification of flavor, Phys. Lett. B **89**, 352 (1980).
[26] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Vacuum configurations for superstrings, Nucl. Phys. **B258**, 46 (1985).
[27] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Strings on orbifolds. 2., Nucl. Phys. **B274**, 285 (1986).
[28] L. E. Ibanez, H. P. Nilles, and F. Quevedo, Orbifolds and Wilson lines, Phys. Lett. B **187**, 25 (1987).
[29] H. Kawai, D. C. Lewellen, and S. H. H. Tye, Construction of fermionic string models in four dimensions, Nucl. Phys. **B288**, 1 (1987).
[30] I. Antoniadis, C. P. Bachas, and C. Kounnas, Four-dimensional superstrings, Nucl. Phys. **B289**, 87 (1987).
[31] D. Gepner, Space-time supersymmetry in compactified string theory and superconformal models, Nucl. Phys. **B296**, 757 (1988).
[32] L. E. Ibanez, J. E. Kim, H. P. Nilles, and F. Quevedo, Orbifold compactifications with three families of $\mathrm{SU}(3)\times \mathrm{SU}(2)\times \mathrm{U}(1)^n$, Phys. Lett. B **191**, 282 (1987).
[33] C. Casas and C. Munoz, Three generation $\mathrm{SU}(3)\times\mathrm{SU}(2)\times \mathrm{U}(1)_Y$ models from orbifolds, Phys. Lett. B **214**, 63 (1988).
[34] S. Chaudhuri, G. Hockney, and J. D. Lykken, Three generations in the fermionic construction, Nucl. Phys. **B469**, 357 (1996).
[35] W. Pokorski and G. G. Ross, Flat directions, string compactification and three generation models, Nucl. Phys. **B551**, 515 (1999).
[36] G. B. Cleaver, A. E. Faraggi, and D. V. Nanopoulos, String derived MSSM and M theory unification, Phys. Lett. B **455**, 135 (1999).
[37] G. B. Cleaver, A. E. Faraggi, and D. V. Nanopoulos, A minimal superstring standard model I: Flat directions, Int. J. Mod. Phys. A **16**, 425 (2001).
[38] G. B. Cleaver, A. E. Faraggi, D. V. Nanopoulos, and J. W. Walker, Phenomenological study of a minimal superstring standard model, Nucl. Phys. **B593**, 471 (2001).
[39] R. Donagi, B. A. Ovrut, T. Pantev, and D. Waldram, Spectral involutions on rational elliptic surfaces, Adv. Theor. Math. Phys. **5**, 93 (2001).
[40] T. Kobayashi, S. Raby, and R-J. Zhang, Searching for realistic 4d string models with a Pati-Salam symmetry: Orbifold grand unified theories from heterotic string compactification on a $\mathbf{Z}_6$ orbifold, Nucl. Phys. **B704**, 3 (2005).
[41] R. Donagi, Y.-H. He, B. A. Ovrut, and T. Pantev, A heterotic standard model, Phys. Lett. B **618**, 259 (2005).
[42] R. Donagi, Y.-H. He, B. A. Ovrut, and R. Reinbacher, The spectra of heterotic standard model vacua, J. High Energy Phys. 06 (2005) 070.
[43] V. Bouchard and R. Donagi, An SU(5) heterotic standard model, Phys. Lett. B **633**, 783 (2006).
[44] V. Braun, Y.-H. He, B. A. Ovrut, and T. Pantev, The exact MSSM spectrum from string theory, J. High Energy Phys. 05 (2006) 043.
[45] R. Blumenhagen, S. Moster, and T. Weigand, Heterotic GUT and standard model vacua from simply connected Calabi-Yau manifolds, Nucl. Phys. **B751**, 186 (2006).
[46] V. Bouchard, M. Cvetic, and R. Donagi, Tri-linear couplings in an heterotic minimal supersymmetric standard model, Nucl. Phys. **B745**, 62 (2006).
[47] R. Blumenhagen, S. Moster, R. Reinbacher, and T. Weigand, Massless spectra of three generation U(N) heterotic string vacua, J. High Energy Phys. 05 (2007) 041.
[48] J. E. Kim, J.-H. Kim, and B. Kyae, Superstring standard model from Z(12-I) orbifold compactification with and without exotics, and effective *R*-parity, J. High Energy Phys. 06 (2007) 034.
[49] A. E. Faraggi, C. Kounas, and J. Rizos, Chiral family classification of fermionic $\mathbf{Z}_2\times\mathbf{Z}_2$ heterotic orbifold models, Phys. Lett. B **648**, 84 (2007).
[50] G. B. Cleaver, In search of the (minimal supersymmetric) standard model string, arXiv:hep-ph/0703027.
[51] C. Munoz, A kind of prediction from string phenomenology: Extra matter at low energy, Mod. Phys. Lett. A **22**, 989 (2007).
[52] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange, and A. Wingerter, Heterotic road to the MSSM with *R* parity, Phys. Rev. D **77**, 046013 (2008).
[53] S. Pakvasa and H. Sugawara, Discrete symmetry and cabibbo angle, Phys. Lett. B **73**, 61 (1978).
[54] G. Segrè and H. A. Weldon, Natural Suppression of Strong P and T Violations and Calculable Mixing Angles in $\mathrm{SU}(2)\times\mathrm{U}(1)$, Phys. Rev. Lett. **42**, 1191 (1979).
[55] K. Nakamura and S. Petcov (Particle Data Group), *Neutrino Masses, Mixing, and Oscillations*, Ref. [56], Sec. 14.
[56] M. Tanabashi *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **98**, 030001 (2018).
[57] P. F. Harrison, D. H. Perkins, and W. G. Scott, Threefold maximal lepton mixing and the solar and atmospheric neutrino deficits, Phys. Lett. B **349**, 137 (1995).
[58] K. Kang, S. K. Kang, J. E. Kim, and P. Ko, Almost maximally broken permutation symmetry for neutrino mass matrix, Mod. Phys. Lett. A **12**, 1175 (1997).
[59] H. Fritzsch and Z. Xing, Lepton mass hierarchy and neutrino oscillations, Phys. Lett. B **372**, 265 (1996).
[60] H. Fritzsch and Z. Xing, Democratic neutrino mixing reexamined, Phys. Lett. B **598**, 237 (2004).
[61] S. Weinberg, Baryon and Lepton Nonconserving Processes, Phys. Rev. Lett. **43**, 1566 (1979).
[62] A more elaborate matrix will be given in J. E. Kim and S-J. Kim, On the progenitor quark mass matrix (to be published).
[63] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, The Heterotic String, Phys. Rev. Lett. **54**, 502 (1985).
[64] J. H. Huh, J. E. Kim, and B. Kyae, $\mathrm{SU}(5)_{\mathrm{flip}}\times\mathrm{SU}(5)'$ from $\mathbf{Z}_{12-I}$, Phys. Rev. D **80**, 115012 (2009).
[65] H. Ishimori *et al.*, *An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists*, Lecture Notes in Physics Vol. 858 (Springer-Verlag, Berlin, 2012).