

Mixing effects of $\Sigma^0 - \Lambda^0$ in Λ_c^+ decays

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(Received 17 September 2019; accepted 19 February 2020; published 10 March 2020)

We analyze the mixing between Σ^0 and Λ^0 based on the baryon masses. We distinguish the contributions from QCD and QED in the baryon mass splittings. We find that the mixing angle between Σ^0 and Λ^0 is $(2.07 \pm 0.03) \times 10^{-2}$, which leads to the decay branching fraction and up-down asymmetry of $\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e$ to be $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e) = (1.5 \pm 0.2) \times 10^{-5}$ and $\alpha(\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e) = -0.86 \pm 0.04$, respectively. Moreover, we obtain that $\Delta\mathcal{B} \equiv \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+) - \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) = (3.8 \pm 0.5) \times 10^{-4}$ and $\Delta\alpha \equiv \alpha(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+) - \alpha(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) = (-1.6 \pm 0.7) \times 10^{-2}$, which should vanish without the mixing.

DOI: [10.1103/PhysRevD.101.054005](https://doi.org/10.1103/PhysRevD.101.054005)

I. INTRODUCTION

The flavor content is one of the cornerstones in particle physics. The hadrons are often categorized and named in terms of their flavor states. For instance, the hadrons in the isospin group, such as pions, share the same name. Since the QCD energy scale is much larger than the mass difference of the hadrons with the same isospin, it is believed that the hadrons also have similar wave functions. The isospin group can be extended to $SU(3)$ by including the strange quark, which is the so-called $SU(3)_F$ flavor symmetry and has been widely used in particle physics.

Based on the $SU(3)_F$ symmetry, the precision for the Gell-Mann-Okubo (GMO) mass formula is around one percent, indicating that the baryons indeed share the similar wave functions. Another well-known mass formula based on $SU(3)_F$ for the octet baryons is the Coleman-Glashow (CG) one [1]. Note that the masses of the octet baryons have been intensively studied in the calculations of the lattice QCD (LQCD) [2–6] as well as the theoretical models with the baryon wave functions [7–13].

The octet baryon states of Σ^0 and Λ^0 have the same quark components of uds . Originally, they are categorized by the isospin property with Σ^0 and Λ^0 being the triplet and singlet states under the $SU(2)_I$ isospin symmetry, respectively. This categorization is based on that the isospin symmetry is much better than $SU(3)_F$. However, both $SU(2)_I$ and

$SU(3)_F$ are not exact, resulting in a possible mixing between Σ^0 and Λ^0 . The physical baryons are made of the mixing of isospin triplet and singlet states. In general, the mixing angle is estimated to be the ratio of the $SU(2)_I$ and $SU(3)_F$ breaking energy scales. Note that the calculation in the LQCD gives the mixing angle $\theta = 0.006 \pm 0.003$ [14].

Recently, the BESIII Collaboration has announced the up-down asymmetries for $\Lambda_c^+ \rightarrow \Sigma\pi$, given by [15]

$$\begin{aligned}\alpha(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) &= -0.73 \pm 0.18, \\ \alpha(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+) &= -0.57 \pm 0.12.\end{aligned}\quad (1)$$

Note that the corresponding branching ratios have been measured to be [16]

$$\begin{aligned}\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) &= (1.24 \pm 0.10)\%, \\ \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+) &= (1.29 \pm 0.07)\%.\end{aligned}\quad (2)$$

However, in the limit of the isospin symmetry, both asymmetries in Eq. (1) and branching ratios in Eq. (2) should be equal. On the other hand, the semileptonic decay of $\Lambda_c^+ \rightarrow \Sigma^0 l^+ \nu_l$ is forbidden since Λ_c^+ and Σ^0 belong to different isospin representations. In this study, we explore the isospin breaking effect in the $\Sigma^0 - \Lambda^0$ mixing and discuss the effects in Λ_c^+ decays.

Our paper is organized as follows. In Sec. II, we introduce the hadron representations under the $SU(3)_F$ flavor group. The mixing effects in Λ_c^+ decays are studied in Sec. III. We present our conclusions in Sec. IV.

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II. HADRON REPRESENTATIONS IN $SU(3)_F$

In terms of $SU(3)_F$, the matrix representations of the octet baryons can be written as [17]

$$(\mathbf{B}_n)_j^i = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda^0 + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix}_{ij}, \quad (3)$$

where the prime \prime denotes the unmixed state. For instance, the proton's matrix representation and state correspond to $(p)_j^i = \delta^{i1}\delta_{j3}$ and $(p)_{j|i}^i = |1^3\rangle$, respectively.

In the standard model, the $SU(3)_F$ symmetry is broken by the quark masses as well as the electromagnetic interaction. The matrix representations of the light quark masses and electric charges of the quark flavors are given as

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} - \frac{1}{3}(m_u + m_d + m_s) \mathbf{1}$$

and $Q = \frac{1}{3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad (4)$

respectively. Note that both M and Q belong to $\mathbf{8}$ under the $SU(3)_F$ group. Consequently, the baryon mass must be the function of M and Q , given by

$$M_B = \langle B|H(M, Q)|B\rangle, \quad (5)$$

where H and $|B\rangle$ are the mass operator and state of the octet baryon, respectively.

Naively, one may write down the baryon mass operator as

$$H(M, Q) = H^0 + H_m^1(M) + H_q^1(Q) + O(H^2), \quad (6)$$

where the superscripts of $n = 0, 1, 2$ stand for the n -order approximations and the subscripts of m and q imply the breaking sources of the $SU(3)_F$ symmetry. However, the second-order correction from the strange quark mass can be the same size as the first-order one from the up and down quark masses, e.g., $(m_s/\mu_H)^2 \sim m_q/\mu_H$, where μ_H is the typical hadronic scale.

A better way to do the approximation is to categorize the breaking effects according to their symmetry properties instead of the sources. We rewrite Eq. (4) as

$$M = m_s T_8 + m_{\bar{q}} T_3 \quad \text{and} \quad Q = \frac{1}{6} T_8 + \frac{1}{2} T_3, \quad (7)$$

with

$$T_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (8)$$

where $m_{\bar{s}} = -(2m_s - m_u - m_d)/6$ and $m_{\bar{q}} = (m_u - m_d)/2$. Here, we have decomposed the matrix representation into two different parts. Note that T_8 is invariant under the isospin transformation, whereas T_3 is not. Accordingly, the baryon mass operator in Eq. (6) is given by

$$\begin{aligned} H(M, Q) &= H^0 + H_m^1(m_s T_8 + m_{\bar{q}} T_3) \\ &\quad + H_q^1\left(\frac{1}{6} T_8 + \frac{1}{2} T_3\right) + O(H^2) \\ &= H^0 + \left(H_m^1(m_s T_8) + H_q^1\left(\frac{1}{6} T_8\right)\right) \\ &\quad + \left(H_m^1(m_{\bar{q}} T_3) + H_q^1\left(\frac{1}{2} T_3\right)\right) + O(H^2) \\ &= H^0 + H_s^1(T_8) + H_l^1(T_3) + O(H^2), \end{aligned} \quad (9)$$

where we have used that the matrix representation is linear and the operators in the parentheses have the same representation, respectively. Since H_s^1 contains the correction from m_s , it is much larger than H_l^1 . On the other hand, the second-order correction in $O(H^2)$ caused by m_s can be the same order as H_l^1 . Explicitly, we have the hierarchy, given by

$$H^0 \gg H_s^1 \gg H_l^1 \sim O(H^2). \quad (10)$$

Note that the correction from m_s is invariant under the isospin transformation. If $O(H^2)$ is neglected in the calculation of H_l^1 , it is only reasonable to deal with the physical quantities, which are not affected by the correction from m_s .

Notice that the baryon wave functions in Eq. (3) are chosen as the eigenvectors of $H^0 + H_s^1$. Explicitly, they have the following properties:

$$\langle B'|H^0 + H_s^1|B\rangle = \langle B'|B\rangle = 0 \quad \text{if } B' \neq B. \quad (11)$$

However, the physical baryon states are the eigenvectors of the full baryon mass operator instead. In general, the isospin breaking term H_l^1 has a nonzero matrix element between Σ^0 and Λ^0 , i.e., $\langle \Sigma^0|H_l^1|\Lambda^0\rangle \neq 0$, while the correction due to m_s has no contribution to this matrix element due to the isospin symmetry. The physical baryon condition is given by

$$\langle B'_p|H|B_p\rangle = 0 \quad \text{if } B'_p \neq B_p, \quad (12)$$

TABLE I. Mass differences (MeV) in the octet baryons and $SU(3)_F$ parametrizations with the experimental data and our results from QCD and QED.

Mass differences	$SU(3)_F$ parameters	Data [16]	QCD	QED
$M_{\Sigma^-} - M_{\Sigma^+}$	$2I_1 - 2I_2$	8.08 ± 0.08	8.59 ± 0.01	-0.51 ± 0.08
$M_{\Sigma^-} - M_{\Sigma^0}$	$I_1 - I_2$	4.81 ± 0.04^a	4.30 ± 0.00	-0.26 ± 0.04
$M_n - M_p$	$-2I_2$	1.29 ± 0.00	2.84 ± 0.00	-1.54 ± 0.00
$M_{\Xi^-} - M_{\Xi^0}$	$2I_1$	6.85 ± 0.21	5.76 ± 0.00	1.04 ± 0.08

^aWe have used the approximation of $M_{\Sigma^0} = M_{\Sigma^0}$.

with [18,19]

$$\begin{aligned} |\Lambda^0\rangle &= \cos\theta|\Lambda^{0'}\rangle - \sin\theta|\Sigma^{0'}\rangle \\ \text{and } |\Sigma^0\rangle &= \cos\theta|\Sigma^{0'}\rangle + \sin\theta|\Lambda^{0'}\rangle, \end{aligned} \quad (13)$$

where P denotes the physical baryon and θ is the mixing angle to be determined through the octet baryon masses.

The baryon masses are evaluated from Eqs. (5) and (9). With the $SU(3)_F$ symmetry described in the beginning of the previous section, we can apply the Wigner-Eckart theorem. Consequently, the matrix element in Eq. (5) is parametrized as

$$\begin{aligned} \langle B'|H^0 + H_S^1|B\rangle \\ = M_0 + S_1\text{Tr}(\overline{\mathbf{B}}'_n T_8 \mathbf{B}_n) + S_2\text{Tr}(\overline{\mathbf{B}}'_n \mathbf{B}_n T_8), \end{aligned} \quad (14)$$

$$\begin{aligned} \langle B'|H_I^1 + O(H^2)|B\rangle \\ = I_1\text{Tr}(\overline{\mathbf{B}}'_n T_3 \mathbf{B}_n) + I_2\text{Tr}(\overline{\mathbf{B}}'_n \mathbf{B}_n T_3) + O(H^2), \end{aligned} \quad (15)$$

where M_0 , S_i , and I_i are the $SU(3)_F$ parameters to be extracted through the experiments, $O(H^2)$ is the higher order correction, and $\overline{\mathbf{B}}_n \equiv \mathbf{B}_n^\dagger$. In fact, Eq. (14) corresponds to the GMO mass formula, while Eq. (15) can give us the CG one. To get a consistent result, the parameters of I_1 and I_2 are determined with the mass differences between the isospin related baryons, while M_0 , S_1 , and S_2 are fixed with the average baryon masses within the same isospin subgroup.

There is one triplet (Σ), two doublets (Ξ and N), and one singlet (Λ^0) under $SU(2)_I$ in the octet baryons. Their experimental mass differences and $SU(3)_F$ parametrizations are summarized in Table I, where the results from QCD and QED are given at the end of this section. We have rounded the experimental data to the second decimal place due to that the $SU(3)_F$ symmetry is only an approximation. The higher precision is not expected.

From Table I, one can easily obtain the CG mass formula, given by

$$(M_{\Sigma^-} - M_{\Sigma^+}) - (M_n - M_p) = M_{\Xi^-} - M_{\Xi^0}, \quad (16)$$

along with an additional one, given by

$$\frac{1}{2}(M_{\Sigma^-} - M_{\Sigma^+}) = M_{\Sigma^-} - M_{\Sigma^0}. \quad (17)$$

If one assumes that the baryon mass operator is invariant under $SU(2)_I$, the CG mass formula is trivial since the baryons within the same isospin subgroup should have the same mass. Consequently, the CG mass formula is often interpreted as the equation between the isospin mass differences. In addition, it can be related to the correction in the U-spin symmetry of the down and strange quarks as well as due to the rearrangement of

$$(M_{\Sigma^-} - M_{\Xi^-}) + (M_p - M_{\Sigma^+}) + (M_{\Xi^0} - M_n) = 0, \quad (18)$$

where (Σ^-, Ξ^-) , (p, Σ^+) , and (Ξ^0, n) belong to the same U-spin subgroup. In fact, Eq. (18) is trivial in the original derivation in Ref. [1] based on the mass operator being only a function of Q , which is invariant under the U-spin transformation, e.g., $M_{\Sigma^-} = M_{\Xi^-}$, $M_p = M_{\Sigma^+}$ and $M_{\Xi^0} = M_n$. Moreover, the CG mass formula can also be related to the correction under the V-spin symmetry of the up and strange quarks with the rearrangement of

$$(M_{\Sigma^-} - M_n) + (M_p - M_{\Xi^-}) + (M_{\Xi^0} - M_{\Sigma^+}) = 0, \quad (19)$$

where (Σ^-, n) , (p, Ξ^-) , and (Ξ^0, Σ^+) have the same V-spin representations. Likewise, Eq. (19) is trivial if the V-spin symmetry is exact in the baryon mass operator. To sum up, the CG mass formula is automatically satisfied if one of the $SU(2)$ subgroups in $SU(3)_F$ is preserved.

From the experimental data of $M_{\Sigma^-} - M_{\Sigma^+}$ and $M_n - M_p$, the parameters I_1 and I_2 in Eq. (15) can be found to be

$$\begin{aligned} I_1 &= (3.40 \pm 0.04) \text{ MeV} \quad \text{and} \\ I_2 &= (-0.65 \pm 0.00) \text{ MeV}. \end{aligned} \quad (20)$$

Let us focus on H_S^1 . To get consistent results cooperated with H_I^1 , we use the average masses in the isospin subgroups, defined as $M_\Sigma = (M_{\Sigma^+} + M_{\Sigma^-})/2$, $M_N = (M_p + M_n)/2$, and $M_\Xi = (M_{\Xi^-} + M_{\Xi^0})/2$. The famous GMO mass formula can be obtained through Eq. (14), read as

$$M_{\Lambda^0} \approx M_{\Lambda^+} = \frac{1}{3}[2(M_{\Xi} + M_N) - M_{\Sigma}]. \quad (21)$$

Experimentally, the left- and right-hand sides of Eq. (21) are given by 1115.68 ± 0.01 and 1107.00 ± 0.07 MeV, respectively. In addition, we get

$$\begin{aligned} M_0 &= (1150.21 \pm 0.04) \text{ MeV}, \\ S_1 &= (84.83 \pm 0.01) \text{ MeV}, \\ S_2 &= (-41.63 \pm 0.04) \text{ MeV} \end{aligned} \quad (22)$$

in Eq. (14), where the experimental data of M_{Σ} , M_N , and M_{Ξ} have been used.

If the QED effect is ignored, the baryon mass depends on the quark mass matrix only, resulting in that

$$M_{\text{QCD}} = M_0 + M_1 \text{Tr}(\bar{\mathbf{B}}'_n \mathbf{M} \mathbf{B}_n) + M_2 \text{Tr}(\bar{\mathbf{B}}'_n \mathbf{B}_n \mathbf{M}) + O(H^2), \quad (23)$$

where M_1 and M_2 are the unknown parameters. By using Eqs. (7), (14), and (15), we obtain that

$$\left(\frac{I_1}{I_2}\right)_{\text{QCD}} = \left(\frac{S_1}{S_2}\right)_{\text{QCD}}, \quad (24)$$

where the subscript of QCD indicates that the QED effect is ignored. Since S_1 and S_2 are dominated by the strange quark mass correction, we can safely approximate Eq. (24) as

$$\begin{aligned} \left(\frac{M_{\Xi^-} - M_{\Xi^0}}{M_n - M_p}\right)_{\text{QCD}} &= -\left(\frac{I_1}{I_2}\right)_{\text{QCD}} = -\left(\frac{S_1}{S_2}\right)_{\text{QCD}} \\ &\approx -\frac{S_1}{S_2} = 2.038 \pm 0.002. \end{aligned} \quad (25)$$

The calculation in the LQCD is indeed consistent with Eq. (25), where the ratio is around $1.6 \sim 2.2$ [2–6]. In reality, I_1/I_2 has the value of -5.25 ± 0.06 , which clearly indicates that $I_1/I_2 \neq (I_1/I_2)_{\text{QCD}}$. Moreover, from Eq. (23), we have

$$\left(\frac{I_1}{S_1}\right)_{\text{QCD}} = \left(\frac{I_2}{S_2}\right)_{\text{QCD}} = \frac{m_{\bar{q}}}{m_{\bar{s}}} = 0.034, \quad (26)$$

where the mass ratios among the light quarks in Ref. [20] have been used. With Eqs. (20), (22), and (26) along with $I_i = (I_i)_{\text{QCD}} + (I_i)_{\text{QED}}$, we can separate the contributions to the mass differences of the isospin breakings in QCD and QED as listed in Table I. The results are fairly close to those in the literature based on LQCD and QED [4–6].

III. Λ_c^+ DEACYS WITH $\Sigma^0 - \Lambda^0$ MIXING

The mixing angle is determined by [18]

$$\tan \theta = \langle \Sigma^0 | H_I^1 | \Lambda^0 \rangle / (M_{\Sigma^0} - M_{\Lambda^0}), \quad (27)$$

where the matrix element is given as

$$\langle \Sigma^0 | H_I^1 | \Lambda^0 \rangle = \frac{1}{\sqrt{3}}(I_1 + I_2) = 1.59 \pm 0.02 \text{ MeV}. \quad (28)$$

By using $I_1 + I_2 = (M_{\Sigma^0} - M_{\Sigma^+}) - (M_n - M_p)$, our formula is the same as the one in Ref. [19], in which only the mixing through the electromagnetic interaction is considered. Consequently, we obtain the mixing angle

$$\theta = (2.07 \pm 0.03) \times 10^{-2}. \quad (29)$$

Although the mixing has little effect on the baryon masses, it plays a significant role in the decays associated with the isospin symmetry. We now study the mixing effects in the semileptonic and nonleptonic charmed baryon decays. In general, the states of Λ_c^+ and Σ_c^+ , corresponding to $I = 0$ and $I = 1$, are also mixed. However, the mixing is suppressed by the charmed quark mass [21]. As a result, it is ignored in this work.

Since Λ_c^+ (Σ^0) is antisymmetric (symmetric) in up and down quarks, the decay channel of $\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e$ without the mixing is forbidden. The ratio between the decay rates of $\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$ and $\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e$ is given by

$$\frac{\Gamma(\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e)}{\Gamma(\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e)} = \tan^2 \theta = (4.3 \pm 0.1) \times 10^{-4}, \quad (30)$$

where we have approximated that $M_{\Sigma^0} \approx M_{\Lambda^0}$ in the kinematic phase space. In addition, the angular distributions of the Σ^0 and Λ^0 modes should be the same. With the experimental data for $\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$ [16] and the mixing angle in Eq. (29), the branching ratio and up-down asymmetry of $\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e$ are given by

$$\begin{aligned} \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e) &= (1.5 \pm 0.2) \times 10^{-5} \\ \text{and } \alpha(\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e) &= -0.86 \pm 0.04, \end{aligned} \quad (31)$$

respectively.

We explore the nonleptonic charmed baryon decays of $\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$ and $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$. If there is no mixing between Λ^0 and Σ^0 , two decays should have the same decay width and up-down asymmetry parameter, given by [22]

$$\Gamma = \frac{p_\Sigma}{8\pi} \left(\frac{(m_{\Lambda_c^+} + m_\Sigma)^2 - m_\pi^2}{m_{\Lambda_c^+}^2} |A|^2 + \frac{(m_{\Lambda_c^+} - m_\Sigma)^2 - m_\pi^2}{m_{\Lambda_c^+}^2} |B|^2 \right),$$

$$\alpha = \frac{2\kappa \text{Re}(A^* B)}{|A|^2 + \kappa^2 |B|^2}, \quad \kappa = \frac{p_\Sigma}{E_\Sigma + m_\Sigma}, \quad (32)$$

respectively, where A and B are associated with the P and S wave amplitudes, and m_i , p_i , and E_i are the mass, momentum, and energy for the i th hadron in the center mass frame, respectively. On the other hand, the P wave amplitude for $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$ can be written as

$$A(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+) = \cos \theta A(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+) + \sin \theta A(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+) = -\cos \theta A(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) + \sin \theta A(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+), \quad (33)$$

where the second equation is given by the isospin symmetry [22]. The S wave amplitude can be given by replacing A with B in Eq. (33). From the experimental values of $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) = (1.24 \pm 0.10)\%$ and $\alpha(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) = -0.73 \pm 0.18$ [15,16], we obtain that¹

$$A(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) = (5.7 \pm 0.4) 10^{-2} G_F \text{ GeV}^2, \\ B(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) = (-7.9 \pm 2.4) 10^{-2} G_F \text{ GeV}^2, \quad (34)$$

with the correlation of $R = 0.73$. Similarly, with the data of $\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+) = (1.30 \pm 0.07)\%$ and $\alpha(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+) = -0.80 \pm 0.11$, we find that¹

$$A(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+) = (-2.9 \pm 0.5) 10^{-2} G_F \text{ GeV}^2, \\ B(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+) = (16.7 \pm 0.9) 10^{-2} G_F \text{ GeV}^2, \quad (35)$$

with the correlation of $R = 0.78$.

According to Eqs. (33)–(35), the isospin breaking effects caused by the mixing are given by

¹There are four different solutions for A and B with given values of \mathcal{B} and α . We have chosen the solution that is consistent with the $SU(3)_F$ analysis [22].

$$\Delta \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma \pi) \equiv \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+) - \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) = (3.8 \pm 0.5) \times 10^{-4},$$

$$\Delta \alpha(\Lambda_c^+ \rightarrow \Sigma \pi) \equiv \alpha(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+) - \alpha(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) = (-1.6 \pm 0.7) \times 10^{-2}, \quad (36)$$

which are consistent with the current experimental data, given by $(5 \pm 12) \times 10^{-4}$ and $(16 \pm 22) \times 10^{-2}$ [16], respectively. Since the data are also consistent with 0, it is clear that future experiments with higher accuracy are needed.

The mixing effects in $\Lambda_c^+ \rightarrow \Sigma \pi$ are in the first order of θ , while those in $\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e$ and $\Lambda_b^0 \rightarrow \Sigma^0 J/\psi$ [23] are in the second one. Clearly, the experiments in $\Lambda_c^+ \rightarrow \Sigma \pi$ are more promising for searching the $\Sigma^0 - \Lambda^0$ mixing.

IV. CONCLUSIONS

We have analyzed the mass differences of the octet baryons. We have identified the contribution from QED in the baryon mass splittings. From the baryon masses we have found that the mixing angle between Σ^0 and Λ^0 is $\theta = (2.07 \pm 0.03) \times 10^{-2}$. The possibility of observing such a mixing in the Λ_c^+ decays has been discussed. In particular, we note that the decay channel of $\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e$ is forbidden if $\theta = 0$. With the mixing, the decay branching ratio and up-down asymmetry in $\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e$ are given by $(1.5 \pm 0.2) \times 10^{-5}$ and -0.86 ± 0.04 , respectively. In the nonleptonic decays, we have demonstrated that the mixing causes slight differences for the physical observables between $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$ and $\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$. Explicitly, we have shown that $\Delta \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma \pi) = (3.8 \pm 0.5) \times 10^{-4}$ and $\Delta \alpha(\Lambda_c^+ \rightarrow \Sigma \pi) = (-1.6 \pm 0.7) \times 10^{-2}$, respectively. Future experimental searches for $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 e^+ \nu_e)$, $\Delta \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma \pi)$, and $\Delta \alpha(\Lambda_c^+ \rightarrow \Sigma \pi)$ are recommended. Nonvanishing values of these observables can be the evidences of the $\Sigma^0 - \Lambda^0$ mixing.

ACKNOWLEDGMENTS

This work was supported in part by National Center for Theoretical Sciences and MoST (Grant No. MoST-107-2119-M-007-013-MY3).

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