

$SU(3)$ vector currents in baryon chiral perturbation theory combined with the $1/N_c$ expansion

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 (Received 1 November 2019; accepted 19 February 2020; published 17 March 2020)

Baryon chiral perturbation theory (BChPT) combined with the $1/N_c$ expansion is applied to the $SU(3)$ vector currents. In terms of the ξ power counting linking the low-energy and $1/N_c$ expansions according to $\mathcal{O}(\xi) = \mathcal{O}(p) = \mathcal{O}(1/N_c)$, the study is carried out to next-to-next-to-leading order, and it includes $SU(3)$ breaking corrections to the $|\Delta S| = 1$ vector charges, charge radii, and magnetic moments and radii. The results are obtained for generic N_c , allowing for investigating the various scalings in N_c .

DOI: 10.1103/PhysRevD.101.054026

I. INTRODUCTION

Vector currents, being intimately related to the flavor $SU(3)$ symmetry of QCD, represent a fundamental probe for hadron structure as well as for the breaking of $SU(3)$ by quark masses. This is particularly interesting for baryons, in which the electromagnetic current for nucleons, known empirically to remarkable accuracy [1], along with the magnetic moments of hyperons allow for an almost complete description of all the $SU(3)$ vector currents to the order in the low-energy expansion considered in the present work. The charged vector currents are relevant in β decays, where both $SU(3)$ breaking in the $|\Delta S| = 1$ charges and weak magnetism are still open problems. To the present level of experimental accuracy in hyperon β decays, there is not sufficient sensitivity to the $SU(3)$ breaking in the charges [2]. The reason is that β decay has a branching fraction of about 10^{-3} , being dominated by the nonleptonic component. Fortunately, lattice QCD is producing results [3–6] which can be compared with the predictions of the approach in the present work. The experimental information on charge form factors is limited to the electric form factors of nucleons and the charge radius of the Σ^- . This is, however, sufficient to predict the rest of the charge radii, the $SU(3)$ breaking effects of which are, at the order of the present calculation, finite nonanalytic in quark masses. The octet baryons' electromagnetic (EM) magnetic moments and nucleons' magnetic radii give

an almost complete prediction for the rest of the currents but for one low-energy constant (LEC) which requires knowledge of at least one weak magnetic moment of a $\Delta S = 1$ current. In the approach followed here, results automatically extend to the vector current observables of the decuplet baryons and to EM transitions, e.g., the M_1 transition $\Delta \rightarrow N\gamma$, most of which remain empirically unknown or poorly known. The study of electric currents in baryon chiral perturbation theory (BChPT) with inclusion of the spin-3/2 baryons dates back a quarter century [7,8], and numerous works have since been produced in various versions of that framework; among those close in spirit to the present one are found in Refs. [9–15], and works with additional constraints imposed by consistency with the $1/N_c$ expansion are those of Refs. [16–21]. The present work formalizes the combination of BChPT and the $1/N_c$ expansion [22] for the vector currents following the rigorous power counting scheme of the ξ expansion [23,24] based on the linking $\mathcal{O}(p) = \mathcal{O}(1/N_c) = \mathcal{O}(\xi)$. The combined framework was first applied to the $SU(3)$ vector charges in Ref. [20], in which the ξ expansion was not strictly implemented; however, for the purpose of calculating the corrections of $SU(3)$ breaking to the vector charges, restricted by the Ademollo-Gatto theorem (AGT), such omission has no very significant effect.¹ Here, a complete study is presented to $\mathcal{O}(\xi^3)$ and $\mathcal{O}(\xi^4)$ (depending on the observable) of the $SU(3)$ vector currents. The present work provides results for generic N_c , permitting in this way sorting out in particular the large N_c behavior of nonanalytic terms in ξ stemming from one-loop corrections, which gives additional understanding, as has been shown for instance in the case of the Gell-Mann-Okubo relation and the σ terms discussed in Refs. [25,26]. The subject of

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¹In Ref. [20], the baryon-Goldstone Boson (GB) vertices included higher-order terms in $1/N_c$.

magnetic moments has been addressed in the context of the $1/N_c$ expansion in works limited to a tree-level expansion in composite operators [27–30] and in works including one-loop corrections in BChPT, Refs. [17–19,21]. In addition to BChPT, dispersive approaches have been implemented [31–33], in which constraints imposed by consistency with the $1/N_c$ expansion have been incorporated [34–38]. Such works naturally give a range of applicability beyond the present one, which is limited up to the form factor radii.

This work is organized as follows. Section II presents the baryon chiral Lagrangians needed for the present work. Section III summarizes the one-loop corrections to the vector currents. Section IV presents the analysis of the vector charges and radii. Section V does the same for the magnetic moments and radii. A summary is presented in Sec. VI. Several Appendixes are included for the benefit of readers intending to implement similar calculations.

II. BARYON CHIRAL LAGRANGIAN

This section summarizes the pieces of the baryon chiral Lagrangian up to $\mathcal{O}(\xi^4)$ relevant to the calculations in this work. The details on the construction of the Lagrangians and the notations are given in Ref. [24], and Appendix B of the present paper displays the building blocks. To ensure the validity of the Okubo-Zweig-Iizuka (OZI) rule for the quark mass dependency of baryon masses, namely, that the non-strange baryon mass dependence on m_s be $\mathcal{O}(N_c^0)$, the following combination of the source χ_+ is defined by [see Eqs. (B2) and (B3) in Appendix B and also Ref. [24] for details]

$$\hat{\chi}_+ \equiv \tilde{\chi}_+ + N_c \chi_+^0, \quad (1)$$

which is $\mathcal{O}(N_c)$ but has dependence on m_s , which is $\mathcal{O}(N_c^0)$ for all states with strangeness $\mathcal{O}(N_c^0)$. For convenience,

a scale Λ is introduced and can be chosen to be a typical QCD scale, in order to render most of the LECs dimensionless. In the calculations, $\Lambda = m_\rho$ will be chosen. The quark mass matrix is defined by $\mathcal{M}_q = m^0 + m^a \frac{\lambda^a}{2}$, where in the physical case, $m^0 = \frac{1}{3}(m_u + m_d + m_s)$, $m^3 = m_u - m_d$, and $m^8 = \frac{1}{\sqrt{3}}(m_u + m_d - 2m_s)$, and the rest of the m^a 's vanish.

Collecting the baryons in a spin-flavor multiplet denoted by \mathbf{B} , and using standard notation for the chiral building blocks (for details, see Appendix B and Ref. [24]), the leading-order (LO) $\mathcal{O}(\xi)$ Lagrangian reads

$$\mathcal{L}_B^{(1)} = \mathbf{B}^\dagger \left(iD_0 - \frac{C_{HF}}{N_c} \hat{S}^2 + \hat{g}_A u^{ia} G^{ia} + \frac{c_1}{2\Lambda} \hat{\chi}_+ \right) \mathbf{B}, \quad (2)$$

where the hyperfine mass shifts are given by the second term, G^{ia} are the spin-flavor generators (see Appendix A), and the axial coupling is at LO $\hat{g}_A = \frac{6}{5} g_A$, with $g_A = 1.2732(23)$ being the nucleon's axial coupling. The relevant terms in the $\mathcal{O}(\xi^2)$ Lagrangian are

$$\mathcal{L}_B^{(2)} = \mathbf{B}^\dagger \left(\frac{c_2}{\Lambda} \chi_+^0 + \frac{C_1^A}{N_c} u^{ia} S^i T^a + \frac{\kappa}{2\Lambda} B_+^{ia} G^{ia} + \dots \right) \mathbf{B}, \quad (3)$$

where the flavor $SU(3)$ electric and magnetic fields are denoted by E_+ and B_+ and given by $E_+^i = F_+^{0i}$ and $B_+^i = \frac{1}{2} \epsilon^{ijk} F_+^{jk}$ [see Eq. (B2) in Appendix B]. The term proportional to κ gives at LO the magnetic moments associated with all vector currents. The $\mathcal{O}(\xi^3)$ and $\mathcal{O}(\xi^4)$ Lagrangians needed for the one-loop renormalization of the vector currents are the following:

$$\begin{aligned} \mathcal{L}_B^{(3)} &= \mathbf{B}^\dagger \left(\frac{g_1}{\Lambda^2} D_i E_{+i}^a T^a + \frac{\kappa_1}{2\Lambda N_c} B_+^{ia} S^i T^a + \dots \right) \mathbf{B} \\ \mathcal{L}_B^{(4)} &= \mathbf{B}^\dagger \left(\frac{1}{N_c \Lambda^2} (g_2 D_i E_{+i}^a S^j G^{ja} + g_3 D_i E_{+j}^a \{S^i, G^{ja}\}^{\ell=2}) + \frac{\kappa_r}{\Lambda^3} D^2 B_+^{ia} G^{ia} \right. \\ &\quad \left. + \frac{1}{2\Lambda^3} (\kappa_2 \chi_+^0 B_+^{ia} G^{ia} + i\kappa_F f^{abc} \chi_+^a B_+^{ib} G^{ic} + \kappa_D d^{abc} \chi_+^a B_+^{ib} G^{ic} + \kappa_3 \chi_+^a B_+^{ia} S^i) \right. \\ &\quad \left. + \frac{1}{2\Lambda N_c^2} (\kappa_4 B_+^{ia} \{\hat{S}^2, G^{ia}\} + \kappa_5 B_+^{ia} S^i S^j G^{ja}) + \dots \right) \mathbf{B}. \end{aligned} \quad (4)$$

The LECs g_1 and g_2 will be determined by charge radii; the term proportional to g_3 gives electric quadrupole moments for decuplet baryons and for transitions between decuplet to octet baryons, which will not be discussed here; and the term proportional to κ_r gives a contribution to magnetic radii ($D^2 B_+ \equiv D_\mu D^\mu B_+$ being the covariant divergence of

the magnetic field). The rest are quark mass and higher order in $1/N_c$ corrections to the magnetic moments.

Throughout, spin-flavor operators in the Lagrangians are scaled by appropriate powers of $1/N_c$ such that all LECs start at zeroth order in N_c . Of course, LECs themselves have an expansion in $1/N_c$, kept implicit, which requires

information for $N_c > 3$ to be determined. In that sense, each Lagrangian term has a leading power in $1/N_c$, which is used to assign its order in the ξ power counting, followed by subleading terms in $1/N_c$ due to the expansion of the corresponding LEC. In addition, each term in the Lagrangian is explicitly chiral invariant, and its expansion in powers of the Goldstone boson fields yields factors $1/F_\pi = \mathcal{O}(1/\sqrt{N_c})$ for each additional factor of a GB field.

For convenience, the following definition is used:

$$\delta\hat{m} \equiv \frac{C_{\text{HF}}}{N_c} \hat{S}^2 - \frac{c_1}{2\Lambda} \hat{\chi}_+. \quad (5)$$

Note that $\delta\hat{m}$ gives rise to mass splittings between baryons which are the $\mathcal{O}(1/N_c)$ hyperfine term in Eq. (2) and the $\mathcal{O}(p^2)$ quark mass term. The $\mathcal{O}(m_q N_c)$ term in $\hat{\chi}_+$ becomes immaterial in the loop calculations as only differences of baryon masses appear for which such terms exactly cancel.

III. ONE-LOOP CORRECTIONS TO CURRENTS

The one-loop corrections to the vector currents are given by the two sets of gauge invariant diagrams A and B in Fig. 1, where the vertices appearing in those diagrams are displayed in Appendix D, Fig. 2. The explicit results are

$$\begin{aligned} V^{\mu a}(A_1) &= i \left(\frac{g_A}{F_\pi} \right)^2 \sum_{n_1, n_2} G^{ib} \mathcal{P}_{n_2} \Gamma^{\mu a} \mathcal{P}_{n_1} G^{jb} \frac{1}{q_0 - \delta m_{n_2} + \delta m_{n_1}} (H^{ij}(p_0 - \delta m_{n_1}, M_b) - H^{ij}(p_0 + q_0 - \delta m_{n_2}, M_b)) \\ V^{\mu a}(A_2) &= \frac{1}{2} \{ \Gamma^{\mu a}, \delta \hat{Z}_{1\text{-loop}} \} \\ V^{\mu a}(A_3) &= \left(\frac{g_A}{F_\pi} \right)^2 f^{abc} \sum_n G^{ib} \mathcal{P}_n G^{jc} H^{ij\mu}(p_0 - \delta m_n, q, M_b, M_c) \\ V^{\mu a}(B_1) &= -\frac{i}{2F_\pi^2} f^{abc} f^{bcd} \Gamma^{\mu d} I(0, 1, M_b^2) \\ V^{\mu a}(B_2) &= g^{\mu 0} \frac{i}{4F_\pi^2} f^{abc} f^{bcd} T^d (q_0^2 K(q, M_b, M_c) + 4q_0 K^0(q, M_b, M_c) + 4K^{00}(q, M_b, M_c)), \end{aligned} \quad (6)$$

where \mathcal{P}_n are projectors onto the corresponding baryon in the loop, p_0 is the residual energy of the initial baryon, q_0 is the incoming energy in the current, and $\Gamma^{\mu a} = g^{\mu 0} T^a + i \frac{\xi}{\Lambda} \epsilon^{0\mu ij} f^{abc} f^{bcd} q^i G^{jd}$ contains both the electric charge and magnetic moment components. The one-loop wave function renormalization factor $\delta \hat{Z}_{1\text{-loop}}$ can be found in Ref. [24], and the loop integrals I , K , K^μ , H^{ij} , and $H^{ij\mu}$ are given in Appendix C. Since the temporal component of the current can only connect baryons with the same spin, q_0 is equal to the $SU(3)$ breaking mass difference between them plus the kinetic energy transferred by the current, which are all $\mathcal{O}(\xi^2)$ or higher and must therefore be neglected in this calculation. In the evaluations, one sets $p_0 \rightarrow \delta m_{\text{in}}$ and $p_0 + q_0 \rightarrow \delta m_{\text{out}}$. In particular, for

diagram A_1 , if it requires evaluation at $q_0 = 0$, such a limit must be taken in the end of the evaluation. The $U(1)$ baryon number current can be used to check the calculation; only diagrams A_{1+2} contribute, and as required, they cancel each other.

For a generic current vertex Γ , the combined UV divergent and polynomial piece of diagrams A_{1+2} can be written as

$$\begin{aligned} \Gamma(A_{1+2})^{\text{poly}} &= \frac{1}{(4\pi)^2} \left(\frac{g_A}{F_\pi} \right)^2 \left(\frac{1}{2} (\lambda_\epsilon + 1) M_{ab}^2 [G^{ia}, [G^{ib}, \Gamma]] \right. \\ &\quad + \frac{1}{3} (\lambda_\epsilon + 2) (2[[G^{ia}, \Gamma], [\delta\hat{m}, [\delta\hat{m}, G^{ia}]]] \\ &\quad \left. + [[\Gamma, [\delta\hat{m}, G^{ia}]], [\delta\hat{m}, G^{ia}]] \right), \end{aligned} \quad (7)$$

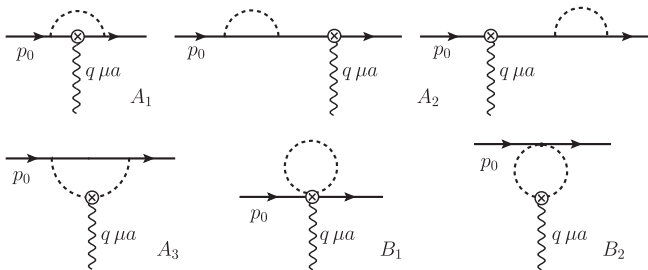


FIG. 1. Diagrams contributing to the one-loop corrections to the vector currents.

where $\lambda_\epsilon = \frac{1}{\epsilon} - \gamma + \log 4\pi$. The first term is proportional to quark masses through the GB mass-square matrix $M_{ab}^2 = m^0 \delta^{ab} + \frac{1}{2} d^{abc} m^c$, and the second involves the baryon hyperfine mass splittings $\delta\hat{m}$ which are $\mathcal{O}(1/N_c)$, and following the strict ξ power counting, the $\mathcal{O}(p^2)$ terms due to $SU(3)$ breaking in $\delta\hat{m}$ are disregarded. The consistency with the $1/N_c$ power counting can be readily checked. Diagrams A_3 and $B_{1,2}$ are separately consistent with the $1/N_c$ power counting. Their polynomial contributions are the following:

$$\begin{aligned}
V^{\mu a}(A_3)^{\text{Poly}} &= -\frac{1}{(4\pi)^2} \left(\frac{\overset{\circ}{g}_A}{F_\pi}\right)^2 \frac{1}{6} i f^{abc} \\
&\quad \times \left(g^{\mu 0} \left(\left(\lambda_\epsilon q^i q^j + \frac{1}{2} (\lambda_\epsilon + 1) q^2 g^{ij} \right) \delta^{bd} - 3 g^{ij} (\lambda_\epsilon + 1) M_{bd}^2 \right) [G^{id}, G^{jc}] \right. \\
&\quad - g^{\mu 0} (\lambda_\epsilon + 2) \left(\frac{1}{2} [G^{ib}, [[G^{ic}, \delta\hat{m}], \delta\hat{m}]] - [[G^{ib}, \delta\hat{m}], [G^{ic}, \delta\hat{m}]] \right) \\
&\quad \left. + g_i^\mu (\lambda_\epsilon + 2) \left(\frac{1}{2} g^{jk} q^i [[G^{kb}, G^{jc}], \delta\hat{m}] + 2 g^{ij} q^k (3 [G^{kb}, [G^{jc}, \delta\hat{m}]] + [[G^{jc}, G^{kb}], \delta\hat{m}]) \right) \right) \\
V^{\mu a}(B_1)^{\text{Poly}} &= \frac{1}{(4\pi)^2} (\lambda_\epsilon + 1) \frac{1}{2F_\pi^2} f^{abd} f^{cde} \Gamma^{\mu e} M_{bc}^2 \\
&= -\frac{1}{(4\pi)^2} \frac{3}{F_\pi^2} (\lambda_\epsilon + 1) g^{\mu 0} B_0 \left(m^0 \Gamma^{\mu a} + \frac{1}{4} d^{abc} m^b \Gamma^{\mu c} \right) \\
V^{\mu a}(B_2)^{\text{Poly}} &= -\frac{1}{(4\pi)^2} \lambda_\epsilon \frac{1}{4F_\pi^2} (g^{\mu 0} \vec{q}^2 + g_i^\mu q^i q_0) T^a - g^{\mu 0} V^{0a}(B_1)^{\text{Poly}}. \tag{8}
\end{aligned}$$

Reduction formulas that can be found in Ref. [25] are used to express the above in a base of irreducible operators, Eqs. (9) and (12) below.

IV. VECTOR CHARGES

In this section, the $SU(3)$ vector current charges and corresponding radii are analyzed. The $SU(3)$ breaking corrections to the charges already presented in Refs. [20,24] are discussed for completeness. At lowest order, the charges are represented by the flavor generators T^a . The one-loop corrections are UV finite at $Q^2 \equiv -q^2 = 0$, and since up to $\mathcal{O}(\xi^3)$ the AGT is satisfied, the corrections to the charges are unambiguously given by UV finite one-loop contributions. Note that the AGT applies to the whole baryon spin-flavor multiplet. On the other hand, at finite Q^2 , the one-loop correction has a UV divergent piece, which is independent of quark masses and is renormalized via the terms g_1 and g_2 in \mathcal{L}_B ; one of them removes the UV divergence (g_1), and the other one is a finite counterterm (g_2).

Combining the polynomial pieces in Eqs. (7) and (8) and using that $[\delta\hat{m}, T^a] = [\delta\hat{m}, \hat{G}^2] = [\delta\hat{m}, G^{ib} T^a G^{ib}] = 0$, one obtains the polynomial loop contributions to vector charges, which are proportional to $Q^2 = \vec{q}^2$,

TABLE I. $SU(3)$ breaking corrections to the $\Delta S = 1$ vector charges. The LQCD results are from Ref. [4].

	$\frac{\delta f_1}{f_1}$	
	One-loop	LQCD
Λp	-0.067(15)	-0.05(2)
$\Sigma^- n$	-0.025(10)	-0.02(3)
$\Xi^- \Lambda$	-0.053(10)	-0.06(4)
$\Xi^- \Sigma^0$	-0.068(17)	-0.05(2)

$$\begin{aligned}
f_1^a(A_{1+2+3})^{\text{poly}} &= \frac{\lambda_\epsilon - 3}{(4\pi)^2} \left(\frac{\overset{\circ}{g}_A}{4F_\pi}\right)^2 Q^2 T^a \\
f_1^a(B_{1+2})^{\text{poly}} &= -\frac{\lambda_\epsilon + 1}{(4\pi)^2} \frac{Q^2}{4F_\pi^2} T^a, \tag{9}
\end{aligned}$$

where $f_1^a \equiv V^{0a}$.

The corrections to the $|\Delta S| = 1$ charges, already discussed in Ref. [20], are evaluated using the physical values $\overset{\circ}{g}_A = \frac{6}{5} \times 1.27$ and $F_\pi = 92$ MeV; however, one needs to be aware that their values are effected by the NLO corrections, leading to a theoretical uncertainty. With the usual notation for those charges [20], evaluating the ratios $\delta f_1/f_1$ in the large N_c limit, one finds that $\delta f_1/f_1 = \mathcal{O}(1/N_c)$. Since the corrections are entirely given by nonanalytic terms in ξ , the naive $1/N_c$ scaling sets in rather slowly at $N_c \sim 20$, emphasizing that the noncommutativity of the low-energy and $1/N_c$ expansions is very important at the physical $N_c = 3$. The results are shown in Table I, in which the errors are estimated from the above-mentioned theoretical uncertainty. The agreement with recent Lattice QCD (LQCD) calculations [4] is encouraging, and further improvement in the precision of those calculations would be very useful.

For the charge radii, the loop contributions are from diagrams A_3 and B_2 , and the renormalization is provided by the LECs g_1 and g_2 in $\mathcal{L}_B^{(3)}$ and $\mathcal{L}_B^{(4)}$, respectively, of which only g_1 is required for canceling the loop UV divergence according to Eq. (9).² As is the case with form factors in ChPT, the charge radii depend logarithmically in the GB masses. They can be determined by fitting to the known electric charge radii of proton, neutron, and Σ^- , or simply fixed using the first two. If one wishes to study also the

²In Ref. [24], the finite term proportional to g_2 was overlooked.

large N_c limit, an assignment at generic N_c of the quark electric charges has to be done. One such an assignment that respects all gauge and gauge-gravitational anomaly cancellations in the Standard Model is given by [39] $\hat{Q} = T^3 + \frac{1}{\sqrt{3}}T^8 + \frac{3-N_c}{6N_c}B$. The last term comes from the baryon number charge B and can be implemented by simply adding to the Lagrangians the corresponding terms with an $SU(3)$ singlet vector source field. This charge operator gives for the states identified with the physical octet and decuplet the same electric charges as the physical ones for any N_c . The analysis of the charge radii in the present framework is revealing: in the strict large N_c limit, one finds that the nonanalytic loop contributions to the T^3 charge radius of nucleons by diagram A_3 is $\mathcal{O}(N_c^0)$, where the contribution is driven by the hyperfine mass splitting term, i.e, for $C_{HF} \rightarrow 0$, the contribution becomes $\mathcal{O}(1/N_c)$, and diagram B_2 gives only contributions $\mathcal{O}(1/N_c)$. For the charge T^8 , the loop contributions are $\mathcal{O}(N_c^0)$. One, however, notes that for the physical π and K meson masses the nonanalytic terms join the large N_c scaling at rather large N_c . The charge radii of the neutral baryons receive only UV finite loop contributions and are renormalized only by the finite g_2 term.

Using the three known charge radii, $g_{1,2}$ are determined modulo the main uncertainty stemming from the value used for $\overset{\circ}{g}_A$. At the renormalization scale $\mu = m_\rho$, using the value of $\overset{\circ}{g}_A \sim 1$ obtained by the analysis of the axial couplings [24], $C_{HF} \sim 200$ MeV, and with $\Lambda = m_\rho$, one finds $g_1 \simeq 1.33$ and $g_2 \simeq 0.74$. g_2 is sensitive to C_{HF} , which can be understood as a result that the nonanalytic contributions to the neutron radius are very important, and thus sensitive to that parameter, while g_1 is not. One also observes that both LECs are crucial for obtaining a good description of the radii. For the used value of μ , the fraction of the loop contribution to the proton's $\langle r^2 \rangle$ is 15%, and 60% for the neutron's one. The short distance contributions are thus very important in both cases. The dominant nonanalytic contributions to the radii are proportional to $\log m_q$, with other nonanalytic terms involving the LEC C_{HF} giving almost negligible contributions, making the results insensitive to it. Table II shows the results for the charge radii of the baryon octet along with the contributions by the Counter Terms (CT). The latter contributions to $\langle r^2 \rangle$ satisfy the exact linear relation, in obvious notation, $a\Lambda + p + \Sigma^+ + \frac{1}{3}(a-4)(n + \Sigma^0 + \Xi^0) + \Sigma^- + \Xi^- = 0$, valid for any a and resulting from the electric charge being a U-spin singlet; it is violated only by finite $SU(3)$ breaking loop contributions. The isotriplet nucleon charge radius is $\mathcal{O}(N_c^0)$, while the isosinglet one receives loop and g_2 contributions $\mathcal{O}(N_c^0)$ and a g_1 contribution $\mathcal{O}(N_c)$, where the $\mathcal{O}(N_c)$ term contribution to the EM charge radius must be canceled by adding to the Lagrangian a finite charge-radius CT proportional to baryon number and weighted

TABLE II. Electric charge radii of octet baryons. The proton and neutron radii are inputs. The proton radius used is the one resulting from the muonic hydrogen Lamb shift [40]. The second column shows the contribution by contact terms $g_{1,2}$ for $\mu = m_\rho$.

	$\langle r^2 \rangle (\text{fm}^2)$		
	Full	CT	Exp
p	0.707	0.596	0.7071(7)
n	-0.116	-0.049	-0.116(2)
Λ	-0.029	-0.024	...
Σ^+	0.742	0.596	...
Σ^0	0.029	0.024	...
Σ^-	0.683	0.548	0.608(156)
Ξ^0	-0.016	-0.049	...
Ξ^-	0.633	0.548	...

according to the electric charge assignment at arbitrary N_c mentioned above.

At the present order in the ξ expansion, the curvature of the form factors, proportional to $\langle r^4 \rangle = 60 \frac{d^2 f_1}{d(Q^2)^2}$, is given by the one-loop nonanalytic terms with contributions that are inversely proportional to quark masses. The curvature is nominally an effect $\mathcal{O}(\xi^4)$ in the form factor, which therefore receives contributions from terms $\mathcal{O}(\xi^6)$ in the Lagrangian, and only in the limit of sufficiently small quark masses will the nonanalytic contributions obtained here be dominant. In the recent work of Ref. [38], the electric charge higher moments have been studied, where t-channel elastic unitarity has been implemented in the EFT along with the constraints of the $1/N_c$ expansion [34–38]. In particular, for the curvature, they find $\langle r^4 \rangle^p = 0.735(35) \text{ fm}^4$ and $\langle r^4 \rangle^n = -0.540(35) \text{ fm}^4$, to be compared with the one-loop contributions found here, 0.032 and -0.021 fm^4 , respectively, roughly a factor 25 smaller in magnitude in each case. Clearly, the description of the curvature must be primarily given by higher-order contact terms, and to the order of the expansion followed here, the failure to account for the curvature limits the present description of charge form factors to the expected range given by the radii, $Q^2 \lesssim 0.05 \text{ GeV}^2$.

V. MAGNETIC MOMENTS

As mentioned earlier, at lowest order, the magnetic moments of all vector currents are given in terms of the single LEC κ . In particular, using the EM current, the LO value of $\frac{\kappa}{\Lambda}$ can be fixed from the proton's magnetic moment μ_p in units of the nuclear magneton μ_N , namely, $e \frac{\kappa}{2\Lambda} = \mu_p = 2.7928 \mu_N$. Also, the M_1 radiative decay width of the Δ at LO is given by

$$\Gamma_{\Delta \rightarrow N\gamma} = \frac{e^2}{9\pi} \left(\frac{\kappa}{\Lambda} \right)^2 \frac{m_N}{m_\Delta} \omega^3, \quad (10)$$

where ω is the photon energy. Using the above result for $\frac{\kappa}{\Lambda}$ gives $\Gamma_{\Delta \rightarrow N\gamma}^{\text{LO}} = 0.38$ MeV, to be compared with the experimental value 0.70 ± 0.06 MeV. In terms of the transition magnetic moment, the LO result is $\mu_{\Delta^+ p} = \frac{2\sqrt{2}}{3}\mu_p$, while the experimental one from Eq. (10) and from the helicity $N - \Delta$ photocouplings [41] are $3.58(10)\mu_N$ and $3.46(3)\mu_N$, respectively. This shows the need for a significant spin-symmetry breaking effect of 30% to be accounted for by the higher-order corrections.

The LO magnetic moment operator G^{ia} is proportional to the LO axial currents, and the next-to-leading-order (NLO) effects stem from quark masses and spin-symmetry breaking. In the strict large N_c limit, those corrections scale as follows: $SU(3)$ breaking corrections $\mathcal{O}((m_s - \hat{m})N_c)$, i.e., the same scaling in N_c as the LO term, and spin-symmetry breaking corrections $\mathcal{O}(1/N_c)$, i.e., $\mathcal{O}(1/N_c^2)$ with respect to the LO term, well known from tree-level analyses in Refs. [42,43].

The experimentally available magnetic moment ratios and the corresponding LO results are represented in Table III. It is evident that there are significant $SU(3)$ breaking effects, which together with the important spin-symmetry breaking observed, in particular in the $\Delta N M_1$ amplitude, indicate the relevance of the next-to-NLO (NNLO) calculation. Note that all weak magnetic moments, i.e., magnetic moments associated with the $\Delta S = 1$ currents, are also fixed at LO, as they are empirically unknown. In the case of the neutron β decay, the weak magnetic term is obtained from the isovector part of the EM magnetic moments of the proton and neutron, which in this case, due to isospin symmetry, is quite accurate. On the other hand, in hyperon beta decay, the effect of weak

TABLE III. LO ratios of magnetic moments.

	Exp	LO
p/n	-1.46	-1.5
Σ^+/Σ^-	-2.12	-3
Λ/Σ^+	-0.25	$-\frac{1}{3}$
p/Σ^+	1.14	1
Ξ^0/Ξ^-	1.92	2
p/Ξ^0	-2.23	-1.5
Δ^{++}/Δ^+	1.4(2.8)	2
Ω^-/Δ^+	-0.75	-1
p/Δ^+	1.03	1
$p/(\Delta^+ p)$	0.78	$\frac{3}{2\sqrt{2}}$
$p/(\Sigma^{*0}\Lambda)$	1.02	$\sqrt{\frac{3}{2}}$
$p/(\Sigma^{*+}\Sigma^+)$	-0.88	$-\frac{3}{2\sqrt{2}}$

magnetism is too small to be at present experimentally accessible. Fortunately, the advent of LQCD calculations of magnetic moments with increasing accuracy will allow the study of weak magnetism.

The one-loop corrections to the magnetic moments are obtained from the spatial components of the vector currents depicted in Fig. 1, in which the contributions stem from diagrams A and B_1 . Diagrams $A_{1,2}$ involve $\Gamma \propto G^{ia}$, which is similar to the axial currents already analyzed in Ref. [24]. The loop contributions to the Q^2 dependence of the magnetic form factors stem from diagram A_3 .

The UV divergencies of the one-loop diagrams contributing to the magnetic moments after the reduction of the corresponding expressions (7) and (8) using a basis of spin-flavor operators read as

$$\begin{aligned}
V_{Mag}^{\mu a}(A_{1+2})^{UV} &= i \frac{\lambda_\epsilon}{(4\pi)^2} \frac{\kappa}{2\Lambda} \left(\frac{g_A}{F_\pi} \right)^2 \epsilon^{ijk} q^j \left(-B_0 \left(\frac{23}{6} m^0 G^{ka} + \frac{11}{24} d^{abc} m^b G^{kc} + \frac{5}{18} m^a S^k \right) \right. \\
&\quad \left. + \frac{2}{3} \left(\frac{C_{HF}}{N_c} \right)^2 \left((N_c(N_c + 6) - 3) G^{ka} + 8\{\hat{S}^2, G^{ka}\} + 8S^k S^m G^{ma} - \frac{11}{2} (N_c + 3) S^k T^a \right) \right) \\
V_{Mag}^{\mu a}(A_3)^{UV} &= i \frac{\lambda_\epsilon}{(4\pi)^2} \left(\frac{g_A}{F_\pi} \right)^2 \frac{C_{HF}}{N_c} \epsilon^{ijk} q^j \left(\frac{N_c + 3}{2} G^{ka} - 2S^k T^a \right) \\
V_{Mag}^{\mu a}(B_1)^{UV} &= -i \frac{\lambda_\epsilon}{(4\pi)^2} \frac{\kappa}{2\Lambda} \frac{1}{F_\pi^2} \epsilon^{ijk} q^j B_0 \left(6m^0 G^{ka} + \frac{3}{2} d^{abc} m^b G^{kc} \right), \tag{11}
\end{aligned}$$

adding up to

$$\begin{aligned}
V_{Mag}^{UV\mu a} &= \frac{i\lambda_\epsilon g_A^j \epsilon^{ijk}}{16\pi^2 F_\pi^2 \Lambda} \left(-\frac{1}{12} \kappa B_0 \left(\left(\frac{11}{4} g_A^2 + 9 \right) m^b G^{kc} d^{abc} + (23g_A^2 + 36) m^0 G^{ka} + \frac{5}{3} g_A^2 m^a S^k \right) \right. \\
&\quad \left. + \frac{C_{HF} g_A^2}{6N_c^2} (2\kappa C_{HF} ((N_c(N_c + 6) - 3) G^{ka} + 8\{\hat{S}^2, G^{ka}\}) + 3\Lambda N_c(N_c + 3) G^{ka} \right. \\
&\quad \left. + 16\kappa S^m G^{ma} S^k - S^k T^a (11\kappa C_{HF}(N_c + 3) + 12\Lambda N_c)) \right). \tag{12}
\end{aligned}$$

TABLE IV. β functions of LECs associated with magnetic moments and radii. The renormalized LECs are defined according to $X = X(\mu) + \frac{\beta_X}{(4\pi)^2} \lambda_c$.

LEC	$\beta \times F_\pi^2$
κ	$\Lambda g_A^2 \frac{C_{HF}}{N_c} (\frac{1}{2}(N_c + 3) + \frac{1}{3}(N_c(N_c + 6) - 3) \frac{\kappa}{\Lambda} \frac{C_{HF}}{N_c})$
κ_1	$-\Lambda g_A^2 C_{HF} (2 + \frac{11}{6}(N_c + 3) \frac{\kappa}{\Lambda} \frac{C_{HF}}{N_c})$
κ_2	$-\Lambda^2 \kappa (3 + \frac{23}{12} g_A^2)$
κ_D	$-\Lambda^2 \kappa (\frac{3}{4} + \frac{11}{48} g_A^2)$
κ_F	0
κ_3	$-\Lambda^2 \kappa \frac{5}{36} g_A^2$
κ_4	$\frac{8}{3} g_A^2 \kappa C_{HF}^2$
κ_5	$\frac{8}{3} g_A^2 \kappa C_{HF}^2$
κ_r	0

The renormalization of the magnetic moments is provided by the Lagrangians with the LECs $\kappa_{D,F,1,\dots,5}$, and the magnetic radii receive only finite one-loop contributions and a finite renormalization by the term κ_r . The β functions of the magnetic LECs resulting from Eq. (12) are shown in Table IV.

For $N_c = 3$, the set of local terms that contribute to the magnetic moments remains linearly independent. If one only considers the EM current, the term proportional to κ_F does not contribute, and for the known magnetic moments together with the information on the M_1 transition $\Delta \rightarrow N\gamma$, one can fit the rest of the LECs. Note that in the absence of information on the $SU(3)$ singlet quark mass m^0 dependence the LEC κ_2 is subsumed into κ , and the lack of knowledge on the $\Delta S = 1$ weak magnetic moments prevents at present a determination of κ_F .

The results of the fits are shown in Table V. The input magnetic moments have errors (much) smaller than the theoretical error of the present calculation, estimated to be given by the magnitude of next-to-NNLO (NNNLO) corrections, about 5% to 10%. Here, the proton and neutron magnetic moments are used as exact inputs giving the following relations between the parameters:

$$\begin{aligned} \kappa_1 &= -19.662 + 6.926 \kappa - 0.833 \left(\kappa_4 + \frac{\kappa_5}{2} \right) + 2.550 \kappa_D \\ \kappa_3 &= -5.136 + 1.648 \kappa - 0.218 \left(\kappa_4 + \frac{\kappa_5}{2} \right) + \kappa_D. \end{aligned} \quad (13)$$

The χ^2 is then given by fitting to the rest of the magnetic moments, where still the errors of the inputs are smaller than the theoretical error. Errors can be assigned to the fitted LECs by defining them in terms of the expected magnitude of the NNNLO corrections. For such an estimation of the parameters' errors, the minimum obtained for the χ^2 per degree of freedom is normalized to unity, or alternatively the experimental inputs are assigned larger

TABLE V. Results from fits to the electric current magnetic moments, in units of the nuclear magneton μ_N . The renormalization scale was set to $\mu = \Lambda = m_\rho$. κ_F requires $\Delta S = 1$ weak magnetic moments to be determined.

LEC $\times \frac{m_N}{\Lambda}$	LO	NNLO	
κ	2.80		2.887
κ_1	0		3.29
κ_2	0		0.00
κ_D	0		0.397
κ_F	0		...
κ_3	0		0.53
κ_4	0		-2.85
κ_5	0		1.05

	μ_{LO}	μ_{NNLO}	μ_{Exp}
p	2.691	Input	2.792847356(23)
n	-1.794	Input	-1.9130427(5)
Σ^+	2.691	2.367	2.458(10)
Σ^0	0.897	0.869	...
Σ^-	-0.897	-0.629	-1.160(25)
Λ	-0.897	-0.611	-0.613(4)
Ξ^0	-1.794	-1.275	-1.250(14)
Ξ^-	-0.897	-0.652	-0.6507(25)
$\Delta^+ p$	2.537	3.65	3.58(10)
$\Sigma^0 \Lambda$	1.553	1.57	1.61(8)
$\Sigma^* \Lambda$	2.197	2.68	2.73(25) ^a
$\Sigma^{*+} \Sigma^+$	-2.537	-2.35	-3.17(36) ^b

	μ_{LO}	μ_{NNLO}	μ_{Exp}
Δ^{++}	5.381	5.962	3.7-7.5
Δ^+	2.691	3.049	2.7(3.6)
Δ^0	0	0.136	...
Δ^-	-2.691	-2.777	...
Σ^{*+}	2.691	3.151	...
Σ^{*0}	0	0.343	...
Σ^{*-}	-2.691	-2.465	...
Ξ^{*0}	0	0.490	...
Ξ^{*-}	-2.691	-2.208	...
Ω	-2.691	-2.005	-2.02(5)

^aEmpirical results are from PDG and Ref. [44].

^bEmpirical results are from PDG and Ref. [45].

errors compatible with the mentioned magnitude of NNNLO corrections, which leads to a similar result. Here, the first procedure is followed. Important correlation is found within the following pairs of LECs: $\kappa - \kappa_D$, and $\kappa_4 - \kappa_5$. The eigenvectors of the correlation matrix of the first pair are $(0.8\kappa + 0.6\kappa_D)$ and $(0.6\kappa - 0.8\kappa_D)$, with respective errors ± 0.004 and ± 0.04 , and of the second pair are $(0.9\kappa_4 + \sqrt{0.19}\kappa_5)$ and $(\sqrt{0.19}\kappa_4 - 0.9\kappa_5)$ with respective errors ± 0.3 and ± 2.1 . This provides the necessary information for the LEC's error analysis.

As mentioned earlier, the $\Delta N\gamma$ amplitude at LO is too small by roughly 30%, a manifestation of an important spin-symmetry breaking effect. The effect receives a small

nonanalytic contribution (at $\mu = m_\rho$), and the contributions from the contact terms are as follows: $\kappa_D: \mathcal{O}((m_s - \hat{m})N_c)$, and $\kappa_4: \mathcal{O}(1/N_c)$. From the fit, one finds a modest contribution from κ_D and a dominant contribution from κ_4 . Since the latter is a $1/N_c^2$ correction with respect to the LO magnetic moment, it seems to be unnaturally large. This is a bit surprising as a similar kind of effect in the ΔN axial vector coupling is actually unnaturally small. This contrast remains to be understood. Finally, a fit where the ΔN transition is not an input shows an enhancement but only by about half of what is needed.

An interesting case is the magnetic moment of Σ^{*0} : all LO and NLO tree-level and quark mass independent contributions vanish, receiving only NNLO tree and loop contributions, which vanish in the $SU(3)$ symmetry limit. On the other hand, the experimental value of the magnetic moment of Σ^- quoted as average by the PDG [40] cannot be described; U-spin symmetry implies that it must be equal to the magnetic moment of the Ξ^- up to NNLO $SU(3)$ breaking by quark masses. The experimental results imply a very large effect which is very difficult to reconcile with the other U-spin multiplets, where the effect is between 12% and 25% per unit of strangeness, while for the pair $\Sigma^- \Xi^-$ case, it is 44%. The present analysis shows that μ_{Σ^-} is an outlier.

One of the early tests of the magnetic moments in $SU(3)$ was provided by the Coleman-Glashow (CG) relation, namely, $\mu_p - \mu_n - \mu_{\Sigma^+} + \mu_{\Sigma^-} + \mu_{\Xi^0} - \mu_{\Xi^-} = 0$. This relation remains valid at tree-level NNLO and receives only a finite correction from the one-loop contributions. Explicit calculation gives the deviation with estimated theoretical error $\Delta_{CG} = 1.09 \pm 0.25 \mu_N$ to be compared with the experimental deviation $0.49 \pm 0.03 \mu_N$, affected, however, by the Σ^- issue.

Finally, the weak interaction magnetic moments for hyperon decays turn out to depend on the LEC κ_F , which does not appear in the EM case. The result for the LECs from the EM case gives the predictions $\mu_{\Sigma^- n} = (0.516 - 0.180 \kappa_F) \frac{g}{2m_N}$ and $\mu_{\Lambda p} = (-1.41 + 0.66 \kappa_F) \frac{g}{2m_N}$, where $g = e/\sin\theta_W$. At LO, one has the large hierarchy $\mu_{\Lambda p}/\mu_{\Sigma^- n} = -\sqrt{27/2}$. A determination of κ_F will require a LQCD calculation.

A. Magnetic radii

The magnetic radii are theoretically very constrained at the order of the present calculation. For all the vector currents and baryons, they are determined only by UV finite loop contributions and the single available finite counterterm fixed by the LEC κ_r . Since only the magnetic radii of the proton and neutron are experimentally known, one can use these to fit that LEC, leading to the results shown in Table VI. The rest of the radii are then predictions which can hopefully be tested in the future with LQCD calculations. Note that the lion's share of the magnetic radii is

TABLE VI. Magnetic radii from a fit to nucleons.

$\kappa_r = -2.63$	$\langle r^2 \rangle (\text{fm}^2)$		
	Exp	Th	Loop
p	0.724	0.718	0.134
n	0.746	0.747	0.179
Σ^+	...	0.766	0.100
Σ^0	...	0.698	0.061
Σ^-	...	0.922	0.189
Λ	...	0.895	0.079
Ξ^0	...	0.872	0.081
Ξ^-	...	0.796	0.035
$\Delta^+ p$...	0.875	0.226

from the short distance terms proportional to κ_r , with the loop contribution from diagram A_3 in Fig. 1 giving up to 20% for the proton, neutron, and Σ^- and less than 10% for the rest.

Finally, a calculation of the curvature of the EM magnetic moments yields $\langle r^4 \rangle^p = 0.38 \text{ fm}^4$ and $\langle r^4 \rangle^n = 0.54 \text{ fm}^4$ to be compared with those obtained in Ref. [38], which are, respectively, 1.72(6) and 2.04(1) fm^4 , leading to a similar assessment as in the case of the electric charge already discussed, although less dramatic.

VI. SUMMARY

This work presented the study of the $SU(3)$ vector currents in baryons based on the combined chiral and $1/N_c$ expansion. It was carried out in the context of the ξ power counting to one loop. This corresponds to a calculation of the charges, magnetic moments, and their radii for both octet and decuplet baryons. The calculations have been provided for generic N_c , which permits an exploration of the behavior of those observables with respect to the number of colors. Only two LECs are needed to determine all $SU(3)$ charge radii, while the magnetic moments need to be renormalized involving eight LECs, of which all but two can be fixed solely in terms of the known EM magnetic moments. Of the two remaining LECs, one needs information about $\Delta S = 1$ weak magnetic moments, and the second requires knowledge of magnetic moments at different values of quark masses, which can be obtained from LQCD calculations. The fits to the magnetic moments indicate that the values of the LECs are within the range of natural magnitude, although there is a puzzling issue, namely, the unnaturally large spin-symmetry breaking required for the description of the ΔN transition magnetic moment. On the other hand, the magnetic radii involve only one LEC, which according to its determination from the proton and neutron magnetic radii, gives the dominant contribution to all magnetic radii. Finally, the curvature of form factors are given at the order of the calculation by nonanalytic terms in m_q , which turn out to be very small, and therefore require for their description an extension of the present work to higher order.

ACKNOWLEDGMENTS

The authors thank Rubén Flores Mendieta and Christian Weiss for very useful discussions. This work was supported by DOE Contract No. DE-AC05-06OR23177, under which JSA operates the Thomas Jefferson National Accelerator Facility, and by the National Science Foundation through Grants No. PHY-1307413, No. PHY-1613951 and No. PHY-1913562.

APPENDIX A: SPIN-FLAVOR ALGEBRA

The $4N_f^2 - 1$ generators of the spin-flavor group $SU(2N_f)$ consist of the three spin generators S^i , the $N_f^2 - 1$ flavor $SU(N_f)$ generators T^a , and the remaining $3(N_f^2 - 1)$ spin-flavor generators G^{ia} . The commutation relations are

$$\begin{aligned}
 [S^i, S^j] &= i\epsilon^{ijk} S^k, & [T^a, T^b] &= if^{abc} T^c, & [T^a, S^i] &= 0, \\
 [S^i, G^{ja}] &= i\epsilon^{ijk} G^{ka}, & [T^a, G^{ib}] &= if^{abc} G^{ic}, \\
 [G^{ia}, G^{jb}] &= \frac{i}{4} \delta^{ij} f^{abc} T^c + \frac{i}{2N_f} \delta^{ab} \epsilon^{ijk} S^k + \frac{i}{2} \epsilon^{ijk} d^{abc} G^{kc}.
 \end{aligned}
 \tag{A1}$$

In spin-flavor representations with N_c indices corresponding to baryons, the generators G^{ia} have matrix

$$\begin{aligned}
 \langle S' S'_3, Y' I' I'_3 | S^m | S S_3, Y I I_3 \rangle &= \delta_{SS'} \delta_{Y Y'} \delta_{I I'} \delta_{I_3 I'_3} \sqrt{S(S+1)} \langle S S_3, 1m | S' S'_3 \rangle \\
 \langle S' S'_3, Y' I' I'_3 | T^{yii_3} | S S_3, Y I I_3 \rangle &= \delta_{SS'} \delta_{S_3 S'_3} \frac{1}{\sqrt{\dim(2S, \frac{1}{2}(N_c - 2S))}} \langle S || T || S \rangle \\
 &\quad \times \left\langle \begin{matrix} (2S, \frac{1}{2}(N_c - 2S)) & (1, 1) \\ Y I I_3 & yii_3 \end{matrix} \middle| \begin{matrix} (2S, \frac{1}{2}(N_c - 2S)) \\ Y' I' I'_3 \end{matrix} \right\rangle_{\gamma=1} \\
 \langle S' S'_3, Y' I' I'_3 | G^{m, yii_3} | S S_3, Y I I_3 \rangle &= \frac{\langle S S_3, 1m | S' S'_3 \rangle}{\sqrt{2S'+1} \sqrt{\dim(2S, \frac{1}{2}(N_c - 2S))}} \\
 &\quad \times \sum_{\gamma=1,2} \langle S' || G || S \rangle_{\gamma} \left\langle \begin{matrix} (2S, \frac{1}{2}(N_c - 2S)) & (1, 1) \\ Y I I_3 & yii_3 \end{matrix} \middle| \begin{matrix} (2S, \frac{1}{2}(N_c - 2S)) \\ Y' I' I'_3 \end{matrix} \right\rangle_{\gamma},
 \end{aligned}
 \tag{A3}$$

where the reduced matrix elements are [here, $p = 2S$, $q = \frac{1}{2}(N_c - 2S)$]

$$\begin{aligned}
 \langle S || T || S \rangle &= \sqrt{\dim(p, q)} C_2(p, q) \\
 &= \frac{\sqrt{(2S+1)(N_c-2S+2)(N_c+2S+4)(N_c(N_c+6)+12S(S+1))}}{4\sqrt{6}} \\
 \langle S' || G || S \rangle_{\gamma=1} &= \begin{cases} \text{if } S = S' + 1: & -\frac{\sqrt{(4S^2-1)((N_c+2)^2-4S^2)((N_c+4)^2-4S^2)}}{8\sqrt{2}} \\ \text{if } S = S' - 1: & -\frac{\sqrt{(4S(S+2)+3)(N_c-2S)(N_c-2S+2)(N_c+2S+4)(N_c+2S+6)}}{8\sqrt{2}} \\ \text{if } S = S': & \text{sign}(N_c - 2S - 0^+) \frac{(N_c+3)(2S+1)\sqrt{S(S+1)(N_c-2S+2)(N_c+2S+4)}}{\sqrt{6N_c(N_c+6)+12S(S+1)}} \end{cases} \\
 \langle S' || G || S \rangle_{\gamma=2} &= -\delta_{SS'} \frac{(2S+1)\sqrt{(N_c-2S)(N_c+2S+6)((N_c+2)^2-4S^2)((N_c+4)^2-4S^2)}}{8\sqrt{2}\sqrt{N_c(N_c+6)+12S(S+1)}}.
 \end{aligned}
 \tag{A4}$$

elements $\mathcal{O}(N_c)$ on states with $S = \mathcal{O}(N_c^0)$. The ground state baryons furnish the totally symmetric irreducible representation of $SU(6)$ with N_c Young boxes, which decomposes into the following $SU(2)_{\text{spin}} \times SU(3)$ irreducible representations: $[S, (p, q)] = [S, (2S, \frac{1}{2}(N_c - 2S))]$, $S = 1/2, \dots, N_c/2$ (assumed N_c is odd). The baryon states can then be denoted by $|SS_3, YII_3\rangle$, where the spin S of the baryon determines its $SU(3)$ multiplet.

The matrix elements of a $SU(2)_{\text{spin}} \times SU(3) \subset SU(6)$ tensor operator between ground state baryons are given by the Wigner-Eckart theorem, with obvious notation,

$$\begin{aligned}
 \langle S' S'_3, R' Y' I' I'_3 | \mathcal{O}_{\tilde{R} \tilde{Y} \tilde{I} \tilde{I}_3}^{\ell \ell_3} | S S_3, R Y I I_3 \rangle \\
 = \frac{1}{\sqrt{2S'+1} \sqrt{\dim R'}} \langle S S_3, \ell \ell_3 | S' S'_3 \rangle \\
 \times \sum_{\gamma} \langle S', R' || \mathcal{O}_{\tilde{R}}^{\ell} || S, R \rangle_{\gamma} \left\langle \begin{matrix} R & \tilde{R} \\ Y I I_3 & \tilde{Y} \tilde{I} \tilde{I}_3 \end{matrix} \middle| \begin{matrix} R' \\ Y' I' I'_3 \end{matrix} \right\rangle_{\gamma},
 \end{aligned}
 \tag{A2}$$

where R represents the $SU(3)$ multiplet of the baryon and γ indicates the possible recouplings in $SU(3)$. Applying this to the matrix elements of the generators of $SU(6)$ one obtains:

APPENDIX B: BUILDING BLOCKS

The fundamental building blocks of the effective theory are the generators of $SU(6)$ detailed in Appendix A and tensor products thereof and the baryon and Goldstone boson fields and sources associated with the different current densities. The GB fields are parametrized as the coordinates of the coset $SU_L(3) \times SU_R(3)/SU(3)$ where here one uses the exponential parametrization

$$u(x) = e^{i\frac{\pi^a T^a}{F_\pi}}, \quad (\text{B1})$$

where π^a are the octet of GBs, T^a are the generators of flavor $SU(3)$, and F_π is the pion decay constant. The baryon fields denoted by \mathbf{B} belong into a multiplet of $SU(6)$, which transforms as the totally symmetric

irreducible representation with N_c indices, which organized in multiplets of spin-flavor $SU(2) \times SU(3)$ contain the multiplets $[S, (p = 2S, q = \frac{1}{2}(N_c - 2S))]$ as mentioned in Appendix A. The effective baryon Lagrangian can be expressed in the usual way as a series of terms which are $SU_L(3) \times SU_R(3)$ invariant (upon introduction of appropriate sources; see, for instance, Ref. [46] for details).

The fields in the effective Lagrangian are the Goldstone bosons parametrized by the unitary $SU(3)$ matrix field u and the baryons given by the symmetric $SU(6)$ multiplet \mathbf{B} . The building blocks for the effective theory consist of low-energy operators composed in terms of the GB fields, derivatives and sources (chiral tensors), and spin-flavor composite operators (spin-flavor tensors).

The low-energy operators are the usual ones, namely,

$$\begin{aligned} D_\mu &= \partial_\mu - i\Gamma_\mu, & \Gamma_\mu &= \Gamma_\mu^\dagger = \frac{1}{2}(u^\dagger(i\partial_\mu + r_\mu)u + u(i\partial_\mu + \ell_\mu)u^\dagger), \\ u_\mu &= u_\mu^\dagger = u^\dagger(i\partial_\mu + r_\mu)u - u(i\partial_\mu + \ell_\mu)u^\dagger, \\ \chi &= 2B_0(s + ip), & \chi_\pm &\equiv u^\dagger\chi u^\dagger \pm u\chi^\dagger u, \\ F_L^{\mu\nu} &= \partial^\mu\ell^\nu - \partial^\nu\ell^\mu - i[\ell^\mu, \ell^\nu], & F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu] \\ F_\pm^{\mu\nu} &\equiv uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u, \end{aligned} \quad (\text{B2})$$

where D_μ is the chiral covariant derivative, s and p are scalar and pseudoscalar sources, and ℓ_μ and r_μ are gauge sources. It is convenient to define the $SU(3)$ singlet and octet components of χ^\pm using the fundamental $SU(3)$ irreducible representation, namely,

$$\begin{aligned} \chi_\pm^0 &= \frac{1}{3}\langle\chi_\pm\rangle \\ \tilde{\chi}_\pm &= \chi_\pm - \chi_\pm^0 = \tilde{\chi}_\pm^a \frac{\lambda^a}{2}. \end{aligned} \quad (\text{B3})$$

Displaying explicitly the quark masses,

$$\chi_+ = 4B_0\mathcal{M}_q + \dots, \quad (\text{B4})$$

where the quark mass matrix \mathcal{M}_q is given in terms of the three quark mass combinations, namely $SU(3)$ singlet, isosinglet, and isotriplet, respectively defined by

$$\begin{aligned} m^0 &= \frac{1}{3}(m_u + m_d + m_s), & m^8 &= \frac{1}{\sqrt{3}}(m_u + m_d - 2m_s), \\ m^3 &\equiv (m_u - m_d). \end{aligned} \quad (\text{B5})$$

Under $SU_L(3) \times SU_R(3)$ chiral transformations, D_μ , u^μ , χ_\pm , and $F_\pm^{\mu\nu}$ transform as $X \rightarrow h(u, L, R)Xh^\dagger(u, L, R)$, where h is the nonlinear realization of the corresponding transformation. Note that all the chiral building blocks when acting on a particular baryon need to be written in terms of the $SU(3)$ generators in the representation of that baryon. In particular, the building blocks discussed here will be written as [neglecting any $SU(3)$ singlet component] $X^a T^a$, where one obtains $X^a = \frac{1}{2}\text{Tr}(\lambda^a X)$, where X is given in the fundamental representation and λ^a is a Gell-Mann matrix.

The leading-order equations of motion are used in the construction of the higher-order terms in the Lagrangian, namely, $iD_0\mathbf{B} = (\frac{C_{\text{MF}}}{N_c}\hat{S}^2 + \frac{c_+}{2\Lambda}\hat{\chi}_+)\mathbf{B}$, and $\nabla_\mu u^\mu = \frac{i}{2}\chi_-$.

APPENDIX C: LOOP INTEGRALS

The one-loop integrals needed in this work are provided here. The definition $d\tilde{k} \equiv d^d k / (2\pi)^d$ is used.

The scalar and tensor one-loop integrals are

$$\begin{aligned}
 I(n, \alpha, \Lambda) &\equiv \int \widetilde{d^d k} \frac{k^{2n}}{(k^2 - \Lambda^2)^\alpha} = i(-1)^{n-\alpha} \frac{1}{(4\pi)^{\frac{d}{2}}} \frac{\Gamma(n + \frac{d}{2})\Gamma(\alpha - n - \frac{d}{2})}{\Gamma(\frac{d}{2})\Gamma(\alpha)} (\Lambda^2)^{n-\alpha+\frac{d}{2}} \\
 I^{\mu_1, \dots, \mu_{2n}}(\alpha, \Lambda) &\equiv \int \widetilde{d^d k} \frac{k_{\mu_1} \dots k_{\mu_{2n}}}{(k^2 - \Lambda^2)^\alpha} = i(-1)^{n-\alpha} \frac{1}{(4\pi)^{\frac{d}{2}}} \frac{1}{4^n n!} \frac{\Gamma(\alpha - n - \frac{d}{2})}{\Gamma(\alpha)} (\Lambda^2)^{n-\alpha+\frac{d}{2}} \sum_{\sigma} g_{\mu_{\sigma_1} \mu_{\sigma_2}} \dots g_{\mu_{\sigma_{2n-1}} \mu_{\sigma_{2n}}} \\
 &= \frac{1}{4^n n!} \frac{\Gamma(\frac{d}{2})}{\Gamma(n + \frac{d}{2})} I(n, \alpha, \Lambda) \sum_{\sigma} g_{\mu_{\sigma_1} \mu_{\sigma_2}} \dots g_{\mu_{\sigma_{2n-1}} \mu_{\sigma_{2n}}}, \tag{C1}
 \end{aligned}$$

where σ are the permutations of $\{1, \dots, 2n\}$.

The Feynman parametrizations needed when heavy propagators are in the loop are

$$\begin{aligned}
 \frac{1}{A_1 \dots A_m B_1 \dots B_n} &= 2^m \Gamma(m+n) \int_0^\infty d\lambda_1 \dots d\lambda_m \int_0^1 d\alpha_1 \dots d\alpha_n \delta(1 - \alpha_1 - \dots - \alpha_n) \\
 &\times \frac{1}{(2\lambda_1 A_1 + \dots + 2\lambda_m A_m + \alpha_1 B_1 + \dots + \alpha_n B_n)^{m+n}}, \tag{C2}
 \end{aligned}$$

where the A_i are heavy particle static propagators denominators and the B_i are relativistic ones.

The integration over a Feynman parameter λ is of the general form

$$J(C_0, C_1, \lambda_0, d, \nu) \equiv \int_0^\infty (C_0 + C_1(\lambda - \lambda_0)^2)^{-\nu+\frac{d}{2}} d\lambda, \tag{C3}$$

which satisfies the recurrence relation:

$$\begin{aligned}
 J(C_0, C_1, \lambda_0, d, \nu) &= \frac{-\lambda_0(C_0 + C_1\lambda_0^2)^{1-\nu+\frac{d}{2}} + (3 + d - 2\nu)J(C_0, C_1, \lambda_0, d, \nu - 1)}{(d - 2\nu + 2)C_0} \\
 J(C_0, C_1, \lambda_0, d, \nu) &= C_0 \frac{d - \nu}{d - 2\nu + 1} J(C_0, C_1, \lambda_0, d, \nu + 1) + \frac{\lambda_0}{d - 2\nu + 1} (C_0 + C_1\lambda_0^2)^{\frac{d}{2}-\nu}. \tag{C4}
 \end{aligned}$$

Integrals with factors of λ in the numerator are obtained by using

$$\begin{aligned}
 J(C_0, C_1, \lambda_0, d, \nu, n = 1) &\equiv \int_0^\infty (\lambda - \lambda_0)^{n=1} (C_0 + C_1(\lambda - \lambda_0)^2)^{-\nu+\frac{d}{2}} d\lambda \\
 &= -\frac{1}{2C_1(\frac{d}{2} + 1 - \nu)} (C_0 + C_1\lambda_0^2)^{\frac{d}{2}+1-\nu} \tag{C5}
 \end{aligned}$$

and the recurrence relations

$$J(C_0, C_1, \lambda_0, d, \nu, n) = \frac{1}{C_1} (J(C_0, C_1, \lambda_0, d, \nu - 1, n - 1) - C_0 J(C_0, C_1, \lambda_0, d, \nu, n - 2)). \tag{C6}$$

For convenience in some of the calculations for the currents, the following integral is defined:

$$\tilde{J}(C_0, C_1, \lambda_0, d, \nu, 1) \equiv J(C_0, C_1, \lambda_0, d, \nu, 1) + \lambda_0 J(C_0, C_1, \lambda_0, d, \nu). \tag{C7}$$

For the calculations in this work, the following integrals are needed at $d = 4 - 2\epsilon$:

$$\begin{aligned}
 J(C_0, C_1, \lambda_0, d, 3) &= \frac{1}{\sqrt{C_0 C_1}} \left(\frac{\pi}{2} + \arctan \left(\lambda_0 \sqrt{\frac{C_1}{C_0}} \right) \right) \\
 J(C_0, C_1, \lambda_0, d, 2) &= \frac{1}{d-3} (\lambda_0(C_0 + C_1\lambda_0^2)^{\frac{d}{2}-2} + (d-4)C_0 J(C_0, C_1, \lambda_0, d, 3)) \\
 J(C_0, C_1, \lambda_0, d, 1) &= \frac{1}{d-1} (\lambda_0(C_0 + C_1\lambda_0^2)^{\frac{d}{2}-1} + (d-2)J(C_0, C_1, \lambda_0, d, 2)). \tag{C8}
 \end{aligned}$$

1. Specific integrals

Here, a summary of relevant one-loop integrals for the calculations in this work is provided for the convenience of the reader.

1) Loop integrals involving only relativistic propagators are

$$\begin{aligned}
I(0, 1, M) &= -\frac{i}{(4\pi)^{\frac{d}{2}}} \Gamma\left(1 - \frac{d}{2}\right) M^{d-2} \\
I(0, 2, M) &= \frac{i}{(4\pi)^{\frac{d}{2}}} \Gamma\left(2 - \frac{d}{2}\right) M^{d-4} \\
K(q, M_a, M_b) &\equiv \int \widetilde{d^d k} \frac{1}{(k^2 - M_a^2 + i\epsilon)((k+q)^2 - M_b^2 + i\epsilon)} = \int_0^1 d\alpha I(0, 2, \Lambda(\alpha)) \\
K^\mu(q, M_a, M_b) &\equiv \int \widetilde{d^d k} \frac{k^\mu}{(k^2 - M_a^2 + i\epsilon)((k+q)^2 - M_b^2 + i\epsilon)} = \int_0^1 d\alpha (\alpha - 1) q^\mu I(0, 2, \Lambda(\alpha)) \\
K^{\mu\nu}(q, M_a, M_b) &\equiv \int \widetilde{d^d k} \frac{k^\mu k^\nu}{(k^2 - M_a^2 + i\epsilon)((k+q)^2 - M_b^2 + i\epsilon)} \\
&= \int_0^1 d\alpha \left((1 - \alpha)^2 q^\mu q^\nu I(0, 2, \Lambda(\alpha)) + \frac{g^{\mu\nu}}{d} I(1, 2, \Lambda(\alpha)) \right), \tag{C9}
\end{aligned}$$

where

$$\Lambda(\alpha) = \sqrt{\alpha M_a^2 + (1 - \alpha) M_b^2 - \alpha(1 - \alpha) q^2}.$$

2) Loop integrals involving one heavy propagator are

$$\begin{aligned}
H(p_0, M) &\equiv \int \widetilde{d^d k} \frac{1}{(p_0 - k_0 + i\epsilon)(k^2 - M^2 + i\epsilon)} \\
&= \frac{2i}{(4\pi)^{\frac{d}{2}}} \Gamma\left(2 - \frac{d}{2}\right) J\left(M^2 - p_0^2, 1, p_0, d, 2\right) \\
H^{ij}(p_0, M) &\equiv \int \widetilde{d^d k} \frac{k^i k^j}{(p_0 - k_0 + i\epsilon)(k^2 - M^2 + i\epsilon)} \\
&= -\frac{i}{(4\pi)^{\frac{d}{2}}} g^{ij} \Gamma\left(1 - \frac{d}{2}\right) J\left(M^2 - p_0^2, 1, p_0, d, 1\right) \\
H^{ij\mu}(p_0, q, M_a, M_b) &\equiv \int \widetilde{d^d k} \frac{k^i (k+q)^j (2k+q)^\mu}{(p_0 - k_0 + i\epsilon)(k^2 - M_a^2 + i\epsilon)((k+q)^2 - M_b^2 + i\epsilon)} \\
&= i \frac{4}{(4\pi)^{\frac{d}{2}}} \int_0^1 d\alpha \left\{ -\frac{1}{2} \Gamma\left(3 - \frac{d}{2}\right) q^i q^j \alpha(1 - \alpha) \right. \\
&\quad \times \left((1 - 2\alpha) q^\mu J(C_0, C_1, \lambda_0, d, 3) - 2g^{\mu 0} \tilde{J}(C_0, C_1, \lambda_0, d, 3, 1) \right) \\
&\quad + \Gamma\left(2 - \frac{d}{2}\right) \left(-(1 - 2\alpha) g^{ij} q^\mu + 2(\alpha g^{\mu i} q^j - (1 - \alpha) g^{\mu j} q^i) \right) J(C_0, C_1, \lambda_0, d, 2) \\
&\quad \left. + 2g^{ij} g^{\mu 0} \tilde{J}(C_0, C_1, \lambda_0, d, 2, 1) \right\}, \tag{C10}
\end{aligned}$$

where

$$\begin{aligned}
 C_0 &= \alpha M_a^2 + (1 - \alpha)M_b^2 - p_0^2 - 2(1 - \alpha)p_0q_0 - (1 - \alpha)(\alpha q^2 + (1 - \alpha)q_0^2) \\
 C_1 &= 1 \\
 \lambda_0 &= p_0 + (1 - \alpha)q_0.
 \end{aligned}
 \tag{C11}$$

The polynomial pieces of the integrals are

$$\begin{aligned}
 H(p_0, M)^{\text{poly}} &= \frac{i}{(4\pi)^2} 2p_0(\lambda_\epsilon + 2) \\
 H^{ij}(p_0, M)^{\text{poly}} &= \frac{i}{(4\pi)^2} \frac{p_0}{3} \left((3M^2 - 2p_0^2)\lambda_\epsilon + 7M^2 - \frac{16}{3}p_0^2 \right) \\
 H^{ij\mu}(p_0, q, M_a, M_b)^{\text{poly}} &= \frac{i}{96\pi^2} (\lambda_\epsilon (g^{ij}(g^{\mu 0}(-3(M_a^2 + M_b^2) + 12p_0(p_0 + q_0) + q^2 + 4q_0^2) - q_0q^\mu) \\
 &\quad - 2q^i(3p_0 + 2q_0)g^{\mu j} + 2q^j((3p_0 + q_0)g^{\mu i} + q^i g^{\mu 0})) \\
 &\quad + g^{ij}(g^{\mu 0}(-3(M_a^2 + M_b^2) + 24p_0(p_0 + q_0) + q^2 + 8q_0^2) - 2q_0q^\mu) \\
 &\quad - 4q^i(3p_0 + 2q_0)g^{\mu j} + 4q^j(3p_0 + q_0)g^{\mu i}),
 \end{aligned}
 \tag{C12}$$

where the UV divergence is given by the terms proportional to $\lambda_\epsilon \equiv 1/\epsilon - \gamma + \log 4\pi$, where $d = 4 - 2\epsilon$.

APPENDIX D: INTERACTION AND VECTOR CURRENT VERTICES NEEDED IN LOOP CALCULATIONS

The interaction and currents vertices needed in the one-loop calculations are given for completeness.

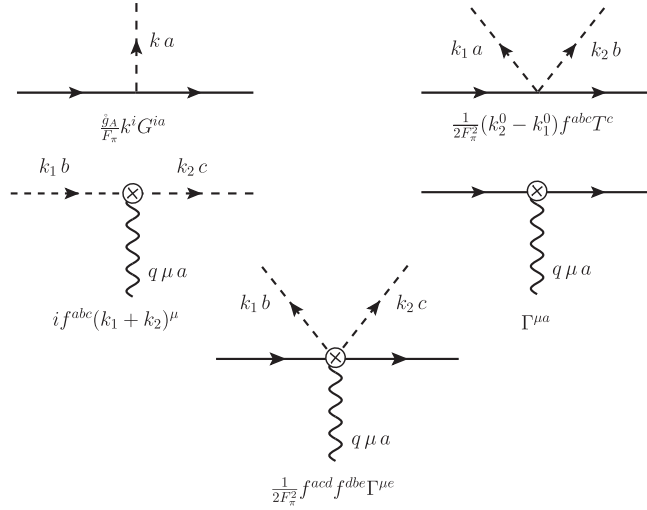


FIG. 2. The vector current vertices indicated with a square are the magnetic ones. The momentum q is incoming, and $\Gamma^{\mu a} = g^{\mu 0} T^a + i \frac{\kappa}{\Lambda} \epsilon^0 \mu ij f^{abc} f^{cbd} q^i G^{jd}$.

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