CKM matrix and FCNC suppression in $SO(5) \times U(1) \times SU(3)$ gauge-Higgs unification

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The Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix and flavor-changing neutral currents (FCNCs) in the quark sector are examined in the grand unified theory inspired $SO(5) \times U(1) \times SU(3)$ gauge-Higgs unification in which the 4D Higgs boson is identified with the Aharonov-Bohm phase in the fifth dimension. Gauge invariant brane interactions play an important role for the flavor mixing in the charged-current weak interactions. The CKM matrix is reproduced except that the up quark mass needs to be larger than the observed one. FCNCs are naturally suppressed as a consequence of the gauge invariance, with a factor of order 10^{-6}. It is also shown that induced flavor-changing Yukawa couplings are extremely small.

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I. INTRODUCTION

The standard model (SM), $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory, has been firmly established at low energies. Yet it is not clear what the observed Higgs boson is. All of the Higgs couplings to other fields and to itself need to be determined with better accuracy in the coming experiments. The fundamental problem is the lack of a principle which regulates the Higgs interactions.

One possible answer is the gauge-Higgs unification in which the Higgs boson is identified with the zero mode of the fifth dimensional component of the gauge potential [1–6]. It appears as a fluctuation mode of the Aharonov-Bohm (AB) phase θ_H in the fifth dimension. As a concrete model, the $SU(3)_C \times SO(5) \times U(1)_X$ gauge theory in the Randall-Sundrum (RS) warped space has been proposed [7–10]. It gives nearly the same phenomenology at low energies as the standard model [10–12]. Deviations of the gauge couplings of quarks and leptons from the SM values are less than 10^{-3} for $\theta_H \sim 0.1$. Higgs couplings of quarks, leptons, W and Z are approximately the SM values times $\cos\theta_H$, the deviation being about 1%. The Kaluza-Klein (KK) mass scale is about $m_{\rm KK} \sim 8$ TeV for $\theta_H \sim 0.1$. The KK excited states contribute, for instance, in intermediate states of the two γ decay of the Higgs boson. Their contribution is finite and very small. The signal strengths of various Higgs decay modes are approximately $\cos^2\theta_H$ times the SM values. The branching fractions of those decay modes are approximately the same as in the SM.

Gauge-Higgs unification predicts Z' bosons, which are the first KK modes of γ, Z, and Z_R [$SU(2)_R$ gauge boson]. Their masses are in the 6–9 TeV range for $\theta_H = 0.11$–0.07 in the model with quark-lepton multiplets introduced in the vector representation of $SO(5)$, which will be referred to as the A model below. Those Z' bosons have broad widths and can be produced at 14 TeV LHC. The current nonobservation of Z' signals puts the limit $\theta_H \lesssim 0.11$. Recently an alternative model with quark-lepton multiplets introduced in the spinor, vector, and singlet representations of $SO(5)$ (referred to as the B model below) has been proposed [13], which can be incorporated in the $SO(11)$ gauge-Higgs grand unification [14,15], Other variants of the fermion content have also been proposed [16]. Implications of gauge-Higgs unification to precision electroweak observables have been investigated. It has been shown that the typical models are consistent with the current measurements.

Distinct signals of the gauge-Higgs unification can be found in e^+e^- collisions [17–20]. Large parity violation appears in the couplings of quarks and leptons to KK gauge bosons, particularly to the Z' bosons. In the A model right-handed quarks and charged leptons have rather large couplings to Z'. The interference effects of Z' bosons can be clearly observed at 250 GeV e^+e^- collisions at the International Linear Collider (ILC). In the process $e^+e^- \to \mu^+\mu^-$ the deviation from the SM amounts to -4% with the electron beam polarized in the right-handed mode by 80% ($P_{e^-} = 0.8$) for $\theta_H \sim 0.09$, whereas there appears negligible deviation with the electron beam polarized in the left-handed mode by 80% ($P_{e^-} = -0.8$). In the forward-backward asymmetry $A_{FB}(\mu^+\mu^-)$ the deviation from the SM becomes -2% for $P_{e^-} = 0.8$. These deviations can be seen at 250 GeV ILC even with 250 fb^{-1} data [21,22]. In the B model the pattern of the polarization dependence is reversed.

So far quarks and leptons in the gauge-Higgs unification models have been incorporated generation by generation so that the flavor mixing among quarks and leptons is left unexplained. In this paper we tackle the flavor mixing in the quark sector [23,24]. We will argue in the B model that the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix in the charged current interaction is reproduced with brane interactions. These brane interactions generally lead to flavor-changing neutral current (FCNC) interactions. It will be shown that the FCNC interactions are naturally suppressed in the gauge-Higgs unification as a consequence of the gauge invariance. The FCNC interaction is suppressed by a factor of $(m_b/m_{\rm KK})^2 \sim 10^{-6}$ where m_b and $m_{\rm KK}$ are the bottom quark mass and the KK mass scale. It is also shown that induced flavor-changing Yukawa interactions are extremely small.

We stress that the natural suppression of FCNC in the gauge-Higgs unification results from the gauge invariance and the orbifold structure, without relying on additional symmetry or mechanisms. We present a rigorous treatment of deriving and evaluating the CKM matrix and Z couplings in the quark sector in the gauge-Higgs unification. We also give a simple explanation in the effective theory of quarks and relevant heavy fields to illuminate the mechanism of suppressing FCNC interactions.

In Sec. II the minimal grand unified theory (GUT) inspired $SU(3)_C \times SO(5) \times U(1)_X$ model of gauge-Higgs unification is described with brane interactions. In Sec. III mass spectra and wave functions of gauge bosons and quarks are derived. Detailed derivation of the mass spectrum and mixing in the down-type quark sector is given. In Sec. IV an effective theory in 4D is formulated for quarks and $SO(5)$ singlet heavy fermion fields. We show how mass terms connecting down quarks and singlet fields lead to flavor mixing. It also illuminates how FCNC interactions are naturally suppressed. In Sec. V we evaluate W and Z couplings of quarks, using the wave functions obtained in Sec. III. The gauge couplings turn out very close to those in the SM. It is confirmed that FCNC interactions are naturally suppressed. Section VI is devoted to a summary and discussions. Basis functions used in the text are summarized in the Appendix.

II. MODEL

The GUT inspired $SU(3)_C \times SO(5) \times U(1)_X (\equiv \mathcal{G})$ gauge-Higgs unification has been introduced in Ref. [13]. It is defined in the Randall-Sundrum (RS) warped space with metric given by

$$ds^2 = g_{MN} dx^M dx^N = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \qquad (2.1)$$

where $M, N = 0, 1, 2, 3, 5$, $\mu, \nu = 0, 1, 2, 3$, $y = x^5$, $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$, $\sigma(y) = \sigma(y + 2L) = \sigma(-y)$, and $\sigma(y) = ky$ for $0 \le y \le L$. In terms of the conformal coordinate $z = e^{ky}$ $(1 \le z \le z_L = e^{kL})$ in the region $0 \le y \le L$

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu + \frac{dz^2}{k^2}\right). \qquad (2.2)$$

The bulk region $0 < y < L$ $(1 < z < z_L)$ is anti-de Sitter (AdS) spacetime with a cosmological constant $\Lambda = -6k^2$, which is sandwiched by the UV brane at $y = 0$ $(z = 1)$ and the IR brane at $y = L$ $(z = z_L)$. The KK mass scale is $m_{\rm KK} = \pi k/(z_L - 1) \simeq \pi k z_L^{-1}$ for $z_L \gg 1$.

Let us denote gauge fields of $SU(3)_C$, $SO(5)$, and $U(1)_X$ by $A_M^{SU(3)_C}$, $A_M^{SO(5)}$, and $A_M^{U(1)_X}$, respectively. The orbifold boundary conditions (BC) are given by

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x, y_j + y) P_j^{-1} \qquad (2.3)$$

for each gauge field where $(y_0, y_1) = (0, L)$. In terms of

$$\begin{aligned} P_{\mathbf{3}}^{SU(3)} &= I_3, \\ P_{\mathbf{4}}^{SO(5)} &= \mathrm{diag}(I_2, -I_2), \\ P_{\mathbf{5}}^{SO(5)} &= \mathrm{diag}(I_4, -I_1), \end{aligned} \qquad (2.4)$$

$P_0 = P_1 = P_{\mathbf{3}}^{SU(3)}$ for $A_M^{SU(3)_C}$ and $P_0 = P_1 = 1$ for $A_M^{U(1)_X}$. $P_0 = P_1 = P_{\mathbf{5}}^{SO(5)}$ for $A_M^{SO(5)}$ in the vector representation and $P_{\mathbf{4}}^{SO(5)}$ in the spinor representation, respectively. $P_{\mathbf{4}}^{SO(5)}$ and $P_{\mathbf{5}}^{SO(5)}$ break $SO(5)$ to $SO(4) \simeq SU(2)_L \times SU(2)_R$. W, Z bosons and γ (photon) are zero modes in the $SO(4)$ part of $A_\mu^{SO(5)}$, whereas the 4D Higgs boson is a zero mode in the $SO(5)/SO(4)$ part of $A_y^{SO(5)}$.

Matter fields are introduced both in 5D bulk and on the UV brane. They are listed in Table I. Quark multiplets $(\mathbf{3}, \mathbf{4})_{\frac{1}{6}}$ and $(\mathbf{3}, \mathbf{1})^{\pm}_{-\frac{1}{3}}$ are introduced in the 5D bulk in three generations. They are denoted as $\Psi^\alpha_{(\mathbf{3},\mathbf{4})}(x, y)$ and $\Psi^{\pm\alpha}_{(\mathbf{3},\mathbf{1})}(x, y)$ $(\alpha = 1, 2, 3)$. $\Psi^\alpha_{(\mathbf{3},\mathbf{4})}$ and $\Psi^{\pm\alpha}_{(\mathbf{3},\mathbf{1})}$ intertwine with each other. These fields obey boundary conditions

TABLE I. $\mathcal{G} = SU(3)_C \times SO(5) \times U(1)_X$ content of matter fields is shown in the GUT inspired model (B model) and previous model (A model). In the A model only $SU(3)_C \times SO(4) \times U(1)_X$ symmetry is preserved on the UV brane so that the $SU(2)_L \times SU(2)_R$ content is shown for brane fields. The B model is analyzed in the present paper.

	B model	A model
Quark	$(\mathbf{3},\mathbf{4})_{\frac{1}{6}}(\mathbf{3},\mathbf{1})^{+}_{-\frac{1}{3}}(\mathbf{3},\mathbf{1})^{-}_{-\frac{1}{3}}$	$(\mathbf{3},\mathbf{5})_{\frac{2}{3}}(\mathbf{3},\mathbf{5})_{-\frac{1}{3}}$
Lepton	$(\mathbf{1},\mathbf{4})_{-\frac{1}{2}}$	$(\mathbf{1},\mathbf{5})_{0}(\mathbf{1},\mathbf{5})_{-1}$
Dark fermion	$(\mathbf{3},\mathbf{4})_{\frac{1}{6}}(\mathbf{1},\mathbf{5})^{+}_{0}(\mathbf{1},\mathbf{5})^{-}_{0}$	$(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$
Brane fermion	$(\mathbf{1},\mathbf{1})_{0}$	$(\mathbf{3},[\mathbf{2},\mathbf{1}])_{\frac{7}{6},\frac{1}{6},-\frac{5}{6}}$ $(\mathbf{1},[\mathbf{2},\mathbf{1}])_{\frac{1}{2},-\frac{1}{2},-\frac{3}{2}}$
Brane scalar	$(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$	$(\mathbf{1},[\mathbf{1},\mathbf{2}])_{\frac{1}{2}}$
Symmetry of brane interactions	$SU(3)_C \times SO(5) \times U(1)_X$	$SU(3)_C \times SO(4) \times U(1)_X$

$$
\begin{aligned}
\Psi^{\alpha}_{(\mathbf{3},\mathbf{4})}(x, y_j - y) &= -P^{SO(5)}_{\mathbf{4}} \gamma^5 \Psi^{\alpha}_{(\mathbf{3},\mathbf{4})}(x, y_j + y), \\
\Psi^{\pm\alpha}_{(\mathbf{3},\mathbf{1})}(x, y_j - y) &= \mp \gamma^5 \Psi^{\pm\alpha}_{(\mathbf{3},\mathbf{1})}(x, y_j + y).
\end{aligned} \tag{2.5}
$$

With (2.5) the parity of quark fields are summarized in Table II with names adopted in the present paper.

The action of each gauge field, $A^{SU(3)_C}_M$, $A^{SO(5)}_M$, or $A^{U(1)_X}_M$, is given by

$$
S^{\rm gauge}_{\rm bulk} = \int d^5x \sqrt{-\det G} \times \left[-{\rm tr}\left(\frac{1}{4} F^{MN} F_{MN} + \frac{1}{2\xi} (f_{\rm gf})^2 + \mathcal{L}_{\rm gh} \right) \right], \tag{2.6}
$$

where $\sqrt{-\det G} = 1/kz^5$. Field strengths are defined by $F_{MN} = \partial_M A_N - \partial_N A_M - ig[A_M, A_N]$ with each 5D gauge coupling constant g. The gauge fixing and ghost terms are taken as

$$
\begin{aligned}
f_{\rm gf} &= z^2 \left\{ \eta^{\mu\nu} \mathcal{D}^{\rm c}_{\mu} A^{\rm q}_{\nu} + \xi k^2 z \mathcal{D}^{\rm c}_{z} \left(\frac{1}{z} A^{\rm q}_{z} \right) \right\}, \\
\mathcal{L}_{\rm gh} &= \bar{c} \left\{ \eta^{\mu\nu} \mathcal{D}^{\rm c}_{\mu} \mathcal{D}_{\nu} + \xi k^2 z \mathcal{D}^{\rm c}_{z} \frac{1}{z} \mathcal{D}_{z} \right\} c,
\end{aligned} \tag{2.7}
$$

TABLE II. Parity assignment (P_0, P_1) of quark multiplets in the bulk. In the third column $G_{22} = SU(2)_L \times SU(2)_R$ content is shown. Brane scalar field $\Phi_{(\mathbf{1},\mathbf{4})}$ is also listed at the bottom for convenience.

Field	$\mathcal{G}$	G_{22}	Left-handed	Right-handed	Name
$\Psi^{\alpha}_{(\mathbf{3},\mathbf{4})}$	$(\mathbf{3},\mathbf{4})_{\frac{1}{6}}$	$[\mathbf{2},\mathbf{1}]$	$(+,+)$	$(-,-)$	$u \;\; c \;\; t$ $d \;\; s \;\; b$
		$[\mathbf{1},\mathbf{2}]$	$(-,-)$	$(+,+)$	$u' \;\; c' \;\; t'$ $d' \;\; s' \;\; b'$
$\Psi^{\pm\alpha}_{(\mathbf{3},\mathbf{1})}$	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$[\mathbf{1},\mathbf{1}]$	$(\pm,\pm)$	$(\mp,\mp)$	$D^{\pm}_{d} D^{\pm}_{s} D^{\pm}_{b}$
$\Phi_{(\mathbf{1},\mathbf{4})}$	$(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$	$[\mathbf{2},\mathbf{1}]$	$\cdots$	$\cdots$	$\Phi_{[\mathbf{2},\mathbf{1}]}$
		$[\mathbf{1},\mathbf{2}]$	$\cdots$	$\cdots$	$\Phi_{[\mathbf{1},\mathbf{2}]}$

where $A_M = A^c_M + A^q_M$. $\mathcal{D}^c_M B = \partial_M B - ig[A^c_M, B]$ and $\mathcal{D}^{c+q}_M B = \partial_M B - ig[A_M, B]$ where $B = A^q_\mu$, A^q_z/z, and c. Only A_z component of $A^{SO(5)}_M$ has nonvanishing classical background A^c_z.

Each fermion multiplet $\Psi(x, y)$ in the bulk has its own bulk-mass parameter c [25] The covariant derivative is given by

$$
\begin{aligned}
\mathcal{D}(c) &= \gamma^A {e_A}^M \left(D_M + \frac{1}{8} \omega_{MBC} [\gamma^B, \gamma^C] \right) - c\sigma'(y), \\
D_M &= \partial_M - ig_S A^{SU(3)}_M - ig_A A^{SO(5)}_M - ig_B Q_X A^{U(1)}_M.
\end{aligned} \tag{2.8}
$$

Here $\sigma' = d\sigma(y)/dy$ and $\sigma'(y) = k$ for $0 < y < L$. g_S, g_A, g_B are $SU(3)_C$, $SO(5)$, $U(1)_X$ gauge coupling constants. The bulk part of the action for the quark multiplets are given by

$$
\begin{aligned}
S^{\rm quark}_{\rm bulk} = \int d^5x \sqrt{-\det G} \sum_{\alpha=1}^{3} \{ & \bar{\Psi}^{\alpha}_{(\mathbf{3},\mathbf{4})} \mathcal{D}(c_\alpha) \Psi^{\alpha}_{(\mathbf{3},\mathbf{4})} \\
& + \bar{\Psi}^{+\alpha}_{(\mathbf{3},\mathbf{1})} \mathcal{D}(c_{D^+_\alpha}) \Psi^{+\alpha}_{(\mathbf{3},\mathbf{1})} + \bar{\Psi}^{-\alpha}_{(\mathbf{3},\mathbf{1})} \mathcal{D}(c_{D^-_\alpha}) \Psi^{-\alpha}_{(\mathbf{3},\mathbf{1})} \\
& - m_{D_\alpha} (\bar{\Psi}^{+\alpha}_{(\mathbf{3},\mathbf{1})} \Psi^{-\alpha}_{(\mathbf{3},\mathbf{1})} + \bar{\Psi}^{-\alpha}_{(\mathbf{3},\mathbf{1})} \Psi^{+\alpha}_{(\mathbf{3},\mathbf{1})}) \},
\end{aligned} \tag{2.9}
$$

where $\bar{\Psi} = i\Psi^\dagger \gamma^0$. The bulk mass parameters of the $SO(5)$ spinor multiplets are denoted as $(c_1, c_2, c_3) = (c_u, c_c, c_t)$ below as each c_α is determined from the mass of each up-type quark. For the $SO(5)$ singlet multiplets we consider the case $c_{D^+_\alpha} = c_{D^-_\alpha} \equiv c_{D_\alpha}$ in the present paper. (An alternative choice $c_{D^+_\alpha} = -c_{D^-_\alpha}$ is also possible. See Ref. [13].)

The action for the brane scalar field $\Phi_{(\mathbf{1},\mathbf{4})}(x)$ is given by

$$
\begin{aligned}
S^{\Phi}_{\rm brane} &= \int d^5x \sqrt{-\det G}\, \delta(y) \{ -(D_\mu \Phi_{(\mathbf{1},\mathbf{4})})^\dagger D^\mu \Phi_{(\mathbf{1},\mathbf{4})} \\
& \quad - \lambda_{\Phi_{(\mathbf{1},\mathbf{4})}} (\Phi^\dagger_{(\mathbf{1},\mathbf{4})} \Phi_{(\mathbf{1},\mathbf{4})} - |w|^2)^2 \}, \\
D_\mu \Phi_{(\mathbf{1},\mathbf{4})} &= \left\{ \partial_\mu - ig_A \sum_{\alpha=1}^{10} A^{\alpha}_{\mu} T^{\alpha} - ig_B Q_X B_\mu \right\} \Phi_{(\mathbf{1},\mathbf{4})}.
\end{aligned} \tag{2.10}
$$

A spinor **4** of $SO(5)$ is decomposed to $[\mathbf{2},\mathbf{1}] \oplus [\mathbf{1},\mathbf{2}]$ of $SO(4) \simeq SU(2)_L \times SU(2)_R$. $\Phi_{(\mathbf{1},\mathbf{4})}$ develops a nonvanishing vacuum expectation value (VEV);

$$\Phi_{(\mathbf{1},\mathbf{4})} = \begin{pmatrix} \Phi_{[\mathbf{2},\mathbf{1}]} \\ \Phi_{[\mathbf{1},\mathbf{2}]} \end{pmatrix}, \qquad \langle \Phi_{[\mathbf{1},\mathbf{2}]} \rangle = \begin{pmatrix} 0 \\ w \end{pmatrix}, \tag{2.11}$$

which reduces the symmetry $\mathcal{G}' = SU(3)_C \times SO(4) \times U(1)_X$ to the SM gauge group $G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$. It is assumed that $w \gg m_{\rm KK}$, which ensures that boundary conditions for the 4D components of gauge fields corresponding to broken generators in the breaking $SU(2)_R \times U(1)_X \to U(1)_Y$ obey effectively Dirichlet conditions at the UV brane for low-lying KK modes [15]. Accordingly the mass of the neutral physical mode of $\Phi_{(\mathbf{1},\mathbf{4})}$ is much larger than $m_{\rm KK}$.

There are brane interactions on the UV brane which are invariant under $\mathcal{G} = SU(3)_C \times SO(5) \times U(1)_X$.

$$S^{\rm int}_{\rm brane} = -\int d^5x \sqrt{-\det G}\,\delta(y) \times \left\{ \sum_{\alpha,\beta} \kappa_{\alpha\beta} \bar{\Psi}^{\alpha}_{(\mathbf{3},\mathbf{4})} \Phi_{(\mathbf{1},\mathbf{4})} \Psi^{+\beta}_{(\mathbf{3},\mathbf{1})} + {\rm H.c.} \right\}, \tag{2.12}$$

where $\kappa_{\alpha\beta}$'s are coupling constants. If only the gauge invariance under $\mathcal{G}'$ were imposed, there would appear additional brane interactions. Instead of (2.12) one would have

$$\sum_{\alpha,\beta} \left\{ \kappa^{(1)}_{\alpha\beta} \bar{\Psi}^{\alpha}_{(\mathbf{3},[\mathbf{2},\mathbf{1}])} \Phi_{(\mathbf{1},[\mathbf{2},\mathbf{1}])} \Psi^{+\beta}_{(\mathbf{3},\mathbf{1})} + \kappa^{(2)}_{\alpha\beta} \bar{\Psi}^{\alpha}_{(\mathbf{3},[\mathbf{1},\mathbf{2}])} \Phi_{(\mathbf{1},[\mathbf{1},\mathbf{2}])} \Psi^{+\beta}_{(\mathbf{3},\mathbf{1})} \right\} + {\rm H.c.} \tag{2.13}$$

in the Lagrangian density. The invariance under $\mathcal{G}$ implies $\kappa^{(1)}_{\alpha\beta} = \kappa^{(2)}_{\alpha\beta}$. For fermion fields we define $\check{\Psi} = z^{-2}\Psi$. With nonvanishing VEV $\langle \Phi_{(\mathbf{1},\mathbf{4})} \rangle \neq 0$, (2.12) generates mass terms

$$S^{\rm fermion}_{\rm brane\ mass} = \int d^5x \sqrt{-\det G}\,\delta(y) \times \left\{ \sum_{\alpha,\beta} 2\mu_{\alpha\beta} \bar{\check{d}}'^{\alpha}_R \check{D}^{+\beta}_L + {\rm H.c.} \right\}, \tag{2.14}$$

where $2\mu_{\alpha\beta} = \sqrt{2}\kappa_{\alpha\beta} w$, $(d'^1, d'^2, d'^3) = (d', s', b')$ and $(D^{+1}, D^{+2}, D^{+3}) = (D^+_d, D^+_s, D^+_b)$. Only the $\kappa^{(2)}_{\alpha\beta}$ part in the decomposition (2.13) gives rise to mass terms. Brane interaction of the form $\bar{\Psi}^{\alpha}_{(\mathbf{3},\mathbf{4})} \Phi_{(\mathbf{1},\mathbf{4})} \Psi^{-\beta}_{(\mathbf{3},\mathbf{1})}$ is possible, which, however, does not yield a mass term as $D^{-\beta}_L|_{y=0} = 0$ due to the BC. It will be shown below that the brane interactions (2.12) lead to the flavor mixing, yielding the CKM matrix in the charged current interactions. We stress that the brane interactions (2.12) respect full $\mathcal{G} = SU(3)_C \times SO(5) \times U(1)_X$ gauge invariance. It may be contrasted to the earlier attempts [23,24] to incorporate flavor mixing in higher dimensional theories where only $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance is respected. We note the same mass terms are generated from (2.13) so that the results obtained below remain valid even with only the $\mathcal{G}'$ invariance imposed on the brane so long as $|\kappa^{(1)}_{\alpha\beta}/\kappa^{(2)}_{\alpha\beta}|$ is not extremely large.

Nonvanishing VEV $\langle \Phi_{(\mathbf{1},\mathbf{4})} \rangle$ also breaks $SU(2)_R \times U(1)_X$ to $U(1)_Y$. $U(1)_Y$ gauge field B^Y_M is given in terms of $SU(2)_R$ gauge fields $A^{a_R}_M$ and $U(1)_X$ gauge field B_M by

$$B^Y_M = s_\phi A^{3_R}_M + c_\phi B_M, \quad c_\phi = \frac{g_A}{\sqrt{g_A^2 + g_B^2}}, \qquad s_\phi = \frac{g_B}{\sqrt{g_A^2 + g_B^2}}, \tag{2.15}$$

where g_A and g_B are gauge couplings in $SO(5)$ and $U(1)_X$, respectively. The 5D $U(1)_Y$ gauge coupling is given by $g^{\rm 5D}_Y = g_A s_\phi$. The 4D $SU(2)_L$ gauge coupling is given by $g_w = g_A/\sqrt{L}$.

The 4D Higgs boson doublet $\phi_H(x)$ is the zero mode contained in the $A_z = (kz)^{-1} A_y$ component;

$$A^{(j5)}_z(x,z) = \frac{1}{\sqrt{k}} \phi_j(x) u_H(z) + \cdots, \quad u_H(z) = \sqrt{\frac{2}{z_L^2 - 1}}\, z, \quad \phi_H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix}. \tag{2.16}$$

Without loss of generality we assume $\langle \phi_1 \rangle, \langle \phi_2 \rangle, \langle \phi_3 \rangle = 0$ and $\langle \phi_4 \rangle \neq 0$, which is related to the AB phase θ_H in the fifth dimension by $\langle \phi_4 \rangle = \theta_H f_H$ where

$$f_H = \frac{2}{g_w} \sqrt{\frac{k}{L(z_L^2 - 1)}}. \tag{2.17}$$

III. MASS SPECTRUM AND WAVE FUNCTIONS

Manipulations are simplified in the twisted gauge [26,27] defined by an $SO(5)$ gauge transformation

$$\tilde{A}_M(x,z) = \Omega A_M \Omega^{-1} + \frac{i}{g_A} \Omega \partial_M \Omega^{-1}, \quad \Omega(z) = \exp\{ i\theta(z) T^{(45)} \}, \quad \theta(z) = \theta_H \frac{z_L^2 - z^2}{z_L^2 - 1}, \tag{3.1}$$

where T^{jk}'s are $SO(5)$ generators and $A_M = 2^{-1/2} \sum_{1\le j<k\le 5} A^{(jk)}_M T^{jk}$. In the twisted gauge the background field vanishes ($\tilde{\theta}_H = 0$) so that all fields satisfy free equations in the RS space in the bulk. Boundary conditions at the UV brane are modified, whereas boundary conditions at the IR brane remain the same as in the original gauge.

A. Gauge fields

The masses of W and Z bosons at the tree level, $m_W = k\lambda_W$ and $m_Z = k\lambda_Z$, are determined by

$$2S(1;\lambda_W)C'(1;\lambda_W) + \lambda_W \sin^2\theta_H = 0,$$
$$2S(1;\lambda_Z)C'(1;\lambda_Z) + (1+s_\phi^2)\lambda_Z \sin^2\theta_H = 0, \tag{3.2}$$

where functions $S(z;\lambda)$ and $C(z;\lambda)$ are given in (A2) and s_ϕ is defined in (2.15). The masses are approximately given by

$$m_W \simeq \sqrt{\frac{k}{L}} z_L^{-1} \sin\theta_H \simeq \frac{\sin\theta_H}{\pi\sqrt{kL}} m_{\rm KK},$$
$$m_Z \simeq \sqrt{1+s_\phi^2}\, m_W. \tag{3.3}$$

s_ϕ is related to the Weinberg angle at the tree level by $\sin\theta_W^0 = s_\phi/\sqrt{1+s_\phi^2}$.

Let us define

$$A_M^{a_L} = \frac{1}{\sqrt{2}}\left(\frac{1}{2}\varepsilon^{abc}A_M^{(bc)} + A_M^{(a4)}\right),$$
$$A_M^{a_R} = \frac{1}{\sqrt{2}}\left(\frac{1}{2}\varepsilon^{abc}A_M^{(bc)} - A_M^{(a4)}\right),$$
$$A_M^{\hat{p}} = A_M^{(p5)}, \tag{3.4}$$

where $a, b, c = 1-3$ and $p = 1-4$. $A_M^{a_L}$ and $A_M^{a_R}$ are gauge fields of $SU(2)_L$ and $SU(2)_R$. For W and Z bosons and photon γ we define

$$\begin{bmatrix} \mathring{W}_\mu(x,z) \\ \mathring{W}_\mu^S(x,z) \end{bmatrix} = \sqrt{k} W_\mu(x) \frac{1}{\sqrt{r_W}} \begin{bmatrix} C(z,\lambda_W) \\ \hat{S}(z,\lambda_W) \end{bmatrix},$$
$$\begin{bmatrix} \mathring{Z}_\mu(x,z) \\ \mathring{Z}_\mu^S(x,z) \end{bmatrix} = \sqrt{k} Z_\mu(x) \frac{1}{\sqrt{r_Z}} \begin{bmatrix} C(z,\lambda_Z) \\ \hat{S}(z,\lambda_Z) \end{bmatrix},$$
$$\mathring{A}_\mu^\gamma = \sqrt{k} A_\mu^\gamma(x) \frac{1}{\sqrt{kL}},$$
$$\hat{S}(z,\lambda) = \frac{C(1,\lambda)}{S(1,\lambda)} S(z,\lambda), \tag{3.5}$$

where

$$r_W = \int_1^{z_L} \frac{dz}{z}\{(1+c_H^2)C(z,\lambda_W)^2 + s_H^2 \hat{S}(z,\lambda_W)^2\},$$
$$r_Z = \int_1^{z_L} \frac{dz}{z}\{[c_\phi^2 + (1+s_\phi^2)c_H^2]C(z,\lambda_Z)^2 + (1+s_\phi^2)s_H^2\hat{S}(z,\lambda_Z)^2\},$$
$$c_H = \cos\theta_H, \qquad s_H = \sin\theta_H. \tag{3.6}$$

Here $W_\mu(x)$, $Z_\mu(x)$, and $A_\mu^\gamma(x)$ represent canonically normalized W, Z, and γ fields, respectively. Note that $\lambda_W z_L, \lambda_Z z_L \ll 1$. For $\lambda z_L \ll 1$, $C(z,\lambda) \sim z_L$ and $\hat{S}(z.\lambda) \sim z_L(1 - z^2/z_L^2)$.

Couplings of W, Z, and γ are obtained by inserting

$$\begin{bmatrix} \tilde{A}_\mu^{1_L} - i\tilde{A}_\mu^{2_L} \\ \tilde{A}_\mu^{1_R} - i\tilde{A}_\mu^{2_R} \\ \tilde{A}_\mu^{\hat{1}} - i\tilde{A}_\mu^{\hat{2}} \end{bmatrix} = \begin{bmatrix} (1+c_H)\mathring{W}_\mu \\ (1-c_H)\mathring{W}_\mu \\ -\sqrt{2}s_H\mathring{W}_\mu^S \end{bmatrix},$$
$$\begin{bmatrix} \tilde{A}_\mu^{3_L} \\ \tilde{A}_\mu^{3_R} \\ \tilde{A}_\mu^{\hat{3}} \\ B_\mu \end{bmatrix} = \frac{\sqrt{1+s_\phi^2}}{\sqrt{2}} \begin{bmatrix} (1+c_H)\mathring{Z}_\mu \\ (1-c_H)\mathring{Z}_\mu \\ -\sqrt{2}s_H\mathring{Z}_\mu^S \\ 0 \end{bmatrix} + \frac{1}{\sqrt{1+s_\phi^2}} \begin{bmatrix} s_\phi \\ s_\phi \\ 0 \\ c_\phi \end{bmatrix} (\mathring{A}_\mu^\gamma - \sqrt{2}s_\phi\mathring{Z}_\mu) \tag{3.7}$$

in the $SO(5)$ gauge fields $\tilde{A}_\mu$ in the twisted gauge and $U(1)_Y$ gauge field B_μ^Y in the action.

B. Up-type quarks

Up, charm, and top quarks are zero modes contained solely in the fields $\Psi_{(\mathbf{3,4})}^\alpha$ and there arises no mixing in generation. The mass spectrum $m_q = k\lambda_q$ ($q = u,\ c,\ t$) is determined by

$$S_L(1;\lambda,c_q)S_R(1;\lambda,c_q) + \sin^2\frac{1}{2}\theta_H = 0. \tag{3.8}$$

Basis functions for fermions, $S_{L/R}(z,\lambda,c)$ and $C_{L/R}(z,\lambda,c)$, are given by (A3). For the first and second generation $|c_u|, |c_c| > \frac{1}{2}$, whereas for the third generation $|c_t| < \frac{1}{2}$. The masses are approximately given by

$$m_{u,c} \sim \pi^{-1}\sqrt{4c_{u,c}^2 - 1}\, z_L^{-|c_{u,c}|+0.5} \sin\frac{1}{2}\theta_H m_{\rm KK},$$
$$m_t \sim \pi^{-1}\sqrt{1-4c_t^2} \sin\frac{1}{2}\theta_H m_{\rm KK}. \tag{3.9}$$

4D fields denoted by $\hat{u}(x)$ appear in the (u, u') components in the 5D fields $\Psi_{(\mathbf{3,4})}^{\alpha=1}(x,z)$. (See Table II.) In the twisted gauge,

$$\begin{aligned}
\begin{bmatrix} \tilde{\check{u}} \\ \tilde{\check{u}}' \end{bmatrix} &= \frac{\sqrt{k}}{\sqrt{r_u}} \left\{ \hat{u}_L(x) \begin{bmatrix} \bar{c}_H C_L(z;\lambda_u,c_u) \\ i\bar{s}_H \hat{S}_L(z;\lambda_u,c_u) \end{bmatrix} + \hat{u}_R(x) \begin{bmatrix} \bar{c}_H S_R(z;\lambda_u,c_u) \\ i\bar{s}_H \hat{C}_R(z;\lambda_u,c_u) \end{bmatrix} \right\}, \\
r_u &= \int_1^{z_L} dz \{ \bar{c}_H^2 C_L(z;\lambda_u,c_u)^2 + \bar{s}_H^2 \hat{S}_L(z;\lambda_u,c_u)^2 \} \\
&= \int_1^{z_L} dz \{ \bar{c}_H^2 S_R(z;\lambda_u,c_u)^2 + \bar{s}_H^2 \hat{C}_R(z;\lambda_u,c_u)^2 \}, \\
\bar{c}_H &= \cos\frac{1}{2}\theta_H, \qquad \bar{s}_H = \sin\frac{1}{2}\theta_H, \\
\hat{S}_L(z;\lambda,c) &= \frac{C_L(1;\lambda,c)}{S_L(1;\lambda,c)} S_L(z;\lambda,c), \qquad \hat{C}_R(z;\lambda,c) = \frac{C_L(1;\lambda,c)}{S_L(1;\lambda,c)} C_R(z;\lambda,c).
\end{aligned} \tag{3.10}$$

The equality of the two expressions for r_u is confirmed with the aid of (3.8). Formulas for charm and top quark fields are obtained by substitution $u \to c, t$.

C. Down-type quarks

Down, strange, and bottom quarks are contained in $\Psi^{\alpha}_{(\mathbf{3},\mathbf{4})}$ and $\Psi^{\pm\alpha}_{(\mathbf{3},\mathbf{1})}$. By the brane interactions (2.12) and (2.14) all three generations mix with each other. In Ref. [13] the mass spectrum is determined in each generation separately. Generalization to the case with mixing is straightforward. We consider the case in which both $\Psi^{+\alpha}_{(\mathbf{3},\mathbf{1})}$ and $\Psi^{-\alpha}_{(\mathbf{3},\mathbf{1})}$ have the same bulk mass parameters $c_{D^+_\alpha} = c_{D^-_\alpha} \equiv c_{D_\alpha}$. Without loss of generality we assume Dirac masses m_{D_α} in (2.9) are real.

For the sake of clarity we adopt vector/matrix notation in the generation space. Fermion fields are expressed in terms of "checked" fields; $\check{\psi} = z^{-2}\psi$. Write

$$\begin{aligned}
&\vec{d} = \begin{pmatrix} \check{d} \\ \check{s} \\ \check{b} \end{pmatrix}, \qquad \vec{d}' = \begin{pmatrix} \check{d}' \\ \check{s}' \\ \check{b}' \end{pmatrix}, \qquad \vec{D}^{\pm} = \begin{pmatrix} \check{D}^{\pm}_d \\ \check{D}^{\pm}_s \\ \check{D}^{\pm}_b \end{pmatrix}, \qquad D^q_{\pm} = \begin{pmatrix} D_\pm(c_u) & & \\ & D_\pm(c_c) & \\ & & D_\pm(c_t) \end{pmatrix}, \\
&D_\pm(c) = \pm\frac{\partial}{\partial z} + \frac{c}{z}, \qquad D^D_{\pm} = \begin{pmatrix} D_\pm(c_{D_d}) & & \\ & D_\pm(c_{D_s}) & \\ & & D_\pm(c_{D_b}) \end{pmatrix}, \\
&\tilde{m}_D = \begin{pmatrix} \tilde{m}_{D_d} & & \\ & \tilde{m}_{D_s} & \\ & & \tilde{m}_{D_b} \end{pmatrix}, \qquad \tilde{m}_{D_\alpha} = \frac{m_{D_\alpha}}{k}, \qquad \mu = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{pmatrix}.
\end{aligned} \tag{3.11}$$

In terms of two-component 4D Lorentz spinors ($\vec{d}_L, \vec{d}_R$ etc.) the equations of motion in the original gauge are given by

$$\begin{aligned}
&\begin{matrix}(a): \\ (b):\end{matrix} \quad \sigma^\mu \partial_\mu \begin{pmatrix} \vec{d}_L \\ \vec{d}'_L \end{pmatrix} - k\hat{D}^q_- \begin{pmatrix} \vec{d}_R \\ \vec{d}'_R \end{pmatrix} = 0, \\
&\begin{matrix}(c): \\ (d):\end{matrix} \quad \bar{\sigma}^\mu \partial_\mu \begin{pmatrix} \vec{d}_R \\ \vec{d}'_R \end{pmatrix} - k\hat{D}^q_+ \begin{pmatrix} \vec{d}_L \\ \vec{d}'_L \end{pmatrix} = \delta(y) 2\mu \begin{pmatrix} 0 \\ \vec{D}^+_L \end{pmatrix}, \\
&(e):\ \sigma^\mu \partial_\mu \vec{D}^+_L - kD^D_- \vec{D}^+_R - \frac{k\tilde{m}_D}{z} \vec{D}^-_R = \delta(y) 2\mu^\dagger \vec{d}'_R, \\
&(f):\ \bar{\sigma}^\mu \partial_\mu \vec{D}^+_R - kD^D_+ \vec{D}^+_L - \frac{k\tilde{m}_D}{z} \vec{D}^-_L = 0, \\
&(g):\ \sigma^\mu \partial_\mu \vec{D}^-_L - kD^D_- \vec{D}^-_R - \frac{k\tilde{m}_D}{z} \vec{D}^+_R = 0, \\
&(h):\ \bar{\sigma}^\mu \partial_\mu \vec{D}^-_R - kD^D_+ \vec{D}^-_L - \frac{k\tilde{m}_D}{z} \vec{D}^+_L = 0.
\end{aligned} \tag{3.12}$$

The μ terms on the right side of the equations come from the brane interaction (2.14). The derivative $\hat{D}^q_\pm$ in Eqs. (a)-(d) represents, in each generation subspace,

$$\hat{D}_\pm(c) = D_\pm(c) \pm i\theta'(z)T^{45},$$
$$T^{45} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{for} \begin{pmatrix} \check{d} \\ \check{d}' \end{pmatrix}, \begin{pmatrix} \check{s} \\ \check{s}' \end{pmatrix}, \begin{pmatrix} \check{b} \\ \check{b}' \end{pmatrix}, \tag{3.13}$$

where $\theta(z)$ is given by (3.1). Note that the mass dimension of each coupling and field is e.g., $[\check{d}_{R/L}] = 2$, $[k] = 1$ and $[\tilde{m}_D] = [\mu] = 0$.

Boundary conditions at the IR brane ($z = z_L$) are, in the original gauge,

$$\begin{cases} \vec{d}_R = 0, \\ D^q_+ \vec{d}_L = 0, \\ D^q_- \vec{d}'_R = 0, \\ \vec{d}'_L = 0, \end{cases} \qquad \begin{cases} \vec{D}^+_R = 0, \\ D^D_+ \vec{D}^+_L = 0, \\ D^D_- \vec{D}^-_R = 0, \\ \vec{D}^-_L = 0. \end{cases} \tag{3.14}$$

Fields in the twisted gauge ($\tilde{\chi}$) are related to those in the original gauge (χ) by

$$\chi = \begin{pmatrix} \cos\frac{1}{2}\theta(z) & -i\sin\frac{1}{2}\theta(z) \\ -i\sin\frac{1}{2}\theta(z) & \cos\frac{1}{2}\theta(z) \end{pmatrix} \tilde{\chi},$$
$$\chi = \begin{pmatrix} \check{d} \\ \check{d}' \end{pmatrix}, \begin{pmatrix} \check{s} \\ \check{s}' \end{pmatrix}, \begin{pmatrix} \check{b} \\ \check{b}' \end{pmatrix}, \tag{3.15}$$

so that all fields in the twisted gauge obey the same boundary conditions as (3.14).

In the twisted gauge all fields in the bulk ($1 < z < z_L$) satisfy free equations with vanishing background field $\tilde{\theta}_H = 0$. General solutions satisfying BC (3.14) are

$$\vec{\tilde{d}}_R = \begin{pmatrix} \alpha_d S_R(z;\lambda,c_u) \\ \alpha_s S_R(z;\lambda,c_c) \\ \alpha_b S_R(z;\lambda,c_t) \end{pmatrix}, \qquad \vec{\tilde{d}}_L = \begin{pmatrix} \alpha_d C_L(z;\lambda,c_u) \\ \alpha_s C_L(z;\lambda,c_c) \\ \alpha_b C_L(z;\lambda,c_t) \end{pmatrix},$$
$$\vec{\tilde{d}}'_R = \begin{pmatrix} \alpha_{d'} C_R(z;\lambda,c_u) \\ \alpha_{s'} C_R(z;\lambda,c_c) \\ \alpha_{b'} C_R(z;\lambda,c_t) \end{pmatrix}, \qquad \vec{\tilde{d}}'_L = \begin{pmatrix} \alpha_{d'} S_L(z;\lambda,c_u) \\ \alpha_{s'} S_L(z;\lambda,c_c) \\ \alpha_{b'} S_L(z;\lambda,c_t) \end{pmatrix},$$

$$\vec{D}^+_R = \begin{pmatrix} a_d \mathcal{S}_{R2}(z;\lambda,c_{D_d},\tilde{m}_{D_d}) + b_d \mathcal{S}_{R1}(z;\lambda,c_{D_d},\tilde{m}_{D_d}) \\ a_s \mathcal{S}_{R2}(z;\lambda,c_{D_s},\tilde{m}_{D_s}) + b_s \mathcal{S}_{R1}(z;\lambda,c_{D_s},\tilde{m}_{D_s}) \\ a_b \mathcal{S}_{R2}(z;\lambda,c_{D_b},\tilde{m}_{D_b}) + b_b \mathcal{S}_{R1}(z;\lambda,c_{D_b},\tilde{m}_{D_b}) \end{pmatrix},$$
$$\vec{D}^+_L = \begin{pmatrix} a_d \mathcal{C}_{L2}(z;\lambda,c_{D_d},\tilde{m}_{D_d}) + b_d \mathcal{C}_{L1}(z;\lambda,c_{D_d},\tilde{m}_{D_d}) \\ a_s \mathcal{C}_{L2}(z;\lambda,c_{D_s},\tilde{m}_{D_s}) + b_s \mathcal{C}_{L1}(z;\lambda,c_{D_s},\tilde{m}_{D_s}) \\ a_b \mathcal{C}_{L2}(z;\lambda,c_{D_b},\tilde{m}_{D_b}) + b_b \mathcal{C}_{L1}(z;\lambda,c_{D_b},\tilde{m}_{D_b}) \end{pmatrix},$$
$$\vec{D}^-_R = \begin{pmatrix} a_d \mathcal{C}_{R1}(z;\lambda,c_{D_d},\tilde{m}_{D_d}) + b_d \mathcal{C}_{R2}(z;\lambda,c_{D_d},\tilde{m}_{D_d}) \\ a_s \mathcal{C}_{R1}(z;\lambda,c_{D_s},\tilde{m}_{D_s}) + b_s \mathcal{C}_{R2}(z;\lambda,c_{D_s},\tilde{m}_{D_s}) \\ a_b \mathcal{C}_{R1}(z;\lambda,c_{D_b},\tilde{m}_{D_b}) + b_b \mathcal{C}_{R2}(z;\lambda,c_{D_b},\tilde{m}_{D_b}) \end{pmatrix},$$
$$\vec{D}^-_L = \begin{pmatrix} a_d \mathcal{S}_{L1}(z;\lambda,c_{D_d},\tilde{m}_{D_d}) + b_d \mathcal{S}_{L2}(z;\lambda,c_{D_d},\tilde{m}_{D_d}) \\ a_s \mathcal{S}_{L1}(z;\lambda,c_{D_s},\tilde{m}_{D_s}) + b_s \mathcal{S}_{L2}(z;\lambda,c_{D_s},\tilde{m}_{D_s}) \\ a_b \mathcal{S}_{L1}(z;\lambda,c_{D_b},\tilde{m}_{D_b}) + b_b \mathcal{S}_{L2}(z;\lambda,c_{D_b},\tilde{m}_{D_b}) \end{pmatrix}. \tag{3.16}$$

The tilde ˜ above each field indicates that it is in the twisted gauge. Note $\vec{\tilde{D}}^\pm = \vec{D}^\pm$. Functions $\mathcal{S}_{R1}(z;\lambda,c,\tilde{m})$ etc. are defined in (A4). The coefficients

$$\vec{\alpha} = \begin{pmatrix} \alpha_d \\ \alpha_s \\ \alpha_b \end{pmatrix}, \quad \vec{\alpha}' = \begin{pmatrix} \alpha_{d'} \\ \alpha_{s'} \\ \alpha_{b'} \end{pmatrix}, \quad \vec{a} = \begin{pmatrix} a_d \\ a_s \\ a_b \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} b_d \\ b_s \\ b_b \end{pmatrix} \tag{3.17}$$

are determined such that BC at $z = 1^+$ $(y = +\epsilon)$ be satisfied.

To find BC at $z = 1^+$, first note that in the y coordinate

$$D_\pm(c) = \frac{e^{-\sigma(y)}}{k}\left\{\pm\frac{\partial}{\partial y} + c\sigma'(y)\right\}. \tag{3.18}$$

Fields $\vec{d}_L$, $\vec{d}'_R$, D^+_L, and D^-_R are parity even at $y = 0$, whereas $\vec{d}_R$, $\vec{d}'_L$, D^+_R, and D^-_L are parity odd. We integrate the equations for parity odd fields, (a), (d), (e), and (h) in (3.12), from $y = -\epsilon$ to $+\epsilon$ to find

$$\vec{d}_R(\epsilon) = 0, \qquad \vec{d}'_L(\epsilon) + \mu\vec{D}^+_L(0) = 0,$$
$$\vec{D}^+_R(\epsilon) - \mu^\dagger\vec{d}'_R(0) = 0, \qquad \vec{D}^-_L(\epsilon) = 0. \tag{3.19}$$

For parity even fields we evaluate the equations (b), (c), (f), and (g) at $y = +\epsilon$, by using (3.19), to find

$$\hat{D}^q_-\vec{d}'_R + \mu\{D^D_-\vec{D}^+_R + \tilde{m}_D\vec{D}^-_R\} = 0, \qquad \hat{D}^q_+\vec{d}_L = 0,$$
$$D^D_+\vec{D}^+_L - \mu^\dagger D^D_+\vec{d}'_L = 0, \quad D^D_-\vec{D}^-_R + \tilde{m}_D\vec{D}^+_R = 0. \tag{3.20}$$

Inserting (3.16) into (3.19) and (3.20), one finds equations for the coefficient vectors in (3.17). The conditions (3.19) and (3.20) are split into two sets, one for left-handed components and the other for right-handed components. The two sets yield equivalent conditions. Making use of the relation (3.15) and equations $D_+(C_L, S_L) = \lambda(S_R, C_R)$, $D_+(\mathcal{C}_{Lj}, \mathcal{S}_{Lj}) = \lambda(\mathcal{S}_{Rj}, \mathcal{C}_{Rj}) - (\tilde{m}/z)(\mathcal{S}_{Lk}, \mathcal{C}_{Lk})$ $[(j,k) = (1,2),(2,1)]$ etc., one finds for the set of left-handed components that

$$\begin{aligned}
&(p_1):\ \bar{c}_H S_R^q \vec{\alpha} - i\bar{s}_H C_R^q \vec{\alpha}' = 0, \\
&(p_2):\ -i\bar{s}_H C_L^q \vec{\alpha} + \bar{c}_H S_L^q \vec{\alpha}' + \mu\{\mathcal{C}_{L2}^D \vec{a} + \mathcal{C}_{L1}^D \vec{b}\} = 0, \\
&(p_3):\ \mathcal{S}_{L1}^D \vec{a} + \mathcal{S}_{L2}^D \vec{b} = 0, \\
&(p_4):\ \mathcal{S}_{R2}^D \vec{a} + \mathcal{S}_{R1}^D \vec{b} - \mu^\dagger\{-i\bar{s}_H S_R^q \vec{\alpha} + \bar{c}_H C_R^q \vec{\alpha}'\} = 0,
\end{aligned} \tag{3.21}$$

where

$$S_R^q = \begin{pmatrix} S_R(1;\lambda,c_u) & & \\ & S_R(1;\lambda,c_c) & \\ & & S_R(1;\lambda,c_t) \end{pmatrix},$$

$$\mathcal{S}_{Rj}^D = \begin{pmatrix} \mathcal{S}_{Rj}(1;\lambda,c_{D_d},\tilde{m}_{D_d}) & & \\ & \mathcal{S}_{Rj}(1;\lambda,c_{D_s},\tilde{m}_{D_s}) & \\ & & \mathcal{S}_{Rj}(1;\lambda,c_{D_b},\tilde{m}_{D_b}) \end{pmatrix}, \tag{3.22}$$

and so on. With the use of (p_1) and (p_3), $\vec{\alpha}'$ and $\vec{b}$ are expressed in terms of $\vec{\alpha}$ and $\vec{a}$, respectively. Then (p_2) and (p_4) become

$$\begin{aligned}
\frac{i}{\bar{s}_H}\{\bar{s}_H^2 C_L^q + \bar{c}_H^2 S_L^q (C_R^q)^{-1} S_R^q\}\vec{\alpha} - \mu\{\mathcal{C}_{L2}^D - \mathcal{C}_{L1}^D (\mathcal{S}_{L2}^D)^{-1}\mathcal{S}_{L1}^D\}\vec{a} = 0, \\
\{\mathcal{S}_{R2}^D - \mathcal{S}_{R1}^D (\mathcal{S}_{L2}^D)^{-1}\mathcal{S}_{L1}^D\}\vec{a} + \frac{i}{\bar{s}_H}\mu^\dagger S_R^q \vec{\alpha} = 0.
\end{aligned} \tag{3.23}$$

All matrices in (3.23) except for μ are diagonal. Eliminating $\vec{a}$, one finds that

$$\begin{aligned}
&K(\lambda) S_R^q \vec{\alpha} = 0, \\
&K(\lambda) = \frac{S_L^q S_R^q + \bar{s}_H^2}{S_R^q C_R^q} + \mu \frac{\mathcal{C}_{L1}^D \mathcal{S}_{L1}^D - \mathcal{C}_{L2}^D \mathcal{S}_{L2}^D}{\mathcal{S}_{R1}^D \mathcal{S}_{L1}^D - \mathcal{S}_{R2}^D \mathcal{S}_{L2}^D}\mu^\dagger.
\end{aligned} \tag{3.24}$$

The mass spectrum $m_n = k\lambda_n$ of down-type quarks is obtained by

$$\det K(\lambda_n) = 0. \tag{3.25}$$

Three lowest roots correspond to m_d, m_s, m_b. In the $\mu \to 0$ limit, the down-quark spectrum is given by $\det(S_L^q S_R^q + \bar{s}_H^2) = 0$, the same formula as for the up-quark spectrum, and the spectrum of $D^\pm$ fields is given by $\det(\mathcal{S}_{R1}^D \mathcal{S}_{L1}^D - \mathcal{S}_{R2}^D \mathcal{S}_{L2}^D) = 0$. As pointed out in Ref. [13], the spectrum for $c_u, c_c > 0$ contains exotic light fermions when $\mu \neq 0$. For this reason we take $c_u, c_c, c_t < 0$. We shall see below that gauge couplings of quarks remain very close to those in the SM for $c_u, c_c, c_t < 0$ as well.

The coefficient vector $S_R^q \vec{\alpha}$ of each down-type quark is an eigenvector of $K(\lambda_n)$ with a zero eignevalue. Once $\vec{\alpha}$ is determined, $\vec{a}$, and $\vec{\alpha}'$ and $\vec{b}$ are determined. Consequently the wave functions in (3.16) are determined, with which all gauge couplings can be evaluated.

IV. EFFECTIVE THEORY OF CKM AND FCNC

Before evaluating the W, Z gauge couplings of quarks by using exact wave functions obtained in Sec. III, it is instructive to write down an effective theory of relevant fields to see how the brane interactions μ lead to flavor mixing and FCNC. The effective theory illuminates also how FCNC interactions are naturally suppressed.

One crucial ingredient for lifting the degeneracy in the masses of up and down quarks is that right-handed component of down quark is mixture of d' and $D_d^\pm$. As confirmed in the next section, dominant part of physical down-type quarks, $(\hat{d}_R, \hat{s}_R, \hat{b}_R)$, are contained in $(D_{dR}^-, D_{sR}^-, D_{bR}^-)$. It also assures that the W boson barely couples to right-handed components of physical up-type quarks as they are contained solely in $\Psi_{(\mathbf{3},\mathbf{4})}^\alpha$.

A. Mass matrix

To simplify expressions, we use the following vector notation for 4D fermion fields in this section:

$$\vec{u} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \qquad \vec{u}' = \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}, \qquad \vec{d} = \begin{pmatrix} d \\ s \\ b \end{pmatrix},$$
$$\vec{d}' = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}, \qquad \vec{D} = \begin{pmatrix} D_d \\ D_s \\ D_b \end{pmatrix}. \tag{4.1}$$

The masses of up-type quarks are generated solely by the Hosotani mechanism. The effective mass terms in four dimensions are written as

$$\mathcal{L}_m^{\rm up} = -\{\vec{u}_L{}^t M_{\rm up} \vec{u}'_R + {\rm H.c.}\},$$
$$M_{\rm up} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}. \tag{4.2}$$

For down-type quarks the effective mass terms are written as

$$\mathcal{L}_m^{\rm down} = -\left\{(\vec{\check{d}}_L{}^t, \vec{\check{D}}_L{}^t)\mathcal{M}_{\rm down}\begin{pmatrix} \vec{d}'_R \\ \vec{D}_R \end{pmatrix} + {\rm H.c.}\right\},$$
$$\mathcal{M}_{\rm down} = \begin{pmatrix} M_{\rm up} & 0 \\ \check{\mu} & \check{m}_D \end{pmatrix},$$
$$\check{\mu} = \begin{pmatrix} \check{\mu}_{11} & \check{\mu}_{12} & \check{\mu}_{13} \\ \check{\mu}_{21} & \check{\mu}_{22} & \check{\mu}_{23} \\ \check{\mu}_{31} & \check{\mu}_{32} & \check{\mu}_{33} \end{pmatrix},$$
$$\check{m}_D = \begin{pmatrix} \check{m}_{D_d} & & \\ & \check{m}_{D_s} & \\ & & \check{m}_{D_b} \end{pmatrix}. \tag{4.3}$$

The Hosotani mechanism generates degenerate masses, the $M_{\rm up}$ term in $\mathcal{M}_{\rm down}$, for the components in $\Psi^\alpha_{(\mathbf{3},\mathbf{4})}$. $D_{\alpha L}$ ($D_{\alpha R}$) is approximately $D^+_{\alpha L}$ ($D^-_{\alpha R}$). $\check{m}_{D_\alpha}$ is a mass generated by m_{D_α} in (2.9). The matrix $\check{\mu}$ represents the brane interactions (2.14). Each element $\check{\mu}_{\alpha\beta}$ is proportional to $(\mu^\dagger)_{\alpha\beta} = \mu^*_{\beta\alpha}$. (Note that $\check{\mu}$ has dimension of mass and that $\check{\mu}$ is not proportional to $\mu^\dagger$ as a matrix).

Mass eigenstates of up-type quarks are gauge eigenstates. However mass eigenstates of down-type quarks are not gauge eigenstates as a result of $\check{\mu}$. $\mathcal{M}_{\rm down}$ can be expressed, in the canonical form, as

$$\mathcal{M}_{\rm down} = \Omega\begin{pmatrix} M_{\rm down} & \\ & M_D \end{pmatrix}\tilde{\Omega}^\dagger, \quad \Omega^\dagger = \Omega^{-1}, \quad \tilde{\Omega}^\dagger = \tilde{\Omega}^{-1},$$
$$M_{\rm down} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}, \quad M_D = \begin{pmatrix} m_{D_1} & & \\ & m_{D_2} & \\ & & m_{D_3} \end{pmatrix}. \tag{4.4}$$

Note $\Omega \neq \tilde{\Omega}$ for $\hat{\mu} \neq 0$. Mass-eigenstates denoted by $\hat{}$ are given by

$$\begin{pmatrix} \vec{\hat{d}}_L \\ \vec{\hat{D}}_L \end{pmatrix} = \Omega^\dagger \begin{pmatrix} \vec{d}_L \\ \vec{D}_L \end{pmatrix}, \qquad \begin{pmatrix} \vec{\hat{d}}_R \\ \vec{\hat{D}}_R \end{pmatrix} = \tilde{\Omega}^\dagger \begin{pmatrix} \vec{d}'_R \\ \vec{D}_R \end{pmatrix},$$
$$\mathcal{L}_m^{\rm down} = -\{\vec{\hat{d}}_L{}^t M_{\rm down}\vec{\hat{d}}_R + \vec{\hat{D}}_L{}^t M_D \vec{\hat{D}}_R + {\rm H.c.}\}. \tag{4.5}$$

All m_{D_α}'s are of $O(m_{\rm KK})$, and much larger than m_d, m_s, and m_b. Unitary matrices Ω and $\tilde{\Omega}$ are decomposed as

$$\Omega = \begin{pmatrix} \Omega_q & \Omega_b \\ \Omega_a & \Omega_D \end{pmatrix}, \qquad \tilde{\Omega}^\dagger = \begin{pmatrix} \tilde{\Omega}_q & \tilde{\Omega}_b \\ \tilde{\Omega}_a & \tilde{\Omega}_D \end{pmatrix}, \tag{4.6}$$

where all Ω_q, $\tilde{\Omega}_q$ etc. are three-by-three matrices. The unitarity of Ω implies that

$$\Omega_q\Omega_q^\dagger + \Omega_b\Omega_b^\dagger = I_3, \qquad \Omega_q^\dagger\Omega_q + \Omega_a^\dagger\Omega_a = I_3,$$
$$\Omega_a\Omega_a^\dagger + \Omega_D\Omega_D^\dagger = I_3, \qquad \Omega_b^\dagger\Omega_b + \Omega_D^\dagger\Omega_D = I_3,$$
$$\Omega_q\Omega_a^\dagger + \Omega_b\Omega_D^\dagger = 0, \qquad \Omega_q^\dagger\Omega_b + \Omega_a^\dagger\Omega_D = 0, \tag{4.7}$$

where I_3 is a three-by-three unit matrix. Similar relations hold for $\tilde{\Omega}$.

B. W couplings

The gauge coupling of $\Psi^\alpha_{(\mathbf{3},\mathbf{4})}(x,z)$ leads to the W coupling

$$\mathcal{L}_W \simeq \frac{g_L^W}{\sqrt{2}} W_\mu \vec{u}_L \Gamma^\mu \vec{d}_L + {\rm H.c.} \tag{4.8}$$

In the next section we will confirm that $g_L^W \sim g_w$ and that couplings of right-handed components are tiny, $g_R^W/g_w \lesssim 10^{-6}$. It follows from (4.5) that the gauge-eigenstate $\vec{d}_L$ is related to the mass-eigenstate $\vec{\hat{d}}_L$ by $\vec{d}_L = \Omega_q\vec{\hat{d}}_L + \Omega_b\vec{\hat{D}}_L$. For up-type quarks $\vec{u}_L = \vec{\hat{u}}_L$. At low energies ($\sqrt{s} \ll m_{D_j}$) the $\hat{D}$ field may be dropped so that

$$\mathcal{L}_W \simeq \frac{g_L^W}{\sqrt{2}} W_\mu \vec{\hat{u}}_L \Gamma^\mu \Omega_q \vec{\hat{d}}_L + {\rm H.c.} \tag{4.9}$$

In other words the CKM matrix is given by

$$V^{\rm CKM} \simeq \Omega_q. \tag{4.10}$$

It should be noted that Ω_q is not unitary in rigorous sense, as $\Omega_q\Omega_q^\dagger = I_3 - \Omega_b\Omega_b^\dagger$.

Equations (4.3) and (4.4) lead to

$$\begin{aligned}
(q_1):&\ \Omega_q M_{\rm down}\tilde{\Omega}_b + \Omega_b M_D \tilde{\Omega}_D = 0,\\
(q_2):&\ \Omega_a M_{\rm down}\tilde{\Omega}_q + \Omega_D M_D \tilde{\Omega}_a = \check{\mu},\\
(q_3):&\ \Omega_q M_{\rm down}\tilde{\Omega}_q + \Omega_b M_D \tilde{\Omega}_a = M_{\rm up},\\
(q_4):&\ \Omega_a M_{\rm down}\tilde{\Omega}_b + \Omega_D M_D \tilde{\Omega}_D = \check{m}_D,
\end{aligned} \tag{4.11}$$

or equivalently

$$\begin{aligned}
(r_1):&\ \Omega_q M_{\rm down} = M_{\rm up}\tilde{\Omega}_q^\dagger,\\
(r_2):&\ \Omega_b M_D = M_{\rm up}\tilde{\Omega}_a^\dagger,\\
(r_3):&\ \Omega_a M_{\rm down} = \check{\mu}\tilde{\Omega}_q^\dagger + \check{m}_D\tilde{\Omega}_b^\dagger,\\
(r_4):&\ \Omega_D M_D = \check{\mu}\tilde{\Omega}_a^\dagger + \check{m}_D\tilde{\Omega}_D^\dagger.
\end{aligned} \tag{4.12}$$

From the relation (q_1) and (r_2) above one finds

$$\Omega_b = -\Omega_q M_{\rm down}\tilde{\Omega}_b\tilde{\Omega}_D^{-1}M_D^{-1} = -M_{\rm up}\tilde{\Omega}_a^\dagger M_D^{-1}. \tag{4.13}$$

In other words the magnitude of each matrix element of Ω_b, denoted as $\|\Omega_b\|$, is

$$\|\Omega_b\| = O\left(\frac{m_q}{m_D}\right)\|\tilde{\Omega}_b\| \ll 1 \tag{4.14}$$

where $m_q = m_d, m_s, m_b$ and $m_D = m_{D_j}$. As $m_b/m_D \sim 10^{-3}$, Ω_q is nearly unitary. As $\Omega_a = -(\Omega_D^\dagger)^{-1}\Omega_b^\dagger\Omega_q$, one sees that $\|\Omega_a\| = O(m_q/m_D)$ as well.

Further (q_2) and (q_4) in (4.11) imply that

$$\check{\mu} \sim \Omega_D M_D \tilde{\Omega}_a, \qquad \check{m}_D \sim \Omega_D M_D \tilde{\Omega}_D. \tag{4.15}$$

The relation (r_1) in (4.12) gives a severe constraint on the mass spectrum. Recall (4.10), which implies that

$$\frac{m_{dk}}{m_{uj}}|V_{jk}^{\rm CKM}| \sim |(\tilde{\Omega}_q^\dagger)_{jk}| < 1 \tag{4.16}$$

where $(m_{d1}, m_{d2}, m_{d3}) = (m_d, m_s, m_b)$ and $(m_{u1}, m_{u2}, m_{u3}) = (m_u, m_c, m_t)$. The observed mean value (magnitude) of $V^{\rm CKM}$ is

$$V_{\rm obs}^{\rm CKM} \sim \begin{pmatrix} 0.974 & 0.224 & 0.004 \\ 0.218 & 0.997 & 0.042 \\ 0.008 & 0.030 & 1.019 \end{pmatrix}. \tag{4.17}$$

The observed $m_u \sim 1.3$ MeV is too small, and the inequality (4.16) is not satisfied for the 11, 12, and 13 elements. Rigorous treatment presented in the previous and next sections also confirms this behavior. In the present paper we tentatively suppose that $m_u \sim 20$ MeV. The issue of small m_u is left for future investigation.

C. Z couplings

For up-type quarks one finds

$$\mathcal{L}_Z^{\rm up} \sim -\frac{g_w}{\cos\theta_W} Z_\mu \left\{ \frac{1}{2}\bar{\vec{\hat{u}}}_L \Gamma^\mu \vec{\hat{u}}_L - \frac{2}{3}\sin^2\theta_W (\bar{\vec{\hat{u}}}_L \Gamma^\mu \vec{\hat{u}}_L + \bar{\vec{\hat{u}}}_R \Gamma^\mu \vec{\hat{u}}_R) \right\}. \tag{4.18}$$

Recall that D_α fields are $SO(5)$ singlet. Z couplings of down-type quarks are given by

$$\mathcal{L}_Z^{\rm down} \sim -\frac{g_w}{\cos\theta_W} Z_\mu \left\{ -\frac{1}{2}\bar{\vec{d}}_L \Gamma^\mu \vec{d}_L + \frac{1}{3}\sin^2\theta_W (\bar{\vec{d}}_L \Gamma^\mu \vec{d}_L + \bar{\vec{D}}_L \Gamma^\mu \vec{D}_L + \bar{\vec{d}}'_R \Gamma^\mu \vec{d}'_R + \bar{\vec{D}}_R \Gamma^\mu \vec{D}_R) \right\}. \tag{4.19}$$

In terms of mass eigenstates in (4.5), Z couplings at low energies are expressed as

$$\begin{aligned}
\mathcal{L}_Z^{\rm down} &\sim -\frac{g_w}{\cos\theta_W} Z_\mu \left\{ -\frac{1}{2}(\bar{\vec{\hat{d}}}_L\, {}^t\Omega_q^\dagger + \bar{\vec{\hat{D}}}_L\, {}^t\Omega_b^\dagger)\Gamma^\mu(\Omega_q \vec{\hat{d}}_L + \Omega_b \vec{\hat{D}}_L) + \frac{1}{3}\sin^2\theta_W (\bar{\vec{\hat{d}}}_L \Gamma^\mu \vec{\hat{d}}_L + \bar{\vec{\hat{D}}}_L \Gamma^\mu \vec{\hat{D}}_L + \bar{\vec{\hat{d}}}_R \Gamma^\mu \vec{\hat{d}}_R + \bar{\vec{\hat{D}}}_R \Gamma^\mu \vec{\hat{D}}_R) \right\} \\
&\sim -\frac{g_w}{\cos\theta_W} Z_\mu \left\{ -\frac{1}{2}\bar{\vec{\hat{d}}}_L \Gamma^\mu \Omega_q^\dagger \Omega_q \vec{\hat{d}}_L + \frac{1}{3}\sin^2\theta_W (\bar{\vec{\hat{d}}}_L \Gamma^\mu \vec{\hat{d}}_L + \bar{\vec{\hat{d}}}_R \Gamma^\mu \vec{\hat{d}}_R) \right\}.
\end{aligned} \tag{4.20}$$

In the first term $\Omega_q^\dagger\Omega_q = I_3 - \Omega_a^\dagger\Omega_a$, and the $\Omega_a^\dagger\Omega_a$ term gives rise to FCNC. However, with the use of the last two relations in (4.7) and the relation (4.13) one sees

$$\Omega_a^\dagger\Omega_a = \Omega_q^\dagger\Omega_b\Omega_b^\dagger(\Omega_q^\dagger)^{-1} = O\left(\frac{m_q^2}{m_D^2}\right) \lesssim 10^{-6}. \tag{4.21}$$

FCNC interactions are naturally suppressed. The FCNC suppression will be confirmed by rigorous treatment in the next section as well.

V. EVALUATION OF GAUGE COUPLINGS

In Sec. III we obtained wave functions of gauge bosons and quarks, with which gauge couplings of quarks can be evaluated. Given the parameters $\mu_{\alpha\beta}$ of the brane interaction (2.14) and the Dirac masses m_{D_α} for the $D_\alpha^\pm$ fields, the bulk mass parameters c_{D_α} are chosen such that the mass spectrum of down-type quarks are reproduced by the condition (3.25). Then the wave functions of all quarks are unambiguously determined. The parameters $\mu_{\alpha\beta}$ need be chosen such that the observed CKM mixing matrix is reproduced. This process, however, is not so trivial.

As inferred in the effective theory formulated in the previous section, consistent solutions are available only when $m_d < m_u$. This behavior has been already recognized in the case of no-mixing in Ref. [13]. In this section we present the detailed results for the W and Z couplings of quarks with typical $\mu_{\alpha\beta}$. It will be seen that a simple form of μ matrix leads to reasonable CKM mixing matrix, though it may not be perfect.

m_Z, $z_L = 10^{10}$, m_t, m_b, m_c, m_s, m_u, and m_d are inputs. The bare Weinberg angle $\sin^2\theta_W^0 = s_\phi^2/(1+s_\phi^2)$ with a given θ_H is determined to fit the LEP1 data for $e^+e^- \to \mu^+\mu^-$ at $\sqrt{s} = m_Z$ [28]. It will be seen below that evaluated gauge couplings turn out very close to those in the SM with $\sin^2\theta_W = 0.2312$. The values for $m_{\rm KK}$, c_u, c_c, c_t etc. with given θ_H are summarized in Table III.

In general nine elements of the brane interaction matrix μ can be complex. Six out of nine phases can be absorbed by redefinition of the fields $\vec{d}\,'_R$ and $\vec{D}_L^+$. Three of them remain as CP violation phases. When all heavy fields such as $\vec{D}^\pm$ are integrated out, only one complex phase survives at the CKM matrix level. In the present paper we consider a real matrix μ, which is parametrized as

$$\mu = U_{12}(\phi_{12})U_{13}(\phi_{13})U_{23}(\phi_{23}) \begin{pmatrix} \mu_1 & & \\ & \mu_2 & \\ & & \mu_3 \end{pmatrix} \times U_{23}(\omega_{23})^\dagger U_{13}(\omega_{13})^\dagger U_{12}(\omega_{12})^\dagger. \tag{5.1}$$

Here $U_{jk}(\phi)$ is a rotation matrix in the jk subspace;

$$U_{12}(\phi) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5.2}$$

As typical values we set $\tilde{m}_{D_d} = \tilde{m}_{D_s} = \tilde{m}_{D_b} = 1$. For the μ matrix, we take $(\mu_1, \mu_2, \mu_3) = (0.1, 0.1, 1)$ as reference values suggested in Ref. [13]. Among the rotation angles in (5.1), ω_{12} is most responsible for the Cabibbo angle. We have explored the parameter space $(\omega_{12}, \omega_{23})$, while keeping $\phi_{jk} = \omega_{13} = 0$. Given μ, the bulk mass parameters of D fields, $(c_{D_d}, c_{D_s}, c_{D_b})$, are determined so as to reproduce (m_d, m_s, m_b). This turns out possible only for appropriate μ. With all the parameters set, wave functions of down-type quarks are determined, and gauge couplings of quarks are evaluated. Sets of typical values of these parameters are tabulated in Table IV. We note that the masses of the first KK excited states of d, s, b quarks turn out around $0.6\, m_{\rm KK}$.

Wave functions of each down-type quark consist of 12 components, (d, d', D_d^+, D_d^-), (s, s', D_s^+, D_s^-), (b, b', D_b^+, D_b^-). Coefficient vectors, $\vec{\alpha}$, $\vec{\alpha}'$, $\vec{a}$ and $\vec{b}$ in (3.16) and (3.17) for $\theta_H = 0.15$ with the parameter set (b) in Table IV are tabulated in Table V. With these coefficients wave functions of left- and right-handed components, $f_{jL}(z)$ and $f_{jR}(z)$, are determined. In Table VI the norm of each component $N_{jL/R} = \int_1^{z_L} dz |f_{jL/R}|^2$ is listed. Note $\sum_j N_{jL} = \sum_j N_{jR} = 1$.

It is seen that the left-handed components of mass eigenstates, $(\hat{d}_L, \hat{s}_L, \hat{b}_L)$, are mostly contained in the original (d, s, b) fields. On the other hand the right-handed components $(\hat{d}_R, \hat{s}_R, \hat{b}_R)$ are distributed among various components. Dominant parts of $\hat{d}_R$ are in $D_s^\pm$, $\hat{s}_R$ in d', $D_d^\pm$, and $D_b^\pm$, and $\hat{b}_R$ in d', $D_d^\pm$, and $D_b^\pm$. The pattern of distribution for the right-handed components depends on the form of the brane interaction, or on the μ matrix.

TABLE III. Values of $m_{\rm KK}$, k, $\sin^2\theta_W^0 = s_\phi^2/(1+s_\phi^2)$, c_u, c_c, c_t are tabulated for $\theta_H = 0.10$, 0.15 and $z_L = 10^{10}$. We set $m_Z = 91.1876$ GeV, $\alpha_{\rm EM}(m_Z) = 1/128$ and $(m_u, m_c, m_t) = (0.020, 0.619, 171.17)$ GeV. The value $m_u > m_d$ has been used for a reason explained in the text.

θ_H	$m_{\rm KK}$ (TeV)	k (GeV)	$\sin^2\theta_W^0$	c_u	c_c	c_t
0.10	12.08	3.84×10^{13}	0.2306	−0.9169	−0.7545	−0.2274
0.15	8.07	2.57×10^{13}	0.2299	−0.9170	−0.7546	−0.2294

TABLE IV. Sets of parameters which yield a reasonable CKM matrix. $(c_{D_d}, c_{D_s}, c_{D_b})$ is determined to give $(m_d, m_s, m_b) = (0.0029, 0.055, 2.89)$ GeV by (3.25). We set $\phi_{jk} = \omega_{13} = 0$ in (5.1) and $\tilde{m}_{D_d} = \tilde{m}_{D_s} = \tilde{m}_{D_b} = 1$.

	θ_H	(μ_1, μ_2, μ_3)	$(\omega_{12}, \omega_{23})$	c_{D_d}	c_{D_s}	c_{D_b}
(a)	0.10	(0.1,0.1,1)	(0.1055,0.0018)	0.520074	0.751360	0.951239
(b)	0.15	(0.1,0.1,1)	(0.1055,0.00198)	0.478059	0.751545	0.955367

TABLE V. Coefficient vectors in (3.17) for wave functions of down-type quarks for $\theta_H = 0.15$ with the parameter set (b) in Table IV are listed. $(\hat{d}, \hat{s}, \hat{b})$ represent mass eigenstates.

	$\vec{\alpha}$	$\vec{\alpha}'$	$\vec{a}$	$\vec{b}$
$\hat{d}$	1.640	$-8.692\times10^{-6}i$	$0.007734i$	$2.207\times10^{-9}i$
	-0.3588	$2.148\times10^{-6}i$	$-0.4697i$	$-1.178\times10^{-7}i$
	1.476×10^{-5}	$-1.520\times10^{-10}i$	$0.005361i$	$1.232\times10^{-9}i$
$\hat{s}$	0.3812	$-3.832\times10^{-5}i$	$0.03452i$	$1.868\times10^{-7}i$
	1.542	$-1.752\times10^{-4}i$	$0.04968i$	$2.362\times10^{-7}i$
	-0.02291	$4.474\times10^{-6}i$	$-0.4376i$	$-1.908\times10^{-6}i$
$\hat{b}$	0.007211	$-3.809\times10^{-5}i$	$0.03431i$	$9.756\times10^{-6}i$
	0.02927	$-1.746\times10^{-4}i$	$0.05525i$	$1.380\times10^{-5}i$
	1.208	$-0.01239i$	$0.4389i$	$1.005\times10^{-4}i$

TABLE VI. Norm of each component of down-type quarks for $\theta_H = 0.15$ with the parameter set (b) in Table IV is listed. $(\hat{d}, \hat{s}, \hat{b})$ represent mass eigenstates. In this table 10^{-13}, for instance, implies order of 10^{-13}.

	$\hat{d}_L$	$\hat{s}_L$	$\hat{b}_L$	$\hat{d}_R$	$\hat{s}_R$	$\hat{b}_R$
d	0.9487	0.0513	10^{-5}	0.0001	0.0022	0.0022
s	0.0513	0.9484	0.0003	10^{-8}	10^{-5}	10^{-5}
b	10^{-10}	0.0004	0.9996	10^{-22}	10^{-13}	10^{-6}
d'	10^{-23}	10^{-19}	10^{-15}	0.0198	0.3856	0.3810
s'	10^{-24}	10^{-18}	10^{-14}	10^{-6}	0.0074	0.0074
b'	10^{-32}	10^{-21}	10^{-10}	10^{-20}	10^{-11}	0.0003
D_d^+	10^{-17}	10^{-13}	10^{-9}	0.0023	0.0465	0.0459
D_s^+	10^{-13}	10^{-12}	10^{-9}	0.3113	0.0035	0.0043
D_b^+	10^{-17}	10^{-10}	10^{-7}	10^{-5}	0.1177	0.1184
D_d^-	10^{-17}	10^{-13}	10^{-9}	0.0025	0.0489	0.0484
D_s^-	10^{-13}	10^{-13}	10^{-9}	0.6639	0.0074	0.0092
D_b^-	10^{-17}	10^{-11}	10^{-7}	0.0001	0.3808	0.3830

A crucial point is that d' component of $\hat{d}_R$, s' component of $\hat{s}_R$, and b' component of $\hat{b}_R$ are all small. As is seen in the following subsection, this property is important to assure vanishingly small W couplings of right-handed quarks.

A. W couplings

The $SO(5)$ gauge potentials can be expanded as

$$A_M = \sum_{a=1}^{3}\{A_M^{a_L}T^{a_L} + A_M^{a_R}T^{a_R} + A_M^{\hat{a}}T^{\hat{a}}\} + A_M^{\hat{4}}T^{\hat{4}}, \tag{5.3}$$

where T^{a_L} and T^{a_R} are $SU(2)_L$ and $SU(2)_R$ generators, respectively. $\{T^{\hat{p}}; p = 1, \ldots, 4\}$ are generators of $SO(5)/SO(4)$. In the spinor representation, for instance,

$$T^{a_L} = \frac{1}{2}\begin{pmatrix}\sigma^a & 0\\ 0 & 0\end{pmatrix}, \quad T^{a_R} = \frac{1}{2}\begin{pmatrix}0 & 0\\ 0 & \sigma^a\end{pmatrix},$$
$$T^{\hat{a}} = \frac{1}{2\sqrt{2}}\begin{pmatrix}0 & i\sigma^a\\ -i\sigma^a & 0\end{pmatrix}, \quad T^{\hat{4}} = \frac{1}{2\sqrt{2}}\begin{pmatrix}0 & I_2\\ I_2 & 0\end{pmatrix} \tag{5.4}$$

where σ^a's and I_2 are Pauli matrices and a two-by-two unit matrix. W boson is contained, in the twisted gauge, in

$$\begin{aligned}\tilde{A}_\mu \Rightarrow\ & \frac{1}{2}\{(\tilde{A}_\mu^{1_L} - i\tilde{A}_\mu^{2_L})(T^{1_L} + iT^{2_L})\\ &+ (\tilde{A}_\mu^{1_R} - i\tilde{A}_\mu^{2_R})(T^{1_R} + iT^{2_R})\\ &+ (\tilde{A}_\mu^{\hat{1}} - i\tilde{A}_\mu^{\hat{2}})(T^{\hat{1}} + iT^{\hat{2}})\} + \text{H.c.}\\ \Rightarrow\ & \frac{1}{2}\{(1 + c_H)\overset{\circ}{W}_\mu(T^{1_L} + iT^{2_L})\\ &+ (1 - c_H)\overset{\circ}{W}_\mu(T^{1_R} + iT^{2_R})\\ &- \sqrt{2}s_H\overset{\circ}{W}{}_\mu^S(T^{\hat{1}} + iT^{\hat{2}})\} + \text{H.c.}\end{aligned} \tag{5.5}$$

Here the expression (3.7) has been inserted. W couplings of quarks come solely from the couplings of $\Psi^\alpha_{(\mathbf{3},\mathbf{4})}$.

$$\begin{aligned}\mathcal{L}_W^{d=4} = -ig_A\int_1^{z_L}\frac{dz}{k}\Big\{&\overset{\circ}{W}_\mu\Big(\frac{1+c_H}{2}\bar{\check{\vec{u}}}\Gamma^\mu\check{\vec{d}} + \frac{1-c_H}{2}\bar{\check{\vec{u}}}'\Gamma^\mu\check{\vec{d}}'\Big)\\ &+ \overset{\circ}{W}{}_\mu^S\Big(-i\frac{s_H}{2}\bar{\check{\vec{u}}}\Gamma^\mu\check{\vec{d}}' + i\frac{s_H}{2}\bar{\check{\vec{u}}}'\Gamma^\mu\check{\vec{d}}\Big)\Big\} + \text{H.c.}\end{aligned} \tag{5.6}$$

Here, as in (3.11), we have denoted as

$$\vec{u} = \begin{pmatrix}\check{u}\\ \check{c}\\ \check{t}\end{pmatrix}, \qquad \vec{u}' = \begin{pmatrix}\check{u}'\\ \check{c}'\\ \check{t}'\end{pmatrix}. \tag{5.7}$$

We use the following notation for wave functions of quarks. 4D quark fields are denoted by a hat $\hat{}$;

$$\begin{pmatrix}\hat{u}_1(x)\\ \hat{u}_2(x)\\ \hat{u}_3(x)\end{pmatrix} = \begin{pmatrix}\hat{u}(x)\\ \hat{c}(x)\\ \hat{t}(x)\end{pmatrix}, \qquad \begin{pmatrix}\hat{d}_1(x)\\ \hat{d}_2(x)\\ \hat{d}_3(x)\end{pmatrix} = \begin{pmatrix}\hat{d}(x)\\ \hat{s}(x)\\ \hat{b}(x)\end{pmatrix}. \tag{5.8}$$

For up-type quarks 5D fields in the twisted gauge are expanded as

$$\begin{aligned}\check{u}_j(x,z) &= \sqrt{k}\{\hat{u}_{jL}(x)f_{Lu_j}^{\hat{u}_j}(z) + \hat{u}_{jR}(x)f_{Ru_j}^{\hat{u}_j}(z)\},\\ \check{u}'_j(x,z) &= \sqrt{k}\{\hat{u}_{jL}(x)f_{Lu'_j}^{\hat{u}_j}(z) + \hat{u}_{jR}(x)f_{Ru'_j}^{\hat{u}_j}(z)\}.\end{aligned} \tag{5.9}$$

With the expression in (3.10), for instance,

$$\begin{aligned}f_{Lu_1}^{\hat{u}_1}(z) &= \bar{c}_H C_L(z;\lambda_u, c_u)/\sqrt{r_u},\\ f_{Lu'_2}^{\hat{u}_2}(z) &= i\bar{s}_H\hat{S}_L(z;\lambda_c, c_c)/\sqrt{r_c}.\end{aligned} \tag{5.10}$$

For down-type quarks 5D fields in the twisted gauge are expanded as

$$\begin{aligned}
\tilde{\check{d}}_j(x,z) &= \sqrt{k}\sum_{m=1}^{3}\{\hat{d}_{mL}(x)f_{Ld_j}^{\hat{d}_m}(z) + \hat{d}_{mR}(x)f_{Rd_j}^{\hat{d}_m}(z)\},\\
\tilde{\check{d}}'_j(x,z) &= \sqrt{k}\sum_{m=1}^{3}\{\hat{d}_{mL}(x)f_{Ld'_j}^{\hat{d}_m}(z) + \hat{d}_{mR}(x)f_{Rd'_j}^{\hat{d}_m}(z)\},\\
\check{D}_j^+(x,z) &= \sqrt{k}\sum_{m=1}^{3}\{\hat{d}_{mL}(x)f_{LD_j^+}^{\hat{d}_m}(z) + \hat{d}_{mR}(x)f_{RD_j^+}^{\hat{d}_m}(z)\},\\
\check{D}_j^-(x,z) &= \sqrt{k}\sum_{m=1}^{3}\{\hat{d}_{mL}(x)f_{LD_j^-}^{\hat{d}_m}(z) + \hat{d}_{mR}(x)f_{RD_j^-}^{\hat{d}_m}(z)\}.
\end{aligned} \tag{5.11}$$

With the expression in (3.16), one finds, for instance,

$$\begin{aligned}
f_{Ld_2}^{\hat{d}_1}(z) &= \alpha_s^{\hat{d}} C_L(z;\lambda_d,c_c),\\
f_{Rd'_3}^{\hat{d}_2}(z) &= \alpha_{b'}^{\hat{s}} C_R(z;\lambda_s,c_t),\\
f_{RD_1^+}^{\hat{d}_3}(z) &= a_d^{\hat{b}}\mathcal{S}_{R2}(z;\lambda_b,c_{D_d},\tilde{m}_{D_d})\\
&\quad + b_d^{\hat{b}}\mathcal{S}_{R1}(z;\lambda_b,c_{D_d},\tilde{m}_{D_d}).
\end{aligned} \tag{5.12}$$

Here $\vec{\alpha}^{\hat{d}_j}$, $\vec{\alpha}'^{\hat{d}_j}$, $\vec{a}^{\hat{d}_j}$, and $\vec{b}^{\hat{d}_j}$ are the coefficient vectors determined for $\hat{d}_j$.

W couplings of quarks are defined by

$$\mathcal{L}_W^{d=4} = \frac{i}{\sqrt{2}} W_\mu \sum_{j,k}\Big\{g_{Ljk}^W \bar{\hat{u}}_{jL}\Gamma^\mu \hat{d}_{kL} + g_{Rjk}^W \bar{\hat{u}}_{jR}\Gamma^\mu \hat{d}_{kR}\Big\} + \text{H.c.} \tag{5.13}$$

Inserting (5.9) and (5.11) into (5.6), one finds

$$\begin{aligned}
\begin{bmatrix} g_{Ljk}^W \\ g_{Rjk}^W \end{bmatrix} &= -ig_w \frac{\sqrt{kL}}{\sqrt{r_W}}\int_1^{z_L} dz \Bigg\{ C(z,\lambda_W)\left(\frac{1+c_H}{2}\begin{bmatrix} f_{Lu_j}^{\hat{u}_j}(z)^* f_{Ld_j}^{\hat{d}_k}(z) \\ f_{Ru_j}^{\hat{u}_j}(z)^* f_{Rd_j}^{\hat{d}_k}(z)\end{bmatrix} + \frac{1-c_H}{2}\begin{bmatrix} f_{Lu'_j}^{\hat{u}_j}(z)^* f_{Ld'_j}^{\hat{d}_k}(z) \\ f_{Ru'_j}^{\hat{u}_j}(z)^* f_{Rd'_j}^{\hat{d}_k}(z)\end{bmatrix}\right)\\
&\quad + \hat{S}(z,\lambda_W)(-i)\frac{s_H}{2}\begin{bmatrix} f_{Lu_j}^{\hat{u}_j}(z)^* f_{Ld'_j}^{\hat{d}_k}(z) - f_{Lu'_j}^{\hat{u}_j}(z)^* f_{Ld_j}^{\hat{d}_k}(z) \\ f_{Ru_j}^{\hat{u}_j}(z)^* f_{Rd'_j}^{\hat{d}_k}(z) - f_{Ru'_j}^{\hat{u}_j}(z)^* f_{Rd_j}^{\hat{d}_k}(z)\end{bmatrix}\Bigg\}.
\end{aligned} \tag{5.14}$$

Let us denote the couplings in the matrix form; $(\hat{g}_L^W)_{jk} = g_{Ljk}^W$ and $(\hat{g}_R^W)_{jk} = g_{Rjk}^W$. $\hat{g}_L^W$ is parametrized as

$$\hat{g}_L^W = g_L^W \hat{V}_{\text{CKM}}, \qquad \det V_{\text{CKM}} = 1. \tag{5.15}$$

$\hat{g}_L^W$ and $\hat{g}_R^W$ are evaluated for the two sets of parameters in Table IV;

$$\begin{aligned}
&(a)\ \theta_H = 0.10:\\
&\quad g_L^W = 0.9978 g_w, \qquad \hat{V}_{\text{CKM}} = \begin{pmatrix} 0.9744 & 0.2245 & 0.0031 \\ -0.2245 & 0.9743 & 0.0134 \\ 9\times10^{-6} & -0.0138 & 1.0002 \end{pmatrix},\\
&\quad \hat{g}_R^W = g_w \begin{pmatrix} 2\times10^{-12} & 8\times10^{-12} & 6\times10^{-12} \\ -1\times10^{-11} & 9\times10^{-10} & 7\times10^{-10} \\ 1\times10^{-13} & -3\times10^{-9} & 1\times10^{-5} \end{pmatrix},\\
&(b)\ \theta_H = 0.15:\\
&\quad g_L^W = 0.9950 g_w, \qquad \hat{V}_{\text{CKM}} = \begin{pmatrix} 0.9737 & 0.2264 & 0.0043 \\ -0.2264 & 0.9736 & 0.0185 \\ 1\times10^{-5} & -0.0190 & 1.0004 \end{pmatrix},\\
&\quad \hat{g}_R^W = g_w \begin{pmatrix} 4\times10^{-12} & 1\times10^{-11} & 2\times10^{-11} \\ -3\times10^{-11} & 2\times10^{-9} & 2\times10^{-9} \\ 4\times10^{-13} & -1\times10^{-8} & 3\times10^{-5} \end{pmatrix}.
\end{aligned} \tag{5.16}$$

We have checked remarkable cancellation among four terms in the right-handed couplings g^W_{Rjk} in (5.14). The resultant $\hat{V}_{\rm CKM}$ is reasonably close to the observed CKM matrix, although the 31 element is still too small. We have evaluated the W couplings of leptons as well. The couplings of left-handed leptons (e,μ,τ) are $(0.997665, 0.997662, 0.997659)g_w$ for $\theta_H = 0.10$, and $(0.994756, 0.994748, 0.994743)g_w$ for $\theta_H = 0.15$. The relative coupling g^W_L to $g^W_{L\,\rm lepton}$ is $g^W_L/g^W_{L\,\rm lepton} = 1.00013$ and 1.00028 for $\theta_H = 0.10$ and 0.15, respectively. The universality holds to high accuracy. The W couplings of right-handed leptons are typically of order $10^{-20}g_w$.

B. Z couplings

Photon γ and Z boson are contained in

$$\tilde{A}_\mu + \frac{g_B}{g_A} Q_X B_\mu \Rightarrow (\tilde{A}^{3_L}_\mu T^{3_L} + \tilde{A}^{3_R}_\mu T^{3_R} + \tilde{A}^{\hat{3}}_\mu T^{\hat{3}}) + \frac{g_B}{g_A} Q_X B_\mu$$
$$\Rightarrow \frac{\sqrt{1+s_\phi^2}}{\sqrt{2}} \Big\{ [(1+c_H)T^{3_L} + (1-c_H)T^{3_R}] \overset{\circ}{Z}_\mu - \sqrt{2} s_H T^{\hat{3}} \overset{\circ}{Z}{}^S_\mu \Big\} + \frac{s_\phi}{\sqrt{1+s_\phi^2}} Q_{\rm EM} (\overset{\circ}{A}{}^\gamma_\mu - \sqrt{2} s_\phi \overset{\circ}{Z}_\mu). \tag{5.17}$$

Here (3.7) and the relation $Q_{\rm EM} = T^{3_L} + T^{3_R} + Q_X$ have been used. Photon couplings are given by

$$\mathcal{L}^{d=4}_\gamma = -i g_A \frac{s_\phi}{\sqrt{1+s_\phi^2}} \int_1^{z_L} \frac{dz}{k} \overset{\circ}{A}{}^\gamma_\mu \Big\{ \frac{2}{3} (\bar{\vec{u}} \Gamma^\mu \vec{u} + \bar{\vec{u}}' \Gamma^\mu \vec{u}') - \frac{1}{3} (\bar{\vec{d}} \Gamma^\mu \vec{d} + \bar{\vec{d}}' \Gamma^\mu \vec{d}' + \bar{\vec{D}}^+ \Gamma^\mu \vec{D}^+ + \bar{\vec{D}}^- \Gamma^\mu \vec{D}^-) \Big\}. \tag{5.18}$$

Inserting (3.5), (5.9) and (5.11) into (5.18), one finds that

$$\mathcal{L}^{d=4}_\gamma = -i g_w \frac{s_\phi}{\sqrt{1+s_\phi^2}} A^\gamma_\mu(x) \int_1^{z_L} dz J^\mu_\gamma(x,z),$$
$$J^\mu_\gamma(x,z) = \frac{2}{3} \sum_{j=1}^3 \Big[\bar{\hat{u}}_{jL} \Gamma^\mu \hat{u}_{jL}(x) \Big\{ f^{\hat{u}_j}_{Lu_j}(z)^* f^{\hat{u}_j}_{Lu_j}(z) + f^{\hat{u}_j}_{Lu'_j}(z)^* f^{\hat{u}_j}_{Lu'_j}(z) \Big\} + (L \to R) \Big]$$
$$- \frac{1}{3} \sum_{\ell,m=1}^3 \Big[\bar{\hat{d}}_{\ell L} \Gamma^\mu \hat{d}_{mL}(x) \sum_{j=1}^3 \Big\{ f^{\hat{d}_\ell}_{Ld_j}(z)^* f^{\hat{d}_m}_{Ld_j}(z) + f^{\hat{d}_\ell}_{Ld'_j}(z)^* f^{\hat{d}_m}_{Ld'_j}(z)$$
$$+ f^{\hat{d}_\ell}_{LD^+_j}(z)^* f^{\hat{d}_m}_{LD^+_j}(z) + f^{\hat{d}_\ell}_{LD^-_j}(z)^* f^{\hat{d}_m}_{LD^-_j}(z) \Big\} + (L \to R) \Big]. \tag{5.19}$$

By making use of orthonormality relations, the z integration can be done to lead to

$$\mathcal{L}^{d=4}_\gamma = -ie A^\gamma_\mu(x) \sum_{j=1}^3 \Big\{ \frac{2}{3} \bar{\hat{u}}_j(x) \Gamma^\mu \hat{u}_j(x) - \frac{1}{3} \bar{\hat{d}}_j(x) \Gamma^\mu \hat{d}_j(x) \Big\}, \qquad e = g_w \sin\theta^0_W, \qquad \sin\theta^0_W = \frac{s_\phi}{\sqrt{1+s_\phi^2}}. \tag{5.20}$$

Z couplings are given by

$$\mathcal{L}^{d=4}_Z = -i g_A \frac{\sqrt{1+s_\phi^2}}{\sqrt{2}} \int_1^{z_L} \frac{dz}{k} \Big\{ \overset{\circ}{Z}_\mu \Big[\frac{1+c_H}{2} (\bar{\vec{u}} \Gamma^\mu \vec{u} - \bar{\vec{d}} \Gamma^\mu \vec{d}) + \frac{1-c_H}{2} (\bar{\vec{u}}' \Gamma^\mu \vec{u}' - \bar{\vec{d}}' \Gamma^\mu \vec{d}') \Big]$$
$$- \overset{\circ}{Z}{}^S_\mu i \frac{s_H}{2} [\bar{\vec{u}} \Gamma^\mu \vec{u}' - \bar{\vec{u}}' \Gamma^\mu \vec{u} - \bar{\vec{d}} \Gamma^\mu \vec{d}' + \bar{\vec{d}}' \Gamma^\mu \vec{d}] \Big\} + i g_A \frac{\sqrt{2} s_\phi^2}{\sqrt{1+s_\phi^2}} \int_1^{z_L} \frac{dz}{k} \overset{\circ}{Z}_\mu J^\mu_\gamma \tag{5.21}$$

where J^μ_γ is given in (5.19). Let us denote Z couplings of quarks as

$$\mathcal{L}^{d=4}_Z = -\frac{i}{\cos\theta^0_W} Z_\mu \Big\{ \sum_j (g^Z_{Lu_ju_j} \bar{\hat{u}}_{jL} \Gamma^\mu \hat{u}_{jL} + g^Z_{Ru_ju_j} \bar{\hat{u}}_{jR} \Gamma^\mu \hat{u}_{jR}) + \sum_{j,k} (g^Z_{Ld_jd_k} \bar{\hat{d}}_{jL} \Gamma^\mu \hat{d}_{kL} + g^W_{Rd_jd_k} \bar{\hat{d}}_{jR} \Gamma^\mu \hat{d}_{kR}) \Big\}. \tag{5.22}$$

The couplings of up-type quarks are diagonal in flavor, but there appear off-diagonal couplings (FCNC) for down-type quarks. Insertion of (3.5), (5.9) and (5.11) into (5.21) leads to

$$
\begin{aligned}
g^Z_{Lu_ju_j} &= g_w \frac{\sqrt{2kL}}{\sqrt{r_Z}} \int_1^{z_L} dz \\
&\times \bigg\{ C(z,\lambda_Z) \bigg(\frac{1+c_H}{4} f^{\hat{u}_j}_{Lu_j}(z)^* f^{\hat{u}_j}_{Lu_j}(z) + \frac{1-c_H}{4} f^{\hat{u}_j}_{Lu'_j}(z)^* f^{\hat{u}_j}_{Lu'_j}(z) - \frac{2}{3}\sin^2\theta_W^0 \Big[f^{\hat{u}_j}_{Lu_j}(z)^* f^{\hat{u}_j}_{Lu_j}(z) + f^{\hat{u}_j}_{Lu'_j}(z)^* f^{\hat{u}_j}_{Lu'_j}(z) \Big] \bigg) \\
&- i\frac{s_H}{2} \hat{S}(z,\lambda_Z) \Big[f^{\hat{u}_j}_{Lu_j}(z)^* f^{\hat{u}_j}_{Lu'_j}(z) - f^{\hat{u}_j}_{Lu'_j}(z)^* f^{\hat{u}_j}_{Lu_j}(z) \Big] \bigg\}, \\
g^Z_{Ld_jd_k} &= g_w \frac{\sqrt{2kL}}{\sqrt{r_Z}} \int_1^{z_L} dz \sum_{\ell=1}^{3} \\
&\times \bigg\{ C(z,\lambda_Z) \bigg(-\frac{1+c_H}{4} f^{\hat{d}_j}_{Ld_\ell}(z)^* f^{\hat{d}_k}_{Ld_\ell}(z) - \frac{1-c_H}{4} f^{\hat{d}_j}_{Ld'_\ell}(z)^* f^{\hat{d}_k}_{Ld'_\ell}(z) + \frac{1}{3}\sin^2\theta_W^0 \Big[f^{\hat{d}_j}_{Ld_\ell}(z)^* f^{\hat{d}_k}_{Ld_\ell}(z) + f^{\hat{d}_j}_{Ld'_\ell}(z)^* f^{\hat{d}_k}_{Ld'_\ell}(z) \\
&+ f^{\hat{d}_j}_{LD^+_\ell}(z)^* f^{\hat{d}_k}_{LD^+_\ell}(z) + f^{\hat{d}_j}_{LD^-_\ell}(z)^* f^{\hat{d}_k}_{LD^-_\ell}(z) \Big] \bigg) + i\frac{s_H}{2} \hat{S}(z,\lambda_Z) \Big[f^{\hat{d}_j}_{Ld_\ell}(z)^* f^{\hat{d}_k}_{Ld_\ell}(z) - f^{\hat{d}_j}_{Ld'_\ell}(z)^* f^{\hat{d}_k}_{Ld_\ell}(z) \Big] \bigg\}. \qquad (5.23)
\end{aligned}
$$

Formulas for $g^Z_{Ru_ju_j}$ and $g^Z_{Rd_jd_k}$ are obtained by the replacement $L \to R$ in each expression.

The Z couplings of down-type quarks are written in the matrix form; $(\hat{g}^Z_{Ld})_{jk} = g^Z_{Ld_jd_k}$ and $(\hat{g}^Z_{Rd})_{jk} = g^Z_{Rd_jd_k}$. One finds for the two sets of parameters in Table IV

(a) $\theta_H = 0.10$:

$$
\begin{pmatrix} g^Z_{Luu} \\ g^Z_{Lcc} \\ g^Z_{Ltt} \end{pmatrix} = \begin{pmatrix} 0.3451 \\ 0.3451 \\ 0.3455 \end{pmatrix} g_w, \quad \begin{pmatrix} g^Z_{Ruu} \\ g^Z_{Rcc} \\ g^Z_{Rtt} \end{pmatrix} = \begin{pmatrix} -0.1538 \\ -0.1538 \\ -0.1534 \end{pmatrix} g_w,
$$

$$
\hat{g}^Z_{Ld} = g_w \begin{pmatrix} -0.4220 & -3\times10^{-7} & -4\times10^{-9} \\ -3\times10^{-7} & -0.4220 & -1\times10^{-7} \\ -4\times10^{-9} & -1\times10^{-7} & -0.4220 \end{pmatrix},
$$

$$
\hat{g}^Z_{Rd} = g_w \begin{pmatrix} 0.0769 & -6\times10^{-7} & -4\times10^{-7} \\ -6\times10^{-7} & 0.0769 & -3\times10^{-6} \\ -4\times10^{-7} & -3\times10^{-6} & 0.0769 \end{pmatrix},
$$

(b) $\theta_H = 0.15$:

$$
\begin{pmatrix} g^Z_{Luu} \\ g^Z_{Lcc} \\ g^Z_{Ltt} \end{pmatrix} = \begin{pmatrix} 0.3441 \\ 0.3441 \\ 0.3449 \end{pmatrix} g_w, \quad \begin{pmatrix} g^Z_{Ruu} \\ g^Z_{Rcc} \\ g^Z_{Rtt} \end{pmatrix} = \begin{pmatrix} -0.1533 \\ -0.1533 \\ -0.1524 \end{pmatrix} g_w,
$$

$$
\hat{g}^Z_{Ld} = g_w \begin{pmatrix} -0.4208 & -7\times10^{-7} & -1\times10^{-8} \\ -7\times10^{-7} & -0.4208 & -4\times10^{-7} \\ -1\times10^{-8} & -4\times10^{-7} & -0.4207 \end{pmatrix},
$$

$$
\hat{g}^Z_{Rd} = g_w \begin{pmatrix} 0.0767 & -1\times10^{-6} & -1\times10^{-6} \\ -1\times10^{-6} & 0.0767 & -7\times10^{-6} \\ -1\times10^{-6} & -7\times10^{-6} & 0.0767 \end{pmatrix}. \qquad (5.24)
$$

Although FCNCs emerge for the down-type quarks, their magnitude is naturally suppressed. FCNCs induce the mixing of neutral mesons ($M = K, B_d, B_s$) at the tree level, yielding $\Delta m_M \sim (m_M f_M^2/3m_Z^2)(\hat{g}^Z_d|_M)^2$ where m_M and f_M are the meson mass and decay constant and $\hat{g}^Z_d|_M$ is the relevant coupling in $\hat{g}^Z_{Ld}$ or $\hat{g}^Z_{Rd}$. Making use of $(m_K, m_{B_d}, m_{B_s}) \sim (0.498, 5.280, 5.367)$ GeV and $(f_K, f_{B_d}, f_{B_s}) \sim (0.156, 0.191, 0.274)$ GeV, one finds, for $\theta_H = 0.10$, $(\Delta m_K, \Delta m_{B_d}, \Delta m_{B_s}) \sim (7\times10^{-20}, 5\times10^{-19}, 6\times10^{-17})$ GeV, which are much smaller than the experimental values $(3.48\times10^{-15}, 3.36\times10^{-13}, 1.17\times10^{-11})$ GeV [29,30].

The gauge invariance guarantees natural suppression of FCNC interactions. This should be contrasted to the previous approaches of Refs. [23,24], in which only $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariance is imposed on the brane. The requirement of the gauge invariance under $\mathcal{G} = SU(3)_C \times SO(5) \times U(1)_X$ restricts the form of brane interactions to (2.12), which yield the specific mass terms of the form (2.14). The resultant FCNCs are suppressed with a factor of order $(m_q/m_D)^2$ ($m_q = m_d, m_s, m_b$) as anticipated from the effective theory developed in Sec. IV. The orbifold boundary condition breaks $SO(5)$ to $SO(4)$ so that one might expect only $\mathcal{G}' = SU(3)_C \times SO(4) \times U(1)_X$ invariance on the brane. As explained earlier, the above conclusion remains valid even with the $\mathcal{G}'$ gauge invariance alone being imposed.

We remark that the relative couplings to $g^W_{L\,\mathrm{lepton}}$ are

$$
\frac{1}{g^W_{L\,\mathrm{lepton}}} \begin{pmatrix} g^Z_{Luu} \\ g^Z_{Ruu} \\ g^Z_{Ldd} \\ g^Z_{Rdd} \end{pmatrix} = \begin{pmatrix} 0.34588 \\ -0.15413 \\ -0.42295 \\ 0.07707 \end{pmatrix}, \qquad \begin{pmatrix} 0.34591 \\ -0.15411 \\ -0.42298 \\ 0.07706 \end{pmatrix} \qquad (5.25)
$$

for $\theta_H = 0.10$, and 0.15, respectively. The values in the SM with $\sin^2\theta_W = 0.2312$ are 0.3458, −0.1541, −0.4229, and 0.0771. The values (5.25) in the gauge-Higgs unification are very close to those in the SM.

C. Yukawa couplings

The flavor mixing in the down-type quarks induces flavor-changing Yukawa couplings. We show that its effect is extremely tiny. The 4D Higgs field $H(x)$ is contained in $A_z^{\hat{4}}$ in the expansion (5.3);

$$\tilde{A}_z = \mathring{H}(x,z)T^{\hat{4}} + \cdots,$$
$$\mathring{H}(x,z) = \frac{1}{\sqrt{k}}H(x)h_H(z) + \cdots, \quad h_H(z) = \sqrt{\frac{2}{z_L^2 - 1}}z. \tag{5.26}$$

Inserting (5.26) into the gauge interaction part of the action, one obtains

$$\begin{aligned} & - ig_A \int_1^{z_L} dz \mathring{H} \sum_{\alpha=1}^{3} \bar{\check{\Psi}}^{\alpha}_{(\mathbf{3},\mathbf{4})} \Gamma^5 T^{\hat{4}} \check{\Psi}^{\alpha}_{(\mathbf{3},\mathbf{4})} \\ &= -\frac{g_A}{2\sqrt{2}} \int_1^{z_L} dz \mathring{H} \{ \vec{\check{u}}_L^\dagger \vec{\check{u}}_R' + \vec{\check{u}}_L'^\dagger \vec{\check{u}}_R + \vec{\check{u}}_R^\dagger \vec{\check{u}}_L' + \vec{\check{u}}_R'^\dagger \vec{\check{u}}_L \\ & \quad + \vec{\check{d}}_L^\dagger \vec{\check{d}}_R' + \vec{\check{d}}_L'^\dagger \vec{\check{d}}_R + \vec{\check{d}}_R^\dagger \vec{\check{d}}_L' + \vec{\check{d}}_R'^\dagger \vec{\check{d}}_L \} \end{aligned} \tag{5.27}$$

where the notation (5.7) has been used. We insert (5.9) and (5.11) into (5.27) and integrate over z. In terms of mass eigenstates (5.8) the Yukawa interactions are written as

$$\begin{aligned} & - iH(x) \bigg\{ \sum_{j=1}^{3} y_{u_j u_j} (\hat{u}_{jL}^\dagger \hat{u}_{jR} - \hat{u}_{jR}^\dagger \hat{u}_{jL}) \\ & \quad + \sum_{j,k=1}^{3} y_{d_j d_k} (\hat{d}_{jL}^\dagger \hat{d}_{kR} - \hat{d}_{kR}^\dagger \hat{d}_{jL}) \bigg\} \end{aligned} \tag{5.28}$$

where the Yukawa couplings are given by

$$\begin{aligned} y_{u_j u_j} &= -i \frac{g_w \sqrt{kL}}{2\sqrt{2}} \int_1^{z_L} dz h_H(z) \Big\{ f_{Lu_j}^{\hat{u}_j}(z)^* f_{Ru_j'}^{\hat{u}_j}(z) \\ & \quad + f_{Lu_j'}^{\hat{u}_j}(z)^* f_{Ru_j}^{\hat{u}_j}(z) \Big\}, \\ y_{d_j d_k} &= -i \frac{g_w \sqrt{kL}}{2\sqrt{2}} \int_1^{z_L} dz h_H(z) \sum_{m=1}^{3} \Big\{ f_{Ld_m}^{\hat{d}_j}(z)^* f_{Rd_m'}^{\hat{d}_k}(z) \\ & \quad + f_{Ld_m'}^{\hat{d}_j}(z)^* f_{Rd_m}^{\hat{d}_k}(z) \Big\}. \end{aligned} \tag{5.29}$$

Note that the Yukawa couplings in the up-type quark sector are diagonal in the generation space, whereas those in the down-type quark sector have nonvanishing off-diagonal elements.

For the two sets of parameters in Table IV one finds

$$\begin{aligned} & (a)\, \theta_H = 0.10: \\ & \quad (y_{uu}, y_{cc}, y_{tt}) = (8.1376\times 10^{-5}, 2.5186\times 10^{-3}, 0.69693), \\ & \quad \hat{y}_d = \begin{pmatrix} 1.1800\times 10^{-5} & -1\times 10^{-16} & -2\times 10^{-13} \\ -2\times 10^{-18} & 2.2378\times 10^{-4} & 1\times 10^{-11} \\ -9\times 10^{-17} & 4\times 10^{-13} & 1.1759\times 10^{-2} \end{pmatrix}. \\ & (b)\, \theta_H = 0.15: \\ & \quad (y_{uu}, y_{cc}, y_{tt}) = (8.1222\times 10^{-5}, 2.5138\times 10^{-3}, 0.69620), \\ & \quad \hat{y}_d = \begin{pmatrix} 1.1777\times 10^{-5} & -3\times 10^{-16} & -9\times 10^{-13} \\ -6\times 10^{-18} & 2.2336\times 10^{-4} & 2\times 10^{-11} \\ -4\times 10^{-16} & 8\times 10^{-13} & 1.1737\times 10^{-2} \end{pmatrix}. \end{aligned} \tag{5.30}$$

Here $(\hat{y}_d)_{jk} = y_{d_j d_k}$. Note that in the evaluation we have used the values $(m_u, m_d) = (20, 2.90)$ MeV for the reason described earlier. The flavor-changing Yukawa couplings are exceedingly small. Splitting of mass Δm_M of neutral mesons ($M = \bar{d}_j d_k, d_j \bar{d}_k, j \neq k$) due to $y_{d_j d_k}$ is estimated to be at most $[m_M/(m_{d_j} + m_{d_k})]^2 (m_M f_M^2/m_H^2)(y_{d_j d_k})^2$, [30] which is much smaller than the observed Δm_M.

The values of the diagonal part of the Yukawa couplings can be understood from the effective theory as well. Recalling that the 4D Higgs field $H(x)$ is the fluctuation mode of the AB phase θ_H, the effective interactions of W, Z and fermion field ψ_f with the Higgs field can be written as [31]

$$\begin{aligned} & \mathcal{L} \sim -\bar{m}_W(\hat{\theta}_H)^2 W_\mu^\dagger W^\mu - \frac{1}{2}\bar{m}_Z(\hat{\theta}_H)^2 Z_\mu Z^\mu - \bar{m}_f(\hat{\theta}_H)\bar{\psi}_f \psi_f, \\ & \hat{\theta}_H(x) = \theta_H + \frac{H(x)}{f_H}. \end{aligned} \tag{5.31}$$

The mass functions are, in good approximation, given by

$$\begin{aligned} & \bar{m}_W(\hat{\theta}_H) \sim a_W \sin\hat{\theta}_H, \\ & \bar{m}_Z(\hat{\theta}_H) \sim a_Z \sin\hat{\theta}_H, \\ & \bar{m}_f(\hat{\theta}_H) \sim \begin{cases} a_f \sin\hat{\theta}_H & \text{in the A model} \\ a_f \sin\frac{1}{2}\hat{\theta}_H & \text{in the B model,} \end{cases} \end{aligned} \tag{5.32}$$

where a_W, a_Z, and a_f are constants. At the tree level $m_W = \bar{m}_W(\theta_H) = \frac{1}{2}g_w f_H \sin\theta_H$, $m_Z = \bar{m}_Z(\theta_H) = m_W/\cos\theta_W^0$ and $m_f = \bar{m}_f(\theta_H)$. Expanding the mass functions in (5.31) around θ_H, one finds the Higgs couplings to be

$$g_{WWH} = \frac{2m_W^2 \cos\theta_H}{f_H \sin\theta_H} = g_w m_W \cos\theta_H,$$
$$g_{ZZH} = \frac{2m_Z^2 \cos\theta_H}{f_H \sin\theta_H} = \frac{g_w m_Z}{\cos\theta_W^0} \cos\theta_H,$$
$$y_f = \begin{cases} \frac{m_f \cos\theta_H}{f_H \sin\theta_H} = \frac{m_f}{v_{\rm SM}} \cos\theta_H & \text{in the A model} \\ \frac{m_f \cos\frac{1}{2}\theta_H}{2f_H \sin\frac{1}{2}\theta_H} = \frac{m_f}{v_{\rm SM}} \cos^2\frac{1}{2}\theta_H & \text{in the B model.} \end{cases} \tag{5.33}$$

Here $v_{\rm SM} = f_H \sin\theta_H$. In other words, compared to the couplings in the SM, the Higgs couplings of W and Z in the gauge-Higgs unification are suppressed by a factor $\cos\theta_H$. The Yukawa couplings of quarks and leptons are suppressed by a factor $\cos\theta_H$ in the A model and by a factor $\cos^2\frac{1}{2}\theta_H$ in the B model.

The diagonal part of the evaluated Yukawa couplings (5.30) are well described by the formula in (5.33). Denoting the couplings in the SM by $y_f^{\rm SM} = m_f/v_{\rm SM}$, one finds

$$(a)\ \theta_H = 0.10\colon\ \cos^2\frac{1}{2}\theta_H = 0.99750$$
$$\left(\frac{y_{uu}}{y_u^{\rm SM}}, \frac{y_{cc}}{y_c^{\rm SM}}, \frac{y_{tt}}{y_t^{\rm SM}}\right) = (0.99758, 0.99758, 0.99826),$$
$$\left(\frac{y_{dd}}{y_d^{\rm SM}}, \frac{y_{ss}}{y_s^{\rm SM}}, \frac{y_{bb}}{y_b^{\rm SM}}\right) = (0.99758, 0.99758, 0.99758).$$
$$(b)\ \theta_H = 0.15\colon\ \cos^2\frac{1}{2}\theta_H = 0.99439$$
$$\left(\frac{y_{uu}}{y_u^{\rm SM}}, \frac{y_{cc}}{y_c^{\rm SM}}, \frac{y_{tt}}{y_t^{\rm SM}}\right) = (0.99456, 0.99456, 0.99607),$$
$$\left(\frac{y_{dd}}{y_d^{\rm SM}}, \frac{y_{ss}}{y_s^{\rm SM}}, \frac{y_{bb}}{y_b^{\rm SM}}\right) = (0.99456, 0.99456, 0.99456). \tag{5.34}$$

The deviation from the SM is rather small.

We would like to add a comment. As explained in Sec. II, the neutral physical scalar of $\Phi_{(\mathbf{1},\mathbf{4})}$ has a large mass ($\gg m_{\rm KK}$) so that its couplings to quarks and leptons at low energies are negligible, playing no role in flavor changing processes.

VI. SUMMARY AND DISCUSSIONS

In this paper we have shown that the flavor mixing in the quark sector can be incorporated in the GUT inspired $SU(3)_C \times SO(5) \times U(1)_X$ gauge-Higgs unification. The brane interactions on the UV brane are responsible both for splitting the mass spectrum between the up-type quarks and down-type quarks and for generating flavor mixing in the charged current (W) interactions. A quite reasonable form of the CKM matrix has been obtained. The mixing, in general, induces FCNC interactions in the Z couplings of quarks. It is shown that the FCNC interactions are naturally suppressed, with a suppression factor of order 10^{-6}. The suppression is a result of the $SU(3)_C \times SO(5) \times U(1)_X$ or $SU(3)_C \times SO(4) \times U(1)_X$ gauge invariance which allows only a certain class of interactions on the UV brane. In addition to presenting rigorous evaluation of the gauge couplings, we have also given an explanation in terms of the effective theory which illustrates how the natural suppression of the FCNC interactions results in the gauge-Higgs unification. The flavor-mixing induces flavor-changing Yukawa couplings as well. We have confirmed that those couplings are extremely small.

There remains an issue to be clarified. In the present model we could obtain a consistent spectrum and mixing only if the up-quark mass m_u were larger than the down-quark mass m_d. With the minimal matter content in the GUT inspired gauge-Higgs unification, m_d necessarily becomes smaller than m_u. One may have an additional field which affects m_u, or may consider the running of quark masses which reverses the order of m_u and m_d at low energies. We leave the issue for future investigation.

In the GUT inspired gauge-Higgs unification we have chosen negative bulk mass parameters. With positive bulk mass parameters there arise exotic light fermions with the same quantum numbers as the down-type quarks. Although negative bulk mass parameters imply that left-handed (right-handed) light quarks are localized near the IR (UV) brane, we have shown that the W and Z couplings of all quarks are very close to those in the SM. This is one of the remarkable properties in the gauge-Higgs unification in the RS space. Similarly negative bulk mass parameters of leptons are preferred to positive ones, as positive ones yield additional light neutral fermions.

The sign of the bulk mass parameters of quarks and leptons can be investigated by e^+e^- collider experiments, as the couplings of quarks and leptons to Z' bosons, namely KK excited states of Z, γ, and Z_R, have large parity violation. It has been shown in the previous A model of $SO(5) \times U(1)$ gauge-Higgs unification that right-handed quarks and leptons have much larger couplings to Z' bosons so that in the process $e^+e^- \to \mu^+\mu^-$, for instance, significant deviation from the SM appears even at 250 GeV at the ILC with 250 fb^{-1} data. If the e^- beam is polarized in the left-handed mode, there would be no deviation from the SM, whereas, if the e^- beam is polarized in the right-handed mode, then there appears large deviation. By changing the polarization of the e^- beam, one can see a distinct pattern of deviation. Similar effects are seen in the forward-backward asymmetry in various processes as well. In the present B model left-handed leptons and quarks have much larger couplings to Z' bosons than right-handed ones. As a consequence the pattern of the dependence on the e^- polarization is reversed in comparison with that in the A model. ILC experiments can provide rich information on underlying physics.

Gauge-Higgs unification is formulated in five or higher dimensions in which the running of gauge couplings is much more rapid than in four dimensions [32] In this paper we have analyzed the W and Z couplings of quarks below the KK mass scale $m_{\rm KK}$. All relations presented in this paper should be understood as those for the energy scale below $m_{\rm KK}$. Above $m_{\rm KK}$ effects of KK modes need to be properly incorporated. Gauge-Higgs unification is a new approach to physics beyond the SM. It may provide a key to solving the problems of dark matter, gauge hierarchy, neutrinos, Higgs couplings, and grand unification as well [33–36]. We will come back to these issues in the future.

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APPENDIX: BASIS FUNCTIONS

We summarize basis functions in the RS space. We define

$$F_{\alpha,\beta}(u,v) \equiv J_\alpha(u)Y_\beta(v) - Y_\alpha(u)J_\beta(v) \tag{A1}$$

where $J_\alpha(x)$ and $Y_\alpha(x)$ are Bessel functions of the first and second kind, respectively. For gauge bosons $C = C(z;\lambda)$ and $S = S(z;\lambda)$ are defined by

$$\begin{aligned} C(z;\lambda) &= +\frac{\pi}{2}\lambda z z_L F_{1,0}(\lambda z, \lambda z_L), \\ C'(z;\lambda) &= +\frac{\pi}{2}\lambda^2 z z_L F_{0,0}(\lambda z, \lambda z_L), \\ S(z;\lambda) &= -\frac{\pi}{2}\lambda z F_{1,1}(\lambda z, \lambda z_L), \\ S'(z;\lambda) &= -\frac{\pi}{2}\lambda^2 z F_{0,1}(\lambda z, \lambda z_L). \end{aligned} \tag{A2}$$

We note that $CS' - SC' = \lambda z$.

For massless fermions we define

$$\begin{aligned} \begin{pmatrix} C_R \\ S_R \end{pmatrix}(z;\lambda,c) &= \mp\frac{\pi}{2}\lambda\sqrt{z z_L}F_{c-\frac{1}{2},c\pm\frac{1}{2}}(\lambda z, \lambda z_L), \\ \begin{pmatrix} C_L \\ S_L \end{pmatrix}(z;\lambda,c) &= \pm\frac{\pi}{2}\lambda\sqrt{z z_L}F_{c+\frac{1}{2},c\mp\frac{1}{2}}(\lambda z, \lambda z_L). \end{aligned} \tag{A3}$$

These satisfy $C_L C_R - S_L S_R = 1$, $C_L(z;\lambda,-c) = C_R(z;\lambda,c)$, and $S_L(z;\lambda,-c) = -S_R(z;\lambda,c)$. For massive fermions such as $D^\pm$ fields with $m_D \neq 0$ we define basis functions

$$\begin{aligned} \begin{pmatrix} \mathcal{C}_{R1} \\ \mathcal{C}_{L1} \end{pmatrix}(z;\lambda,c,\tilde{m}) &= \begin{pmatrix} C_R \\ C_L \end{pmatrix}(z;\lambda,c+\tilde{m}) + \begin{pmatrix} C_R \\ C_L \end{pmatrix}(z;\lambda,c-\tilde{m}), \\ \begin{pmatrix} \mathcal{C}_{R2} \\ \mathcal{C}_{L2} \end{pmatrix}(z;\lambda,c,\tilde{m}) &= \begin{pmatrix} S_R \\ S_L \end{pmatrix}(z;\lambda,c+\tilde{m}) - \begin{pmatrix} S_R \\ S_L \end{pmatrix}(z;\lambda,c-\tilde{m}), \\ \begin{pmatrix} \mathcal{S}_{R1} \\ \mathcal{S}_{L1} \end{pmatrix}(z;\lambda,c,\tilde{m}) &= \begin{pmatrix} S_R \\ S_L \end{pmatrix}(z;\lambda,c+\tilde{m}) + \begin{pmatrix} S_R \\ S_L \end{pmatrix}(z;\lambda,c-\tilde{m}), \\ \begin{pmatrix} \mathcal{S}_{R2} \\ \mathcal{S}_{L2} \end{pmatrix}(z;\lambda,c,\tilde{m}) &= \begin{pmatrix} C_R \\ C_L \end{pmatrix}(z;\lambda,c+\tilde{m}) - \begin{pmatrix} C_R \\ C_L \end{pmatrix}(z;\lambda,c-\tilde{m}). \end{aligned} \tag{A4}$$

These functions satisfy various relations which are summarized in Appendix B of Ref. [13].

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