Obtaining a scalar fifth force via a symmetry-breaking coupling between the scalar field and matter

Hai-Chao Zhang[*]

Key Laboratory for Quantum Optics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China

(Received 19 October 2018; accepted 16 January 2020; published 12 February 2020)

A matter-coupled scalar field model is presented to obtain a scalar fifth force when the constraint of the current cosmological constant is satisfied. The interaction potential energy density between the scalar field and matter has a symmetry-breaking form with two potential wells. We prove that the cosmological constant is a value of the scalar field's self-interaction potential energy density at the minimum of the effective matter-density-dependent potential energy density. The effective potential is a sum of the interaction potential and the self-interaction potential of the scalar field. The scalar field can stably sit at the minimum; then the time-dependent cosmological "constant" behaves like a constant. The scheme does not conflict with chameleon no-go theorems. However, since the quintessence is trapped by one of the interaction potential wells, the observed cosmic acceleration can be accounted for by the scalar field. The scalar field is also extrapolated to account for inflation at the inflationary era of the Universe. In this era matter fluid is relativistic, and the interaction potential wells vanish. The unconfined quintessence therefore dominates the evolution of the Universe. We conclude that Planck 2018 results favor the closed space of the Universe. The reasons for this are the measured value of the current Hubble constant and the observation of a concave potential in the framework of single-field inflationary models. By invoking a pseudopotential in the inflationary era, the concave feature can be attributed to the pseudopotential, although the self-interaction potential is a convex function. The pseudopotential is defined by a sum of the self-interaction potential and the energy density scale of the curvature of the Universe. The positive curvature leads to the concave feature of the pseudopotential. Within the constraints of the cosmological constants, including the maximum cosmological constant in the inflationary era, we find that the strength of the fifth force is large compared with gravity. Due to the short range of the interaction, the local test of gravity is satisfied. The coupling coefficient denoting the force strength is inversely proportional to the ambient density, while the interaction range is inversely proportional to the square root of the density. For the current matter density $\sim 10^{-27}$ kg/m^3 of the Universe, the corresponding interaction range is ~ 5 μm and the coupling coefficient is $\sim 10^{31}$. Since the fifth force is localized in an extreme thin shell, experiments might be designed so that the test objects can pass through the thin shell.

DOI: 10.1103/PhysRevD.101.044020

I. INTRODUCTION

The acceleration of the cosmic expansion has now been firmly established [1,2], and the cosmological parameters are constrained at the subpercent level [3,4]. A possible origin of this repulsive gravitational effect is that of new scalar fields coupling to matter [5–10]. According to quantum field theory, the coupled scalar fields could produce new fifth forces [11–16]. However, this setting still lacks specification for how to depict the fifth forces in a precise mathematical mode with the constraint of the cosmological observations and laboratory experiments, such as the cosmological constant, the ratio of matter density to the total energy density in the Universe, the precision measurements of hydrogenic energy levels [17], etc. Since the fifth forces have not yet been observed in the laboratory [18] or in solar system experiments, modified gravity models—such as scalar field theories, including chameleon [6], symmetron [8,9], and dilaton [10]—introduce screening mechanisms to suppress the coupling strength and/or the interaction range through dense environments. The initial motivation for introducing scalar fields to understand dark energy, especially to naturally obtain the cosmological constant, remain open questions in physics.

The idea of quintessence for solving the problem of the cosmological constant is that the potential energy of a single scalar field dynamically relaxes with time [19–22]. The argument is that, since our Universe is old enough, the cosmological constant becomes smaller than its "natural" value in the Planck energy scale [23–26]. Owing to its

[*]zhanghc@siom.ac.cn

dynamic property [27–29], this scenario is considered as one of the possible models to overcome Weinberg's no-go theorem [30,31]. However, the problem is as follows [32]: If the dark energy evolves slowly on the cosmological time scale, the requisite potential of the scalar field is often regarded as very shallow. The shallowness implies that the mass of the scalar field is smaller than $\hbar H_0 \sim 10^{-33}$ eV, with H_0 denoting the current Hubble constant. Then the scalar field leads to a long-range interaction [33] if it couples to ordinary matter. The absence of an observable interaction and the constraint of the equivalence principle imply the existence of some suppressing mechanism. One even note that the scalar field does not couple to baryons but only to dark matter.

The other natural value of the cosmological constant is zero [5]. The coupled scalar field theory argues that the coupling of the scalar field to matter may lead to its potential energy dynamically evolving from zero to the observed cosmological constant [10,34]. For a chameleonlike scalar field (e.g., symmetron and varying-dilaton), chameleon no-go theorems [35,36] have been proven based on the assumption that the strength of the chameleonlike force is comparable to gravity. Unfortunately, for chameleon no-go theorems, a misleading corollary is extensively accepted—that a chameleonlike scalar field cannot account for the cosmic acceleration except as some form of dark energy [35–37]. This seems to imply that the chameleonlike model cannot simultaneously screen and drive dark energy.

The misleading corollary results from an additional requirement that the scalar field should mediate a long-range interaction in low-density regions [38] (e.g., the current density of the Universe). The restrictive requirement may come from the consideration that the field should be able to enact cosmological effects (such as the acceleration of the cosmic expansion) through the long-range interaction. From the ΛCDM model [39], we know that it is the cosmological constant that drives the Universe acceleration rather than any long-range force [40]. When a scalar field is used to explain the cosmological constant, it is worth noting that the potential density of the scalar field links to the cosmological constant rather than a long-range force. Although the existence of a scalar field will generate a scalar fifth force, the effect of the potential density is not equivalent to the effect of the scalar fifth force. Therefore, the requirement that the scalar field should be light to mediate a long-range interaction is not necessary.

The other recognition also takes effect in deriving the misleading corollary. One of the chameleon no-go theorems precludes the possibility of self-acceleration over the last Hubble time. However, this preclusion cannot be used to rule out the other chameleonlike model to mimic the cosmological constant if quintessence or vacuum energy can naturally emerge in the model.

We need to pay much more attention to the following: the extremely small fifth force in the current precision tests of gravity [38], the cosmological constant, and inflation of the Universe. If the fifth force really exists, the lesser effect of the observable interaction [38] means an extremely short interaction range and/or an extremely low strength.

There is much literature about evading the chameleon no-go theorems. It has been suggested to use a symmetry-breaking self-interaction potential as a phase transition switch and another scalar field to drive dark energy [41]. By introducing scalar-field-dependent masses of neutrinos [34], it has been proven that the potential of the scalar field becomes positive (from its initial zero value) to drive the Universe's acceleration. But, why does not the mechanism of mass-varying neutrinos apply to baryons? The reason may be the same as mentioned above: to avoid gravitational problems such as a long-range fifth force. This discrimination indicates that the equivalence principle no longer holds. By applying Gaussian potentials and their asymptotic behavior [34], however, the adiabatic instability [42] can be avoided. There exists an adiabatic regime in which the dark energy scalar field instantaneously tracks the minimum of its effective potential [43]. However, the adiabatic regime is always subject to an instability if the coupling strength is much larger than the gravitation, although the instability can be evaded at weaker couplings [42]. The screening effect in the chameleonlike models efficiently suppresses the strength of the scalar force so as to be in agreement with precision tests of gravity [5].

In this paper a symmetry-breaking interaction is introduced to keep the minimum of the effective potential nearly invariant and to alleviate the adiabatic instability problem in the most cosmic epochs, except for the inflationary era. The adiabatic instability is one of the most important features in inflation. In order to derive a mathematical expression for the scalar fifth force under the constraint of the cosmological constant, it is necessary to use a symmetry-breaking coupling function rather than using symmetry breaking in the self-interaction potential as in [41,44]. The symmetry-breaking interaction between matter and the scalar field can localize a vacuum expectation value (VEV) of the scalar field in the effective potential minimum. The effective potential is a sum of the interaction potential and the self-interaction potential [8,9]. The parameters in the model are determined by using Planck 2018 results [3,45] and the naturalness of the theory.

It should be emphasized here that the theory of the chameleonlike scalar field has introduced a very important and crucial concept [8,14]: a scalar-field-independent energy density of matter. Furthermore, it has potentially introduced a corresponding field-independent pressure and has proven a conservation law of energy density. The conservation law is one of the foundations of this paper. Since matter couples to the scalar field, the energy density of matter in the Universe no longer conserves itself. Therefore, this scalar-field-independent matter density should be introduced to reflect a conserved quantity, such as a nonrelativistic particle number in the Universe. Only the new conserved quantity is included in the model and distinguished from the real physics energy density of matter, so we can use the results of astronomical observations to fit the parameters of the model. The real physics energy density of matter includes both the scalar-field-independent density and the energy density coupling with the scalar field.

To be clear, a scalar-field-independent but temperature-dependent equation of state for matter is also introduced and discussed. We regard the equation of state as a hypothesis which needs to be further confirmed by cosmological observations. The setting with the symmetry-breaking coupling function not only drives dark energy without adding a cosmological constant to the self-interaction potential but also suppresses the interaction range of the fifth force to satisfy the local tests of gravity. The force strength and the interaction range are dependent on the ambient matter density. The force strength is considerably larger compared with gravity under an ultrahigh-vacuum environment, which makes it possible to detect the fifth force in the laboratory. For the further constraint of the scalar fifth force model, we extrapolate the scalar field to drive inflation at the inflationary era of the Universe.

For the closed space in our scenario, the Universe will contract in the future, and the Universe will become hotter and hotter. The ultrahigh-frequency oscillation of the scalar field behaves like a pressureless fluid and rapidly enhances the contraction, which may be called deflation. With the temperature increasing, the interaction between matter and the scalar field will approach a decoupling phase. The vigorous scalar field will climb up along its self-interaction potential to its maximum value when the kinetic energy is exhausted and then roll down from the maximum to cause the Universe's rapid growth. As a result, a systematic description of the cosmic acceleration expansion at the present epoch and the very rapid expansion at the inflationary epoch is possible. However, in the current literature [4] the most probable candidate of the self-interaction potential might be a concave shape, while the self-interaction potential used in our model is a convex one. This paradox results from the assumption in the literature that $\dot{\phi}$ does not pass through zero (not change sign) during inflation in deriving the parametrization of the self-interaction potential $V(\phi)$ [46–52], where ϕ denotes the scalar field and the overdot indicates the derivative with respect to cosmic time. The result of the parametrization of $V(\phi)$ depends on the initial value of $\dot{\phi}$. One often chooses either $\dot{\phi} > 0$ or $\dot{\phi} < 0$ throughout, which is obviously not valid for the case with a turning point from the climbing-up phase to the rolling-down one. The concave feature means that the curvature of the Universe plays an important role in the inflationary era.

This paper is organized as follows. In Sec. II, the technical preliminaries are listed. The expression of the fifth force is reviewed. Both a scalar-field-independent matter density and a scalar-field-independent equation of state for matter are introduced. The temperature dependence of the equation of state is also discussed. In Sec. III, the acceleration equation of the Universe is rewritten in the scalar field coupling case, and the cosmological constant is described by a special value of the self-interaction potential density of the scalar field. For characterizing the fact that the cosmological constant is nearly fixed with the dynamical model, the symmetry-breaking interaction potential is introduced and discussed. In addition, a negative damping motion is presented, which collects energy in the scalar field during the contraction of the Universe. In this section, we also distinguish the adiabatic condition and the oscillation condition. In Sec. IV, the important parameter of the setting and the current matter density of the Universe are determined by using the model with the current astronomical observation data. It is proven in this section that the cosmological constant is nearly fixed as long as the matter density is large enough. When the matter density becomes extremely small due to the expansion, the cosmological constant is proportional to the square of the density. Comparing the total energy density of the Universe to the critical density calculated with the Hubble constant in the Planck 2018 results, the Universe might be a closed universe. The maximum radius of the Universe and some transition redshifts are also calculated. In Sec. V, we show why our setting can avoid the physical corollary of chameleon no-go theorems and the overshooting problem. Interestingly, our model does not conflict with the no-go theorems, at least mathematically. But the model can break through their unreasonable corollary that the chameleonlike scalar field cannot account for the cosmic acceleration. When the symmetry-breaking coupling with matter is used, the appropriate value of the self-interaction potential can be easily acquired and then drive the acceleration. At the end of the Sec. V, the problem of the zero-point energy density is discussed briefly. In Sec. VI, the contraction of the Universe is discussed. The minimum radius of the Universe is estimated to be falling in a large range. The Universe might undergo a climbing-up and rolling-down process near the minimum radius. In this section, by introducing a pseudopotential, which is a sum of the self-interaction potential density of the scalar field and the energy density scale of the curvature of the Universe, the feature of the observed concave potential is explained as an attribute of the pseudopotential. Then, the feature implies a closed space. In Sec. VII, the screening effect and the strength of the scalar fifth force are discussed. It is shown that the matter-coupled scalar field model satisfies the constraint of the precision measurements of hydrogenic energy levels [17] due to the density-dependent screening effect. Approximate expressions of the scalar fifth force are derived in this section, which can help experimental physicists design experiments to test the scalar fifth force. Conclusions are presented in Sec. VIII.

II. TECHNICAL PRELIMINARIES

Consider the action governing the dynamics of a scalar field as follows [34,42]:

$$S = \frac{1}{\hbar^3 c^4}\int d^4x\sqrt{-g}\left[\frac{\hbar^3 c^7}{16\pi G}R - \frac{1}{2}\hbar^2 c^2 \partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + \sum_i S_i(g_{\mu\nu}A^2(\phi), \psi_i), \tag{1}$$

where ϕ is the scalar field with self-interaction potential $V(\phi)$ and ψ_i denotes matter fields, such as the spinor field. The coupling between the scalar field and ψ_i is given by the conformal coupling $A^2(\phi)g_{\mu\nu}$, where the coupling function $A(\phi) > 0$. Since the forces relate to the spatial gradient of some potential, one may work in Newtonian gauge with the perturbed line element about Minkowskian space-time as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -(1+2\Phi)c^2dt^2 + (1-2\Psi)d\vec{x}^2, \tag{2}$$

where the metric potentials Φ and Ψ are space dependent but time independent. For the source of a static, pressureless, nonrelativistic matter distribution, apart from the Newtonian force, a test particle is subject to a new fifth force [5,15,16,36]:

$$\vec{a} = -c^2\nabla \ln A(\phi) = -c^2\frac{A_{,\phi}(\phi)}{A(\phi)}\nabla\phi. \tag{3}$$

Be careful not to confuse the unfortunate notation $\vec{a}$ for the acceleration of a test particle and $a(t)$ for the scale factor of the Universe. The scalar fifth force is strongly dependent on the form of the coupling function $A(\phi)$, besides the gradient of the scalar field. The mathematical expression of $A(\phi)$ will be speculated, and the physical parameters of $A(\phi)$ will be given based on the constraint of the cosmological constant in Sec. III B. The validness of the scheme is tested in the rest of this paper.

Astronomical observations have not found that dark energy evolves with time [3]. Consequently, if dark energy originates from a dynamic scalar field, the most probable candidate of $A(\phi)$ might be a symmetry-breaking form. The symmetry-breaking shape for $A(\phi)$ can localize the VEV of the scalar field, which will be discussed in detail in Sec. III. In order to infer the form of the coupling function from the constraints of the cosmological observation data, one can consider a homogeneous, isotropic universe with a scale factor $a(t)$ described by the line element

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{4}$$

where the values $K = 1, 0,$ or -1 correspond to closed, flat, or open spaces, respectively. Variation of the action (1) with respect to the metric yields the Friedmann equation [34,42]

$$H^2 = \frac{8\pi G}{3}\left[\sum_i \rho_i A^{1-3w_i}(\phi) + \frac{1}{\hbar^3 c^5}\left(V(\phi) + \frac{\hbar^2}{2}\dot{\phi}^2\right)\right] - \frac{Kc^2}{a^2}, \tag{5}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter which defines the cosmic expansion rate, G is the gravitational constant, overdots indicate derivatives with respect to cosmic time t, i denotes several species of noninteracting perfect fluids of matter sources, ρ_i is scalar-field-independent matter density [5–10,31,36], and its equation of state is

$$w_i \equiv \frac{p_i}{\rho_i c^2}, \tag{6}$$

with p_i being the pressure of the fluid component. According to statistical mechanics, both the energy density $\rho_i c^2$ and the pressure are functions of the system temperature T. Then, the equation of state w_i is T dependent, i.e., $w_i(T)$. It is worth noting that, regardless of whether the temperature value is large or not, the equation of state must be calculated by relativistic statistical mechanics so that both thermal energy and rest energy are included [34,39,53]. Then, we can conclude that for dust, including cold dark matter (CDM) [39], $w_i = 0$; for radiations and relativistic particles, $w_i = 1/3$; in general, $0 \le w_i(T) \le 1/3$. As temperature increases continuously, we see that $w_i(T)$ gradually approaches $1/3$ from 0, and the final value of $w_i(T) = 1/3$ will result in the decoupling of the scalar field to matter. It should be emphasized that the choice of ρ_i and p_i is independent of the scalar field and satisfies the conservation law:

$$\dot{\rho}_i = -3H\rho_i(1+w_i). \tag{7}$$

In Eq. (7), both the number density and the corresponding entropy are conserved. The number of particles (or the distribution numbers for energy levels) is not altered, but the masses of the particles (or the energy eigenvalues) are shifted due to the coupling of matter to the scalar field. Consequently, ρ_i (p_i) denotes the mass densities (pressures) in the decoupled cases, such as $w_i = 1/3$ or $A(\phi) = 1$. Thus, Eq. (7) shows that the corresponding entropy is conserved. Actually, Eq. (7) is assumed to be valid not only for nonrelativistic particles but also for relativistic particles (see also Appendix B 2). To distinguish between the expansion and the contraction by the Hubble parameter, we rewrite Eq. (5) as

$$\frac{\dot{a}}{a} \equiv H^{\pm} = \pm\left(\frac{8\pi G}{3}\left[\sum_i \rho_i A^{1-3w_i}(\phi) + \frac{1}{\hbar^3 c^5}\left(V(\phi) + \frac{\hbar^2}{2}\dot{\phi}^2\right)\right] - \frac{Kc^2}{a^2}\right)^{1/2}. \tag{8}$$

Then, H^+ and H^- denote the expansion and the contraction of the Universe, respectively. Variation of the action (1) with respect to ϕ gives [34,42]

$$\hbar^2\ddot{\phi} + 3H^{\pm}\hbar^2\dot{\phi} + V_{\mathrm{eff},\phi}(\phi) = 0, \tag{9}$$

where the subscript ", ϕ" denotes a partial derivative with respect to ϕ; the effective potential density is

$$V_{\mathrm{eff}}(\phi) = V(\phi) + V_{\mathrm{int}}, \tag{10}$$

with

$$V_{\mathrm{int}} \equiv \sum_i \rho_i \hbar^3 c^5 [A^{1-3w_i}(\phi) - 1] \tag{11}$$

indicating the interaction with matter [8]. From Eq. (9), one can deduce that when the Universe contracts, the scalar field may grow rapidly. The details will be discussed in Sec. VI. According to statistical mechanics, as the temperature approaches infinity, the equation of state approaches $1/3$ and then the interaction potential vanishes. In contrast, for perfect fluid with cold but extremely dense matter, the value $A(\phi_{\mathrm{min}})$ of a symmetry-breaking coupling function shown in the next section at the minimum of the effective potential tends to 1, and the interaction potential also vanishes [see Eqs. (19b) and (22a)].

In summary, the scalar field is required to account for the observed cosmic acceleration so as to logically obtain the fifth force from the coupled scalar field. To achieve this goal, a scalar-field-independent matter density and the corresponding conservation law are introduced in a definitionlike manner. The conservation law states that both the number of particles and the corresponding entropy are conserved regardless of whether matter couples to the scalar field or not. Since the masses of the particles are shifted due to the coupling, the real physics matter density depends on the scalar field, and the corresponding entropy is no longer conserved due to the T dependence of w_i (see also Appendix B 1). When a new degree of freedom (d.o.f.) is introduced, it is necessary to add accordingly a new energy form. Here, ρ_i and the corresponding conservation law are introduced to reflect the aspect of the scalar-field independence of matter.

III. A QUARTIC SELF-INTERACTION POTENTIAL ENERGY DENSITY WITH A SYMMETRY-BREAKING INTERACTION TO MAKE THE UNIVERSE ACCELERATE

It has been assumed that only when the scalar field leads to a long-range fifth force [10,33] can it represent dark energy evolving on cosmological timescales. However, the requirement of the long range of the interaction is not necessary, which will be discussed in Sec. V. In this section we introduce a symmetry-breaking interaction between the scalar field and matter to localize the minimum of the effective potential; then the fifth force is very short ranged. Our setting differs from that of [32,41,42,44,54] in which the broken symmetry is only related to the self-interaction potential of the scalar field; then adiabatic instability occurs or reacts very sensitively to the changes of the background density [15,37,42,55]. It will be proven in this section that a value of the self-interaction potential around the minimum of the effective potential acts a constantlike dark energy (or, equivalently, the cosmological constant) due to the symmetry-breaking interaction. The symmetry-breaking coupling also results in a density-dependent and short ranged fifth force, which will be discussed in Sec. VII.

A. Driving cosmic acceleration via the coupled scalar field

From Eqs. (5) and (9), the acceleration equation of the Universe is obtained as

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3\hbar^3 c^5}\left[2V(\phi) - 2\hbar^2\dot{\phi}^2 - \sum_i \rho_i (1+3w_i)\hbar^3 c^5 A^{1-3w_i}(\phi)\right]. \tag{12}$$

Since Eq. (12) gives one of our foundations, it is derived in detail in Appendix A. It is noteworthy that w_i, the equation of state for matter, is temperature dependent. With the temperature growing, the coupling of matter to the scalar field decreases. For pressureless matter sources $w_i = 0$, the acceleration of the Universe becomes

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3\hbar^3 c^5}[2V(\phi) - 2\hbar^2\dot{\phi}^2 - \rho\hbar^3 c^5 A(\phi)], \tag{13}$$

where $\rho = \sum_i \rho_i$. We emphasize again that ρ is a decoupled total matter density which is independent of the scalar field, and the total physics matter density in the pressureless case should be $\rho A(\phi)$, which includes the interaction energy of matter with the scalar field. The energy exchange between the scalar field and matter is discussed in detail in Appendix B.

Equation (12) shows that the self-interaction potential energy density $V(\phi)$ of the scalar field drives the accelerating expansion of the Universe, while both the kinetic energy density of the scalar field and any form of energy density of matter lead to a decelerating expansion. From Eq. (9) one sees that the evolution of the scalar field is a damping oscillation in the expansion period of the Universe ($H \equiv H^+ > 0$). Therefore, if the scalar field evolves to the minimum of the effective potential and can stably sit at the minimum, one might obtain a cosmological constant. If we substitute the field value ϕ_{min} at the minimum into Eqs. (5) and (13), and neglect the kinetic energy term of the scalar field, we get simple expressions for the Friedmann equation (5) and the acceleration equation (13) as follows:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left[\frac{V(\phi_{\rm min})}{\hbar^3 c^5} + \rho A(\phi_{\rm min})\right] - \frac{Kc^2}{a^2}, \quad (14a)$$

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\left[\frac{2V(\phi_{\rm min})}{\hbar^3 c^5} - \rho A(\phi_{\rm min})\right]. \quad (14b)$$

We compare Eq. (14b) with the acceleration of the Universe in the ΛCDM model [39,40] as

$$\frac{\ddot{a}}{a} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3}\rho_{\rm m}, \quad (15)$$

where Λ and $\rho_{\rm m}$ are the cosmological constant and the real physics matter density, respectively. One can see that the value of the self-interaction potential at $\phi_{\rm min}$ acts as the cosmological constant,

$$\Lambda = \frac{8\pi G V(\phi_{\rm min})}{\hbar^3 c^7}, \quad (16)$$

rather than the minimum of the effective potential as used previously [10,18]. Also, the mass density $\rho_{\rm m}$ of matter is equal to $\rho A(\phi_{\rm min})$ and becomes ϕ dependent, i.e.,

$$\rho_{\rm m} = \rho A(\phi_{\rm min}). \quad (17)$$

That the value of the self-interaction potential at the minimum plays the role of dark energy has also been demonstrated through a post-Newtonian approximation [56].

Physicists often use $\Lambda_{\rm E}$, the energy scale of the dark energy density $\Lambda_{\rm E}^4$, to describe the cosmological constant, which is defined by

$$\Lambda_{\rm E}^4 \equiv \frac{\Lambda \hbar^3 c^7}{8\pi G} \equiv V(\phi_{\rm min}). \quad (18)$$

Since the scalar field always tends towards the minimum of the effective potential due to the positive damping coefficient of $3H^+ > 0$ in the expansion period of the Universe, obtaining the cosmological constant is strongly dependent on the adiabatic condition that guarantees the stability of the scalar field sitting at the minimum of the effective potential. Consequently, large effective masses of the scalar field and nearly invariant minimums of the effective potential are necessary. This can be achieved by invoking a symmetry-breaking coupling function, which will be shown in Sec. III B.

B. Symmetry-breaking coupling function

In order to obtain the fifth force under the cosmic constraints, we choose a quartic self-interaction potential and a symmetry-breaking coupling function as follows:

$$V(\phi) = \frac{\lambda}{4}\phi^4, \quad (19a)$$

$$A(\phi) = 1 + \frac{1}{4{M_1}^4 c^8}(\phi^2 - {M_2}^2 c^4)^2, \quad (19b)$$

where M_1, M_2 are parameters with mass dimension and λ is a dimensionless parameter. Both the self-interaction potential and the coupling function have $\mathbf{Z}_2$ $(\phi \to -\phi)$ symmetry. All the parameters above can be determined by the constraints of the cosmological observations and the theoretical naturalness (a detailed discussion is given in Appendix C). A concise display of the parameters is shown in Eq. (20):

$$\lambda = \frac{1}{6}, \qquad M_1 = \frac{M_2}{2^3},$$
$$M_2 = 4.96168\ {\rm meV}/c^2 = 8.845 \times 10^{-39}\ {\rm kg}. \quad (20)$$

According to Eqs. (10) and (19), the effective potential energy density has a very simple form in the case of $w_i = 0$ as follows:

$$V_{\rm eff}(\phi) = \frac{\lambda}{4}\phi^4 + \frac{\rho\hbar^3}{4{M_1}^4 c^3}(\phi^2 - {M_2}^2 c^4)^2. \quad (21)$$

The effective potential density versus the scalar field is shown in Fig. 1.

1. The λ-dependent minima and the λ-independent effective mass

The two degenerate minima of the effective potential of Eq. (21) and the effective mass around the minima are obtained as follows (see Appendix C):

$$\phi_{\rm min} = \pm\left(\frac{\rho\hbar^3 {M_2}^2 c^4}{\lambda {M_1}^4 c^3 + \rho\hbar^3}\right)^{1/2}, \quad (22a)$$

$$m_{\rm eff}^2 \equiv \frac{V_{{\rm eff},\phi\phi}(\phi_{\rm min})}{c^4} = \frac{2\rho\hbar^3 {M_2}^2}{{M_1}^4 c^3}. \quad (22b)$$

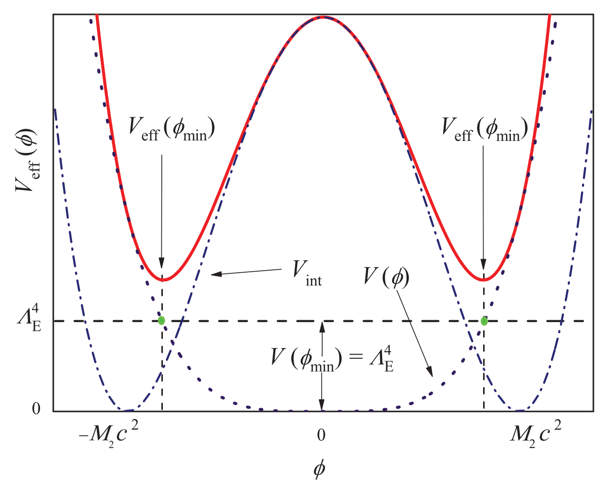

FIG. 1. The effective potential $V_{\rm eff}(\phi)$ (solid curve) is the sum of a scalar potential $V(\phi)$ (dotted curve) and a matter-density-dependent interaction term $V_{\rm int}$ (dot-dashed curve). The value $V(\phi_{\rm min})$ (dashed line) of the self-interaction potential at one of the symmetry-breaking vacuums $\phi_{\rm min}$ acts as the cosmological constant to drive late-time cosmic acceleration.

The scalar field has to choose only one of the minima; then the $\mathbf{Z}_2$ symmetry is spontaneously broken.

From Eqs. (22), one sees that the effective mass of the scalar field is not dependent on λ, but $\phi_{\min}$ is dependent on λ. The two properties are the main reason for the choice shown in Eq. (19). These properties show that the self-interaction potential of the scalar field has nothing to do with the effective mass but can move the position of the minimum of the effective potential. These important properties guarantee that the observed cosmic acceleration stems entirely from the scalar field rather than any static vacuum energy, which will be discussed at the end of Sec. V. The λ-independent effective mass in the general case of $w_i \neq 0$ is obtained in Appendix C 2.

2. The condition of adiabatic tracking

One sees that through Eq. (22), the scalar field can adiabatically track the minimum of the effective potential. The changing rate $\dot{\phi}_{\min}/\phi_{\min}$ of the minimum position due to the change of the matter density can be described by Eq. (C10) in Appendix C. The adiabatic condition guarantees that, if the field is initially at the minimum, it will follow the minimum adiabatically during the later evolution. Since the reciprocal of $|3H/2|$ is the characteristic time of the evolution of the Universe, the adiabatic condition can be expressed as follows:

$$\left|\frac{\dot{\phi}_{\min}}{\phi_{\min}}\right| \leq \left|\frac{3H}{2}\right|. \tag{23}$$

The smaller the changing rate of the minimum position, the stronger the stability of the scalar field sitting at the minimum. For a pressureless matter source the scalar field in our scheme can adiabatically follow the minimum, which is proven by Eq. (C10) in Appendix C.

If the field is not initially at the minimum, one should take into account the oscillation condition. The response time for the scalar field to adjust itself to the position of the minimum is characterized by $1/\omega_c$, with the Compton frequency $\omega_c \equiv m_{\mathrm{eff}} c^2/\hbar$. The decay time for the evolution of the scalar field is characterized by $2/(3H)$. In general cases, the Compton frequency is considerably larger than the Hubble expansion rate, and the oscillation condition

$$\omega_c \geq \left|\frac{3H}{2}\right| \tag{24}$$

is satisfied. For example, the energy scale of the Compton frequency in the present matter density of the Universe is estimated to be $\hbar\omega_{c0} \sim 60$meV, which is about 26 times the cosmological constant. The energy scale of the Hubble expansion rate at the present time is $\hbar H_0 \sim 10^{-33}$ eV. However, it is worth noting that the oscillation condition and the adiabatic condition cannot be satisfied in the inflationary era when the Hubble rate, the effective mass of the scalar field, and the energy density of the Universe vary in an extreme way. Particularly, Eq. (22) is no longer valid in the inflationary era since the assumption of the equation of state $w_i = 0$ is invalid.

3. The Compton wavelength of the scalar field

The Compton wavelength of the scalar field is defined by $\lambda\!\!\!^{-}_c \equiv \hbar/(m_{\mathrm{eff}} c)$, which describes the interaction range between matter and the scalar field. Using Eqs. (20) and (22), the Compton wavelength is obtained as

$$\lambda\!\!\!^{-}_c[\mathrm{m}] = \frac{1.648 \times 10^{-19}}{(\rho[\mathrm{kg/m^3}])^{1/2}}. \tag{25}$$

The larger the ambient matter density, the shorter the Compton wavelength of the coupled scalar field. This interaction range is so short that it is difficult to detect even in low-density empty space. A further discussion is given in Sec. VII.

4. The negative-damping oscillation of the scalar field

Considering the scalar field around the minimum to obtain an approximation for the equation of motion, the effective potential can be expanded as $V_{\mathrm{eff}}(\phi) = V_{\mathrm{eff}}(\phi_{\min}) + V_{\mathrm{eff},\phi}(\phi_{\min})(\phi - \phi_{\min}) + m_{\mathrm{eff}}^2 c^4 (\phi - \phi_{\min})^2/2$; then the equation of motion (9) becomes

$$\ddot{\phi} + 3H^{\pm}\dot{\phi} + \omega_c^2(\phi - \phi_{\min}) = 0, \tag{26}$$

which describes a damped (negative-damped) oscillation for $H^+ > 0$ ($H^- < 0$). The damped (negative-damped) oscillation can be classified into three cases:

(a) $|3H/2| > \omega_c$ overdamping (over-negative-damping);
(b) $|3H/2| = \omega_c$ critically damping (critically negative-damping);
(c) $|3H/2| < \omega_c$ underdamping (under-negative-damping).

The absolute value symbol is used because the damping coefficient $3H < 0$ in the contraction phase of the Universe. The oscillation frequency of the scalar field around the minimum is equal to $\sqrt{\omega_c^2 - 9H^2/4}$, which is less than the Compton frequency ω_c. Figure 2 shows a schematic sketch of the curves of motion of the scalar field in the cases of (a) the expansion and (b) the contraction of the Universe, respectively.

The negative-damping oscillation of the scalar field can absorb energy from the gravitational field in a negatively damped manner during the contraction process of the Universe if the contraction really occurs. Thus, the negative-damping oscillation is different from forced oscillations. The negative-damping oscillation of the scalar field must result in the explosion of the Universe due to the exponentially growing oscillation magnitude.

However, if the scalar field initially sits at the minimum, i.e., the oscillation magnitude is zero, the question is what activates the oscillation. When the adiabatic condition of

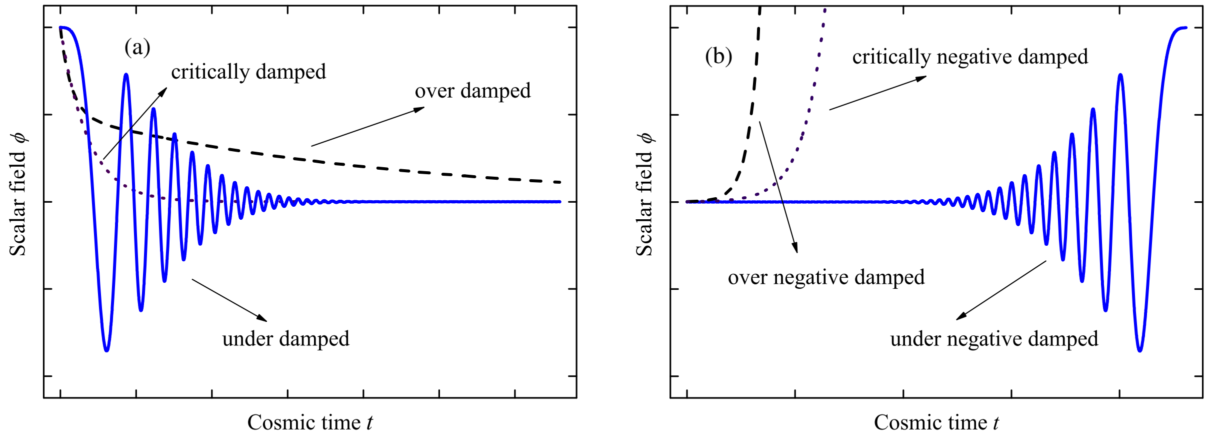

FIG. 2. The schematic sketch of the motion of the scalar field. (a) The curves of motion of the scalar field correspond to the expansion of the Universe. The scalar field experiences three types of damped oscillation: overdamping, critically damping, and underdamping. The underdamped oscillation will last for a long time until the adiabatic condition shown in Eq. (23) is satisfied. When the adiabatic condition is satisfied, the scalar field will quickly decay to the minimum and stay there. (b) The curves of motion of the scalar field correspond to the contraction of the Universe. The scalar field experiences three types of negative damped oscillation: under-negative-damping, critically negative-damping, and over-negative-damping. In the case of under-negative-damping, the scalar field can sit at one of the minima of the effective potential until the adiabatic condition is broken, which will be discussed in Sec. VI. When the temperature increases, the scalar field will finally decouple with matter and shift to the case of over-negative-damping. It is worth noting that neither graph shows the minimum moving over time. The moving rate of the minimum determines whether the adiabatic condition is satisfied.

Eq. (23) is satisfied, the scalar field will stay at the minimum. With the temperature of the Universe becoming hotter and hotter, the adiabatic condition no longer holds, and the oscillation is triggered by the quick movement of $\phi_{\rm min}$, which will be discussed in Sec. VI.

5. The cosmological constant in quintessence form pinned by the symmetry-breaking coupling

Since the adiabatic tracking always holds for $w_i = 0$, the cosmological constant can be finely defined by Eq. (18) and can be obtained in our scheme by Eq. (19a) as follows:

$$\Lambda_{\rm E}^4 \equiv V(\phi_{\rm min}) = \frac{\lambda}{4}\phi_{\rm min}^4. \tag{27}$$

If the symmetry is not broken, i.e., $M_2 = 0$ in Eq. (19b), one immediately obtains $\phi_{\rm min} = 0$ and $m_{\rm eff} = 0$ by using Eq. (22); then the cosmological constant goes to zero. In this sense, the nonzero cosmological constant seems to stem from a symmetry-breaking interaction between the scalar field and matter. However, if $\lambda = 0$, from Eq. (27) one also obtains zero. Indeed, our setting is essentially a quintessence model, in which the quintessence is trapped at the bottom of the interaction potential well. When the matter density is large enough, the value of the self-interaction potential pinned by the symmetry-breaking interaction approaches a constant. Whether or not the localization works well is left to Sec. IV.

C. Summary

Since the adiabatic condition is satisfied in our setting, the value of the quartic self-interaction potential at the minimum of the effective potential can be localized stably by the symmetry-breaking interaction potential. It has been proven that the value acts as the cosmological constant. When the density of matter is large enough, the value of the self-interaction potential indeed approaches a constant.

In the scheme, the effective mass of the scalar field has two important characteristics: (1) The mass, in general, is large enough so that the adiabatic instability can be suppressed. (2) The mass is unrelated to the self-interaction potential so that the zero-point energy can be canceled out; then the fine-tuning is avoided in deriving the cosmological constant, which will be discussed in Sec. V D. Although it has nothing to do with the effective mass, the self-interaction potential moves the position of the minimum of the effective potential. Thus, the observed cosmic acceleration can be ascribed entirely to the scalar field rather than any static vacuum energy.

In addition, the negative-damping oscillation of the scalar field in this section is introduced for the contraction process of the Universe.

IV. APPLICATION OF THE MODEL TO THE EXPANSION OF THE UNIVERSE ($H^+ > 0$)

We now test our setting for the following: to explain the current astronomical observations, to extrapolate backward in early time, and to predict the future trends of the Universe.

A. Quantitative comparison with some important astronomical observations

One can see from Eq. (20) that only one parameter of M_2 needs to be determined by experimental data.

The parameter of M_2 is chosen so as to satisfy both the values of the current cosmological constant and the current ratio of the energy density of matter to the total energy density of the Universe. After the determination of M_2, we will test whether or not the setting works well.

1. Determining the adjustable parameter M_2

The ratio of matter density to the total mass density is

$$\Omega_{\rm m} \equiv \frac{\rho_{\rm m}}{\rho_{\rm tot}}, \tag{28}$$

where the physics matter density $\rho_{\rm m} = \sum_i \rho_{{\rm m}i}$ with $\rho_{{\rm m}i} = \rho_i A^{1-3w_i}(\phi)$, and the total mass density is a sum of the physics matter density and the mass density of the scalar field. We can see from the Friedmann equation (5) why the real physics matter density is $\rho_{\rm m}$ rather than ρ_i or $\sum_i \rho_i$. For pressureless matter sources $w_i = 0$, the physics matter density can be written as $\rho_{\rm m} = \rho A(\phi)$ with $\rho = \sum_i \rho_i$. The scalar field mass density is defined by $\rho_\phi = V(\phi)/(\hbar^3 c^5) + \dot{\phi}^2/(2\hbar c^5)$ [20], which can also be seen from the Friedmann equation (5). When the scalar field adiabatically follows the minimum of the effective potential, the kinetic energy of the scalar field can be neglected and the scalar field mass density becomes $\rho_\phi = V(\phi_{\rm min})/(\hbar^3 c^5)$. Thus, Eq. (28) becomes

$$\Omega_{\rm m} = \frac{\rho \hbar^3 c^5 A(\phi_{\rm min})}{\rho \hbar^3 c^5 A(\phi_{\rm min}) + V(\phi_{\rm min})}. \tag{29}$$

Substituting the current astronomical observation data [3],

$$\Omega_{\rm m0} = 31.58\%, \tag{30a}$$

$$\Lambda_{\rm E0} = 2.239 \text{ meV},$$
$$(\Lambda_0 = 4.24 \times 10^{-66} \text{ eV}^2 = 1.089 \times 10^{-52} \text{ m}^{-2}), \tag{30b}$$

into Eqs. (29) and (27) together with (22a), we obtain simultaneous equations. Noting the expressions shown in Eq. (20) and regarding M_2 as an undetermined parameter, we solve the simultaneous equations to derive M_2 and the current matter (including CDM) density of the Universe as follows:

$$M_2 = 4.96168 \text{ meV}/c^2 = 8.845 \times 10^{-39} \text{ kg}, \tag{31a}$$

$$\rho_{\rm m0} \equiv \rho_0 A_0(\phi_{\rm min}) = 2.69271 \times 10^{-27} \text{ kg/m}^3. \tag{31b}$$

Here the subscript 0 is the current time. The corresponding scalar-field-independent matter density is

$$\rho_0 = 2.68026 \times 10^{-27} \text{ kg/m}^3, \tag{32}$$

which is smaller than the real physics matter density $\rho_{\rm m0}$. The reason for this is that the physics matter density includes the interaction energy between matter and the scaler field.

The total energy density of the Universe is then obtained as follows:

$$\rho_{\rm tot0} = \rho_0 A_0(\phi_{\rm min}) + \frac{V_0(\phi_{\rm min})}{\hbar^3 c^5}$$
$$= 8.52665 \times 10^{-27} \text{ kg} \cdot \text{m}^{-3}. \tag{33}$$

Therefore, by using the current values of $\Omega_{\rm m0}$ and Λ_0, we obtain not only the free parameter M_2 but also all the forms of the current energy density of the Universe.

2. The effective equation of state for the scalar field in the present era

The effective equation of state for the coupled scalar field in the present era is estimated by Eq. (B20) in Appendix B to be

$$w_{\rm eff0} \equiv \frac{p_{\rm eff0}}{\rho_{\rm eff0} c^2} = \frac{-V_0(\phi_{\rm min})}{V_{\rm eff0}(\phi_{\rm min})} = -0.998. \tag{34}$$

This value is slightly larger than the result $w_0 = -1.03 \pm 0.03$ shown in [3], but it is slightly smaller than the result $w = -0.80^{+0.09}_{-0.11}$ shown in [57]. The small differences may result from the fact that the models used in the literature [3,57] are not the same as the model in this paper.

3. The two transition redshifts in the past and the future

We can now calculate the transition redshifts by letting the acceleration in Eq. (14b) equal zero. Although the geometry curvature K appears in the Friedmann equation, it disappears in the acceleration equation of the Universe. Consequently, the transition redshifts associated with the zero acceleration are independent of the curvature of the Universe. Since the redshift is defined by $1 + z = a_0/a(t)$, with $a(t)$ the scale factor of the Universe at cosmic time t and a_0 the current value [39], the scalar-field-independent matter density in the pressureless case can be expressed via Eq. (7) as follows:

$$\rho = \rho_0 (1 + z)^3. \tag{35}$$

The physical significance of Eq. (35) is that the particle number of the Universe is not altered during its expansion. However, it is worth noting that, in general, $\rho_{\rm m} \neq \rho_{\rm m0}(1 + z)^3$ due to the interaction energy between matter and the scalar field (see also Appendix B). Substituting both $\ddot{a} = 0$ and Eq. (35) into Eq. (14b), we derive two solutions for the transition redshift, which mark the transition time of the Universe expansion from deceleration to acceleration and vice versa.

The transition redshift corresponding to the deceleration-acceleration transition in the Universe's past is

$$z_{\mathrm{past}} = 0.634478, \tag{36}$$

which is consistent with [3,45,58–60]. At this transition time, the scalar-field-independent matter density $\rho_{\mathrm{past}} = 1.1703 \times 10^{-26}\ \mathrm{kg/m^3}$, which is slightly smaller than the corresponding physics matter density $\rho_{\mathrm{mpast}} = 1.1706 \times 10^{-26}\ \mathrm{kg/m^3}$. The corresponding cosmological constant is obtained by Eqs. (22a) and (27) as $\Lambda_{\mathrm{Epast}} = 2.241\ \mathrm{meV}$ or, equivalently, $\Lambda_{\mathrm{past}} = 1.1092 \times 10^{-52}\ \mathrm{m^{-2}}$. The effective equation of state for the coupled scalar field is estimated by Eq. (B20) to be $w_{\mathrm{eff}} = -1$.

Another transition redshift that corresponds to the next transition of acceleration-deceleration is obtained as follows:

$$z_{\mathrm{future}} = -0.8977287, \tag{37}$$

which will occur in the future. The scalar-field-independent matter density $\rho_{\mathrm{future}} = 2.867 \times 10^{-29}\ \mathrm{kg/m^3}$ is considerably smaller than the corresponding physics matter density $\rho_{\mathrm{mfuture}} = 1.3058 \times 10^{-27}\ \mathrm{kg/m^3}$. This means that, with the density decreasing, the interaction potential energy between matter and the scalar field will increase due to the symmetry-breaking coupling function. Of course, one also finds that the cosmological constant will decrease when the matter density decreases. The corresponding cosmological constant is $\Lambda_{\mathrm{Efuture}} = 1.295\ \mathrm{meV}$ ($\Lambda_{\mathrm{future}} = 1.219 \times 10^{-53}\ \mathrm{m^{-2}}$). The corresponding effective equation of state for the coupled scalar field is estimated by Eq. (B20) to be $w_{\mathrm{eff}} = -0.334$.

4. The nearly fixed cosmological constant before the present era

If matter density increases in the pressureless case, the interaction potential energy between matter and the scalar field will decrease and finally approach zero. According to Eqs. (19a) and (22a), when ρ approaches infinity, the interaction potential Eq. (11) will vanish due to $A(\phi_{\mathrm{min}}) = 1$. But the density-dependent cosmological constant will increase and finally approach a constant. In other words, when the density is large enough, the cosmological constant obtained by Eq. (27), together with Eq. (22a), is nearly density independent, which is a desired result. In this sense, the cosmological constant becomes a constant. For example, when $\rho \to \infty$, one has

$$\Lambda_{\mathrm{E}} = 2.242\ \mathrm{meV}, \quad (\Lambda = 1.093 \times 10^{-52}\ \mathrm{m^{-2}}), \tag{38a}$$

$$w_{\mathrm{eff}} = -1. \tag{38b}$$

The limit towards infinity does not represent any physical process: it is a mathematical construction that depicts a nearly fixed value of Λ in the Universe's past [61]. In fact, $\Lambda_{\mathrm{E}} \simeq 2.242\ \mathrm{meV}$ always holds as long as $\rho \gg \lambda M_1{}^4 c^3/\hbar^3 \sim 10^{-30}\ \mathrm{kg/m^3}$. This implies that an actual time variable of the cosmological constant in the matter-coupled scalar field model behaves as a real constant before the present era. Figure 3 shows the cosmological constant versus the matter density.

Due to the broken symmetry of the interaction between the scalar field and matter, the value of the self-interaction potential at the minimum of the effective potential is pinned at a nearly fixed value if the density of matter is large enough. The minimum is mainly determined by three factors: the self-interaction potential, the shape of the coupling function, and the density of matter. These remind us to choose the appropriate coupling shape so that the minimum is insensitive to the change of matter density for large density. The requirements of adiabatic stability also imply that the effective mass of the scalar field should be large enough, which results in a very short-range fifth force in a wide region of ambient matter density (see Sec. VII). It is the symmetry-breaking coupling function that plays a pivotal role in suppressing the scalar gradient force effect and in promoting the dark energy role of the scalar field in a matter environment.

B. Space-curvature-dependent future of the expansion of the Universe

The nearly fixed cosmological constant before the present era has been proven above. We now return to the case of

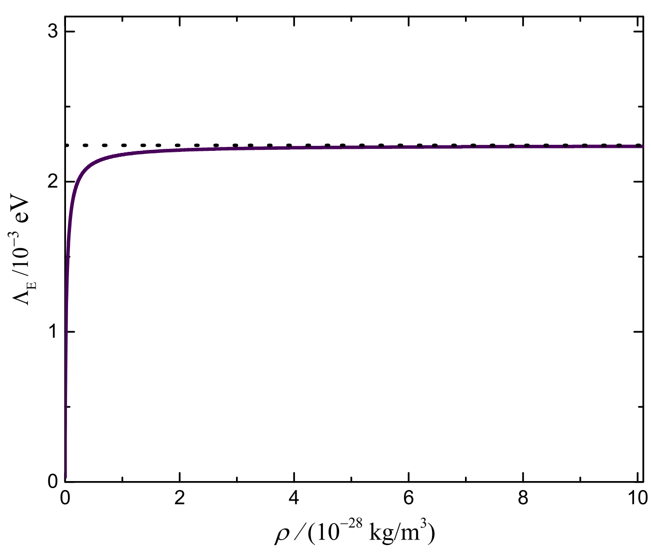

FIG. 3. The cosmological constant versus the density of matter. The density-dependent cosmological constant obtained by Eq. (27) becomes nearly density independent when the density is large enough, regardless of the Universe being flat, open, or closed. However, for flat and open spaces, the cosmological constant approaches zero when the density of matter approaches zero. For closed spaces, the minimum cosmological constant is $\Lambda_{\mathrm{Emin}} = 8.664 \times 10^{-8}\ \mathrm{meV}$ corresponding to the maximum radius of the Universe shown in Eq. (46a).

matter density decreasing due to the expansion of the Universe. The Universe will switch to a decelerating expansion status according to the acceleration equation of the Universe. However, in order to find whether the expansion will continue in the distant future, one has to consider the curvature of the Universe.

1. $\Lambda \propto \rho^2$ long after the present era

With the density decreasing further in the future, for example, when $\rho \ll \lambda {M_1}^4 c^3/\hbar^3$, it can be easily obtained from Eqs. (16), (22), and (27) that

$$\Lambda \simeq \left(\frac{2\pi G\hbar^3 {M_2}^4}{\lambda c^5 {M_1}^8}\right)\rho^2. \tag{39}$$

Therefore, Λ will reach a region where it decreases faster than matter density does in the future. In other words, $\ddot{a} < 0$ will occur according to Eq. (14b) as long as

$$2V(\phi_{\mathrm{min}}) < \rho_{\mathrm{m}}\hbar^3 c^5. \tag{40}$$

Consequently, the Universe will switch to a decelerating expansion status. It is clear that the self-interaction potential of the scalar field causes the accelerating expansion, while matter density ρ_{m} decreases the acceleration. Although both $\Lambda \to 0$ and $\rho_{\mathrm{m}} \to 0$ when $\rho \to 0$, the different convergence rates result in the next decelerating expansion after the transition redshift of z_{future}.

What would happen next? Does the Universe keep expanding forever or switch to a contracting? This cannot be solved by the acceleration equation (14b) alone. Applying the current Hubble constant H_0 to the Friedmann equation (14a), the question may be answered. Due to the Hubble tension [62,63], however, another criterion is needed, which will be shown in Sec. VI.

2. Flat space

Let us discuss the flat space first, i.e., $K = 0$. In this case, since only the combination $\dot{a}/a$ appears in the Friedmann equation (14a), one is free to rescale $a(t)$ as one chooses. For example, one can choose $a_0 = 1$ at the present time, which means that the physical coordinate system coincides with the comoving one at the present time. The scalar-field-independent matter density in the pressureless case can be expressed via Eq. (7) as follows:

$$\rho = \frac{a_0^3 \rho_0}{a^3}, \tag{41}$$

where the value of the current scalar-field-independent matter density ρ_0 has been estimated as shown by Eq. (32). Substituting (41) into Eqs. (22a) and (27), and then substituting both into the Friedmann equation (14a), we obtain a complicated differential equation about the scale factor in time.

However, we can study the asymptotic behavior for the late-time evolution of the Universe. When the matter density becomes smaller and smaller with the expansion so that $\phi_{\mathrm{min}}^2 \ll M_2^2 c^4$, one finds that the solution is $a(t) \propto t^{2/3}$, and then $\rho(t) \propto 1/t^2$, which is the same as the traditional matter-dominated solution in flat space. The Universe expands forever, but the Hubble parameter decreases with time as $H = 2/(3t)$, meaning that it will infinitely approach zero as the time approaches infinity. Substituting $K = 0$ and the total density Eq. (33) into the Friedmann equation (14a), one can calculate the current Hubble constant for flat space as follows:

$$H_0(K = 0) = 67.376\ \mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{42}$$

This value is close to the measured value of the current Hubble constant [3,45], such as $H_0 = 67.36\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ [3] or $H_0 = 67.8\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ [45]. The Hubble tension is not completely resolved [62,63]. Although the Universe is close to a flat space at the present time, it still has the possibility of being positive or negative curvature space. However, a closed space is favored by the observed feature of a concave potential [3], which will be discussed in Sec. VI B. In addition, since the measured Hubble constant corresponds to the Jordan frame, it should be transformed into the Einstein frame due to the calculation performed here in the Einstein frame. The relationship between the two frames will be discussed in Sec. V A.

3. Open space

In the case of $K \neq 0$, one often rescales the scale factor $a(t)$ by setting $|K| = 1$ as shown in Eq. (4). When this rescaling is used, the freedom to set $a_0 = 1$ in flat space is lost, and the scale factor has definite meaning in the physical scale, e.g., curvature radius.

For the negative curve space $K = -1$ [45], one can deduce from the Friedmann equation (14a) that the Universe expands forever, but the Hubble parameter decreases with time and will infinitely approach zero as the time approaches infinity. These conclusions are the same as that in flat space. The asymptotic behavior of the Universe evolution in the future is also the same as the traditional matter-dominated solution in open space, i.e., $a \propto t$.

Besides, it is very clear from Eq. (14a) that $\dot{a}$, the expansion speed of the Universe, exceeds the speed of light forever in this situation. This means that there is an infinite space of the Universe that cannot be observed. Thus, it should be stressed that, essentially, the expansion based on the cosmological principle results from the entire homogeneous energy density and the corresponding pressure in the Universe. Due to the homogeneity, there is no gradient force driving the expansion. Because of the entirety, all of the density and pressure contribute their effect to the expansion. More specifically, the expansion does not involve any gradient force and is not related to the Compton wavelength of the scalar field.

4. Closed space

We now discuss the case of $K = 1$. If we use the current Hubble constant [3] $H_0 = 67.36\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, the critical density can be estimated as

$$\rho_c \equiv \frac{3H_0{}^2}{8\pi G} = 8.52261\ \mathrm{kg\cdot m^{-3}}. \tag{43}$$

Since the total density ρ_{tot} shown by Eq. (33) is slightly larger than the critical density ρ_c, the Universe might be closed, i.e., $K = 1$, and the current radius of the Universe is derived from Eq. (14a) as

$$a_0 = 6.305 \times 10^{27}\ \mathrm{m} = 204.3\ \mathrm{Gpc}. \tag{44}$$

This value satisfies the constraint condition $a_0 > 81$ Gpc shown in the Planck 2018 results (X. Constraints on inflation) [4].

From the Friedmann equation (14a), one can easily deduce that the Universe must go through infinite cycles of two stages: contraction and expansion. At the end of the decelerating expansion, the Universe will reach its maximum radius a_{max}, where the expansion speed $\dot{a} = 0$. Substituting $\dot{a} = 0$ into Eq. (14a) (noticing that the equation is only valid in the pressureless case), we get a redshift

$$z = -0.9999985281 \tag{45}$$

corresponding to the maximum radius of the Universe. The maximum radius, the corresponding matter density, and the cosmological constant are given, respectively, as follows:

$$a_{max} \equiv \frac{a_0}{1+z} = 4.283 \times 10^{33}\ \mathrm{m}, \tag{46a}$$

$$\rho_{min} \equiv \rho_0(1+z)^3 = 8.547 \times 10^{-45}\ \mathrm{kg\cdot m^{-3}}, \tag{46b}$$

$$\Lambda_{Emin} = 8.664 \times 10^{-8}\ \mathrm{meV}. \tag{46c}$$

The corresponding physics matter density is $\rho_{mmin} = 8.761 \times 10^{-42}\ \mathrm{kg/m^3}$, which is about 3 orders of magnitude larger than the scalar-field-independent matter density shown in Eq. (46b). The cosmological constant corresponding to the maximum radius should be the minimum value in all of the permissible values of the cosmological constant, that is, $\Lambda_{min} = 2.44 \times 10^{-82}\ \mathrm{m^{-2}}$. Therefore, the situation of $\Lambda = 0$ does not exist. The effective equation of state for the coupled scalar field in this case is estimated by Eq. (B20) to be $w_{eff} = -1.494 \times 10^{-15}$.

After the expansion of $\dot{a} = 0$, the Universe will begin its contraction [61], which will be discussed in Sec. VI. However, for completeness, the sketch of the whole evolution, including both the expansion and the contraction of the Universe, is presented here in Fig. 4. Figure 4(a) shows a sketch of the expansion speed versus the cosmic time. Correspondingly, the sketch of the scalar field versus the cosmic time during the evolution of the Universe is plotted schematically in Figs. 4(b)–4(d). The current value of the scalar field is chosen to be positive in Figs. 4(b)–4(d). However, the next new inflation due to the contraction may occur at either the same sign or opposite sign with the current choice of the sign of the scalar field. Figure 4(b) denotes both the next inflation and the second acceleration occurring at $\phi > 0$. Figures 4(c) and 4(d) denote the next inflation occurring at $\phi < 0$, with the second acceleration occurring at $\phi < 0$ and $\phi > 0$, respectively. Since the potential density of the scalar field is equal to $\phi^4/4!$, the three cases of Figs. 4(b)–4(d) present the same results of the Universe's evolution.

C. Summary

Using the only free parameter in the symmetry-breaking interaction model, the nearly fixed cosmological constant is obtained. The cosmological constant is estimated to be $\Lambda_E \simeq 2.242$ meV before the present era, which is slightly larger than the current value $\Lambda_E = 2.239$ meV. Long after the present era, the cosmological constant is proven to be proportional to the square of the density of matter. Therefore, the energy density of the scalar field will decrease faster than that of matter, and the Universe will shift into a decelerating expansion. Based on the current observational precision of the Hubble constant, one can deduce that the Universe is a nearly flat space in the present era. Due to the Hubble tension, it is necessary to invoke another criterion to judge the curvature of the Universe. The criterion will be shown in Sec. VI, and the positive curvature is preferred. For a closed Universe, the expansion will stop, and the Universe will then contract.

The problem of how to reconcile our scheme with chameleon no-go theorems will be discussed in Sec. V.

V. EVADING THE PHYSICAL COROLLARY OF CHAMELEON NO-GO THEOREMS

The main purpose of this paper is to find out how to obtain a scalar fifth force under the requirement that the model should reflect the measured value of the cosmological constant as much as possible. The model should not add a cosmological constant to drive cosmic acceleration since any force involves a spatial gradient and the effect of the added constant disappears in the differentiation. This reminds us that it is the energy density of the scalar field that drives cosmic acceleration rather than the spatial gradient fifth force.

It is well known that the cosmological constant model provides the simplest explanation of cosmic acceleration. Apparently, there is no local fifth force in this model, albeit

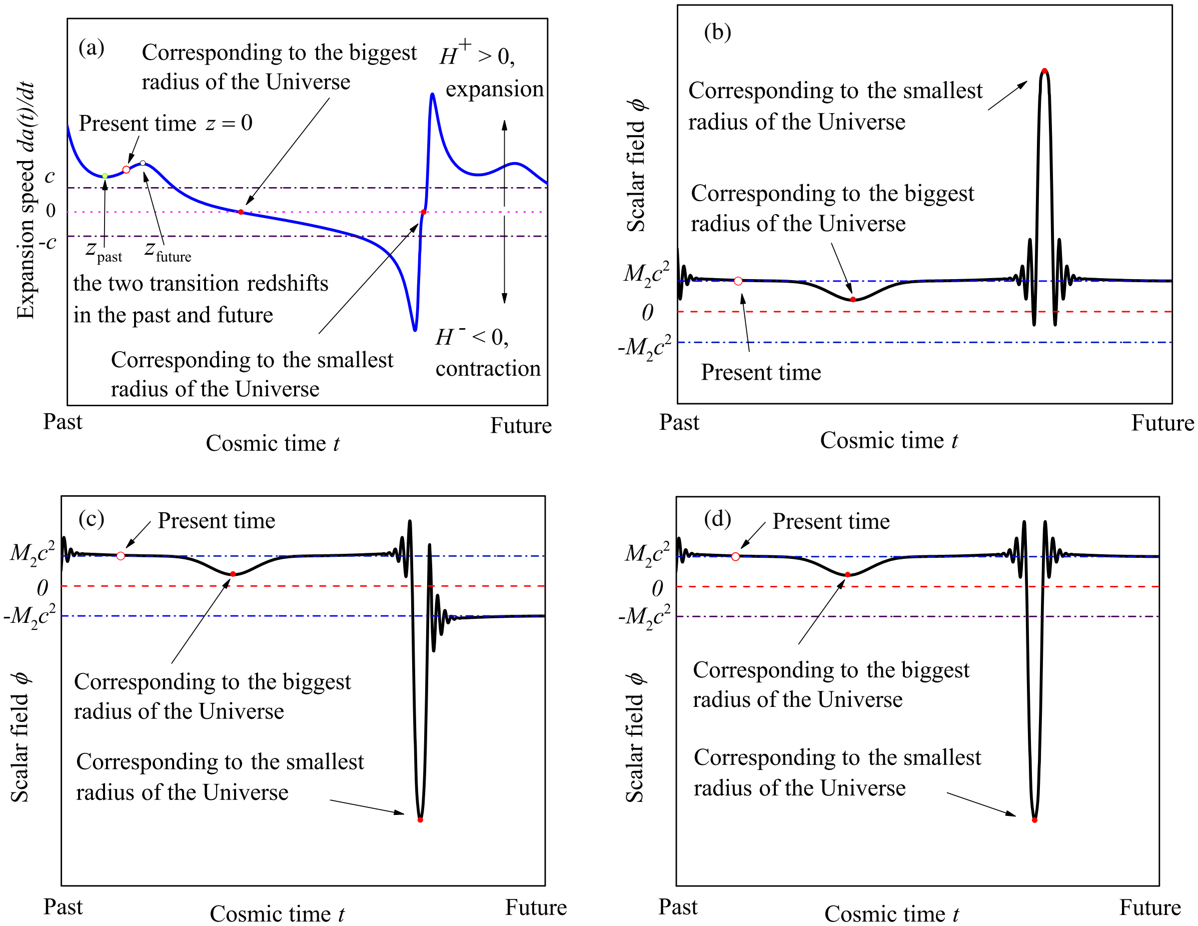

FIG. 4. The schematic sketch of the Universe's evolution with time. (a) The sketch of the expansion speed of the Universe versus the cosmic time from the present to the future. Note that c is the speed of light. For acceleration expansion, $\ddot{a} > 0$ and $\dot{a} > 0$; for decelerating expansion, $\ddot{a} < 0$ and $\dot{a} > 0$; for accelerating contraction, $\ddot{a} < 0$ and $\dot{a} < 0$; for deceleration contraction, $\ddot{a} > 0$ and $\dot{a} < 0$; for the biggest radius of the Universe, $\ddot{a} < 0$ and $\dot{a} = 0$; for the smallest radius of the Universe, $\ddot{a} > 0$ and $\dot{a} = 0$; for the transitions between deceleration and acceleration expansion (contraction), $\ddot{a} = 0$; and such a pattern repeats itself. (b) The sketch of the scalar field versus cosmic time during the evolution of the Universe for both the next new inflation and the second accelerating expansion occurring at $\phi > 0$. (c) The sketch of the scalar field versus the cosmic time during the evolution of the Universe for both the next new inflation and the second accelerating expansion occurring at $\phi < 0$. (d) The sketch of the scalar field versus cosmic time during the evolution of the Universe for the next new inflation $\phi < 0$ but with the second accelerating expansion still occurring at $\phi > 0$. The current value of the scalar field is chosen to be positive in all of the situations of (b)–(d). Since the potential density of the scalar field is equal to $\phi^4/4!$, all of the cases present the same results of the Universe's evolution.

the nature of its negative pressure guarantees acceleration of the Universe. One does not regard this as a satisfactory solution based on the viewpoint of quantum field theory. When the vacuum expectation value of a conventional quantum field theory is used to mimic the cosmological constant, however, Weinberg's no-go theorem occurs [30], which states that no tuning of the corresponding energy density can be achieved naturally.

Scalar field theories of dark energy, such as quintessence with a time-dependent energy density but without coupling to matter [19,22,27,28], may be able to circumvent Weinberg's no-go theorem. If the scalar field does not couple to matter, i.e., $A(\phi) = 1$, our model mentioned above becomes one of the typical quintessence forms. To mimic a cosmological constant, the scalar field must be in a very slow-roll state so that its kinetic energy is negligible. This requires that the Compton frequency of the scalar field is smaller than the Hubble rate, i.e., $m_\phi c^2 < \hbar H_0 \sim 10^{-33}$ eV, with m_ϕ being the mass of the scalar field. In this case, the scalar fifth force does not appear, but the scalar field is still used to act as the dark energy field. The drawback of the models is that the mass of the scalar field is too small. In any case, one can see that the Universe's accelerating expansion is independent of any gradient force of the scalar field. Thus, the interaction range of the scalar field does not play an important role in the observed cosmic acceleration.

In scalar-tensor theories, such as the chameleon [6] and symmetron [9] models, there is a coupling of the scalar field to matter, and then scalar fifth forces appear. Unfortunately, a corollary based on chameleon no-go theorems states that the chameleonlike fields cannot drive the observed cosmic acceleration [35,36]. A detailed analysis of the conclusion is given below.

A. Self-acceleration problem

The acceleration equation (14b) clearly shows that acceleration will take place when $2V(\phi_{\rm min}) > \rho_{\rm m}\hbar^3 c^5$. The acceleration is caused by the stable value of $2V(\phi_{\rm min})$, with $\phi_{\rm min}$ being the minimum of the effective potential. The coupling function indirectly affects the acceleration through the matter-density-dependent interaction potential.

Let us check the possibility of self-acceleration in our scheme. We need to introduce the Jordan frame, which, indeed, has been implied in the second term on the right-hand side of the action equation (1). The Jordan-frame metric $g^J_{\mu\nu}$ is related to the Einstein-frame metric $g_{\mu\nu}$ by the positive coupling function $A(\phi)$ as follows [35,36]:

$$g^J_{\mu\nu} = A^2(\phi) g_{\mu\nu}. \tag{47}$$

Self-accelerating theories attempt to attribute the observed (Jordan-frame) cosmic acceleration to the self-acceleration [35]. That is, the cosmic acceleration stems entirely from the conformal transformation shown in Eq. (47). The literature [35] has proven that this is impossible. This is one of the chameleon no-go theorems. Later in this subsection, we show that our model does not conflict with this no-go theorem, albeit the scalar field can account for the observed cosmic acceleration, as has been shown in Sec. IV.

To obtain the observable quantities in the Jordan frame, we need to use the following translation between the Einstein and Jordan frames:

$$a^J(t^J) = A(\phi)a(t), \tag{48a}$$

$$dt^J = A(\phi)dt, \tag{48b}$$

where a^J and t^J are the scale factor and the cosmic time in the Jordan frame, respectively.

Suppose that the scalar field is stable at the minimum of the effective potential; then the coupling function is completely determined by the density of matter due to $A(\phi) = A[\phi_{\rm min}(\rho)]$. In our scheme, when $w_i = 0$, the function $\phi_{\rm min}(\rho)$ has been shown as Eq. (22a). Following [35], in the case of a pressureless matter source, w can easily obtain that

$$\dot{a}^J = \dot{a} - 3\dot{a}\frac{d\ln A}{d\ln\rho}, \tag{49a}$$

$$\ddot{a}^J = \frac{\ddot{a}}{A}\left(1 - 3\frac{d\ln A}{d\ln\rho}\right) + \frac{9\dot{a}^2}{Aa}\frac{d^2\ln A}{(d\ln\rho)^2}, \tag{49b}$$

where ρ is scalar-field-independent matter density, $\dot{a}^J \equiv da^J/dt^J$, and $\ddot{a}^J \equiv d^2a^J/(dt^J)^2$, respectively. Apparently, if the expansion speed in the Einstein frame is equal to zero, i.e., $\dot{a} = 0$, the expansion speed in the Jordan frame is also equal to zero, i.e., $\dot{a}^J = 0$. However, if the acceleration in the Einstein frame is equal to zero, i.e., $\ddot{a} = 0$, the acceleration in the Jordan frame is no longer equal to zero, i.e., $\ddot{a}^J \neq 0$. This no-zero acceleration in the Jordan frame can be regarded as self-acceleration, i.e., a genuine modified gravity effect.

The existence of the self-acceleration implies that the transition redshifts calculated in Sec. IV should be corrected because cosmological observations are implicitly performed in the Jordan frame. To obtain the correction to the transition redshifts, one should use $\ddot{a}^J = 0$ rather than $\ddot{a} = 0$. The calculation of the transition redshifts can still be performed in the Einstein frame. After obtaining the transition redshifts z in the Einstein frame, one can use the following translation to obtain the transition redshift z^J in the Jordan frame,

$$1 + z^J = \frac{A_0}{A}(1 + z), \tag{50}$$

where $1 + z^J = a_0{}^J/a^J$ and the subscript 0 marks the current time. The Hubble parameter should also be converted into the appropriate frame. It is worth noting that the previous calculation in the last section is indeed in the Einstein frame. When the measured Hubble parameter (in the Jordan frame $H^J = \dot{a}^J/a^J$) is used in the calculation, it should be transformed into the Einstein frame as $H = \dot{a}/a$, which is ignored in the last section. According to Eqs. (19b) and (22a), the coupling function $A(\phi_{\rm min})$ is nearly fixed to the value of 1 in the Universe's past of due to $\rho \gg \lambda M_1{}^4 c^3/\hbar^3 \sim 10^{-30}$ kg/m^3. Therefore, the correction to the Hubble parameter related to the different frames can be approximately neglected [64]. We may introduce a notation $C(\rho)$ to mark the part of the self-acceleration in Eq. (49b) as follows:

$$C(\rho) = \frac{9\dot{a}^2}{Aa}\frac{d^2\ln A}{(d\ln\rho)^2}. \tag{51}$$

Of course, the second term in the parentheses on the right-hand side of Eq. (49b) can also contribute to the so-called self-acceleration. In any case, when the density of matter is larger than the current matter density, from Eq. (19b), together with Eq. (22a), one can easily obtain that $A(\phi_{\rm min}) \cong 1$. Therefore, self-acceleration is approximated to zero, and then the Jordan- and Einstein-frame metrics are indistinguishable. Substituting Eqs. (19b) and (22a) into Eq. (51), one can also obtain that $C(\rho = 4.5\times10^{-29}\ {\rm kg\cdot m^{-3}}) = 0$, and $C(\rho) > 0$ ($C(\rho) < 0$) when $\rho > 4.5\times10^{-29}\ {\rm kg\cdot m^{-3}}$ ($\rho < 4.5\times10^{-29}\ {\rm kg\cdot m^{-3}}$). If the matter density is small enough, self-acceleration (self-deceleration) occurs. Figure 5(a) shows that the value of

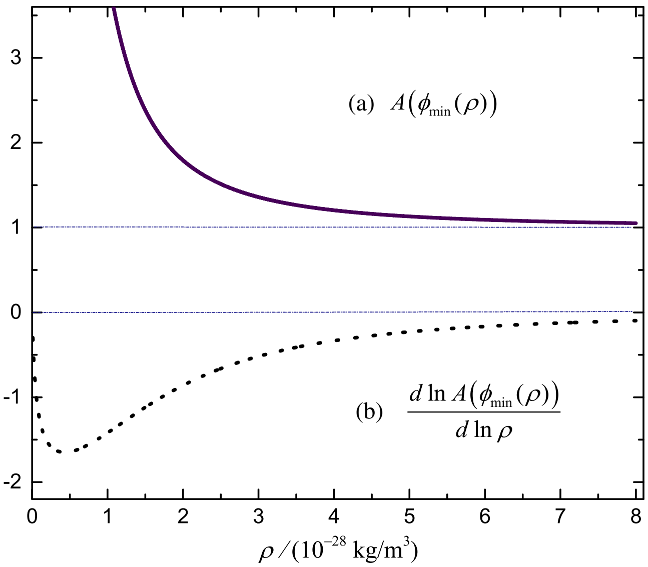

FIG. 5. (a) The matter-density-dependent value of the coupling function at the minimum of the effective potential. When the density of matter is large enough, e.g., larger than the current matter density of the Universe, the value of the coupling function is almost equal to 1, and the corresponding self-acceleration is nearly equal to zero. (b) The coefficient $(d\ln A)/(d\ln\rho)$ varies with the density of matter in the pressureless case. Since the coefficient is always negative, the cosmological observation value of the expansion speed (performed in the Jordan frame) is larger than the corresponding value obtained in the Einstein frame. The derivative of the coefficient with respect to the density of matter can roughly reflect the effect of self-acceleration. If the derivative is positive, the self-acceleration is positive in the Jordan frame, even if the corresponding acceleration in the Einstein frame is zero, and vice versa. When the density of matter is large enough, e.g., larger than the current matter density of the Universe, the coefficient and its derivative, with respect to the density of matter, are nearly equal to zero. Therefore, Jordan- and Einstein-frame metrics are indistinguishable before the current era for the pressureless fluid of matter sources.

the coupling function at the minimum varies with the density of matter in the pressureless case. Figure 5(b) describes the trend of self-acceleration varying with the density of matter.

From Eq. (49a), together with Fig. 5(b), one sees that $\dot{a}^J > \dot{a}$ due to $(d\ln A)/(d\ln\rho) < 0$. From Fig. 5, with the Universe expanding, the self-acceleration marked by Eq. (51) will occur in the future and then turn to self-deceleration, with the matter density decreasing further. Before the present time, since the coupling function nearly keeps a constant value of 1 (which is equivalent to the statement of $\Delta A/A \ll 1$ in [35]), the self-acceleration vanishes. Clearly, our scheme coincides with the second chameleon no-go theorem [35]. Unfortunately, based on the chameleon no-go theorem, a misleading corollary is deduced, which states that the chameleonlike scalar field cannot impact cosmological observations [35]. Apparently, the corollary conflicts with our scheme. By adopting a symmetry-breaking coupling function, the cosmological constant has been obtained in Sec. IV since the chameleon-like scalar field in our proposal is indeed also a quintessence field. The vanishing of self-acceleration does not mean that the quintessence effect of the scalar field must vanish. One cannot deduce the above corollary from the almost zero-self-acceleration. There is no convincing no-go theorem that hinders the establishment of a chameleonlike model to mimic the cosmological constant.

B. Overshooting problem

Since the minimum of the effective potential changes with time, a characteristic time has been introduced in Sec. III B to describe whether the minimum moves quickly or slowly. The changing rate of the minimum position has been naturally defined by $\dot{\phi}_{\min}/\phi_{\min}$. If the rate is smaller than the damping rate of $|3H(t)/2|$, with $H(t)$ being the Hubble parameter, the scalar field can adiabatically follow the minimum of the effective potential; on the contrary, overshooting must occur. The overshooting problem in the chameleonlike model is sometimes regarded as another no-go theorem, although it is not explicitly mentioned in [35,36].

In our scheme, it has been demonstrated that $|\dot{\phi}_{\min}/\phi_{\min}| \le |3H/2|$ [see Eq. (C10) in Appendix C]. The higher the density of matter, the smaller the changing rate of $\phi_{\min}$. This means that the scalar field sits more stably at the minimum in a higher density of matter. The transition redshift denoting the deceleration-acceleration transition has been estimated in Sec. IV. At the transition redshift, the matter density $\rho_{\mathrm{past}} = 1.17 \times 10^{-26}\ \mathrm{kg/m^3}$. That is, when matter density $\rho > \rho_{\mathrm{past}}$, the Universe is in the decelerating expansion phase, and vice versa. Near the transition density, the scalar field sits very stably at the time-dependent minimum due to $\rho_{\mathrm{past}} \gg \lambda M_1{}^4 c^3/\hbar^3 \sim 10^{-30}\ \mathrm{kg/m^3}$. We conclude that the large effective mass around the minimum results in the small changing rate of the minimum position and suppresses the possibility of overshooting.

Now, we explain why the overshooting occurs in the symmetron model [8], one of the chameleonlike models, in which the self-interaction is a symmetry-breaking potential and the coupling function is not a symmetry-breaking one, i.e.,

$$V(\phi) = -\frac{\mu^2 c^4}{2}\phi^2 + \frac{\lambda}{4}\phi^4, \tag{52a}$$

$$A(\phi) = 1 + \frac{1}{2M^2c^4}\phi^2. \tag{52b}$$

One can easily obtain the minima of their effective potential as follows:

$$\phi_{\min} = \begin{cases} 0 & \rho \geq \frac{\mu^2 M^2 c^3}{\hbar^3} \\ \pm[\frac{1}{\lambda}(\mu^2 c^4 - \frac{\rho \hbar^3 c}{M^2})]^{1/2} & \rho < \frac{\mu^2 M^2 c^3}{\hbar^3}. \end{cases} \quad (53)$$

The corresponding effective masses around the minima are

$$m_{\mathrm{eff}}^2 = \begin{cases} \frac{\rho \hbar^3}{M^2 c^3} - \mu^2 & \rho \geq \frac{\mu^2 M^2 c^3}{\hbar^3} \\ 2(\mu^2 - \frac{\rho \hbar^3}{M^2 c^3}) & \rho < \frac{\mu^2 M^2 c^3}{\hbar^3}. \end{cases} \quad (54)$$

It should be emphasized that the effective mass equals zero at the critical value of matter density.

The literature [8] has attempted to mimic the deceleration-acceleration transition redshift in the recent past by a phase transition when the effective mass vanishes. Apparently, the vanishing undoubtedly causes a serious overshooting. When the effective mass is far smaller than the Hubble rate in the mass scale, the stability of the minimum is too fragile to resist perturbations. A little energy from the perturbations can result in a quick moving speed of the minimum.

Substituting Eqs (52) and (53) into Eq. (C9b) in Appendix C, one has

$$\frac{\dot{\phi}_{\min}}{\phi_{\min}} = H \cdot \frac{3\hbar^3 \rho}{M^2 m_{\mathrm{eff}}^2 c^3}. \quad (55)$$

Since the effective mass $m_{\mathrm{eff}}^2 \to 0$ at the critical value of matter density, Eq. (55) gives the changing rate $\dot{\phi}_{\min}/\phi_{\min} \to \infty$. Thus, the minimum of the effective potential moves too quickly for the scalar field to follow it adiabatically during the expansion of the Universe. In no-adiabatic tracking cases, when the Compton frequency of the scalar field is smaller than the damping rate of $|3H/2|$, it undergoes an overdamped evolution; when the Compton frequency is larger than the damping rate, the scalar field undergoes underdamped oscillations. These two phenomena are what have been shown in the literature [8] near the transition redshift. In addition, when the symmetry-breaking potential (52a) is chosen as a quintessence field, its value at the minima is too small to drive cosmic acceleration.

The symmetronlike generalized potential and coupling function shown in [8] cannot avoid the overshooting problem because the effective mass still vanishes at the critical value of matter density.

C. Compton wavelength problem

At the beginning of this section, we have shown, by way of example, that the observed cosmic acceleration is ascribed to the self-interaction potential density of the scalar field rather than any form of the spatial gradient of the scalar field. Although the gradient may contribute its effect to the Universe locally, its resulting contribution to the cosmic acceleration should be zero. The reason for this is that the positive values of the gradient have to be offset by the corresponding negative values; otherwise the cosmological principle would be violated.

In our scheme, as long as the matter density is large enough, the minimum of the effective potential of the scalar field becomes almost matter-density independent [see Eq. (C7a) in Appendix C]. In this case of large matter density, even if the matter density varies with space, the minimum almost keeps the same value. Thus, dark energy behaves like the cosmological constant not only in the temporal scale but also in the spatial scale. Unlike the minimum, however, the corresponding effective mass of the scalar field is strongly dependent on the density of matter and is very large in the general case. The very large mass guarantees the stability of the minimum to perturbations. The large effective mass means that the Compton wavelength of the scalar field is short ranged, and the effects of the corresponding fifth force are considerably suppressed, which will be discussed in Sec. VII.

Although the symmetry-breaking coupling function has been used to explain the cosmological constant and will be used to explain the inflation of the Universe in Sec. VI, a seriously misleading problem of the short interaction range of the scalar field needs to be presented. Unfortunately, the problem cannot be clearly resolved by some mathematical equations, as we have done above, because it essentially results from the misunderstanding of the physical concept related to the accelerating expansion. It has always been considered that the scalar field must be light if it is to address the cosmological-constant problem [38]. This questionable viewpoint stems from two requirements. One results from the case in which the scalar field does not couple with matter, i.e., the traditional quintessence situation. A shallow potential is required so that the evolution of the scalar field can satisfy the slow-roll condition. This requirement is unreasonably extrapolated to the coupling case [38]. We call this the shallow-potential requirement. In the other case, the scalar field should mediate a long-range interaction so as to explain the acceleration expansion of the Universe [65–67]. We call the second requirement the long-range-interaction requirement.

The long-range-interaction requirement is not the same as the shallow-potential one. In the quintessence situation where the light scalar field does not interact directly with matter, the shallow potential guarantees that the quintessence field can roll down slowly; then its kinetic energy can be neglected. Thus, as long as the self-interaction potential is large enough, the quintessence model can mimic the cosmological constant. In the quintessence model, there is no long-range interaction. This also implies that a long-range interaction is not a necessary condition to explain the cosmological constant with a scalar field. Apparently, the shallow-potential requirement is not necessary in our scheme because the value of the self-interaction potential can now be localized by the symmetry-breaking coupling.

Therefore, in the rest of the section, we focus on whether or not the long-range-interaction requirement is necessary.

1. The Compton wavelength of the scalar field

We now review the second chameleon no-go theorem. The theorem is an upper bound on the chameleon Compton wavelength at the present cosmological density [35,36], which is given as follows:

$$\lambdabar_c \equiv \frac{\hbar}{m_{\rm eff} c} \lesssim \text{Mpc}. \tag{56}$$

In our model, the Compton wavelength is estimated to be about 5 μm at the present cosmological density, which of course satisfies the constraint denoted by Eq. (56). According to [35], any cosmological observable probing linear scales should see no deviation from general relativity in our model due to the short range. From the mathematical point of view, our model does not conflict with the chameleon theorems. Based on chameleon theorems, however, the literature [35] claims that chameleons have a negligible effect on the linear growth of structure and cannot account for the observed cosmic acceleration except as some form of dark energy. This creates a paradox since the cosmological constant is obtained in our chameleonlike setting.

How can one deduce the wrong physical corollary using the correct chameleon theorems? It is because of the misunderstanding of the concept of the energy density and pressure. One confuses the effect of the energy density and pressure with that of their gradient. Many works suggest that it is the long-range interaction that drives the accelerating expansion. However, as pointed out by [40], there are no forces in a homogeneous universe because the density and pressure are the same everywhere. To supply a force, some gradient is required. Energy density and pressure do not contribute any force to help the expansion along. It is the density and pressure that drive the Universe's accelerating expansion. One should not confuse the acceleration of the Universe's expansion with the acceleration of a test particle in a scalar field, where the force originates from the spatial gradient and which will be discussed in Sec. VII. It is worth noting that the fifth force is not the same as the pressure gradient force, albeit the fifth force is also a gradient force. A concrete example of the pressure gradient force is the buoyant force, while the scalar fifth force is a fundamental force.

The acceleration of the Universe is determined by all of the density and pressure, including the observable and nonobservable parts of the Universe, which has been discussed in Sec. IV B 3. The observable Universe is defined by a region with a radius that light can travel through during the lifetime of the Universe. Even the fastest light cannot establish the causal relationship in all parts of the Universe. Of course, a light scalar field cannot achieve it, either, due to its nonzero mass. It is unreasonable and unrealistic to suggest that using a light scalar field mediates a long-range interaction, and it is also in disagreement with the current precision tests of gravity that there is no evidence of the long-range fifth force. The requirement that the scalar field should be light is not necessary because the acceleration of the Universe is essentially related to the density and pressure of matter, the potential energy density, and the kinetic energy density of the scalar field. Only the potential energy density of the scalar field drives the Universe's accelerating expansion. There is no evidence that the interaction range plays an important role in the expansion. This conclusion can be derived directly from the acceleration equation (12) of the Universe, in which there is no term related to the Compton wavelength of the scalar field. The Compton wavelength of the scalar field is related to the second derivative of the effective potential with respect to the scalar field, while the acceleration of the Universe expansion is related to the self-interaction potential itself.

However, another long Compton wavelength is needed in our scheme, which will be discussed in the following.

2. The Compton wavelength of dark matter

The matter-density-dependent cosmological constant in the symmetry-breaking coupling model requires matter to permeate all of the Universe's space according to Eqs. (22) and (27). Due to the nature of the asymptote shown in Fig. 3, a locally concentrative distribution of matter cannot enhance the cosmological constant further, but the presence of voids that are completely empty of matter may lower the cosmological constant. The observed spatial independent cosmological constant implies that the distribution region of the complete voids is smaller if they exist and dark matter should be cold and/or fuzzy in the present time.

Since the rapid motion of relativistic particles may destroy the inhomogeneous seed structures generated in inflation, the current dark matter model assumes that dark matter gas is cold [39,68–70]. That is to say, its thermal velocity is negligible with respect to the Hubble flow [70]. But the mass m of its particles is not determined [70–72], which is widely ranging, for example, from the so-called axion, $\sim 10^{-6} - 10^{-4}$ eV [71], to weakly interacting massive particles (WIMPs), $\sim 10^2$–10^3 GeV [73]. The requirement that matter permeates all of space is not incompatible with the CDM model. When dark matter gas is cold, the thermal de Broglie wavelength $h/\sqrt{3mk_{\rm B}T}$, with Planck's constant h and Boltzmann's constant $k_{\rm B}$, of the particles can be large due to the small values of the root-mean-square speed in the low temperature T, and there will be a large extension of the wave functions for the particles. The colder the dark matter gas, the more notable the quantum effect of the gas. Thus, the dark matter wave can permeate the Universe everywhere.

Fuzzy dark matter is also permissible [74,75]. Its gas corresponds to ultralight particles, such as the mass of dark matter particles, $\sim 10^{-22}$ eV [74,75]. If the ultralight particles are fermions, the Fermi energy of the Fermi gas is ultrahigh, and the gas is in a state of complete quantum degeneracy, although it may also be a relativistic gas [53]. Thus, when a structure forms, it will be stable and nearly undisturbed by collision of another particle due to the Pauli principle. If they are bosons, since the characteristic temperature is inversely proportional to the particle mass [53], the characteristic temperature will be extremely high. Thus, when a structure forms, it will be a stable Bose-Einstein condensation. The lighter the particles, the more notable the quantum effect of the gas.

Of course, as time goes on, a nearly void distribution region may occur and even exceed the concentrative distribution region of matter, which will decrease the cosmological constant. Consequently, the Universe's expansion rate will decelerate and shift to a contraction phase. A detailed dark matter model that matches the scalar fifth force model needs further investigation.

3. Relationship between the cosmological constant and dark matter

As the isotropic microwave background has the same temperature, there exists light dark matter that distributes the space almost homogeneously and isotropically, at least at the Universe scale. As we have demonstrated, the cosmological constant can be obtained by the scalar field via the symmetry-breaking coupling to matter. Apparently, like the isotropic temperature of the microwave background, the current cosmological constant still needs an inflation era. The reason for this is given below.

Although the late-time acceleration is ascribed to the self-interaction potential of the scalar field in our setting, only light dark matter can localize the value of the self-interaction potential through the symmetry-breaking coupling. If most of the space is completely empty and dark matter is cluster distributed in space, the cosmological constant will be so small since the larger matter density cannot further enhance the cosmological constant. To acquire a cosmological constant, a homogeneous background is needed for dark matter. The homogeneous distribution of dark matter can be established during the inflationary era because different regions of the Universe are able to interact and move towards thermodynamics equilibrium. That is to say, temperature, pressure, and density of matter have the same values everywhere in the inflationary era.

If there is no light dark matter but only agglomerate ordinary matter, the current cosmological constant cannot be obtained from our setting. Our model is dependent on both the scalar field and dark matter with light mass. The light dark matter helps the Universe generate the homogeneous cosmological constant through the scalar field and the symmetry-breaking coupling to matter. The property of light mass [76–78] guarantees that dark matter fills the space everywhere. Even if dark matter itself is not very homogeneous, the cosmological constant is still spatially homogeneous and time independent, as long as the density of dark matter is large enough, i.e., $\rho \gg \lambda M_1{}^4 c^3/\hbar^3 \sim 10^{-30}\ \mathrm{kg/m^3}$.

If one wants a light mass field as a medium to permeate all the space of the Universe, it should be dark matter. One can say that the scalar field, though its Compton wavelength is short at the present cosmological density, makes a cosmological impact mediated by light dark matter. In this sense, the light dark matter seems to play a role to mediate a long-range interaction that does not really exist. In our setting, both of the Compton wavelengths indeed vary with time: The Compton wavelength of the scalar field will become longer and longer as the Universe expands, while the Compton wavelength of dark matter will increase, to some extent, limited by the value of the symmetry-breaking coupling function at $\phi = 0$.

Recently, fuzzy dark matter has become a topic of interest [79,80]. This means that the requirement of ultralight particles for dark matter is not a disadvantage of our setting.

D. The zero-point energy problem

Essentially, Weinberg's no-go theorem results from the problem of the zero-point energy of quantum field theory [30,31]. In order to evade Weinberg's no-go theorem, it is possible to use a new dynamical scalar field [19,20,31] (e.g., quintessence), instead of the assumption of the vacuum energy density, to mimic the cosmological constant. Therefore, we introduce a space-time-dependent scalar field that preserves Lorentz invariance to mimic a material-free dynamical vacuum instead of the traditional vacuum in quantum field theory. When the kinetic energy density of the scalar field is much less than its self-interaction potential density, the potential density is defined as a cosmological constant.

Of course, even if a new d.o.f. is introduced, one still faces the problem of the traditional zero-point energy. Since we require that the new d.o.f. accounts entirely for the cosmic acceleration, the zero-energy problem must be avoided in our scheme. The reason is that the lowest energy density of the system in question is always divergent whether the new d.o.f. is invoked or not. We call the scalar field here a dynamical vacuum to distinguish it from the lowest energy state of quantum field theory. Although the scalar field sits at the minimum of the effective potential in the case of pressureless matter fluid, the definition of the cosmological constant [see Eq. (16)] makes the dynamical vacuum different from the vacuum of quantum field theory. In Sec. VI, we discuss the case in which the scalar field departs from the minimum. Therefore, the dynamical vacuum does not always correspond to the lowest energy state of the system. Even so, we have to face the zero-energy problem since the current cosmological constant is related to the minimum of the effective potential.

The absence of the value of the static vacuum energy density in our scheme indicates that some counteracting mechanism is hidden and used in the model. The counteracting mechanism requires that the cosmological constant is related to the difference of the energy density values of the system in some parameter space rather than the zero-point energy density itself. The parameter can be naturally defined by a so-called self-coupling coefficient, which marks the strength of the self-interaction of the scalar field. In the self-interaction potential shown in Eq. (19a), λ can act as a parameter, in spite of the fact that its value has been determined in our scheme.

We now discuss how the scheme can avoid the zero-point energy problem. We assume that the scalar field has been sitting at the minimum of the effective potential. Without the self-interaction potential, the minimum is completely fixed as a constant. When the self-interaction is considered, the minimum is shifted, but the effective mass around the minimum keeps the same value as in the absence of the self-interaction. We can see from Eq. (22b) that m_{eff} does not depend on λ, which is a very important character to guarantee the validity of the following discussion. Note that $\lambda = 0$ denotes the absence of the self-interaction of the scalar field. Thus, if we apply the second quantization and discuss the lowest energy states of the coupled scalar field in both cases of $\lambda = 0$ and $\lambda \neq 0$, the biggest difference between the two cases is the λ-dependent lowest states. The λ dependence of $\phi_{\min}$ has been shown clearly in Eq. (22a). Since both the equation of motion for the scalar field and the effective mass around the minimum are just related to the partial derivative of potentials with respect to the scalar field, one naturally deduces that the choice of the zero potential energy should be arbitrary. However, in the traditional quintessence model, the self-interaction potential needs to be small or zero at the value of ϕ where $V_{,\phi}(\phi) = 0$ [81]. Weinberg argues that theories of quintessence offer no explanation as to why this should be the case [81].

Let us give a brief explanation of our model. Assume that the first derivative of the self-interaction potential has a λ-dependent form as follows:

$$V_{,\phi}(\phi) = \lambda f_{,\phi}(\phi). \tag{57}$$

If one wants to discuss the case without the self-interaction potential, one can let $\lambda = 0$, which corresponds to a completely conformal transformation theory. When the scalar field is added to the physical system, the minimum of the effective potential should be shifted. Then the value of the self-interaction potential corresponding to the minimum appears, which indeed is the definition of the cosmological constant when the scalar field sits at the minimum, i.e.,

$$\Lambda_{\mathrm{E}}^4 = \int_{\phi_{\min}(\lambda=0)}^{\phi_{\min}(\lambda\neq0)} V_{,\phi}(\phi_{\min}) d\phi_{\min}. \tag{58}$$

Thus, the self-interaction potential has a general form as follows:

$$V(\phi) = V_0 + \lambda f(\phi), \tag{59}$$

where V_0 is an integration constant. If V_0 has any observable effect, it must be the other form of energy. Consequently, if one genuinely wants to use the coupled scalar field to entirely describe the cosmological constant without any other form of energy to drive the cosmic acceleration, one must choose $V_0 = 0$. That is, the self-interaction potential should have the form $V(\phi) = \lambda f(\phi)$. Otherwise, the other form of energy will contribute to dark energy. This is the main reason that we have chosen $V(\phi) = \lambda\phi^4/4$ without V_0.

In brief, the strong constraint that the coupled scalar field should account entirely for the observed cosmic acceleration leads to the following mathematical requirements: The cosmological constant should vanish if there is no scalar field, i.e., $\phi = 0$; the cosmological constant should also vanish if there is no self-interaction potential in the presence of the scalar field, i.e., $\lambda = 0$.

Of course, the scalar field does not need to stay in the lowest energy state according to its equation of motion. Due to the expansion, however, the scalar field can be damped to sit at the lowest energy state. In this situation, the cosmological constant is defined by the part of the energy-density difference between the two cases of the lowest energy state: one with the self-interaction potential and the other without the self-interaction potential. Since the effective masses are the same in both cases, the parts of the zero-point energy density exactly cancel each other out in the subtraction. The left part in the subtraction is a sum of the two expectation values in the lowest energy state of the system with the self-interaction. The two expectation values include both the expectation value of the interaction potential energy density with matter and the expectation value of the self-interaction potential energy density of the scalar field. It is the expectation value of the self-interaction potential density that acts as the cosmological constant due to the scalar field stably sitting at the minimum.

E. Summary

The scalar field can entirely account for the cosmic acceleration. Our model does not conflict with chameleon no-go theorems, at least mathematically. But our model conflicts with the corollary that a chameleonlike scalar field cannot account for the observed cosmic acceleration. The corollary gives a misleading conclusion for the following reasons. First, the negligible self-acceleration shown by one of the no-go theorems *only* implies that a (narrowly defined) appropriate value of the self-interaction potential of the scalar field is needed to drive the Universe's accelerating expansion. It does not imply that none of the chameleonlike scalar fields can entirely explain the cosmological constant. Second, the short chameleon Compton wavelength shown by the other no-go theorem also misleadingly implies that chameleonlike fields have a negligible effect on the large scale of the Universe. This misleading corollary results from the assumption that only a long-range force mediated by the scalar field can impact the cosmic acceleration. However, it is the pressure of the scalar field that drives the cosmic acceleration rather than any force from the pressure gradient or from the field gradient. Third, the overshooting problem in the symmetron model can be avoided in our setting due to the heavy effective mass of the coupled scalar field.

At the end of this section, we show briefly that the zero-point energy density exactly cancels out in our scheme, and the expectation value of the self-interaction potential in the lowest energy state acts as the cosmological constant. Thus, Weinberg's no-go theorem is circumvented.

VI. APPLICATION OF THE MODEL TO THE CONTRACTION OF THE UNIVERSE ($H^- < 0$)

We have obtained the cosmological constant, which is dependent on the adiabatic condition. In the pressureless fluid of the matter source, the adiabatic condition always holds. In this section, we discuss the unavailability of the adiabatic condition of the scalar field when the matter source becomes hotter and hotter due to the contraction of the Universe.

In Sec. IV, we deduced that the Universe might be closed, i.e., $K = 1$. In the closed space, the Universe must pass through both $\dot{\phi} = 0$ and $H = 0$, which is schematically shown in Fig. 4. Consequently, if the scalar field that accounts for the late-time acceleration is used as an inflation field, one of the slow-roll conditions $|\ddot{\phi}| \ll 3|H\dot{\phi}|$ is apparently not satisfied. In fact, in the slow-roll approximation in inflation, one often requires that neither $\dot{\phi}$ nor the Hubble parameter passes through zero [46]. Therefore, we discuss the unavailability of the slow-roll approximation in this section.

A. The mechanism of the quasicyclic Universe

The end of the expansion that has been discussed in Sec. IV is also the beginning of the contraction. After the expansion stops ($\dot{a} = 0$, $\ddot{a} < 0$) in the closed Universe, the contraction process will start and the nonrelativistic matter density will increase. The scalar field can adiabatically follow the minimum of the effective potential at the initial stage of the contraction period. In this stage, the value of the self-interaction potential at $\phi_{\rm min}$ still acts as the cosmological constant. With the density of nonrelativistic matter increasing, the cosmological constant approaches a fixed value, which has been estimated by using $\rho \to \infty$ to be $\Lambda_{\rm E} = 2.242$ meV $\quad (\Lambda = 1.093 \times 10^{-52}\ {\rm m}^{-2})$. However, $\rho \to \infty$ for $w_i = 0$ is just a mathematical construction. In the contraction process, the parameter w_i of the equation-of-matter state must depart from zero due to the collision of matter particles.

1. Start of the negative-damping evolution of the scalar field

With further contraction, violent particle collisions occur, and the temperature becomes higher and higher. As a result, the parameter of the equation of state for matter fluid is no longer equal to zero. The better stability of the minimum of the scalar field at the higher matter density gradually becomes weaker and weaker until entering an unstable stage, which can be seen by Eq. (C17) in Appendix C.

In this nonadiabatic tracking case, with the density of matter increasing further, the kinetic energy density of the scalar field has to increase much more quickly to store the redundant part of the scalar field energy increased during the contraction process. The field can no longer remain stable at the minimum, which can be described by the negative damping oscillation shown in Figs. 2(b) and 4(b)–4(d). The negative damping, which exponentially grows the magnitude of the oscillation rather than attenuating the magnitude, is one of the most prominent characteristics of the contraction process. It is worth noting that, although the adiabatic condition of $|\dot{\phi}_{\rm min}/\phi_{\rm min}| \leq |3H/2|$ no longer holds, the oscillation condition of $|3H/2| \leq \omega_{\rm c}$ still holds when the oscillation is activated by the quick movement of $\phi_{\rm min}$. Thus, the start of the under-negative-damping oscillation stems from the adiabatic instability. With matter fluid approaching the extremely relativistic case of $w_i \to 1/3$, the effective mass of the scalar field approaches zero, which can be seen by Eq. (C14) in Appendix C. Since both the adiabatic condition and the oscillation condition are no longer satisfied in the relativistic case, over-negative damping occurs.

2. Energy exchange between the scalar field and matter

Let us now analyze the energy exchange among the scalar field, matter, and gravitational field more mathematically. In the absence of any coupling to the scalar field, both the density and the temperature of matter would grow due to the contraction of the Universe, and it can be regarded as adiabatic compression for matter systems according to thermodynamics. However, the presence of interactions between the scalar field and matter allows for the conversion of the scalar field energy density and matter energy density. By using Eqs. (7)–(9), the coupled equations are easily obtained as follows (the detailed derivation is shown in Appendix B):

$$\dot{\rho}_\phi + 3H^-\left(\rho_\phi + \frac{p_\phi}{c^2}\right) = -\sum_i \rho_i \frac{dA^{1-3w_i}(\phi)}{dt}, \tag{60a}$$

$$\dot{\rho}_{\rm m} + 3H^-\left(\rho_{\rm m} + \frac{p_{\rm m}}{c^2}\right) = \sum_i \rho_i \frac{dA^{1-3w_i}(\phi)}{dt}. \tag{60b}$$

In Eq. (60),

$$\rho_\phi = \frac{V(\phi)}{\hbar^3 c^5} + \frac{\dot{\phi}^2}{2\hbar c^5}, \tag{61a}$$

$$p_\phi = -\frac{V(\phi)}{\hbar^3 c^3} + \frac{\dot{\phi}^2}{2\hbar c^3} \tag{61b}$$

are the scalar field energy density and the pressure of the scalar field, respectively [20]. And

$$\rho_{\mathrm{m}} = \sum_i \rho_i A^{1-3w_i}(\phi), \tag{62a}$$

$$p_{\mathrm{m}} = \sum_i p_i A^{1-3w_i}(\phi) \tag{62b}$$

are the mass density and pressure of the matter fluid, respectively. If only pressureless matter sources $w_i = 0$ are considered, matter energy density and pressure can be simplified as $\rho_{\mathrm{m}} = \rho A(\phi)$ and $p_{\mathrm{m}} = pA(\phi)$, respectively. The superscript "−" on the right-hand side of the Hubble parameter H in Eq. (60) is used to emphasize that the Hubble parameter describes a contraction process of the Universe. If the adiabatic condition is not satisfied due to the increase of temperature, the scalar field will undergo a negative-damping motion in the contraction process. It is the contraction in the adiabatic instability case that can drive the negative-damping motion of the scalar field. Equation (60) is different from Eq. (4.88) in Ref. [31] because the total energy density is defined by $\rho + \rho_\phi$ and is considered to be conserved, as shown in Eqs. (4.82) and (4.83) in that work. The total energy should be $\rho_{\mathrm{m}} + \rho_\phi$ rather than $\rho + \rho_\phi$, which is discussed in Appendix B.

In the contraction process, besides the increase of the energy density of both matter and the scalar field due to the contraction, there also exists a complicated energy exchange between matter and the scalar field as shown in Eq. (60). In the adiabatic instability case, the energy exchange is more complicated and can be roughly classified into two types: One corresponds to the high-frequency oscillations of the scalar field in the case of under-negative damping, which can cause the rapid energy exchanges between matter and the scalar field and, accordingly, obtain much more energy from matter heated by the contraction and the oscillating field; the other corresponds to the exponential growth magnitude in the cases of negative-damping oscillations (such as over-negative-damping evolution, critically negative damping oscillations, and the exponential growth profile of the curve of under-negative-damping oscillations).

During high-frequency oscillations of the scalar field, the attractive gravitational effect grows rapidly. If we average both the kinetic energy and the self-interaction potential of the scalar field in Eqs. (8) and (12) over a period of time that is large compared with the period of oscillation, it can be easily proven that the high-frequency oscillating field corresponds to a nearly pressureless fluid [39]. Thus, with the magnitude growing exponentially, a rapid contraction occurs. Consequently, not only is the adiabatic condition not satisfied, but also the oscillation condition is no longer satisfied due to the huge Hubble parameter and the extremely small mass of the scalar field. In this case, the field will experience an over-negative damping motion with huge kinetic energy density. It is well known that, when the kinetic energy density is far larger than the potential energy, the parameter of the equation of state for the scalar field approaches 1 [see Eq. (B22)]. The scalar field will now generate an attractive gravitational effect that is more effective than matter; then a more dramatic contraction will occur, which may be called deflation in contrast with inflation.

The scalar field can also be used to drive the inflation of the Universe, which will be discussed in Sec. VI B.

3. *Minimum radius of the Universe corresponding to the maximum of the scalar field*

The contraction heats matter and causes energy to transfer from matter to the scalar field. When $w_i(T) \to 1/3$, matter particles become relativistic, and matter fluid decouples from the scalar field. After the decoupling is completed, the Universe continues contracting. The vigorous scalar field will climb up along its self-interaction potential to the maximum value $V(\phi_{\max})$, with $\phi_{\max} > 0$ [one could equivalently consider the case $\phi_{\max} < 0$ as shown in Figs. 4(c) and 4(d)] and then roll down from the maximum. It should be emphasized that the maximum value $V(\phi_{\max})$ exceeds the initial maximum of the kinetic energy density that corresponds to the zero value of the self-interaction potential of the scalar field. The reason is that the scalar field can also acquire additional energy from the gravitational field due to the contraction. If the scalar field passes through its zero value, overshooting would occur due to the zero mass of the scalar field. Since the Hubble rate at this time approaches huge values, this very light mass marks a over-negative-damped evolution of the scalar filed. The higher the Hubble parameter, the stronger the scalar field absorbing energy from the gravitational field.

During the contracting process, there exists a transition from accelerating contraction ($\dot{a} < 0$, $\ddot{a} < 0$) to deceleration contraction ($\dot{a} < 0$, $\ddot{a} > 0$) due to the increase of the potential of the scalar field. Then the Universe gradually approaches the end of the decelerating contraction. At the end of the decelerating contraction, the Universe will reach its minimum radius corresponding to $\dot{a} = 0$ and $\ddot{a} > 0$. Substituting $\dot{a} = 0$ and $w_i = 1/3$ into Eq. (5) and assuming that the scalar field is close to the maximum of its self-interaction potential, i.e., $V(\phi_{\max})$ with $\dot{\phi} = 0$, the minimum radius of the Universe can be estimated by

$$a_{\rm min} = \left(\frac{3\hbar^3 c^7}{8\pi G(V(\phi_{\rm max}) + \hbar^3 c^5 \rho_{\rm max})}\right)^{1/2}. \quad (63)$$

The scalar-field-independent matter density $\rho_{\rm max} = \sum_i \rho_i$ will also reach its highest value due to the minimum volume of the Universe. However, when the Universe departs from its minimum volume, the energy density of matter decreases more rapidly than the potential energy density of the scalar field. Thus, to roughly estimate the minimum radius of the Universe, one may neglect the matter density in Eq. (63), which gives

$$a_{\rm min} = \left(\frac{3\hbar^3 c^7}{8\pi G V(\phi_{\rm max})}\right)^{1/2}. \quad (64)$$

Substituting $\dot{\phi} = 0$ and $w_i = 1/3$ into Eq. (12), the accelerating expansion of the Universe at its minimum radius is given by

$$\frac{\ddot{a}_{\rm min}}{a_{\rm min}} = \frac{8\pi G}{3\hbar^3 c^5}(V(\phi_{\rm max}) - \hbar^3 c^5 \rho_{\rm max}). \quad (65)$$

According to the astronomical observation and the big bang nucleosynthesis calculation, we can deduce the decoupling temperature as $T \geq 0.1$ MeV [82]. Therefore, the scalar field will approach its maximum of $|\phi_{\rm max}| \geq$ 0.1 MeV and correspondingly climb to the maximum of its self-interaction potential density of $V(\phi_{\rm max}) \geq (0.1\ {\rm MeV})^4/4!$ because of $V(\phi) = \phi^4/4!$ shown in Eq. (19a). The upper bound of the scalar field is plausibly assumed as the reduced Planck energy $M_{\rm Pl} c^2 = 2.4 \times 10^{18}$ GeV with $M_{\rm Pl} \equiv (\hbar c/8\pi G)^{1/2}$, and the corresponding self-interaction potential density is $V(\phi_{\rm max}) = (2.4 \times 10^{18}\ {\rm GeV})^4/4!$. Correspondingly, the minimum radius of the Universe is estimated from Eq. (64) to be

$$10^{-33}\ {\rm m} \leq a_{\rm min} \leq 10^{12}\ {\rm m}. \quad (66)$$

When the scalar field achieves its maximum value, the equation of state $w_\phi \equiv p_\phi/(\rho_\phi c^2)$ for the scalar field equals -1 due to $\dot{\phi}^2 = 0$.

4. Maximum of the cosmological constant for relativistic matter fluid

Comparing Eq. (12) with the acceleration of the Universe in the ΛCDM model, the maximum of the self-interaction potential acts as the maximum of the cosmological constant due to the zero kinetic energy of the scalar field, i.e.,

$$\Lambda^4_{\rm Emax} \equiv V(\phi_{\rm max}) = \frac{1}{4!}\phi^4_{\rm max}. \quad (67)$$

In addition, $\rho_{\rm m} = \rho$ is obtained due to $w_i = 1/3$, which indicates that matter particle masses are no longer affected by the scalar field. This conclusion is natural because $w_i = 1/3$ describes the decoupling case for the scalar field and matter. It is interesting to note one aspect of the coupling case $w_i = 0$. According to Eq. (22a), when the matter density is so large that $|\phi_{\rm min}|$ reaches $M_2 c^2$, $A(\phi_{\rm min})$ then approaches 1. As a result, $\rho_{\rm m} = \rho$ almost completely holds in the coupling case, which implies that matter particle masses are hardly affected by the scalar field. The larger the matter density, the less the influence of the scalar field on the particle mass. This does not mean that the interaction does not play an important role in this density case. It localizes the scalar field around $|\phi_{\rm min}|$. The confinement strength can be described by the curvature of the effective potential shown in Eq. (22b). Therefore, the confinement strength depends on the density of matter. The larger the matter density, the stronger the confinement. The curvature of the effective potential around the minimum is not related to the self-interaction potential as mentioned in Sec. III B 1. However, in the case of $w_i = 1/3$ here, when the scalar field climbs to the highest value along its self-interaction potential, the unstable point also corresponds to a curvature which is completely dependent on the self-interaction potential. One cannot confuse the two types of masses of the scalar field.

Only in the case where the kinetic energy is considerably smaller than the potential energy of the scalar field can we introduce a cosmological constant. In the decoupling case of $w_i = 1/3$, when the scalar field climbs to its highest potential value, it must return to a rolling-down phase along the self-interaction potential curve. Then the maximum cosmological constant is not a stable value (see Fig. 4). In the coupling case of $w_i = 0$, however, the scalar field is trapped in the minimum of the effective potential. Since the role of the cosmological constant is played by the value of the self-interaction potential at the minimum, which is shown by Eq. (18), it becomes stable and appears as a constant during the Universe's evolution.

Unlike the stable cosmological constant for $w_i = 0$, we cannot completely determine the maximum cosmological constant for $w_i = 1/3$ due to the lack of decoupling parameter.

5. The quasicyclic universe

When the constraints of the cosmological constant are satisfied, the symmetry-breaking coupling function between matter and the scalar field is determined in obtaining the fifth force. For the sake of completeness, the scheme has also been extended to the cosmic evolution, which leads to a quasicyclic universe. There are many alternative scenarios to the standard inflationary paradigm, such as bouncing cosmologies [83] and cyclic universes [84,85]. However, our quasicyclic model corresponding to a regular scalar field differs from not only the bouncing cosmologies but also the cyclic universes. To realize a contracting phase, many bouncing and cyclic models violate the null energy condition $\rho_\phi c^2 + p_\phi > 0$ [83–85]. For example, a negative potential energy is introduced in [84], while a ghost field with negative kinetic energy is introduced in [85].

B. Biggest challenge in the scheme

Until now, there have been no real challenges specific to the scheme. However, when the scalar field is used to drive the inflation of the Universe, the biggest challenge is encountered. On the one hand, the quartic form of the self-interaction potential of the scalar field in our scheme is a convex function, i.e., $V_{,\phi\phi}(\phi) > 0$ shown in Eq. (19a). On the other hand, according to [4], in the framework of standard single-field inflationary models with Einstein gravity, the most probable candidate shape of the self-interaction might be a concave potential, i.e., $V_{,\phi\phi}(\phi) < 0$. In this sense, it seems impossible to use our scheme to drive the inflation of the Universe. The observed data are strongly dependent on the assumption of the slow-roll parameters. However, the slow-roll parameters are not always valid in our scheme. We analyze the possibility that the "observed concave behavior" can be explained by the self-interaction potential with a convex property.

1. *Solutions to equations of motion near the neighborhood of the minimum radius*

It is worth noting that the method of using the scalar field as a time variable [39,46,47] will not work here due to the assumption that $\dot{\phi}$ does not pass through zero. In our models, the repulsive gravitational effect starts at the beginning of the decelerating contraction process of the Universe. When the potential energy of the scalar field gradually grows and exceeds the sum of matter energy and the kinetic energy of the field in the climbing-up process along the self-interaction potential curve, the Universe shifts from accelerating contraction to decelerating contraction. Unfortunately, there has been no investigation of this climbing-up model at the present time, to our knowledge. With the temperature increasing, from an under-negative-damping oscillation around one of the minima of the effective potential, the scalar field shifts to a climbing-up phase. After reaching the highest value along the self-interaction potential, the scalar field will roll down. Near the minimum radius of the Universe, there does not exist a stable minimum of the effective potential. Thus, the stable vacuum expectation value of the scalar field in the effective potential minimum is then meaningless. Dynamical vacuum energy has been used in the previous section and can be used to characterize the time-variable value of the self-interaction potential of the scalar field, which accounts for both the late-time acceleration and inflation. Figure 6 shows the sketch of the dynamical vacuum energy appearing in the climbing-up and rolling-down models of the scalar field.

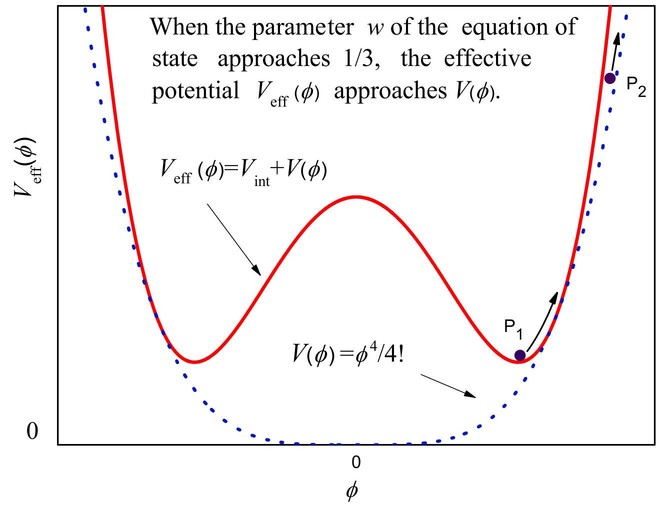

FIG. 6. The sketch of the dynamical vacuum energy that appears in the climbing-up process of the scalar field along the self-interaction potential. With the temperature increasing during the contraction process of the Universe, the scalar field becomes vigorous and ultimately decouples from matter. In the coupling case, the scalar field can oscillate around one of the minima P_1 of the effective potential. With the Universe contracting further, the vigorous scalar field decouples from matter and climbs up along its self-interaction potential (see also P_2), which results in the repulsive gravitational effect.

Near the neighborhood of the minimum radius of the Universe, since $V(\phi_{\rm max}) \gg \hbar^2\dot{\phi}^2/2 + \sum_i \rho_i \hbar^3 c^5$, Eqs. (8) and (9) become

$$\frac{\dot{a}}{a} \equiv H^\pm = \pm\left(\frac{c^2}{a_{\rm min}^2} - \frac{c^2}{a^2}\right)^{1/2}, \tag{68a}$$

$$\hbar^2\ddot{\phi} + \frac{1}{3!}\phi_{\rm max}^3 = 0. \tag{68b}$$

Therefore, the evolutions of the radius and the scalar field with cosmic time are, respectively, obtained as

$$a(t) = a_{\rm min}\cosh\left[\frac{c(t-t_{\rm c})}{a_{\rm min}}\right], \tag{69a}$$

$$\phi(t) = \phi_{\rm max} - \frac{1}{12}\phi_{\rm max}^3\left(\frac{t-t_{\rm c}}{\hbar}\right)^2, \tag{69b}$$

where $t_{\rm c}$ stands for the shift time from contracting to expanding and corresponding to the minimum radius of the Universe. It is the self-interaction potential of the scalar field that drives the Universe's evolution rather than the Hubble parameter since the Hubble parameter equals zero at the minimum radius of the Universe.

Apparently, one of the slow-roll conditions $|\ddot{\phi}| \ll 3|H\dot{\phi}|$ in the literature, such as [39,49], is not satisfied and is not necessary due to the zero Hubble rate. Because the next rapid expansion always follows the last rapid contraction of the Universe, one does not need to worry about the amount of inflation being sufficient or not. The slow-roll condition, however, is satisfied at the time when the kinetic energy becomes nearly the same value as the potential energy of the scalar field. Thus, whether the slow-roll condition is valid or not is determined by the time of the expansion process.

Near the shift time t_c, both the scale factor and the scalar field have time-reversal symmetry. The characteristic time of the scalar field near its maximum is $\tau_\phi \sim \hbar/|\phi_{\max}|$, while the characteristic time of the cosmic scale factor near its minimum is $\tau_a \sim a_{\min}/c$. Thus, $\tau_a/\tau_\phi \sim M_{\rm pl}c^2/|\phi_{\max}|$. If $|\phi_{\max}| = M_{\rm pl}c^2$, both of the characteristic times are the same as the Planck time. If $|\phi_{\max}|$ equals the calculation temperature of big bang nucleosynthesis, 0.1 MeV [82], the two characteristic times become larger. But, the characteristic time of the cosmic scale factor near its minimum value is considerably larger than that of the scalar field. On the face of it, this seems to mean that the Universe will remain in the phase of the minimum radius for a longer time compared to the time it stays in the maximum of the scalar field. However, noticing that the Universe's radius near the minimum increases or decreases in the form of exponential growth, one may conclude that a slow evolution of the scalar field corresponds to a fast evolution of the Universe's radius in this case. Therefore, the total time period near the minimum radius of the Universe is not very different for the two physical quantities. In the language of inflation, it is the slow evolution of the scalar field that drives the fast inflation of the Universe. Unfortunately, the property of slow evolution of the scalar field is always extrapolated to such an extent that $|\ddot{\phi}|$ needs to be neglected, i.e., $|\ddot{\phi}| \ll 3|H\dot{\phi}|$, which is apparently not the same as the case near the maximum value of the scalar field, i.e., $|\ddot{\phi}| \gg 3|H\dot{\phi}| = 0$. Thus, one should be careful in applying the traditional slow-roll conditions to the closed space.

The solutions described by Eqs. (69a) and (69b) can be rewritten as useful alternative forms by using the scalar field as an independent variable. Since the traditional slow-roll conditions are not always valid due to the zero Hubble rate, the slow-roll parameters are not appropriate for describing the inflation for the closed Universe. Notice that the method shown in [46] cannot be used because it assumes that $\dot{\phi}$ does not change sign during inflation. We have the case that both $\dot{\phi}$ and H pass through zero. Substituting Eq. (69b) into Eq. (69a) to eliminate the time t, the ϕ-dependent $a(\phi)$ near the minimum radius of the Universe can be obtained as follows:

$$a(\phi) = a_{\min}\cosh\left[\frac{\hbar c}{a_{\min}}\left(\frac{12(\phi_{\max}-\phi)}{\phi_{\max}^3}\right)^{1/2}\right],\quad \text{for } |\phi| \le |\phi_{\max}|. \tag{70}$$

Substituting Eq. (70) into Eq. (68a) and squaring it, one has

$$H^2(\phi) = \frac{c^2}{a_{\min}^2}\tanh^2\left[\frac{\hbar c}{a_{\min}}\left(\frac{12(\phi_{\max}-\phi)}{\phi_{\max}^3}\right)^{1/2}\right],\quad \text{for } |\phi| \le |\phi_{\max}|. \tag{71}$$

Apparently, $H^2(\phi)$ is a concave function of ϕ. It is worth noting that these solutions are valid if $V(\phi_{\max}) \gg \hbar^2\dot{\phi}^2/2 + \sum_i \rho_i \hbar^3 c^5 \approx \hbar^2\dot{\phi}^2/2$. Since the value of the self-interaction potential corresponding to the minimum radius of the Universe is extremely large, they are indeed good approximations around the minimum.

2. The pseudopotential density

To investigate inflation using our scheme, we neglect the density of matter and introduce a pseudopotential in the Friedmann equation. Equation (5) is then rewritten as follows:

$$H^2 = \frac{8\pi G}{3\hbar^3 c^5}\left[\left(V_{\rm pse}(\phi) + \frac{\hbar^2}{2}\dot{\phi}^2\right)\right], \tag{72}$$

where the pseudopotential

$$V_{\rm pse}(\phi) \equiv V(\phi) - \frac{3\hbar^3 c^7}{8\pi G}\frac{1}{a^2(\phi)} \tag{73}$$

with $K = 1$. Therefore, the pseudopotential plays the role of the real potential in Eq. (1) of Ref. [46] in the inflation of the Universe. In the traditional treatment for inflation, however, it is always assumed that the space is flat [46].

3. A short review of slow-roll parameters

The first Hubble slow-roll parameter and the $(n+1)$st Hubble slow-roll parameter are given, respectively, as follows [39,46,47]: $\epsilon_1 = -\dot{H}/H^2$ and $\epsilon_{n+1} = \dot{\epsilon}_n/H\epsilon_n$ with $n \ge 1$. Apparently, the Hubble slow-roll parameters are just valid when the radius of the Universe is large enough to render the curvature term small, but the term cannot be neglected completely [see Eq. (A7) in Appendix A]. Noticing Eq. (A7) and neglecting the effect of matter in the equation, one can easily see that only the positive curvature of the Universe can give a positive value of $\dot{H}$ as long as $\hbar^2\dot{\phi}^2/2 < \hbar^3 c^7/[8\pi G a^2(\phi)]$. However, this does not mean that inflation can only occur in the closed space since inflation is described by $\ddot{a}$ rather than by $\dot{H}$, although the closed space is of interest in this section only. If enough inflation has occurred so that $3\hbar^3 c^7/[4\pi G a^2(\phi)] - 2\hbar^2\dot{\phi}^2 \ll V(\phi)$ (in fact, this means that the minimum radius is large enough to hold the inequality in small kinetic energies), the Hubble slow-roll parameters are good approximations. However, the curvature term must not vanish. If the curvature term completely vanishes, there is no difference between the pseudopotential and the self-interaction potential.

When the pseudopotential is introduced, the traditional slow-roll parameters for the real potential of the scalar field should be replaced by the pseudopotential. Thus, the first pseudopotential slow-roll parameter and the second pseudopotential slow-roll parameter are given, respectively, as follows:

$$\epsilon_V = \frac{M_{\rm Pl}^2 c^4}{2}\left(\frac{V_{{\rm pse},\phi}}{V_{\rm pse}}\right)^2, \quad (74a)$$

$$\eta_V = \frac{M_{\rm Pl}^2 c^4 V_{{\rm pse},\phi\phi}}{V_{\rm pse}}. \quad (74b)$$

4. The pseudopotential with a concave property

We now demonstrate how the convex potential we adopted can have a concave property. Substituting Eq. (71) into Eq. (72) and neglecting the kinetic energy density of the scalar field, one has

$$V_{\rm pse}(\phi) \simeq \frac{3\hbar^3 c^5}{8\pi G} H^2(\phi). \quad (75)$$

Thus, $V_{\rm pse}(\phi)$ is really a concave function of ϕ due to the concave property of $H^2(\phi)$, although $V(\phi)$ is a convex function. A further detailed investigation using the convex potential to analyze the concave behavior is needed.

In fact, the conclusion can be directly obtained from the definition of the pseudopotential marked by Eq. (73). The minimum value of the pseudopotential corresponds to the minimum radius of the Universe, which can be demonstrated by substituting Eq. (70) into Eq. (73). In this sense, the observed concave property implies that the Universe is a closed space. Planck evidence for a closed Universe has also been shown very recently in [86] from the presence of an enhanced lensing amplitude in cosmic microwave background power spectra.

Figure 7 shows the rolling down and climbing up of the scalar field along the pseudopotential and the self-interaction potential, respectively, near the maximum value of the scalar field.

C. Summary

Using the quartic self-interaction potential and the symmetry-breaking interaction potential, the quasicyclic Universe model is obtained. We have introduced the pseudopotential to describe the evolution of the Universe near its minimum radius. The pseudopotential is the sum of the self-interaction potential density and the energy density scale of the positive curvature of the Universe. The observed potential concave property is explained by the pseudopotential rather than the self-interaction potential only. Thus, the prediction of the closed space is favored by the observation data.

VII. SCALAR FIFTH FORCE

We have determined the parameters of the symmetry-breaking interaction model under the constraints of the cosmological constant. Since the cosmological constant is based on the homogeneous and isotropic assumption, the gradient terms in the scalar equation of motion are ignored. If ordinary agglomerate matter couples to the scalar field in the same way as dark matter does, the gradient force cannot be ignored and might be tested in the laboratory, provided that the experiments are designed properly. In this section, we discuss the characteristics of the fifth force, such as its strength, interaction range, thin shell, and saturation effects.

In order to obtain the modified geodesic equation for nonrelativistic matter particles in the case of the

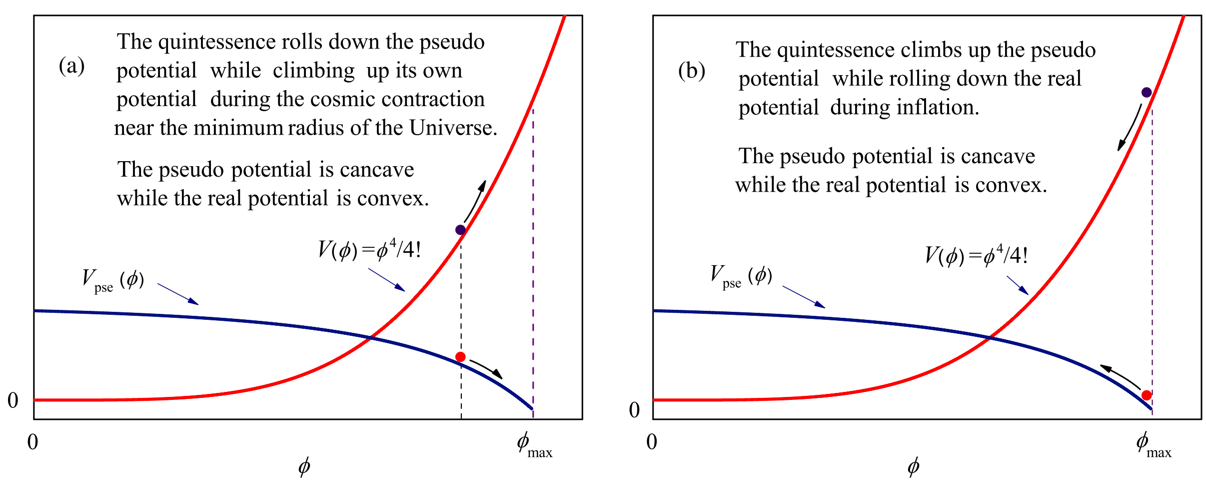

FIG. 7. The self-interaction potential and the corresponding pseudopotential near the maximum value of the scalar field. (a) The case of the contraction of the Universe near the minimum radius of the Universe; (b) the case of the expansion near the minimum radius of the Universe, i.e., inflation.

inhomogeneity and anisotropy, instead of the line element of Eq. (4), we use a linearly perturbed form, Eq. (2), with two potentials Φ and Ψ in the weak-gravitational-field limit. Then, one gets the scalar field fifth force shown in Eq. (3). Since the fifth force occurs in the extremely small scale compared with the scale factor of the Universe, it is not necessary to introduce the time-variable scale factor in Eq. (2). In the equation of motion for the scalar field, the generally covariant d'Alembertian operator $D^{\mu}D_{\mu}$, where D_{μ} is the covariant derivative with respect to the Einstein metric, can be approximated to the common form as $D^{\mu}D_{\mu} = \nabla^2 - c^{-2}\partial_t^2$ in the weak gravitational field [87]. Thus, to compute the scalar field in the case of the weak gravitational field, it is sufficient to use the Minkowskian line element as $ds^2 = -c^2dt^2 + d\vec{x}^2$. For a static, space-variable density of a nonrelativistic matter source, the scalar equation of motion that results from the action (1) is given as follows:

$$\hbar^2 c^2 \nabla^2 \phi = V_{\mathrm{eff},\phi}(\phi). \tag{76}$$

A. Klein-Gordon equation for a massive scalar field

In this subsection, we solve one of the two biggest challenges—that the upper bound of the linear coupling coefficient is about 10^{14} [17]. The other challenge of the observed potential concave feature has been solved in Sec. VI B.

In order to distinguish the roles of the homogeneous ambient density and the space-variable source density that generates the spatial gradient of the scalar field, we may imagine a scalar field profile induced by the source embedded in the medium of background density ρ_{b}. Since the scalar-field-independent matter mass density is frequently used in this section, it is often called matter density or density for conciseness. In the homogeneous background, $\nabla^2\phi = 0$, and the equilibrium value of the scalar field is $\phi_{\mathrm{b}} = \phi_{\min}(\rho_{\mathrm{b}})$, corresponding to a minimum of the effective potential. Assuming the density of the source $\rho = \rho_{\mathrm{b}} + \delta\rho$, but not assuming $\delta\rho \ll \rho_{\mathrm{b}}$, we can still expand the field around the background value $\phi = \phi_{\mathrm{b}} + \delta\phi$. The reason is that matter density in laboratory experiments is always larger than the current density of the Universe. Correspondingly, the scalar field is almost fixed, and $\delta\phi \ll \phi_{\mathrm{b}}$ [see also Fig. 3, and Eqs. (22a) and (84)]. An equation of motion for a massive scalar field from Eq. (76) is then obtained as follows:

$$\left(\nabla^2 - \frac{m_{\mathrm{eff}}^2 c^2}{\hbar^2}\right)\delta\phi = A_{,\phi}(\phi_{\mathrm{b}})\hbar c^3 \delta\rho, \tag{77}$$

where the effective mass is

$$m_{\mathrm{eff}}^2 \equiv m_{\mathrm{eff}}^2(\phi_{\mathrm{b}}) + \frac{A_{,\phi\phi}(\phi_{\mathrm{b}})\delta\rho\hbar^3 c^5}{c^4} \tag{78}$$

with $m_{\mathrm{eff}}^2(\phi_{\mathrm{b}}) = V_{\mathrm{eff},\phi\phi}(\phi_{\mathrm{b}})/c^4$. If $\delta\rho \ll \rho_{\mathrm{b}}$, then $m_{\mathrm{eff}}^2 = m_{\mathrm{eff}}^2(\phi_{\mathrm{b}})$. However, in general, the density of the source is always larger than that of the background, i.e., $\delta\rho \gg \rho_{\mathrm{b}}$. Consequently, $m_{\mathrm{eff}}^2 \gg m_{\mathrm{eff}}^2(\phi_{\mathrm{b}})$ in the source region. This means that the equation of motion is different from the usual Klein-Gordon equation in which the value of the mass is the same everywhere. If the masses in the equation of motion for the scalar field vary in space, the linear superposition principle is not valid.

1. Matter-density-dependent interaction range

From the Klein-Gordon equation (77), the interaction range is naturally defined by $\lambda\!\!\!^{-}_{\mathrm{c}} \equiv \hbar/(m_{\mathrm{eff}}c)$. The density-dependent interaction range has been estimated by Eq. (25), which can be rewritten with the background density ρ_{b} as an approximate expression in the following:

$$\lambda\!\!\!^{-}_{\mathrm{cb}}[\mathrm{m}] \approx \frac{1.648 \times 10^{-19}}{(\rho_{\mathrm{b}}[\mathrm{kg/m^3}])^{1/2}}. \tag{79}$$

Apparently, in the common density, the interaction range is so short that the fifth force is suppressed from local tests of gravity [65–67].

We know the four fundamental interactions well: the gravitational, electromagnetic, strong, and weak interactions. The gravitational and electromagnetic ones produce long-range forces. The strong and weak ones produce forces at subatomic distances and govern nuclear interactions. The fifth force discussed here has a density-dependent interaction range. Even in the extreme vacuum space, such as in the case of the current matter density of the Universe, the interaction range is estimated to be about 5 μm. It is worth noting that the short-range force is not responsible for the Universe's acceleration. The force results from the space inhomogeneity of the matter distribution. It is the pressure of the scalar field that drives the Universe's acceleration.

2. Matter-density-dependent coupling coefficient β

Substituting $\phi = \phi_{\mathrm{b}} + \delta\phi$ into Eq. (3), the acceleration of a test particle due to the scalar field becomes

$$\vec{a} = -c^2 \frac{A_{,\phi}(\phi)}{A(\phi)}\nabla\delta\phi \approx -c^2 \frac{A_{,\phi}(\phi_{\mathrm{b}})}{A(\phi_{\mathrm{b}})}\nabla\delta\phi. \tag{80}$$

The coefficient $A_{,\phi}(\phi)/A(\phi)$ defines a coupling coefficient $\beta(\phi)$ to characterize the strength of the fifth force [5,10,17,37], i.e.,

$$\beta(\phi) \equiv M_{\mathrm{Pl}} c^2 \frac{A_{,\phi}(\phi)}{A(\phi)} \approx M_{\mathrm{Pl}} c^2 \frac{A_{,\phi}(\phi_{\mathrm{b}})}{A(\phi_{\mathrm{b}})}. \tag{81}$$

Since $\beta(\phi)$ corresponds to the slope of the coupling function $A(\phi)$, it only describes the linear part of the nonlinear coupling function [17]. Due to the nonlinear property of $A(\phi)$, the coupling coefficient $\beta(\phi)$ is not a constant, and it varies with ϕ. For the symmetry-breaking coupling function of Eq. (19b), the coupling coefficient is derived in Appendix C in Eq. (C19). For density far larger than the current matter density of the Universe, the density-dependent coupling coefficient is estimated from Eqs. (C19), (19b), and (22a) as

$$|\beta| \approx \frac{1.15\times 10^4}{\rho_{\mathrm{b}}[\mathrm{kg/m^3}]}. \tag{82}$$

For density smaller than the current matter density of the Universe, Eq. (82) is not valid. A complete expression for β is shown by Eq. (C19). In addition, a mass scale M_{m} is often used to describe the coupling strength [17], which is defined by

$$M_{\mathrm{m}} \equiv \frac{M_{\mathrm{pl}}}{|\beta|}. \tag{83}$$

Note that M_{m} is not a constant either due to the nonlinear property of $A(\phi)$.

Obviously, the absolute value of β can be far bigger than $O(1)$ in a wide density region. Strong coupling to matter does not mean that it must not satisfy experimental constraints, which has been discussed in [88]. For lower density in the local environment, the magnitude of the fifth force is much larger than gravity. Figure 8 shows the coupling β versus the ambient density of matter.

The huge values of β may mislead one to think that the interaction should be easily experienced, and this contradicts the absence of an observable interaction. The coupling β has a shortcoming that it just describes the linear property of the nonlinear coupling function. This shortcoming can be partly remedied by the parameter of the Compton wavelength of the scalar field. The Compton wavelength estimated by Eq. (79) is, in general, very short, which leads to the absence of an observable interaction.

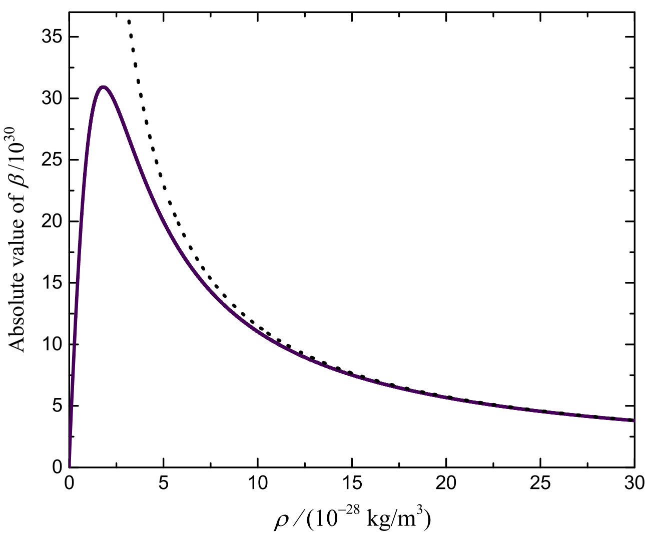

FIG. 8. The coupling $|\beta|$ versus the ambient density of matter. The solid curve corresponds to a complete (and a little complicated) expression of Eq. (C19), while the dotted curve corresponds to an approximation of Eq. (82) to estimate the order of magnitude of the coupling. When the density of matter approaches zero, the approximate expression is no longer valid.

3. The constraint on β from the precision measurements

The authors of Ref. [17] have claimed to derive the "upper" bound of $|\beta|$. In fact, Ref. [17] reported the equivalent "lower" bound of $M_{\mathrm{m}} > 10^4\ \mathrm{GeV}/c^2$ from the precision measurements of hydrogenic energy levels. It is a strong constraint. But it should be emphasized that, since the coupling coefficient is dependent on ambient density, the description "lower" is not appropriate, especially in the case of low ambient density.

We now explain the constraint using our scheme. For hydrogen atoms, the mass density of the electron cloud around the atomic nucleus can be estimated as $\rho_{\mathrm{b}} \sim m_{\mathrm{e}}/(4\pi r_0^3/3) \approx 1.468\ \mathrm{kg/m^3}$, where $r_0 = 5.29\times 10^{-11}$ m and m_{e} are the Bohr radius and electron mass, respectively. The mass density of the electron cloud should screen the scalar field perturbation by the pointlike density of the atomic nucleus. Substituting the background density of the electron cloud into Eq. (82), the coupling coefficient is estimated to be $|\beta| \approx 7.836\times 10^3$. By using Eq. (83) the mass scale of the coupling is obtained as $M_{\mathrm{m}} \approx 3\times 10^{14}\ \mathrm{GeV}/c^2$, which satisfies the constraint of $M_{\mathrm{m}} > 10^4\ \mathrm{GeV}/c^2$ [17]. These satisfactory results are due to the large ambient density of $1.468\ \mathrm{kg/m^3}$.

B. Screening effects on the scalar fifth force

To detect the fifth force, one must uncover the shield on the scalar field. Of course, all the screening effects to shield the fifth force essentially originate from the symmetry-breaking interaction between the scalar field and matter. We now discuss the two important screening mechanisms.

1. Saturation effect at a high density

Besides the short interaction range, the fifth force is also suppressed by the saturation effect discussed in the following. The symmetry-breaking interaction potential acts like a trap that confines the value of the scalar field falling in the range of either $(0, M_2c^2)$ or $(-M_2c^2, 0)$ in the static situation, where $M_2 = 4.96168\ \mathrm{meV}/c^2$. As an example we choose the case of VEV > 0 (one can also discuss the case of VEV < 0). From Eq. (22), one can see that $\phi_{\min}$ is almost independent of the density of matter and approaches M_2c^2 as long as the matter density is large enough, i.e.,

$$\rho \gg \frac{\lambda {M_1}^4 c^3}{\hbar^3} \simeq 5.722\times 10^{-30}\ \mathrm{kg/m^3}. \tag{84}$$

No matter how violently the matter density varies in space, as long as it is large enough, the fifth force denoted by Eq. (80) vanishes due to the scalar field keeping almost the same value. This may be called the saturation effect. Therefore, the strength of the fifth force cannot be further enhanced by continuously increasing the density of sources. It is unnecessary to use high density metal as a source to induce the scalar field. But for detecting the fifth force in the laboratory, an ultrahigh vacuum is necessary.

2. Thin shell of the scalar fifth force

In an unbounded homogeneous background, the scalar field always equals $\phi_{\min}$ at any space point, and the acceleration on a test particle due to the scalar field interaction is zero. When a test particle traveling in one medium with matter density ρ_1 impinges on another medium with a different matter density ρ_2, it experiences a scalar fifth force. For simplicity, we assume that the boundary surface is the plane $z = 0$, and the test particle travels in the $+z$ direction from the left region 1 of the boundary surface to the right region 2. It is helpful to gain intuition on how the fifth force is localized near the interface between the two mediums. The region is called a thin shell [89–92]. Inside medium 1 the scalar field $\phi = \phi_{\min 1}$ and inside medium 2 the scalar field $\phi = \phi_{\min 2}$; the gradient along the z direction near the interface can be roughly estimated as

$$\frac{d\phi}{dz} \sim \frac{\phi_{\min 2} - \phi_{\min 1}}{\lambdabar_{c1} + \lambdabar_{c2}}. \tag{85}$$

Thus, an asymptotic solution to the scalar field equation of motion can be as follows:

$$\phi(z) = \frac{\phi_{\min 2}}{2}\left(1 + \tanh\frac{z}{\zeta}\right) + \frac{\phi_{\min 1}}{2}\left(1 - \tanh\frac{z}{\zeta}\right). \tag{86}$$

Then, the gradient of ϕ along the $+z$ direction is

$$\nabla\phi \equiv \frac{d\phi}{dz}\hat{z} = \frac{\phi_{\min 2} - \phi_{\min 1}}{2\zeta}\operatorname{sech}^2\left(\frac{z}{\zeta}\right)\hat{z}, \tag{87}$$

where $\hat{z}$ is the unit vector of the coordinate z and ζ denotes an effective range of the fifth force. If the effective fifth force length ζ is defined by

$$\zeta = \lambdabar_{c2} + \lambdabar_{c1}, \tag{88}$$

with $\lambdabar_{c2}$ and $\lambdabar_{c1}$ corresponding to the Compton wavelengths for ρ_2 and ρ_1, respectively, then near the surface, $d\phi/dz \sim (\phi_{\min 2} - \phi_{\min 1})/(2\zeta)$, as expected in Eq. (85), except for the factor 2.

The fifth force mainly appears in a thin-shell region around the boundary surface. The thin-shell effect suppresses the scalar fifth force away from the boundary surface. Figure 9 depicts the thin-shell region through both (a) $\phi(z)$ and (b) $d\phi(z)/dz$. The thin-shell length can be defined by the effective range ζ of the fifth force.

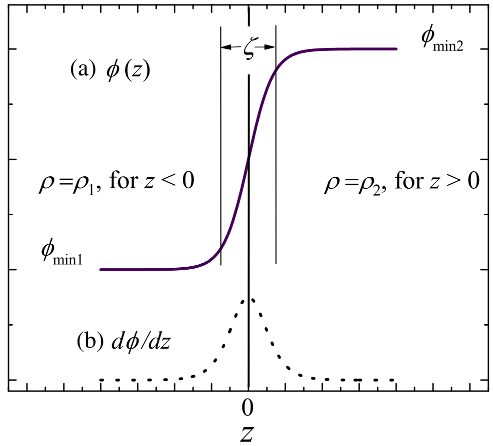

FIG. 9. The thin shell near the boundary surface $z = 0$. (a) $\phi(z)$ versus z. (b) $d\phi(z)/dz$ versus z. Here, ζ denotes the thin-shell length. The fifth force is strongly localized in space, with a maximum on the boundary surface $z = 0$ and falling rapidly to zero for $|z| > \zeta$.

All in all, the peak magnitude of the gradient shown in Eq. (87) is proportional to the difference of $(\phi_{\min 2} - \phi_{\min 1})$. Due to the saturation effect, the largest difference $(\phi_{\min 2} - \phi_{\min 1})$ is M_2c^2, which cannot increase further. Although the peak magnitude of the gradient is inversely proportional to the thin-shell length ζ, a narrow thin shell makes a sharp decay of the fifth force away from the boundary surface. However, if experiments are properly designed in which test objects are able to pass through the thin shell [18] and performed in the ultrahigh vacuum [93], the fifth force might be detected due to the bigger value of the coupling β.

C. Approximate solution in the one-dimensional case

Now, we estimate the magnitude order of the scalar fifth force more quantitatively in the case where the density difference between the source and background is much smaller than the homogeneous background, that is, $\delta\rho \ll \rho_b$. In this case, the effective mass of the coupled scalar field can be seen as the same value in all of space, i e., $m^2_{\rm eff} = m^2_{\rm eff}(\phi_b)$, and Eq. (77) is indeed a Klein-Gordon equation. Therefore, the linear superposition principle is valid. Supposing that the source is homogeneously filled in the region between the two planes $z = -z_0$ and $z = z_0$,

$$\delta\rho(z) = \begin{cases} \delta\rho_0 & |z| \le z_0 \\ 0 & |z| > z_0, \end{cases} \tag{89}$$

with z_0 and $\delta\rho_0$ positive values, the solution to Eq. (77) can be easily obtained as follows:

$$\delta\phi(z) = A_{,\phi}(\phi_b)\hbar c^3 \int_{-\infty}^{+\infty} \delta\rho(\eta) g(z;\eta) d\eta$$
$$= A_{,\phi}(\phi_b)\hbar c^3 \delta\rho_0 \int_{-z_0}^{+z_0} g(z;\eta) d\eta, \ \text{for}\ \delta\rho_0 \ll \rho_b, \quad (90)$$

where the Green function

$$g(z;\eta) = -\frac{\lambdabar_{cb}}{2} \exp\left(-\frac{|z-\eta|}{\lambdabar_{cb}}\right) \quad (91)$$

with the Compton wavelength $\lambdabar_{cb}$ corresponding to the ambient density ρ_b. The scalar fifth force along the $+z$ direction can be obtained by Eq. (80) as follows:

$$a(z) = -\frac{\beta(\phi_b)}{M_{Pl}} \frac{d\delta\phi(z)}{dz}. \quad (92)$$

1. Estimation of the value of the thin shell

Since the interaction range calculated by Eq. (79) is extremely small in the common background density of the laboratory case, we consider another extreme case where the interaction range can achieve about the order of magnitude of 10 μm so that one can gain a quantitative picture. In addition, although the extreme background vacuum might not be achieved in the laboratory, it is reasonable that in the dilute gas of atoms the space among the atoms must be extreme vacuums, which can be used to design the test experiment for the scalar fifth force.

Of course, if dark matter permeates everywhere, the current minimum density should be $\simeq 2.7 \times 10^{-27}$ kg · m^{-3}, and the corresponding interaction range is 3×10^{-6} m, which is too small to draw a clearly illustrated graph. To clearly display the character of the fifth force, we choose a lower value of the background density of $\rho_b = 2 \times 10^{-29}$ kg · m^{-3} (the corresponding Compton wavelength $\lambdabar_{cb} = 3.684 \times 10^{-5}$ m). We also choose $\delta\rho_0 = 1 \times 10^{-31}$ kg · m^{-3} $\ll \rho_b$ and $z_0 = 2 \times 10^{-4}$ m to calculate $\delta\phi(z)$ and $a(z)$. The calculated results of $\delta\phi(z)$ and $a(z)$ are shown in Figs. 10(a) and 10(b), respectively.

One can see that two thin-shell regions appear on the two boundary surfaces in Fig. 10. The maximum value of $\delta\phi(z)$ in Fig. 10(a) appears at the center plane $z = 0$ of the source, which is denoted by $\delta\phi_{max}$; the maxima of $|a(z)|$ in Fig. 10(b) appear at the two boundary surfaces of $|z| = 2 \times 10^{-4}$ m. The positive (negative) value of the force describes its movement along the $+z$ ($-z$) direction, and the maximum value at $z = -z_0$ is denoted by a_{max} in Fig. 10(b).

The source region acts as an attractive trap due to $\delta\rho_0 > 0$. Oppositely, if $\delta\rho_0 < 0$, the region acts like a repulsive barrier. Whether the length of the source region is larger than the Compton wavelength $\lambdabar_{cb}$ or not, the fifth force is indeed equal to zero on the center plane of $z = 0$.

2. Maximum value of the fifth force at the boundary surface

As long as $\delta\rho_0 \ll \rho_b$, both $\delta\phi_{max} \equiv \delta\phi(z=0)$ and $a_{max} \equiv a(z=-z_0)$ are proportional to $\delta\rho_0$ described by Eqs. (90) and (92). When $\delta\rho_0 > \rho_b$, Eqs. (90) and (92) are no longer valid. However, it can be deduced that $\delta\phi_{max}$ cannot exceed M_2c^2 since the value of the scalar field falls in the range of $(0, M_2c^2)$. The gradient of $\delta\phi(z)$ can be estimated by $\delta\phi_{max}/\lambdabar_{cb}$. Then the gradient cannot exceed $M_2c^2/\lambdabar_{cb}$. Consequently, one can deduce that a_{max} cannot exceed $M_2c^2/(M_m\lambdabar_{cb})$ even if $\delta\rho_0 \to +\infty$.

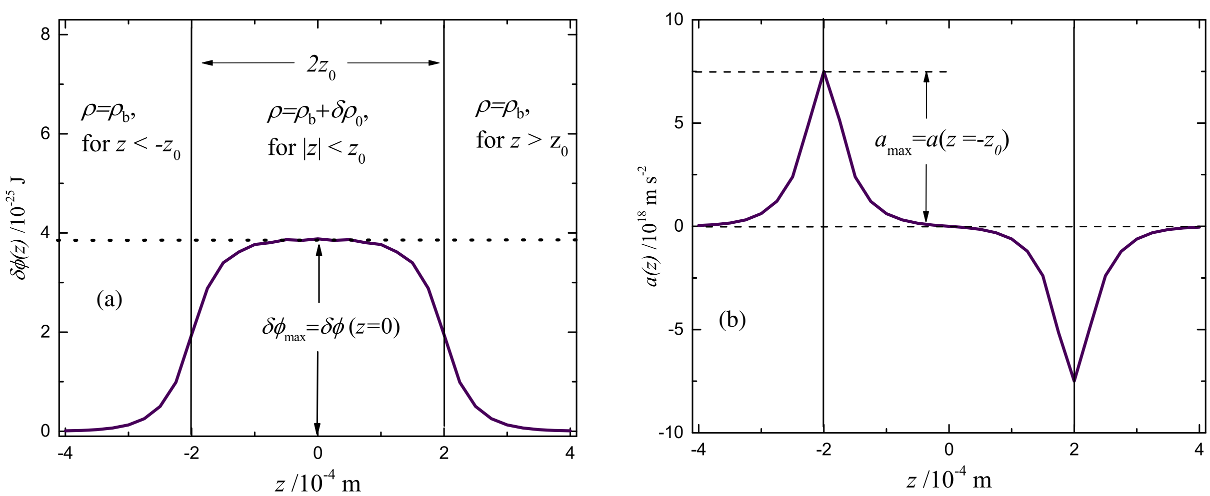

FIG. 10. (a) The scalar field difference $\delta\phi(z)$ along the z direction. The maximum difference appears at the $z = 0$ plane. (b) The scalar fifth force along the z direction. The two maxima of the absolute value of the fifth force are at the two boundary surfaces $z = -2 \times 10^{-4}$ m and $z = 2 \times 10^{-4}$ m, respectively, but the directions of the two forces are opposite. Since the Compton wavelength is too short in the case of large matter density, we plot the pictures in the very small density of matter. That is, $\rho_b = 2 \times 10^{-29}$ kg · m^{-3} and $\delta\rho_0 = 1 \times 10^{-31}$ kg · m^{-3} $\ll \rho_b$, respectively.

We now return again to the case of $\delta\rho_0 \ll \rho_b$. Both $\delta\phi_{max}$ and a_{max} are dependent on the background density ρ_b, but they do not increase with the increase of the background density. To see the density dependence trend, we fix $\delta\rho_0 = 1 \times 10^{-31}$ kg $\cdot$ m$^3 \ll \rho_b$ and plot the curves of $\delta\phi_{max}$ and a_{max} versus ρ_b. The background density dependence of $\delta\phi_{max}$ and a_{max} is shown in Figs. 11(a) and 11(b), respectively.

The curve of $\delta\phi_{max}$ versus ρ_b is a monotonically decreasing function shown in Fig. 11(a). However, the curve of a_{max} versus ρ_b is a concave function shown in Fig. 11(b), and the maximum point appears at $\rho_b \approx 5 \times 10^{-30}$ kg/m^3. When the background density $\rho_b \to 0$ or $\rho_b \to \infty$, the fifth force rapidly approaches zero.

3. Estimation expression of the average fifth force in the thin shell

To get an intuitive picture of how the scalar fifth force is dependent on the background density in the case of $\delta\rho_0 \ll \rho_b$, we derive a concise approximation expression. Since the force is localized in the thin shell, we now consider an average acceleration in this region rather than the maximum value of the acceleration discussed above. The average gradient of the scalar field along the z direction is roughly estimated as follows:

$$\left\langle \frac{d\phi}{dz} \right\rangle \sim \frac{\phi_{min}(\rho_b + \delta\rho_0) - \phi_{min}(\rho_b)}{\lambda\!\!\!^{-}_{cb}}, \tag{93}$$

where the density-dependent ϕ_{min} is described by Eq. (22a). From Eq. (92), one gets the average acceleration in the thin shell as follows:

$$\bar{a} \sim \frac{|\beta(\rho_b)|}{M_{Pl}} \left\langle \frac{d\phi}{dz} \right\rangle. \tag{94}$$

For $\delta\rho_0 \ll \rho_b$, Eq. (94) becomes

$$\bar{a} \sim \frac{|\beta(\rho_b)|}{M_{Pl}} \frac{d\phi_{min}}{d\rho}\bigg|_{\rho=\rho_b} \frac{\delta\rho_0}{\lambda\!\!\!^{-}_{cb}}. \tag{95}$$

For $\rho_b \gg 10^{-30}$ kg/m^3 and $\delta\rho_0 \ll \rho_b$, using Eqs. (22a), (79), and (82), we get a concise expression as

$$\bar{a}[\mathrm{m/s^2}] \sim \frac{3.668 \times 10^{-20}}{(\rho_b[\mathrm{kg/m^3}])^{5/2}} \delta\rho_0[\mathrm{kg/m^3}]. \tag{96}$$

It can be seen that the fifth force decreases rapidly as the ambient density increases. Although the condition of $\rho_b \gg 10^{-30}$ kg/m^3 can always be easily satisfied in the laboratory, $\delta\rho_0 \ll \rho_b$ cannot be easily achieved.

4. Estimation expression of the fifth force away from a solid surface

In general, when $\delta\rho_0 \geq \rho_b$, Eqs. (95) and (96) are no longer valid. However, one can still use Eqs. (93) and (94) to roughly estimate the average acceleration in the thin shell. In the most experimental designs, the source objects are solid. The test objects cannot pass through the region of the thin shell but can only sense the fifth force away from the surface. Supposing that the boundary surface is the plane $z = z_0$, and noticing Eq. (91), the acceleration of a test object to linear order from the action (1) is then given by

$$a(z) = a(z_0)e^{-|z-z_0|/\lambda\!\!\!^{-}_{cb}}. \tag{97}$$

The average acceleration in the thin shell can be defined by

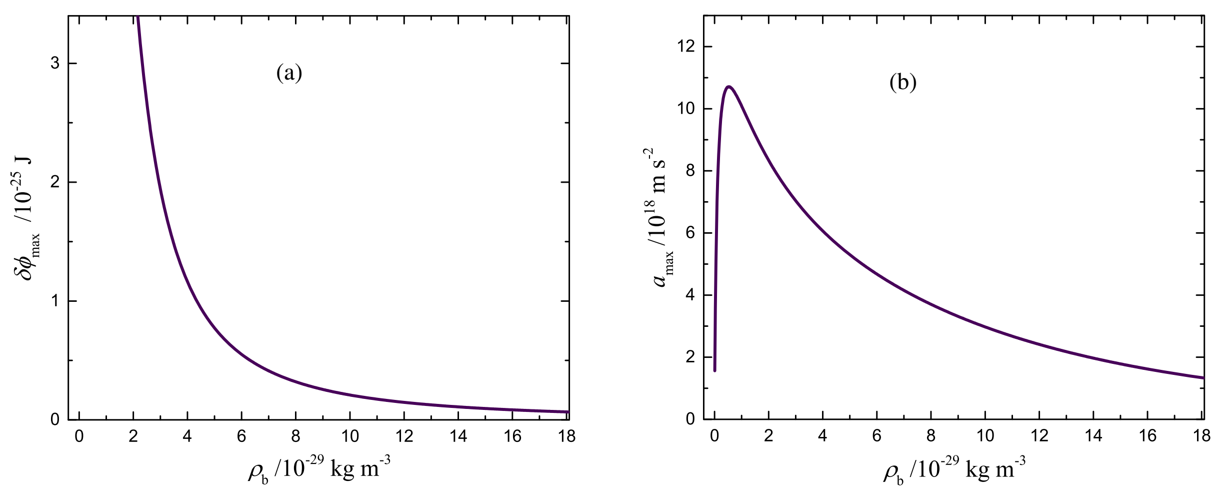

FIG. 11. (a) $\delta\phi_{max}$ versus ρ_b. This is a monotonically decreasing function. When $\delta\rho_0$ is fixed, the difference of the scalar field deep inside and outside of the source decreases rapidly with the ambient density increasing. (b) a_{max} versus ρ_b. This is a concave function. When $\delta\rho_0$ is fixed, the acceleration on a test particle approaches zero if the ambient density approaches either zero or infinity.

$$\bar{a} \equiv \frac{1}{\lambda_{\rm cb}} \int_{z_0}^{z_0+\lambda_{\rm cb}} a(z)dz = a(z_0)(1 - e^{-1}) = 0.632a(z_0). \qquad (98)$$

Noticing Eqs. (93) and (94), Eq. (97) then becomes

$$a(z) \sim -1.582 \frac{\beta(\rho_{\rm b})}{M_{\rm Pl}} \frac{\phi_{\rm min}(\rho_{\rm b} + \delta\rho_0) - \phi_{\rm min}(\rho_{\rm b})}{\lambda_{\rm cb}} e^{-|z-z_0|/\lambda_{\rm cb}}. \qquad (99)$$

Both the saturation factor of $[\phi_{\rm min}(\rho_{\rm b} + \delta\rho_0) - \phi_{\rm min}(\rho_{\rm b})]$ and the exponential attenuation factor of $\exp(-\frac{|z-z_0|}{\lambda_{\rm cb}})$ suppress the scalar fifth force. Thus, the fifth force is too small to be detected [38,79]. Apparently, the main screening factor in detecting the fifth force is the short-ranged interaction.

Since the fifth force mainly appears in the thin-shell region, it might be detected if the designed experiments allow test objects to pass through the region of the thin shell. To obtain a huge value of the coupling coefficient $\beta(\rho_{\rm b})$, an extremely high background vacuum in the laboratory is necessary. If dark matter permeates all of the space of the Universe, the current minimum density should be as small as $\sim 10^{-27}$ kg · m^{-3}, and the upper bound of the force range is ~ 5 μm.

D. Summary

The scalar fifth force is investigated in our model; in particular, some concise approximate expressions of the fifth force are deduced. Under the requirements that the coupled scalar field must account for the observed cosmic acceleration, the interaction range of the fifth force is extremely short, even at the current density of the Universe. Therefore, the local test of gravity is satisfied. To test the force in the laboratory, experiments may be designed that allow test objects to pass through the thin shell of the source.

We have also explained the so-called upper bound of $|\beta| \sim 10^{14}$ from the precision measurements of hydrogenic energy levels. It is worth noting that this value is not an upper bound due to the matter-density dependence of $|\beta|$. Note that $|\beta|$ can acquire very huge values in the extremely low density of matter. For the case of the precision measurements of hydrogenic energy levels, the calculated value of our model is $\sim 10^4$, which is considerably smaller than the upper bound of $\sim 10^{14}$. Because the mass density of the electron cloud around the atomic nucleus is used in our model, the satisfactory result implies that both ordinary matter (e.g., the electron) and dark matter can couple to the scalar field in the same manner.

VIII. DISCUSSIONS AND CONCLUSIONS

A. Equivalence principle, Mach's principle, and the Copernican principle

Since the scalar field couples universally to all matter fields as shown in the action (1), the weak equivalence principle holds. In terms of Jordan frame variables, the action (1) describes a Brans-Dicke-type-like scalar-tensor theory with a field-dependent Brans-Dicke parameter. In the original work [87], the Brans-Dicke parameter is chosen as a constant. Since neither the self-interaction potential nor the symmetry-breaking interaction potential is inserted specifically in their original action, the model describes a long-range interaction and cannot provide a cosmological constant to drive the Universe's accelerating expansion in the present time.

Based on the equivalence principle, Brans and Dicke introduced a scalar-tensor combination inducing their gravitational field to incorporate Mach's principle into general relativity [87]. According to Mach, the inertial forces observed in a laboratory may originate in distant matter that is accelerated relative to the laboratory [87,94]. The action (1) combined with Eq. (19) may be seen as a Machian model. Since the equation of motion for the scalar field is determined by a sum of the self-interaction and the interaction potential with matter, the scalar field no longer appears to be a long-range field as in the original Brans-Dicke model. The interaction range is dependent on the ambient mass density, and the range is very short in the common matter density. But the coupled scalar field can indirectly influence matter by a global effect of driving the Universe's expansion since the Einstein tensor is determined by both the energy-momentum tensor and the scalar field. In addition, the scalar field can also directly influence matter by the short-range interaction if the local gradient of the scalar field is present.

It is well known that the principle of a homogeneous and isotropic universe is a spatial embodiment of the Copernican principle. The extended Copernican principle indicates further that there is no special space-time position in the Universe. That is, not every part or every era of the Universe is the center or the origin. However, suitable initial conditions are required for inflation to start in the current cosmological theory [82,95]. In general, a flat potential of the inflaton is also required so that a sufficient amount of inflation will be obtained. This starting origin of time is at odds with the extended Copernican principle [96]. The quasicyclic model discussed in this paper may resolve the incompatibility with the extended Copernican principle and avoid fine-tuning in choosing initial conditions in inflation.

When the coupled scalar field is invoked to successfully drive the Universe's acceleration, a strong fifth force appears. However, the very short interaction ranges and the screening effects enable our Brans-Dicke-type-like model in the calculation to converge to the results of general relativity. Although the coupling to ordinary matter should not lead to observable long-range forces, it may result in observable short-range forces. The strength of the fifth forces is dependent on the ambient density, and the density dependence is a concave curve. In both extremely opposite cases, that is, when the mass density approaches zero or infinity, the fifth forces vanish.

B. Conclusions and outlook

In order to obtain the scalar fifth force, we have required that the scalar field must entirely account for the observed cosmic acceleration. This requirement leads to the symmetry-breaking coupling that can localize the value of the self-interaction potential to act as the cosmological constant. Since the interaction potential between matter and the scalar field is directly proportional to the particle number density of matter, the localization gives the nearly fixed cosmological constant as long as the number density is large enough in the nonrelativistic matter case.

Our scheme does not conflict with chameleon no-go theorems, at least mathematically. As has been pointed out in the original literature [35], any model that purports to explain the cosmic acceleration and passes solar system tests must be doing so using some form of quintessence or vacuum energy. Our model in fact uses a coupled scalar field as quintessence, but only the quintessence is pinned via a symmetry-breaking coupling.

As for Weinberg's no-go theorem, since our scheme uses dynamic quintessence rather than static vacuum energy, the no-go theorem is circumvented. In our model, the self-interaction potential of the scalar field has nothing to do with the effective mass around the minimum of the effective potential but can move the position of the minimum. These properties guarantee that the observed cosmic acceleration stems entirely from the coupled scalar field rather than any stable vacuum energy.

The scheme has also been applied to the relativistic matter case (e.g., during inflation). In this case, matter decouples completely with the scalar field, and the evolution of the Universe is mainly dominated by the scalar field. The pseudopotential density has been introduced as the sum of the self-interaction potential energy density of the scalar field and the energy density scale of the curvature of the Universe. It is the pseudopotential that plays the role of the inflaton potential in the inflationary slow-roll approximation rather than only the self-interaction potential itself. Thus, the observed concave potential feature naturally belongs to the pseudopotential and is in favor of the closed space of the Universe.

When the cosmic constraints are satisfied, the scheme predicts that the magnitude of the fifth force is considerably larger than gravity, especially for lower density in the local environment. However, the fifth force is suppressed into a very short interaction range. Tests of gravity are then satisfied. A typical interaction range is estimated to be about 5 μm for the current matter density of the Universe.

The fifth force might be significantly detectable, provided that experiments are designed that allow test particles to pass through the thin-shell region of sources or at least allow test particles to approach the thin shell as close as possible.

ACKNOWLEDGMENTS

We acknowledge discussions with J. N. Zhang and C. Liu. This work was supported by Science Challenge Project, Grant No. TZ2018003, and the National Natural Science Foundation of China through Grant No. 11604348.

APPENDIX A: OBTAINING THE ACCELERATION EQUATION OF THE UNIVERSE

We now derive the acceleration equation of the Universe in detail from the Friedmann equation shown as Eq. (5). For the simplification, the abbreviated notation $B_i(\phi)$ is introduced through $B_i(\phi) \equiv A^{1-3w_i}(\phi)$ at the beginning in deriving equations. The final results are obtained by using $A^{1-3w_i}(\phi)$ instead of $B_i(\phi)$. The Friedmann equation (5) in a Friedmann-Robertson-Walker (FRW) metric then becomes

$$\begin{aligned} H^2 &\equiv \frac{\dot{a}^2}{a^2} \\ &= \frac{8\pi G}{3\hbar^3 c^5}\left[\sum_i \rho_i \hbar^3 c^5 B_i(\phi) + \left(V(\phi) + \frac{\hbar^2}{2}\dot{\phi}^2\right)\right] - \frac{Kc^2}{a^2}, \end{aligned} \tag{A1}$$

where the values $K = 1, 0$, or -1 correspond to closed, flat, or open spaces, respectively. Differentiating Eq. (A1) with respect to time, one has

$$\begin{aligned} 2H\dot{H} &\equiv 2H\left(\frac{\ddot{a}}{a} - H^2\right) \\ &= \frac{8\pi G}{3\hbar^3 c^5}\left\{\sum_i \rho_i \hbar^3 c^5 B_{i,\phi}(\phi)\dot{\phi} + \sum_i \dot{\rho}_i \hbar^3 c^5 B_i(\phi) \right. \\ &\quad \left. + [V_{,\phi}(\phi)\dot{\phi} + \hbar^2 \ddot{\phi}\dot{\phi}]\right\} + \frac{2Kc^2\dot{a}}{a^3}. \end{aligned} \tag{A2}$$

Using Eq. (9), i.e.,

$$\hbar^2\ddot{\phi} + 3H\hbar^2\dot{\phi} + V_{,\phi}(\phi) + \sum_i \rho_i \hbar^3 c^5 B_{i,\phi}(\phi) = 0, \tag{A3}$$

one has

$$2H\dot{H} = \frac{8\pi G}{3\hbar^3 c^5}\left[-3H\hbar^2\dot{\phi}^2 + \sum_i \dot{\rho}_i \hbar^3 c^5 B_i(\phi)\right] + 2H\frac{Kc^2}{a^2}. \tag{A4}$$

Substituting the conservation law of Eq. (7), i.e.,

$$\dot{\rho}_i = -3H\left(\rho_i + \frac{p_i}{c^2}\right), \tag{A5}$$

into Eq. (A4), one has

$$2H\dot{H} = \frac{8\pi G}{3\hbar^3 c^5}\left[-3H\hbar^2\dot{\phi}^2 - 3H\sum_i\left(\rho_i + \frac{p_i}{c^2}\right)\hbar^3 c^5 B_i(\phi)\right] + 2H\frac{Kc^2}{a^2}. \tag{A6}$$

Again, we need to emphasize that ρ_i and p_i are independent of the scalar field, and the satisfaction of the conservation law is a definitionlike choice or constraint. This conservation law of Eq. (A5) shows that the corresponding entropy is conserved. The number of particles is not altered, but the masses of the particles are shifted due to the coupling of the particles to the scalar field. In other words, ρ_i and p_i denote the matter mass density and pressure in the decoupled case with the scalar field, but $\rho_i A^{1-3w_i}(\phi)$ describes the physics matter density. Thus, only the expression $p_i A^{1-3w_i}(\phi)$ can be used to describe the physics pressure of the perfect fluid, so the parameter of the equation of state for matter is independent of the scalar field [see Eq. (A11)].

Removing the common factor $2H$ in Eq. (A6), we have

$$\dot{H} = \frac{8\pi G}{3\hbar^3 c^5}\left[-\frac{3}{2}\hbar^2\dot{\phi}^2 - \frac{3}{2}\sum_i\left(\rho_i + \frac{p_i}{c^2}\right)\hbar^3 c^5 B_i(\phi)\right] + \frac{Kc^2}{a^2}. \tag{A7}$$

Since $\ddot{a}/a \equiv \dot{H} + H^2$, from Eqs. (A1) and (A7), one has

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3\hbar^3 c^5}\left[2V(\phi) - 2\hbar^2\dot{\phi}^2 - \sum_i\left(\rho_i + \frac{3p_i}{c^2}\right)\hbar^3 c^5 B_i(\phi)\right]. \tag{A8}$$

Noticing the equation of state, Eq. (6), i.e., $w_i \equiv p_i/(\rho_i c^2)$, and introducing a coupled matter mass density for each species of matter as

$$\rho_{\mathrm{m}i} = \rho_i A^{1-3w_i}(\phi) \equiv \rho_i B_i(\phi), \tag{A9}$$

which includes the coupling energy with the scalar field, and a corresponding coupled pressure of the perfect fluid as

$$p_{\mathrm{m}i} = p_i A^{1-3w_i}(\phi) \equiv p_i B_i(\phi), \tag{A10}$$

which includes the coupling pressure with the scalar field, we immediately have

$$w_i \equiv \frac{p_i}{\rho_i c^2} = \frac{p_{\mathrm{m}i}}{\rho_{\mathrm{m}i} c^2}. \tag{A11}$$

We expect that $\rho_{\mathrm{m}i}(p_{\mathrm{m}i})$ denotes the real physics matter density (physics pressure). Although $\rho_{\mathrm{m}i}(p_{\mathrm{m}i})$ is ϕ dependent, from Eq. (A11) we can see that the parameter w_i of the equation of state for matter is really independent of the scalar field as we desired.

Replacing $B_i(\phi)$ by $A^{1-3w_i}(\phi)$, Eq. (A8) can be rewritten as Eq. (12), i.e.,

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3\hbar^3 c^5}\left[2V(\phi) - 2\hbar^2\dot{\phi}^2 - \sum_i \rho_{\mathrm{m}_i}(1+3w_i)\hbar^3 c^5\right]. \tag{A12}$$

In addition, Eq. (A1) or Eq. (5) can also be rewritten as follows:

$$\begin{aligned} H^2 &= \frac{8\pi G}{3}\left[\sum_i \rho_{\mathrm{m}i} + \frac{1}{\hbar^3 c^5}\left(V(\phi) + \frac{\hbar^2}{2}\dot{\phi}^2\right)\right] - \frac{Kc^2}{a^2} \\ &= \frac{8\pi G}{3}\left[\rho_{\mathrm{m}} + \frac{1}{\hbar^3 c^5}\left(V(\phi) + \frac{\hbar^2}{2}\dot{\phi}^2\right)\right] - \frac{Kc^2}{a^2}, \end{aligned} \tag{A13}$$

where $\rho_{\mathrm{m}} = \sum_i \rho_{\mathrm{m}i}$ denotes the total scalar-field-dependent matter density.

In this appendix, it has also been shown through Eq. (A11) that the equation of state for matter is temperature dependent but free of the scalar field. The property of ϕ independence is very important to get Eq. (C15) in Appendix C 2.

APPENDIX B: CONSERVATION LAWS

When the scalar field couples with matter, the energy exchange between the scalar field and matter occurs. Thus, the equation of state for the scalar field should be modified. In this appendix, the average energy of an individual matter particle will also be introduced as a supplement to the conservation law of Eq. (A5).

1. Coupled equations

The basic assumption of the conservation law of Eq. (A5), and its equivalent form of Eq. (7), is that both ρ_i and w_i are independent of the scalar field, with i denoting several species of noninteracting perfect fluids of matter sources. However, this does not mean that the energy density of matter is independent of the scalar field. The energy exchange between matter and the scalar field is discussed as follows: To avoid complications, we derive the rest of the conservation laws by using $B_i(\phi)$ instead of $A^{1-3w_i}(\phi)$. We have introduced the scalar-field-dependent matter density and the scalar-field-dependent pressure shown as Eqs. (A9) and (A10), respectively. Since i-species matter density $\rho_{\mathrm{m}i} = \rho_i B_i(\phi)$, when differentiating with respect to time, we obtain

$$\dot{\rho}_{\mathrm{m}i} = \dot{\rho}_i B_i(\phi) + \rho_i \dot{B}_i(\phi). \tag{B1}$$

Substituting the conservation law of Eq. (A5) into Eq. (B1), we get

$$\dot{\rho}_{\mathrm{m}i} = -3H\left(\rho_i + \frac{p_i}{c^2}\right) B_i(\phi) + \rho_i \dot{B}_i(\phi). \tag{B2}$$

Noticing Eqs. (A9) and (A10), we then get

$$\dot{\rho}_{\mathrm{m}i} + 3H\left(\rho_{\mathrm{m}i} + \frac{p_{\mathrm{m}i}}{c^2}\right) = \rho_i \dot{B}_i(\phi). \tag{B3}$$

Summing over all species matter i in Eq. (B3), we get

$$\dot{\rho}_{\mathrm{m}} + 3H\left(\rho_{\mathrm{m}} + \frac{p_{\mathrm{m}}}{c^2}\right) = \sum_i \rho_i \dot{B}_i(\phi), \tag{B4}$$

where $\rho_{\mathrm{m}} = \sum_i \rho_{\mathrm{m}i}$ and $p_{\mathrm{m}} = \sum_i p_{\mathrm{m}i}$, respectively. Equation (B4) is different from the conservation law of Eq. (A5). The difference results from the interaction between matter and the scalar field.

Since the energy density of the scalar field is defined by $\hbar^3 c^5 \rho_\phi = V(\phi) + \dot{\phi}^2 \hbar^2/2$, differentiating this equation gives

$$\dot{\rho}_\phi \hbar^3 c^5 = V_{,\phi}(\phi)\dot{\phi} + \hbar^2 \dot{\phi}\ddot{\phi}. \tag{B5}$$

Substituting Eq. (A3) into Eq. (B5) and noticing the pressure definition of the scalar field $\hbar^3 c^3 p_\phi = -V(\phi) + \dot{\phi}^2 \hbar^2/2$, it follows that

$$\dot{\rho}_\phi \hbar^3 c^5 = -3H(\hbar^3 c^5 \rho_\phi + \hbar^3 c^3 p_\phi) - \sum_i \rho_i \hbar^3 c^5 \dot{B}_i(\phi). \tag{B6}$$

Then, removing the common factor $\hbar^3 c^5$ in Eq. (B6), we have

$$\dot{\rho}_\phi + 3H\left(\rho_\phi + \frac{p_\phi}{c^2}\right) = -\sum_i \rho_i \dot{B}_i(\phi). \tag{B7}$$

Introducing the interaction energy density between i-species matter and the scalar field by $\rho_{i\mathrm{int}} = \rho_i[B_i(\phi) - 1]$ and the corresponding interaction pressure by $p_{i\mathrm{int}} = p_i[B_i(\phi) - 1]$, and noticing the conservation equation (A5), one has

$$\dot{\rho}_{i\mathrm{int}} + 3H\left(\rho_{i\mathrm{int}} + \frac{p_{i\mathrm{int}}}{c^2}\right) = \rho_i \dot{B}_i(\phi). \tag{B8}$$

Summing over all species matter i to Eq. (B8), we get

$$\dot{\rho}_{\mathrm{int}} + 3H\left(\rho_{\mathrm{int}} + \frac{p_{\mathrm{int}}}{c^2}\right) = \sum_i \rho_i \dot{B}_i(\phi), \tag{B9}$$

where $\rho_{\mathrm{int}} = \sum_i \rho_{i\mathrm{int}}$ and $p_{\mathrm{int}} = \sum_i p_{i\mathrm{int}}$, respectively.

The sum of Eqs. (B7) and (B9) gives

$$\dot{\rho}_{\mathrm{eff}} + 3H\left(\rho_{\mathrm{eff}} + \frac{p_{\mathrm{eff}}}{c^2}\right) = 0, \tag{B10}$$

with the effective energy density of the scalar field $\rho_{\mathrm{eff}} = \rho_\phi + \rho_{\mathrm{int}}$ and the effective pressure of the scalar field $p_{\mathrm{eff}} = p_\phi + p_{\mathrm{int}}$, respectively. Thus, the effective energy of the scalar field is conserved. Equation (B10) implies that, for a coupled scalar field, the effective equation of state becomes

$$w_{\mathrm{eff}} \equiv \frac{p_{\mathrm{eff}}}{\rho_{\mathrm{eff}} c^2}. \tag{B11}$$

Summing over all species matter i to Eq. (A5), we get

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) = 0, \tag{B12}$$

where $\rho = \sum_i \rho_i$ and $p = \sum_i p_i$, respectively. The conserved energy density implies that the corresponding equation of state can be introduced, i.e., $w \equiv p/(\rho c^2)$. If we assume that all the matter species of noninteracting perfect fluids have the same parameter w_i, we have $p = w_i \rho c^2$. We prefer to use w_i instead of w to emphasize species of matter.

The sum of Eqs. (B12) and (B10), or the sum of Eqs. (B7) and (B4), results in a conservation equation of the total energy density as

$$\dot{\rho}_{\mathrm{total}} + 3H\left(\rho_{\mathrm{total}} + \frac{p_{\mathrm{total}}}{c^2}\right) = 0, \tag{B13}$$

where $\rho_{\mathrm{total}} \equiv \rho + \rho_\phi + \rho_{\mathrm{int}} \equiv \rho + \rho_{\mathrm{eff}} \equiv \rho_{\mathrm{m}} + \rho_\phi$, $p_{\mathrm{total}} \equiv p + p_\phi + p_{\mathrm{int}} \equiv p + p_{\mathrm{eff}} \equiv p_{\mathrm{m}} + p_\phi$. The total energy definition is different from Eq. (4.82) in Ref. [31] because its total energy density is defined by $\rho + \rho_\phi$ and is considered to be conserved as shown in Eq. (4.83) in the literature. The total energy should be $\rho_{\mathrm{m}} + \rho_\phi$ rather than $\rho + \rho_\phi$. Now, using $A^{1-3w_i}(\phi)$ instead of $B_i(\phi)$, Eq. (B4) becomes

$$\dot{\rho}_{\mathrm{m}} + 3H\left(\rho_{\mathrm{m}} + \frac{p_{\mathrm{m}}}{c^2}\right) = \sum_i \rho_i \frac{dA^{1-3w_i}(\phi)}{dt}, \tag{B14}$$

where $\rho_{\mathrm{m}} = \sum_i \rho_i A^{1-3w_i}(\phi)$ is the physics density of matter, and $p_{\mathrm{m}} = \sum_i p_i A^{1-3w_i}(\phi)$ is the physics pressure of matter fluid, respectively. In addition, Eq. (B7) becomes

$$\dot{\rho}_\phi + 3H\left(\rho_\phi + \frac{p_\phi}{c^2}\right) = -\sum_i \rho_i \frac{dA^{1-3w_i}(\phi)}{dt}. \quad \text{(B15)}$$

Equations (B14) and (B15) describe the energy exchange between matter and the scalar field, while Eq. (B13) describes the total energy conservation law. Since $dA^{1-3w_i}(\phi)/dt = (1-3w_i)A^{-3w_i}\dot{A} + (-3\dot{w}_i)A^{1-3w_i}\ln A$, from the equation, one can get the physical image of the energy transfer between matter and the scalar field as follows: The first term on the right-hand side of the equation is related to the work done on the system of matter by the scalar field; the second term on the right-hand side is related to the entropy variation of the system of matter due to the temperature dependence of the equation of state for the matter fluid.

At this time, one can see that it is necessary and important to introduce the scalar-field-independent density of matter and the corresponding conservation law of Eq. (A5).

2. The average energy of an individual particle

We now analyze further the meaning of the conservation law of Eq. (7), i.e., Eq. (A5). Its solution can be easily obtained as follows:

$$\rho_i = \frac{\rho_{i0} a_0^{3(w_i+1)}}{a^{3(w_i+1)}}, \quad \text{(B16)}$$

with the subscript 0 marking the current time. Since the number of matter particles is not altered, the number density n_i that changes with the volume expansion is given as follows:

$$n_i = \frac{n_0 a_0^3}{a^3}. \quad \text{(B17)}$$

If an average energy $\langle \varepsilon_{ci} \rangle$ per particle is introduced, one has

$$\rho_i = n_i \langle \varepsilon_{ci} \rangle. \quad \text{(B18)}$$

The average energy of an individual particle corresponds to the thermal scalar-field-independent Compton energy of the matter particle. The ratio of Eq. (B16) to Eq. (B17) gives

$$\langle \varepsilon_{ci} \rangle = \langle \varepsilon_{ci} \rangle_0 \frac{a_0^{3w_i}}{a^{3w_i}}, \quad \text{(B19)}$$

with $\langle \varepsilon_{ci} \rangle_0 = \rho_{i0}/n_{i0}$. Equation (7) shows that both the number density and the corresponding entropy are conserved. When the coupling to the scalar field is introduced, one has to describe not only the scalar-field-independent number density but also the scalar-field-dependent energy density of matter. Without the scalar field coupling, these two densities are essentially the same. Therefore, invoking Eq. (7) is by no means an expedient measure.

3. The effective equation of state

Based on Eq. (B11), we now discuss several important values of the effective equation of state for the scalar field as follows: For $w_i = 0$ and $\dot{\phi} = 0$, one has

$$w_{\text{eff}} = \frac{-V(\phi_{\min})}{V(\phi_{\min}) + \rho\hbar^3 c^5 [A(\phi_{\min}) - 1]}. \quad \text{(B20)}$$

When matter density is large enough, i.e., $A(\phi_{\min}) \approx 1$, one sees $w_{\text{eff}} \approx -1$, which mimics the cosmological constant before the current time in the pressureless case of the matter source.

For $w_i = 1/3$, i.e., $A^{1-3w_i} = 1$, and assuming that the scalar field has climbed along its self-interaction potential to the highest point where its kinetic energy is zero, from Eq. (B11) one has

$$w_{\text{eff}} = \frac{-V(\phi_{\max})}{V(\phi_{\max})} = -1, \quad \text{(B21)}$$

which mimics the largest cosmological constant during the inflation era. When the kinetic energy density is far larger than the potential energy density, i.e., $\hbar^2\dot{\phi}^2/2 \gg V(\phi)$, from Eq. (B11) one has

$$w_{\text{eff}} = 1, \quad \text{(B22)}$$

which means that the scalar field can generate a great deceleration effect.

Thus, in general, $-1 \leq w_{\text{eff}} \leq 1$. When the scalar field oscillates around the minimum at high frequency in the case of $|3H/2| < \omega_c$, the time average gives $w_{\text{eff}} = 0$, which means that the scalar field behaves as the pressureless matter fluid. In the case of $|3H/2| > \omega_c$, the over (negative) damped evolution of the scalar field occurs when the scalar field decouples with matter due to the ultrahigh temperature, and the parameter w_{eff} varies with time between -1 and 1.

APPENDIX C: THE λ-INDEPENDENT EFFECTIVE MASS AND THE CHANGING RATE OF $\dot{\phi}_{\min}/\phi_{\min}$

When both the quartic self-interaction potential and the symmetry-breaking coupling function are chosen, it will be proven in this appendix that the effective mass around the minimum of the effective potential is independent of the self-interaction potential of the scalar field. The λ-independent property is very important to circumvent Weinberg's no-go theorem, which has been demonstrated in Sec. V D. Since the condition of the scalar field adiabatically following the minimum of the effective potential is essential in obtaining the cosmological constant, the changing rate of $\dot{\phi}_{\rm min}/\phi_{\rm min}$ will also be discussed in this appendix.

1. The case of $w_i = 0$

For the pressureless fluid of matter sources $w_i = 0$, the effective potential is the sum of

$$V_{\rm eff}(\phi) = V(\phi) + V_{\rm int}, \tag{C1}$$

where

$$V(\phi) = \frac{\lambda}{4}\phi^4 \tag{C2}$$

and

$$V_{\rm int} = \rho\hbar^3 c^5[A(\phi) - 1] = \frac{\rho\hbar^3 c^5}{4{M_1}^4 c^8}(\phi^2 - {M_2}^2 c^4)^2. \tag{C3}$$

Note that $A(\phi)$ with a symmetry-breaking shape is described by Eq. (19b). The first derivative and the second derivative of the effective potential with respect to the scalar field are

$$V_{{\rm eff},\phi}(\phi) = \left(\lambda + \frac{\rho\hbar^3 c^5}{{M_1}^4 c^8}\right)\phi^3 - \left(\frac{\rho\hbar^3 c^5 {M_2}^2}{{M_1}^4 c^4}\right)\phi, \tag{C4}$$

$$V_{{\rm eff},\phi\phi}(\phi) = 3\left(\lambda + \frac{\rho\hbar^3 c^5}{{M_1}^4 c^8}\right)\phi^2 - \left(\frac{\rho\hbar^3 c^5 {M_2}^2}{{M_1}^4 c^4}\right). \tag{C5}$$

Let the first derivative of the effective potential equal zero, i.e.,

$$V_{,\phi} + \rho\hbar^3 c^5 A_{,\phi} = 0. \tag{C6}$$

Then, the extrema are obtained as

$$\phi^2_{\rm min} = \frac{\rho\hbar^3 {M_2}^2 c^4}{\lambda {M_1}^4 c^3 + \rho\hbar^3}, \tag{C7a}$$

$$\phi_{\rm max} = 0, \tag{C7b}$$

and the effective mass about the minima is

$$m^2_{\rm eff} \equiv \frac{V_{{\rm eff},\phi\phi}(\phi_{\rm min})}{c^4} = \frac{2\rho\hbar^3 {M_2}^2}{{M_1}^4 c^3}. \tag{C8}$$

The $\mathbf{Z}_2$ symmetry is spontaneously broken as the scalar field chooses one of the minima of the effective potential. The field values corresponding to the minima of the effective potential are nearly the same as that of the minima of the coupling function in the Universe's past due to the coupled interaction and larger matter density. However, the difference is that the minima of the effective potential are density dependent, while those of the coupling function are density independent. This means that the minima of the effective potential will depart from the fixed minima of the coupling function in the future.

We choose $\lambda = 1/6$ so that the coefficient in Eq. (C2) can naturally be written as $1/(4!)$. Besides the consideration of the theoretical naturalness, the choice of the value of the parameters M_2 is determined by fitting the cosmological constant. The range of the ratio $M_1/M_2 \geq 4$ is obtained with the following requirements: The ratio should not only match the range of the current cosmological scale factor under the constraint of the approximate current density of the Universe, but it can also correspond to a Compton wavelength of the scalar field that is as large as possible. The larger the ratio is, the shorter the wavelength is. As a concrete example, the ratio M_1/M_2 is selected in this paper to be a slightly larger integer of 2^3 instead of the lower bound of 2^2. Of course, the other choice is not forbidden as long as the inequality of $M_1/M_2 \geq 4$ is satisfied. Indeed, we can let $M_1/M_2 = 2^n$ and regard n as another adjustable parameter. Through calculations, we can find that $n = 2$ to $n = 4$ all work well in obtaining the cosmological constant, which means that the symmetry-breaking model is insensitive to the selection of the parameter. However, the range of the fifth interaction shown in Eq. (C8) is sensitive to the selection of the parameter. Since the main purpose of this paper is to display the symmetry-breaking coupling function, the detailed determination of M_1/M_2 will be discussed elsewhere.

Due to the change of the matter density, the minimum position of the effective potential will change. The changing rate can be defined by $\dot{\phi}_{\rm min}/\phi_{\rm min}$. Differentiating Eq. (C6) with respect to time t, and using the conservation law of Eq. (B12), assuming the pressureless matter source $p = 0$, one has

$$\frac{\dot{\phi}_{\rm min}}{\phi_{\rm min}} = -H \cdot \frac{3V_{,\phi}(\phi_{\rm min})}{\phi_{\rm min} m^2_{\rm eff} c^4} \tag{C9a}$$

$$= H \cdot \frac{3c\hbar^3\rho A_{,\phi}(\phi_{\rm min})}{\phi_{\rm min} m^2_{\rm eff}}. \tag{C9b}$$

Therefore, the changing rate of the minimum position of the effective potential can be expressed by the Hubble parameter.

Using Eq. (C7a), Eq. (C9) becomes

$$\frac{\dot{\phi}_{\rm min}}{\phi_{\rm min}} = -\frac{3H\lambda M_1^4 c^3}{2(\lambda M_1^4 c^3 + \rho\hbar^3)}. \tag{C10}$$

The negative sign on the right-hand side of the equation describes the fact that the trend of the changing rate is opposite to that of the Hubble rate. For $\rho \gg \lambda {M_1}^4 c^3/\hbar^3 \sim 10^{-30}\ {\rm kg/m^3}$, the absolute value of the changing rate is much less than that of the Hubble rate. In general, one has

$$\left|\frac{\dot{\phi}_{\min}}{\phi_{\min}}\right| < \left|\frac{3H}{2}\right|. \tag{C11}$$

Here, $|3H/2|$ denotes the damping rate in the oscillation equation of the scalar field. Thus, for a pressureless matter source the scalar field can adiabatically follow the minimum [see also the adiabatic condition of Eq. (23)].

2. The general case of $w_i \neq 0$

We now consider the general case of $w_i \neq 0$. In this case the value of the scalar field at the minimum cannot be described by a simple analytic expression. One can only give the region $0 \le \phi_{\min}^2 \le M_2^2 c^4$, which is the same as the case of $w_i = 0$. Since the self-interaction potential of Eq. (C2) is a quartic form, the effective mass of the scalar field around the minimum can be derived only from the interaction potential shown by Eq. (11), i.e.,

$$m_{\mathrm{eff}}^2 = \frac{-2V_{\mathrm{int},\phi\phi}(\phi = 0)}{c^4}. \tag{C12}$$

For the sake of simplicity, we assume that all the species of noninteracting perfect fluids of matter sources have the same parameter w_i. Thus, Eq. (11) becomes

$$V_{\mathrm{int}} = \rho\hbar^3 c^5 [A^{1-3w_i}(\phi) - 1] \tag{C13}$$

with $\rho = \sum_i \rho_i$. Combining Eqs. (C12) and (C13) with the symmetry-breaking coupling function of Eq. (19b), one has

$$m_{\mathrm{eff}}^2 = \frac{2\rho\hbar^3 M_2^2 (1-3w_i)}{M_1^4 c^3}\left(1 + \frac{M_2^4}{4M_1^4}\right)^{-3w_i}. \tag{C14}$$

When the fluid of the matter source becomes relativistic, i.e., $w_i \to 1/3$, the effective mass around the minimum approaches zero. In the contraction process, since the matter fluid evolves from norelativistic to relativistic, the scalar field must experience chirped under-negative-damping oscillations from a higher frequency to a lower one and then shift to an over-negative-damping motion. In the expansion process, the evolution of the Universe is reversed. To describe whether the scalar field is able to adiabatically follow the minimum or not, by using the same method as used in deriving Eq. (C9) and noticing the ϕ independence of w_i denoted by Eq. (A11), the changing rate of the minimum position of the effective potential is obtained as follows:

$$\frac{\dot{\phi}_{\min}}{\phi_{\min}} = -3H(1+w_i)\frac{V_{,\phi}(\phi_{\min})}{\phi_{\min} m_{\mathrm{eff}}^2 c^4} \tag{C15a}$$

$$= \frac{3H\rho(1+w_i)\hbar^3 c(1-3w_i)A^{-3w_i}(\phi_{\min})A_{,\phi}(\phi_{\min})}{\phi_{\min} m_{\mathrm{eff}}^2}. \tag{C15b}$$

Substituting

$$V_{,\phi}(\phi_{\min}) = \lambda\phi_{\min}^3, \tag{C16a}$$

$$A_{,\phi}(\phi_{\min}) = \frac{\phi_{\min}(\phi_{\min}^2 - M_2^2 c^4)}{M_1^4 c^8}, \tag{C16b}$$

into Eqs. (C15a) and (15b), respectively, one has

$$\frac{\dot{\phi}_{\min}}{\phi_{\min}} = \frac{-3H(1+w_i)\lambda\phi_{\min}^2}{m_{\mathrm{eff}}^2 c^4}, \tag{C17a}$$

$$= \frac{3H(1+w_i)(1+\frac{M_2^4}{4M_1^4})^{3w_i}}{2A^{3w_i}(\phi_{\min})}\frac{(\phi_{\min}^2 - M_2^2 c^4)}{M_2^2 c^4}. \tag{C17b}$$

Thus, with w_i increasing from 0 to $1/3$, the evolution of the scalar field must undergo a stage where $|\dot{\phi}_{\min}/\phi_{\min}|$ is larger than the damping rate $|3H/2|$. In this stage, the scalar field is not able to sit stably at the minimum. In the case of $w_i = 0$, from Eq. (C10) one can see that the stability of the minimum is enhanced as the density of matter increases. In the case of $w_i \neq 0$, the stability of the minimum cannot be enhanced further with the increase in the density of matter. In fact, the stability even decays as the matter density increases due to the decoupled effect between the scalar field and matter at the extreme temperature. The fragile stability of the adiabatic can lead to a great result when the effective mass of the scalar field around the minimum approaches zero. For example, in the contraction process, the magnitude of the scalar field will grow exponentially in the over-negative-damping case. This is in sharp contrast to the case when the Universe is at the maximum radius. At that time, the Hubble rate is zero, but the effective mass of the scalar field is not zero, though the mass is extremely small. Both the adiabatic tracking of the minimum and the oscillation condition can always hold in the case of $w_i = 0$. The two conditions cannot hold forever in the case of $w_i \neq 0$ (see also Sec. VI A 1). The periodically vibrational state of the scalar field only appears when the oscillation condition is satisfied, but the adiabatic condition is not satisfied (see caption of Fig. 2), which corresponds to the overshooting phenomenon in the under-(negative-)damped case. When neither condition is satisfied, the field can neither adiabatically follow the minimum nor oscillate around the minimum, which corresponds to the overshooting phenomenon in the over-(negative-)damped case (see also Sec. V B).

In addition, it is worth noting that the mass of the scalar field corresponding to the minimum radius of the Universe (the maximum absolute value of the scalar field) is not related to any minimum of the effective potential. The mass is essentially the self-mass of the scalar field since only the self-interaction potential is considered. The evolution of the scalar field now belongs to over-(negative-)damped motion. In fact, it does not make sense to talk about the adiabatic following some minimum in this case. One often uses so-called slow-roll conditions in this case. However, the requirement of the slow-roll conditions is not always reasonable (see also Sec. VI B 1).

3. The coupling coefficient β

The coupling coefficient β is dependent on the ambient density, which corresponds to the linear part of the coupling function $A(\phi)$. Using Eqs. (81) and (19b), one has

$$\beta(\phi) = M_{\rm Pl}c^2 \frac{4\phi(\phi^2 - {M_2}^2c^4)}{4{M_1}^4c^8 + (\phi^2 - {M_2}^2c^4)^2}. \quad \text{(C18)}$$

Therefore, the coupling coefficient β is not a constant, but it depends on ϕ. At $\phi_{\rm min}$, the coupling coefficient β is

$$\beta(\phi_{\rm min}) = M_{\rm Pl}c^2 \frac{4\phi_{\rm min}(\phi_{\rm min}^2 - {M_2}^2c^4)}{4{M_1}^4c^8 + (\phi_{\rm min}^2 - {M_2}^2c^4)^2}. \quad \text{(C19)}$$

Thus, $\beta(\phi_{\rm min}) > 0$ if $\phi_{\rm min} < 0$, and $\beta(\phi_{\rm min}) < 0$ if $\phi_{\rm min} > 0$. Consider a space-variable source density embedded in a homogeneous ambient density $\rho_{\rm b}$; the coupling coefficient $\beta(\phi)$ can be approximated to $\beta(\phi_{\rm b})$ with $\phi_{\rm b} = \phi_{\rm min}(\rho_{\rm b})$. In fact, one often uses $\beta = \beta(\phi_{\rm b})$ to denote the coupling coefficient.

The value of the coupling coefficient β denotes the coupling strength between matter and the scalar field. For a lower density of matter in the local environment, the coupling strength is much larger than gravitational. This means that the scalar fifth force might be detectable provided that future experiments are designed properly. If $M_2 = 0$, one has $\phi_{\rm min} = 0$ and $\beta(\phi_{\rm min}) = 0$ corresponding to a zero fifth force at $\phi_{\rm min}$.

APPENDIX D: FUNDAMENTAL PHYSICAL CONSTANTS

Some of the physical constants we used in our calculations are as follows: the speed of light in vacuum $c = 2.99792458 \times 10^8$ m $\cdot$ s^{-1}, the gravitational constant $G = 6.67430 \times 10^{-11}$ m$^3 \cdot$ kg$^{-1} \cdot$ s^{-2}, Planck's constant $h = 6.62607015 \times 10^{-34}$ J $\cdot$ s, and the elementary charge $e = 1.602176624 \times 10^{-19}$ C.

We do not use the units $c = \hbar = 1$, but we use the International System of Units in which c and $\hbar$ appear explicitly. The purpose of this is not only to compare the calculation with observation data but also for the mark of the quantum effect at the cosmic scale. Quantum mechanics is often regarded as a theory for the microscale.

[1] S. Perlmutter *et al.* (Supernova Cosmology Project Collaboration), Discovery of a supernova explosion at half the age of the Universe and its cosmological implications, Nature (London) **391**, 51 (1998).

[2] A. G. Riess *et al.* (Supernova Search Team Collaboration), Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astron. J. **116**, 1009 (1998).

[3] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, arXiv:1807.06209v1.

[4] Y. Akrami *et al.* (Planck Collaboration), Planck 2018 results. X. Constraints on inflation, arXiv:1807.06211v1.

[5] A. Joyce, B. Jain, J. Khoury, and M. Trodden, Beyond the cosmological standard model, Phys. Rep. **568**, 1 (2015).

[6] J. Khoury and A. Weltman, Chameleon Fields: Awaiting Surprises for Tests of Gravity in Space, Phys. Rev. Lett. **93**, 171104 (2004).

[7] P. Brax, C. van de Bruck, A. C. Davis, J. Khoury, and A. Weltman, Detecting dark energy in orbit—The cosmological chameleon, Phys. Rev. D **70**, 123518 (2004).

[8] K. Hinterbichler, J. Khoury, A. Levy, and A. Matas, Symmetron cosmology, Phys. Rev. D **84**, 103521 (2011).

[9] K. Hinterbichler and J. Khoury, Symmetron Fields: Screening Long-Range Forces through Local Symmetry Restoration, Phys. Rev. Lett. **104**, 231301 (2010).

[10] P. Brax, A. C. Davis, B. Li, H. A. Winther, and G.-B. Zhao, Systematic simulations of modified gravity: Symmetron and dilaton models, J. Cosmol. Astropart. Phys. 10 (2012) 002.

[11] P. Brax, Atomic interferometry test of dark energy, Phys. Rev. D **94**, 104069 (2016).

[12] A. Upadhye, Dark energy fifth forces in torsion pendulum experiments, Phys. Rev. D **86**, 102003 (2012).

[13] S. S. Gubser and J. Khoury, Scalar self-interactions loosen constraints from fifth force searches, Phys. Rev. D **70**, 104001 (2004).

[14] J. Khoury and A. Weltman, Chameleon cosmology, Phys. Rev. D **69**, 044026 (2004).

[15] A. Padilla, E. Platts, D. Stefanyszyn, A. Walters, A. Weltmanb, and T. Wilsona, How to avoid a swift kick in the chameleons, J. Cosmol. Astropart. Phys. 03 (2016) 058.

[16] C. Burrage, E. J. Copelanda, and E. A. Hinds, Probing dark energy with atom interferometry, J. Cosmol. Astropart. Phys. 03 (2015) 042.

[17] P. Brax and C. Burrage, Atomic precision tests and light scalar couplings, Phys. Rev. D **83**, 035020 (2011).

[18] H.-C. Zhang, Experimental demonstration of a fifth force due to chameleon field via cold atoms, arXiv: 1702.03050v1. The conclusion about the experimental demonstration for the fifth force has not yet been accepted. One of the purposes of this paper is to establish a theoretical model to explain the experimental data quantitatively

instead of using the chameleon model in the work listed in this reference.
[19] P. J. E. Peebles and B. Ratra, The cosmological constant and dark energy, Rev. Mod. Phys. **75**, 559 (2003).
[20] B. Ratra and P. J. E. Peebles, Cosmological consequences of a rolling homogeneous scalar field, Phys. Rev. D **37**, 3406 (1988).
[21] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Cosmological Imprint of an Energy Component with General Equation of State, Phys. Rev. Lett. **80**, 1582 (1998).
[22] I. Zlatev, L. Wang, and P. J. Steinhardt, Quintessence, Cosmic Coincidence, and the Cosmological Constant, Phys. Rev. Lett. **82**, 896 (1999).
[23] K. Coble, S. Dodelson, and J. A. Frieman, Dynamical lambda models of structure formation, Phys. Rev. D **55**, 1851 (1997).
[24] J. A. Frieman, C. T. Hill, and R. Watkins, Late-time cosmological phase transitions: 1. Particle-physics models and cosmic evolution, Phys. Rev. D **46**, 1226 (1992).
[25] S. M. Carroll, W. H. Press, and E. L. Turner, The cosmological constant, Annu. Rev. Astron. Astrophys. **30**, 499 (1992).
[26] P. J. E. Peebles and B. Ratra, Cosmology with a time variable cosmological constant, Astrophys. J. **325**, L17 (1988).
[27] J. R. Swaney and R. J. Scherrer, Quadratic approximation for quintessence with arbitrary initial conditions, Phys. Rev. D **91**, 123525 (2015).
[28] R. J. Scherrer and A. A. Sen, Thawing quintessence with a nearly flat potential, Phys. Rev. D **77**, 083515 (2008).
[29] L. Amendola, Coupled quintessence, Phys. Rev. D **62**, 043511 (2000).
[30] S. Weinberg, The cosmological constant problem, Rev. Mod. Phys. **61**, 1 (1989).
[31] P. Brax, What makes the Universe accelerate? A review on what dark energy could be and how to test it, Rep. Prog. Phys. **81**, 016902 (2018).
[32] P. Brax, A.-C. Davis, B. Li, H. A. Winther, and G.-B. Zhao, Systematic simulations of modified gravity: Symmetron and dilaton models, J. Cosmol. Astropart. Phys. 10 (2012) 002.
[33] S. M. Carroll, Quintessence and the Rest of the World: Suppressing Long-Range Interactions, Phys. Rev. Lett. **81**, 3067 (1998).
[34] H. M. Sadjadi and V. Anari, Mass varying neutrinos, symmetry breaking, and cosmic acceleration, Phys. Rev. D **95**, 123521 (2017).
[35] J. Wang, L. Hui, and J. Khoury, No-Go Theorems for Generalized Chameleon Field Theories, Phys. Rev. Lett. **109**, 241301 (2012).
[36] J. Khoury, Chameleon field theories, Classical Quantum Gravity **30**, 214004 (2013).
[37] P. Brax, A.-C. Davis, B. Li, and H. A. Winther, Unified description of screened modified gravity, Phys. Rev. D **86**, 044015 (2012).
[38] M. Jaffe, P. Haslinger, V. Xu, P. Hamilton, A. Upadhye, B. Elder, J. Khoury, and H. Müller, Testing sub-gravitational forces on atoms from a miniature in-vacuum source mass, Nat. Phys. **13**, 938 (2017).
[39] D. H. Lyth and A. R. Liddle, *The Primordial Density Perturbation Cosmology, Inflation and the Origin of Structure* (Cambridge University Press, Cambridge, 2009), pp. 60, 315, 316, 319.
[40] A. R. Liddle, *An Introduction to Modern Cosmology* (John Wiley & Sons, West Sussex, 2015), pp. 27, 55.
[41] K. Bamba, R. Gannouji, M. Kamijo, S. Nojiri, and M. Sami, Spontaneous symmetry breaking in cosmos: The hybrid symmetron as a dark energy switching device, J. Cosmol. Astropart. Phys. 07 (2013) 017.
[42] R. Bean, E. E. Flanagan, and M. Trodden, Adiabatic instability in coupled dark energy/dark matter models, Phys. Rev. D **78**, 023009 (2008).
[43] S. Das, P. S. Corasaniti, and J. Khoury, Super-acceleration as signature of dark sector interaction, Phys. Rev. D **73**, 083509 (2006).
[44] H. Mohseni Sadjadi and V. Anari, Cosmic acceleration and de Sitter expansion in hybrid mass varying neutrino model, J. Cosmol. Astropart. Phys. 10 (2018) 036.
[45] E. Macaulay *et al.* (DES Collaboration), First cosmological results using type Ia supernovae from the dark energy survey: Measurement of the Hubble constant, Mon. Not. R. Astron. Soc. **486**, 2184 (2019). This value of the Hubble constant is close to the value reported in [3]. In our paper, since the high precision ratio of matter density to the dark energy density given by the Planck Collaboration has been chosen in obtaining the matter density, using a specific value of the Hubble constant from the Planck Collaboration [3] in the discussion of space curvature is more reasonable than using that from another Hubble constant, such as that from the DES Collaboration. Although one can use the measured Hubble constant from the DES Collaboration together with the matter density deduced from the Planck Collaboration to discuss the space curvature (and one probably obtains an open universe), the reliability of the open universe is questionable because the calculation parameters from the different measurement systems may lead to an inconsistency problem.
[46] A. R. Liddle, P. Parsons, and J. D. Barrow, Formalizing the slow-roll approximation in inflation, Phys. Rev. D **50**, 7222 (1994).
[47] E. J. Copeland, E. W. Kolb, A. R. Liddle, and J. E. Lidsey, Reconstructing the inflaton potential: In principle and in practice, Phys. Rev. D **48**, 2529 (1993).
[48] E. W. Kolb and S. L. Vadas, Relating spectral indices to tensor and scalar amplitudes in inflation, Phys. Rev. D **50**, 2479 (1994).
[49] S. Dodelson, W. H. Kinney, and E. W. Kolb, Cosmic microwave background measurements can discriminate among inflation models, Phys. Rev. D **56**, 3207 (1997).
[50] H. P. de Oliveira and C. A. Terrero-Escalante, Problems for observing the inflaton potential, J. Cosmol. Astropart. Phys. 01 (2006) 024.
[51] P. Adshead and R. Easther, Constraining inflation, J. Cosmol. Astropart. Phys. 10 (2008) 047.
[52] D. S. Salopek and J. R. Bond, Nonlinear evolution of long-wavelength metric fluctuations in inflationary models, Phys. Rev. D **42**, 3936 (1990).
[53] D. ter Haar, *Elements of Statistical Mechanics* (Butterworth-Heinemann, Oxford, 1995), p 250.
[54] A. Upadhye, Symmetron Dark Energy in Laboratory Experiments, Phys. Rev. Lett. **110**, 031301 (2013).

[55] P. Brax and A. Upadhye, Chameleon fragmentation, J. Cosmol. Astropart. Phys. 02 (**2014**) 018.

[56] X. Zhang, W. Zhao, H. Huang, and Y. f. Cai, Post-Newtonian parameters and cosmological constant, Phys. Rev. D **93**, 124003 (2016).

[57] T. M. C. Abbott *et al.* (DES Collaboration), Cosmological Constraints from Multiple Probes in the Dark Energy Survey, Phys. Rev. Lett. **122**, 171301 (2019).

[58] O. Farooq, F. R. Madiyar, S. Crandall, and B. Ratra, Hubble parameter measurement constraints on the redshift of the deceleration-acceleration transition, dynamical dark energy, and space curvature, Astrophys. J. **835**, 26 (2017).

[59] J. V. Cunha and J. A. S. Lima, Transition redshift: New kinematic constraints from supernovae, Mon. Not. R. Astron. Soc. **390**, 210 (2008).

[60] J. F. Jesus, R. F. L. Holanda, and S. H. Pereira, Model independent constraints on transition redshift, J. Cosmol. Astropart. Phys. 05 (2018) 073.

[61] If we extrapolate back to the early cosmological time when matter density is extremely high, a corollary is that the system temperature is extremely high and matter is decoupled to the scalar field. The nearly fixed cosmological constant is no longer valid since the interaction potential trapping vanishes. The Universe's evolution in the early period can be described by inflation models. For flat and open spaces, since the Hubble parameter cannot equal zero [see Eq. (5)], one of the slow-roll conditions $|\ddot{\phi}| \ll 3|H\dot{\phi}|$ is reasonable as in [39,49]. Thus, we do not need to analyze these two cases. However, for closed space, since the Hubble parameter can pass through zero, the slow-roll condition above is not always valid, and it needs further investigation. The detailed discussion is shown in Sec. VI.

[62] V. Poulin, T. L. Smith, T. Karwal, and M. Kamionkowski, Early Dark Energy Can Resolve the Hubble Tension, Phys. Rev. Lett. **122**, 221301 (2019).

[63] K. Vattis, S. M. Koushiappas, and A. Loeb, Dark matter decaying in the late Universe can relieve the H_0 tension, Phys. Rev. D **99**, 121302(R) (2019).

[64] One should carefully distinguish the presentation of the Hubble constant in both the Jordan and the Einstein frame. Although the two frames can perhaps be used to relieve the Hubble tension, it is not an appropriate time to discuss this interesting issue. The discussion may lead to more confusion due to the model-dependent determination of some parameters.

[65] A. D. Rider, D. C. Moore, C. P. Blakemore, M. Louis, M. Lu, and G. Gratta, Search for Screened Interactions Associated with Dark Energy below the 100 μm Length Scale, Phys. Rev. Lett. **117**, 101101 (2016).

[66] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle, and H. E. Swanson, Tests of the Gravitational Inverse-Square Law below the Dark-Energy Length Scale, Phys. Rev. Lett. **98**, 021101 (2007).

[67] P. Hamilton, M. Jaffe, P. Haslinger, Q. Simmons, and H. Müller, and J. Khoury, Atom-interferometry constraints on dark energy, Science **349**, 849 (2015).

[68] P. James and E. Peebles, Dark matter, Proc. Natl. Acad. Sci. U.S.A. **112**, 12246 (2015).

[69] S. Tremaine and J. E. Gunn, Dynamical of Role of Light Neural Leptons in Cosmology, Phys. Rev. Lett. **42**, 407 (1979). It places a lower limit of mass ≥ 1 MeV.

[70] G. R. Blumenthal, S. M. Faber, J. R. Primack, and M. J. Rees, Formation of galaxies and large-scale structure with cold dark matter, Nature (London) **311**, 517 (1984).

[71] G. Bertone and D. Hooper, History of dark matter, Rev. Mod. Phys. **90**, 045002 (2018).

[72] M. R. Buckley and A. H. G. Peter, Gravitational probes of dark matter physics, Phys. Rep. **761**, 1 (2018).

[73] G. Bertone, The moment of truth for WIMP dark matter, Nature (London) **468**, 389 (2010).

[74] W. Hu, R. Barkana, and A. Gruzinov, Fuzzy Cold Dark Matter: The Wave Properties of Ultralight Particles, Phys. Rev. Lett. **85**, 1158 (2000).

[75] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper, and J. March-Russell, String axiverse, Phys. Rev. D **81**, 123530 (2010).

[76] P. Mocz *et al.*, First Star-Forming Structures in Fuzzy Cosmic Filaments, Phys. Rev. Lett. **123**, 141301 (2019).

[77] W. A. Terrano, E. G. Adelberger, C. A. Hagedorn, and B. R. Heckel, Constraints on Axionlike Dark Matter with Masses Down to 10^{-23} eV$/c^2$, Phys. Rev. Lett. **122**, 231301 (2019).

[78] K. Nagano, T. Fujita, Y. Michimura, and I. Obata, Axion Dark Matter Search with Interferometric Gravitational Wave Detectors, Phys. Rev. Lett. **123**, 111301 (2019).

[79] D. O. Sabulsky, I. Dutta, E. A. Hinds, B. Elder, C. Burrage, and E. J. Copeland, Experiment to Detect Dark Energy Forces Using Atom Interferometry, Phys. Rev. Lett. **123**, 061102 (2019).

[80] P. W. Graham, D. E. Kaplan, J. Mardon, S. Rajendran, W. A. Terrano, L. Trahms, and T. Wilkason, Spin precession experiments for light axionic dark matter, Phys. Rev. D **97**, 055006 (2018).

[81] S. Weinberg, The cosmological constant problems, Proceedings of the Dark Matter 2000, arXiv:astro-ph/0005265v1.

[82] K. A. Olive, Inflation, Phys. Rep. **190**, 307 (1990).

[83] R. Brandenberger and P. Peter, Bouncing cosmologies: Progress and problems, Found. Phys. **47**, 797 (2017); S. Nojiri, S. D. Odintsov, and V. K. Oikonomou, Modified gravity theories on a nutshell: Inflation, bounce and late-time evolution, Phys. Rep. **692**, 1 (2017).

[84] P. J. Steinhardt and N. Turok, A cyclic model of the Universe, Science **296**, 1436 (2002).

[85] S. Alexander, S. Cormack, and M. Gleiser, A cyclic universe approach to fine tuning, Phys. Lett. B **757**, 247 (2016).

[86] E. Di Valentino, A. Melchiorri, and J. Silk, Planck evidence for a closed universe and a possible crisis for cosmology, Nat. Astron., https://doi.org/10.1038/s41550-019-0906-9.

[87] C. Brans and R. H. Dicke, Mach's principle and relativistic theory of gravitation, Phys. Rev. **124**, 925 (1961).

[88] D. F. Mota and D. J. Shaw, Evading equivalence principle violations, cosmological, and other experimental constraints in scalar field theories with a strong coupling to matter, Phys. Rev. D **75**, 063501 (2007).

[89] K. Jones-Smith and F. Ferrer, Detecting Chameleon Dark Energy via an Electrostatic Analogy, Phys. Rev. Lett. **108**, 221101 (2012).

[90] K. Hinterbichler and J. Khoury, Screening Long-Range Forces through Local Symmetry Restoration, Phys. Rev. Lett. **104**, 231301 (2010).
[91] C. Burrage, A. Kuribayashi-Coleman, J. Stevenson, and B. Thrussell, Constraining symmetron fields with atom interferometry, J. Cosmol. Astropart. Phys. 12 (2016) 041.
[92] P. Brax, A.-C. Davis, and R. Jha, Neutron stars in screened modified gravity: Chameleon versus dilaton, Phys. Rev. D **95**, 083514 (2017).
[93] T. Schuldt *et al.*, Design of a dual species atom interferometer for space, Exp. Astron. **39**, 167 (2015).
[94] R. H. Dicke, Mach's principle and invariance under transformation of units, Phys. Rev. **125**, 2163 (1962).
[95] A. D. Linde, Eternally existing self-reproducing chaotic inflation universe, Phys. Lett. B **175**, 395 (1986).
[96] R. Dong, W. H. Kinney, and D. Stojkovic, Symmetron inflation, J. Cosmol. Astropart. Phys. 01 (2014) 021.