Causality, unitarity, and indefinite metric in Maxwell-Chern-Simons extensions

Ricardo Avila,[1,*] Jose R. Nascimento,[2,†] Albert Yu. Petrov,[2,‡] Carlos M. Reyes,[1,§] and Marco Schreck[3,∥]

[1]*Departamento de Ciencias Básicas, Universidad del Bío-Bío, 3800708 Chillán, Chile*
[2]*Departamento de Física, Universidade Federal da Paraíba, 58051-970 João Pessoa, Paraíba, Brazil*
[3]*Departamento de Física, Universidade Federal do Maranhão, 65080-805 São Luís, Maranhão, Brazil*

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We canonically quantize $(2+1)$-dimensional electrodynamics including a higher-derivative Chern-Simons term. The effective theory describes a standard photon and an additional degree of freedom associated with a massive ghost. We find the Hamiltonian and the algebra satisfied by the field operators. The theory is characterized by an indefinite metric in the Hilbert space that brings up questions on causality and unitarity. We study both of the latter fundamental properties and show that microcausality as well as perturbative unitarity up to one-loop order are conserved when the Lee-Wick prescription is employed.

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I. INTRODUCTION

The concept of an indefinite metric in a Hilbert space plays a fundamental role in the formulation of relativistic quantum field theory. Dirac was the first to show how an indefinite metric arises in quantum electrodynamics and proposed how to deal with its probability interpretation [1]. One can mention two reasons for Dirac's suggestion. On the one hand, any finite representation of a noncompact group—the Lorentz group included—leads to a state space endowed with an indefinite metric. On the other hand, the commutator of two vector field operators reads

$$[A_\mu(x), A_\nu(y)] = \mathrm{i}\eta_{\mu\nu} D(x-y), \tag{1}$$

with the scalar commutator function D and the Minkowski metric $\eta_{\mu\nu}$. The difference in the signs of the metric components η_{00} and η_{ii} induces an indefinite metric in the corresponding state space; see, in particular, Heisenberg's contribution in the list of references [2–4].

Gupta and Bleuler used this concept within the covariant quantization of electrodynamics. The Gupta-Bleuler formalism shows that the unphysical degrees of freedom are eliminated by imposing the weak Lorentz condition on the Hilbert space. Much of the motivation for studying indefinite metric theories comes from the theory of gravitation, where the nonrenormalizability of the Einstein-Hilbert action forces one to consider the possibility of modified gravity theories. Some of them also introduce indefinite metrics in the Hilbert space [5–7].

The most notorious drawback of indefinite-metric theories is the possibility of negative probabilities leading to the loss of unitarity. Unitarity in this context has been studied extensively for the past decades. In the 1960s, Lee and Wick, being attracted by the idea of reconciling the divergencies in quantum electrodynamics (QED) without spoiling unitarity, constructed a modified electrodynamics with an indefinite metric. Their theory, which is known as the Lee-Wick model [8,9], is a modified electrodynamics including a massive boson field associated with negative metric components. One characteristic of the propagator of their theory is that it contains complex conjugate pairs of additional poles, which are called Lee-Wick poles.

The Lee-Wick model is also obtained by introducing a higher-derivative term into the Lagrangian [10]. In this model, perturbative unitarity of the S matrix has been successfully implemented via the Cutkosky-Landshoff-Olive-Polkinghorne prescription in which a pair of Lee-Wick poles cancel each other out in cut diagrams [11]. Several approaches have provided a deeper understanding of many physical aspects of Lee-Wick models in recent years [12–15]. In fact, investigations aimed at providing finiteness in quantum field theory have not stopped, reaching diverse application within nonlocal quantum gravity; see, e.g., [16–18] and higher-derivative gravity extensions studied even earlier [5].

Basically, the loss of unitarity occurs due to the negative contribution of the residue of the ghost field to scattering

*raavilavi@gmail.com
†jroberto@fisica.ufpb.br
‡petrov@fisica.ufpb.br
§creyes@ubiobio.cl
∥marco.schreck@ufma.br

cross sections. In this case the cutting equations provided by the optical theorem cannot be satisfied. It was demonstrated that one can modify the definition of the internal product in the Hilbert space in order to cope with the unitarity problem. However, this approach leads to theories characterized by non-Hermitian Hamiltonians, i.e., they exhibit a nonstandard time evolution. However, Bender and collaborators found that such Hamiltonians have real eigenvalues when they are symmetric under PT transformations [19]. Scenarios of this kind have attracted an exceeding amount of interest, see, e.g., [20,21] where non-Hermitian Hamiltonians are discussed, too.

Another motivation for the interest in indefinite-metric theories originated from gravity where it was demonstrated that adding higher-derivative terms allows for gravity to be renormalizable [5]. This fact implied active studies of renormalization of R^2 gravity and other higher-derivative gravity theories (see, e.g., [22] and references therein). Nevertheless, it was realized soon that this kind of improvement of the renormalization behavior inevitably leads to ghosts. From the formal viewpoint, their presence can be explained as follows. Consider the example of a propagator $\frac{1}{k^2(k^2+m^2)}$ occurring in a fourth-derivative theory. A simple transformation shows that this propagator describes two particles: a massless and a massive one. The propagator of the latter carries a negative sign, whereby the massive particle corresponds to a free scalar field with possibly negative energy. Even if the energy in the theory can be bounded from below due to a redefinition of vacuum, unitarity, upon the presence of interactions, is expected to be broken (see [23,24] for more detailed explanations).

Furthermore, more problems related to the consistent quantum description of higher-derivative theories were discussed in [13,14,25]. In the latter papers, it was claimed that these problems actually arise due to differences between the behaviors of the theory in Minkowski spacetime and its counterpart in Euclidean space. At the same time, it was argued in [26] that in certain cases the ghosts are "benign" so that the theory turns out to be perturbatively unitary, with the vacuum being perturbatively stable. Therefore, the problem of ghosts must be considered separately for any higher-derivative theory.

An interesting example of a higher-derivative extension of QED containing dimension-5 operators was proposed by Myers and Pospelov [27]. The higher-derivative term in its Lagrangian, called the Myers-Pospelov term, involves explicit Lorentz symmetry breaking, so that for some special choice of the Lorentz-breaking preferred four-vector, higher time derivatives do not arise, whereupon unitarity breaking is avoided. In case an indefinite metric occurs, one can apply the Lee-Wick prescription to show that unitarity is conserved [28–30]. According to the latter, all negative-norm states are removed from the asymptotic Hilbert space. This procedure will turn out to be fruitful in the analysis that we intend to carry out in the current paper.

A further interesting Lorentz-breaking modification of QED is the higher-derivative Carroll-Field-Jackiw-like term exhibiting a similar behavior (both of these terms were shown to be generated perturbatively at the one-loop level, whereby the corresponding contributions are finite, see [31]). In a different context, though, the possibility of Lorentz violation due to an indefinite metric was pointed out several years ago by Nakanishi [32,33].

Therefore, to understand the physical impact of effective higher-derivative extensions of QED, it is important to check how such terms affect unitarity. To do so, though, it is reasonable to investigate a simplified model first, that is, $(2+1)$-dimensional QED with an additive higher-derivative Chern-Simons (CS) term, which does not involve Lorentz symmetry breaking. Some classical issues related to this theory such as the nature and behavior of degrees of freedom were analyzed earlier in [34]. Its canonical formulation was discussed in [35] and the perturbative generation of the higher-derivative CS term was carried out in [36]. Here, we intend to elaborate on the aspects of microcausality and unitarity of this theory.

The structure of the paper looks as follows. In Sec. II, we introduce the classical action and the propagator of our theory and write down the classical field equations, the dispersion equation, and its solutions. Furthermore, we decompose the higher-derivative theory into a standard one involving degrees of freedom associated with a three-component photon field and a second contribution in terms of a Proca ghost field. We then find the polarization vectors for the photon and the massive ghost as well as their stress tensors. In Sec. III, we canonically quantize the theory, construct the field operators such that they satisfy the expected algebra, and analyze the constraint structure in combination with finding the Hamiltonian. In Sec. IV, we verify tree-level unitarity of our theory and we also study perturbative unitarity at one-loop level. Section V states a final summary and discussion of our results. Appendix A contains details of the derivation of Dirac brackets and the Dirac formalism that reduces second-class constraints to zero. Appendix B explains how to express the Hamiltonian of the theory in terms of creation and annihilation operators. Appendix C delivers detailed computations of the nonzero equal-time commutators satisfied by the field operators. Finally, Appendix D provides a summary of the most important properties of a Dirac theory in $(2+1)$ dimensions.

II. HIGHER-DERIVATIVE MAXWELL-CHERN-SIMONS THEORY

In this section, we present the higher-derivative CS term coupled to the Maxwell Lagrangian in $(2+1)$ dimensions. The theory describes a standard photon and a massive mode at high energies associated with a ghost. To show this, we apply a linear transformation to the higher-derivative Lagrangian, decoupling it into a sum of two standard-derivative parts. We find the polarization vectors and connect their sum with the propagator, which simplifies the study of unitarity in Sec. IV.

A. The (2 + 1)-dimensional model

Our starting point consists of a Lagrangian that is the sum of the standard Maxwell term and the higher-derivative CS extension in $(2+1)$ dimensions [34], given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g}{2}\epsilon^{\alpha\beta\gamma}(\Box A_\alpha)(\partial_\beta A_\gamma) + \mathcal{L}_{\mathrm{GF}}, \quad (2)$$

where $\Box = \partial^\mu\partial_\mu$ is the d'Alembertian and g is a small constant with inverse mass dimension. We will see that the inverse of g is related to a mass scale. Thus, it is assumed that $g > 0$. Furthermore, $\mathcal{L}_{\mathrm{GF}}$ is a covariant gauge-fixing term inversely proportional to the arbitrary gauge-fixing parameter ξ,

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2. \quad (3)$$

We take the metric convention $\eta_{\mu\nu} = \mathrm{diag}(+,-,-)$, and our definition of the Levi-Civita symbol is based on $\epsilon^{012} = \epsilon_{012} = \epsilon^{12} = \epsilon_{12} = 1$. Hence, all Lorentz indices run over 0,1,2.

In our study we do not consider the usual single-derivative CS term for the sake of simplicity, since we aim at keeping track of the higher-derivative contribution. We note that the CS term is suppressed above some energy scale in comparison to our higher-derivative term. In principle, though, it is natural to expect that it would not render the physics essentially different. Nevertheless, the complete analysis of unitarity and, especially, of the Dirac algebra of constraints would be much more involved if the CS term were present. Therefore, we discard it in our analysis.

We note in passing that a $(2+1)$-dimensional Lorentz-violating electromagnetism involving higher-derivative terms was derived in [37] from the electromagnetic sector of the nonminimal Standard Model extension [38] via a procedure known as dimensional reduction (see, e.g., [39,40]). The second contribution in Eq. (2) can be mapped onto the third one in $\mathcal{L}_{(1+2)}$ of [37] via suitable partial integrations.

The treatment of systems in classical mechanics described by higher-derivative Lagrangians was initiated by Ostrogradsky in his seminal paper [41]. Subsequent scientific papers reviewing and extending his original ideas are [42–44], where this list is not claimed to be exhaustive. One of the central results of these works is that an application of the Hamilton principle leads to a modified set of Euler-Lagrange equations. An analogous development of the formalism in the context of higher-derivative field theory can be found, e.g., in [45]. For the particular field theory defined by Eq. (2), it is sufficient to restrict these generalized Euler-Lagrange equations to

$$-\partial_\kappa\partial_\lambda\frac{\partial\mathcal{L}}{\partial(\partial_\kappa\partial_\lambda A_\sigma)} + \partial_\rho\frac{\partial\mathcal{L}}{\partial(\partial_\rho A_\sigma)} - \frac{\partial\mathcal{L}}{\partial A_\sigma} = 0. \quad (4)$$

They lead to the modified Maxwell equations

$$\partial_\rho F^{\rho\sigma} + g\epsilon^{\sigma\beta\gamma}\Box\partial_\beta A_\gamma + \frac{1}{\xi}\partial^\sigma(\partial\cdot A) = 0. \quad (5)$$

Now, contracting Eq. (5) with ∂_σ yields

$$\frac{1}{\xi}\Box(\partial\cdot A) = 0. \quad (6)$$

Hence, by imposing suitable boundary conditions at infinity it follows that $\partial\cdot A = 0$ can be set.

Now, let us rewrite the Lagrangian (2) as

$$\mathcal{L} = \frac{1}{2}A_\mu\left[\Box\eta^{\mu\nu} - \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu + g\epsilon^{\mu\beta\nu}\partial_\beta\Box\right]A_\nu, \quad (7)$$

yielding the equations of motion for the gauge field:

$$\left[\Box\eta^{\mu\nu} - \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu + g\epsilon^{\mu\beta\nu}\partial_\beta\Box\right]A_\nu(x) = 0. \quad (8)$$

Transforming the latter to the momentum representation with $\mathrm{i}\partial_\mu = p_\mu$, we write

$$S^{\mu\nu}(p)A_\nu(p) = 0, \quad (9a)$$

with

$$S^{\mu\nu}(p) = p^2\left[\eta^{\mu\nu} - \left(1-\frac{1}{\xi}\right)\frac{p^\mu p^\nu}{p^2} - \mathrm{i}g\epsilon^{\mu\beta\nu}p_\beta\right]. \quad (9b)$$

The propagator $P_{\mu\nu}$ follows from inverting the operator $S^{\mu\nu}$, giving

$$P_{\mu\nu}(p) = -\frac{G_{\mu\nu}(\xi,p)}{p^2(1-g^2p^2)}, \quad (10a)$$

where

$$G_{\mu\nu}(\xi,p) = \eta_{\mu\nu} - [1-\xi(1-g^2p^2)]\frac{p_\mu p_\nu}{p^2} + \mathrm{i}g\epsilon_{\mu\beta\nu}p^\beta. \quad (10b)$$

The conventions have been chosen such that the propagator satisfies

$$S^{\mu\nu}(p)P_{\nu\rho}(p) = -\delta^{\mu}{}_{\rho}. \tag{11}$$

Considering the pole structure of the propagator (10) and defining $g \equiv M^{-1}$, we decompose the denominator as

$$\frac{M^2}{p^2(p^2 - M^2)} = -\frac{1}{p^2} + \frac{1}{p^2 - M^2}, \tag{12}$$

where the second contribution has a residue whose sign is opposite that of the first contribution. Hence, it can be associated with a ghost. The dispersion relations are given by the propagator poles with respect to p_0. Determining the poles yields the modes corresponding to a photon and a massive gauge field given by

$$\omega(\vec{p}) = \omega_p = |\vec{p}|, \tag{13a}$$

$$\Omega(\vec{p}) = \Omega_p = \sqrt{\vec{p}^2 + M^2}, \tag{13b}$$

respectively.

Let us write down the energy-momentum tensor of our theory. It is clear that it is a sum of two contributions. The first is the energy-momentum tensor for electrodynamics in $(2+1)$ dimensions whose symmetric form is the well-known Belinfante tensor equal to

$$T^{\mu\nu}_{\text{Bel}} = F^{\mu}{}_{\lambda}F^{\lambda\nu} + \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}. \tag{14}$$

The second is connected to the higher-derivative Chern-Simons (HDCS) theory, whose symmetric form was found explicitly in [34]. So we merely quote the result, which is

$$T^{\mu\nu}_{\text{HDCS}} = g[(\epsilon^{\mu\alpha\beta}F^{*\nu} + \epsilon^{\nu\alpha\beta}F^{*\mu})\partial_\alpha F^*_\beta - \eta^{\mu\nu}\epsilon^{\alpha\beta\gamma}F^*_\alpha\partial_\beta F^*_\gamma], \tag{15}$$

where $F^*_\alpha = \frac{1}{2}\epsilon_{\alpha\mu\nu}F^{\mu\nu}$ is the dual of the field strength tensor $F^{\mu\nu}$.

B. Decoupling the ghost

Here we make explicit the two types of fields described by the Lagrangian (2). We define the new fields as

$$\bar{A}_\mu = \frac{1}{\sqrt{2}}(A_\mu + gF^*_\mu), \tag{16a}$$

$$G_\mu = \frac{g}{\sqrt{2}}F^*_\mu, \tag{16b}$$

in terms of the dual tensor F^*_μ defined under Eq. (15) and the original photon field A_μ.

Considering Eqs. (16a) and (16b), we find the identities

$$-\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} = -\frac{1}{8}F_{\mu\nu}F^{\mu\nu} - \frac{g}{2}\left(\partial_\mu A_\nu + \frac{g}{2}\partial_\mu F^*_\nu\right)(\partial^\mu F^{*\nu} - \partial^\nu F^{*\mu}) \tag{17a}$$

and

$$-\frac{1}{4}F^*_\mu F^{*\mu} = -\frac{1}{8}F_{\mu\nu}F^{\mu\nu}, \tag{17b}$$

where $\bar{F}_{\mu\nu} = \partial_\mu\bar{A}_\nu - \partial_\nu\bar{A}_\mu$ is the field strength tensor associated with the new field of Eq. (16a).

Now, by adding both equations, performing suitable integrations by parts, and using the (unmodified) homogeneous Maxwell equation $\partial_\mu F^{*\mu} = 0$ in $(2+1)$ dimensions, we can rewrite the first part of the Lagrangian (2) as

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g}{2}\epsilon^{\alpha\beta\gamma}(\Box A_\alpha)(\partial_\beta A_\gamma) = -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} - \frac{g^2}{4}F^*_\mu\left(\frac{1}{g^2} + \Box\right)F^{*\mu}. \tag{18}$$

Using the definition (16b) and $\partial_\mu A^\mu = \sqrt{2}\partial_\mu\bar{A}^\mu$ allows us to write the higher-derivative Lagrangian as the sum

$$\mathcal{L} = -\frac{1}{4}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} - \frac{1}{\xi}(\partial_\mu\bar{A}^\mu)^2 + \frac{1}{2}\partial_\mu G_\nu\partial^\mu G^\nu - \frac{1}{2}M^2 G_\mu G^\mu, \tag{19}$$

where the higher derivatives have been absorbed into the new fields. The first part of the new Lagrange density describes a photon with a gauge-fixing term and the second part corresponds to a Proca field theory involving a mass scale of the order of $M \sim g^{-1}$. As the coupling constant g of the modification is assumed to be small, the latter mass scale M is supposed to be large. The Proca field theory presumably describes a ghost dominating the regime of high energies.

C. Polarization vectors

Now that the theory has been decomposed into two decoupled standard-derivative parts associated with the fields of Eqs. (16a) and (16b), our next step is to find the polarization vectors. First, they are crucial for the computation of the Hamiltonian in terms of creation and annihilation operators. Second, they are needed to construct the tensor structure in the equal-time commutation relations of the field operators. Last but not least, the propagator can be expressed in terms of the polarization vectors, which will be helpful to prove the validity of the optical theorem.

To begin with, consider the following orthogonal basis of $(2+1)$-dimensional Minkowski spacetime that involves the three real vectors

$$e^{(0)\mu} = \frac{1}{\sqrt{p^2}} p^\mu, \tag{20a}$$

$$e^{(1)\mu} = \frac{1}{\sqrt{G}} \epsilon^{\mu\beta\gamma} p_\beta n_\gamma, \tag{20b}$$

$$e^{(2)\mu} = -\frac{1}{\sqrt{p^2}} \epsilon^{\mu\beta\gamma} p_\beta e^{(1)}_\gamma = \frac{1}{\sqrt{p^2 G}} (p^2 n^\mu - p^\mu (p\cdot n)), \tag{20c}$$

where $G = (p\cdot n)^2 - p^2 n^2$ and n^μ is an auxiliary three-vector. The three-vectors $e^{(a)}_\mu$ are normalized according to

$$e^{(a)} \cdot e^{(b)} = g_{ab}, \tag{21}$$

with $a = 0$, 1, 2 and $g_{ab} = \mathrm{diag}(1,-1,-1)$. Although g_{ab} formally corresponds to the Minkowski metric in $(2+1)$ dimensions, we use another symbol here, as the indices of this object are not Lorentz indices, but merely the labels of the vectors introduced before. In order to ensure $G > 0$, we will take $p^2 > 0$ and choose n^μ as a timelike vector.

Furthermore, these vectors satisfy the completeness relation

$$\sum_{a,b=0}^{2} g_{ab} e^{(a)}_\mu e^{(b)}_\nu = \eta_{\mu\nu}. \tag{22}$$

However, note that the above basis is not suitable to describe the photon field due to the denominator depending on $\sqrt{p^2}$. To construct suitable polarization vectors for photons we will proceed differently in Sec. III A.

Moreover, one can check that the $e^{(a)}_\mu$ fulfill the relations

$$\epsilon^{\mu\beta\gamma} p_\beta e^{(2)}_\gamma = \sqrt{p^2} e^{(1)\mu}, \tag{23a}$$

$$\epsilon^{\mu\beta\gamma} p_\beta e^{(1)}_\gamma = -\sqrt{p^2} e^{(2)\mu}. \tag{23b}$$

With the real basis $\{e^{(a)}\}$ at hand, we look for a complex basis $\{\varepsilon^{(\lambda)}\}$ diagonalizing the operator $S^\mu{}_\nu(p)$ of Eq. (9b). Our intention is to relate the propagator to the sum of polarization tensors formed from the vectors of $\{\varepsilon^{(\lambda)}\}$. This particular method was introduced in [46] and applied in the context of the Maxwell-Chern-Simons-like theory in $(3+1)$ dimensions. We adopt it to the theory of Eq. (2), as it turned out to be valuable for checking the validity of the optical theorem. Hence, considering Eq. (9b) we demand that these vectors fulfill

$$S^\mu{}_\nu(p)\varepsilon^{(\lambda)\nu}(p) = \Lambda_\lambda(p)\varepsilon^{(\lambda)\mu}(p), \tag{24}$$

with the new label $\lambda \in \{0,+,-\}$ and the eigenvalue $\Lambda_\lambda(p)$ of the polarization mode λ.

We now define the complex basis as follows:

$$\varepsilon^{(0)\mu} = e^{(0)\mu}, \tag{25a}$$

$$\varepsilon^{(+)\mu} = \frac{e^{(2)\mu} + \mathrm{i}e^{(1)\mu}}{\sqrt{2}}, \tag{25b}$$

$$\varepsilon^{(-)\mu} = \frac{e^{(2)\mu} - \mathrm{i}e^{(1)\mu}}{\sqrt{2}}. \tag{25c}$$

The $\pm$ modes are orthogonal to the momentum, that is, $p\cdot\varepsilon^{(\pm)} = 0$. By using Eqs. (21) and (23a) one can show that

$$\varepsilon^{(\lambda)} \cdot \varepsilon^{(\lambda')*} = g_{\lambda\lambda'}, \tag{26a}$$

$$\epsilon^{\mu\beta\sigma} p_\beta \varepsilon^{(\pm)}_\sigma = \mp\, \mathrm{i}\sqrt{p^2}\varepsilon^{(\pm)\mu}, \tag{26b}$$

with $g_{\lambda\lambda'} = \mathrm{diag}(1,-1,-1)$. Note that the latter matrix again corresponds to the Minkowski metric in $(2+1)$ dimensions. As its indices are the labels of the vectors $\{\varepsilon^{(\lambda)}\}$, we denote it by $g_{\lambda\lambda'}$.

Indeed, it is not difficult to show that the vectors of Eq. (25) diagonalize $S^{\mu\nu}$, i.e.,

$$S^\mu{}_\nu(p)\varepsilon^{(0)\nu} = \Lambda_0(p)\varepsilon^{(0)\mu}, \tag{27a}$$

$$S^\mu{}_\nu(p)\varepsilon^{(+)\nu} = \Lambda_+(p)\varepsilon^{(+)\mu}, \tag{27b}$$

$$S^\mu{}_\nu(p)\varepsilon^{(-)\nu} = \Lambda_-(p)\varepsilon^{(-)\mu}, \tag{27c}$$

where the eigenvalues are given by

$$\Lambda_0(p) = \frac{p^2}{\xi}, \tag{28a}$$

$$\Lambda_+(p) = p^2\left(1 - g\sqrt{p^2}\right), \tag{28b}$$

$$\Lambda_-(p) = p^2\left(1 + g\sqrt{p^2}\right). \tag{28c}$$

The dispersion relations of our theory follow from requiring that the product of eigenvalues vanish,

$$\prod_{\lambda=0,\pm} \Lambda_\lambda(p) = \frac{1}{\xi}(p^2)^3(1 - g^2 p^2) = 0, \tag{29}$$

giving the dispersion relations of Eqs. (13a) and (13b) for the photon and massive ghost mode, respectively. Hence, the vectors of the basis $\{\varepsilon^{(\lambda)}\}$ are solutions of the field equations when they are evaluated on shell. Therefore, they can be interpreted as polarization vectors.

From these relations, it is possible to show that

$$\varepsilon_\mu^{(\pm)}\varepsilon_\nu^{(\pm)*} = -\frac{1}{2}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \pm \mathrm{i}\frac{\epsilon_{\mu\beta\nu}p^\beta}{\sqrt{p^2}}\right) \tag{30}$$

or

$$\varepsilon_\mu^{(\pm)}\varepsilon_\nu^{(\pm)*} = \frac{1}{2}(e_{\mu\nu} \pm \mathrm{i}\epsilon_{\mu\nu}), \tag{31}$$

where we have defined the tensors $e_{\mu\nu}$ and $\epsilon_{\mu\nu}$ by

$$e_{\mu\nu} \equiv e_\mu^{(1)}e_\nu^{(1)} + e_\mu^{(2)}e_\nu^{(2)} = -\eta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}, \tag{32a}$$

$$\epsilon_{\mu\nu} \equiv e_\mu^{(1)}e_\nu^{(2)} - e_\nu^{(1)}e_\mu^{(2)} = -\frac{1}{\sqrt{p^2}}\epsilon_{\mu\beta\nu}p^\beta. \tag{32b}$$

Now, to make contact with the propagator $P_{\mu\nu}$ of Eq. (10a) via the relation [46]

$$P_{\mu\nu} = -\sum_{\lambda,\lambda'=0,\pm} g_{\lambda\lambda'}\frac{\varepsilon_\mu^{(\lambda)}\varepsilon_\nu^{(\lambda')*}}{\Lambda_\lambda}, \tag{33}$$

we consider the sum over two-tensors formed from the polarization vectors.

First, we investigate the transverse part and perform the sum over the $\pm$ modes. Based on the eigenvalues of Eqs. (28a), (28b), and (28c) and the finding of Eq. (30), we have

$$\frac{\varepsilon_\mu^{(+)}\varepsilon_\nu^{(-)}}{\Lambda_+} + \frac{\varepsilon_\mu^{(-)}\varepsilon_\nu^{(+)}}{\Lambda_-} = -\frac{1}{p^2(1-g^2p^2)}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} + \mathrm{i}g\epsilon_{\mu\beta\nu}p^\beta\right). \tag{34}$$

Next, by adding the mode labeled with $\lambda = 0$ we obtain

$$\frac{\varepsilon_\mu^{(0)}\varepsilon_\nu^{(0)}}{\Lambda_0} - \frac{\varepsilon_\mu^{(+)}\varepsilon_\nu^{(-)}}{\Lambda_+} - \frac{\varepsilon_\mu^{(-)}\varepsilon_\nu^{(+)}}{\Lambda_-} = \frac{1}{p^2(1-g^2p^2)}\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} + \mathrm{i}g\epsilon_{\mu\beta\nu}p^\beta\right) + \frac{\xi p_\mu p_\nu}{(p^2)^2}, \tag{35}$$

to finally arrive at

$$-\sum_{\lambda,\lambda'=0,\pm} g_{\lambda\lambda'}\frac{\varepsilon_\mu^{(\lambda)}\varepsilon_\nu^{(\lambda')*}}{\Lambda_\lambda} = -\frac{1}{p^2(1-g^2p^2)} \times\left[\eta_{\mu\nu} - (1-\xi(1-g^2p^2))\frac{p_\mu p_\nu}{p^2} + \mathrm{i}g\epsilon_{\mu\beta\nu}p^\beta\right]. \tag{36}$$

The latter is just the propagator of Eq. (10). Hence, the method introduced in [46,47] turns out to work in the context of the $(2+1)$-dimensional theory defined by Eq. (2), as well.

III. CANONICAL QUANTIZATION

In this section, we quantize the higher-derivative theory starting from the extended symplectic structure provided by the Ostrogradsky formalism [41–44] applied to the context of higher-derivative field theories [45]. The theory of Eq. (2) has constraints that modify the canonical Poisson brackets rendering its quantization more involved. We compute the Hamiltonian by choosing a particular vacuum state and show that the theory is stable, but the associated Hilbert space is endowed with an indefinite metric. We prove that in spite of the presence of negative-norm states, which can be interpreted as ghosts, causality is preserved in the theory.

A. Constrained Hamiltonian formulation

We consider the Lagrangian (2) for $\xi = 1$ and after some integration by parts we arrive at

$$\mathcal{L} = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu + \frac{1}{2}g\epsilon^{\mu\beta\gamma}\Box A_\mu \partial_\beta A_\gamma. \tag{37}$$

The variational methods of higher-derivative theories [41–45] are applied to obtain the canonical conjugated momenta to both A_μ and $\dot{A}_\mu$. They are given by

$$P^\mu = \frac{\partial\mathcal{L}}{\partial\dot{A}_\mu} - \frac{\partial\Pi^\mu}{\partial t}, \tag{38a}$$

$$\Pi^\mu = \frac{\partial\mathcal{L}}{\partial\ddot{A}_\mu}, \tag{38b}$$

respectively. The higher-order Hamiltonian follows from an extended Legendre transformation,

$$H = \int \mathrm{d}^2x(P^\mu(x)\dot{A}_\mu(x) + \Pi^\mu(x)\ddot{A}_\mu(x) - \mathcal{L}(x)), \tag{39}$$

and the canonical Poisson brackets for the extended phase space are

$$\{A_\mu(t,\vec{x}),P_\nu(t,\vec{y})\}=\eta_{\mu\nu}\delta^{(2)}(\vec{x}-\vec{y}),\tag{40a}$$

$$\{\dot{A}_\mu(t,\vec{x}),\Pi_\nu(t,\vec{y})\}=\eta_{\mu\nu}\delta^{(2)}(\vec{x}-\vec{y}),\tag{40b}$$

where the remaining ones vanish.

Applying these formulas to the specific Lagrangian (37) one finds

$$P^\mu=-\dot{A}^\mu-\frac{g}{2}\epsilon^{\mu0\gamma}\Box A_\gamma-\frac{g}{2}\epsilon^{\mu\beta\gamma}\partial_\beta\dot{A}_\gamma,\tag{41a}$$

$$\Pi^\mu=\frac{g}{2}\epsilon^{\mu\beta\gamma}\partial_\beta A_\gamma.\tag{41b}$$

After one inserts them into Eq. (39), the Hamiltonian reads

$$H=\int \mathrm{d}^2x\left(-\frac{1}{2}\dot{A}_\mu\hat{U}^{\mu\nu}\dot{A}_\nu+\frac{1}{2}A_\mu\hat{U}^{\mu\nu}\nabla^2A_\nu+\frac{g}{2}\epsilon^{ij}\dot{A}_i\Box A_j\right),\tag{42a}$$

where we have defined the tensor operator

$$\hat{U}^{\mu\nu}=\hat{U}^{\mu\nu}(\partial)=\eta^{\mu\nu}+g\epsilon^{\mu\beta\nu}\partial_\beta.\tag{42b}$$

Recall the Levi-Civita symbol in $(2+1)$ dimensions defined below Eq. (3).

In order to quantize the theory, as usual, one postulates equal-time commutation relations on the phase space variables:

$$[A_\mu(t,\vec{x}),P_\nu(t,\vec{y})]=\mathrm{i}\eta_{\mu\nu}\delta^{(2)}(\vec{x}-\vec{y}),\tag{43a}$$

$$[\dot{A}_\mu(t,\vec{x}),\Pi_\nu(t,\vec{y})]=\mathrm{i}\eta_{\mu\nu}\delta^{(2)}(\vec{x}-\vec{y}),\tag{43b}$$

where all the others are defined to vanish.

However, for constrained systems, the above commutators are not always possible to satisfy [48]. For instance, taking the derivative $\frac{g}{2}\epsilon^{\mu\beta\gamma}\partial_\beta$ of the first field of the commutator

$$[A_\mu(t,\vec{x}),\dot{A}_\nu(t,\vec{y})]=0,\tag{44}$$

producing $\Pi(t,x)$, gives a relation incompatible with the commutator of Eq. (43b). Therefore, the canonical structure of constraints has to be taken into consideration in order to modify the Poisson brackets consistently. Some work in this direction has already been carried out; see the formulation of first- and second-class constraints for the higher-derivative Maxwell-Chern-Simons theory in [35,49,50]. In the latter papers, the Dirac approach has been implemented and the reduced Hamiltonian has been obtained successfully with second-class constraints strongly imposed to zero. The Dirac brackets together with the reduced Hamiltonian neatly reproduce the equations of motion.

Here, in order to implement quantization we will follow an alternative method. We will quantize the fields such that they satisfy the second-class constraints automatically via their expansion in terms of plane waves. That is, in addition to requiring that the plane waves propagate with energy ω_p of Eq. (13a) and Ω_p of Eq. (13b), respectively, we choose the polarization vectors such that the fields satisfy the equations of motion and the second-class constraints in the Dirac formalism; see Appendix A. Then, we expect the fields $A_\mu(t,\vec{x})$ and $\dot{A}_\mu(t,\vec{x})$ together with their canonical conjugate momenta to reproduce the Dirac algebra. We verify this property for each relevant field operator in Appendix C. Notice, though, that the field $A_\mu(t,\vec{x})$ cannot be considered physical in the sense of propagating degrees of freedom independent of the gauge-fixing parameter ξ. In Lorenz gauge, there is still the unphysical polarization vector associated with the mode $\lambda=0$.

Let us consider the decomposition of our gauge field A_μ in terms of the photon and massive ghost field of Eqs. (16a) and (16b) as follows:

$$A_\mu(x)=\bar{A}_\mu(x)+G_\mu(x).\tag{45}$$

By inserting the decomposition into the equation of motion (8) with $\xi=1$ and considering the on-shell condition for the photon, $\Box\bar{A}_\mu=0$, we arrive at

$$(\eta^{\mu\nu}+g\epsilon^{\mu\beta\nu}\partial_\beta)G_\nu=0.\tag{46}$$

By taking the derivative ∂_μ of Eq. (46), one has

$$\partial\cdot G=0.\tag{47}$$

Considering all these conditions, we can write the photon field operator as

$$\bar{A}_\mu(x)=\int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\sum_{\lambda=0,1,2}\frac{1}{2\omega_p}\left[a_{\vec{p}}^{(\lambda)}\bar{e}_\mu^{(\lambda)}(p)\mathrm{e}^{-\mathrm{i}p\cdot x}+a_{\vec{p}}^{(\lambda)\dagger}\bar{e}_\mu^{(\lambda)*}(p)\mathrm{e}^{\mathrm{i}p\cdot x}\right]_{p_0=\omega_p},\tag{48}$$

with suitable annihilation and creation operators $a_{\vec{p}}^{(\lambda)}=a^{(\lambda)}(\vec{p})$ and $a_{\vec{p}}^{(\lambda)\dagger}=a^{(\lambda)\dagger}(\vec{p})$, respectively, for the mode λ. The polarization vectors are chosen as

$$\bar{e}_\mu^{(\lambda)}(p)=\left(\eta_{\mu\nu}-\frac{3g^2}{8}p_\mu p_\nu+\frac{\mathrm{i}g}{2}\epsilon_{\mu\beta\nu}p^\beta\right)\bigg|_{p_0=\omega_p}v^{(\lambda)\nu}(p),\tag{49a}$$

where

$$v^{(0)\mu}(p)=n^\mu,\tag{49b}$$

$$v^{(1)\mu}(p) = \frac{\epsilon^{\mu\beta\gamma}p_\beta n_\gamma}{(p\cdot n)}\bigg|_{p_0=\omega_p}, \tag{49c}$$

$$v^{(2)\mu}(p) = \epsilon^{\mu\beta\gamma}n_\beta v^{(1)}_\gamma(p) = \frac{p^\mu - n^\mu(p\cdot n)}{(p\cdot n)}\bigg|_{p_0=\omega_p}, \tag{49d}$$

with a timelike auxiliary vector n_μ. Note the bar on top of the symbol in Eq. (49a) to distinguish these vectors from the basis $\{e^{(a)}_\mu\}$ introduced in Eq. (20). One can check that the latter form an orthonormal basis, i.e.,

$$v^{(\lambda)}_\mu v^{(\lambda')\mu} = g^{\lambda\lambda'}. \tag{50}$$

Also, they satisfy the relation

$$\sum_{\lambda,\lambda'} g_{\lambda\lambda'}\bar{\varepsilon}^{(\lambda)}_\mu(p)\bar{\varepsilon}^{(\lambda')*}_\nu(p) = T_{\mu\nu}(p)|_{p_0=\omega_p}, \tag{51a}$$

where we defined

$$T_{\mu\nu}(p) \equiv G_{\mu\nu}(\xi=1,p) = \eta_{\mu\nu} - g^2 p_\mu p_\nu + \mathrm{i}g\epsilon_{\mu\beta\nu}p^\beta, \tag{51b}$$

using Eq. (10b). According to Eq. (46) and the orthogonality condition of Eq. (47), we write the ghost field operator as

$$G_\mu(x) = \int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\frac{1}{2\Omega_p}\Big[b_{\vec{p}}\bar{\varepsilon}^{(+)}_\mu(\vec{p})\mathrm{e}^{-\mathrm{i}p\cdot x} + b^\dagger_{\vec{p}}\bar{\varepsilon}^{(+)*}_\mu(\vec{p})\mathrm{e}^{\mathrm{i}p\cdot x}\Big]_{p_0=\Omega_p}, \tag{52}$$

with another set of annihilation and creation operators $b_{\vec{p}} = b(\vec{p})$ and $b^\dagger_{\vec{p}} = b^\dagger(\vec{p})$, respectively. Furthermore, we defined the polarization vector $\bar{\varepsilon}^{(+)}_\mu = \sqrt{2}\varepsilon^{(+)}_\mu$ in terms of the one introduced in Eq. (25). It may be convenient to make use of the property

$$\bar{\varepsilon}^{(+)}_\mu\bar{\varepsilon}^{(+)*}_\nu = -T_{\mu\nu}(p)|_{p_0=\Omega_p}, \tag{53}$$

which is equivalent to Eq. (31). The relation $p^2 = g^{-2}$ was employed to arrive at the latter result. We impose the following algebra on the annihilation and creation operators for the photon and ghost field:

$$\Big[a^{(\lambda)}_{\vec{p}}, a^{(\lambda')\dagger}_{\vec{k}}\Big] = -(2\pi)^2 g_{\lambda\lambda'}2\omega_p\delta^{(2)}(\vec{p}-\vec{k}), \tag{54a}$$

$$\Big[b_{\vec{p}}, b^\dagger_{\vec{k}}\Big] = -(2\pi)^2 2\Omega_p\delta^{(2)}(\vec{p}-\vec{k}). \tag{54b}$$

Replacing the fields in Eq. (42a) by the field operators of Eqs. (48) and (52) and using the algebra of Eqs. (54a) and (54b) and the properties of the polarization vectors, we find the following Hamiltonian:

$$H = -\frac{1}{4}\int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\Big[\sum_{\lambda,\lambda'}g_{\lambda\lambda'}\Big(a^{(\lambda)}_{\vec{p}}a^{(\lambda')\dagger}_{\vec{p}} + a^{(\lambda)\dagger}_{\vec{p}}a^{(\lambda')}_{\vec{p}}\Big) + \Big(b_{\vec{p}}b^\dagger_{\vec{p}} + b^\dagger_{\vec{p}}b_{\vec{p}}\Big)\Big]. \tag{55}$$

We give more details of this derivation in Appendix B.

By defining the vacuum as the state annihilated by the operators,

$$a^{(\lambda)}_{\vec{p}}|0\rangle = b_{\vec{p}}|0\rangle = 0, \tag{56}$$

for all λ, we can define the number operators associated with the photon and the ghost:

$$N_{\bar{A},\lambda} = -g_{\lambda\lambda}a^{(\lambda)\dagger}_{\vec{p}}a^{(\lambda)}_{\vec{p}}, \tag{57a}$$

$$N_G = -b^\dagger_{\vec{p}}b_{\vec{p}}. \tag{57b}$$

Indeed, the above number operators satisfy the standard relations

$$\Big[N_{\bar{A},\lambda}, a^{(\lambda')}_{\vec{p}}\Big] = -a^{(\lambda)}_{\vec{p}}\delta_{\lambda\lambda'}, \qquad \Big[N_{\bar{A},\lambda}, a^{(\lambda')\dagger}_{\vec{p}}\Big] = a^{(\lambda)\dagger}_{\vec{p}}\delta_{\lambda\lambda'}, \tag{58a}$$

$$[N_G, b_{\vec{p}}] = -b_{\vec{p}}, \qquad \Big[N_G, b^\dagger_{\vec{p}}\Big] = b^\dagger_{\vec{p}}. \tag{58b}$$

We define n-particle states as usual by subsequently applying creation operators on the vacuum state:

$$|n_{\bar{A},\lambda}\rangle = \frac{1}{\sqrt{n_{\bar{A},\lambda}!}}(a^{(\lambda)\dagger}_{\vec{p}})^{n_{\bar{A},\lambda}}|0\rangle, \qquad |n_G\rangle = \frac{1}{\sqrt{n_G!}}(b^\dagger_{\vec{p}})^{n_G}|0\rangle, \tag{59}$$

where $n_{\bar{A},\lambda}$ is the eigenvalue of the number operator of Eq. (57a) for a state of n photons of fixed polarization λ. In an analog manner, n_G is the eigenvalue of the number operator of Eq. (57b) for a state of n ghosts. The metric η in the state space is given by the scalar product of such n-particle states [9,51]. For photons, $\langle n_{\bar{A},0}|n_{\bar{A},0}\rangle = (-1)^{n_{\bar{A},0}}$ for the $\lambda = 0$ mode and $\langle n_{\bar{A},k}|n_{\bar{A},k}\rangle = 1$ for the remaining ones with $k = 1, 2$. For ghosts, it holds that $\langle n_G|n_G\rangle = (-1)^{n_G}$. Thus, we see that the states with an odd occupation number of ghosts have a negative norm. The metric for the photon can be written as $\eta_{A,\lambda} = (-g_{\lambda\lambda})^{N_{\bar{A},\lambda}}$ with $g_{\lambda\lambda'}$ given under Eq. (26) and that for the ghost reads $\eta_G = (-1)^{N_G}$. Hence, our theory exhibits an indefinite metric in the Fock space of the ghost states. It is clear that the same problem occurs for the $\lambda = 0$ mode of the photon, but this behavior is expected and can be dealt with by the usual Gupta-Bleuler method.

In order to remove the vacuum energy, the normal-ordered Hamiltonian is introduced:

$$:H: = \frac{1}{2}\int \frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\left(\sum_{\lambda,\lambda'} - g_{\lambda\lambda'}N_{\bar{A},\lambda\lambda'} + N_G\right). \tag{60}$$

The latter is positive definite, except for the usual $\lambda = 0$ mode of the photon again, which must be treated with the Gupta-Bleuler formalism. Note that the ghost does lead to issues with the positive definiteness of the Hamiltonian.

B. Feynman propagator

The next step is to derive the Feynman propagator at the level of field operators for the theory based on Eq. (2) with $\xi = 1$. We employ its definition as the vacuum expectation value of the time-ordered product of field operators at different spacetime points x and y. Hence,

$$D^F_{\mu\nu}(x-y) = \theta(x_0-y_0)D^{(+)}_{\mu\nu}(x-y) + \theta(y_0-x_0)D^{(-)}_{\mu\nu}(x-y), \tag{61a}$$

with

$$D^{(+)}_{\mu\nu}(x-y) = \langle 0|A_\mu(x)A_\nu(y)|0\rangle, \tag{61b}$$

$$D^{(-)}_{\mu\nu}(x-y) = \langle 0|A_\nu(y)A_\mu(x)|0\rangle, \tag{61c}$$

and the Heaviside step function $\theta(x)$. Using the decomposition of Eq. (45), we define

$$D^F_{\mu\nu}(x-y) = D^{(1)F}_{\mu\nu}(x-y) + D^{(2)F}_{\mu\nu}(x-y), \tag{62}$$

where the first part,

$$D^{(1)F}_{\mu\nu}(x-y) = \theta(x_0-y_0)D^{(1)(+)}_{\mu\nu}(x-y) + \theta(y_0-x_0)D^{(1)(-)}_{\mu\nu}(x-y), \tag{63a}$$

is the Feynman propagator for photons with

$$D^{(1)(+)}_{\mu\nu}(x-y) = \langle 0|\bar{A}_\mu(x)\bar{A}_\nu(y)|0\rangle, \tag{63b}$$

$$D^{(1)(-)}_{\mu\nu}(x-y) = \langle 0|\bar{A}_\nu(y)\bar{A}_\mu(x)|0\rangle. \tag{63c}$$

Furthermore, the second part is the Feynman propagator of the ghost and it reads

$$D^{(2)F}_{\mu\nu}(x-y) = \theta(x_0-y_0)D^{(2)(+)}_{\mu\nu}(x-y) + \theta(y_0-x_0)D^{(2)(-)}_{\mu\nu}(x-y), \tag{64a}$$

where

$$D^{(2)(+)}_{\mu\nu}(x-y) = \langle 0|G_\mu(x)G_\nu(y)|0\rangle, \tag{64b}$$

$$D^{(2)(-)}_{\mu\nu}(x-y) = \langle 0|G_\nu(y)G_\mu(x)|0\rangle. \tag{64c}$$

Notice that crossed terms such as $\langle 0|\bar{A}_\mu(x)G_\nu(y)|0\rangle$ have been set to zero, since the corresponding field operators commute.

Inserting the field operators of Eqs. (48) and (52), we arrive at

$$D^{(1)(+)}_{\mu\nu}(z) = -\int \frac{\mathrm{d}^2\vec{p}}{(2\pi)^2 2\omega_p}\sum_{\lambda,\lambda'} g_{\lambda\lambda'}\bar{\varepsilon}^{(\lambda)}_\mu(p)\bar{\varepsilon}^{(\lambda')*}_\nu(p)\mathrm{e}^{-\mathrm{i}p\cdot z}, \tag{65a}$$

$$D^{(1)(-)}_{\mu\nu}(z) = -\int \frac{\mathrm{d}^2\vec{p}}{(2\pi)^2 2\omega_p}\sum_{\lambda,\lambda'} g_{\lambda\lambda'}\bar{\varepsilon}^{(\lambda)}_\nu(p)\bar{\varepsilon}^{(\lambda')*}_\mu(p)\mathrm{e}^{\mathrm{i}p\cdot z}, \tag{65b}$$

for the photon and

$$D^{(2)(+)}_{\mu\nu}(z) = -\int \frac{\mathrm{d}^2\vec{p}}{(2\pi)^2 2\Omega_p}\bar{\varepsilon}^{(+)}_\mu(p)\bar{\varepsilon}^{(+)*}_\nu(p)\mathrm{e}^{-\mathrm{i}p\cdot z}, \tag{66a}$$

$$D^{(2)(-)}_{\mu\nu}(z) = -\int \frac{\mathrm{d}^2\vec{p}}{(2\pi)^2 2\Omega_p}\bar{\varepsilon}^{(+)}_\nu(p)\bar{\varepsilon}^{(+)*}_\mu(p)\mathrm{e}^{\mathrm{i}p\cdot z}, \tag{66b}$$

for the ghost with $z^\mu = x^\mu - y^\mu$. To obtain these results, we have used the algebra of Eqs. (54a) and (54b).

In the photon sector, we apply Eq. (51a) to express the sum over polarization tensors in terms of the tensor $T_{\mu\nu}$ of Eq. (51b). This leads to

$$D^{(1)F}_{\mu\nu}(z) = -\int \frac{\mathrm{d}^2\vec{p}}{(2\pi)^2 2\omega_p}\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{z}}[\theta(z_0)T_{\mu\nu}(\vec{p})\mathrm{e}^{-\mathrm{i}\omega_p z_0} + \theta(-z_0)T_{\nu\mu}(-\vec{p})\mathrm{e}^{\mathrm{i}\omega_p z_0}]. \tag{67}$$

Furthermore, in the ghost sector, we take advantage of relation (53) to carry out the analogous steps:

$$D^{(2)F}_{\mu\nu}(z) = \int \frac{\mathrm{d}^2\vec{p}}{(2\pi)^2 2\Omega_p}\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{z}}[\theta(z_0)T_{\mu\nu}(\vec{p})\mathrm{e}^{-\mathrm{i}\Omega_p z_0} + \theta(-z_0)T_{\nu\mu}(-\vec{p})\mathrm{e}^{\mathrm{i}\Omega_p z_0}]. \tag{68}$$

Now, we consider the following representation of the Heaviside function given by

$$\theta(z_0) = \frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\tau\frac{\mathrm{e}^{-\mathrm{i}\tau z_0}}{\tau+\mathrm{i}\epsilon}, \tag{69}$$

where $\epsilon = 0^+$ is an infinitesimal, positive parameter. With the latter representation, we can cast the photon propagator into the form

$$D^{(1)F}_{\mu\nu}(z) = -\frac{\mathrm{i}}{2\pi}\int \frac{\mathrm{d}^2\vec{p}}{(2\pi)^2 2\omega_p}\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{z}} \times\left[\int_{-\infty}^{\infty}\mathrm{d}\tau\frac{\mathrm{e}^{-\mathrm{i}(\omega_p+\tau)z_0}}{\tau+\mathrm{i}\epsilon}T_{\mu\nu}(\vec{p}) + \int_{-\infty}^{\infty}\mathrm{d}\tau\frac{\mathrm{e}^{\mathrm{i}(\omega_p+\tau)z_0}}{\tau+\mathrm{i}\epsilon}T_{\nu\mu}(-\vec{p})\right]. \tag{70}$$

Making a change of variables $p_0 = \tau + \omega_p$ and $-p_0 = \tau + \omega_p$ in the first and second integral, respectively, we have

$$D^{(1)F}_{\mu\nu}(z) = -\mathrm{i}\int \frac{\mathrm{d}^2\vec{p}}{(2\pi)^3 2\omega_p}\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{z}} \times \int_{-\infty}^{\infty}\mathrm{d}p_0\,\mathrm{e}^{-\mathrm{i}p_0 z_0}\left[\frac{T_{\mu\nu}(\vec{p},p_0)}{p_0-\omega_p+\mathrm{i}\epsilon} - \frac{T_{\nu\mu}(-\vec{p},-p_0)}{p_0+\omega_p-\mathrm{i}\epsilon}\right] = -\mathrm{i}\int_{C_F}\frac{\mathrm{d}^3p}{(2\pi)^3}\mathrm{e}^{-\mathrm{i}p\cdot z}\frac{T_{\mu\nu}(p)}{p^2+\mathrm{i}\epsilon}. \tag{71}$$

To formulate the final form of the photon propagator, we benefited from the property $T_{\nu\mu}(-\vec{p},-p_0) = T_{\mu\nu}(\vec{p},p_0)$. Furthermore, we have written the integral over p_0 as a contour integral in the complex p_0 plane. The contour C_F is closed in the lower half plane for positive energies and in the upper half plane for negative energies. It is passed through in counterclockwise direction. By evaluating the ghost part in a similar way, we obtain

$$D^{(2)F}_{\mu\nu}(z) = \mathrm{i}\int \frac{\mathrm{d}^2\vec{p}}{(2\pi)^3 2\Omega_p}\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{z}} \times \int_{-\infty}^{\infty}\mathrm{d}p_0\,\mathrm{e}^{-\mathrm{i}p_0 z_0}\left[\frac{T_{\mu\nu}(\vec{p},p_0)}{p_0-\Omega_p+\mathrm{i}\epsilon} - \frac{T_{\nu\mu}(-\vec{p},-p_0)}{p_0+\Omega_p-\mathrm{i}\epsilon}\right] = \mathrm{i}\int_{C_F}\frac{\mathrm{d}^3p}{(2\pi)^2}\mathrm{e}^{-\mathrm{i}p\cdot z}\frac{T_{\mu\nu}(p)}{p^2-g^{-2}+\mathrm{i}\epsilon}, \tag{72}$$

by writing the integral over p_0 as another contour integral along the same contour C_F introduced before. Adding the contributions of Eqs. (71) and (72) results in

$$D^F_{\mu\nu}(z) = -\mathrm{i}\int_{C_F}\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{T_{\mu\nu}(p)}{(p^2+\mathrm{i}\epsilon)(1-g^2p^2-\mathrm{i}\epsilon)}\mathrm{e}^{-\mathrm{i}p\cdot z}, \tag{73}$$

where the infinitesimal parameter ϵ is only kept at linear order. In momentum space the Feynman propagator with the iϵ prescription is

$$D^F_{\mu\nu}(p) = -\frac{G_{\mu\nu}(\xi=1,p)}{(p^2+\mathrm{i}\epsilon)(1-g^2p^2-\mathrm{i}\epsilon)}, \tag{74}$$

where we have used Eq. (51b). The latter can be generalized to arbitrary ξ. By inserting $M = g^{-1}$, we reformulate it as

$$D^F_{\mu\nu}(\xi,p) = \frac{M^2 G_{\mu\nu}(\xi,p)}{(p^2+\mathrm{i}\epsilon)(p^2-M^2+\mathrm{i}\epsilon)}, \tag{75}$$

which corresponds to the inverse $P_{\mu\nu}$ of Eq. (10) for $\epsilon \mapsto 0$.

C. Microcausality

Two spacetime points that cannot be connected by a light signal (or a signal propagating with lower velocity) are called causally disconnected. In a theory with Lorentz symmetry intact, such a set of spacetime points is separated by a spacelike interval. When Lorentz symmetry is violated, the causal structure is not simply determined by the Minkowski metric, but directly by the propagation velocity of the field operator under consideration, i.e., the interval need not necessarily be spacelike. As Lorentz symmetry is preserved for our theory, its causal structure is, indeed, based on the Minkowski metric.

Now, field operators evaluated at such a set of spacetime points can be considered independent of each other, i.e., they should commute. If the latter is the case, microcausality is guaranteed for the theory under investigation. To prove microcausality for the theory defined by Eq. (2), we start with the basic commutator of field operators at the points x and y:

$$D_{\mu\nu}(x-y) = [A_\mu(x), A_\nu(y)]. \tag{76}$$

A direct calculation starting from Eq. (45) provides

$$[\bar{A}_\mu(x),\bar{A}_\nu(y)] = \int\frac{\mathrm{d}^2\vec{p}\,\mathrm{d}^2\vec{k}}{(2\pi)^4 4\omega_p\omega_k}\sum_{\lambda,\lambda'}\Big(\bar{e}^{(\lambda)}_\mu(\vec{p})\bar{e}^{*(\lambda')}_\nu(\vec{k})\left[a^{(\lambda)}_{\vec{p}}, a^{(\lambda')\dagger}_{\vec{k}}\right]\mathrm{e}^{-\mathrm{i}p\cdot x+\mathrm{i}k\cdot y} + \bar{e}^{*(\lambda)}_\mu(\vec{p})\bar{e}^{(\lambda')}_\nu(\vec{k})\left[a^{(\lambda)\dagger}_{\vec{p}}, a^{(\lambda')}_{\vec{k}}\right]\mathrm{e}^{\mathrm{i}p\cdot x-\mathrm{i}k\cdot y}\Big) \tag{77}$$

and

$$[G_\mu(x), G_\nu(y)] = \int\frac{\mathrm{d}^2\vec{p}\,\mathrm{d}^2\vec{k}}{(2\pi)^4 4\Omega_p\Omega_k}(\varepsilon^{(+)}_\mu(\vec{p})\varepsilon^{(+)*}_\nu(\vec{k})[b_{\vec{p}}, b^\dagger_{\vec{k}}]\mathrm{e}^{-\mathrm{i}p\cdot x+\mathrm{i}k\cdot y} + \varepsilon^{(+)*}_\mu(\vec{p})\varepsilon^{(+)}_\nu(\vec{k})[b^\dagger_{\vec{p}}, b_{\vec{k}}]\mathrm{e}^{\mathrm{i}p\cdot x-\mathrm{i}k\cdot y}). \tag{78}$$

Hence, it is important to study the commutator for the photon and the ghost separately, as the corresponding field operators are independent of each other. By using the algebra of Eq. (54) and the properties of the polarization vectors of Eqs. (51a) and (53), we arrive at

$$D_{\mu\nu}(x-y) = -\int \frac{d^2\vec{p}}{(2\pi)^2}\frac{1}{2\omega_p}(T_{\mu\nu}(\vec{p},\omega_p)e^{-ip\cdot(x-y)} - T_{\nu\mu}(\vec{p},\omega_p)e^{ip\cdot(x-y)}) + \int \frac{d^2\vec{p}}{(2\pi)^2}\frac{1}{2\Omega_p}(T_{\mu\nu}(\vec{p},\Omega_p)e^{-ip\cdot(x-y)} - T_{\nu\mu}(\vec{p},\Omega_p)e^{ip\cdot(x-y)}), \tag{79}$$

where we employed the tensor $T_{\mu\nu}$ of Eq. (51b). We define $z = x - y$ and perform a change of variables $\vec{p} \to -\vec{p}$ in the second contribution above to obtain

$$D_{\mu\nu}(z) = -\int \frac{d^2\vec{p}}{(2\pi)^2}\frac{e^{i\vec{p}\cdot\vec{z}}}{2\omega_p}(T_{\mu\nu}(\vec{p},\omega_p)e^{-i\omega_p z_0} - T_{\nu\mu}(-\vec{p},\omega_p)e^{i\omega_p z_0}) + \int \frac{d^2\vec{p}}{(2\pi)^2}\frac{e^{i\vec{p}\cdot\vec{z}}}{2\Omega_p}(T_{\mu\nu}(\vec{p},\Omega_p)e^{-i\Omega_p z_0} - T_{\nu\mu}(-\vec{p},\Omega_p)e^{i\Omega_p z_0}). \tag{80}$$

Since $T_{\nu\mu}(-\vec{p}, p_0) = T_{\mu\nu}(\vec{p}, -p_0)$, we can introduce another contour integral in the complex p_0 plane along a contour C that encircles all poles in counterclockwise direction:

$$D_{\mu\nu}(z) = i\int \frac{d^2\vec{p}}{(2\pi)^2}\int_C \frac{dp_0}{2\pi}\left(\frac{T_{\mu\nu}(\vec{p}, p_0)}{(p_0+\omega_p)(p_0-\omega_p)} - \frac{T_{\mu\nu}(\vec{p}, p_0)}{(p_0+\Omega_p)(p_0-\Omega_p)}\right)e^{-ip\cdot z}. \tag{81}$$

Note that the contour C is different from the contour C_F that we defined in the context of the Feynman propagator in Sec. III B. Therefore,

$$D_{\mu\nu}(z) = i\int_C \frac{d^3p}{(2\pi)^3}\frac{T_{\mu\nu}(p)}{p^2(1-g^2p^2)}e^{-ip\cdot z}. \tag{82}$$

To prove that this expression vanishes outside the light cone, that is, for $(x-y)^2 < 0$, we can perform a Lorentz transformation of the coordinates to a frame where $x^0 - y^0 = 0$ and compute the integral in this new frame. Thus, we focus on the integral over p_0,

$$I_{\mu\nu} = \int_C dp_0 \frac{T_{\mu\nu}(p)}{p^2(1-g^2p^2)} = -\frac{1}{g^2}\int_C dp_0 \frac{T_{\mu\nu}(p)}{(p_0-\omega_p)(p_0+\omega_p)(p_0-\Omega_p)(p_0+\Omega_p)}, \tag{83}$$

whose result is given by

$$-g^2 I_{\mu\nu} = \left[\frac{T_{\mu\nu}(\vec{p},\omega_p)}{2\omega_p(\omega_p^2-\Omega_p^2)} - \frac{T_{\mu\nu}(\vec{p},-\omega_p)}{2\omega_p(\omega_p^2-\Omega_p^2)}\right] + \left[\frac{T_{\mu\nu}(\vec{p},\Omega_p)}{2\Omega_p(\Omega_p^2-\omega_p^2)} - \frac{T_{\mu\nu}(\vec{p},-\Omega_p)}{2\Omega_p(\Omega_p^2-\omega_p^2)}\right]. \tag{84}$$

Now we employ the explicit form of the tensor $T_{\mu\nu}$ in Eq. (51b). The terms proportional to $\eta_{\mu\nu}$ cancel for each contribution enclosed in parentheses as well as those proportional to $g^2 p_i p_j$ and $ig\epsilon_{0ij}p^i$. The only terms that survive are proportional to $g^2 p_0 p_i$ and $ig\epsilon_{i0j}p_0$. However, these cancel due to the identity

$$\frac{\omega_p}{\omega_p(\omega_p^2-\Omega_p^2)} + \frac{\Omega_p}{\Omega_p(\Omega_p^2-\omega_p^2)} = 0, \tag{85}$$

whereupon $I_{\mu\nu} = 0$ and $D_{\mu\nu} = 0$ in the particular frame considered. Lorentz invariance allows us to generalize this finding to an arbitrary frame. We conclude that the theory is microcausal, since the commutator of two field operators vanishes when they are evaluated at causally disconnected spacetime points.

IV. PERTURBATIVE UNITARITY

In the previous sections we have seen that the theory defined by (2) develops an indefinite metric in the Hilbert space of states due to higher-time derivatives present in the Lagrangian. This metric is responsible for negative-norm states and could possibly induce a violation of unitarity. As a consequence of this, the normal probabilistic interpretation of quantum theory would be undermined.

Unitarity can be investigated in various ways. A reasonable method for a free theory is to study the condition of reflection positivity [52]. However, in the presence of interactions, computations based on the optical theorem in perturbation theory [53] are better under control. In this context, imaginary parts of forward-scattering amplitudes are compared to cross sections of processes corresponding to cut Feynman diagrams. In the forthcoming subsections we check the validity of unitarity of the theory via reflection positivity and the optical theorem.

A. Reflection positivity

Reflection positivity is a property of a scalar two-point function in Euclidean space that guarantees the validity of unitarity of the corresponding free field theory in Minkowski spacetime. It is primarily used in the context of lattice gauge theory, but also found application in proofs of unitarity for Lorentz-violating theories [see, e.g., [54–56] for applications to Maxwell-Chern-Simons theory in $(3+1)$ dimensions, modified Maxwell theory, and higher-derivative theories of fermions].

To check the validity of reflection positivity for our theory, we will make some simplifications as follows. Let us consider the combination of poles in the scalar propagator function

$$K(p_0,\vec{p}) = \frac{M^2}{p^2(p^2-M^2)}, \tag{86}$$

whose form is taken from Eq. (10a). We can rearrange the latter as

$$K(p_0,\vec{p}) = -\frac{1}{p^2} + \frac{1}{p^2-M^2}. \tag{87}$$

Now we go to Euclidean space by means of the replacement $p_0 \to \mathrm{i}p_3$,

$$K(p_0,\vec{p}) \mapsto K_E(p_3,\vec{p}) = \frac{1}{p_E^2} - \frac{1}{p_E^2+M^2}. \tag{88}$$

The weak version of reflection positivity requires that the one-dimensional Fourier transform of the latter Euclidean propagator function with respect to p_3 be non-negative. Computing this Fourier transform leads to

$$\begin{aligned} K_E(x_3,|\vec{p}|) &= \int_{-\infty}^{\infty} \mathrm{d}p_3 K_E(p_3,|\vec{p}|)\mathrm{e}^{-\mathrm{i}p_3x_3} \\ &= \int_{-\infty}^{\infty} \mathrm{d}p_4 \frac{\mathrm{e}^{-\mathrm{i}p_3x_3}}{p_3^2+\vec{p}^2} - \int_{-\infty}^{\infty} \mathrm{d}p_3 \frac{\mathrm{e}^{-\mathrm{i}p_3x_3}}{p_3^2+\vec{p}^2+M^2} \\ &= \pi\left[\frac{\exp(-|x_3||\vec{p}|)}{|\vec{p}|} - \frac{\exp\left(-|x_3|\sqrt{\vec{p}^2+M^2}\right)}{\sqrt{\vec{p}^2+M^2}}\right]. \end{aligned} \tag{89}$$

We see that the latter expression is non-negative for all momentum magnitudes $|\vec{p}|$ (see Fig. 1). However, it should be noted that the condition of reflection positivity refers to the scalar part of the two-point function only. Also, it does not take into account interactions. Therefore, reflection

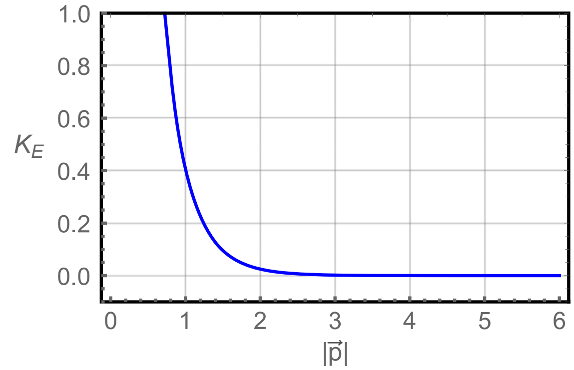

FIG. 1. Plot of the function $K_E(x_3,|\vec{p}|)$ of Eq. (89) for $x_3=2$ and $M=2$ as a function of $|\vec{p}|$.

positivity does not provide a complete understanding of unitarity when the tensor structure of the two-point function and interactions are taken into consideration.

To check the validity of unitarity more thoroughly, it is wise to go beyond this technique and, for instance, use the optical theorem. In the next section, we give an example in which a study of the optical theorem with the complete structure of poles and polarization vectors is indispensable.

B. Electron-positron annihilation at tree level

Our intention is to check the perturbative validity of the optical theorem for the theory defined by Eq. (2). To do so, we have to couple the modified photon theory to standard Dirac fermions in $(2+1)$ dimensions, i.e., we will consider a modified QED in three dimensions (QED_3). A summary on a theory of Dirac spinors in $(2+1)$ dimensions is given in Appendix D. We then write the total Lagrange density as

$$\mathcal{L}_{\mathrm{tot}} = \mathcal{L} + \mathcal{L}_{\psi,\gamma}, \tag{90a}$$

$$\mathcal{L}_{\psi,\gamma} = \bar{\psi}[\gamma^\mu(\mathrm{i}\partial_\mu - eA_\mu) + m]\psi, \tag{90b}$$

with $\mathcal{L}$ given by Eq. (2). Here, e is the electric charge, m the fermion mass, ψ a four-component Dirac spinor, and γ^μ the set of three Dirac matrices of Eq. (D5). Note again that Lorentz indices run over 0,1,2.

The optical theorem establishes a connection between the forward-scattering amplitude of a particular particle physics process and the decay rates or total cross sections of processes that are obtained by cutting the Feynman diagram of the forward-scattering amplitude into two pieces. We will study processes at tree level and one-loop order that involve the gauge-field propagator (10a) of the theory [57,58]. Let us start with the polarized forward scattering annihilation process of electron-positron pairs, $\mathrm{e}^+\mathrm{e}^- \to \mathrm{e}^+\mathrm{e}^-$ of Fig. 2. The corresponding amplitude is given by

$$\mathrm{i}\mathcal{M}_F = \bar{v}(p_2)(-\mathrm{i}e\gamma^\mu)u(p_1)(\mathrm{i}D^F_{\mu\nu}(\xi,q))\bar{u}(p_1)(-\mathrm{i}e\gamma^\nu)v(p_2), \tag{91}$$

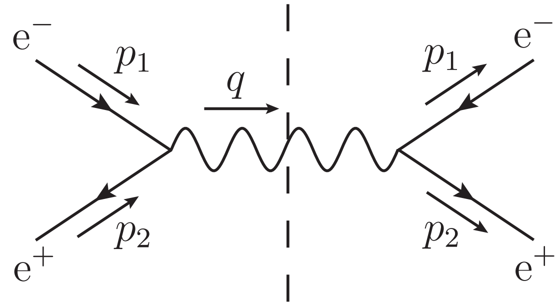

FIG. 2. Polarized forward-scattering electron-positron annihilation where a cut of the gauge-field propagator is indicated by the dashed line. The three-momenta of the incoming particles are p_1, p_2, where the three-momentum of the intermediate modified photon is denoted as q.

with the Feynman propagator of Eq. (75) and $q = p_1 + p_2$. Particle and antiparticle spinors of a particular spin projection are denoted as $u(p)$ and $v(p)$, respectively, and correspond to those of Eqs. (D14) and (D15). Considering polarized scattering is not crucial for the verification of the optical theorem, though. It just simplifies the expressions, as the polarizations of the incoming and outgoing particles need not be averaged or summed over. Note also that we suppress the spin index for external spinors. Now, we can write

$$\mathcal{M}_F = -e^2 \mathcal{M}^\mu(p_1, p_2) D^F_{\mu\nu}(\xi, q) \mathcal{M}^{\dagger\nu}(p_1, p_2), \tag{92}$$

where

$$\mathcal{M}^\mu(p_1, p_2) = \bar{v}(p_2)\gamma^\mu u(p_1), \tag{93}$$

$$\mathcal{M}^{\dagger\nu}(p_1, p_2) = \bar{u}(p_1)\gamma^\nu v(p_2). \tag{94}$$

The process that results from cutting the diagram of the forward-scattering amplitude into two pieces is the production of a modified photon by an electron-positron pair. In contrast to what happens in standard QED, the cross section of this process is not necessarily equal to zero due to energy-momentum conservation. The reason is the presence of the massive ghost, which can render the process possible. In this case, the condition of energy conservation can be evaluated in the center-of-mass frame: $|\vec{p}_1| = |\vec{p}_2| = 1/2g$. Therefore, it will be sufficient to prove unitarity by considering the contributions to the imaginary part (or discontinuity) of the amplitude for the massive ghost.

In the forward-scattering amplitude of Eq. (92) an integral over the three-momentum q of the intermediate state can be introduced that is canceled again by the three-dimensional δ function of total energy-momentum conservation (which is equivalent to energy-momentum conservation at each vertex):

$$\mathcal{M}_F = -e^2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \mathcal{M}^\mu D^F_{\mu\nu}(\xi, q) \times \mathcal{M}^{\dagger\nu}(2\pi)^3 \delta^{(3)}(p_1 + p_2 - q). \tag{95}$$

By inserting the Feynman propagator of Eq. (75), we have

$$\mathcal{M}_F = -e^2 M^2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{\mathcal{M}^\mu G_{\mu\nu}(\xi, q) \mathcal{M}^{\dagger\nu}}{(q^2 + \mathrm{i}\epsilon)(q^2 - M^2 + \mathrm{i}\epsilon)} \times (2\pi)^3 \delta^{(3)}(p_1 + p_2 - q). \tag{96}$$

As the photon propagator is coupled to a conserved external current and energy-momentum is conserved at the vertex, we can use the Ward identity to get rid of all terms in the propagator proportional to this momentum: $q_\mu \mathcal{M}^\mu = 0$. Doing so allows for instating the tensor $T_{\mu\nu}$ of Eq. (51b).

It is valuable to recall that

$$\frac{M^2}{(q^2 + \mathrm{i}\epsilon)(q^2 - M^2 + \mathrm{i}\epsilon)} = -\frac{1}{q^2 + \mathrm{i}\epsilon} + \frac{1}{q^2 - M^2 + \mathrm{i}\epsilon}. \tag{97}$$

By making use of the latter, we can decompose the denominator into two parts:

$$\mathcal{M}_F = e^2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \left[\frac{\mathcal{M}^\mu T_{\mu\nu}(q) \mathcal{M}^{\dagger\nu}}{q^2 + \mathrm{i}\epsilon} - \frac{\mathcal{M}^\mu T_{\mu\nu}(q) \mathcal{M}^{\dagger\nu}}{q^2 - M^2 + \mathrm{i}\epsilon} \right] \times (2\pi)^3 \delta^{(3)}(p_1 + p_2 - q). \tag{98}$$

Now we insert the expression for $T_{\mu\nu}$ in terms of the polarization vectors given in Eqs. (51a) and (53) and obtain

$$\mathcal{M}_F = e^2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \left[\frac{\sum_{\lambda,\lambda'} (\mathcal{M}^\mu \bar{\varepsilon}^{(\lambda)}_\mu) g_{\lambda\lambda'} (\mathcal{M}^{\dagger\nu} \bar{\varepsilon}^{(\lambda')*}_\nu)}{q^2 + \mathrm{i}\epsilon} + \frac{(\mathcal{M}^\mu \bar{\varepsilon}^{(+)}_\mu)(\mathcal{M}^{\dagger\nu} \bar{\varepsilon}^{(+)*}_\nu)}{q^2 - M^2 + \mathrm{i}\epsilon} \right] \times (2\pi)^3 \delta^{(3)}(p_1 + p_2 - q). \tag{99}$$

Since it is not possible to satisfy energy-momentum conservation and the dispersion relation for the photon at the same time, the first contribution is zero. We are then left with

$$\mathcal{M}_F = e^2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{|\mathcal{M}^\mu \bar{\varepsilon}^{(+)}_\mu(q)|^2}{(q_0 + \Omega_q - \mathrm{i}\epsilon)(q_0 - \Omega_q + \mathrm{i}\epsilon)} \times (2\pi)^3 \delta^{(3)}(p_1 + p_2 - q). \tag{100}$$

We perform the integration over q_0 by defining the center-of-mass energy $\sqrt{s} = p^0_1 + p^0_2$ and exploit the property of the δ function. This leads to

$$\mathcal{M}_F(s) = e^2 \int \frac{\mathrm{d}^2\vec{q}}{(2\pi)^3} \frac{|\mathcal{M}^\mu \bar{\varepsilon}_\mu^{(+)}(\Omega_q, \vec{q})|^2}{(\sqrt{s}+\Omega_q - \mathrm{i}\epsilon)(\sqrt{s}-\Omega_q+\mathrm{i}\epsilon)} \times (2\pi)^3 \delta^{(2)}(\vec{p}_1 + \vec{p}_2 - \vec{q}). \quad (101)$$

The imaginary part of the amplitude can be evaluated based on the identity

$$\lim_{\epsilon\to 0^+} \frac{1}{x \pm \mathrm{i}\epsilon} = \mathcal{P}\left(\frac{1}{x}\right) \mp \mathrm{i}\pi\delta(x), \quad (102)$$

where $\mathcal{P}$ denotes the principal value. We also consider

$$\frac{2\Omega_q}{(\sqrt{s}+\Omega_q-\mathrm{i}\epsilon)(\sqrt{s}-\Omega_q+\mathrm{i}\epsilon)} = \frac{1}{\sqrt{s}-\Omega_q+\mathrm{i}\epsilon} - \frac{1}{\sqrt{s}+\Omega_q-\mathrm{i}\epsilon}. \quad (103)$$

The result is

$$\mathrm{Im}(\mathcal{M}_F(s)) = -e^2 \int \frac{\mathrm{d}^2\vec{q}}{(2\pi)^3} |\mathcal{M}^\mu \bar{\varepsilon}_\mu^{(+)}(\Omega_q,\vec{q})|^2 (2\pi)^3 \delta^{(2)}(\vec{p}_1+\vec{p}_2-\vec{q}) \times \frac{\pi}{2\Omega_q}[\delta(\sqrt{s}-\Omega_p)+\delta(\sqrt{s}+\Omega_q)]. \quad (104)$$

The second δ function in Eq. (104) involves a nonzero contribution coming from the possibility of negative energies. This can be seen in the following way. From the definition of the Feynman propagator one has

$$D^F_{\mu\nu}(z_0,\vec{z}) = \theta(z_0) D^{(+)}_{\mu\nu}(z_0,\vec{z}) + \theta(-z_0) D^{(-)}_{\mu\nu}(z_0,\vec{z}). \quad (105)$$

Performing a coordinate Poincaré transformation, for instance, a constant time translation that adds a constant purely timelike three-vector to z such that $z_0 \to -z_0$, one has

$$D^F_{\mu\nu}(-z_0,\vec{z}) = \theta(-z_0) D^{(+)}_{\mu\nu}(-z_0,\vec{z}) + \theta(z_0) D^{(-)}_{\mu\nu}(-z_0,\vec{z}). \quad (106)$$

The interpretation is that negative energies occur in the opposite flow of time. This is precisely the reason why we include the second δ function in Eq. (104). In the literature, the latter is sometimes represented by a cut with a shaded region indicating the corresponding direction of energy flow [14].

Finally, we can write

$$2\mathrm{Im}(\mathcal{M}_F(s)) = -e^2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} |\mathcal{M}^\mu \bar{\varepsilon}_\mu^{(+)}(q)|^2 (2\pi)^3 \delta^{(3)}(p_1+p_2-q) \times (2\pi)\delta(q^2-M^2)[\theta(q_0)+\theta(-q_0)], \quad (107)$$

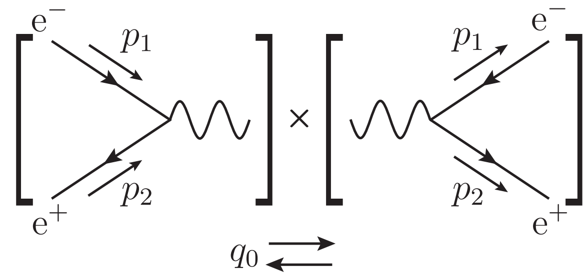

FIG. 3. After cutting the photon propagator in the diagram of Fig. 2, the sum over intermediate states in both directions of the energy flow is considered.

which represents the sum of diagrams with energy flow in the positive and negative direction as represented in Fig. 3.

Now we come to the crucial point in the analysis where we must introduce some of the ideas developed by Lee and Wick. As a first observation, the negative global sign in Eq. (107) may threaten unitarity since the left-hand side of the latter equation, which is related to the cross section, is positive definite. To overcome this problem, we apply the Lee-Wick prescription that removes the negative-metric states from the asymptotic Hilbert space. Furthermore, as long as the energy of the incoming state is low enough, the argument of $\delta(q^2 - M^2)$ is impossibly different from zero due to the large mass of the ghost. The latter will simply not be excited under this condition. Then unitarity is guaranteed in a direct way just as in the standard case [8,9,11].

C. Compton scattering at one-loop level

Our next step is to study unitarity when virtual ghosts arise in loop diagrams. We analyze the optical theorem for the (polarized) Compton scattering process of Fig. 4. The forward-scattering amplitude at one-loop level for this process in the extended Maxwell-Chern-Simons theory in $(2+1)$ dimensions given by the Lagrangian (2) reads

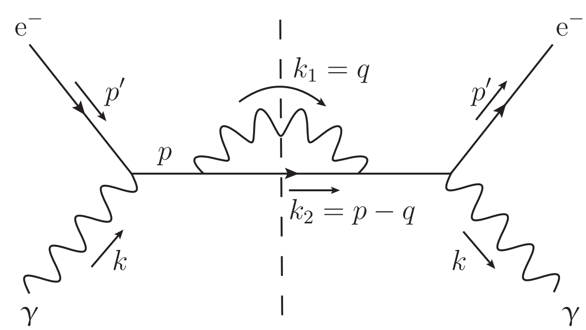

FIG. 4. Forward Compton scattering with one-loop correction of the fermion propagator included. The cut of both propagators is indicated by a dashed line. The external three-momenta are given by k and p'.

$$\mathrm{i}\mathcal{M} = \epsilon_\beta^{(\lambda)*}(k)\bar{u}(p')(-\mathrm{i}e\gamma^\beta)\left(\frac{\mathrm{i}(\not{p}+m)}{p^2-m^2+\mathrm{i}\epsilon}\right)(-\mathrm{i}e\gamma^\mu)\int\frac{\mathrm{d}^3q}{(2\pi)^3}\frac{\mathrm{i}(\not{p}-\not{q}+m)}{(p-q)^2-m^2+\mathrm{i}\epsilon}$$
$$\times(\mathrm{i}D^F_{\mu\nu}(q))(-\mathrm{i}e\gamma^\nu)\left(\frac{\mathrm{i}(\not{p}+m)}{p^2-m^2+\mathrm{i}\epsilon}\right)(-\mathrm{i}e\gamma^\alpha)u(p')\epsilon_\alpha^{(\lambda)}(k), \tag{108}$$

where the external electrons and photons are considered polarized. The fermion propagator for the $(2+1)$-dimensional Dirac theory of Eq. (D12) has been inserted. For simplicity, we choose the particular gauge-fixing parameter $\xi = 1$ and employ the Feynman propagator of Eq. (74). We introduce the following shorthand notation for expressions formed from external spinors and polarization vectors:

$$J_1^{(\lambda)}(p',k) = \epsilon_\beta^{*(\lambda)}(k)\bar{u}(p')\gamma^\beta, \tag{109a}$$

$$J_2^{(\lambda)}(p',k) = \gamma^\alpha u(p')\epsilon_\alpha^{(\lambda)}(k), \tag{109b}$$

and rewrite the denominators of Eq. (108) in terms of the poles. We also work in the center-of-mass frame where $\vec{p} = \vec{0}$ and use Eq. (97) to obtain

$$\mathrm{i}\mathcal{M} = -e^4 J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2+\mathrm{i}\epsilon}\right)\gamma^\mu\int\frac{\mathrm{d}^3q}{(2\pi)^3}\frac{\not{p}-\not{q}+m}{(q_0-p_0-E_q+\mathrm{i}\epsilon)(q_0-p_0+E_q-\mathrm{i}\epsilon)}$$
$$\times T_{\mu\nu}(q)\left[\frac{1}{(q_0-\omega_q+\mathrm{i}\epsilon)(q_0+\omega_q-\mathrm{i}\epsilon)}-\frac{1}{(q_0-\Omega_q+\mathrm{i}\epsilon)(q_0+\Omega_q-\mathrm{i}\epsilon)}\right]\gamma^\nu\left(\frac{\not{p}+m}{p^2-m^2+\mathrm{i}\epsilon}\right)J_2^{(\lambda)}(p',k). \tag{110}$$

Let us decompose the amplitude into a sum of amplitudes via

$$\mathcal{M} = \mathcal{M}^{(1)} + \mathcal{M}^{(2)}, \tag{111a}$$

with

$$\mathrm{i}\mathcal{M}^{(1)} = -e^4 J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2+\mathrm{i}\epsilon}\right)\gamma^\mu\int\frac{\mathrm{d}^2\vec{q}\mathrm{d}q_0}{(2\pi)^3}\frac{\not{p}-\not{q}+m}{(q_0-p_0-E_q+\mathrm{i}\epsilon)(q_0-p_0+E_q-\mathrm{i}\epsilon)}$$
$$\times\frac{T_{\mu\nu}(q)}{(q_0-\omega_q+\mathrm{i}\epsilon)(q_0+\omega_q-\mathrm{i}\epsilon)}\gamma^\nu\left(\frac{\not{p}+m}{p^2-m^2+\mathrm{i}\epsilon}\right)J_2^{(\lambda)}(p',k) \tag{111b}$$

and

$$\mathrm{i}\mathcal{M}^{(2)} = e^4 J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2+\mathrm{i}\epsilon}\right)\gamma^\mu\int\frac{\mathrm{d}^2\vec{q}\mathrm{d}q_0}{(2\pi)^3}\frac{\not{p}-\not{q}+m}{(q_0-p_0-E_q+\mathrm{i}\epsilon)(q_0-p_0+E_q-\mathrm{i}\epsilon)}$$
$$\times\frac{T_{\mu\nu}(q)}{(q_0-\Omega_q+\mathrm{i}\epsilon)(q_0+\Omega_q-\mathrm{i}\epsilon)}\gamma^\nu\left(\frac{\not{p}+m}{p^2-m^2+\mathrm{i}\epsilon}\right)J_2^{(\lambda)}(p',k). \tag{111c}$$

Our next step is to integrate over the complex variable q_0 by using the residue theorem and closing the contour in the lower half plane of the complex q_0 plane. Each integrand has two contributing poles leading to four poles $q_0 = z_i$ $(i = 1\ldots4)$, in total. For the first integrand we have

$$z_1 = p_0 + E_q - \mathrm{i}\epsilon, \tag{112a}$$

$$z_2 = \omega_q - \mathrm{i}\epsilon, \tag{112b}$$

where E_q is the dispersion relation (D13) of a massive fermion in $(2+1)$ dimensions. The poles of the second integrand are given by

$$z_3 = p_0 + E_q - \mathrm{i}\epsilon, \tag{113a}$$

$$z_4 = \Omega_q - \mathrm{i}\epsilon. \tag{113b}$$

We then arrive at

$$\mathcal{M}^{(1)} = -e^4 J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2}\right)\gamma^\mu \times \int \frac{\mathrm{d}^2\vec{q}}{(2\pi)^2}(\not{p}-\not{q}+m)T_{\mu\nu}(q) \times (\mathrm{Res}(z_1)+\mathrm{Res}(z_2))\gamma^\nu\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(\lambda)}(p',k) \tag{114}$$

and

$$\mathcal{M}^{(2)} = e^4 J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2}\right)\gamma^\mu \times \int \frac{\mathrm{d}^2\vec{q}}{(2\pi)^2}(\not{p}-\not{q}+m)T_{\mu\nu}(q) \times (\mathrm{Res}(z_3)+\mathrm{Res}(z_4))\gamma^\nu\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(\lambda)}(p',k), \tag{115}$$

with the residues

$$\mathrm{Res}(z_1) = \frac{-1}{2E_q(p_0+E_q-\omega_q)(p_0+E_q+\omega_q-\mathrm{i}\epsilon)}, \tag{116a}$$

$$\mathrm{Res}(z_2) = \frac{-1}{2\omega_q(p_0+E_q-\omega_q)(p_0-E_q-\omega_q+\mathrm{i}\epsilon)} \tag{116b}$$

and

$$\mathrm{Res}(z_3) = \frac{-1}{2E_q(p_0+E_q-\Omega_q)(p_0+E_q+\Omega_q-\mathrm{i}\epsilon)}, \tag{117a}$$

$$\mathrm{Res}(z_4) = \frac{-1}{2\Omega_q(p_0+E_q-\omega_q)(p_0-E_q-\Omega_q+\mathrm{i}\epsilon)}, \tag{117b}$$

where we have rescaled the parameter ϵ and set $\epsilon \to 0$ where it is not important.

Our amplitude $\mathcal{M}$ of Eq. (110) considered as an analytic function of the complex variable q_0 has a branch cut along the real axis. In order to extract the imaginary part of the diagram we will compute the imaginary parts of the residues by using the identity (102). We obtain

$$\mathrm{Im}(\mathrm{Res}(z_1)) = \frac{\pi\delta(p_0+\omega_p+E_q)}{4\omega_q E_q}, \tag{118a}$$

$$\mathrm{Im}(\mathrm{Res}(z_2)) = \frac{\pi\delta(p_0-\omega_p-E_q)}{4\omega_q E_q}, \tag{118b}$$

$$\mathrm{Im}(\mathrm{Res}(z_3)) = \frac{\pi\delta(p_0+\Omega_p+E_q)}{4\Omega_q E_q}, \tag{118c}$$

$$\mathrm{Im}(\mathrm{Res}(z_4)) = \frac{\pi\delta(p_0-\Omega_p-E_q)}{4\Omega_q E_q}. \tag{118d}$$

We can then write the imaginary parts of the amplitudes as

$$\mathrm{Im}(\mathcal{M}^{(1)}) = -e^4 J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2}\right)\gamma^\mu \int \frac{\mathrm{d}^2\vec{q}}{(2\pi)^3}(\not{p}-\not{q}+m)T_{\mu\nu}(q) \times \frac{(2\pi)\pi}{4\omega_q E_q}[\delta(p_0-\omega_p-E_q)+\delta(p_0+\omega_p+E_q)]\gamma^\nu\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(\lambda)}(p',k), \tag{119}$$

and in the same way

$$\mathrm{Im}(\mathcal{M}^{(2)}) = e^4 J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2}\right)\gamma^\mu \int \frac{\mathrm{d}^2\vec{q}}{(2\pi)^3}(\not{p}-\not{q}+m)T_{\mu\nu}(q) \times \frac{(2\pi)\pi}{4\Omega_q E_q}[\delta(p_0-\Omega_p-E_q)+\delta(p_0+\Omega_p+E_q)]\gamma^\nu\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(\lambda)}(p',k). \tag{120}$$

Now, we define

$$q = k_1, \tag{121a}$$

$$p - q = k_2, \tag{121b}$$

and use energy conservation expressed by the δ functions $\delta(p_0 \pm \Omega_p \pm E_q)$ and $\delta(p_0 \pm \omega_p \pm E_q)$. Furthermore, we employ the relation

$$\int \frac{\mathrm{d}^2\vec{q}}{(2\pi)^3} = \int \frac{\mathrm{d}^2\vec{k}_1}{(2\pi)^3} \int \frac{\mathrm{d}^2\vec{k}_2}{(2\pi)^3} (2\pi)^3 \delta^{(2)}(\vec{p} - \vec{k}_1 - \vec{k}_2), \tag{122}$$

to write the integrals over the spatial momentum components as integrals over three-momenta:

$$2\mathrm{Im}(\mathcal{M}^{(1)}) = -e^4 J_1^{(\lambda)}(p', k)\left(\frac{\not{p} + m}{p^2 - m^2}\right)\gamma^\mu \left\{ \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3} (\not{k}_2 + m) T_{\mu\nu}(k_1) \left[\frac{2\pi\delta(k_1^0 - \omega_{k_1}) 2\pi\delta(k_2^0 - E_{k_2})}{(2\omega_{k_1})(2E_{k_2})} + \frac{2\pi\delta(k_1^0 + \omega_{k_1}) 2\pi\delta(k_2^0 + E_{k_2})}{(2\omega_{k_1})(2E_{k_2})} \right] (2\pi)^3 \delta^{(3)}(p - k_1 - k_2) \right\} \gamma^\nu \left(\frac{\not{p} + m}{p^2 - m^2}\right) J_2^{(\lambda)}(p', k) \tag{123}$$

and

$$2\mathrm{Im}(\mathcal{M}^{(2)}) = e^4 J_1^{(\lambda)}(p', k)\left(\frac{\not{p} + m}{p^2 - m^2}\right)\gamma^\mu \left\{ \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3} (\not{k}_2 + m) T_{\mu\nu}(k_1) \left[\frac{2\pi\delta(k_1^0 - \Omega_{k_1}) 2\pi\delta(k_2^0 - E_{k_2})}{(2\Omega_{k_1})(2E_{k_2})} + \frac{2\pi\delta(k_1^0 + \Omega_{k_1}) 2\pi\delta(k_2^0 + E_{k_2})}{(2\Omega_{k_1})(2E_{k_2})} \right] (2\pi)^3 \delta^{(3)}(p - k_1 - k_2) \right\} \gamma^\nu \left(\frac{\not{p} + m}{p^2 - m^2}\right) J_2^{(\lambda)}(p', k). \tag{124}$$

Recall the relations (51a) and (53) for the gauge polarization vectors. Furthermore, we apply the completeness relation (D17a) for standard particle spinors in $(2+1)$ dimensions to this particular case, i.e.,

$$\sum_s u^{(s)}(k_2) \bar{u}^{(s)}(k_2) = \not{k}_2 + m, \tag{125}$$

where the sum runs over the spin projection s of the fermion in the former loop. Note that this spinor index is kept explicitly. We can then write

$$2\mathrm{Im}(\mathcal{M}^{(1)}) = -\sum_{s,\lambda',\lambda''} \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3} \left\{ -\mathrm{i}e^2 J_1^{(\lambda)}(p', k)\left(\frac{\not{p} + m}{p^2 - m^2}\right)\gamma^\mu u^{(s)}(k_2) \bar{\varepsilon}_\mu^{(\lambda')}(k_1) \right\} \times g_{\lambda'\lambda''} \left\{ \mathrm{i}e^2 \bar{\varepsilon}_\nu^{(\lambda'')*}(k_1) \bar{u}^{(s)}(k_2) \gamma^\nu \left(\frac{\not{p} + m}{p^2 - m^2}\right) J_2^{(\lambda)}(p', k) \right\} 2\pi\delta(k_1^2) 2\pi\delta(k_2^2 - m^2)(2\pi)^3 \delta^{(3)}(p - k_1 - k_2) \times [\theta(k_1^0)\theta(k_2^0) + \theta(-k_1^0)\theta(-k_2^0)] \tag{126}$$

and

$$2\mathrm{Im}(\mathcal{M}^{(2)}) = \sum_s \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3} \left\{ \mathrm{i}e^2 J_1^{(\lambda)}(p', k)\left(\frac{\not{p} + m}{p^2 - m^2}\right)\gamma^\mu u^{(s)}(k_2) \bar{\varepsilon}_\mu^{(+)}(k_1) \right\} \times \left\{ \mathrm{i}e^2 \bar{\varepsilon}_\nu^{(-)}(k_1) \bar{u}^{(s)}(k_2) \gamma^\nu \left(\frac{\not{p} + m}{p^2 - m^2}\right) J_2^{(\lambda)}(p', k) \right\} 2\pi\delta(k_1^2 - M^2) 2\pi\delta(k_2^2 - m^2)(2\pi)^3 \delta^{(3)}(p - k_1 - k_2) \times [\theta(k_1^0)\theta(k_2^0) + \theta(-k_1^0)\theta(-k_2^0)]. \tag{127}$$

In this way we obtain

$$2\mathrm{Im}(\mathcal{M}^{(1)}) = -\sum_{s,\lambda',\lambda''}\int\frac{\mathrm{d}^3k_1}{(2\pi)^3}\int\frac{\mathrm{d}^3k_2}{(2\pi)^3}\left\{-\mathrm{i}e^2J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(\lambda')}(k_2,k_1)\right\}$$
$$\times g_{\lambda'\lambda''}\left\{\mathrm{i}e^2J_1^{(\lambda'')}(k_2,k_1)\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(\lambda)}(p',k)\right\}2\pi\delta(k_1^2-M^2)2\pi\delta(k_2^2-m^2)(2\pi)^3\delta^{(3)}(p-k_1-k_2)$$
$$\times[\theta(k_1^0)\theta(k_2^0)+\theta(-k_1^0)\theta(-k_2^0)] \tag{128}$$

and

$$2\mathrm{Im}(\mathcal{M}^{(2)}) = \sum_{s}\int\frac{\mathrm{d}^3k_1}{(2\pi)^3}\int\frac{\mathrm{d}^3k_2}{(2\pi)^3}\left\{\mathrm{i}e^2J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(+)}(k_2,k_1)\right\}\left\{\mathrm{i}e^2J_1^{(-)}(k_2,k_1)\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(\lambda)}(p',k)\right\}$$
$$\times 2\pi\delta(k_1^2-M^2)2\pi\delta(k_2^2-m^2)(2\pi)^3\delta^{(3)}(p-k_1-k_2)[\theta(k_1^0)\theta(k_2^0)+\theta(-k_1^0)\theta(-k_2^0)], \tag{129}$$

where

$$J_2^{(+)}(k_2,k_1) = \gamma^\mu u^{(s)}(k_2)\bar{\varepsilon}_\mu^{(+)}(k_1), \tag{130a}$$

$$J_1^{(-)}(k_2,k_1) = \bar{\varepsilon}_\nu^{(-)}(k_1)\bar{u}^{(s)}(k_2)\gamma^\nu. \tag{130b}$$

Let us define

$$g_{\lambda'\lambda''}\mathcal{M}_1^{(\lambda')}\mathcal{M}_1^{(\lambda'')\dagger} = g_{\lambda'\lambda''}\left(e^2J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(\lambda')}(k_2,k_1)\right)\left(e^2J_1^{(\lambda'')}(k_2,k_1)\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(\lambda)}(p',k)\right) \tag{131}$$

and

$$\mathcal{M}_2^{(+)}\mathcal{M}_2^{(-)\dagger} = \left(e^2J_1^{(\lambda)}(p',k)\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(+)}(k_2,k_1)\right)\left(e^2J_1^{(-)}(k_2,k_1)\left(\frac{\not{p}+m}{p^2-m^2}\right)J_2^{(\lambda)}(p',k)\right). \tag{132}$$

Thus, we can express both imaginary parts as

$$2\mathrm{Im}(\mathcal{M}^{(1)}) = -\sum_{s,\lambda',\lambda''}\int\frac{\mathrm{d}^3k_1}{(2\pi)^3}\int\frac{\mathrm{d}^3k_2}{(2\pi)^3}g_{\lambda'\lambda''}\mathcal{M}_1^{(\lambda')}\mathcal{M}_1^{(\lambda'')\dagger}2\pi\delta(k_1^2)2\pi\delta(k_2^2-m^2)$$
$$\times(2\pi)^3\delta^{(3)}(p-k_1-k_2)[\theta(k_1^0)\theta(k_2^0)+\theta(-k_1^0)\theta(-k_2^0)] \tag{133}$$

and

$$2\mathrm{Im}(\mathcal{M}^{(2)}) = -\sum_{s}\int\frac{\mathrm{d}^3k_1}{(2\pi)^3}\int\frac{\mathrm{d}^3k_2}{(2\pi)^3}\mathcal{M}_2^{(+)}\mathcal{M}_2^{(-)\dagger}2\pi\delta(k_1^2-M^2)2\pi\delta(k_2^2-m^2)$$
$$\times(2\pi)^3\delta^{(3)}(p-k_1-k_2)[\theta(k_1^0)\theta(k_2^0)+\theta(-k_1^0)\theta(-k_2^0)]. \tag{134}$$

The sum in Eq. (133) runs over the spin projection of the fermion and the polarization of the photon. Both particles were put on shell by cutting the diagram of the forward-scattering amplitude (see Fig. 5) into two pieces. Note that the right-hand side of Eq. (134) is zero, as the δ function does not provide a contribution due to the large mass scale M. This behavior is precisely the effect of the Lee-Wick prescription according to which the negative-norm states are removed from the asymptotic Hilbert space just as in the tree-level analysis [see Eq. (107) and the subsequent paragraph]. So, we conclude that the optical theorem and, therefore, unitarity continue being valid at one-loop order, as well.

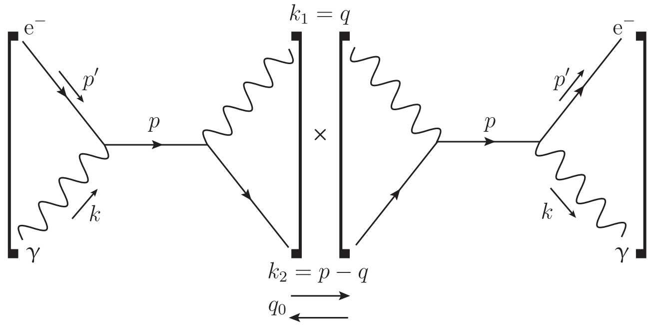

FIG. 5. Sum over intermediate states and energy flow in the cut Compton diagram of Fig. 4.

V. CONCLUSIONS AND OUTLOOK

In this paper, we considered a higher-derivative Chern-Simons-type modification of electrodynamics in $(2+1)$ dimensions. We decomposed the Lagrangian of the model into a physical and a ghost sector and obtained the polarization vectors for the corresponding modes. In addition, the propagator of the theory was computed and it was demonstrated how it can be expressed in terms of the polarization vectors. Based on these findings, we performed the canonical quantization of the theory and studied its perturbative unitarity at both tree level and one-loop order by checking the validity of the optical theorem.

Throughout this paper, we explicitly demonstrated that reflection positivity, known as a sufficient condition for unitarity, is satisfied. As the latter requirement applies to a free field theory only, we were interested in understanding unitarity when taking interactions into account. Hence, we coupled our theory to standard Dirac fermions in $(2+1)$ spacetime dimensions and evaluated the optical theorem for particular scattering processes. This analysis of unitarity revealed inconsistencies due to negative contributions at the pole of the ghost, as one should expect. However, by using the Lee-Wick prescription we demonstrated that unitarity is conserved at both tree level and one-loop order. The method of removing contributions from ghosts from the in- and out-states clearly provided this result. We applied the usual cutting rules of Feynman diagrams and amplitudes to guarantee the validity of the optical theorem. It was necessary to assume that the ghost mass is high enough, perhaps of the order of the Planck mass. It is expected that the situation at higher order in perturbation theory will not be very different.

It is also reasonable to expect that these results can be generalized naturally to the four-dimensional case where the higher-derivative Chern-Simons-like term breaks Lorentz symmetry. Some preliminary studies of unitarity in this alternative theory have been carried out in [57]. They are complemented by the analysis performed in our latest work [59].

Moreover, our opinion is that the results obtained here could serve as a base to explicitly define classes of higher-derivative theories consistent with the requirement of unitarity. In particular, our methodology could be useful for studies of various higher-derivative extensions of gravity including the Lorentz-breaking ones. We hope that this methodology will help to solve the problem of formulating a perturbatively consistent gravity model.

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APPENDIX A: DIRAC FORMALISM

We follow the Dirac procedure to reduce second-class constraints from the higher-derivative theory based on the Lagrangian (2) to zero [35] and to find the Dirac brackets. From Eqs. (41a) and (41b), we have four primary second-class constraints,

$$\chi_0(t,\vec{x}) = \Pi^0(t,\vec{x}) - \frac{g}{2}\epsilon^{ij}\partial_i A_j(t,\vec{x}), \tag{A1a}$$

$$\chi_1(t,\vec{x}) = P_0(t,\vec{x}) + \dot{A}_0(t,\vec{x}) + \frac{g}{2}\epsilon^{ij}\partial_i \dot{A}_j(t,\vec{x}), \tag{A1b}$$

$$\varphi^i(t,\vec{x}) = \Pi^i(t,\vec{x}) + \frac{g}{2}\epsilon^{ij}\dot{A}_j(t,\vec{x}) - \frac{g}{2}\epsilon^{ij}\partial_j A_0(t,\vec{x}). \tag{A1c}$$

The nonvanishing elements of the algebra are

$$\{\chi_1(t,\vec{x}),\chi_0(t,\vec{y})\} = \delta^{(2)}(\vec{x}-\vec{y}), \tag{A2a}$$

$$\{\varphi^i(t,\vec{x}),\chi_1(t,\vec{y})\} = -g\epsilon^{ij}\partial_j\delta^{(2)}(\vec{x}-\vec{y}), \tag{A2b}$$

$$\{\varphi^i(t,\vec{x}),\varphi^j(t,\vec{y})\} = g\epsilon^{ij}\delta^{(2)}(\vec{x}-\vec{y}). \tag{A2c}$$

The convention we use is that the derivatives act on the first set of spatial variables named $\vec{x}$, in general. To begin, let us introduce the notation $\varphi_A = (\chi_0, \chi_1, \varphi^i)$, with $A = \bar{0}, \bar{1}, 1, 2$ and $i = 1, 2$. The matrix of the second-class constraints will be denoted by

$$C_{AB}(t;\vec{x},\vec{y}) = \{\varphi_A(t,\vec{x}), \varphi_B(t,\vec{y})\}. \tag{A3}$$

From Eq. (A2c) we have

$$C_{AB} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & -g\partial_2 & g\partial_1 \\ 0 & -g\partial_2 & 0 & g \\ 0 & g\partial_1 & -g & 0 \end{bmatrix} \delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(A4)}$$

The inverse matrix is (where the δ function is not inverted)

$$C^{-1}_{AB} = \begin{bmatrix} 0 & 1 & -\partial_1 & -\partial_2 \\ -1 & 0 & 0 & 0 \\ -\partial_1 & 0 & 0 & -1/g \\ -\partial_2 & 0 & 1/g & 0 \end{bmatrix} \delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(A5)}$$

The nonzero components are

$$C^{-1}_{\bar{0}\,\bar{1}}(\vec{x},\vec{y}) = -C^{-1}_{\bar{1}\,\bar{0}}(\vec{x},\vec{y}) = \delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A6a)}$$

$$C^{-1}_{\bar{0}i}(\vec{x},\vec{y}) = C^{-1}_{i\bar{0}}(\vec{x},\vec{y}) = -\partial_i\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A6b)}$$

$$C^{-1}_{ij}(\vec{x},\vec{y}) = -\frac{1}{g}\epsilon^{ij}\delta^{(2)}(\vec{x}-\vec{y}), \quad i,j = 1,2. \quad \text{(A6c)}$$

The Dirac brackets are defined by

$$\{X,Y\}^* = \{X,Y\} - \{X,\varphi_A\}C^{-1}_{AB}\{\varphi_B,Y\}. \quad \text{(A7)}$$

We promote the Dirac algebra to the equal-time commutators satisfied by the fields and obtain

$$[A_0(t,\vec{x}),\dot{A}_0(t,\vec{y})] = -\mathrm{i}\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A8a)}$$

$$[A_0(t,\vec{x}),P_0(t,\vec{y})] = \mathrm{i}\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A8b)}$$

$$[A_0(t,\vec{x}),P^i(t,\vec{y})] = \frac{\mathrm{i}g}{2}\epsilon^{ij}\partial_j\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A8c)}$$

$$[\dot{A}_i(t,\vec{x}),\dot{A}_j(t,\vec{y})] = -\frac{\mathrm{i}}{g}\epsilon^{ij}\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A8d)}$$

$$[\dot{A}_i(t,\vec{x}),\dot{A}_0(t,\vec{y})] = -\mathrm{i}\partial_i\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A8e)}$$

$$[\dot{A}_i(t,\vec{x}),P_0(t,\vec{y})] = \frac{\mathrm{i}}{2}\partial_i\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A8f)}$$

$$[\dot{A}_i(t,\vec{x}),P^j(t,\vec{y})] = \frac{\mathrm{i}g}{2}\epsilon^{jk}\partial_i\partial_k\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A8g)}$$

$$[\dot{A}_i(t,\vec{x}),\Pi^j(t,\vec{y})] = \frac{\mathrm{i}}{2}\delta_{ij}\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A8h)}$$

$$[P_0(t,\vec{x}),\Pi^i(t,\vec{y})] = \frac{\mathrm{i}g}{4}\epsilon^{ij}\partial_j\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A8i)}$$

$$[\Pi_0(t,\vec{x}),P^i(t,\vec{y})] = -\frac{\mathrm{i}g}{2}\epsilon^{ij}\partial_j\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(A8j)}$$

$$[\Pi^i(t,\vec{x}),\Pi^j(t,\vec{y})] = -\frac{\mathrm{i}g}{4}\epsilon^{ij}\delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(A8k)}$$

Note that the momentum P^μ has been changed in comparison to that employed in Ref. [35] and, consequently, we have obtained a different algebra.

APPENDIX B: THE HAMILTONIAN

The current section delivers a detailed demonstration on how the Hamiltonian of the theory given by Eq. (2) can be expressed in terms of creation and annihilation operators. We consider the Hamiltonian (42a) written as

$$H = H_{\bar{A}} + H_G, \quad \text{(B1)}$$

where by using the decomposition (45) we have

$$H_{\bar{A}} = \int \mathrm{d}^2x\left(-\frac{1}{2}\dot{\bar{A}}_\mu(\hat{U}^{\mu\nu})\dot{\bar{A}}_\nu + \frac{1}{2}\bar{A}_\mu(\hat{U}^{\mu\nu})\nabla^2\bar{A}_\nu\right), \quad \text{(B2)}$$

$$H_G = \frac{g}{2}\int \mathrm{d}^2x\epsilon^{ij}\dot{G}_i\Box G_j. \quad \text{(B3)}$$

Above, we have applied the equation of motion (46) for the ghost and $\Box\bar{A}_\mu = 0$ for the photon.

Let us define

$$H^{\mathrm{kin}}_{\bar{A}} = -\frac{1}{2}\int \mathrm{d}^2x\dot{\bar{A}}_\mu\hat{U}^{\mu\nu}\dot{\bar{A}}_\nu, \quad \text{(B4)}$$

$$H^{\mathrm{pot}}_{\bar{A}} = \frac{1}{2}\int \mathrm{d}^2x\bar{A}_\mu\hat{U}^{\mu\nu}\nabla^2\bar{A}_\nu. \quad \text{(B5)}$$

Inserting the photon field operator of Eq. (48), the first contribution reads

$$H^{\mathrm{kin}}_{\bar{A}} = \frac{1}{8}\int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\sum_{\lambda,\lambda'}\Big[a^{(\lambda)}_{\vec{p}}a^{(\lambda')}_{-\vec{p}}\bar{e}^{(\lambda)}_\mu(\vec{p})U^{\mu\nu}_{-\vec{p}}\bar{e}^{(\lambda')}_\nu(-\vec{p})e^{-2\mathrm{i}\omega_{\vec{p}}x_0} - a^{(\lambda)}_{\vec{p}}a^{(\lambda')\dagger}_{\vec{p}}\bar{e}^{(\lambda)}_\mu(\vec{p})U^{\mu\nu*}_{\vec{p}}\bar{e}^{(\lambda')*}_\nu(\vec{p}) - a^{(\lambda)\dagger}_{\vec{p}}a^{(\lambda')}_{\vec{p}}\bar{e}^{(\lambda)*}_\mu(\vec{p})U^{\mu\nu}_{\vec{p}}\bar{e}^{(\lambda')}_\nu(\vec{p})$$
$$+ a^{(\lambda)\dagger}_{\vec{p}}a^{(\lambda')\dagger}_{-\vec{p}}\bar{e}^{(\lambda)*}_\mu(\vec{p})U^{\mu\nu*}_{-\vec{p}}\bar{e}^{(\lambda')*}_\nu(-\vec{p})e^{2\mathrm{i}\omega_{\vec{p}}x_0}\Big], \quad \text{(B6)}$$

where $U^{\mu\nu}_{\vec{p}} = (\eta^{\mu\nu} - \mathrm{i}g\epsilon^{\mu\beta\nu}p_\beta)_{p_0=\omega_p}$ corresponds to Eq. (42b) in momentum space.

In the same way,

$$H_{\bar{A}}^{\rm pot} = -\frac{1}{8}\int \frac{d^2\vec{p}}{(2\pi)^2}\sum_{\lambda,\lambda'}\frac{\vec{p}^2}{\omega_{\vec{p}}^2}\Big[a_{\vec{p}}^{(\lambda)}a_{-\vec{p}}^{(\lambda')}\bar{e}_\mu^{(\lambda)}(\vec{p})U_{-\vec{p}}^{\mu\nu}\bar{e}_\nu^{(\lambda')}(-\vec{p})e^{-2i\omega_{\vec{p}}x_0} + a_{\vec{p}}^{(\lambda)}a_{\vec{p}}^{(\lambda')\dagger}\bar{e}_\mu^{(\lambda)}(\vec{p})U_{\vec{p}}^{\mu\nu*}\bar{e}_\nu^{(\lambda')*}(\vec{p})$$
$$+ a_{\vec{p}}^{(\lambda)\dagger}a_{\vec{p}}^{(\lambda')}\bar{e}_\mu^{(\lambda)*}(\vec{p})U_{\vec{p}}^{\mu\nu}\bar{e}_\nu^{(\lambda')}(\vec{p}) + a_{\vec{p}}^{(\lambda)\dagger}a_{-\vec{p}}^{(\lambda')\dagger}\bar{e}_\mu^{(\lambda)*}(\vec{p})U_{-\vec{p}}^{\mu\nu*}\bar{e}_\nu^{(\lambda')*}(-\vec{p})e^{2i\omega_{\vec{p}}x_0}\Big]. \tag{B7}$$

We see that the first and last terms vanish due to the global factor $1-\vec{p}^2/\omega_p^2$, while the other terms pick up a factor of $1+\vec{p}^2/\omega_p^2 = 2$. We arrive at

$$H_{\bar{A}} = -\frac{1}{4}\int \frac{d^2\vec{p}}{(2\pi)^2}\sum_{\lambda,\lambda'}\eta_{\lambda\lambda'}(a_{\vec{p}}^{(\lambda)}a_{\vec{p}}^{(\lambda')\dagger} + a_{\vec{p}}^{(\lambda)\dagger}a_{\vec{p}}^{(\lambda')}), \tag{B8}$$

where we have used

$$\bar{e}_\mu^{(\lambda)}U_{\vec{p}}^{\mu\nu*}\bar{e}_\nu^{*(\lambda')} = g^{\lambda\lambda'} \tag{B9}$$

and its complex conjugate.

For the ghost part we insert the ghost field operator of Eq. (52) and obtain

$$H_G = \frac{g}{8}\int \frac{d^2\vec{p}}{(2\pi)^2}\frac{i\epsilon^{ij}}{g^2\Omega_p}\Big[b_{\vec{p}}b_{\vec{p}}^\dagger\bar{\varepsilon}_i^{(+)}(\vec{p})\bar{\varepsilon}_j^{(+)*}(\vec{p}) - b_{\vec{p}}^\dagger b_{\vec{p}}\bar{\varepsilon}_i^{(+)*}(\vec{p})\bar{\varepsilon}_j^{(+)}(\vec{p})\Big], \tag{B10}$$

where we have used that $p^2 = 1/g^2$ as well as

$$\epsilon^{ij}\bar{\varepsilon}_i^{(+)}(\vec{p})\bar{\varepsilon}_j^{(+)}(-\vec{p}) = 0, \tag{B11a}$$

since

$$\bar{\varepsilon}_k^{(+)}(-\vec{p}) = -\bar{\varepsilon}_k^{(+)}(\vec{p}), \tag{B11b}$$

for $k = 1, 2$. We then arrive at

$$H_G = -\frac{1}{4}\int \frac{d^2\vec{p}}{(2\pi)^2}(b_{\vec{p}}b_{\vec{p}}^\dagger + b_{\vec{p}}^\dagger b_{\vec{p}}), \tag{B12}$$

where we have also employed

$$\epsilon^{ij}\bar{\varepsilon}_i^{(\pm)}\bar{\varepsilon}_j^{(\pm)*}|_{p_0=\Omega_p} = \pm 2ig\Omega_p. \tag{B13}$$

This proves our expression (55).

APPENDIX C: EXTENDED EQUAL-TIME COMMUTATORS

In this section we intend to compute the equal-time commutators for the field operators that emerge from field theory of higher derivatives defined by Eq. (2). Consider the basic commutator

$$[A_\mu(x), A_\nu(y)] = [\bar{A}_\mu(x), \bar{A}_\nu(y)] + [G_\mu(x), G_\nu(y)], \tag{C1a}$$

with

$$[\bar{A}_\mu(x), \bar{A}_\nu(y)] = -\int \frac{d^2\vec{p}}{(2\pi)^2}\frac{1}{2\omega_p}(T_{\mu\nu}(p)e^{-ip\cdot(x-y)} - T_{\nu\mu}(p)e^{ip\cdot(x-y)})_{p_0=\omega_p}, \tag{C1b}$$

$$[G_\mu(x), G_\nu(y)] = \int \frac{d^2\vec{p}}{(2\pi)^2}\frac{1}{2\Omega_p}(T_{\mu\nu}(p)e^{-ip\cdot(x-y)} - T_{\nu\mu}(p)e^{ip\cdot(x-y)})_{p_0=\Omega_p}. \tag{C1c}$$

To derive the Dirac commutators we work directly with the field operators of Eqs. (48) and (52). Our strategy will be as follows:

(a) We consider the basic commutator (C1a) and construct the various elements in phase space by applying the different operators on the fields.

(b) For a commutator containing $\Box A_\mu(t,\vec{x})$ we use the identities $\Box\bar{A}_\mu(t,\vec{x}) = 0$ and $\Box G_\mu(t,\vec{x}) = -\frac{1}{g^2}G_\mu(t,\vec{x})$.

(c) Whenever an integral involves momentum variables we use the relation $p_\mu = i\partial_\mu$, whereupon derivatives can be extracted from the integral.

(d) To treat derivatives for the second variable ∂_i^y, we integrate by parts to produce ∂_i^x, whereby an additional minus sign occurs.

(e) We assume that the spatial derivatives ∂_i act on the first variable $\vec{x}$ of δ functions in all final expressions.

1. Commutator $[A_0(t,\vec{x}), \dot{A}_0(t,\vec{y})]$

With the previous rules in mind and to demonstrate our technique explicitly we apply a first time derivative ∂_{y_0} to the basic commutator (C1a):

$$[A_\mu(x), \partial_{y_0} A_\nu(y)] = -\int \frac{d^2\vec{p}}{(2\pi)^2} \frac{i}{2} (T_{\mu\nu} e^{-ip\cdot(x-y)} + T_{\nu\mu} e^{ip\cdot(x-y)})_{p_0=\omega_p} + \int \frac{d^2\vec{p}}{(2\pi)^2} \frac{i}{2} (T_{\mu\nu} e^{-ip\cdot(x-y)} + T_{\nu\mu} e^{ip\cdot(x-y)})_{p_0=\Omega_p}. \quad \text{(C2)}$$

We set both times equal, $x_0 = y_0 = t$, and change $\vec{p} \to -\vec{p}$ in the second term of each contribution. We then obtain

$$[A_\mu(t,\vec{x}), \dot{A}_\nu(t,\vec{y})] = -\int \frac{d^2\vec{p}}{(2\pi)^2} \frac{i}{2} (T_{\mu\nu}(\vec{p}) + T_{\nu\mu}(-\vec{p})) e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + \int \frac{d^2\vec{p}}{(2\pi)^2} \frac{i}{2} (T_{\mu\nu}(\vec{p}) + T_{\nu\mu}(-\vec{p})) e^{i\vec{p}\cdot(\vec{x}-\vec{y})}. \quad \text{(C3)}$$

In the following calculations we implicitly consider the dependence on ω_p and Ω_p of the expressions in parentheses above. For the indices $\mu = 0$ and $\nu = 0$, we have

$$[A_0(t,\vec{x}), \dot{A}_0(t,\vec{y})] = -\int \frac{d^2\vec{p}}{(2\pi)^2} \frac{i}{2} (2 - 2g^2\omega_p^2) e^{i\vec{p}\cdot(\vec{x}-\vec{y})} + \int \frac{d^2\vec{p}}{(2\pi)^2} \frac{i}{2} (2 - 2g^2\Omega_p^2) e^{i\vec{p}\cdot(\vec{x}-\vec{y})}, \quad \text{(C4)}$$

where we have used $T_{00}(\vec{p}) + T_{00}(-\vec{p}) = 2 - 2g^2 p_0$. Adding both terms yields

$$[A_0(t,\vec{x}), \dot{A}_0(t,\vec{y})] = \int \frac{d^2\vec{p}}{(2\pi)^2} i g^2 (\omega_p^2 - \Omega_p^2) e^{i\vec{p}\cdot(\vec{x}-\vec{y})}, \quad \text{(C5)}$$

and since

$$\omega_p^2 - \Omega_p^2 = -\frac{1}{g^2}, \quad \text{(C6)}$$

one arrives at the first commutator (A8a):

$$[A_0(t,\vec{x}), \dot{A}_0(t,\vec{y})] = -i\delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(C7)}$$

2. Commutator $[A_0(t,\vec{x}), P_0(t,\vec{y})]$

Here we compute an unmodified commutator by using our method. Recall Eq. (41a) and write

$$P_0(t,\vec{y}) = -\dot{A}_0(t,\vec{y}) - \frac{g}{2}\epsilon^{ij}\partial_i \dot{A}_j(t,\vec{y}). \quad \text{(C8)}$$

We get

$$[A_0(t,\vec{x}), P_0(t,\vec{y})] = [A_0(t,\vec{x}), -\dot{A}_0(t,\vec{y}) - \frac{g}{2}\epsilon^{ij}\partial_i \dot{A}_j(t,\vec{y})]. \quad \text{(C9)}$$

The second commutator is zero, i.e.,

$$[A_0(t,\vec{x}), \dot{A}_j(t,\vec{y})] = 0, \quad \text{(C10)}$$

and using the result (C7) we arrive at

$$[A_0(t,\vec{x}), P_0(t,\vec{y})] = i\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(C11)}$$

which gives Eq. (A8b).

3. Commutator $[A_0(t,\vec{x}), P^i(t,\vec{y})]$

It follows from (41a) that the spatial momentum components read

$$P^i = -\dot{A}^i + \frac{g}{2}\epsilon^{ik}\Box A_k + \frac{g}{2}\epsilon^{ik}\ddot{A}_k - \frac{g}{2}\epsilon^{ik}\partial_k \dot{A}_0. \quad \text{(C12)}$$

Then

$$[A_0(t,\vec{x}), P^i(t,\vec{y})] = \Big[A_0(t,\vec{x}), -\dot{A}^i(t,\vec{y}) + \frac{g}{2}\epsilon^{ik}\Box A_k(t,\vec{y}) + \frac{g}{2}\epsilon^{ik}\ddot{A}_k(t,\vec{y}) - \frac{g}{2}\epsilon^{ik}\partial_k \dot{A}_0(t,\vec{y})\Big]. \quad \text{(C13)}$$

We take into account that the first commutator is zero; see Eq. (C10). Furthermore, we employ $\Box\bar{A}_k(t,\vec{y}) = 0$ and $\Box G_k(t,\vec{y}) = -\frac{1}{g^2}G_j(t,\vec{y})$ in the second to arrive at

$$[A_0(t,\vec{x}), P^i(t,\vec{y})] = -\frac{1}{2g}\epsilon^{ik}[G_0(t,\vec{x}), G_k(t,\vec{y})] + \frac{g}{2}\epsilon^{ik}[A_0(t,\vec{x}), \ddot{A}_k(t,\vec{y})] + \frac{g}{2}\epsilon^{ik}\partial_k[A_0(t,\vec{x}), \dot{A}_0(t,\vec{y})], \quad \text{(C14)}$$

where the final spatial derivative has been integrated by parts.

One can show that

$$[G_0(t,\vec{x}), G_k(t,\vec{y})] = -i g^2 \partial_k \delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(C15)}$$

and also

$$[A_0(t,\vec{x}), \ddot{A}_k(t,\vec{y})] = i\partial_k \delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(C16)}$$

Substituting these expressions into Eq. (C14), we obtain

$$[A_0(t,\vec{x}),P^i(t,\vec{y})] = -\frac{1}{2g}\epsilon^{ik}(-\mathrm{i}g^2\partial_k\delta^{(2)}(\vec{x}-\vec{y})) + \frac{g}{2}\epsilon^{ik}(\mathrm{i}\partial_k\delta^{(2)}(\vec{x}-\vec{y})) + \frac{g}{2}\epsilon^{ik}\partial_k(-\mathrm{i}\delta^{(2)}(\vec{x}-\vec{y})). \quad \text{(C17)}$$

The last two terms cancel and we arrive at

$$[A_0(t,\vec{x}),P^i(t,\vec{y})] = \frac{\mathrm{i}g}{2}\epsilon^{ik}\partial_k\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(C18)}$$

which is Eq. (A8c).

4. Commutator $[\dot{A}_i(t,\vec{x}),\,\dot{A}_j(t,\vec{y})]$

To derive Eq. (A8d), it follows from Eq. (C1a) that

$$[\dot{A}_i(t,\vec{x}),\dot{A}_j(t,\vec{y})] = -\int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\frac{\omega_p}{2}(T_{ij}(\vec{p})-T_{ji}(-\vec{p}))e^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} + \int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\frac{\Omega_p}{2}(T_{ij}(\vec{p})-T_{ji}(-\vec{p}))e^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})}, \quad \text{(C19)}$$

and applying the definition (51b) we find

$$T_{ij}(\vec{p})-T_{ji}(-\vec{p}) = -2\mathrm{i}g\epsilon_{ij}p_0. \quad \text{(C20)}$$

Thus, we have

$$[\dot{A}_i(t,\vec{x}),\dot{A}_j(t,\vec{y})] = -\int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}(-\mathrm{i}g\epsilon_{ij}\omega_p^2)e^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} + \int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}(-\mathrm{i}g\epsilon_{ij}\Omega_p^2)e^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} = \mathrm{i}g\epsilon_{ij}\int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}(\omega_p^2-\Omega_p^2)e^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})}, \quad \text{(C21)}$$

and finally,

$$[\dot{A}_i(t,\vec{x}),\dot{A}_j(t,\vec{y})] = -\frac{\mathrm{i}}{g}\epsilon_{ij}\delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(C22)}$$

5. Commutator $[\dot{A}_i(t,\vec{x}),\,\dot{A}_0(t,\vec{y})]$

Repeating the calculations performed in Appendix C 1 we find

$$[\dot{A}_i(t,\vec{x}),\dot{A}_0(t,\vec{y})] = -\int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\frac{\omega_p}{2}(T_{i0}(\vec{p})-T_{0i}(-\vec{p}))e^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} + \int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\frac{\Omega_p}{2}(T_{i0}(\vec{p})-T_{0i}(-\vec{p}))e^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})}. \quad \text{(C23)}$$

Using

$$[T_{i0}(\vec{p})-T_{0i}(-\vec{p})]_{p_0=\omega_p} = -2g^2\omega_p p_i, \quad \text{(C24)}$$

we can write

$$[\dot{A}_i(t,\vec{x}),\dot{A}_0(t,\vec{y})] = -\int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\frac{\omega_p}{2}(-2g^2\omega_p p_i)e^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} + \int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}\frac{\Omega_p}{2}(-2g^2\Omega_p p_i)e^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})}. \quad \text{(C25)}$$

Therefore,

$$[\dot{A}_i(t,\vec{x}),\dot{A}_0(t,\vec{y})] = \int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}g^2p_i(\omega_p^2-\Omega_p^2)e^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})} = -\int\frac{\mathrm{d}^2\vec{p}}{(2\pi)^2}p_ie^{\mathrm{i}\vec{p}\cdot(\vec{x}-\vec{y})}. \quad \text{(C26)}$$

By employing $p_i = \mathrm{i}\partial_i$, we arrive at

$$[\dot{A}_i(t,\vec{x}),\dot{A}_0(t,\vec{y})] = \mathrm{i}\partial_i\delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(C27)}$$

6. Commutator $[\dot{A}_i(t,\vec{x}),\,P_0(t,\vec{y})]$

We have

$$[\dot{A}_i(t,\vec{x}),P_0(t,\vec{y})] = \left[\dot{A}_i(t,\vec{x}),-\dot{A}_0(t,\vec{y})-\frac{g}{2}\epsilon^{mk}\partial_m\dot{A}_k(t,\vec{y})\right] = -[\dot{A}_i(t,\vec{x}),\dot{A}_0(t,\vec{y})]+\frac{g}{2}\epsilon^{mk}\partial_m[\dot{A}_i(t,\vec{x}),\dot{A}_k(t,\vec{y})]. \quad \text{(C28)}$$

These commutators have been found in Appendixes C 4 and C 5 and after inserting their results we obtain

$$[\dot{A}_i(t,\vec{x}),P_0(t,\vec{y})] = \mathrm{i}\partial_i\delta^{(2)}(\vec{x}-\vec{y})+\frac{g}{2}\epsilon^{mk}\partial_m\left(\frac{-\mathrm{i}}{g}\epsilon_{ik}\delta^{(2)}(\vec{x}-\vec{y})\right). \quad \text{(C29)}$$

Therefore,

$$[\dot{A}_i(t,\vec{x}),P_0(t,\vec{y})] = \frac{\mathrm{i}}{2}\partial_i\delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(C30)}$$

7. Commutator $[\dot{A}_i(t,\vec{x}),\, P^j(t,\vec{y})]$

Consider

$$[\dot{A}_i(t,\vec{x}), P^j(t,\vec{y})] = \Big[\dot{A}_i(t,\vec{x}), -\dot{A}^j(t,\vec{y}) + \frac{g}{2}\epsilon^{jk}\Box A_k(t,\vec{y}) + \frac{g}{2}\epsilon^{jk}\ddot{A}_k(t,\vec{y}) - \frac{g}{2}\epsilon^{jk}\partial_k\dot{A}_0(t,\vec{y})\Big]. \tag{C31}$$

Hence,

$$\begin{aligned}[\dot{A}_i(t,\vec{x}), P^j(t,\vec{y})] = &-[\dot{A}_i(t,\vec{x}), \dot{A}^j(t,\vec{y})] \\ &+ \Big[\dot{A}_i(t,\vec{x}), \frac{g}{2}\epsilon^{jk}\Box A_k(t,\vec{y})\Big] \\ &+ \Big[\dot{A}_i(t,\vec{x}), \frac{g}{2}\epsilon^{jk}\ddot{A}_k(t,\vec{y})\Big] \\ &- \Big[\dot{A}_i(t,\vec{x}), \frac{g}{2}\epsilon^{jk}\partial_k\dot{A}_0(t,\vec{y})\Big]. \end{aligned} \tag{C32}$$

We get

$$\begin{aligned}[\dot{A}_i(t,\vec{x}), P^j(t,\vec{y})] = &\frac{\mathrm{i}}{g}\epsilon_i{}^j\delta^{(2)}(\vec{x}-\vec{y}) \\ &- \frac{1}{2g}\epsilon^{jk}[\dot{G}_i(t,\vec{x}), G_k(t,\vec{y})] \\ &+ \frac{g}{2}\epsilon^{jk}[\dot{A}_i(t,\vec{x}), \ddot{A}_k(t,\vec{y})] \\ &+ \frac{g}{2}\epsilon^{jk}\partial_k[\dot{A}_i(t,\vec{x}), \dot{A}_0(t,\vec{y})], \end{aligned} \tag{C33}$$

where we have also used Eq. (C22).

Since

$$\begin{aligned}[\dot{G}_i(t,\vec{x}), G_k(t,\vec{y})] &= -\mathrm{i}(\eta_{ik} + g^2\partial_i\partial_k)\delta^{(2)}(\vec{x}-\vec{y}), \\ [\dot{A}_i(t,\vec{x}), \ddot{A}_k(t,\vec{y})] &= \frac{\mathrm{i}}{g^2}(\eta_{ik} + g^2\partial_i\partial_k)\delta^{(2)}(\vec{x}-\vec{y}), \end{aligned} \tag{C34}$$

and with Eq. (C27), we write

$$\begin{aligned}&[\dot{A}_i(t,\vec{x}), P^j(t,\vec{y})] \\ &= \frac{\mathrm{i}}{g}\epsilon_i{}^j\delta^{(2)}(\vec{x}-\vec{y}) - \frac{1}{2g}\epsilon^{jk}[-\mathrm{i}(\eta_{ik} + g^2\partial_i\partial_k)\delta^{(2)}(\vec{x}-\vec{y})] \\ &\quad + \frac{g}{2}\epsilon^{jk}\Big[\frac{\mathrm{i}}{g^2}(\eta_{ik} + g^2\partial_i\partial_k)\delta^{(2)}(\vec{x}-\vec{y})\Big] \\ &\quad + \frac{g}{2}\epsilon^{jk}\partial_k[-\mathrm{i}\partial_i\delta^{(2)}(\vec{x}-\vec{y})]. \end{aligned} \tag{C35}$$

We see that the first, second, and fourth terms cancel and are left with the result

$$[\dot{A}_i(t,\vec{x}), P^j(t,\vec{y})] = \frac{\mathrm{i}g}{2}\epsilon^{jk}\partial_i\partial_k\delta^{(2)}(\vec{x}-\vec{y}). \tag{C36}$$

8. Commutator $[\dot{A}_i(t,\vec{x}),\, \Pi^j(t,\vec{y})]$

Inserting the field operators, we have

$$[\dot{A}_i(t,\vec{x}), \Pi^j(t,\vec{y})] = \Big[\dot{A}_i(t,\vec{x}), -\frac{g}{2}\epsilon^{jk}\dot{A}_k(t,\vec{y}) + \frac{g}{2}\epsilon^{jk}\partial_k A_0(t,\vec{y})\Big], \tag{C37}$$

which is equal to

$$\begin{aligned}[\dot{A}_i(t,\vec{x}), \Pi^j(t,\vec{y})] = &-\frac{g}{2}\epsilon^{jk}[\dot{A}_i(t,\vec{x}), \dot{A}_k(t,\vec{y})] \\ &- \frac{g}{2}\epsilon^{jk}\partial_k[\dot{A}_i(t,\vec{x}), A_0(t,\vec{y})]. \end{aligned} \tag{C38}$$

The second commutator is zero and after using Eq. (C22) we find

$$[\dot{A}_i(t,\vec{x}), \Pi^j(t,\vec{y})] = -\frac{g}{2}\epsilon^{jk}\Big[-\frac{\mathrm{i}}{g}\epsilon_{ik}\delta^{(2)}(\vec{x}-\vec{y})\Big]. \tag{C39}$$

Therefore, our result is

$$[\dot{A}_i(t,\vec{x}), \Pi^j(t,\vec{y})] = \frac{\mathrm{i}}{2}\delta^{ij}\delta^{(2)}(\vec{x}-\vec{y}). \tag{C40}$$

9. Commutator $[\dot{A}_0(t,\vec{x}),\, P^i(t,\vec{y})]$

Here we compute one commutator which gives zero. We start with

$$[\dot{A}_0(t,\vec{x}), P^i(t,\vec{y})] = \Big[\dot{A}_0(t,\vec{x}), -\dot{A}^i(t,\vec{y}) + \frac{g}{2}\epsilon^{ij}\Box A_j(t,\vec{y}) + \frac{g}{2}\epsilon^{ik}\ddot{A}_k(t,\vec{y}) - \frac{g}{2}\epsilon^{ik}\partial_k\dot{A}_0(t,\vec{y})\Big], \tag{C41}$$

which yields

$$\begin{aligned}&[\dot{A}_0(t,\vec{x}), P^i(t,\vec{y})] \\ &= -[\dot{A}_0(t,\vec{x}), \dot{A}^i(t,\vec{y})] - \frac{1}{2g}\epsilon^{ik}[\dot{G}_0(t,\vec{x}), G_k(t,\vec{y})] \\ &\quad + \frac{g}{2}\epsilon^{ik}[\dot{A}_0(t,\vec{x}), \ddot{A}_k(t,\vec{y})] + \frac{g}{2}\epsilon^{ik}\partial_k[\dot{A}_0(t,\vec{x}), \dot{A}_0(t,\vec{y})]. \end{aligned} \tag{C42}$$

The last term is zero and so

$$[\dot{A}_0(t,\vec{x}),P^i(t,\vec{y})] = -[\dot{A}_0(t,\vec{x}),\dot{A}^i(t,\vec{y})] - \frac{1}{2g}\epsilon^{ik}[\dot{G}_0(t,\vec{x}),G_k(t,\vec{y})] + \frac{g}{2}\epsilon^{ik}[\dot{A}_0(t,\vec{x}),\ddot{A}_k(t,\vec{y})]. \quad \text{(C43)}$$

We need the three elements

$$[\dot{A}_0(t,\vec{x}),\dot{A}_i(t,\vec{y})] = -\mathrm{i}\partial_i\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(C44a)}$$

$$[\dot{G}_0(t,\vec{x}),G_k(t,\vec{y})] = -\mathrm{i}g\epsilon_{km}\partial^m\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(C44b)}$$

$$[\dot{A}_0(t,\vec{x}),\ddot{A}_k(t,\vec{y}))] = \frac{\mathrm{i}}{g}\epsilon_{km}\partial^m\delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(C44c)}$$

Inserting the latter results gives

$$[\dot{A}_0(t,\vec{x}),P^i(t,\vec{y})] = -[\mathrm{i}\partial_i\delta^{(2)}(\vec{x}-\vec{y})] - \frac{1}{2g}\epsilon^{ik}[-\mathrm{i}g\epsilon_{km}\partial^m\delta^{(2)}(\vec{x}-\vec{y})] + \frac{g}{2}\epsilon^{ik}\left[\frac{\mathrm{i}}{g}\epsilon_{km}\partial^m\delta^{(2)}(\vec{x}-\vec{y})\right]. \quad \text{(C45)}$$

Therefore, our result is

$$[\dot{A}_0(t,\vec{x}),P^i(t,\vec{y})] = 0. \quad \text{(C46)}$$

10. Commutator $[P_0(t,\vec{x}),\ \Pi^i(t,\vec{y})]$

We start with

$$[P_0(t,\vec{x}),\Pi^i(t,\vec{y})] = \left[-\dot{A}_0(t,\vec{x}) - \frac{g}{2}\epsilon^{mr}\partial_m\dot{A}_r(t,\vec{x}), -\frac{g}{2}\epsilon^{ik}\dot{A}_k(t,\vec{y}) + \frac{g}{2}\epsilon^{ik}\partial_k A_0(t,\vec{y})\right] \quad \text{(C47)}$$

or, which is the same,

$$[P_0(t,\vec{x}),\Pi^i(t,\vec{y})] = \frac{g}{2}\epsilon^{ik}[\dot{A}_0(t,\vec{x}),\dot{A}_k(t,\vec{y})] + \frac{g}{2}\epsilon^{ik}\partial_k[\dot{A}_0(t,\vec{x}),A_0(t,\vec{y})] + \frac{g^2}{4}\epsilon^{mr}\partial_m\epsilon^{ik}[\dot{A}_r(t,\vec{y}),\dot{A}_k(t,\vec{y})]. \quad \text{(C48)}$$

Hence, from the previous results of Eqs. (C27), (C7), and (C22) one has

$$[P_0(t,\vec{x}),\Pi^i(t,\vec{y})] = \frac{g}{2}\epsilon^{ik}[-\mathrm{i}\partial_k\delta^{(2)}(\vec{x}-\vec{y})] + \frac{g}{2}\epsilon^{ik}\partial_k[\mathrm{i}\delta^{(2)}(\vec{x}-\vec{y})] + \frac{g^2}{4}\epsilon^{mr}\partial_m\epsilon^{ik}\left[\frac{-\mathrm{i}}{g}\epsilon_{rk}\delta^{(2)}(\vec{x}-\vec{y})\right]. \quad \text{(C49)}$$

The first and second terms cancel each other and we arrive at

$$[P_0(t,\vec{x}),\Pi^i(t,\vec{y})] = \frac{\mathrm{i}g}{4}\epsilon^{im}\partial_m\delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(C50)}$$

11. Commutator $[\Pi_0(t,\vec{x}),\ P^i(t,\vec{y})]$

We have

$$[\Pi_0(t,\vec{x}),P^i(t,\vec{y})] = \left[\frac{g}{2}\epsilon^{lm}\partial_l A_m(t,\vec{x}), -\dot{A}^i(t,\vec{y}) + \frac{g}{2}\epsilon^{ij}\Box A_j(t,\vec{y}) + \frac{g}{2}\epsilon^{ik}\ddot{A}_k(t,\vec{y}) - \frac{g}{2}\epsilon^{ik}\partial_k\dot{A}_0(t,\vec{y})\right]. \quad \text{(C51)}$$

The only nonzero contributions are

$$[\Pi_0(t,\vec{x}),P^i(t,\vec{y})] = -\frac{1}{4}\epsilon^{lm}\partial_l\epsilon^{ij}\left[G_m(t,\vec{x}),G_j(t,\vec{y})] + \frac{g^2}{4}\epsilon^{lm}\partial_l\epsilon^{ik}[A_m(t,\vec{x}),\ddot{A}_k(t,\vec{y})\right]. \quad \text{(C52)}$$

We need

$$[G_m(t,\vec{x}),G_j(t,\vec{y})] = -\mathrm{i}g\epsilon_{mj}\delta^{(2)}(\vec{x}-\vec{y}), \quad \text{(C53a)}$$

$$[A_m(t,\vec{x}),\ddot{A}_k(t,\vec{y})] = \frac{\mathrm{i}}{g}\epsilon_{mk}\delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(C53b)}$$

Inserting the previous commutators results in

$$[\Pi_0(t,\vec{x}),P^i(t,\vec{y})] = -\frac{1}{4}\epsilon^{lm}\partial_l\epsilon^{ij}[-\mathrm{i}g\epsilon_{mj}\delta^{(2)}(\vec{x}-\vec{y})] + \frac{g^2}{4}\epsilon^{lm}\partial_l\epsilon^{ik}\left[\frac{\mathrm{i}}{g}\epsilon_{mk}\delta^{(2)}(\vec{x}-\vec{y})\right], \quad \text{(C54)}$$

and so

$$[\Pi_0(t,\vec{x}),P^i(t,\vec{y})] = -\frac{\mathrm{i}g}{2}\epsilon^{ij}\partial_j\delta^{(2)}(\vec{x}-\vec{y}). \quad \text{(C55)}$$

12. Commutator $[P^i(t,\vec{x}),\, P^j(t,\vec{y})]$

Now we compute a difficult commutator, which we prove to be zero in accordance with the classical result using the constraints and the Dirac approach. We take advantage of the previous findings. Consider

$$[P^i(t,\vec{x}), P^j(t,\vec{y})] = \Big[P^i(t,\vec{x}), -\dot{A}^j(t,\vec{y}) + g\epsilon^{jr}\ddot{A}_r(t,\vec{y}) - \frac{g}{2}\epsilon^{jr}\vec{\nabla}^2 A_r(t,\vec{y}) - \frac{g}{2}\epsilon^{jr}\partial_r\dot{A}_0(t,\vec{y})\Big]. \tag{C56}$$

We rewrite the latter commutator as follows:

$$\begin{aligned}[P^i(t,\vec{x}), P^j(t,\vec{y})] = &-[P^i(t,\vec{x}), \dot{A}^j(t,\vec{y})] \\ &+ g\epsilon^{jr}[P^i(t,\vec{x}), \ddot{A}_r(t,\vec{y})] \\ &- \frac{g}{2}\epsilon^{jr}\vec{\nabla}^2[P^i(t,\vec{x}), A_r(t,\vec{y})] \\ &+ \frac{g}{2}\epsilon^{jr}\partial_r[P^i(t,\vec{x}), \dot{A}_0(t,\vec{y})].\end{aligned} \tag{C57}$$

The individual commutators read

$$[P^i(t,\vec{x}), A_r(t,\vec{y})] = -\mathrm{i}\eta^i{}_r\delta^{(2)}(\vec{x}-\vec{y}), \tag{C58a}$$

$$[P^i(t,\vec{x}), \dot{A}^j(t,\vec{y})] = -\frac{\mathrm{i}g}{2}\epsilon^{ik}\partial^j\partial_k\delta^{(2)}(\vec{x}-\vec{y}), \tag{C58b}$$

$$[P^i(t,\vec{x}), \dot{A}_0(t,\vec{y})] = 0. \tag{C58c}$$

After some calculation we also find

$$[P^i(t,\vec{x}), \ddot{A}_r(t,\vec{y})] = -\mathrm{i}\left(\frac{1}{2}\partial^i\partial_r + \eta^i{}_r\vec{\nabla}^2\right)\delta^{(2)}(\vec{x}-\vec{y}). \tag{C59}$$

Inserting all the previous contributions leads to

$$\begin{aligned}[P^i(t,\vec{x}), P^j(t,\vec{y})] = &\frac{\mathrm{i}g}{2}\epsilon^{ik}\partial^j\partial_k\delta^{(2)}(\vec{x}-\vec{y}) \\ &- \frac{\mathrm{i}g}{2}\epsilon^{jk}\partial^i\partial_k\delta^{(2)}(\vec{x}-\vec{y}) \\ &+ \frac{\mathrm{i}g}{2}\epsilon^{ij}\vec{\nabla}^2\delta^{(2)}(\vec{x}-\vec{y}).\end{aligned} \tag{C60}$$

Indeed, considering each case for i, j separately, one can check that

$$[P^i(t,\vec{x}), P^j(t,\vec{y})] = 0. \tag{C61}$$

13. Commutator $[\Pi^i(t,\vec{x}),\, \Pi^j(t,\vec{y})]$

For this last commutator we have

$$\begin{aligned}[\Pi^i(t,\vec{x}), \Pi^j(t,\vec{y})] = \Big[&-\frac{g}{2}\epsilon^{im}\dot{A}_m(t,\vec{x}) + \frac{g}{2}\epsilon^{im}\partial_m A_0(t,\vec{x}), \\ &-\frac{g}{2}\epsilon^{jk}\dot{A}_k(t,\vec{y}) \\ &+\frac{g}{2}\epsilon^{jk}\partial_k A_0(t,\vec{y})\Big].\end{aligned} \tag{C62}$$

Due to Eq. (C10), the only contribution different from zero is

$$[\Pi^i(t,\vec{x}), \Pi^j(t,\vec{y})] = \frac{g^2}{4}\epsilon^{im}\epsilon^{jk}[\dot{A}_m(t,\vec{x}), \dot{A}_k(t,\vec{y})], \tag{C63}$$

and we have

$$[\Pi^i(t,\vec{x}), \Pi^j(t,\vec{y})] = \frac{g^2}{4}\epsilon^{im}\epsilon^{jk}\left[-\frac{\mathrm{i}}{g}\epsilon_{mk}\delta^{(2)}(\vec{x}-\vec{y})\right]. \tag{C64}$$

Finally, we find

$$[\Pi^i(t,\vec{x}), \Pi^j(t,\vec{y})] = -\frac{\mathrm{i}g}{4}\epsilon^{ij}\delta^{(2)}(\vec{x}-\vec{y}). \tag{C65}$$

With this final result at hand, we conclude the computation of the equal-time commutators.

APPENDIX D: DIRAC THEORY IN (2 + 1) DIMENSIONS

In the current section we would like to review the properties of a Dirac theory in $(2+1)$ dimensions that are important for our work. The latter is based on the Lorentz algebra $\mathfrak{so}(1,2)$, which involves three generators: two boosts K^1, K^2 and a single rotation L^3. We obtain the corresponding generators as

$$K^1 = \mathrm{i}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad K^2 = \mathrm{i}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad L^3 = \mathrm{i}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{D1}$$

The latter satisfy the algebra

$$[L^3, K^1] = \mathrm{i}K^2, \qquad [L^3, K^2] = -\mathrm{i}K^1, \qquad [K^1, K^2] = -\mathrm{i}L^3. \tag{D2}$$

By forming appropriate linear combinations of these generators,

$$X = K^1 + \mathrm{i}K^2, \quad Y = -(K^1 - \mathrm{i}K^2), \quad Z = 2L^3, \tag{D3}$$

we obtain the Lie algebra $\mathfrak{sl}(2,\mathbb{R})$:

$$[Z, X] = 2X, \qquad [Z, Y] = -2Y, \qquad [X, Y] = Z. \tag{D4}$$

Therefore, we conclude that $\mathfrak{so}(1,2) \simeq \mathfrak{sl}(2,\mathbb{R})$.

The first possibility of constructing a Dirac theory in $(2+1)$ dimensions is to work with an irreducible spinor representation for which the Dirac matrices correspond to the Pauli matrices (multiplied by appropriate factors) and the spinors have two components only. An alternative is to propose a reducible spinor representation with three (4×4) Dirac matrices and four-component spinors. We follow the latter possibility and choose the Dirac matrices as

$$\gamma^0 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} -\mathrm{i}\sigma^1 & 0 \\ 0 & \mathrm{i}\sigma^1 \end{pmatrix},$$
$$\gamma^2 = \begin{pmatrix} -\mathrm{i}\sigma^2 & 0 \\ 0 & \mathrm{i}\sigma^2 \end{pmatrix}. \tag{D5}$$

Note that we can define generators

$$\tilde{Z} = \gamma^0, \quad \tilde{X} = \frac{1}{2}(\gamma^1 + \mathrm{i}\gamma^2), \quad \tilde{Y} = -\frac{1}{2}(\gamma^1 - \mathrm{i}\gamma^2), \tag{D6}$$

which satisfy

$$[\tilde{Z}, \tilde{X}] = 2\tilde{X}, \qquad [\tilde{Z}, \tilde{Y}] = -2\tilde{Y}, \qquad [\tilde{X}, \tilde{Y}] = \tilde{Z}, \tag{D7}$$

showing that these new generators also form a representation of $\mathfrak{sl}(2,\mathbb{R})$. Furthermore, the Dirac matrices of Eq. (D5) obey the Clifford algebra in $(2+1)$ dimensions,

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \tag{D8}$$

where $\eta^{\mu\nu}$ is the $(2+1)$-dimensional Minkowski metric. It is clear that all Lorentz indices run from $0\ldots 2$. The Dirac equation is now given by

$$\mathcal{D}(\partial)\psi = 0, \qquad \mathcal{D}(\partial) = \mathrm{i}\partial_\mu\gamma^\mu - m = \mathrm{i}\not{\partial} - m, \tag{D9}$$

with the Dirac operator $\mathcal{D}(\partial)$ acting on a four-component spinor $\psi = \psi(x)$. Transforming the Dirac equation to momentum space provides

$$\mathcal{D}(p)\tilde{\psi} = 0, \qquad \mathcal{D}(p) = \not{p} - m, \tag{D10}$$

with the Fourier-transformed spinor $\tilde{\psi} = \psi(p)$. The inverse $S(p)$ of the Dirac operator in momentum space (multiplied by i) corresponds to the propagator:

$$\mathrm{i}S(p) = \frac{\mathrm{i}(\not{p}+m)}{p^2 - m^2}, \quad S(p)\mathcal{D}(p) = \mathcal{D}(p)S(p) = 1. \tag{D11}$$

The Feynman propagator for fermions is obtained as usual by means of the $\mathrm{i}\epsilon$ prescription:

$$\mathrm{i}S^F(p) = \frac{\mathrm{i}(\not{p} + m)}{p^2 - m^2 + \mathrm{i}\epsilon}. \tag{D12}$$

Requiring that the determinant of the Dirac operator vanish for nontrivial solutions leads to the positive fermion energy

$$E(\vec{p}) = E_p = \sqrt{\vec{p}^2 + m^2}. \tag{D13}$$

Solving the Dirac equation subsequently provides the following particle spinors:

$$u^{(1)} = (p^1 + \mathrm{i}p^2)\begin{pmatrix} 1/\sqrt{E_p - m} \\ \mathrm{i}\sqrt{E_p - m}/(p^1 - \mathrm{i}p^2) \\ 0 \\ 0 \end{pmatrix},$$
$$u^{(2)} = \begin{pmatrix} 0 \\ 0 \\ (p^1 - \mathrm{i}p^2)/\sqrt{E_p + m} \\ \mathrm{i}\sqrt{E_p + m} \end{pmatrix}. \tag{D14}$$

On the other hand, the antiparticle spinors are given by

$$v^{(1)} = (p^1 + \mathrm{i}p^2)\begin{pmatrix} 1/\sqrt{E_p + m} \\ \mathrm{i}\sqrt{E_p + m}/(p^1 - \mathrm{i}p^2) \\ 0 \\ 0 \end{pmatrix},$$
$$v^{(2)} = \begin{pmatrix} 0 \\ 0 \\ (p^1 - \mathrm{i}p^2)/\sqrt{E_p - m} \\ \mathrm{i}\sqrt{E_p - m} \end{pmatrix}. \tag{D15}$$

These spinors are normalized such that

$$u^{(s)\dagger}u^{(t)} = 2E_p\delta^{s,t}, \qquad v^{(s)\dagger}v^{(t)} = 2E_p\delta^{s,t}. \tag{D16}$$

We define the Dirac conjugated spinors as $\bar{u}^{(s)} = u^{(s)\dagger}\gamma^0$ and $\bar{v}^{(s)} = v^{(s)\dagger}\gamma^0$ and derive the completeness relations

$$\sum_s u^{(s)}(p)\bar{u}^{(s)}(p) = \not{p} + m, \tag{D17a}$$

$$\sum_s v^{(s)}(p)\bar{v}^{(s)}(p) = \not{p} - m. \tag{D17b}$$

They formally correspond to those in $(3+1)$-dimensional Dirac theory.

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