Evaluation of pion-nucleon sigma term in Dyson-Schwinger equation approach of QCD

Jing-Hui Huang,* Ting-Ting Sun,† and Huan Chen‡

School of Mathematics and Physics, China University of Geosiciences, Lumo Road 388, 430074 Wuhan, China

(Received 18 October 2019; accepted 18 February 2020; published 6 March 2020)

We calculate the variation of the chiral condensate in medium with respect to the quark chemical potential and evaluate the pion-nucleon sigma term via the Hellmann-Feynman theorem. The variation of the chiral condensate in medium is obtained by solving the truncated Dyson-Schwinger equation for the quark propagator at finite chemical potential, with various ansätz for the quark-gluon vertex and gluon propagator. We obtain the value of the sigma term $\sigma_{\pi N} = 62(1)$ MeV, where the (1) represents the systematic error due to our different ansätz for the quark-gluon vertex and gluon propagator. Our result favors a relatively large value and is rather consistent with the recent data obtained by analyzing pion-nucleon scattering and pionic-atom experiments.

DOI: 10.1103/PhysRevD.101.054007

I. INTRODUCTION

The pion-nucleon sigma term $\sigma_{\pi N}$ is of fundamental importance for understanding the chiral symmetry breaking effects in the nucleon [1,2] and the origin of the mass of observable matter [3,4]. Recently, special attention has been paid to $\sigma_{\pi N}$, since it is also significant in searches for the Higgs boson, supersymmetric particles, cold dark matter [5–7], and *CP* violation [8,9]. $\sigma_{\pi N}$ can be obtained indirectly in experiments, such as pion-nucleon scattering or pionic-atom experiments [10–13]. Several recent analyses [11–13] gave 50 MeV $< \sigma_{\pi N} <$ 70 MeV, which is relatively larger than the widely used value $\sigma_{\pi N} \simeq 45$ MeV [14]. In particular, Refs. [11,12] gave a value around 60 MeV with quite small error bars. Theoretically, the pion-nucleon sigma term could be calculated in chiral perturbation theory [15–18], lattice QCD [19–24], the Dyson-Schwinger equation (DSE) approach of QCD [25–27] and various other models [28–31]. However, theoretical results vary widely with different methods. Notably, the values from lattice QCD are around 30 to 40 MeV, which are much smaller than those from the above experimental analyses. Conversely, some other works gave relatively large values, even up to 80 MeV [29] or 95 MeV [27]. Thus, further effort is needed in the theoretical calculations of the sigma term. In this work, we evaluate the pion-nucleon sigma term in the DSE approach of QCD, via the Hellmann-Feynman theorem.

Theoretically, the pion-nucleon sigma term $\sigma_{\pi N}$ is usually written via the Hellmann-Feynman theorem as

$$\sigma_{\pi N} = m_q \frac{\partial M_N}{\partial m_q}, \tag{1}$$

where M_N is the nucleon mass and m_q is the average current-quark mass for u and d quarks (see Sec. III for details).

It has been known that the nucleon mass M_N comes almost entirely from the dynamical chiral symmetry breaking (DCSB) (see, e.g., Ref. [3]). It has also been known that the DSEs of QCD provide a natural approach to investigate the DCSB and the chiral symmetry restoration in vacuum (see, e.g., Refs. [32–35]), in hot medium (see, e.g., Refs. [36–41]), and in cold dense matter (see, e.g., Refs. [42–47]), as well as the properties of hadrons (see, e.g., Refs. [48–54]).

Inspired by the above-mentioned successes of the DSE approach, we restudy the pion-nucleon sigma term in the DSE approach with both the widely used rainbow approximation and the Ball-Chiu vertex [55,56] for the effective quark-gluon vertex, and two different infrared-dominant models for the effective interaction.

The paper is organized as follows. In Sec. II the truncation scheme of the DSE for the quark propagator in vacuum and at finite chemical potential is given. In Sec. III we briefly describe the method for evaluating the pion-nucleon sigma term $\sigma_{\pi N}$ via the DCSB in medium (more explicitly, the chiral condensate in medium). Then, the numerical results are given in Sec. IV. Finally, we summarize our work and conclude with a brief remark in Sec. V.

II. DYSON-SCHWINGER EQUATION FOR THE QUARK PROPAGATOR

Our calculation is based on the quark propagator at finite chemical potential $S(p;\mu)$, which satisfies the Dyson-Schwinger equation

*jinghuihuang@cug.edu.cn
†tingtingsun@cug.edu.cn
‡huanchen@cug.edu.cn

$$S(p;\mu)^{-1} = Z_2(i\gamma\cdot\tilde{p}+m_q) + Z_1 g^2(\mu)\int\frac{d^4q}{(2\pi)^4} \times D_{\rho\sigma}(k;\mu)\frac{\lambda^a}{2}\gamma_\rho S(q;\mu)\Gamma^a_\sigma(q,p;\mu), \quad (2)$$

where $\tilde{p} = (\vec{p}, p_4 + i\mu)$, $k = p - q$, $D_{\rho\sigma}(k;\mu)$ is the full gluon propagator, $\Gamma^a_\sigma(q,p;\mu)$ is the dressed quark-gluon vertex, Z_1 is the renormalization constant for the quark-gluon vertex, and Z_2 is the quark wave-function normalization constant. The general structure of the quark propagator at finite chemical potential can be written as

$$S^{-1}(p;\mu) = i\vec{\gamma}\cdot\vec{p}A(\vec{p}^2, p_4;\mu) + i\gamma_4\tilde{p}_4 C(\vec{p}^2, p_4;\mu) + B(\vec{p}^2, p_4;\mu), \quad (3)$$

where $A(\vec{p}^2, p_4;\mu)$ and $B(\vec{p}^2, p_4;\mu)$, $C(\vec{p}^2, p_4;\mu)$ are scalar functions of p^2 and p_4, while in vacuum $A(\vec{p}^2, p_4;\mu=0) = C(\vec{p}^2, p_4;\mu=0) = A_0(p^2)$, $B(\vec{p}^2, p_4;\mu) = B_0(p^2)$, and

$$S^{-1}(p) = i\gamma\cdot pA(p^2) + B(p^2). \quad (4)$$

We solve Eq. (2) with models of the gluon propagator and the quark-gluon vertex, which describe meson properties in vacuum well in the symmetry-preserving Dyson-Schwinger equation and Bethe-Salpeter equation (BSE) schemes (see, e.g., Refs. [57–60]). In vacuum, they are usually taken as

$$Z_1 g^2 D_{\rho\sigma}(k)\Gamma^a_\sigma(q,p) = \mathcal{G}(k^2)D^0_{\rho\sigma}(k)\frac{\lambda^a}{2}\Gamma_\sigma(p,q), \quad (5)$$

where $D^0_{\rho\sigma}(k) = \frac{1}{k^2}[\delta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2}]$ is the Landau-gauge free gauge-boson propagator, $\mathcal{G}(k^2)$ is a model effective interaction, and $\Gamma_\sigma(q,p)$ is the effective quark-gluon vertex. Since the chemical potential only appears explicitly in the DSE of the quark propagator, its effects on the gluon propagator and quark-gluon vertex are indirect. We expect these effects at lower order, except in the Ball-Chiu (BC) vertex ansätz [55,56], where we introduce the quark-gluon vertex modification via the nonperturbative Ward-Takahashi identity. The extended form of the BC vertex at finite chemical potential was given in Ref. [43],

$$\begin{aligned} i\Gamma^{BC}_\sigma(q,p;\mu) = {} & i\Sigma_A(q,p;\mu)\gamma^\perp_\sigma + i\Sigma_C(q,p;\mu)\gamma^\parallel_\sigma \\ & + (\tilde{q}+\tilde{p})_\sigma\Big[\frac{i}{2}\gamma^\perp\cdot(\tilde{q}+\tilde{p})\Delta_A(q,p;\mu)\Big] \\ & + (\tilde{q}+\tilde{p})_\sigma\Big[\frac{i}{2}\gamma^\parallel\cdot(\tilde{q}+\tilde{p})\Delta_C(q,p;\mu) \\ & + (\tilde{q}+\tilde{p})_\sigma\Delta_B(q,p;\mu)\Big], \end{aligned} \quad (6)$$

where $\gamma^\parallel = (\vec{0}, \gamma_4)$, $\gamma^\perp = \gamma - \gamma^\parallel$, $F = A, B, C$, and

$$\Sigma_F(q,p;\mu) = \frac{1}{2}[F(\vec{q}^2, q_4;\mu) + F(\vec{p}^2, p_4;\mu)],$$

$$\Delta_F(q,p;\mu) = \frac{F(\vec{q}^2, q_4;\mu) - F(\vec{p}^2, p_4;\mu)}{\tilde{q}^2 - \tilde{p}^2}.$$

As a comparison, we also investigate the rainbow (RB) approximation for the vertex:

$$\Gamma^{\rm RB}_\sigma(q,p;\mu) = \gamma_\sigma. \quad (7)$$

For the model effective interaction we employ two infrared-dominant models, denoted as the "GS" and "QC" models, which only express the long-range behavior of the renormalization-group-improved Maris-Tanday model [61] and the Qin-Chang (QC) model [62,63]. Though the ultraviolet parts of the models ensure the correct perturbative behavior, they are not essential in describing nonperturbative physics, e.g., the spectrum of light mesons [52,62]. Herein we neglect them since our work is based on the competition between the chiral condensate and quark number density in the infrared region [44]. The two models are expressed as

$$\mathcal{G}^{\rm GS}(k^2) = \frac{4\pi^2}{\omega^6}Dk^2{\rm e}^{-k^2/\omega^2}, \quad (8)$$

$$\mathcal{G}^{\rm QC}(k^2) = \frac{8\pi^2}{\omega^4}D{\rm e}^{-k^2/\omega^2}. \quad (9)$$

Equations (8) and (9) deliver an ultraviolet-finite model gap equation. Hence, the regularization mass scale can be removed to infinity and the renormalization constants can be set to 1. For the corresponding ansätz at finite chemical potential, we follow that in Ref. [43], neglecting the dependence of the effective interaction $\mathcal{G}$ and the gluon propagator on the chemical potential at low densities.

There are only two main parameters in our model: D and ω. We choose the set of values that can fit meson properties in vacuum well [57] or fit the chiral condensate and pion decay constant f_π in vacuum approximately. We use the approximate formula for calculating f_π which is accurate to within 5% in the chiral limit with the rainbow approximation [64,65].

$$\begin{aligned} f_\pi^2 = \int\frac{ds}{8\pi^2}N_c sB(s)^2[& \sigma(s)_V^2 - 2\sigma(s)_S\sigma(s)'_S - 2s\sigma(s)_V\sigma(s)'_V \\ & - s\sigma(s)_S\sigma(s)''_S + s(\sigma(s)'_S)^2 \\ & - s^2(\sigma(s)_V\sigma(s)''_V - (\sigma(s)'_V)^2)], \end{aligned} \quad (10)$$

with the primes denoting differentiation with respect to $s = p^2$, and

$$\sigma_V = \frac{A(p^2)}{p^2A(p^2) + B(p^2)}, \quad (11)$$

$$\sigma_S = \frac{B(p^2)}{p^2 A(p^2) + B(p^2)}. \tag{12}$$

III. THE PION-NUCLEON SIGMA TERM

It has been known that the pion-nucleon sigma term can be determined by the chiral susceptibility $\frac{\partial M_N}{\partial m_q}$ and the current-quark mass in the form of Eq. (1). However, it is very complicated to calculate the nucleon mass M_N, which depends on the four-dimensional Poincaré-invariant Faddeev equations in the DSE approach of QCD, and the results are still too robust enough to get the dependence of nucleon mass on the current-quark mass [19,26]. Meanwhile, it is oversimplified to regard the nucleon as three noninteracting constituent quarks [25,66]. Therefore, we do not perform the calculation from Eq. (1) directly.

It has been well known that, in the QCD Hamiltonian $\hat{H}_{\rm QCD}$, the mass term $\hat{H}_{\rm mass}$ is

$$\hat{H}_{\rm mass} = \int d^3x (m_u \bar{u}u + m_d \bar{d}d + \cdots), \tag{13}$$

where u and d denote up and down quarks with current-quark masses m_u and m_d, respectively, and $\cdots$ denotes the contributions from heavier quarks. It is useful to reorganize the up- and down-quark contributions to $\hat{H}_{\rm mass}$ in order to isolate the isospin-breaking effects. Defining $\bar{q}q = \frac{1}{2}(\bar{u}u + \bar{d}d)$ and $m_q = \frac{1}{2}(m_u + m_d)$, Eq. (13) can be rewritten as

$$\hat{H}_{\rm mass} = \int d^3x [2m_q \bar{q}q + \frac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d) + \cdots]. \tag{14}$$

Making use of the Hellmann-Feynman theorem, one obtains (see, for example, Ref. [67])

$$\begin{aligned} 2m_q \langle\Psi| \int d^3x \bar{q}q |\Psi\rangle &= m_q \langle\Psi| \frac{d\hat{H}_{\rm mass}}{dm_q} |\Psi\rangle \\ &= m_q \frac{d}{dm_q} E_\Psi, \end{aligned} \tag{15}$$

where $|\Psi\rangle$ represents a normalized eigenvector of the QCD Hamiltonian and E_Ψ stands for the energy of the state $|\Psi\rangle$.

Considering the case in which $|\Psi\rangle$ is the state of hadron matter at rest with baryon number density n_B, and also the case of the vacuum state, one has

$$2m_q [\langle\bar{q}q\rangle_n - \langle\bar{q}q\rangle_0] = m_q \frac{d\epsilon}{dm_q}, \tag{16}$$

where ϵ is the energy density of the baryon matter and can be written as

$$\epsilon = M_N n_B + \delta\epsilon, \tag{17}$$

where $\delta\epsilon$ denotes the contributions from the kinetic energy of baryons and baryon-baryon interactions. $\delta\epsilon$ is of high order in density and is empirically small at low densities: the binding energy per nucleon at the nuclear matter saturation density is only 16 MeV. Therefore, neglecting $\delta\epsilon$ and implementing Eq. (16), one obtains

$$2m_q [\langle\bar{q}q\rangle_n - \langle\bar{q}q\rangle_0] = m_q \frac{dM_n}{dm_q} n_B = \sigma_{\pi N} n_B. \tag{18}$$

Replacing the baryon number density n_B with the quark number density $n_q = 3n_B$, we obtain the linear dependence of the variation of the chiral condensate on the quark number density,

$$[\langle\bar{q}q\rangle_n - \langle\bar{q}q\rangle_0] = \frac{\sigma_{\pi N}}{6m_q} n_q = k n_q, \tag{19}$$

where k is the slope which can be obtained from the linear fitting of the relation in Eq. (19). Conversely, we can evaluate the pion-nucleon sigma term as

$$\sigma_{\pi N} = 6m_q k = 6m_q \frac{\langle\bar{q}q\rangle_n - \langle\bar{q}q\rangle_0}{n_q}. \tag{20}$$

For the light u and d quarks, we can take advantage of the Gell-Mann–Oakes–Renner relation [68], which is accurate within 5% [69]:

$$m_\pi^2 f_\pi^2 = -2m_q \langle\bar{q}q\rangle_0^0, \tag{21}$$

where $m_\pi = 138$ MeV and $f_\pi = 93$ MeV have been well established in experiments, and $\langle\bar{q}q\rangle_0^0$ is the chiral condensate in the chiral limit (represented by the superscript 0) in vacuum. We can then obtain

$$\sigma_{\pi N} = 3k \left(-\frac{m_\pi^2 f_\pi^2}{\langle\bar{q}q\rangle_0^0} \right), \tag{22}$$

with which the pion-nucleon sigma term $\sigma_{\pi N}$ can be evaluated from the chiral condensate in vacuum and that in medium.

The quark number density n_q can be calculated from the quark propagator at finite chemical potential, $S(p;\mu)$, with the definition

$$n_q = N_c N_f Z_2 \mathrm{Tr}[-\gamma_4 S(p;\mu)]. \tag{23}$$

For light quarks, we can further approximate the chiral condensate in Eq. (19) with that in the chiral limit, which can be well defined from the quark propagator in the chiral limit

$$-\langle\bar{q}q\rangle_n^0 = N_c Z_2 Z_m \mathrm{Tr}[S^0(p;\mu)], \tag{24}$$

TABLE I. Parameters and some characteristic numerical results (dimensional quantities are in units of MeV). DES1, DSE2, DSE3, and DSE4 are in the chiral limit $m_q = 0$, while DSE5 investigates the chiral condensate beyond the chiral limit; see the text for details.

DSE	Vertex	Interaction	ω	D	$-\langle\bar{q}q\rangle_0^{1/3}$	m_q	k	S_k	$\sigma_{\pi N}$
DSE1	RB	GS	500	1.00	252	5.2	1.95	0.03	61
DSE2	RB	QC	678	1.10	253	5.2	2.04	0.03	63
DSE3	BC	GS	500	0.50	258	4.7	2.22	0.01	63
DSE4	BC	QC	678	0.55	256	4.7	2.21	0.01	62
DSE5	RB	GS	500	1.00	–	5.2	1.94	0.05	61

where Tr represents the trace in color and Dirac space and integration in momentum space, and Z_2 and Z_m are renormalization constants for the quark wave function and quark mass, respectively.

To be more accurate in the cases of physical u, d, and even s quarks, one can take a current-quark mass that better fits the meson properties obtained from the BSE. However, Eq. (24) is divergent in the case of a finite current-quark mass. Although different subtraction points have been introduced to give finite values, it is still an open question to define the chiral condensate from the quark propagator with a finite quark mass; see, for example, Refs. [70,71]. Fortunately, we only need the variation of the chiral condensate in medium, which is independent of a fixed subtraction point. Therefore, we also investigate the variation of the chiral condensate in medium with a finite current-quark mass, defined as

$$\Delta\langle\bar{q}q\rangle_n^{m_q} = \langle\bar{q}q\rangle_n^{m_q} - \langle\bar{q}q\rangle_0^{m_q} = Z_2 Z_m \mathrm{Tr}[S(p;\mu) - S(p;\mu=0)], \quad (25)$$

where the quark propagator is calculated with a finite current-quark mass m_q.

IV. NUMERICAL CALCULATIONS AND RESULTS

To carry out the numerical calculations, we need the parameters D and ω in the effective interaction. Usually the parameters are determined by fitting meson properties with the BSE approach. The parameters and some characteristic results at $\mu = 0$ are listed in Table I. DSE1 represents the results with the rainbow approximation and the GS model. DSE2 represents the results with the rainbow approximation and the QC model. DSE3 represents the results with the BC vertex and the GS model. DSE4 represents the results with the BC vertex and the QC model. DSE5 represents the results with the rainbow approximation and the GS model, but the variation of the chiral condensate in medium is calculated with Eq. (25) beyond the chiral limit.

The parameters of DSE1 and DSE3 are taken from Ref. [57]. The parameters of DSE2 are obtained by fitting the pion decay constant $f_\pi = 93$ MeV with Eq. (10) and the chiral condensate in vacuum $-\langle\bar{q}q\rangle_0$. The parameter d of DSE4 is obtained by fitting the chiral condensate $-\langle\bar{q}q\rangle_0$ with the same ω as in DSE2. In DSE5, we take the same parameters as in DSE1.

With the above-determined parameters and the ansätz described in the last section, we solve the Dyson-Schwinger equation of the quark propagator and calculate the chemical potential dependence of the chiral condensate and the quark number density. The obtained results in the chiral limit are illustrated in Fig. 1.

Figure 1 shows that when the quark chemical potential $\mu < M_1$, where M_1 is the first constituent-mass-like pole of the quark propagator, the chiral condensate keeps the same value as that in vacuum (i.e., at $\mu = 0$) and the quark number density remains zero, i.e., the system remains the same as that in the vacuum and no matter emerges [43]. When $\mu > M_1$, the quark number density becomes nonzero and simultaneously the chiral condensate decreases gradually. This indicates that dynamical chiral symmetry is partially restored in the medium at low density [44,72]. With the above results, we get the variation of the chiral condensate $\Delta\langle\bar{q}q\rangle_n$ in the medium with respect to the quark number density n_q of the system. The obtained result is displayed in Fig. 2.

Although the chemical potential dependence of the quark number density and chiral condensate is model dependent

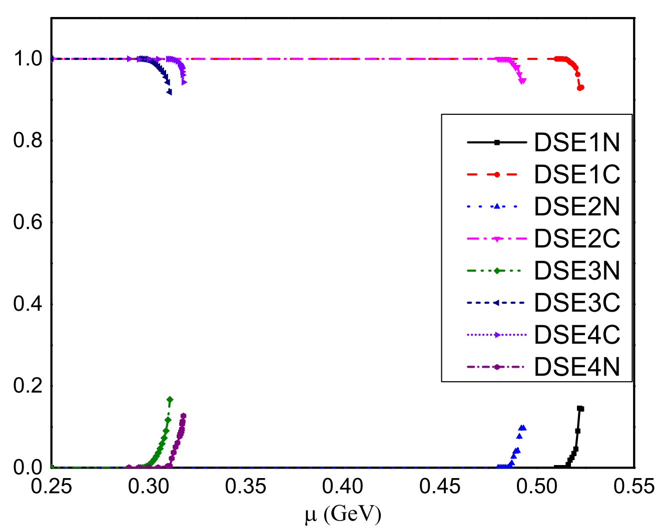

FIG. 1. Quark chemical potential dependence of the quark number density (DSE1N, DSE2N, DSE3N, DSE4N) (scaled with the saturation density $n_s = 3n_{B,s} \approx 3 \times 0.16\ \mathrm{fm}^{-3} \approx 0.0038\ \mathrm{GeV}^3$) and chiral condensate (DSE1C, DSE2C, DSE3C, DSE4C) (scaled with the values in vacuum).

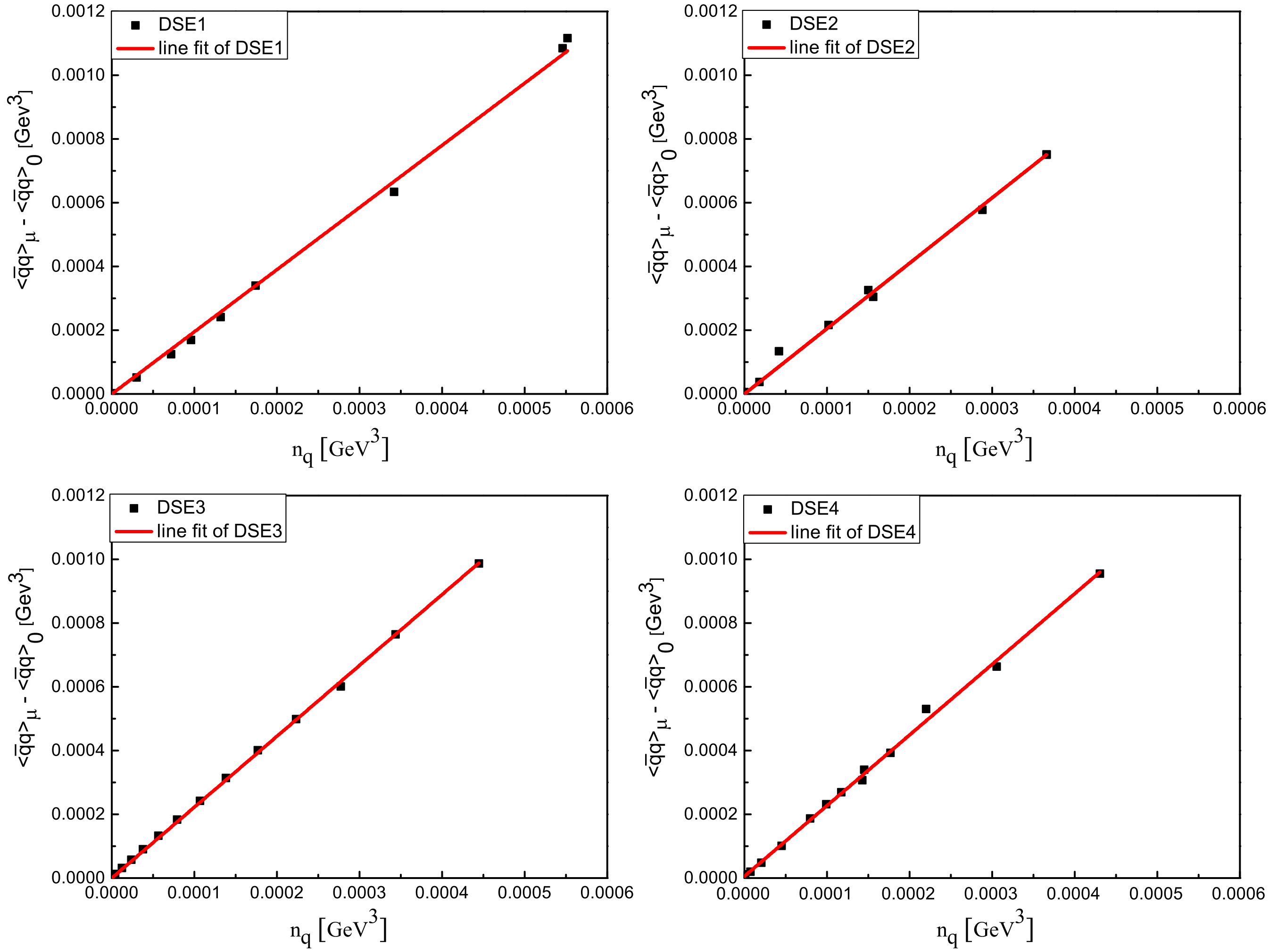

FIG. 2. Variation of the chiral condensate in medium with respect to the quark number density.

and looks complicated, Fig. 2 exhibits a linear relation [in Eq. (19)] between the variation of the chiral condensate and the baryon number density. We linearly fit the dependence of the variation of the chiral condensate (y) on the quark number density (x) as

$$y = kx, \tag{26}$$

with

$$k = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}, \tag{27}$$

where $\bar{y} = n^{-1}\sum_{i=1}^{n} y_i$ is the sample average of the y_i, and likewise for $\bar{x}$. The standard error S_k is given as

$$S_k = \sqrt{\frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{(n-2)\sum_{i=1}^{n}(x_i - \bar{x})^2}}, \tag{28}$$

where $\hat{y}_i = kx_i$. The fitted values of the slope and corresponding values of the sigma term are listed in Table I. Remarkably, the slopes and the corresponding results for $\sigma_{\pi N}$ quantitatively depend very weakly on the choice of the ansätz for the quark-gluon vertex and effective interaction.

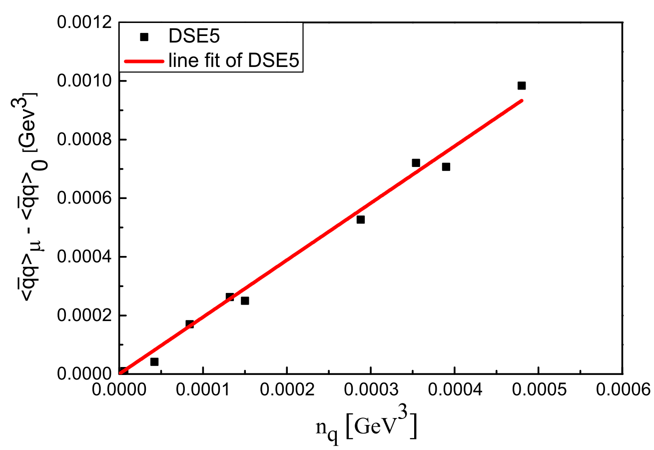

FIG. 3. Variation of the chair chiral condensate with respect to the quark number density, with the model DSE5 beyond chiral limit.

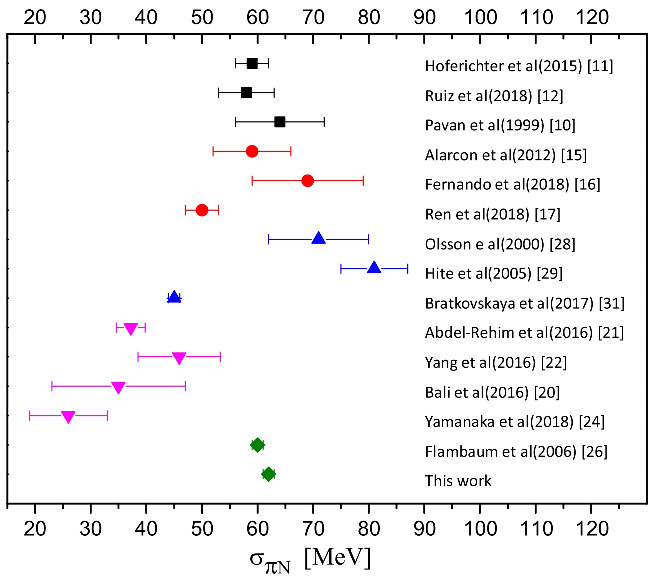

FIG. 4. Comparison of our result for the pion-nucleon sigma term $\sigma_{\pi N}$ with those in the literature, from π-N scattering data (black) [10–12], chiral perturbation theory (red) [15–17], lattice QCD (magenta) [20–22,24], various other models (blue) [28,29,31], and the DSE (green) approach [26].

In DSE5 we investigate the effect of a finite current-quark mass with Eq. (25), and one can find from Fig. 3 and Table I that the results for k and the sigma term are not sensitive to such a change. Therefore, we can be quite confident of the chiral limit approximation (22) for the pion-nucleon sigma term and the validation of Eq. (25) in the case of a finite current-quark mass.

With the above results, we estimate that the pion-nucleon sigma term $\sigma_{\pi N}$ is about $6k \approx 12$ times the current-quark mass m_q. This leads to a larger value of $\sigma_{\pi N}$ than the result given in Ref. [25], which estimated that $\sigma_{\pi N}$ in the chiral limit in the vacuum is $9/2$ times m_q. Finally, we obtain the pion-nucleon sigma term $\sigma_{\pi N} = 62(1)$ MeV, where the (1) represents the systematic error due to our different ansätz for the quark-gluon vertex and gluon propagator. Comparing our results with recent experimental data and theoretical results in Fig. 4, one can note that our present result is remarkably consistent with the recent experimental results.

V. SUMMARY AND REMARK

In summary, using the Dyson-Schwinger equation approach of QCD, we calculated the chiral condensate in strong-interaction matter at low density, and then evaluated the pion-nucleon sigma term $\sigma_{\pi N}$ via the Hellmann-Feynman theorem. In this work, we adopted various ansätz for the gluon propagator and quark-gluon vertex, and found that our evaluated value for the pion nucleon sigma term depends on the model very weakly. We obtained the result $\sigma_{\pi N} = 62(1)$ MeV in the Dyson-Schwinger equation approach of QCD. Our results are rather consistent with the relatively large value given in recent experimental analyses.

In solving the Dyson-Schwinger equation of the quark propagator, we adopted models of the effective interaction (gluon propagator) that are independent of the quark chemical potential. However, the gluon propagator should depend on the quark chemical potential via the quark-loop diagram in its vacuum polarization. It is reasonable to expect this to be a lower-order effect on the pion-nucleon sigma term. However, the situation may be different when calculating the sigma term for the strange quark σ_s, which is important for dark matter searches [7]. Since the strange quark chemical potential is zero at low baryon number densities, such an effect is the leading-order effect for the variation of the strange chiral condensate. It is then necessary to consider this effect when calculating σ_s. We could investigate it to calculate σ_s and further improve our results for $\sigma_{\pi N}$. On the other hand, the more sophisticated quark-gluon vertex has also been established (e.g., Refs. [50,73,74]). Calculating $\sigma_{\pi N}$ and σ_s with such a quark-gluon vertex is also interesting, and related work is in progress.

ACKNOWLEDGMENTS

We acknowledge financial support from the National Natural Science Foundation of China (Grant Nos. 11475149 and U1738130). We are also in debt to Yu-xin Liu at Peking University for his stimulating discussions.

[1] K. D. Bednar, I. C. Cloët, and P. C. Tandy, Phys. Lett. B **782**, 675 (2018).
[2] V. Dmitrasinovic, H.-X. Chen, and A. Hosaka, Phys. Rev. C **93**, 065208 (2016).
[3] I. C. Cloet and C. D. Roberts, Prog. Part. Nucl. Phys. **77**, 1 (2014).
[4] H. Leutwyler, *Proc. Sci.*, CD15 (2015) 022.
[5] F. Bishara, J. Brod, B. Grinstein, and J. Zupan, J. High Energy Phys. 11 (2017) 059.
[6] A. Bottino, F. Donato, N. Fornengo, and S. Scopel, Phys. Rev. D **78**, 083520 (2008).
[7] J. Giedt, A. W. Thomas, and R. D. Young, Phys. Rev. Lett. **103**, 201802 (2009).
[8] N. Yamanaka, B. K. Sahoo, N. Yoshinaga, T. Sato, K. Asahi, and B. P. Das, Eur. Phys. J. A **53**, 54 (2017).
[9] K. Yanase, N. Yoshinaga, K. Higashiyama, and N. Yamanaka, Phys. Rev. D **99**, 075021 (2019).

[10] M. M. Pavan, R. A. Arndt, I. I. Strakovsky, and R. L. Workman, PiN Newslett. **15**, 118 (1999).
[11] M. Hoferichter, J. Ruiz de Elvira, B. Kubis, and U.-G. Meißner, Phys. Rev. Lett. **115**, 092301 (2015).
[12] J. R. de Elvira, M. Hoferichter, B. Kubis, and U.-G. Meißner, J. Phys. G **45**, 024001 (2018).
[13] E. Friedman and A. Gal, Phys. Lett. B **792**, 340 (2019).
[14] J. Gasser, H. Leutwyler, and M. E. Sainio, Phys. Lett. B **253**, 252 (1991).
[15] J. M. Alarcon, J. M. Camalich, and J. A. Oller, Phys. Rev. D **85**, 051503(R) (2012).
[16] I. P. Fernando, J. M. Alarcon, and J. L. Goity, Phys. Lett. B **781**, 719 (2018).
[17] X.-L. Ren, X.-Z. Ling, and L.-S. Geng, Phys. Lett. B **783**, 7 (2018).
[18] J. R.de Elvira, M. Hoferichter, B. Kubis, and U. G. Meißner, *Proc. Sci.*, Hadron2017 (2018) 141.
[19] H. Ohki, H. Fukaya, S. Hashimoto, T. Kaneko, H. Matsufuru, J. Noaki, T. Onogi, E. Shintani, and N. Yamada, Phys. Rev. D **78**, 054502 (2008).
[20] G. S. Bali, S. Collins, D. Richtmann, A. Schafer, W. Soldner, and A. Sternbeck, Phys. Rev. D **93**, 094504 (2016).
[21] A. Abdel-Rehim, C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, C. Kallidonis, G. Koutsou, and A. V. Aviles-Casco, Phys. Rev. Lett. **116**, 252001 (2016).
[22] Y.-B. Yang, A. Alexandru, T. Draper, J. Liang, and K.-F. Liu, Phys. Rev. D **94**, 054503 (2016).
[23] L. Varnhorst (Budapest-Marseille-Wuppertal), Nucl. Part. Phys. Proc. **300–302**, 107 (2018).
[24] N. Yamanaka, S. Hashimoto, T. Kaneko, and H. Ohki, Phys. Rev. D **98**, 054516 (2018).
[25] L. Chang, Y.-X. Liu, and H. Guo, Phys. Rev. D **72**, 094023 (2005).
[26] V. V. Flambaum, A. Holl, P. Jaikumar, C. D. Roberts, and S. V. Wright, Few Body Syst. **38**, 31 (2006).
[27] N. Yamanaka, S. Imai, T. M. Doi, and H. Suganuma, Phys. Rev. D **89**, 074017 (2014).
[28] M. G. Olsson, Phys. Lett. B **482**, 50 (2000).
[29] G. E. Hite, W. B. Kaufmann, and R. J. Jacob, Phys. Rev. C **71**, 065201 (2005).
[30] F. Huang, F.-G. Cao, and B.-Q. Ma, Phys. Rev. D **76**, 114016 (2007).
[31] E. L. Bratkovskaya, A. Palmese, W. Cassing, E. Seifert, T. Steinert, and P. Moreau, J. Phys. Conf. Ser. **878**, 012018 (2017).
[32] C. D. Roberts and B. H. J. McKellar, Phys. Rev. D **41**, 672 (1990).
[33] L. Chang, Y.-X. Liu, M. S. Bhagwat, C. D. Roberts, and S. V. Wright, Phys. Rev. C **75**, 015201 (2007).
[34] K.-L. Wang, S.-X. Qin, Y.-X. Liu, L. Chang, C. D. Roberts, and S. M. Schmidt, Phys. Rev. D **86**, 114001 (2012).
[35] C. S. Fischer and J. Luecker, Phys. Lett. B **718**, 1036 (2013).
[36] C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45**, S1 (2000).
[37] S.-X. Qin, L. Chang, H. Chen, Y.-X. Liu, and C. D. Roberts, Phys. Rev. Lett. **106**, 172301 (2011).
[38] F. Gao, J. Chen, Y.-X. Liu, S.-X. Qin, C. D. Roberts, and S. M. Schmidt, Phys. Rev. D **93**, 094019 (2016).
[39] F. Gao and Y.-X. Liu, Phys. Rev. D **94**, 094030 (2016).
[40] F. Gao and Y.-X. Liu, Phys. Rev. D **94**, 076009 (2016).
[41] C. S. Fischer, Prog. Part. Nucl. Phys. **105**, 1 (2019).
[42] W. Yuan, H. Chen, and Y.-X. Liu, Phys. Lett. B **637**, 69 (2006).
[43] H. Chen, W. Yuan, L. Chang, Y.-X. Liu, T. Klahn, and C. D. Roberts, Phys. Rev. D **78**, 116015 (2008).
[44] H. Chen, W. Yuan, and Y.-X. Liu, J. Phys. G **36**, 064073 (2009).
[45] H. Chen, M. Baldo, G. F. Burgio, and H. J. Schulze, Phys. Rev. D **84**, 105023 (2011).
[46] H. Chen, J. B. Wei, and H. J. Schulze, Eur. Phys. J. A **52**, 291 (2016).
[47] Z. Bai, H. Chen, and Y.-x. Liu, Phys. Rev. D **97**, 023018 (2018).
[48] P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12**, 297 (2003).
[49] L. Chang and C. D. Roberts, Phys. Rev. Lett. **103**, 081601 (2009).
[50] L. Chang, Y.-X. Liu, and C. D. Roberts, Phys. Rev. Lett. **106**, 072001 (2011).
[51] S.-X. Qin, L. Chang, Y.-X. Liu, C. D. Roberts, and D. J. Wilson, Phys. Rev. C **85**, 035202 (2012).
[52] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, and C. S. Fischer, Prog. Part. Nucl. Phys. **91**, 1 (2016).
[53] F. Gao, L. Chang, and Y.-X. Liu, Phys. Lett. B **770**, 551 (2017).
[54] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, and C. S. Fischer, Prog. Part. Nucl. Phys. **91**, 1 (2016).
[55] J. S. Ball and T.-W. Chiu, Phys. Rev. D **22**, 2542 (1980).
[56] J. S. Ball and T.-W. Chiu, Phys. Rev. D **22**, 2550 (1980).
[57] L. Chang and C. D. Roberts, Phys. Rev. Lett. **103**, 081601 (2009).
[58] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84**, 1232 (1951).
[59] C. S. Fischer and R. Williams, Phys. Rev. Lett. **103**, 122001 (2009).
[60] D. Binosi, L. Chang, J. Papavassiliou, and C. D. Roberts, Phys. Lett. B **742**, 183 (2015).
[61] P. Maris and P. C. Tandy, Phys. Rev. C **60**, 055214 (1999).
[62] R. Alkofer, P. Watson, and H. Weigel, Phys. Rev. D **65**, 094026 (2002).
[63] S. X. Qin, L. Chang, Y. X. Liu, C. D. Roberts, and D. J. Wilson, Phys. Rev. C **84**, 042202(R) (2011).
[64] D. W. McKay, H. J. Munczek, and B.-L. Young, Phys. Rev. D **37**, 195 (1988).
[65] T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D **35**, 774 (1987).
[66] H. Fujii and Y. Tsue, Phys. Lett. B **357**, 199 (1995).
[67] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, Phys. Rev. C **45**, 1881 (1992).
[68] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).
[69] M. Jamin, Phys. Lett. B **538**, 71 (2002).
[70] P. Maris and C. D. Roberts, Phys. Rev. C **56**, 3369 (1997).
[71] S. J. Brodsky, C. D. Roberts, R. Shrock, and P. C. Tandy, Phys. Rev. C **82**, 022201(R) (2010).
[72] L. Chang, H. Chen, B. Wang, W. Yuan, and Y.-X. Liu, Phys. Lett. B **644**, 315 (2007).
[73] S.-X. Qin, L. Chang, Y.-X. Liu, C. D. Roberts, and S. M. Schmidt, Phys. Lett. B **722**, 384 (2013).
[74] C. Tang, F. Gao, and Y.-X. Liu, Phys. Rev. D **100**, 056001 (2019).